A core-selecting auction for portfolio's packages

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Abstract

We introduce the "local-global" approach for a divisible portfolio, and perform an equilibrium analysis for two variants of core-selecting auctions. Our main novelty is extending the Nearest-VCG pricing rule in a dynamic two-round setup, mitigating bidders' free-riding incentives and further reducing the sellers' costs. The two-round setup admits an information-revelation mechanism that may offset the "winner's curse", and it is in accord with the existing iterative procedure of combinatorial auctions. With portfolio trading becoming an increasingly important part of investment strategies, our mechanism contributes to increasing interest in portfolio auction protocols.

Keywords - Package auction · VCG payments · Portfolio Trading

JEL Classification - D44 · D47 · G11

1 Introduction

Portfolio Auctions (PA) have recently been in the limelight due to their rapid growth in fixed-income markets. The growth approximates 5% of total market trading volume based on the recent estimates of TRACE[1]. The changes in the market behavior have been driven by the various developments in ETFs and the surge of algorithmic trading, which has facilitated the vertical slicing of portfolios[2]. The automated execution protocol for portfolio trading by ICE Bonds Portfolio Auction and Tradeweb's portfolio trading platform are examples of the current investment strategies in using auctions for trading portfolios.

More commonly known as basket or program trading, it has been in the investment landscape for a while and has accounted for 50% to 60% of the total daily trading volume at the NYSE[3]. This portfolio management strategy allows broker-dealers to bid fast for a bundle of trades executed as a single transaction and operates in two formats: principal and agency trading[4].

In this paper, we focus on *principal* trade in which a broker undertakes the risk price on behalf of the asset manager and executes the portfolio at an agreed price plus a commission fee. After acquiring the portfolio, the broker must attempt to minimize any deficit caused by the actual execution versus the agreed price with the asset manager (a price equal or better in the market).

The impact of this investment strategy for asset managers is vast, especially in periods of high volatility where cost savings and speedy risk transfer are im-

[1] https://www.finra.org/filing-reporting/trace

[2] When considering liquidity management a portfolio can be sliced horizontally or vertically. A slice of liquid assets is described as slicing the portfolio horizontally, while a proportional slice on the entire portfolio is considered as vertical slicing.

[3] In periods with high volatility, it can reach up to 90%. Since 2012, the NYSE no longer publishes weekly program trading reports.

[4] In principal trading, the brokerage completes a customer's trade with their own inventory and can profit from the bid-ask spread, while in the agency trading the brokerage locates a counterparty to a customer's trade willing to purchase or sell the security for the same price as the counterparty.

perative. Even though the asset managers always have the option to execute those trades at their discretion, using a portfolio auction trading tool enables them to have access to multiple liquidity providers simultaneously. In addition, this strategy reduces any information leakage and guarantees execution efficiency - an efficient way to deal with large and complex transactions. From the broker's perspective, this strategy is an opportunity to gain access to new order flows that might match their inventory or facilitate new business. The challenge is to bid low enough to win the auction and at the same time to cover the assumed price risk from the execution [Padilla and Van Roy, 2012].

Currently, portfolio auctions take place as a single round first-price sealed-bid auction, where the entire portfolio is allocated to a single winning broker. One of the caveats in this design is that it can cause a weak demand since some brokers might have no preference in possessing the whole portfolio but have a higher valuation for a part of it. Thus, the asset manager might suffer an increased cost for the portfolio's execution. Another issue is that it does not provide bidders with valuable information to avoid aggressive bidding, thereby the "winner's curse".

Motivated by this context and other real world auctions, here we suggest a package auction for portfolios. We divide the portfolio into packages for a discrete number of brokers with different valuations. We characterize those who compete for the packages as "local" brokers and those who bid for the whole portfolio as "global" brokers. Each of the brokers is aware of the individual securities included in the portfolio and takes long positions in all securities.

The auction occurs in two rounds. In the first round, the asset manager performs simultaneous sealed-bid first-price auctions (a) for "local" brokers who compete for each package separately, and (b) for "global" brokers who compete for the aggregate portfolio. The first round qualifies one global broker and one local broker for each package to participate in the second round. Then, the asset manager reveals information for the qualified bids of the first round. This attribute in the design provides a solution to brokers' information asymmetries by revealing valuations and avoiding jump-bidding [Perry et al., 2000].

In the second round, the local brokers jointly compete for the whole portfolio against the global, who values packages as perfect complements[5]. If the coalition of local brokers submits a bid lower than that of global, the coalition wins, and the fees are awarded to each local broker based on a pricing rule that ensures a *core* outcome.

In our setup, the *core* attributes the set of payoff vectors, that correspond to the portfolio's allocation, where no better outcome exists both for the asset manager and the local brokers. Any payoff vector in the core is *bidder-optimal* if there is no other payoff vector that *Pareto* improves upon it. In other words, the coalition of local brokers is unblocked and feasible [Day and Milgrom, 2008]. Hence, our core-selecting auction provides a framework where the payoff of brokers is on the *bidder-optimal-frontier*, and may reduce significantly the costs of the asset manager [Milgrom, 2004].

2 Related Literature

Most literature in package auctions (or combinatorial auctions) focuses on developing fast heuristics to solve the complex winner's determination problem

[5]The utility of the whole portfolio has a higher utility than the sum of the utilities for the individual packages [Cramton et al., 2006].

[De Vries and Vohra, 2003; Rothkopf et al., 1998], while economists focus on specific properties by using simplified theoretical models.

The underpinnings of package auctions can be traced to the seminal paper of Vickrey [1961], in which each bidder is asked to pay an amount equal to the externalities he exerts on the competing bidders. Vickrey shows that this payment rule motivates bidders to submit a "bid" according to the actual demand schedules, regardless of the bids made by others. It is easy to think that a Vickrey auction could generate an efficient outcome for package auctions due to its appealing property of incentive compatibility. Nevertheless, in practice, the Vickrey auction is never used because this mechanism can lead to low payoffs for the auctioneer, even if bids are high enough [Milgrom, 2007]. Also, the Vickrey pricing is determined by a non-monotonic function of broker's values in the sense that an increase in the number of brokers can reduce equilibrium revenues for the asset manager up to zero. Thus, brokers can use profitably "shill bidding" to increase competition in order to finally charge higher fees [Ausubel and Milgrom, 2002, 2006].

The existing literature alleviates the aforementioned shortcomings by proposing alternative procedures. Ausubel and Milgrom [2002] developed a mechanism called the ascending proxy auction, while, Xia et al. [2004] reviewed several pricing schemes for incentive-compatibility and ascertained that the revelation of the losing bids may reduce the value of the prices relatively to winning bids. Day and Milgrom [2008] and Day and Cramton [2012] suggested a new cluster of payment rules for core-selecting auctions with respect to the reported values.

Recently, Ausubel and Baranov [2020] provide a theoretical justification for the use of core-selecting auctions. They propose an incomplete-information setting in which bidders' values are correlated and analyze the equilibrium under a "local-global" approach. They found that in environments with positive correlations, core-selecting auctions can be significantly closer to the true core than the VCG outcome[6]. To our knowledge, Krishna and Rosenthal [1996] were the first to explore an independent private value setting[7] for the simultaneous sale of multiple items in the "local-global" setting.

The problem that arises in the "local-global" setting is when a coalition wins and the VCG outcome is not in the core [8]. Then, the closer the bidder-optimal frontier gets to VCG pricing, the fewer incentives for misreporting [Ausubel and Milgrom, 2002; Day and Milgrom, 2008]. Day and Raghavan [2007] and Day and Cramton [2012] find alternative payment rules which minimize bidder's incentives for this strategic manipulation. This rule is called the *Nearest-VCG*, which is a point in the bidder-optimal-frontier where the maximum deviation from VCG pricing is minimized.

Erdil and Klemperer [2010] have proposed a new class of pricing rules for core-selecting package auctions focusing on the marginal incentives to deviate from "truthful bidding". The idea is to select a point in the bidder-optimal frontier that is close in a reference point. Motivated by their suggestion, we construct a new payment rule, the Dynamic-Nearest-VCG, using an endogenous reference point suggested by the brokers' own strategic bidding behavior.

From a different perspective, it has been developed a vast literature on iterative combinatorial auctions[9]. One of the merits of this approach, is their

[6]Goeree and Lien [2016] showed that core-selecting auctions concerning true values do not exist.

[7]Rosenthal and Wang [1996] extended the setting with common values.

[8]If the VCG outcome is in the core, it is the unique bidder-optimal allocation.

[9]A multiple-round bidding process at which the auctioneer releases information regarding

ease of deployment. It allows bidders to learn about rivals' valuation, and it is the most popular combinatorial auction format used in practice. For example, the FCC has used only multi-round formats for its auction design [Pekeč and Rothkopf, 2003]. Also, auction designs which allow the creation of synergies through bidder-determined combinations[10] can lead to economically significant outcomes bypassing any computational complexities [Park and Rothkopf, 2005].

The Combinatorial Clock Auction [Ausubel et al., 2017; Bichler et al., 2013; Cramton et al., 2009] in the spectrum auctions is relevant to our model. The initial stage in a CCA design is an open round (clock phase) where bidders demand the desired bundle at each price, allowing information revelation. In the second stage, bidders bid their true preferences in a sealed-bid round with Vickrey prices. Levin and Skrzypacz [2016] identify a weakness in the equilibrium outcome of CCA since bidders in the clock phase have the opportunity to exaggerate their bids and gain a surplus in the supplementary round through types' revelation. In the CCA, bidders learn approximate prices during the clock phase while in our sealed-bid design the information disclosure is concrete for those who participate in the second-round and bidder specific. Nevertheless, in the initial clock phase bidders' incentives might be distorted and efficiency is not ensured. Our aim is to restore incentives for truthful biding in the initial round and yield efficient outcomes.

Our paper proceeds as follows. Section 3 presents the model and describes

the provisional winners and the actual prices, at the end of each round. Bidders obtain information regarding the bids of their rivals and can modify their bids in the following rounds [Parkes, 2006].

[10]Bidders decide what is biddable and what combinations are important to them from economic perspective.

the mechanism for the two rounds. Section 4 derives to the intuitive form of the optimality conditions and analyses the equilibrium. Finally, section 5 discusses the results and concludes.

3 Model

An asset manager sells $m > 2$ securities packaged in a divisible portfolio $\Theta \in \mathbb{R}^m_+$. A set of risk-neutral brokers, $\mathbb{N} = \{1, \ldots, n\}$ are competing[11] for a portfolio Θ. There are two types of brokers who participate in the auction, a set of *local* brokers denoted by the generic element $\ell \in \mathbb{L}$, and a set of *global* brokers $g \in \mathbb{G}$, where $\mathbb{L}, \mathbb{G} \subset \mathbb{N}$ are disjoint and $\mathbb{L} \cup \mathbb{G} = \mathbb{N}$.

We assume that Θ is divided in a finite set of q packages θ_j, with $j = \{1, \ldots, q\}$, such as $\theta_j \in \mathbb{R}^m_+$ and $\Theta = \sum\limits_{j=1}^{q} \theta_j$. Each broker $i \in \mathbb{N}$ observes a private signal $s_i \in \mathbb{S}$ about the value of the Θ or θ_j and a public signal $z \in \mathbb{Z}$ for the aggregate characteristics of portfolio. Information $(\mathbf{s}, z)$, where $\mathbf{s} = (s_\ell, s_{-\ell}, s_g)$, $\forall \ell \in \mathbb{L}$ and $g \in \mathbb{G}$, is distributed according to a continuous i.i.d. function $F_\ell(\cdot)$, with the density function $f_\ell(\cdot) > 0$ for *local* brokers, and $F_g(\cdot)$, with $f_g(\cdot) > 0$ for *global* brokers respectively.

All brokers form expectations for the percentage change of the securities' prices given by the vector,

$$\mathbb{E}\Big[\frac{\Delta p}{p^*}|s_i, z\Big] = \left(\frac{p^*_k - \mathbb{E}[p_k|s_i, z]}{p^*_k}\right)_{k \leq m},$$

where the vector $p^* \in \mathbb{R}^m_+$ includes the agreed exercise price for m securities, and the vector $p \in \mathbb{R}^m_+$ the anticipated price of m securities when delivered. Both random vectors are conditional on the signal received and the available

[11] Without loss of generality, we assume no entry costs for participating brokers.

public information. Evidently, when $\mathbb{E}[\frac{\Delta p}{p^*}|s_i, z] > 0$ brokers anticipate to incur a loss. Also, we denote by $\omega_j = \dfrac{p^* \cdot \theta_j}{p^* \cdot \Theta}$, the weight of θ_j's value over Θ with $\omega_j \in (0, 1]$.

3.1 Mechanism

The auction takes place in two rounds $t = 1, 2$. Each broker $i \in \mathbb{N}$ submits consecutively a single[12] bid (fee) in basis points $\phi_i^t \in [0, 1]$, with $\phi_i^1(s_i, z)$ to be the first-round bid and $\phi_i^2(s_i, z')$ be the second-round bid for z' an update in public information. The fee is calculated ad valoren on the portfolios value. The standard tie-breaking rule (in which the winner is selected at random) applies to both rounds.

First round. The asset manager initiates $q+1$ simultaneous sealed first-price auctions for the q packages and the aggregate portfolio Θ. Each local $\ell \in \mathbb{L}$ competes for the package θ_j that he is interested in and receives no extra utility from owning more than one package. Accordingly, each global $g \in \mathbb{G}$ competes for the aggregate Θ and receives no utility from owning a single package. The qualified brokers for the next round are q *local* winners and one *global* winner with the lowest bids.

Second round. At the outset, the asset manager, updates the available public information to z', by revealing the winning bids of the previous round. Then, the qualified "local" winners of the first round, defined as $\mathbb{Q} \subset \mathbb{L}$ with cardinality $|\mathbb{Q}| = q$ i.e. the number of local packages, jointly compete against the qualified "global" winner g.

In all cases, the second-round bids are bounded from above by the first-

[12] For simplicity we restrict our analysis to this class of auctions.

round bidding ($\phi_i^2 \leq \phi_i^1$, $\forall i \in \mathbb{N}$). This round follows the rules of core-selecting auctions with two possible outcomes[13]: the global broker g wins all packages as Θ when $\phi_g^2 < \sum_{i \in \mathbb{Q}} \omega_i \phi_i^2$, and each local broker i wins one package θ_j if $\phi_g^2 > \sum_{i \in \mathbb{Q}} \omega_i \phi_i^2$.

The payoff of a local broker i, who wins a package θ_j with m securities for the charged commission $c_i \in \mathbb{R}_+$ is given by:

$$\pi_i(\phi^2 | s_i, z) = \mathbb{E}\big[(\theta_j \cdot p^*) \cdot c_i - \theta_j \cdot \mathbb{E}[\Delta p]\big] \cdot \mathbf{1}_{\{\sum\limits_{i \in \mathbb{Q}} \omega_i \phi_i^2 < \phi_g^2\}} \tag{1}$$

where the last term is an indicator function for $\sum_{i \in \mathbb{Q}} \omega_i \phi_i^2 < \phi_g^2$, when the whole portfolio Θ is assigned to local brokers for execution.

For each package θ_j the expected payoff of each local broker i results from the charged commission upon the trading value ($\theta_j \cdot p^* \cdot c_i$) minus the potential losses from the price variation ($\theta_j \cdot \mathbb{E}[\Delta p(s_i, z)]$), if exists. We denote the private valuation of each local broker i for θ_j with $\alpha_i = \mathbb{E}\Big[\dfrac{\theta_j \cdot \Delta p}{\theta_j \cdot p^*} | s_i, z\Big]$, with $\alpha_i \in \mathbb{R}$.

On the other hand, if the global broker wins the aggregate portfolio Θ, for $\sum_{i \in \mathbb{Q}} \omega_i \phi_i^2 > \phi_g^2$, he charges $C = \sum_{i \in \mathbb{Q}} \omega_i \phi_i^2$ and follows a similar payoff function. We denote the private valuation for Θ of each global broker g with $\upsilon = \mathbb{E}\Big[\dfrac{\Theta \cdot \Delta p}{\Theta \cdot p^*} | s_g, z\Big]$, with $\upsilon \in \mathbb{R}$. The broker-optimal-frontier, for any local broker i, is satisfied when $\sum_{i \in \mathbb{Q}} c_i = \phi_g^2$.

The VCG pricing function $c(\phi_\ell^2, \phi_g^2)$, where $\phi_\ell^2 = (\phi_1^2, \ldots, \phi_q^2)$, is given by:

$$c(\phi_\ell^2, \phi_g^2) = \begin{cases} (c_1^V, \ldots, c_q^V, 0) & \text{if } \phi_g^2 > \sum\limits_{i \in \mathbb{Q}} \omega_i \phi_i^2 \ , \\ (0, \ldots, 0, C) & \text{if } \phi_g^2 < \sum\limits_{i \in \mathbb{Q}} \omega_i \phi_i^2 \ . \end{cases} \tag{2}$$

[13]Without loss of generality ties are resolved.

where $c_i^V = \max \left\{ 0, \dfrac{\phi_g^2 - \sum\limits_{j \neq i} \omega_j \phi_j^2}{\omega_i} \right\}$.

Respectively, the core-selecting pricing rule is given by:

$$c(\phi_\ell^2, \phi_g^2) = \begin{cases} (c_1, \dots, c_q, 0) & \text{if } \phi_g^2 > \sum\limits_{i \in \mathbb{Q}} \omega_i \phi_i^2 \ , \\ \\ (0, \dots, 0, C) & \text{if } \phi_g^2 < \sum\limits_{i \in \mathbb{Q}} \omega_i \phi_i^2 \ . \end{cases} \tag{3}$$

such that $c_i \in [\phi_i^2, c_i^V]$ with $\sum\limits_{i \in \mathbb{Q}} c_i \leq \phi_g^2$ and $C \in \left[\phi_g^2, \sum\limits_{i \in \mathbb{Q}} \omega_i \phi_i^2\right]$.

If the VCG outcome is in the core, no broker has an incentive to deviate from his truthful preferences and it is the only selected Pareto-dominant outcome [Ausubel and Milgrom, 2002]. However, when the VCG is outside the core, a different pricing rule is necessary if we are to minimize the incentives for deviation [Day and Raghavan, 2007].

In the following, we present the two core-selecting pricing rules: the Nearest-VCG rule [Day and Cramton, 2012] and the Dynamic-Nearest-VCG, a slight modification of the former that we introduce to accommodate our two-round set up. In both cases, the global broker receives a fee equal to $\sum\limits_{i \in \mathbb{Q}} \omega_i \phi_i^2$ upon winning. Whereas if local brokers win, they apportion ϕ_g^2 as follows:

1. **Nearest-VCG rule**

   This pricing approach was firstly introduced by Day and Raghavan [2007] and Day and Cramton [2012]. The fundamental notion is to select a point in the broker-optimal-frontier which will minimize the euclidean distance from the VCG outcome [Ausubel and Baranov, 2020]. For a finite set of locals the payments for the weighted-packages are divided into:

$$c_i(\phi^2_\ell, \phi^2_g) = (\omega_1 c^V_1 - \Delta_1, \ldots, \omega_q c^V_q - \Delta_q, 0), \tag{4}$$

where $\Delta_i = \big(\sum_{i\in\mathbb{Q}} \omega_i c^V_i - \phi^2_g\big)\omega_i$ is the minimum downward correction on the VCG outcome that corresponds to each local broker i.

2. **Dynamic-NVCG (D-NVCG) rule**

   This rule selects a vector of fees in the *broker-optimal-frontier* determined by local brokers' first-round bidding. The rationale is that overbidding incentives in the first round are penalized for deviating from the VCG pricing.

   Suppose $\mathbb{Q} = \mathbb{Q}^u \cup \mathbb{Q}^d$ and $\mathbb{Q}^u \cap \mathbb{Q}^d = \varnothing$, where $\mathbb{Q}^u = \{j \in \mathbb{Q} | \phi^1_j > c^V_j\}$ and $\mathbb{Q}^d = \{i \in \mathbb{Q} | \phi^1_i \leq c^V_i\}$. Then, for any bidder i, the final fees of all bidders are readjusted by $\epsilon_i = \omega_i(\phi^1_i - c^V_i)$.

$$c_i(\phi^2_\ell, \phi^2_g) = \begin{cases} \big[\omega_i c^V_i - \Delta_i\big] + \omega_i \dfrac{\sum\limits_{j\in\mathbb{Q}^u} \epsilon_j}{\sum\limits_{i\in\mathbb{Q}^d} \omega_i} & \text{if } \phi^1_i \leq c^V_i \\ \big[\omega_i c^V_i - \Delta_i\big] - \epsilon_i & \text{if } \phi^1_i > c^V_i \end{cases} \tag{5}$$

   where $\Delta_i = \big(\sum_{i\in\mathbb{Q}} \omega_i c^V_i - \phi^2_g\big)\omega_i$.

Each bidder with $\phi^1_i > c^V_i$ will receive a downward adjustment on the Nearest-VCG pricing equal to the deviation ϵ_i, while a bidder with $\phi^1_i \leq c^V_i$ will be rewarded for his strategy in the first round with an increase in the Nearest-VCG fee.

3.2 Examples

We provide two examples for the implementation of the payment rules.

Example 1 Assume an asset manager demands liquidity for a portfolio Θ. He decides to divide the portfolio into two packages: the first package is θ_1 with a weighted-value $\omega_1 = 0.6$ over the nominal value of Θ, and the second package is θ_2 with a weighted-value $\omega_2 = 0.4$ over the nominal value of Θ. The qualified winners of the first round are: the local broker 1 for the package θ_1, the local broker 2 for the package θ_2 and the global broker g for portfolio Θ.

In the second round, lets suppose that brokers submit the following bids in basis points:

Local 1		Local 2		Global
$\phi^1_{\ell_1}$	$\phi^2_{\ell_1}$	$\phi^1_{\ell_2}$	$\phi^2_{\ell_2}$	ϕ^2_g
27	25	19	10	22

Since the aggregate bid of the local brokers equals to $\omega_1\phi^2_{\ell_1} + \omega_2\phi^2_{\ell_2} = 19$, they win. Thus, the asset manager assigns to the local brokers to execute the portfolio Θ jointly.

At this point, the question that arises is how much the asset manager will have to pay each local broker to execute each assigned package. If the asset manager applies the VCG pricing rule from equation (2), he will have to pay the local broker 1 with $c^V_1 = 30$ and broker 2 with $c^V_2 = 17.5$. However, for the asset manager their total payment $\omega_1 c^V_1 + \omega_2 c^V_2$, would be higher than if the portfolio was assigned to the global broker. Thus, the total payment of the two locals must not exceed the biding ϕ^2_g of the global bidder ("second-price" rule).

Figure 1 maps the payoff vectors for which the coalition of locals is not blocked. Following equation (3), the core corresponds to $c_1 \in [25, 30]$ for local broker 1, $c_2 \in [10, 17.5]$ for local broker 2, and $C \in [19, 22]$ for the global.

One can readily notice that the constraints defining upwardly the core are simply the tie-breaking bids. Suppose that the local broker 1 bids $\phi_1^2 > 30$. This outcome would be blocked by the global's bid $\phi_g^2 = 22$ and broker 2's bid $\phi_2^2 = 10$. The same applies if the local broker 2 bids $\phi_2^2 > 17.5$. The lower bounds on the local brokers' pricing are their bids, consistent with the assumption of individual rationality.

Using the Nearest-VCG pricing rule from equation (4), we will minimize the distance from the VCG pricing rule to obtain an outcome that will be included in the core intervals. In the core interval, local brokers' payoff is maximized at *broker-optimal-frontier* where the tie-break occurs. Thus, the asset manager will pay broker 1 with a commission $c_1 = 16.2$ (that is $\omega_1 c_1^V = 18$ and $\Delta_1 = 1.8$) and broker 2 with a commission $c_2 = 5.8$ (that is $\omega_2 c_2^V = 7$ and $\Delta_2 = 1.2$).

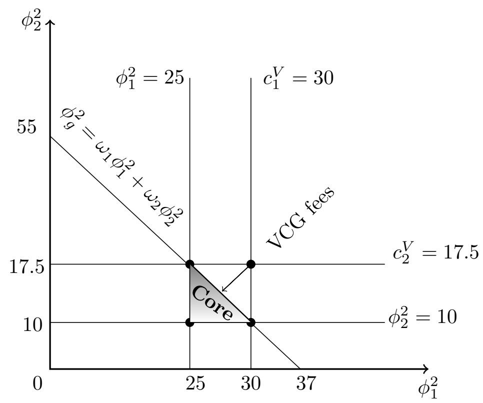

Figure 1: Core point closest to VCG payments

One shortcoming of the Nearest-VCG pricing rule is that has been designed for single-round auctions without encompassing the bidding behavior of the previous round. With the Dynamic-Nearest-VCG, incentives for bidding close to truthful valuations in the first round are rewarded, while those who misreport are "punished" by receiving a lower commission fee when the auction ends.

For instance, the local broker 2 submits $\phi_2^1 = 19$ in the first round. This bid qualifies him for the second round, yet in the second round, he has a larger interval $19 \leq \phi_2^2 \leq 10$ to reduce. By the information release at the interim, each broker is updated for the prices' estimates of others. According to equation (5) the asset manager will pay the Nearest-VCG prices minus any deviation between first-round bidding and the VCG outcome, for broker 2 $c_2 = 5.2$ and for broker 1 $c_1 = 16.8$.

In the next example, we illustrate the pricing rules for $\ell > 2$ local brokers, again the bids are quoted in basis points:

Example 2 Assume that the qualified winners for the second round are 5 local brokers who compete against 1 global. Table 1 presents each local broker i's bid for each package θ_j with a weight ω_j, respectively. Since $\sum\limits_{i \in \mathbb{Q}} \omega_i \phi_i^2 < \phi_g^2$, with $\sum\limits_{i \in \mathbb{Q}} \omega_i \phi_i^2 = 20$ and $\phi_g^2 = 25$, the asset manager assigns the portfolio's execution to local brokers. The VCG outcome c_i^V is calculated for each local broker i based on equation (2) and presented in the relevant column.

Local Brokers	Weights ω_i	Bids $\omega_i\phi_i^1$	$\omega_i\phi_i^2$	VCG $\omega_i c_i^V$	Core Interval	Nearest VCG	Dynamic NVCG
1	0.15	5.1	3	8	[3, 8]	5	5.3
2	0.1	2.4	2	7	[2, 7]	5	5.2
3	0.4	9.6	4	9	[4, 9]	1	0.4
4	0.2	10.2	5	10	[5, 10]	6	5.8
5	0.15	7.2	7.2	11	[6, 11]	8	8.3
Global		28	25		[22, 25]		

Table 1: Example 2 with 5 local bidders and 1 global

Similarly to the previous example, for the local broker i, a fee higher than $\phi_i^2 > c_i^V$ is blocked by the coalition of locals given that others submit a bid equal to ϕ_{-i}^2 and the global's bid $\phi_g^2 = 25$. The broker-optimal-frontier is satisfied for $\sum_{i=1}^{5} \omega_i\phi_i^2 = 25$ maximizing the local brokers' pay-offs, for every core interval defined by equation (3).

With Nearest-VCG pricing rule from equation (4) the asset managers pays a commission fee to the local broker 1 equal to 5 ($\omega_1 c_1^V = 8$, $\Delta_1 = 3$), i.e., the local broker i receives 3 basis points of the commission fee less compared to the VCG rule. For the local brokers 2,3,4,5, the Nearest-VCG fees are presented in Table 1.

In this example, we see that two local brokers have submitted a higher fee in the first round: local broker 3 with $\omega_3\phi_3^1 > \omega_3 c_3^V$ and local broker 4 with $\omega_4\phi_4^1 > \omega_4 c_4^V$, respectively. The suggested pricing rule restricts brokers from manipulating the outcome of the auction.

Applying the Dynamic-Nearest-VCG pricing rule from equation (5) for the

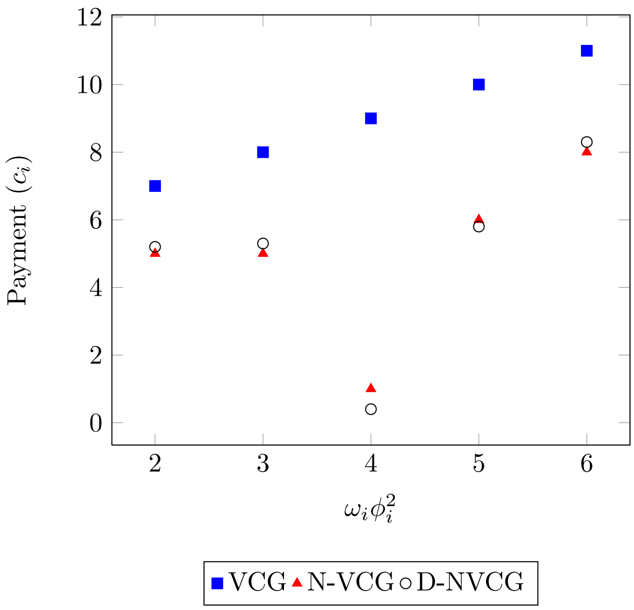

Figure 2: Pricing Rules for Package Bidding

local brokers 3 and 4 who have bidden excessively in the first round, the asset manager will pay the local broker 3 a commission equal to 0.4 ($\epsilon_3 = 0.6$) and the local broker 4 a commission equal to 5.8 ($\epsilon_4 = 0.2$). Both local brokers 3 and 4 will bear an extra cost for their misreporting incentives in the first round.

Those local brokers who bid prudently in the first round will receive an increase in the commission fee. Specifically, the local broker 1 will receive a fee equal to 5.3 ($\sum_{j \in \mathbb{Q}^u} \epsilon_j = 0.8$ and $\sum_{i \in \mathbb{Q}^d} \omega_i = 0.4$). The same rule works for local brokers 2 and 5. In Figure 2, we can observe that the Dynamic-Nearest VCG pricing rule can yield higher fees to those locals who bid close to their truthful valuations in the first round.

4 Analysis

We start our analysis by characterizing our mechanism in the second round as pivotal. This means that the fee received by any local broker i is equal to the loss imposed on other locals by adjusting ϕ_ℓ^2 to attribute i's values. We define a bid ϕ_i^2 submitted by broker i as *pivotal*, if and only if $\phi_g^2 = \sum_{i \in Q} \omega_i \phi_i^2$ holds and if for any $\gamma > 0$, a bid $\phi_i^2 - \gamma$ attributes a non-empty package θ_j, while $\phi_i^2 + \gamma$ yields the null package.

A broker is pivotal if his report changes the auction outcome, in comparison to excluding the broker or attributing the null report to him, then the auction satisfies the pivotal pricing property [Ausubel and Baranov, 2020; Milgrom, 2004]. In a local-global setting, this property is satisfied for any core-selecting auction.

Lemma 1 (Ausubel and Baranov [2020])**.** *Every core selecting auction satisfies the pivotal pricing property.*

It is not hard to prove that the Dynamic-NVCG pricing rule results in allocations that belong to the broker-optimal-frontier and minimizes any misreporting incentives.

Lemma 2. *Any auction with a Dynamic-NVCG pricing rule $c(\phi_\ell^2, \phi_g^2)$ is a core-selecting auction.*

Proof. From equation (5), the total sum of local brokers' commission is:

$$\sum_{i \in \mathbb{Q}^d} \omega_i \left[c_i^V + \frac{\sum_{j \in \mathbb{Q}^u} \epsilon_j}{\sum_{i \in \mathbb{Q}^d} \omega_i} \right] - \sum_{i \in \mathbb{Q}^d} \Delta_i + \sum_{j \in \mathbb{Q}^u} [\omega_j c_j^V - \epsilon_j] - \sum_{j \in \mathbb{Q}^u} \Delta_j$$

$$= \sum_{i \in \mathbb{Q}} \omega_i c_i^V - \sum_{i \in \mathbb{Q}} \Delta_i = \phi_g^2$$

Consequently, the Dynamic-NVCG rule always lies on the broker-optimal frontier. □

We do not disregard the Nearest-VCG, but instead, we are using it as a touchstone to improve incentive compatibility further and reduce the degree of manipulation freedom in a two-stage framework [Parkes et al., 2001]. The following Proposition explains why the Dynamic-NVCG rule is optimal for distributing the commission of local brokers when there are perverse incentives by some brokers in the first round.

Proposition 1. *For any local bidder i bidding above $\mathbb{E}[c_i^V|s_i, z]$ in the first round is always a weakly dominated strategy.*

Proof. Suppose not. Then for any broker i, a bidding strategy $\phi_i^{1\prime} \leq \mathbb{E}[c_i^V|s_i, z]$ is weakly dominated by $\phi_i^1 > \mathbb{E}[c_i^V|s_i, z]$. By substitution in equation (1) the pricing rule of (5) for $\pi_i(\phi_i^2|s_i, z') \geq \pi_i'(\phi_i^2|s_i, z')$ we have:

$$\theta_j \cdot p^* \left(\omega_i c_i^V - \Delta_i - \epsilon_i\right) \geq \theta_j \cdot p^* \left(\omega_i c_i^V - \Delta_i + \omega_i \frac{\epsilon_i}{\sum\limits_{i \in Q^d} \omega_i}\right)$$

By solving this inequality we result $\phi_i^1 \leq c^V$. Thus, any bidding in the first round above VCG price is a weakly dominated strategy. □

The following Assumption 1 imposes continuity and monotonicity for all pricing rules, and this will simplify our analysis.

Assumption 1. *For any winning bidder i and any bidding vector $(\phi_1^2, \ldots, \phi_q^2, \phi_g^2)$, the pricing function $c_i(\phi_1^2, \ldots, \phi_q^2, \phi_g^2)$ is continuous in all bids, differentiable and non-decreasing in bidder's i bid.*

Bosshard et al. [2017] have proved that the N-VCG rule does not always satisfy the non-decreasing condition. This seems to be not the case in our mechanism.

In the VCG mechanism, brokers bid their valuation truthfully, and it is their weakly dominant strategy with no surplus. The following lemma suggests the global broker has a weakly dominant strategy in the second round.

Lemma 3. *Suppose that Assumption 1 is satisfied. Then, for the restricted second-round auction and for* $\upsilon(s_g, z') > 0$, $\phi_g^2 = \min\{\phi_g^1, \upsilon(s_g, z')\}$ *is a weakly dominant strategy for the global broker.*

Proof. By design no broker can bid in the second round higher than his fist-round bid. For the VCG mechanism, it is a weakly dominant strategy for the global bidder to bid his "valuation", which in our case equals to $\upsilon(s_g, z')$. The result follows directly. $\square$

Whereas for the local bidders, it is always worse off to bid lower than their expected losses, $\alpha_i(s_i, z') > 0$.

Lemma 4. *Suppose that Assumption 1 and the pivotal pricing property are satisfied. Then, for each local broker i any bid* $\phi_i^2 \in \big[0, \min\{\phi_i^1, \alpha_i(s_i, z')\}\big)$ *is a weakly dominated strategy.*

Proof. For an arbitrary broker i, let $\alpha_i(s_i, z') \geq \phi_i^1$. Then, for any strategy $\phi_i^2 \leq \phi_i^1$ is trivially weakly dominated and obtains negative surplus. Suppose now that for broker i, it is $\alpha_i(s_i, z') < \phi_i^1$. For the local broker i with $\hat{\phi}_i^2 = \alpha_i(s_i, z')$ and $\hat{\phi}_i^2 > \phi_i^2$, we prove that bidding ϕ_i^2 is weakly dominated. By *Assumption* 1, it will always result in $c_i(\phi_\ell^2, \phi_g^2) \leq c_i(\hat{\phi}_\ell^2, \phi_g^2)$, and from the pivotal pricing property, it follows:

$$\mathbb{E}\big[\theta_j \cdot p^* \cdot c_i(\phi_\ell^2, \phi_g^2) - \theta_j \cdot \Delta p(s,z')\big] \leq \mathbb{E}\big[\theta_j \cdot p^* \cdot c_i(\hat{\phi_\ell^2}, \phi_g^2) - \theta_j \cdot \Delta p(s,z')\big]$$
$$\leq \mathbb{E}\big[\theta_j \cdot p^* \cdot \frac{\theta_j \cdot \Delta p(s,z')}{\theta_j \cdot p^*} - \theta_j \cdot \Delta p(s,z')\big] = 0$$

Thus, any $\hat{\phi_i^2} > \phi_i^2$ is weakly dominated. □

Suppose now that all local brokers $j \neq i$ bid according to the profile $(\phi_j^{2*})_{j \neq i}$. Let $H_i \equiv H_i\big[\phi_i^2, \alpha_i(s_i, z')\big]$ be the probability of winning for a local bidder i who bids $\phi_i^2 \in [\alpha_i(s_i,z'), \phi_i^1\big]$, and its marginal probability $h_i \equiv h_i\big[\phi_i^2, \alpha_i(s_i,z')\big]$:

$$H_i = \Pr\big(\omega_i \phi_i^2 + \sum_{j \neq i} \omega_j \phi_j^{2*} \leq \upsilon(s_g, z') \big| \alpha_i(s_i,z')\big)$$

$$h_i = \frac{\partial H_i\big(\phi_i^2, \alpha_i(s_i,z')\big)}{\partial \phi_i^2} \tag{6}$$

Also, we denote the expected commission fee of each local bidder i with $c_i \equiv C_i\big(\phi_i^2, \alpha_i(s_i,z')\big)$ and with $MC_i \equiv MC_i\big(\phi_i^2, \alpha_i(s_i,z')\big)$ the expected marginal commission when each local broker i bids $\phi_i^2 \in \big[\alpha_i(s_i,z'), \phi_i^1\big]$.

$$\begin{aligned} C_i &= \mathbb{E}\big[c_i\big(\phi_i^2, \sum_{j \neq i} \phi_j^{2*}, \upsilon(s_g,z')\big) \big| \alpha_i(s_i,z')\big] \\ MC_i &= \mathbb{E}\Bigg[\frac{\partial c_i\big(\phi_i^2, \sum_{j \neq i} \phi_j^{2*}, \upsilon(s_g,z')\big)}{\partial \phi_i^2} \Big| \alpha_i(s_i,z')\Bigg] \end{aligned} \tag{7}$$

The expected marginal commission expresses any change in the expected commission arising by the incremental increase in the bidding ϕ_i^2. For instance, if brokers anticipate a loss in the expected prices, they will counterbalance their payoff by moving their bid upwardly.

Next, we define the first-order optimality conditions for the local broker's maximization problem on the steps of Ausubel and Baranov [2020].

Proposition 2. *Under Assumption 1 and the pivotal pricing property, the optimality condition for choosing* $0 < \phi_i^2 \leq \phi_i^1$ *for a local bidder i is given by:*

$$MC_i = \Bigg(\alpha_i(s_i, z') - \phi_i^2\Bigg)h_i. \tag{8}$$

Proof. We apply the optimality condition on the expected payoff of equation (1) upon the probability of winning

$$E[\pi_i(\phi_i^2|s_i, z')] = \Bigg[\theta_j \cdot p^* \cdot c_i - \theta_j \cdot \mathbb{E}[\Delta p|s_i, z']\Bigg]H_i$$

with $0 \leq \phi_i^2 \leq \phi_i^1$

$$\begin{aligned}\frac{\partial E[\pi_i(\phi_i^2|s_i, z')]}{\partial \phi_i^2} &= \big[\theta_j \cdot p^* \cdot \frac{\partial c_i}{\partial \phi_i^2}\big] \cdot H_i + \Bigg[\theta_j \cdot p^* \cdot c_i - \theta_j \cdot \mathbb{E}[\Delta p|s_i, z']\Bigg] \cdot h_i \\ &= \theta_j \cdot p^* MC_i + \theta_j \cdot p^* \cdot c_i \cdot h_i - \theta_j \cdot \mathbb{E}[\Delta p|s_i, z'] \cdot h_i = 0\end{aligned}$$

By Lemma 4, ϕ_i^2 is always nonnegative. Due to *Assumption* 1 and the pivotal pricing property the following is in effect:

$$c_i \equiv c_i\big(\omega_i\phi_i^2, \sum_{j\neq i}\omega_j\phi_j^{2*}, \phi_i^2 + \sum_{j\neq i}\omega_j\phi_j^{2*}\big) = \phi_i^2$$

Thus, it is easily to conclude that:

$$MC_i = \theta_j \cdot \mathbb{E}[\Delta p|s_i, z'] \cdot h_i - \theta_j \cdot p^* \cdot \phi_i^2 \cdot h_i$$
$$MC_i = \left(\frac{\theta_j \cdot \mathbb{E}[\Delta p|s_i, z']}{\theta_j \cdot p^*} - \phi_i^2 \right) \cdot h_i$$

□

Intuitively, if $MC_i < 0$ it means that $\alpha_i(s_i, z') < \phi_i^2$, and broker i is not included among the winners. Otherwise, if $\phi_i^2 < \alpha_i(s_i, z')$, broker i will have to increase his bidding fee to reach the optimal payoff where $MC_i = 0$.

Theorem 1. *For each pricing rule, it exists an equilibrium where the bidding function of each broker is given by:*

(a) for the NVCG rule

$$\phi_i^2 = \begin{cases} \alpha_i(s_i, z') - \sigma_i \omega_i^2 (q-1) & \text{,if } \alpha_i > 0 \\ 0 & \text{,if } \alpha_i \leq 0 \end{cases} \tag{9}$$

(b) for the D-NVCG rule

$$\phi_i^2 = \begin{cases} \alpha_i(s_i, z') - \sigma_i \omega_i^2 (q-1) & \text{,if } \phi_i^1 > c_i^V \text{ and } \alpha_i > 0 \\ \alpha_i(s_i, z') - \sigma_i \omega_i^2 \left[\frac{\ell}{\sum\limits_{i \in Q^d} \omega_i} + (q-1) \right] & \text{,if } \phi_i^1 \leq c_i^V \text{ and } \alpha_i > 0 \\ 0 & \text{,if } \alpha_i \leq 0 \end{cases} \tag{10}$$

where $\dfrac{1}{\sigma_i} \equiv \dfrac{h_i}{H_i}$ *is a reverse hazard rate and with* ℓ *to be the number of local bidders with* $\phi_i^1 > \phi_i^V$.

Proof. The optimality conditions are given by equation (8).

(a) From equation (4) of the *Nearest-VCG rule*, the expected marginal commission of equation (7) for a broker i is:

$$
\begin{aligned}
MC_i &= [\omega_i c^V - \Delta_i]' H_i \\
&= \omega_i^2 (q-1) H_i
\end{aligned}
$$

Replacing the above to (8) we result to the equilibrium bid:

$$
\phi_i^2 = \frac{\theta_j \cdot \mathbb{E}[\Delta p | s_i, z']}{\theta_j \cdot p^*} - \omega_i^2 (q-1) \frac{H_i}{h_i}
$$

(b) For the D-NVCG if $\phi_i^1 > \phi_i^V$ the equilibrium bidding is similar to the *Nearest-VCG* of equation (9). However, if there are ℓ number of locals with $\phi_j^1 > \phi_j^V$ and bidder's i bid in the first round is $\phi_i^1 \leq \phi_i^V$, then the expected marginal commission from equation (5) is given by:

$$
MC_i = \omega_i^2 \left[\frac{\ell}{\sum\limits_{i \in Q^d} \omega_i} + (q-1) \right] H_i
$$

By substitution in the optimality conditions of equations (8) we conclude:

$$
\phi_i^2 = \frac{\theta_j \cdot \mathbb{E}[\Delta p | s_i, z']}{\theta_j \cdot p^*} - \omega_i^2 \left[\frac{\ell}{\sum\limits_{i \in Q^d} \omega_i} + (q-1) \right] \frac{H_i}{h_i}
$$

□

In Theorem 1 we proved the existence of equilibrium for Nearest-VCG and the Dynamic-Nearest-VCG. In both cases, we have shown that when the portfolio is sliced in many packages, the equilibrium bid of a winning broker is negatively affected by the number of packages and their size in the overall portfolio.

5 Conclusions

This research has studied a stylized model for a portfolio's auction, which is divided into packages. We design a "local-global" environment with a finite set of locals, each one interested in a single package. We introduce a dynamic setup for the Nearest-VCG pricing rule, which conforms to a multiple-round auction. This new pricing setup aligns brokers' incentives to lower bids mitigating the free-riding opportunities of the first round. Using an endogenous reference rule for the expected VCG pricing outcome, the brokers are motivated to submit bids close to their truthful valuations in the first round, squeezing execution costs downwardly.

Additionally, we proved that our mechanism allows the asset manager to engage many brokers in the auction process, resulting in lower transaction costs. The information update at the interim for others' valuations mitigates the "winner's curse" [Zarpala and Voliotis, 2021]. Also, it increases broker's trust in the sense that the auction's rules have been followed.

Finally, we propose a simple iterative mechanism that provides transparency in the auction process to eliminate the complexity of the winner's determination problem and keeps the brokers' problems manageable, incorporating their strategic incentives.

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