Corruption Networks*

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Abstract

This paper studies how an agent's propensity to accept bribes depends on the organizational structure modeled with a *monitoring network*. In *hierarchies*, bribe taking is risker if others accept more bribes, for it is then easier for a corruption investigation to trace through bribe transactions to locate bribe takers. The opposite happens in flat, *two-layer networks*, as the subordinates who offer bribes are then better protected from being caught. In equilibrium, a denser monitoring network always deters agents from accepting bribes. I use this model to point out a corruption identification problem and propose a remedy to it.

Keywords: *Network Formation, Corruption, Monitoring*
JEL Classification: *D73, D85*

1 Introduction

Corruption is a huge and persistent problem. An IMF study (Mauro & Driscoll, 1997) shows that a 2.38-unit increase in the corruption index (on a 0-10 scale) decreases the investment rate by over 4%, and the per capita GDP by at least 0.5%. More recently, in 2014, the World Economic Forum and the UNODC conduct a survey on 814 millennials across five continents and find that 72.1% people believe "corruption is holding [their] country back."

To study corruption, it is important to be mindful of its social context. Consider the uncovering of rampant corruption in the NYPD by the Knapp Commission in the 1970s. It

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was established right after the police officer Frank Serpico made a brave revelation in The New York Times.[1] After a careful investigation lasting several years, the Knapp Commission was able to gather testimonies from dozens of witnesses (some were corrupt themselves) and confirm the presence of systematic corruption in the NYPD, implicating policemen from all layers accustomed to receive favors from criminals and ordinary people, while also covering up for each other's bribe taking routines.[2]

An important question to ask in such an environment is whether corruption stimulates or curbs individual corrupt behaviors, and how it depends on the organizational structure. The instance below illustrates what an organizational structure is.

> *In Nov. 2011, a robbery at Peizhong Bai's villa accidentally uncovered that this chairman of Shanxi Coking Coal Group had embezzled ￥50 million. The case was originally closed in haste. However, years later, it was dug out by the Standing Committee of the National People's Congress (NPCSC) and prosecuted again. This time several officials and businessmen involved were arrested, among whom were four police officers from the local Public Security Bureau (PSB) initially in charge of the robbery case, including even the Deputy Chief Laiwei Dai. These police officers were accused of under-reporting Bai's amount of loss in the robbery in exchange for bribes counted in millions. Notably, one of them, Yongping Li, also bribed his superior Dai (Kong, 2016).*

One can identify two networks in this example. First, since the PSB and the NPCSC separately monitor Bai, their relationship forms a "V structure" (as shown in figure 1a). Another network is the hierarchy consisting of the Deputy Chief Dai, his subordinate Li, and the criminal Bai that Li monitors (figure 1b).

Inspired by this example, I consider a group of agents connected by a directed *monitoring network*, each link pointing from a *supervisor* to a *subordinate*. Per supervision pair, the supervisor can costlessly verify if the subordinate is *criminal* and, conditional on observing a criminal subordinate, decides whether to accept her bribe offer and in return withhold the crime report. All bribe acceptance decisions are made simultaneously.

An agent is *caught* by external law enforcement if she is directly detected to be criminal, reported by a supervisor, or discovered through accepting a subordinate's bribe. A caught agent is obliged to pay a fixed punishment cost and surrender all accepted bribes. Hence, while each additional bribe acceptance brings in constant marginal value, it imposes

[1] See Wikipedia (`https://en.wikipedia.org/wiki/Knapp_Commission` and `https://en.wikipedia.org/wiki/FrankSerpico`) for a detailed description of this series of events.

[2] There exist rich documentations on systematic police corruption (see, for instance, Punch (2000, 2009) and Verma (1999)), which is both widespread and recurrent. The example mentioned here was by no means the only corruption scandal about the NYPD. In fact, such scandals broke out every 20 years (1895, 1913, 1932, 1954, 1973 and 1994). Over 20 years after the 1970s incident, the Mollen Commission was summoned only to discover pervasive corruption in the NYPD regenerated in novel forms (Punch, 2000, pp. 306-308).

(a) V Structure (b) Hierarchy

Figure 1

increasing marginal cost. An agent thus employs a cutoff bribe acceptance strategy, that is, accepting bribes up to an optimal desired number of bribes.

The structure of the monitoring network is the key to how others' bribe acceptance behaviors translate into an agent's bribe taking risk, thereby determining her bribe acceptance propensity. In the first result, I analyze two canonical network structures (generalizations of the two three-agent networks in figure 1) and demonstrate that they offer contrasting implications on that. One is *hierarchies*, the other is *two-layer networks* where the agents are divided into pure supervisors and pure subordinates. In a hierarchy, an agent's bribe taking risk is only affected by the actions of those lower down the tree. The more bribes they accept, the easier it is for a corruption investigation to percolate up to reach the agent, should one lower-rank opponent be directly caught as criminal.[3] Therefore, corruption renders everyone more susceptible to detection, decreasing an agent's propensity to accept bribes. The opposite occurs in a two-layer network: corrupt supervisors tend to cover up subordinates' crimes, creating a safer environment for co-supervisors to accept bribes.[4] In other words, in hierarchies, agents' bribe acceptance behaviors exhibit *strategic substitutability*; yet in two-layer networks, they demonstrate *strategic complementarity*.[5] As a result, while hierarchies feature equilibrium uniqueness, multiple equilibria can exist in a two-layer network.

My second result is that in equilibrium, a denser monitoring network deters agents from accepting bribes, as formalized in Proposition 2. I argue that though it holds for any (random, acyclic) network, the intuitions are very different across the two network structures mentioned above. When a hierarchical monitoring network becomes denser, an agent has more subordinates and hence receives more bribe offers, enabling her to accept more bribes. However, since corruption in hierarchies features strategic substitutability,

[3] In figure 1b, Dai was investigated and caught because his subordinate Li was caught through accepting a bribe from Bai. He would have been safe had Li instead duly reported Bai's crime.

[4] In figure 1a, the PSB's corruption was discovered because the NPCSC reported Bai honestly. It would have remained hidden had the NPCSC been corrupt as well.

[5] In a model extension with *incorruptible* agent types, strategic substitutability/complementarity manifests itself in that depending on the network structure, *corruptible* agents' propensity to accept bribes falls or rises with the proportion of incorruptible agents (Section 5.1, Proposition 4).

bribe taking becomes riskier and thus less appealing. As for a denser two-layer network, since a subordinate is monitored by more supervisors, each receiving more bribe offers and so accepting a smaller proportion of them, she is more vulnerable to crime detection. Therefore, bribe taking carries higher risk and is thus less attractive.

This model also has empirical implications. Specifically, I argue that the *corruption measure* defined based on the number of bribe taking cases detected by law enforcements is non-monotonic in and thus an unreliable indicator of the actual *corruption level*. It is because the *corruption detection rate* is endogenous and depends on agents' bribe acceptance decisions, thus rampant corruption may accompany weak detection, leading to meagre observed corruption cases (likewise, the opposite can also happen). This discovery casts doubt on the validity of corruption measures constructed with law enforcement data in evaluating anti-corruption policies. For instance, employing such measures, mixed empirical results are obtained as to whether higher salaries for public officials curb or aggravate corruption.[6]

The baseline model assumes for simplicity that everyone is criminal and offers bribes to their supervisors. In Section 5.2, I extend the model by letting some of the agents to be initially noncriminal (*innocent*). This richer model allows me to explore innocent agents' propensities to accept bribes as well as how the density of criminals affects agents' bribe acceptance decisions. Compared to a criminal agent, an innocent agent is more reluctant to accept bribes, as corruption exposes her to the risk of being caught by external law enforcement.

The extended model provides two results pertaining to the two network structures with regards how the prevalence of crime affects corruption. First, in a special case of hierarchies where each agent monitors at most a subordinate (a *linear line*), one is less inclined to accept bribes when there are more criminals. Indeed, as criminal agents are not only more ready to accept bribes but also bribe their supervisors, their vaster presence makes a corruption investigation easier to percolate up and endanger a bribe taker. Second, in a two-layer network, while more criminals among the supervisors encourages an agent to accept bribes, a greater number of criminals among the subordinates produces the opposite effect. On one hand, if one's co-supervisors are more likely to be criminal, thus corrupt, her subordinates are better protected from crime detection, which encourages her to accept bribes. On the other hand, an increase in criminal subordinates boosts up the supply of bribe offers, each then is less likely to be accepted. Hence, a subordinate is more vulnerable to crime detection, rendering bribe taking less attractive. These outcomes echo my first result that the relationship between agents' propensities to accept bribes depends on the network structure.

[6] While evidence exists that corruption is alleviated if public officials are better paid, as shown by Goel and Rich (1989) and Goel and Nelson (1998) for the US and by Schulze et al. (2016) for Russia, Alt and Lassen (2012) find no obvious relationship between corruption and public officials' salaries using the same US dataset. Moreover, in a case study on the Mississippi state, Karahan et al. (2006) find that corruption is more rampant when public officials' salaries rise.

Literature Review

This paper is related to the contract theory literature on *collusion* between a supervisor and a monitored agent. Employing a three-layer hierarchical model, Tirole (1986) first identifies the difficulty for a principal to preclude a delegated supervisor from colluding with an agent, as it may be jointly beneficial for the supervisor to receive the agent's bribe in exchange for underreporting his productivity.[7] Recognizing that, Tirole imposes *coalition incentive constraints* upon the principal to study the optimal contract design in a collusion-free environment. Later, several papers follow this track without radically changing the setup and the approach, such as Kofman and Lawarree (1993) and Faure-Grimaud et al. (2003).

Outside the contract theory sphere, a few applied theory papers discuss optimal policies in the presence of such collusion. Chander and Wilde (1992) and Besley and McLaren (1993) adopt the context of tax collection, where a taxpayer can bribe an auditor into underreporting his tax evasion. Similar mechanism is studied by Mookherjee and Png (1995) between a pollution inspector and a polluting factory. More recently, Ortner and Chassang (2018) puts forward a principal-monitor-agent model where the monitor is supposed to report the agent's crime to the principal but may gloss over it if sufficiently bribed by the agent. They emphasize randomizing the monitor's wage to introduce information asymmetry between the potentially collusive parties, thus elevating the cost of bribing.

These papers generally assume an exogenous probability at which the monitor is discovered to be corrupt, which I endogenize with a monitoring network. Consequently, while the only anti-corruption policies at their disposal are compensations and fines, I am able to explore how the network structure dictates corrupt monitors' risks of being caught, thus altering people's bribe acceptance decisions.

More closely related are two network formation papers. Both center on the role of a network in spreading the risk of detection by an external force among a group of agents, and analyze individual network formation choices in this setting.

Baccara and Bar-Isaac (2008) study the optimal *information structure* in a terrorist organization in the form of an endogenous directed network. In their model, each agent chooses the set of colleagues to whom he reveals his identity. While disclosing personal information helps build internal trust and thus improves group efficiency, it renders an agent more vulnerable to external threat – once the anti-terrorist agency detects an agent, it also detects those he holds information on. It turns out that in equilibrium, only small information structures – singletons, binary cells or two-tier hierarchies – are likely to form, as revealing personal information to one colleague induces sufficient trust, and so larger components do

[7] Empirical evidence of collusion has been found under various social contexts. For instance, Duflo et al. (2013) confirm the existence of collusion between polluting plants and pollution auditors in India, which Bandiera et al. (2021) corroborate between procurement officers and their monitors in the Pakistanian government.

not justify the additional risk they bring. This idea is echoed by the strategic substitutability in our hierarchical networks, where more corruption among one's subordinates increases the risk of accepting bribes. But unlike in their information sharing networks, our corruption networks can also exhibit strategic complementarity: indeed, in two-layer networks, more corruption among one's co-supervisors lowers the bribe acceptance risk.

Acemoglu et al. (2016) consider a setting where a network of agents is threatened by external cyberattack. The attack is contagious: should one agent be located by the hacker, all the agents in the same component are caught as well. They explore how an agent's optimal security investment (costly investment that reduces the probability one is directly attacked) depends upon the network structure. Though most of the paper deals with an exogenous network, one section endogenizes it by allowing an agent to unilaterally sever any of his links. As a link generates positive payoff, the agent optimally preserves as many neighbors as possible while minimizing the size of the component he is attached to. Just like in Baccara and Bar-Isaac (2008), this result demonstrates strategic substitutability. In comparison, as my model also features strategic complementarity, a large corruption network can sometimes be conducive to link formation, i.e., bribe acceptance. Another distinction is that while they make the network observable to the agents, I consider an imperfect information environment where each agent only knows her own degrees. Consequently, instead of respecting the entire network structure, agents base their optimal decisions on network parameters. This way I can compare their behaviors across structurally different networks.

As this paper points out the non-monotonic relationship between the actual corruption level and corruption measures obtained from law enforcement data, it also contributes to the applied economics literature on corruption where such measures are extensively used. To cite a few examples, Meier and Holbrook (1992) and Glaeser and Saks (2006) adopt the number of convicted corrupt individuals to study corruption in the US; Dong and Torgler (2013) and Zakharov (2019) utilize the number of registered corrupt cases to measure corruption in China and Russia, respectively. My structural approach can help obtain more accurate measures of the underlying corruption in the future.

The paper is organized as follows. Section 2 delineates the baseline model and the techniques I use to generate the monitoring network. Section 3 provides the equilibrium analysis. In 3.1, I derive agents' best response, and discuss how the network structure determines the relationship between their propensities to accept bribes. In 3.2, I derive the equilibria, prove their existence, and perform comparative statics on network parameters. Section 4 evaluates the performance of a commonly used corruption measure and raises concerns for its validity. Section 5 considers two model extensions. 5.1 introduces in incorruptible, criminal agents to reiterate the results in 3.1 as comparative statics. 5.2 lets some agents be innocent, and discusses the equilibrium derivation process, equilibrium existence proof, and comparative statics on the crime rate for this extended model. Section 6 concludes.

2 Model

2.1 Primitives

A set of criminal agents[8] are connected by a directed monitoring network, each edge representing a crime monitoring relationship pointing from the supervisor to the subordinate. Per monitoring pair (edge), the supervisor has observed the subordinate's crime, who has offered a bribe with fixed value $b > 0$ back. The supervisor chooses either to *accept* it, or to decline it and *report* the subordinate's crime to the law enforcement agency. All bribe acceptance decisions are made simultaneously.

There are three ways through which an agent can get caught: (i) being reported by at least one supervisor; (ii) being directly caught as criminal by the law enforcement agency, which happens on each agent with independent probability $q \in (0, 1)$; (iii) being detected through accepting bribes: once an agent is caught, the law enforcement agency initiates an investigation over her supervisors; any of them who has accepted the agent's bribe is detected independently with probability $\eta \in (0, 1]$.

A caught agent pays a punishment cost $c > 0$ and has all the bribes confiscated, including those she accepted from her subordinates and those she offered to her supervisors but were declined[9]. Hence, if an agent with $l \geq 0$ supervisors accepts $n \geq 0$ bribes, her payoff is $(-c - lb)$ if she is caught and $(n - l)b$ if not.

All the parameters mentioned above are public knowledge, as well as the network parameters to be introduced in the next section. Regarding the monitoring network, each agent only knows how many supervisors and subordinates she herself has, that is, her own in- and out-degrees.

2.2 Monitoring Network

This section constructs a random, acyclic monitoring network. Let π be an agent's joint distribution of in- and out-degrees with finite support. The in- and out-degree distributions are thus $\lambda = \int \pi(\cdot, k)dk$ and $\mu = \int \pi(l, \cdot)dl$. Denote by $\hat{\lambda}$ the in-degree distribution for any of an agent's subordinates, and by $\hat{\mu}$ the out-degree distribution for any of her supervisors. By definition, $\hat{\lambda}(l) = \lambda(l)l/\mathbb{E}_\lambda(\tilde{l}),\ \hat{\mu}(k) = \mu(k)k/\mathbb{E}_\mu(\tilde{k})\ \forall l, k.$

We generate the network with the *branching process*:[10]

[8] In Section 5.2, we will take one more step towards realism by allowing some of the agents to be innocent.

[9] When a supervisor reports a subordinate, she also submits the bribe the subordinate offers to the law enforcement agency.

[10] The conventional branching process deals with undirected networks. To obtain directed networks, we innovate on the technique to allow the network to grow in both directions, as will be illustrated in the detailed procedure below.

1. Start with a node, create its l supervisors and k subordinates according to π.

2. (a) If no new node is generated in the last step, terminate;

   (b) otherwise, for each new subordinate, create its $\hat{l}$ new supervisors from $(\hat{\lambda}-1)$, then create its k subordinates from $\mu(\cdot|l = \hat{l}+1)$; for each new supervisor, create its $\hat{k}$ new subordinates from $(\hat{\mu}-1)$, then create its l supervisors from $\lambda(\cdot|k = \hat{k}+1)$.[11]

3. Repeat step 2.

In this paper, we are particularly interested in hierarchies (the left panel of figure 2) and two-layer networks (the right panel of figure 2). The former is constructed by specifying $\operatorname{supp} \lambda = \{0, 1\}$, the latter by setting $\pi(l, k) = 0$ if $l, k > 0$.

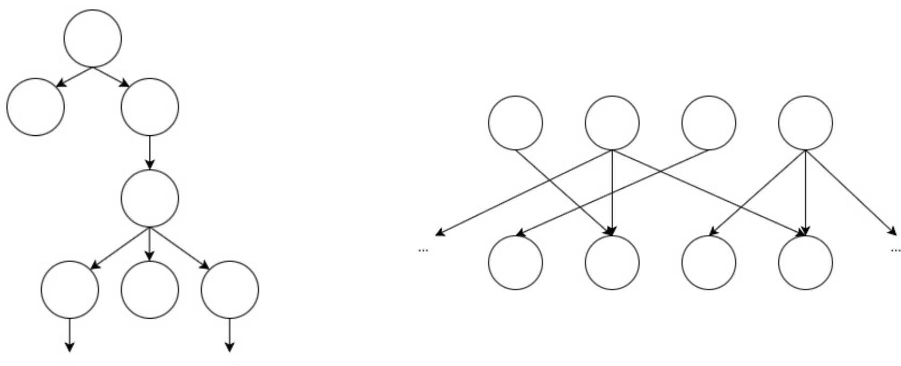


Figure 2: Hierarchy (Left) and Two-Layer Network (Right)

Lastly, the following assumption is imposed such that the network does not grow unbounded and is well-defined.

Assumption 1. *i* ***(finiteness)*** $\max\left\{\mathbb{E}_{\hat{\mu}}(k) + \mathbb{E}_{\pi}(l|k \geq 1),\ \mathbb{E}_{\hat{\lambda}}(l) + \mathbb{E}_{\pi}(k|l \geq 1)\right\} < 2.$

ii ***(consistency)*** $\mathbb{E}_{\mu}(k) = \mathbb{E}_{\lambda}(l)$.

(i) ensures that the expected size of the network is finite: for each node, conditional on its having one link, the number of new links generated for it is strictly less than 1. (ii) states that the network's expected in- and out-degree must be the same, for only then is it properly defined.[12]

[11] In this step, neighbors' degree distributions $\langle\hat{\lambda}, \hat{\mu}\rangle$ are employed to adjust for the *friendship paradox*. In Appendix a.1, we define the friendship paradox and justify the usage of $\langle\hat{\lambda}, \hat{\mu}\rangle$ in the branching process.

[12] See Appendix a.2 for a detailed explanation on why Assumption 1(ii) is necessary.

3 Equilibrium Analysis

The ultimate goal of this section is to answer how the monitoring network shapes agents' propensities to accept bribes. Specifically, we will contrast the relationships between agents' bribe acceptance decisions across hierarchies and two-layer networks and explore how the equilibria are controlled by network parameters.

3.1 Best Response

First, we solve for the optimal bribe acceptance decision of a random agent in the monitoring network. Suppose she has $l \geq 0$ supervisors, and $k > 0$ subordinates (an agent with no subordinate does not make strategic decisions) who have offered bribes to her. We consider which of these k bribe offers she optimally accepts.

This decision boils down to how many of the k bribe offers to accept, which can be summarized with one variable. This is because all bribe offers have the same value b; and accepting each of them brings to the agent identical risk of being caught, for the agent can only observe her own degrees (similarly, the risk of being reported by each of the l supervisors is equal).

Define the risk variables as follows:

Definition 1. ***Lower risk*** *p is the probability an agent is caught through accepting any given subordinate's bribe;* ***upper risk*** *r is the probability an agent is reported by any given supervisor.*

They are endogenous and will be derived from network parameters and agents' strategies.

We temporarily disregard how many subordinates the agent has and just think about how many bribes she desires to accept given no upper limit. To solve for that, we first need to know her *expected utility* of accepting $n \in \mathbb{N}$ bribes. Since the monitoring network is acyclic, the risk coming from each link is independent. Therefore, if the agent accepts n bribes, the probability she stays safe from being caught is $(1-q)(1-r)^l(1-p)^n$. Thus her expected utility is:

$$\begin{aligned} U^l(n) &= (1-q)(1-r)^l(1-p)^n \cdot (n-l)b \\ &\quad + \Big[1-(1-q)(1-r)^l(1-p)^n\Big] \cdot (-lb-c) \end{aligned} \tag{1}$$

Only when not caught can the agent keep the accepted bribes nb and avoid the punishment cost c; lb is the bribes she offers to her supervisors.

Denote by $\hat{n} \equiv \arg\max U^l(n)$ the agent's *desired number of bribes.* The following lemma presents an important property of the expected utility function $U^l(n)$ which allows $\hat{n}$ to be represented in a simple form.

Lemma 1. *The expected utility function $U^l(n)$ is quasi-concave; moreover, an agent's desired number of bribes $\hat{n}$ is independent of her degrees l, k and the upper risk r, it is a nonincreasing correspondence of the lower risk p.*

$U^l(n)$ is quasi-concave because while the marginal benefit of a bribe is constant at b, its marginal cost increases with the number of accepted bribes – each additional bribe acceptance brings the same confiscation risk p to all bribes already accepted. To see why the agent's desired number of bribes $\hat{n}$ is independent of her upper degree l and the upper risk r, notice that she is subject to the same reporting risk $\left[1-(1-r)^l\right]$ regardless of how many bribes she accepts; and the bribes she hands to her supervisors lb are predetermined sunk cost. The number of subordinates the agent has k only poses a capacity constraint on how many bribes she can accept and hence is irrelevant to how many she desires. The formal proof is supplied in Appendix b.

Lemma 1 implies that all agents desire the same number(s) of bribes, which we will derive as a correspondence of the lower risk p. For that, we first compute the *cutoff lower risk* $p_{n(n+1)}$ that makes an agent indifferent between accepting n and $(n+1)$ bribes by equalizing the expected utilities $U^l(n)$ and $U^l(n+1)$:

$$p_{n(n+1)} = \frac{b}{c+(n+1)b}$$

Since $p_{n(n+1)}$ strictly decreases with n, the number of bribes an agent desires $\hat{n}$ is a nonincreasing correspondence of the lower risk p (the riskier bribe taking is, the fewer bribes one accepts):

$$\hat{n}(p) = \begin{cases} 0, & p \in (p_{01}, 1] \\ n, & p \in (p_{n(n+1)}, p_{(n-1)n}) \\ \{n, n+1\}, & p = p_{n(n+1)} \\ \infty, & p = 0 \end{cases} \tag{2}$$

$\hat{n}$ is nondecreasing in the bribe value b, as an agent desires more bribes when they are more valuable.

(2) implies that at optimum, an agent mixes at most between accepting two adjacent numbers of bribes. Therefore, to capture mixed strategies, it is enough to extend the variable n to the real line: let $n \in \mathbb{R}_+$ stand for accepting $\lfloor n \rfloor$ bribes with probability $(\lfloor n \rfloor + 1 - n)$ and $(\lfloor n \rfloor + 1)$ bribes with probability $(n - \lfloor n \rfloor)$. For example, $n = 3.6$ means accepting 3 bribes with probability 0.4 and 4 bribes with probability 0.6. Next, extend the desired number of bribes correspondence $\hat{n}(p)$ to the real line accordingly by letting $\hat{n}(p) = [n, n+1]$ whenever $p = p_{n(n+1)}$ for some n.

Since an agent with k subordinates only receives k bribe offers, the optimal number(s) of bribes she actually accepts is $\min\left\{k, \hat{n}(p)\right\}$. As shown in figure 3, variations in agents' optimal strategies come solely from their different capacity constraints (in this figure and all subsequent analyses, the punishment cost is normalized to 1).

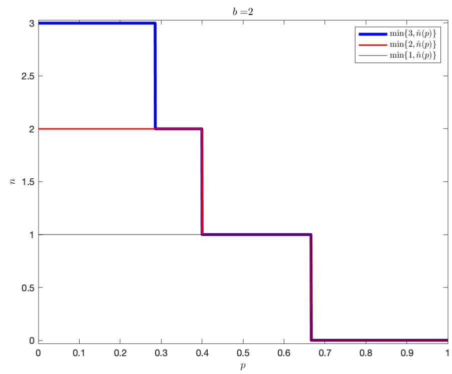


Figure 3: Optimal Strategies for Agents with Different Numbers of Subordinates ($k = 1, 2, 3$)[13]

In principle, the probabilities with which an indifferent agent (whose capacity constraint is not binding) mixes between accepting $n \in \mathbb{N}$ and $(n+1)$ bribes could depend on her degrees. In the spirit of symmetric equilibria we assume they do not by imposing the following assumption:

Assumption 2. ***(Strong Symmetry Assumption)*** *There is* $n \in \mathbb{R}_+$ *such that an agent with* k *subordinates accepts* $\min\{k, n\}$ *bribes.*

Assumption 2 allows us to conveniently express an equilibrium with a one-dimensional variable n (henceforth referred to as agents' *propensity to accept bribes*). Besides, it is without loss of generality.[14]

Risk Functions

Having mapped the lower risk p to agents' propensity to accept bribes n through the desired number of bribes correspondence $\hat{n}(p)$, we now do the reverse by defining the lower and upper risk functions: $P(n), R(n) \in [0, 1]$. They crucially rely on the network structure.

Due to information asymmetry, an agent treats her subordinates equally and randomly selects a set of bribe offers to accept. We thus obtain the upper risk – the probability an

[13]Parameter values: $b = 2$.

[14]To establish the generality of *strongly symmetric equilibria*, we define a *symmetric equilibrium* as an equilibrium at which agents with the same upper and lower degrees (l, k) play the same mixed strategy, and show that given a set of parameters, any symmetric equilibrium shares the same upper risk and expected utilities with one and only one strongly symmetric equilibrium. We relegate the formal statement, its proof, and an illustrative example to Appendix c.1.

agent is reported by any given supervisor – as follows:

$$R(n) = \sum_{k \geq 1} \Big(1 - \frac{n \wedge k}{k}\Big) \hat{\mu}(k), \tag{3}$$

where $\big(1 - (n \wedge k)/k\big)$ is the probability that a supervisor with k subordinates declines the agent's bribe offer and reports her.

Given $R(n)$, the lower risk function $P(n)$ is recursively defined[15] by the following equation:

$$\begin{aligned} p = \eta \cdot \sum_{k} \sum_{l \geq 1} \bigg[& 1 - (1-q)\big(1 - R(n)\big)^{l-1} \\ & \cdot (1-p)^{\lfloor n \wedge k \rfloor} \big(1 - (n \wedge k - \lfloor n \wedge k \rfloor) p\big) \bigg] \hat{\lambda}(l) \mu(k|l). \end{aligned} \tag{4}$$

An agent is caught through accepting any given subordinate's bribe if and only if that subordinate is caught, and their bribe transaction is consequently detected (with probability η). Specifically, $\big[1-(1-q)(1-R(n))^{l-1}(1-p)^{\lfloor n \wedge k \rfloor}\big(1-(n \wedge k - \lfloor n \wedge k \rfloor)p\big)\big]$ is the probability a subordinate with degrees (l, k) is caught conditional on the agent's having accepted her bribe. It generalizes the similar expression in equation (1) to mixed strategies.

Strategic Substitutability/Complementarity

We now discuss whether an agent optimally acts against (strategic substitutability) or follows (strategic complementarity) her opponents' bribe acceptance decisions in two network structures – hierarchies and two-layer networks (see figure 2). We show that as each network class features a distinctive driving force, they provide opposite answers to this question.

In a hierarchy, the lower risk function $P(n)$ is strictly **increasing**[16], so an agent accepts **fewer** bribes when her opponents accept **more**, exhibiting strategic substitutability. Intuitively, if a subordinate of an agent accepts more bribes, the chance she gets caught rises, and so does the risk of accepting her bribe. This effect is further intensified by risk transmission through the *collusive subnetwork*[17], namely, the fact that an indirect subordinate accepts more bribes also increases the agent's bribe taking risk.[18]

In a two-layer network, the lower risk function $P(n)$ is strictly **decreasing**[16], so an agent accepts **more** bribes when her opponents accept **more**, exhibiting strategic complementar-

[15] $P(n)$ is well-defined because the monitoring network is finite, as ensured by Assumption $1(i)$.

[16] More precisely, $P(n)$ is strictly monotone on $[0, \bar{k}]$ and stays constant on $[\bar{k}, \infty)$, where $\bar{k} \equiv \operatorname{supp} \mu$ is the largest number of bribes an agent can accept.

[17] The *collusive subnetwork* is derived from the monitoring network by preserving only the links where bribe transactions successfully occur.

[18] To understand that, let us pick three agents in the network. Suppose agent 1 monitors and colludes with agent 2, who monitors and colludes with agent 3. When 3 accepts more bribes, 2 becomes more likely to get caught through colluding with her. As a result, the risk for 1 to accept 2's bribe also increases.

ity, This is because when one's co-supervisors accept more bribes, the risk her subordinates are reported, thus caught, falls. Hence, accepting bribes becomes less dangerous.[19]

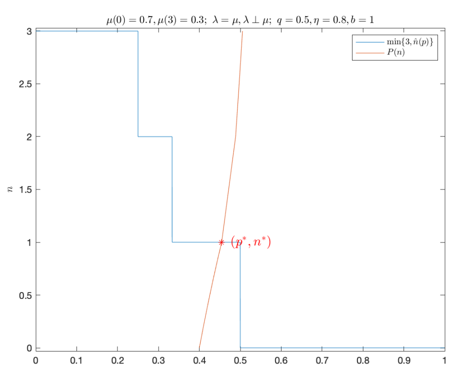
While the two opposite forces – strategic substitutability/complementarity – are demonstrated with best responses here, with slight modifications on the model, they can be easily transformed into equilibrium results. See Section 5.1.

3.2 Equilibrium

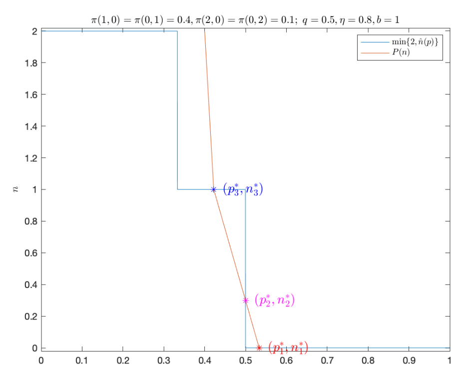
Fixing a set of parameters, the equilibria are found at the intersecting points of the lower risk function $P(n)$ and the desired number of bribes correspondence $\hat{n}(p)$. The *Intermediate Value Theorem* then implies:

Proposition 1. *An equilibrium $n^* \in \mathbb{R}_+$ exists.*

The shape of the lower risk function $P(n)$ has implications on equilibrium multiplicity. In a hierarchy, since $P(n)$ is strictly increasing while the desired number of bribes correspondence $\hat{n}(p)$ is weakly decreasing, they must intersect exactly once, producing a unique equilibrium (see figure 4a). On the contrary, in a two-layer network, the strictly decreasing lower risk function $P(n)$ may cross the desired number of bribes correspondence $\hat{n}(p)$ more than once. Figure 4b illustrates that in this case, we may have multiple equilibria.[20]



(a) Equilibrium for a Hierarchy[21]



(b) Equilibria for a Two-Layer Network[22]

[19]We have analyzed two special cases. In a more general network, both forces are present: an agent's action is substitutable to the actions of those "lower than" her, and complementary to the actions of those "parallel to" her. Hence, the lower risk function $P(n)$ is generally non-monotonic.

[20]It suggests corruption can be stable at different levels, as pointed out by various studies on police corruption: in a generally corrupt environment, a new police officer would be easily coaxed into conforming to the corrupt norm. Thus corruption maintains prevalent over the long run. However, in a generally incorrupt environment, threatened by the high exposure rate, he would be deterred from corruption at the first place. Hence integrity sustains.

Comparative Statics

We now study how agents' equilibrium propensity to accept bribes n^* is influenced by the *density*[23] of the monitoring network. First, we look at how the lower risk function $P(n)$ and the desired number of bribes correspondence $\hat{n}(p)$ are shifted by model parameters. It constitutes an intermediate step to the comparative statics, for the equilibria are found at their intersecting points.

Lemma 2. *The desired number of bribes correspondence $\hat{n}(p)$ only depends on the bribe value b and shifts right when b increases; the lower risk function $P(n)$ shifts right when the network contagion rate η, the external monitoring success rate q, or the network degree distributions $\langle \lambda, \mu \rangle$ increase.*

Notice that λ and μ are connected via Assumption 1(ii). Here we let them increase in the MLRP (*monotone likelihood ratio property*) manner, as it has the important characteristic that for any two degree distributions λ, λ' and their counterparts adjusted for the friendship paradox $\hat{\lambda}, \hat{\lambda}'$, $\lambda' \succeq_{MLRP} \lambda$ implies $\hat{\lambda}' \succeq_{MLRP} \hat{\lambda}$.[24]

To tackle the equilibrium multiplicity problem, we restrict attention to the largest bribe acceptance propensity n^*, which is the one that maximizes any agent's expected utility (see Appendix d). Lemma 2 implies that n^* decreases with the network contagion rate η, the external monitoring success rate q, and increases with the bribe value b.

Proposition 2. *In equilibrium, if the network degree distributions $\langle \lambda, \mu \rangle$* ***increase****, agents' bribe acceptance propensity n^* is* ***reduced****.*

While Proposition 2 holds for all networks, the underlying intuition depends on the network structure. To gain more insight into it, consider first a hierarchy. When the network becomes denser, an agent has more subordinates and so can accept more bribes – the capacity constraint is relaxed. She is thus more likely to get caught through bribe taking, which in turn makes her bribe riskier to accept. Hence, agents tend to accept fewer bribes. Mathematically, increasing out-degree distribution μ raises the average number of bribe an agent accepts,[25] so the lower risk function $P(n)$ shifts right, pushing down agents' propensity to accept bribes n^*.

Next, we look at a two-layer network. When the network becomes denser, not only is a subordinate monitored by more supervisors, but her risk of getting reported through each

[14]Parameter values: $\mu(0) = 0.7, \mu(3) = 0.3$; $\lambda = \mu, \lambda \perp \mu$; $q = 0.5, \eta = 0.8, b = 1$. In this network, an agent has at most 3 subordinates and so can accept at most 3 bribes. Hence, to find the equilibrium, it suffices to truncate the desired number of correspondence $\hat{n}(p)$ at 3. We will apply the similar truncation to all following graphs.

[22]Parameter values: $\pi(1, 0) = \pi(0, 1) = 0.4, \pi(2, 0) = \pi(0, 2) = 0.1$; $q = 0.5, \eta = 0.8, b = 1$.

[23]A monitoring network becomes denser if both of its in- and out-degree distributions $\langle \lambda, \mu \rangle$ increase. See the next paragraph for details.

[24]See Board and Meyer-ter-Vehn (2021) for other ways of treating comparative statics on network parameters when the friendship paradox is present.

[25]It strictly increases only if some agents have binding capacity constraints before the change.

link rises. To see that, notice now a supervisor has more subordinates and so receives more bribe offers. She thus accepts a smaller proportion of them.[26] Again, a subordinate becomes more likely to get caught, deterring supervisors from accepting bribes. At a mathematical level, increases in the degree distributions $\langle \lambda, \mu \rangle$ shifts the lower risk function $P(n)$ right, resulting in decrease in agents' propensity to accept bribes n^*.

The same applies for any network, for it must preserve the characteristics of both hierarchies and two-layer networks, namely, a subordinate of an agent can get caught either through accepting bribes (as manifested in hierarchies), or through being reported by others (as manifested in two-layer networks). As the network becomes denser, both events are more likely to happen, making bribe taking less appealing.

4 Identifying Corruption

In Section 3.2, we have analyzed agents' propensity to accept bribes n^*. Though n^* reflects how many bribes one desires to take, it does not speak to the prevalence of corruption. This is because an agent's bribe acceptance behavior is limited by how many bribe offers she receives. To study the actual *corruption level*, we define $\kappa^* \equiv \mathbb{E}_\mu\big[\min\{k, n^*\}\big]$ – the expected number of bribes an agent accepts.

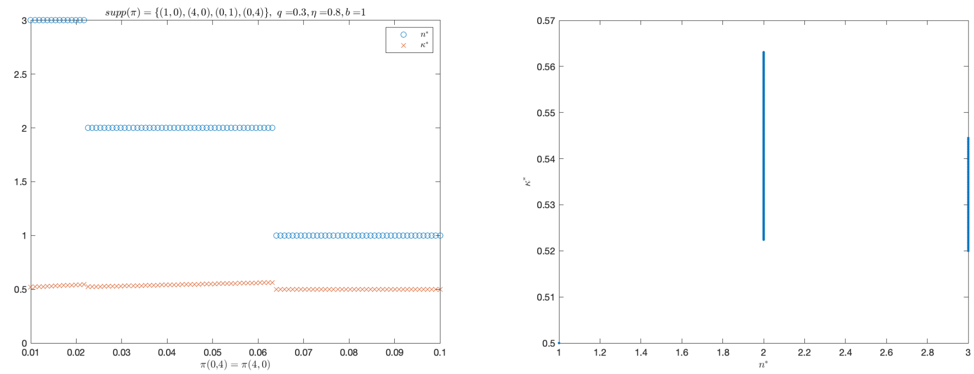
While agents' bribe acceptance propensity n^* falls with the density of the monitoring network (Proposition 2), the same does not necessarily apply for the corruption level κ^*, as it is confounded by the direct increase in the out-degree distribution μ. In other words, although densifying the monitoring network facilitates corruption detection and thus deters people from accepting bribes, it simply creates more bribe taking opportunities. Hence, whether it reduces the corruption level is ambiguous.

In particular, since the desired number of bribes correspondence $\hat{n}(p)$ is a step function, when the network becomes denser, agents' propensity to accept bribes n^* either drops or stays constant. In both two-layer networks (figure 5a) and hierarchies (figure 6a), a rise in the network density that does not affect the bribe acceptance propensity n^* (blue) raises the corruption level κ^* (orange) through relaxing agents' capacity constraints. Yet increases in the network density that do reduce the bribe acceptance propensity n^* also lower the corruption level κ^* – it is obvious for two-layer networks, as the bribe acceptance propensity n^* drops discontinuously; in a hierarchy, the corruption level κ^* falls continuously along with n^*, for agents' reduced propensity to accept bribes always dominates their expanded freedom in doing so.[27]

[26] For example, fix $n = 1$. If each supervisor has one subordinate, then all bribes are accepted. Hence the upper risk $r = 0$. Now let each supervisor monitor two subordinates, then she randomly accepts one out of the two bribes. r thus rises to $1/2$.

[27] In Appendix e, we formalize this result for hierarchies (Proposition 10) and generalize it to any network where the in- and out-degrees are mutually independent ($\lambda \perp \mu$).

[28] Parameter values: $\operatorname{supp} \pi = \big\{(1,0),(4,0),(0,1),(0,4)\big\}$; $q = 0.3, \eta = 0.8, b = 1$. In this example, the



(a) n^*, κ^* on the Network Density (b) κ^* on n^*

Figure 5: Comparing n^* with κ^* when Varying the Network Density in a Two-Layer Network[28]

We now delve into the identification of the corruption level κ^*, which can be interpreted as the per person number of corruption cases, where a *corruption case* refers to an incident of successful bribe exchange. Since it is hard to observe κ^* in reality, many empirical studies instead adopt the per person number of corruption cases **detected** by the law enforcement agency (denoted by $\hat{\kappa}^*$) to measure corruption.[30] As the probability each corruption case is detected is given by the lower risk p^* (henceforth referred to as the *detection rate*), we have

$$\hat{\kappa}^* = p^* \cdot \kappa^*. \tag{5}$$

An immediate problem of the *corruption measure* $\hat{\kappa}^*$ is underestimation ($\hat{\kappa}^* < \kappa^*$), as corruption detection is rarely perfect ($p^* < 1$). This issue is widely acknowledged in applied economics literature and formally addressed by Kato and Sato (2014).[31] Another problem is the non-monotonicity of the corruption measure $\hat{\kappa}^*$ on the corruption level κ^*, so that the former does not even capture the trend of the latter:

Proposition 3. *The corruption measure $\hat{\kappa}^*$ is not generally monotonic in the corruption level κ^*: a change in exogenous parameter values can raise (lower) κ^* and lower (resp., raise) $\hat{\kappa}^*$.*

We prove Proposition 3 by examples. Figure 7a illustrates what happens in a hierarchy

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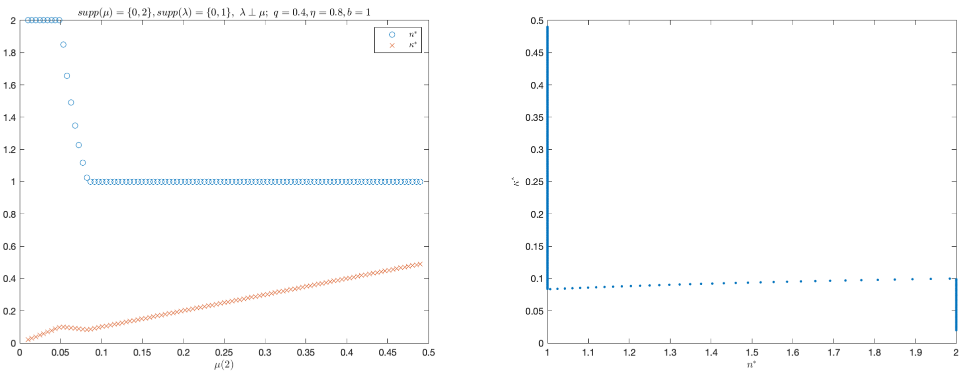
network finiteness assumption (Assumption 1(i)) is violated. But it is innocuous for two-layer networks.

[29]Parameter values: $\operatorname{supp}\mu = \{0, 2\}, \operatorname{supp}\lambda = \{0, 1\}$, $\lambda \perp \mu$; $q = 0.4, \eta = 0.8, b = 1$.

[30]Such measures are particularly popular in studying corruption in non-US countries. To cite a few examples, see Dong and Torgler (2013), Kato and Sato (2014), Schulze et al. (2016), Mocetti and Orlando (2019), Zakharov (2019), etc..

[31]They estimate the "reporting rate of corruption" and divide the observed number of corruption cases by it to obtain a relatively unbiased corruption measure.



(a) n^*, κ^* on the Network Density

(b) κ^* on n^*

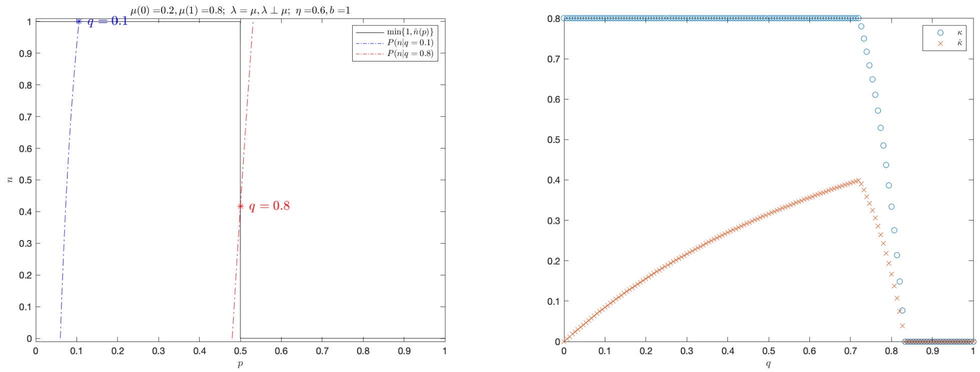
Figure 6: Comparing n^* with κ^* when Varying the Network Density in a Hierarchy[29]

when the external monitoring success rate q is enhanced from 10% to 80%. In this case, since one's subordinate is more likely to be caught directly, it becomes more dangerous to accept bribes, i.e., the lower risk function $P(n)$ shifts right. Hence, agents tend to accept fewer bribes (n^* drops), resulting in reduction in the corruption level κ^* from 0.8 to 0.33. However, thanks to the higher detection rate p^*, the corruption measure $\hat{\kappa}^*$ rises from 0.08 to 0.17. This example is consistent with the finding in Goel and Nelson (2011) that the corruption conviction rate in the US is positively and significantly correlated with law enforcement strength.

More generally, figure 7b plots the corruption level κ^* (blue) and corruption measure $\hat{\kappa}^*$ (orange) against the external monitoring success rate q when it varies between 0 and 1. We can see that only when the corruption level κ^* strictly decreases with q does the corruption measure $\hat{\kappa}^*$ follow its trend, for the detection rate p^* is constant in this range; otherwise, if the corruption level κ^* stays constant, the detection rate p^* rises with q, and so does the corruption measure $\hat{\kappa}^*$.

To cite another example, consider changing the bribe value b in a two-layer network (it is equivalent to varying the punishment cost c). In figure 8a, when the bribe value b increases from 0.324 to 0.38, accepting bribes becomes more attractive, shifting the desired number of bribes correspondence $\hat{n}(p)$ right. Thus, agents' propensity to accept bribes n^* rises from 1 to 2, as well as the corruption level κ^* from 0.5 to 0.6. Nevertheless, since the lower risk function $P(n)$ is decreasing (strategic complementarity), the detection rate p^* drops with the bribe acceptance propensity n^*, resulting in falling corruption measure $\hat{\kappa}^*$ from 0.122 to 0.12. In a more general manner, figure 8b demonstrates the non-monotonic relationship between the corruption level κ^* (blue) and the corruption measure $\hat{\kappa}^*$ (orange) when the

[32]Parameter values: $\mu(0) = 0.2, \mu(1) = 0.8$; $\lambda = \mu, \lambda \perp \mu$; $\eta = 0.6, b = 1$.

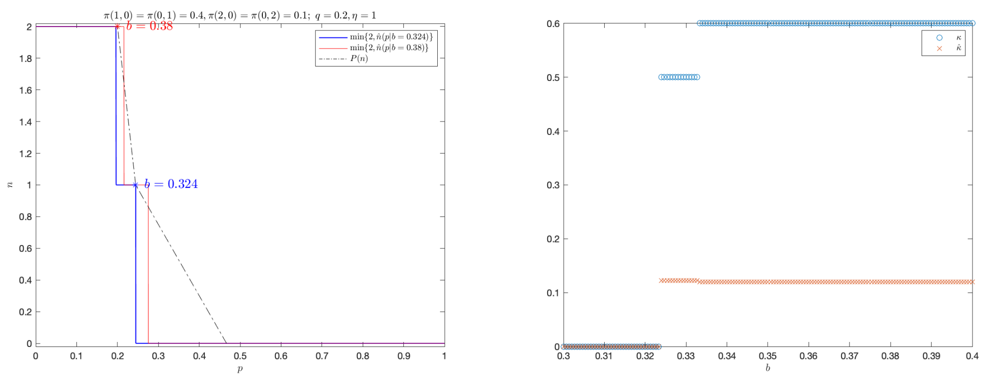


(a) Equilibria at Different q's

(b) $\kappa^*, \hat{\kappa}^*$ on q

Figure 7: Discrepancies between κ^* and $\hat{\kappa}^*$ when Varying q in a Hierarchy[32]

bribe value b varies on a larger scale.



(a) Equilibria at Different b's

(b) $\kappa^*, \hat{\kappa}^*$ on b

Figure 8: Discrepancies between κ^* and $\hat{\kappa}^*$ when Varying b in a Two-Layer Network[33]

Lastly, when network parameters change, while the corruption measure $\hat{\kappa}^*$ follows exactly the same trend as the corruption level κ^* in both hierarchies and two-layer networks (see figure 5a for the "discrete jumps" in two-layer networks and figure 6a for the "zigzag pattern" in hierarchies), wrong prediction can still arise around the turning points.[34] For instance, in a hierarchy where one has either 0 or 2 subordinates, if we increase the probability of having 2 subordinates, $\mu(2)$, from 0.044 to 0.087, the corruption level κ^* drops from 0.0872

[33] Parameter values: $\pi(1,0)=\pi(0,1)=0.4, \pi(2,0)=\pi(0,2)=0.1;\ q=0.2, \eta=1$.

[34] We use a hypothetical example to illustrate the intuition. In a hierarchy, suppose we keep increasing the network density and find three equilibrium points along the way: $(\kappa^*, \hat{\kappa}^*) = (0.4, 0.2), (0.43, 0.21), (0.41, 0.18)$, where (0.43,0.21) is a local maximum, then the surrounding points (0.4,0.2) and (0.41,0.18) form an instance of non-monotonicity.

to 0.0868, whereas the corruption measure $\hat{\kappa}^*$ rises from 0.0289 to 0.0290.

The measurement exercise fails because the detection rate p^* is endogenous: the probability a corruption case is detected depends crucially upon agents' strategic choices on crime reporting and bribe taking (in this model, they are summarized with one action).

One tentative approach to solve this problem is to trim off the endogenous part of the corruption measure $\hat{\kappa}^*$. Suppose we know the sources of detection for all observed corruption cases, that is, given any observed corruption case, we know whether the briber is (1) reported by a supervisor, (2) detected directly by the law enforcement agency, or (3) caught through accepting bribes (notice that the three cases are not mutually exclusive). Then we can construct the *per person number of corruption cases detected through source (2)* (denoted by $\hat{\kappa}^*_e$) as a measure for the corruption level κ^*.

Define the *probability any given corruption case is detected through source (2)*:

$$p_e = \eta q.$$

Then

$$\hat{\kappa}^*_e = p_e \cdot \kappa^* = \eta q \cdot \kappa^*.$$

Since p_e is exogenous, so long as the law enforcement strength (q, η) is controlled for,[35] the new measure $\hat{\kappa}^*_e$ correctly captures the trend of the corruption level κ^*.[36]

5 Extensions

5.1 A Model with Corruptible and Incorruptible Agents

In Section 3.1, we show that while agents' bribe acceptance propensities are strategically substitutable in hierarchies, they are strategically complementary in two-layer networks. This section reformulates the discovery as comparative statics.

We achieve it by making modest extensions on the baseline model. Suppose now an agent is *corruptible* with independent probability $\gamma \in (0, 1]$ and *incorruptible* with probability $(1 - \gamma)$. A corruptible agent makes strategic bribe acceptance decisions; an incorruptible agent is a crazy type that never accepts bribes.[37]

[35] For details on how it is typically addressed in applied economics research, see, for instance, Goel and Rich (1989), Schulze et al. (2016), and Mocetti and Orlando (2019).

[36] The term "law enforcement agency" should not be taken at face value. More precisely, it refers to any anticorruption agency exerting influence on but sufficiently independent from the object of study. For instance, if we are interested in studying police corruption, this term fails to apply as the police system itself is a law enforcement agency. Instead, to circumvent the endogeneity problem, we can focus on those corruption cases independently discovered by an external investigative commission, i.e., they are detected neither thanks to a whistleblower nor through tracing up bribe transactions.

[37] Equilibria are derived in the same way as those for the baseline model, though the risk functions $P(n), R(n)$ are slightly different, as displayed in Appendix f.1.

Proposition 4. *In equilibrium, when the density of corruptible agents γ* ***increases****, the bribe acceptance propensity n^** ***decreases*** *in a hierarchy, and* ***increases*** *in a two-layer network.*

Proof: When corruptible agents' density γ rises, in a hierarchy, the lower risk function $P(n)$ shifts right, lowering the bribe acceptance propensity n^*; in a two-layer network, the upper risk function $R(n)$ decreases, hence $P(n)$ shifts left, elevating the bribe acceptance propensity n^*.

Q.E.D.

5.2 A Model with Criminal and Innocent Agents

Each agent is criminal with independent, publicly known probability $s \in (0, 1)$, so the chance she is innocent is $(1 - s)$. An agent is *guilty* if she is either criminal or *corrupt* − having accepted at least one bribe. Guilty agents are subject to conviction by the external law enforcement agency and offer bribes to their supervisors.

Agents simultaneously choose how many bribes to accept maximally $n \geq 0$. The game then clears starting from the agents with no subordinate.

5.2.1 Best Response

Criminal agents' incentives are the same as before. Their expected utilities remain unaltered (equation (1)), and so do their optimal strategy correspondences depending only on the lower risk p (figure 3).

In comparison, innocent agents are less inclined to accept bribes, as keeping away from corruption protects them against being caught. An innocent agent's optimal strategy can be derived from that of a criminal agent with the same degrees. Suppose they both have l supervisors and $j > 0$ guilty subordinates. Their expected utilities when accepting any positive number of bribes coincide; if accepting no bribe, the innocent agent is not guilty and so derives strictly higher expected utility than the criminal agent does $\big(0 > U^l(0) = (1 - q)(1 - r)^l c - (lb + c)\big)$. Hence, while the criminal agent accepts $\min\{j, n\}$ bribes at optimum ($n \in \mathbb{R}_+$ is the bribe acceptance propensity), the innocent agent accepts either 0 or $\min\{j, n\}$ bribes depending on which option generates larger payoff.[38]

This binary choice can be captured by a convenient expression. Suppose the bribe acceptance propensity $n \geq 1$. Since the expected utility function U^l is quasi-concave (Lemma 1), before the optimal number n is reached, the more bribes one accepts, the higher her payoff is. Construct the *cutoff policy* $\underline{j}^l \in \{1, ..., \lfloor n \rfloor, \infty\}$ to be the smallest number of bribes an

[38]We insist on the Strong Symmetry Assumption, that is, all criminal agents and innocent ones likely to accept bribes have the same bribe acceptance propensity $n \in \mathbb{R}_+$. See Assumption 3(i) in Appendix c.2 for the formal statement.

innocent agent with l supervisors is willing to accept. Given that her utility of accepting no bribe is zero, $\underline{j}^l$ must be the smallest number of bribes that generates nonnegative expected utility:

$$\begin{cases} U^l(\underline{j}^l) \geq 0 \text{ and } U^l(\underline{j}^l - 1) < 0 & \text{if } \underline{j}^l < \infty \\ U^l(\lfloor n \rfloor) < 0 & \text{if } \underline{j}^l = \infty \end{cases} \tag{6}$$

So the optimal strategy for an innocent agent with l supervisors and $j > 0$ guilty subordinates is to accept $\min\{j, n\}$ bribes if $\underline{j}^l \leq \min\{j, n\}$ and to decline all bribes otherwise. Notice that when the bribe acceptance propensity $n < 1$, $\underline{j}^l = \infty$ and thus the agent optimally accepts no bribe, for the expected utility of strictly mixing between accepting 0 and 1 bribe must be negative $\big(U^l(1) = U^l(0) < 0\big)$.

The following lemma presents some illustrative properties of the cutoff policy $\underline{j}^l$:

Lemma 3. *i An innocent agent's cutoff policy $\underline{j}^l$ **increases** in the number of supervisors she has l and the lower and upper risks p, r.*

ii An innocent agent strictly prefers not to accept bribes if the number of her supervisors is at least the same as that of her guilty subordinates ($l \geq j > 0$).

To see (i), notice that if an innocent agent has more supervisors – l gets larger, then once she becomes corrupt, not only does she need to pay more bribes, but she is also more likely to get reported. Similarly, increasing upper risk r elevates her chance of getting reported through each link. Both deter her from engaging in corruption. Larger lower risk p makes accepting bribes riskier and thus less attractive. (ii) can be understood with a simple reasoning: if an innocent agent decides to engage in corruption, her most optimistic outcome is to accept all the bribes while remaining safe from being caught. If she has more supervisors than guilty subordinates ($l \geq j > 0$), it gives her nonpositive payoff $(j - l)b \leq 0$. Since there is a positive chance of getting caught (the external monitoring success rate $q > 0$), the agent would rather stay away from corruption.

Lemma 3(i) implies that compared with a criminal agent whose bribe acceptance decision only depends on her (direct and indirect) subordinates' and co-supervisors' behaviors, an innocent agent bases the decision on her supervisors' actions as well – the more inclined they are to accept bribes, the less likely the innocent agent is reported should she engage in corruption, and the more she tends to do so (the cutoff policy $\underline{j}^l$ decreases when the upper risk r gets smaller), suggesting a new complementary force between agents' strategies.

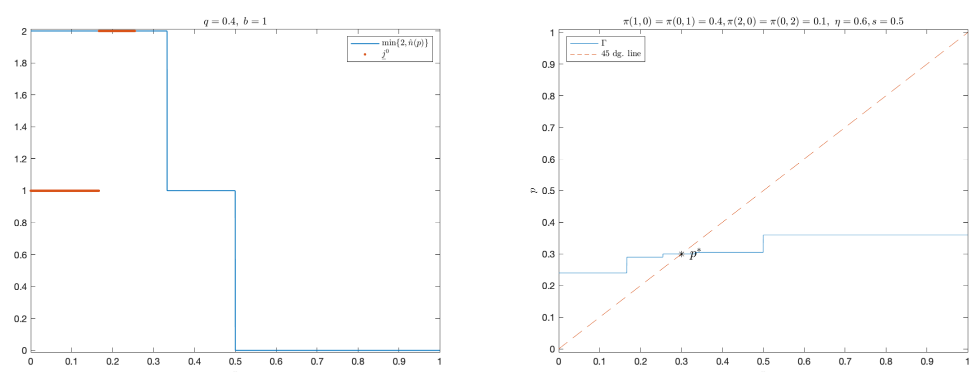
5.2.2 Equilibrium

Since only guilty agents offer bribes, we prune non-guilty agents from the network. Let g be the probability any given subordinate of an agent is guilty, and μ_g be the distribution of the number of guilty subordinates a random agent has. g and μ_g are interdependent and can be jointly solved as functions of agents' strategies $\langle n, (\underline{j}^l)_l \rangle$. Given these statistics, we can then define the risk functions $P\big(n, (\underline{j}^l)_l\big), R\big(n, (\underline{j}^l)_l\big) \in [0,1]$.[39]

Since agents' optimal strategies depend on both risk variables (p, r), in finding the equilibria, it is necessary to employ a more advanced technique: we construct a self-map $\Gamma : (p, r) \mapsto (p, r)$ and show that the equilibria are found at its fixed points: $(p^*, r^*) \in \Gamma(p^*, r^*)$. Specifically, given the lower and upper risks (p, r), we solve for all optimal strategies with the form $\langle n, (\underline{j}^l)_l \rangle$ through the desired number of bribes correspondence $\hat{n}(p)$ and condition (6), which are then mapped back to the risk variables (p, r) using the risk functions P, R.

For the self-map Γ to be well-defined, we need to incorporate in innocent agents' mixed strategies between accepting zero and some positive number of bribes: if at some lower and upper risks (p, r) an innocent agent is indifferent between these two choices, that is, if $U^l\big(\lfloor \hat{n}(p) \wedge j \rfloor\big)|_{p,r} = 0$ for some l, j, we collect all the induced optimal mixed strategies[40] and input them into the risk functions P, R.[41] Thus the self-map Γ generates a continuous range of outputs.

Figure 9 illustrates how the equilibrium is reached in a two-layer network. In this case, since the supervisors who make bribe acceptance decisions are not monitored, their optimal strategies and thus the self-map Γ are defined on the lower risk p alone. 9a depicts the optimal strategies. Innocent agents' cutoff policy $\underline{j}^0$ (red) increases with the lower risk p, for they are more reluctant to engage in corruption when accepting bribes is more dangerous. If p is sufficiently large, $\underline{j}^0$ goes to infinity, indicating they never accept bribes. 9b visualizes the self map $\Gamma : p \mapsto p$ (blue) and characterizes the equilibrium p^* at its fixed point.



(a) Optimal Strategies

(b) Self-Map Γ and the Equilibrium

Figure 9: Equilibrium for a Two-Layer Network[42]

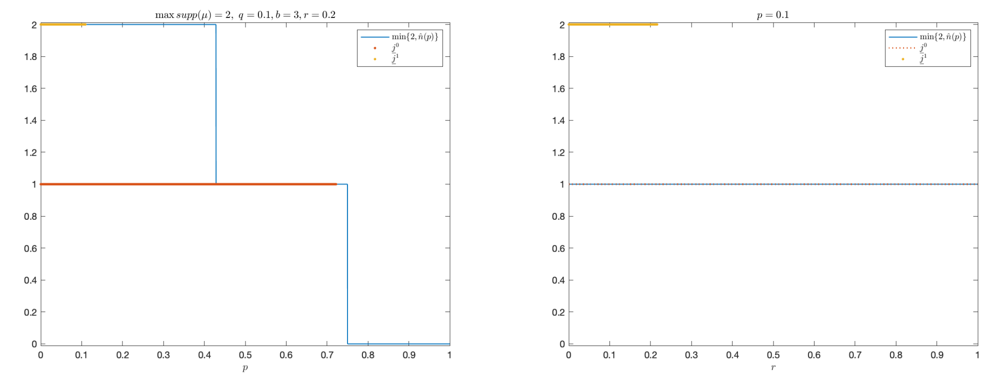
[39] For the formal expressions of the network statistics $\langle g, \mu_g \rangle$ and risk functions P, R, see Appendix f.2.

[40] As part of the Strong Symmetry Assumption, we impose that all innocent agents indifferent between accepting bribes and not mix them with the same probabilities (Assumption 3(ii) in Appendix c.2).

[41] In accordance, the domain of the risk functions P, R is expanded.

[42] Parameter values: $\pi(1,0) = \pi(0,1) = 0.4, \pi(2,0) = \pi(0,2) = 0.1$; $q = 0.4, \eta = 0.6, b = 1, s = 0.5$.

Figure 10 demonstrates agents' optimal strategies in a hierarchy. Now, since innocent, subordinate agents base their bribe acceptance decisions on both the lower and upper risks (p, r), in each of 10a and 10b, we fix one risk variable and plot the optimal strategies on the other. As reflected in both graphs, among the innocent agents, compared with those at the top, the subordinate agents incur the additional risk of being reported and thus adopt a more stringent cutoff policy ($\underline{j}^1 \geq \underline{j}^0$, $\underline{j}^1$ in yellow, $\underline{j}^0$ in red). This result is consistent with Lemma 3(i) that the cutoff policy $\underline{j}^l$ increases with the number of supervisors one has l.



(a) Optimal Strategies on p (Fixing $r = 0.2$)　(b) Optimal Strategies on r (Fixing $p = 0.1$)

Figure 10: Optimal Strategies in a Hierarchy[43]

The self-map Γ is upper hemicontinuous, convex and closed at each point (p, r), thus equilibrium existence is proved with *Kakutani's Fixed Point Theorem.*

Proposition 5. *An equilibrium exists.*

5.2.3 Comparative Statics on the Crime Rate

In this model, corruption is triggered by crime, hence a natural question to ask is how the prevalence of crime influences agents' bribe acceptance decisions. We focus on two network structures – *linear lines* (a special case of hierarchies where each agent has at most one subordinate) and two-layer networks[44] – and show that changes in the *crime rate s* have different implications within and across networks.

We analyze both agents' *propensities to accept bribes* measured with the equilibrium strategies $\langle n^*, (\underline{j}^{l*})_l \rangle$, and the corruption level κ^* defined again as the expected number of bribes an agent accepts.[45] A linear line produces a unique equilibrium,[46] yet multiple

[43] Parameter values: $\max \operatorname{supp} \mu = 2$; $q = 0.1, b = 3$.

[44] In Appendix c.2, we argue that even in this extended model, it is without loss of generality to restrict attention to strongly symmetric equilibria in a two-layer network.

[45] See Appendix f.2 for the formal expression of the *corruption level* κ.

[46] In a linear line, the lower risk function P depends only on criminal agents' bribe acceptance propensity n (see Footnote 48). Since $P(n)$ is increasing in a hierarchy, we have equilibrium uniqueness.

equilibria can exist for a two-layer network. We thus select the largest equilibrium as before.[47]

Unlike hierarchies in general, only top agents' cutoff policy $\underline{j}^{0*}$ matters in linear lines, as innocent, subordinate agents never accept bribes – they have weakly more supervisors than guilty subordinates (Lemma 3(ii)). The irrelevance of subordinate agents' cutoff policies is necessary in producing clear-cut results, justifying our restriction.

In a linear line, in equilibrium, when the crime rate s **increases**, agents' propensities to accept bribes **fall**: n^* decreases, $\underline{j}^{0*}$ increases. This is because when there are more criminals who offer and (may) accept bribes, the collusive subnetwork expands, making a corruption investigation easier to percolate up to endanger a bribe taker. Thus, agents are deterred from accepting bribes.[48] This result echoes the strategic substitutability introduced in Section 3.1: in a hierarchy, an agent is less inclined to accept bribes if her opponents are more corrupt.

The comparative statics on the corruption level κ^* is ambiguous, as it is confounded by two forces that counteract the reduction in agents' bribe acceptance propensities – more criminals induce more bribe taking opportunities, relaxing agents' capacity constraints; besides, criminal agents who are more prone to bribe taking now constitute a larger population.

Now look at two-layer networks. We distinguish between the crime rate among supervisors and that among subordinates, for they generate different predictions.

Proposition 6. *In a two-layer network, in equilibrium, when supervisors' crime rate **increases**, both agents' propensities to accept bribes and the corruption level **rise**: n^* increases, $\underline{j}^{0*}$ decreases, κ^* increases; when subordinates' crime rate **increases**, agents' propensities to accept bribes **fall**: n^* decreases, $\underline{j}^{0*}$ increases.*

Proof: When supervisors' crime rate increases, the upper risk function R shifts down, and so does the lower risk function P and, thus, the self-map $\Gamma : p \mapsto p$. Hence, the equilibrium lower risk p^* falls, implying larger bribe acceptance propensities and, consequently, larger corruption level κ^*. The opposite happens when subordinates' crime rate increases, leading to smaller bribe acceptance propensities.

Q.E.D.

[47] The largest equilibrium is the one with the smallest lower risk p^*, which must accompany the largest n^* (the desired number of bribes correspondence $\hat{n}(p)$ is nonincreasing) and the smallest cutoff policy $\underline{j}^{0*}$ (the smaller the lower risk p^* is, the more incentivized innocent agents are to accept bribes). Extending Proposition 9 in Appendix d, we claim that in a two-layer network, the largest equilibrium maximizes any agent's expected utility out of all equilibria. The proof is rather similar except that the upper risk r^* is now irrelevant to supervisors' expected utilities, and innocent supervisors have the option to decline all bribes and achieve 0 payoff. Multiple equilibria at the smallest lower risk p^* can occur if both criminal and innocent agents employ mixed strategies, though they share the same expected utilities and are thus virtually equivalent. In this case, we make the further refinement by selecting the one with the largest n^*.

[48] Mathematically, since innocent, subordinate agents never accept bribes, the lower risk function P depends only on criminal agents' bribe acceptance propensity n. When the crime rate s increases, $P(n)$ shifts right, resulting in smaller n^* and larger lower risk p^*, which in turn raises innocent, top agents' cutoff policy $\underline{j}^{0*}$.

Intuitively, if there are more criminals among supervisors who tend more to accept bribes, criminal subordinates are less likely to be reported. Hence, accepting bribes from them is less risky and thus more attractive. This result is reminiscent of the strategic complementarity, namely, in a two-layer network, one had better accept more bribes if her co-supervisors are more corrupt. This increase in agents' propensity to accept bribes, together with the rising population of criminal supervisors more susceptible to bribe taking, elevates the corruption level κ^*.

In contrast, if there are more criminals among subordinates, more bribes are offered and thus fewer are accepted. As a result, it is more dangerous and thus less appealing to accept criminal subordinates' bribes. Nevertheless, just like in a linear line, more bribe offers also relax supervisors' capacity constraints. So it is hard to say whether the corruption level κ^* rises or falls.

6 Conclusion

In this paper, I study the bribe acceptance decisions of corruptible monitors when they are placed in a monitoring network that propagates bribe taking risk. All networks are in between two extreme cases – hierarchies and two-layer networks – that predict opposite relationships between an agent's bribe taking risk and her opponents' bribe acceptance behaviors. In equilibrium, a denser monitoring network elevates agents' bribe taking risk, thus deterring them from accepting bribes. Interestingly, as it also induces more bribe taking opportunities, changes in the corruption level may not agree with shifts in individuals' bribe acceptance propensity.

For simplicity, we assume away bribers' strategic decisions, cyclic networks (which allow mutual monitoring), and the possibility of being caught through bribing a supervisor. All are meaningful directions future research could advance in. Besides, it remains to be tested whether this model can be implemented in empirical studies, and whether the issue it uncovers on corruption measurement is evident in reality.

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Appendix

a Monitoring Network

a.1 Friendship Paradox

In this section, we justify the introduction of neighbors' degree distributions $\langle\hat{\lambda},\hat{\mu}\rangle$ in the branching process. First, notice that a random monitoring network exhibits *friendship paradox*, namely, compared with an agent, a subordinate of an agent has in expectation more supervisors, i.e., larger in-degree, and a supervisor of an agent has in expectation more subordinates, i.e., larger out-degree. In the branching process, since we generate each new node by attaching it to a preexisting link, it is by nature a neighboring node (except for the first node), and so when we draw its degrees, we must adjust for the friendship paradox to ensure that in the generated network, a random node's degrees meet the specified joint degree distribution π. As the network is directed, this requires us to separately deal with a new subordinate (who already has an in-link) and a new supervisor (who already has an out-link) by creating the in-links of the former from a random subordinate's in-degree distribution $\hat{\lambda}$, and the out-links of the latter from a random supervisor's out-degree distribution $\hat{\mu}$.

For derivations of neighbors' degree distributions, and formal treatments on how the friendship paradox is related to the branching process, see Jackson (2010) and Sadler (2020).

a.2 Consistency Assumption

Here we justify Assumption 1(ii). Suppose it is violated, i.e., the expected in-degree does not equal the expected out-degree: $\mathbb{E}_\lambda(l) \neq \mathbb{E}_\mu(k)$. We show that the resulting random network does not follow the specified degree distributions $\langle\lambda,\mu\rangle$ through a special case. Assume the degree distributions satisfy $\lambda(0) = 1$, $\mu(1) = 0.9$, $\mu(0) = 0.1$ and $\lambda \perp \mu$. Applying the branching process, there is a positive chance that we obtain a finite chain of nodes, where except for the top node, every node has one in-link. Therefore, although the specified expected in-degree is $\mathbb{E}_\lambda(l) = 0$, the actual expected in-degree is positive, indicating the network does not follow the chosen degree distributions.

This problem comes up because in the branching process, we generate new directed links at each step, creating correlations between the numbers of in- and out-links in the network – whenever we generate an out-link, it must also serve as an in-link for some node in the network, and vice versa. Therefore, if the chosen degree distributions exhibit smaller expected in-degree than expected out-degree: $\mathbb{E}_\lambda(l) < \mathbb{E}_\mu(k)$, then the actual expected number of in-links for the realized network is adjusted up by the "superfluous" out-links and so larger than $\mathbb{E}_\lambda(l)$, and the actual expected number of out-links is smaller than $\mathbb{E}_\mu(k)$. Similar logics apply if we set $\mathbb{E}_\lambda(l) > \mathbb{E}_\mu(k)$. As a result, to ensure the chosen degree distributions correctly reflect the realized network, we must equalize $\mathbb{E}_\lambda(l)$ and $\mathbb{E}_\mu(k)$.[49]

b Proof of Lemma 1

Suppose the lower and upper risks $p, r < 1$. We first show that the expected utility function $U^l(n)$ extended on $\mathbb{R}_+$ is quasi-concave. For that, we compute its derivative:

$$\frac{\partial U^l(n)}{\partial n} = (1-q)(1-p)^n(1-r)^l\big[b + (nb+c)\log(1-p)\big].$$

If $p = 0$, it is positive, and so $U^l(n)$ is an increasing function. When $p > 0$, if $c \geq -b/\log(1-p)$, it is nonpositive, thus $U^l(n)$ is nonincreasing; otherwise, it is positive when $n < -1/\log(1-p) - c/b$ and negative when $n > -1/\log(1-p) - c/b$, and so $U^l(n)$ first increases, then decreases. In any case, $U^l(n)$ is quasi-concave.

We now derive an agent's desired number of bribes $\hat{n}$, which furnishes the proof of the rest of Lemma 1. Since the expected utility function $U^l(n)$ is quasi-concave, accepting any $n \in \mathbb{N}_+$ bribes is optimal iff $U^l(n) \geq \max\big\{U^l(n+1), U^l(n-1)\big\}$, which yields

$$\frac{b}{c+(n+1)b} \leq p \leq \frac{b}{c+nb}$$

Similarly, declining all bribes is optimal iff $U^l(0) \geq U^l(1)$, which gives $p \geq b/(c+b)$. These conditions suggest that the desired number of bribes $\hat{n}$ is irrelevant of the degrees k, l and the upper risk r, and characterize it as a nonincreasing correspondence of the lower risk p.

Lastly, we incorporate the left-out boundary cases into the desired number of bribes correspondence $\hat{n}(p)$. Specifically, we show that adding the condition $\hat{n}(1) = 0$ is sufficient to ensure that $\hat{n}(p)$ is well-defined on $[0,1]$. Suppose the lower risk $p = 1$. If the upper risk $r < 1$, an agent optimally desires no bribe. This is because while accepting any positive number of bribes would result in conviction with certainty, a strictly positive chance of not getting caught is ensured if she instead refrains from bribe taking. If the upper risk $r = 1$, while a monitored agent is indifferent between accepting any numbers of bribes, as she is certainly caught anyway; those with no supervisor and at least one subordinate (who must exist in a non-trivial, acyclic monitoring network) still desire no bribe at optimum, for their decisions are independent of the upper risk r. We thus let $\hat{n}(1) = 0$. Since the upper risk $r = 1$ only if no one accepts any bribe, $\hat{n}(1) = 0$ correctly predicts any agent's choice. By a similar logic, the desired number of bribes correspondence $\hat{n}(p)$ captures all cases where $p < 1$ and $r = 1$.

[49] Violating the consistency assumption is innocuous in some cases where the network grows unbounded, that is, Assumption 1(i) is not satisfied. To see that, set $\lambda(1) = \mu(2) = 1$. Then we obtain an "infinite hierarchy" where each agent has exactly one supervisor and two subordinates, consistent with the given degree distributions.

Q.E.D.

c Strong Symmetry Assumption

c.1 Baseline Model

In this section, we divide the set of mixed-strategy *symmetric equilibria* into different groups, each including one and only one *strongly symmetric equilibrium* defined in Assumption 2. We then show that all equilibria in the same group share the same expected utility for any agent, such that it is without loss of generality to select only the strongly symmetric equilibrium.

Define a symmetric equilibrium by $\mathbf{n}^* \equiv (n^*_{lk})$, where $n^*_{lk} \in \mathbb{R}_+$ is the number of bribes an agent with l supervisors and k subordinates accepts. We only consider the symmetric equilibria where some agents adopt strictly mixed strategies (pure-strategy symmetric equilibria are strongly symmetric), and classify them in the following way:

Definition 2. *An* ***equivalence class*** $\mathcal{C}(m, x_m)$ *is the set of symmetric equilibria* $\mathbf{n}^*$ *for which the associated lower risk* $p^* = p_{m(m+1)}$ *and* $\mathbb{E}_\pi[n^*_{lk}|k > m] = x_m \in (m, m+1)$.

In words, at any equilibrium in an equivalence class $\mathcal{C}(m, x_m)$, agents are indifferent between accepting m and $(m+1)$ bribes, and the expected number of bribes agents with relaxed capacity constraints accept is x_m. $\mathcal{C}(m, x_m)$ must include a unique strongly symmetric equilibrium $n^* = x_m$.

The following proposition establishes the similarity between all equilibria in an equivalence class, justifying our sole selection of the strongly symmetric one.

Proposition 7. *All equilibria in the same equivalence class entail the same upper risk and thus the same expected utilities.*

Proof: We first show that all equilibria in an equivalence class $\mathcal{C}(m, x_m)$ share the same upper risk. Consider any equilibrium $\mathbf{n}^* \in \mathcal{C}(m, x_m)$. Its upper risk is expressed as:[50]

$$\begin{aligned}
R(\mathbf{n}^*) &= \sum_l \sum_{k \geq 1} \Big(1 - \frac{n^*_{lk}}{k}\Big) \hat{\mu}(k) \lambda(l|k) \\
&= \sum_l \sum_{k \geq 1} \frac{k - n^*_{lk}}{k} \cdot \frac{k\mu(k)}{\bar{k}} \lambda(l|k) \quad \big(\bar{k} \equiv \mathbb{E}_\mu(\tilde{k})\big) \\
&= \sum_l \sum_{k > m} \frac{k - n^*_{lk}}{k} \cdot \frac{k\mu(k)}{\bar{k}} \lambda(l|k) \quad (n^*_{lk} = k \ \forall k \leq m) \\
&= \frac{\sum_{k>m} k\mu(k) - x_m \mathbb{E}_\pi\big[\mathbb{1}\{k > m\}\big]}{\bar{k}}
\end{aligned}$$

[50] This expression is adapted from the upper risk function $R(n)$ defined by equation (3).

Since it only depends on (m, x_m), all equilibria in the equivalence class $\mathcal{C}(m, x_m)$ share the same upper risk. Because they also share the same lower risk $p^* = p_{m(m+1)}$, and agents optimize their bribe acceptance choices in equilibrium, any agent must derive the same expected utility at any equilibrium in $\mathcal{C}(m, x_m)$.

Q.E.D.

Here we present a numerical example where multiple equilibria exist in an equivalence class. Consider a hierarchy with out-degree distributions given by $\mu(0) = 1/2$ and $\mu(1) = \mu(2) = 1/4$. Set parameter values $\eta = 1$, $q = 1/2$, $b = 4/3$, $c = 1$. This game has an equivalence class $\mathcal{C}(0, 1/2)$, which includes, for instance, three equilibria: $(n^*_{01}, n^*_{11}, n^*_{02}, n^*_{12}) = (1, 1, 0, 0)$, $(0, 0, 1, 1)$ and $(1/2, 1/2, 1/2, 1/2)$ (apparently, $n^*_{00} = n^*_{10} = 0$), where the last one is strongly symmetric. It is easy to verify that they share the same lower risk $p^* = p_{01} = 4/7$ and upper risk $r^* = 1/3$. Consequently, agents' expected utilities are also the same across them: -1/2 for those with no supervisor and -2 for those with one supervisor.

c.2 Model with Criminal and Innocent Agents

This section presents the Strong Symmetry Assumption for the extended model introduced in Section 5.2, and show that it is without loss of generality to restrict attention to strongly symmetric equilibria in a two-layer network – the main focus of study in our comparative statics.

Assumption 3. *In a strongly symmetric equilibrium,*

i there is $n^* \in \mathbb{R}_+$ such that an agent with $j > 0$ guilty subordinates accepts either $\min\{j, n^*\}$ or 0 bribe;

ii all innocent agents indifferent between accepting bribes and not mix them with the same probabilities.

To establish the generality of strongly symmetric equilibria in a two-layer network, we first group symmetric equilibria[51] in the following way:

Definition 3. *An **equivalence class** $\mathcal{C}(p^*)$ is the set of symmetric equilibria with the same lower risk p^*.*

Since in a two-layer network, supervisors who make bribe acceptance decisions are not monitored, their expected utilities are irrelevant to the upper risk r. Thus all equilibria in the same equivalence class $\mathcal{C}(p^*)$ produce the same expected utility for any agent, indicating that they are virtually equivalent. Hence, the following proposition implies that it is without loss of generality to focus only on strongly symmetric equilibria:

[51]A symmetric equilibrium is an equilibrium in which criminal (innocent) agents with the same numbers of supervisors and guilty subordinates (l, j) adopt the same mixed strategy.

Proposition 8. *In a two-layer network, each equivalence class $\mathcal{C}(p^*)$ contains a strongly symmetric equilibrium.*

Proof: In a two-layer network, subordinates do not make bribe acceptance decisions, so any given subordinate of an agent is guilty if and only if she is criminal: $g = s$, and the distribution of the number of guilty subordinates an agent has is fixed at μ_s. Hence, the more prone agents are to accept bribes, the smaller the upper risk r is, and the smaller the lower risk p is. Therefore, among the optimal symmetric strategies at some equilibrium lower risk p^*, the function P that maps from agents' strategies to the lower risk p obtains its maximum $p_M \geq p^*$ (minimum $p_m \leq p^*$) when any innocent agent indifferent between accepting bribes and not declines all bribes (resp., accepts bribes) with certainty, and all criminal agents and innocent ones who accept bribes desire $\min \hat{n}(p^*)$ $\big($resp., $\max \hat{n}(p^*)\big)$ bribes. Since both the maximizer and the minimizer are strongly symmetric, if they do not coincide $\big($otherwise, they define the unique element of the equivalence class $\mathcal{C}(p^*)\big)$, we can easily construct a continuum of optimal strongly symmetric strategies at p^* for which they are the two boundary points and on which the mapping P is continuous.[52] Hence, the *Intermediate Value Theorem* suggests that p^* must be achievable by some optimal strongly symmetric strategies in this range. By definition, it belongs to the equivalence class $\mathcal{C}(p^*)$.

Q.E.D.

d Equilibrium Selection

In the baseline model, in face of equilibrium multiplicity, we select the largest equilibrium. More precisely, denote by N^* the set of all equilibria given certain parameter values. The largest equilibrium $n_M^* = \max N^*$. Here we justify this selection by showing the following:

Proposition 9. *The largest equilibrium n_M^* maximizes any agent's expected utility out of all the equilibria in N^*.*

Proof: When there are multiple equilibria, pick any two $n_1^*, n_2^* \in N^*$ that satisfy $n_1^* > n_2^*$. We want to show that any agent obtains higher expected utility at n_1^* than at n_2^*. Denote the lower and upper risks at n_i^* by p_i^*, r_i^*. Since the desired number of bribes correspondence $\hat{n}(p)$ is nonincreasing, we have $p_1^* \leq p_2^*$. Moreover, the strictly decreasing upper risk function $R(n)$ implies $r_1^* < r_2^*$. Therefore, the expected utility for an agent with l supervisors and k

[52] For this construction, restriction to a subset of the optimal strongly symmetric strategies at p^* is only necessary if the two types of mixed strategies can coexist: criminal agents may mix strictly between accepting two adjacent numbers of bribes, i.e., $\min \hat{n}(p^*) < \max \hat{n}(p^*)$ and some agents' capacity constraints do not bind at $\max \hat{n}(p^*)$; and some innocent agent may mix strictly between accepting bribes and not – there exists some j such that $U^0(\lfloor \hat{n}(p^*) \wedge j \rfloor)|_{p^*} = 0$. In this case, we reduce the space to the optimal strongly symmetric equilibria that can be summarized with one variable $r \in [0, 1]$: all innocent agents indifferent between accepting bribes and not accept bribes with probability r and decline all bribes with probability $(1-r)$, and all agents likely to accept bribes desire $\max \hat{n}(p^*)$ bribes with probability r and $\min \hat{n}(p^*)$ bribes with probability $(1 - r)$.

subordinates satisfies $U^l\big(\lfloor n_1^* \wedge k \rfloor\big)|_{p_1^*, r_1^*} \geq U^l\big(\lfloor n_2^* \wedge k \rfloor\big)|_{p_1^*, r_1^*} \geq U^l\big(\lfloor n_2^* \wedge k \rfloor\big)|_{p_2^*, r_2^*}$, such that the larger equilibrium n_1^* generates larger expected utility than n_2^* does. The first inequality holds because n_1^* is the optimal choice at lower risk p_1^*. The second is true for $p_1^* \leq p_2^*$ and $r_1^* < r_2^*$.

Q.E.D.

e Comovement of n^* and κ^*

In this section, we formalize the comovement of the equilibrium bribe acceptance propensity n^* and corruption level κ^* in a hierarchy in the baseline model, and generalize it to any network with mutually independent in- and out-degrees.

Proposition 10. *In a hierarchy, in equilibrium, if the bribe acceptance propensity n^* strictly drops with a local increase in the out-degree distribution μ, so does the corruption level κ^*.*

Proof: Given a local increase in the out-degree distribution μ, the lower risk function $P(n)$ shifts right, causing the bribe acceptance propensity n^* to drop. Observe that it strictly drops only if the lower risk p^* is fixed at some cutoff $p_{m(m+1)} \in (0, 1)$ where $m \in \mathbb{N}$ and $n^* \in (m, m + 1]$. Therefore, rearranging equation (4) that defines the lower risk function $P(n)$, we obtain:

$$\sum_{k>m} n^* \mu(k) = \frac{1}{p^*} \sum_{k \leq m} (1 - p^*)^{k-m} \mu(k) + \frac{mp^* + 1}{p^*} \sum_{k>m} \mu(k) + C, \tag{7}$$

$$\text{where} \quad C = \frac{p^* - \eta}{\eta(1 - q)p^*(1 - p^*)^m}$$

Since the corruption level $\kappa^* = \mathbb{E}_\mu\big[n^* \wedge k\big] = \sum_{k \leq m} k\mu(k) + \sum_{k>m} n^* \mu(k)$, plugging (7) in, it can be expressed as:

$$\kappa^* = \mathbb{E}_\mu\big[g(k)\big] + C,$$

$$\text{where} \quad g(k) = \mathbb{1}\{k \leq m\} \cdot \Big(k + \frac{(1 - p^*)^{k-m}}{p^*}\Big) + \mathbb{1}\{k > m\} \cdot \Big(m + \frac{1}{p^*}\Big)$$

It is easy to verify that the function $f(k) \equiv k + (1 - p^*)^{k-m}/p^*$ is strictly decreasing on $[0, m]$ and $f(m) = m + 1/p^*$, such that $g(k)$ is decreasing. Hence, a local MLRP increase in the out-degree distribution μ (which implies an FOSD increase) decreases the corruption level κ^*. It strictly decreases because $\mu(k) > 0$ for some $k \leq m$. To see that, notice that agents' bribe acceptance propensity n^* strictly drops with the out-degree distribution μ only if some agents have binding capacity constraints, such that they indeed accept more bribes when offered more, rendering bribe acceptance more dangerous.

Q.E.D.

Proposition 10 can be generalized to any network where the in- and out-degrees are mutually independent: $\lambda \perp \mu$. Formally, if some equilibrium bribe acceptance propensity n^* (not necessarily the largest one) satisfies $n^* \in (m, m+1)$ for some $m \in \mathbb{N}$ and $P'(n^*) > 0$, then given a local increase in the out-degree distribution μ, both n^* and the corresponding corruption level κ^* fall. κ^* falls strictly whenever n^* does.

The bribe acceptance propensity n^* falls because the lower risk function $P(n)$ shifts right when the out-degree distribution μ increases.[53] To see why the corruption level κ^* falls, notice that, just like in a hierarchy, it can be expressed as $\kappa^* = \mathbb{E}_\mu\big[g(k)\big] + C(n^*)$, where $g(k)$ is the same as before, and

$$C(n^*) = \frac{p^* - \eta}{\eta(1-q)p^*(1-p^*)^m \mathbb{E}_{\hat{\lambda}}\big[1 - R(n^*)\big]^{l-1}}$$

Since the upper risk function $R(n)$ is decreasing and the lower risk p^* is by definition smaller than the contagion rate η, $C(n^*) \leq 0$ falls given a smaller bribe acceptance propensity n^*. The rest of the proof follows that for Proposition 10.

f Expressions for Extended Models

f.1 Model with Corruptible and Incorruptible Agents

Risk Functions $P(n), R(n)$

The *upper risk function* $R(n)$ is:

$$R(n) = \sum_{k \geq 1} \left\{ (1-\gamma) + \gamma\Big(1 - \frac{n \wedge k}{k}\Big) \right\} \hat{\mu}(k).$$

For a supervisor with k subordinates, with probability $(1-\gamma)$, she never accepts bribes and so reports the agent with certainty; with probability γ, she accepts n bribes, thus the chance of being reported by her is $\big(1 - (n \wedge k)/k\big)$.

The *lower risk function* $P(n)$ is recursively defined by:

$$p = \eta \cdot \sum_k \sum_{l \geq 1} \bigg\{ 1 - (1-q)\big(1 - R(n)\big)^{l-1} \Big[(1-\gamma) + \gamma(1-p)^{\lfloor n \wedge k \rfloor}\big(1 - (n \wedge k - \lfloor n \wedge k \rfloor)p\big) \Big] \bigg\} \hat{\lambda}(l)\mu(k|l).$$

An agent is caught through accepting a subordinate's bribe if the subordinate is caught and their bribe transaction is detected (with chance η). Consider a subordinate with degrees

[53] For this result, the assumption $P'(n^*) > 0$ – the lower risk function $P(n)$ is strictly increasing at n^* – is necessary. Otherwise, if $P'(n^*) < 0$, then n^* rises when $P(n)$ shifts right.

(l,k). Conditional on the agent's having accepted her bribe, if she is incorruptible $\big($with chance $(1-\gamma)\big)$, she remains safe with probability $(1-q)\big(1-R(n)\big)^{l-1}$; otherwise, if she is corruptible (with chance γ), since she may be caught through accepting bribes, her chance of being safe is further discounted by $(1-p)^{\lfloor n\wedge k\rfloor}\big(1-(n\wedge k-\lfloor n\wedge k\rfloor)p\big)$.

As in the baseline model, the finiteness of the monitoring network $\big($Assumption $1(i)\big)$ ensures that the lower risk function $P(n)$ is well-defined.

f.2 Model with Criminal and Innocent Agents[54]

Statistics for the Network of Guilty Agents $\langle g,\mu_g\rangle$

The probability any given subordinate of an agent is guilty g and the distribution of the number of guilty subordinates a random agent has μ_g are jointly solved by the following two equations:

$$g = s + (1-s)\sum_{j}\sum_{l\geq 1}\mathbb{X}\{j\geq \underline{j}^l\}\hat{\lambda}(l)\mu_g(j|l), \tag{8}$$

$$\mu_g(j) = \sum_{k\geq j} C_j^k g^j(1-g)^{k-j}\mu(k) \quad \forall j. \tag{9}$$

$\mu_g(\cdot|l)$ is the distribution of the number of guilty subordinates an agent has conditional on her having l supervisors, so $\sum_j\sum_{l\geq 1}\mathbb{X}\{j\geq \underline{j}^l\}\hat{\lambda}(l)\mu_g(j|l)$ is the probability an innocent subordinate accepts bribes. If an agent has k subordinates, $j\leq k$ of them are guilty with probability $C_j^k g^j(1-g)^{k-j}$.

Since the monitoring network is finite $\big($Assumption $1(i)\big)$, (8) and (9) pin down a unique solution $\langle g,\mu_g\rangle$.

Risk Functions P,R

The *upper risk function* R is expressed as:

$$R\big(n,(\underline{j}^l)_l\big) = \sum_{l}\sum_{j\geq 1}\left\{s\Big(1-\frac{n\wedge j}{j}\Big)+(1-s)\Big(1-\mathbb{X}\{j\geq \underline{j}\}\frac{n\wedge j}{j}\Big)\right\}\hat{\mu}_g(j)\lambda_g(l|j).$$

$\hat{\mu}_g$ is the distribution of the number of guilty subordinates any supervisor of a guilty agent has, which can be derived from μ_g by adjusting for the friendship paradox. $\lambda_g(\cdot|j)$ is an agent's upper degree distribution conditional on her having j guilty subordinates. While a criminal supervisor of a guilty agent who has degrees (l,j) reports the agent with probability $\big(1-(n\wedge j)/j\big)$, an innocent one reports her with probability $\big(1-\mathbb{X}\{j\geq \underline{j}\}(n\wedge j)/j\big)$.

[54] For ease of exposition, these expressions disregard the likelihood for innocent agents to mix between accepting zero and some positive number of bribes. To incorporate in such mixed strategies, modifications are necessary.

The *lower risk function* $P\big(n, (\underline{j}^l)_l\big)$ is recursively defined by:

$$p = \frac{\eta}{g} \sum_j \sum_{l \geq 1} \bigg\{ \Big[s + (1-s)\mathbb{X}\{j \geq \underline{j}^l\} \Big] \Big[1 - (1-q)\Big(1 - R\big(n, (\underline{j}^l)_l\big)\Big)^{l-1} \cdot (1-p)^{\lfloor n \wedge j \rfloor} \big(1 - (n \wedge j - \lfloor n \wedge j \rfloor)p\big) \Big] \bigg\} \hat{\lambda}(l) \mu_g(j|l).$$

It measures the probability a subordinate of an agent is caught and their bribe transaction is consequently detected (with probability η) conditional on the subordinate's being guilty (with probability g) and the agent's having accepted her bribe. Specifically, for a subordinate with degrees (l, j), $\big[s + (1-s)\mathbb{X}\{j \geq \underline{j}^l\}\big]$ is the probability she is guilty, and $\big[1 - (1-q)\big(1 - R(n, (\underline{j}^l)_l)\big)^{l-1}(1-p)^{\lfloor n \wedge j \rfloor}\big(1 - (n \wedge j - \lfloor n \wedge j \rfloor)p\big)\big]$ is the probability she is caught conditional on her being guilty and the agent's having accepted her bribe.

Just like in the baseline model, the lower risk function $P\big(n, (\underline{j}^l)_l\big)$ is well-defined due to the finiteness of the monitoring network $\big($Assumption $1(i)\big)$.

Corruption Level κ

By definition, the *corruption level* κ – the expected number of bribes an agent accepts – is given by:

$$\kappa = \mathbb{E}_{\pi_g}\bigg[\Big(s + (1-s)X\{n \geq \underline{j}^l\} \Big)(n \wedge j) \bigg].$$

π_g is the joint distribution of the numbers of supervisors and guilty subordinates an agent has, with marginals $\langle \lambda, \mu_g \rangle$. While a criminal agent with degrees (l, j) accepts $(n \wedge j)$ bribes, an innocent one accepts $(n \wedge j)$ bribes if $n \geq \underline{j}^l$ and 0 otherwise.