Compliance with Policy Measures and Network Games

Abstract

We construct a Bayesian network game to study individuals' compliance (or lack thereof) with public health mandates, such as social-distancing measures against Covid. Agents form their networks to minimize cognitive dissonance that arises from the mismatch between compliance behaviors implied by their ideologies and circumstances, and those of peers in their networks. When agents' ideologies are immune to the outside influence in the form of interaction with neighbors in their social networks and exogenous shocks such as political polarization, one giant connected component emerges, which maintains communications open and behaviors stable at the initial distribution. However, when we introduce exogenous shocks to the ideologies of select few agents, referred to here as the "political elites", we find that given that individuals place sufficient weight on the actions of their peers when choosing their own behaviors, two disparate network communities emerge that partition the network and action space into two, which reinforces the polarizing force of the exogenous shock in further alienating the two communities from each other. We arrive at the same conclusion if we allow individuals to adjust their ideologies over time in maximizing their utility.

1 Introduction

The Covid-19 pandemic has highlighted many challenges in implementing effective public health mandates. A notable barrier to progress toward a successful curtailment of the pandemic has been the lack of compliance with mandates, such as social distancing and mask-wearing, on political, ideological, and social grounds. In this paper, we construct a dynamic network games model in which individuals' behaviors manifested through their compliance (or lack thereof) with a set of public health policy measures. Each agent's action is determined by the weighted average of the implied action values from their ideologies, circumstances such as socioeconomic and demographic backgrounds, and peers in their network. To build intuition, we first investigate the evolution of aggregate compliance behaviors and network structures when individuals' ideologies are fixed at some initial distribution, which we assume to be symmetric. Then, we allow exogenous shocks to impact ideologies of certain individuals we call the "political elites", who then affects the ideologies of their the neighbors through their interactions. We show that this in turn affects the decisions on the compliance with policy mandates. This effect of social networks on ideologies can incorporate the increasingly visible signs of political polarization taking place both at the level of political elites and media, as well as the general public. We show that in the absence of exogenous shocks to ideologies, the distribution of compliance behaviors is uniform with all individuals belonging to one giant connected component. However, with exogenous shocks affecting even a small minority of the population, such as polarization, two disconnected network communities emerge, which in turn exacerbate the polarizing force of the exogenous shock without any communication taking place between them.

2 Notation

We use $\langle x, y \rangle$ to denote the inner product of vectors x and y in the (not necessarily finite) Euclidean space, and $\odot$ to denote component-wise vector multiplication. We use italics to refer to vectors and boldface capital letters to refer to matrices. $\mathit{1}^k$ denotes the k-vector of all 1s, $\mathit{1}^k_a$ denotes the k-vector of as in all entries, and $I_A(x)$ denotes the indicator function such that it evaluates to 1 if $x \in A$ and 0 otherwise. For any vector v, v_{-i} denotes v with the i-th component removed and v_S refers to a subset of v defined by $\{j | j \in S \cap v\}$. For any $N \in \mathbb{N}$, $[N]$ represents the set $\{1, 2, ..., N\}$.

3 The Model

We consider a society of population n and let subscripts $i, j \in [n]$ denote typical members of the society. At time $t = 0$ nature endows each agent i with characteristics X_i^0, such as demographic and socioeconomic circumstances, and ideologies Y_i^0, such as political party affiliation. For simplicity, we assume that $X_i^t, Y_i^t \in [-1,1]^m, \forall t$ are sampled from the Gaussian distribution[1] but are known only privately to the individual i. Each individual is identified with a node in a network where links between individuals reflect relationships. Let $\phi_i^t \in \mathbb{R}_+^m$ satisfying $\sum_i \phi_i^t = 1$ denote the vector where each component represents the extent of individual i's "emotional attachment" to the corresponding issue in Y_i^t, and $\rho_i^t \in \mathbb{R}_+^{n-1}$ with $\frac{1}{n-1}\sum_i \rho_i^t = 1$ refer to the weight given by i to each individual in the society that they consider in making policy compliance decisions. We call the set of individuals for whom $\rho_i^t > 0$ individual i's *community* and denote it by C_i^t. When it is clear from context, the superscript t is omitted from the variables.

Suppose that at time $t = 0$ the government imposes K policies to curtail the pandemic with its enforcement largely dependent on citizens' voluntary compliance. We assume that every public health measure entails certain limitations on civil liberties and democratic processes, which is where conflict with individuals' ideologies Y may potentially arise. In each period $t \in [T]$, agent i makes two simultaneous decisions:

1 Whether to comply with each of the K policies, such as mask wearing and vaccination;

2 Whether to maintain the link with individual $j, \forall j \in C_i$, and for all $j \notin C_i$, whether to initiate a link with j, incurring a cost of $c > 0$.

Note that c is an increasing function and takes non-negative integers as arguments and that we assume, without loss of generality, that it represents both the cost incurred in maintaining an existing as well as initiating a new link. We can think of c as time spent attending meetings, reading correspondence, and engaging in conversations and take $0 = c(0) < c(1) < 1$.

First, we consider the binary action space where individuals' compliance options are defined in $A := \{-1,1\}^K$ with K representing the dimension of the pandemic policy space. Suppose agent i's action on the mandate of policy $k \in [K]$ at time t is observable to other agents and is expressed:

$$a_{ik}^t \in \begin{cases} -1 & \textit{if} \text{ do not comply with mandate} \\ 1 & \textit{if} \text{ comply with mandate} \end{cases} \tag{3.1}$$

Similarly, the actions for the rest of the society are denoted by the $(n-1)$-vector $a^t := (a_i^t)_{i\in[n-1]}$ and the actions for those in agent i's community are expressed $a_{C_i}^t$. Agents' linking decisions are denoted by the $n \times n$ adjacency matrix

$$G^t := (g_{ij}^t)_{i,j\in[n]} = \begin{cases} 0 & \textit{if} \text{ i=j or there is no link between i and j} \\ 1 & \textit{if} \text{ there is a link between i and j} \end{cases}$$

and $G^0 = (0)_{ij}$. Let G_{-i} denote the network with all of i's links removed. Together, they form agent i's strategy $s_i : [-1,1]^m \times [-1,1]^m \times \{0,1\}^{n-1} \to a_i^K \times (g_{ij})_{j\neq i}$. Denote $s := s_i \cup s_{-1}$ and the set of all strategies of i by $\mathcal{S}_i$ and $\mathcal{S} := \mathcal{S}_1 \times \mathcal{S}_2 \times \cdots \mathcal{S}_n$. Given a^t, each individual has utility of the following form:

$$u_i^t(s_i^t, s_{-1}^t) = \alpha a_i^t \langle (X_i^t, Y_i^t), (\beta, \phi_i^t)\rangle + (1-\alpha) a_i^t \langle a^t, \rho_i^t\rangle - c(|C_i^t|) \tag{3.2}$$

where $\alpha \in [0,1]$. $\langle (X_i^t, Y_i^t), (\beta, \phi_i^t)\rangle$ represents the compliance behavior implied by individual i's characteristics and ideologies such that $\mathbb{E}\langle (X_i^0, Y_i^0), (\beta, \phi_i^0)\rangle = 0$, and $|C_i^t|$ denotes the cardinality of i's community. The second term on the right-hand side in (3) refers to the peer effect. Let $P_i^t(a_{-i}|X_i^t, Y_i^t, a_{-i}^{t-1})$ be

[1] We assume Gaussian distribution for tractability but any symmetric distribution would suffice for our results.

agent i's beliefs about its neighbors' actions at time t and $\Theta(a^t, a^{t-1}, X_i^t, Y_i^t)$ be the probability distribution over a^t induced by P^t. For $t = 0$, assume $P(1|X_i^0, Y_i^0) = P(-1|X_i^0, Y_i^0) = 1/2$. Then, individual i's expected utility is

$$\mathbb{E}u_i^t(s_i, s_{-1}) = \sum_{a \in A} \Big[\alpha a_i^t \langle (X_i^t, Y_i^t), (\beta, \phi_i^t) \rangle + (1-\alpha)\langle \rho_i^t a^t, a_i^t \rangle - c(|C_i^t|)\Big] \Theta(a^t, a^{t-1}, X_i^t, Y_i^t) \tag{3.3}$$

where the summation is taken over realizations of the other $n-1$ agents actions conditional on a_i.

This setup induces the game $\Gamma := \langle n, \{S_i\}_i, \{u_i\}_i \rangle$.

Next, we recall some definitions from Mas-Colell et al. (1995) and Bala and Goyal (2000) that we will use throughout this paper.

Definition 3.1. *s_i is called a best response of agent i to s_{-i} if*

$$u_i(s_i, s_{-i}) \geqslant u_i(s_i', s_{-i}), \forall s_i' \in \mathcal{S}_i \tag{3.4}$$

And the set of all best responses of i is denoted $B_i(s_{-1})$.

Definition 3.2. *G is called a Nash network if*

$$s_i \in B_i(s_{-1}), \forall i \in [n] \tag{3.5}$$

where $s_i := (a_i^K, G_i)$ and $G = (G_i)_i \forall i$.

We adapt from Galeotti et al. (2010) the notions of strategic complements and substitutes based on our action space and the utility function above.

Definition 3.3. *Utility functions are said to exhibit strategic complements if, for all $a_i > a_i'$ and $1_{a_{N_i}} > 1_{a'_{N_i}}$, and assuming uniform node degrees d,*

$$u(a_i, a_{N_i}) - u(a_i', a_{N_i}) \geqslant u(a_i', a_{N_i}') - u(a_i', a_{N_i}') \tag{3.6}$$

and strategic substitutes if Equation 3.6 holds with the inequality in the other direction.

Related to strategic substitutes and complements is the notion of externalities.

Definition 3.4. *Utility functions are said to exhibit positive externalities if*

$$u(a_i, a_{N_i}) \geqslant u(a_i, a_{N_i}') \tag{3.7}$$

and negative externalities if

$$u(a_i, a_{N_i}) < u(a_i, a_{N_i}') \tag{3.8}$$

4 Results

4.1 Symmetric Case with Ideologies Fixed at Y^0

First, we consider symmetric games with ideologies fixed at Y_i^0 for all individuals i. In this setting, an individual's decision of whether to add another individual to their network depends solely on the marginal benefit-cost analysis. Suppose $K = 1$ and time $t = 0$. Since the random variables X_{-i}, Y_{-i} are not observable by agent $i, \forall i$, and we can consider the case $a_{-i} =$ at time 0, each agent i bases their compliance decision solely on X_i, Y_i. Thereafter, agents maintain and initiate links with all others whose expected compliance behavior matches their own as long as the following condition is satisfied.

Lemma 4.1. For $t > 0$ and $K = 1$, each agent i forms a link with j if and only if

$$\langle a_i^t, \rho_j^t p_j^t a_j^t \rangle (1-\alpha) > c \tag{4.1}$$

where $p_j^t = P(a_j^t | X_i^t, Y_i^t, a_j^{t-1})$.

Proof. The proof follows immediately by observing that the hypothesis is equivalent to requiring that the net payoff from forming the link with j is positive, that is, in (3.3), $(1-\alpha)\langle \rho_i^t a^t, a_i^t \rangle - c(|C_i^t|) > 0$. ■

In the scenario where $a_{-i}^0 = \emptyset$, due to the symmetry in the distribution of X and Y, for each $k \in [K]$, we must have for sufficiently large n, we have

$$|\{a_{i,k} = a_{j,k} | \forall j \in [n]\}| = |\{a_{i,k} \neq a_{j,k} | \forall j \in [n]\}| \tag{4.2}$$

Then, starting $t = 1$, as long as each agent i places sufficient weight on the actions of their network, they will form a link with j as long as the condition in lemma 4.1 is satisfied.

Note that if $c > (1-\alpha)\rho_j p_j$, then no individual would form links with anyone else, leading to a discrete network with n 0-degree nodes. However, if individual j's action is very important to individual i's decision on whether to comply (i.e., $\rho_j \approx 1$) and j's past action predicts perfectly their future action $(i.e., a_j^t = a_j^{t-1})$, then the condition becomes $c < 1 - \alpha$, which in the case where the individual merely follows their peers (i.e., $\alpha = 0$) would surely be satisfied. But such a setup would result in a sparse network with all 1-degree nodes without any noticeable community structure.

For the more general case when an individual cares about actions of multiple others in their network (i.e., $\rho_j \ll 1, \forall j \in C_i$) and their own ideologies and circumstances (i.e., $\alpha > 0$), then the policy must be multi-faceted with $K \gg 1$ to satisfy the condition 4.1. More concretely,

Lemma 4.2. Assume the setup in Lemma 4.1 with the exception of the dimension of a_i. Suppose instead that the dimension of the action vector is greater than 1. Then, the necessary and sufficient condition in (4.1 that the number of policy mandates on which the actions of i and j agree must be greater than the quantity $\frac{c}{(1-\alpha)(\rho_j p_j)}$.

Proof. Notice that this is a rearrangement of the condition in Equation 4.1 in Lemma 4.1, and observing that the term $\langle a_i^t, a_j^t \rangle$ is an inner product of -1 and $+1$. ■

Lemma 4.3. When the ideologies are fixed at Y^0, there is exactly 1 connected component in a Nash network.

Proof. Recall that since Y^0 is assumed to be distributed symmetrically, we have the condition in (4.2). This implies that for every individual i, the cardinality $|C_i| := |\{j | j > \frac{c}{(1-\alpha)(\rho_j p_j)}\}|$ is the same for all i. No agent can improve their utility by changing their behavior on any of the K policies since the distribution of a_i is uniform. ■

4.2 Variable Ideologies due to Political Polarization

The American public has become more politically polarized over the past two decades with Democrats and Republicans increasingly divided ideologically and antipathetic toward each other. The most ideologically oriented are also the most politically active, while the more moderate majority at the center are disengaged. Politicians have adopted more extreme stances to cater to the politically active at both ends of the political spectrum. The Covid-19 pandemic has highlighted many aspects of this polarization, given its outsized presence on the political agenda. There is abundant evidence in the literature suggesting that polarization occurs at the elite level first and more sharply Enders (2021). In addition, Layman et al. (2006) proposes that political activists are a main spring of elite polarization since ideologically more extreme individuals are more politically engaged, exerting considerable influence at the elite level.

The key insight we want to show in this section is that individuals' ideologies Y can change over time due to interaction with political elites in their network, which in turn can lead to the polarization of actions. Recall that individuals' objective is to maximize their expected utility as shown in Equation 3.3, where Y_i^0 is initially assigned according to some symmetric distribution. Consider first the set E of select few political elites, such as politicians and candidates for political offices. We consider exogenous shocks to their ideologies at time $t = 1$ such that the implied action is lopsided to either "total compliance" or "total noncompliance", i.e., $a_i, \forall i \in E \in \{1_K, -1_K\}$. Given that the individuals $i \in E$ are political elites, one distinguishing feature of i's utility function is that the cost c of initiating and maintaining relationships is much lower than those that are not in E and the role of C_i in influencing i's compliance behavior is also much lower, which allows the elites to form a much larger network, i.e., $\alpha_i \lll \alpha_j, c_i \lll c_j, i \in E, j \notin E$, and their degrees are much larger than the mean $deg_i \ggg \mathbb{E}(deg)_j$. In fact, we suppose that the utility of the political elites is increasing in not only the size of their network community, but also in the degree of dissimilarity between their action vector and those of their neighbors. Without loss of generality, we let $c = 0$. More concretely, the political elites' utility has the following form:

$$\mathbb{E}u_i^t(s_i, s_{-1}) = \sum_{a \in C_i} \Big[\alpha\langle a^t, a_i^t\rangle) - (1-\alpha)\langle a^t, a_i^t\rangle)\Big]\Theta(a^t, a^{t-1}, X_i^t, Y_i^t) \tag{4.3}$$

where $i \in E$ and the summation is taken over realizations of the other agents' actions in the network community conditional on a_i. The first value in the square bracket is the weighted sum over the action vectors of like-minded followers, while the second term is the weighted sum of the dissimilarity score with each follower in the community. We present the following main result of our paper.

Theorem 4.1. *Suppose some fixed number m of individuals, called the "political elites", experience exogenous shocks to their ideologies akin to the "Democrat vs. Republican" political polarization, and are characterized by the utility function of the form in Equation 4.3. Then, within a finite number of time iterations, there will be exactly 2 communities that are disconnected from each other in a Nash network.*

Proof. When individual j who is not a political elite comes in contact with an elite i, two things happen. First, by being in j's network, the elite's action has an immediate effect on the compliance decision. More importantly, through persuasive rhetoric and political correspondence, the weight that j places on i as captured by ρ_{ij} will be high, making i's influence on j's decision on compliance behavior much stronger. For those individuals who base their compliance decision more on their neighbors' actions than on their ideologies and circumstances, i.e., those with relatively lower values of α, the change in their behaviors would be immediate. On the other hand, those with a stronger basis on their ideologies, i.e., those with higher α values would experience the cognitive dissonance arising from the mismatch between the compliance action implied by their ideologies and circumstances, and that implied by the actions from the political elites in their networks. Allowing the ideologies of these individuals to slowly change, we can see that they would be able to increase their utility by adjusting their ideologies such that the implied action more closely matches that implied by their neighbors, in particular the political elites. ■

our final result shows that political elites are strategic substitutes in a sense that their presence in individuals' networks reduce the difference in utilities resulting from different compliance behaviors. In addition, political elites are negative externalities.

Lemma 4.4. Political elites are strategic substitutes and negative externalities to rest of the individuals.

Proof. The claim is made obvious if one considers the action profile of the political elites, which makes it less likely that a given individual's action profile would yield a high inner product. ■

Bibliography

Venkatesh Bala and Sanjeev Goyal. A noncooperative model of network formation. *Econometrica*, 68(5): 1181–1229, 2000.

Adam M Enders. Issues versus affect: How do elite and mass polarization compare? *The Journal of Politics*, 83(4):1872–1877, 2021.

Andrea Galeotti, Sanjeev Goyal, Matthew O Jackson, Fernando Vega-Redondo, and Leeat Yariv. Network games. *The review of economic studies*, 77(1):218–244, 2010.

Geoffrey C Layman, Thomas M Carsey, and Juliana Menasce Horowitz. Party polarization in american politics: Characteristics, causes, and consequences. *Annu. Rev. Polit. Sci.*, 9:83–110, 2006.

Andreu Mas-Colell, Michael Dennis Whinston, Jerry R Green, et al. *Microeconomic theory*, volume 1. Oxford university press New York, 1995.