Corporate Financing and Investment Decisions When Equity Issuance Reveals Firms' Information to Investors

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Abstract

I study a two-stage infinite signaling game, in which firms can issue debt or equity to finance sequentially arriving investment projects. When management's first-stage decision can change investors' beliefs and consequently impact the second-stage security issuance, its optimal choice differs significantly from the strict debt-equity preference in a comparable one-stage model. I discuss a refinement concept that restricts the set of separating equilibria by requiring that the low type firm has no incentive to mimic the high type firm's actions. In equilibrium, a dynamic pecking order arises, suggesting that the information friction can solely explain various observed corporate financing behaviors.

Keywords: dynamic pecking order theory, infinite signaling game, adverse selection, equilibrium refinement
JEL: G32, C73, D82

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1 Introduction

How do firms finance their investment projects in a market with frictions? Since Myers and Majluf (1984), the capital structure literature has constantly debated the validity of the pecking order theory. Despite the insightful argument about the adverse selection cost, the information friction has been challenged by much of the empirical evidence.[1] Specifically, unlike the model prediction, equity financing has rarely been used as a last resort, and issuing activities are typically conducted during good times, e.g., high stock valuation, rather than under duress (Fama and French, 2005). In this paper I propose a two-stage dynamic model to reconcile the classical pecking order theory with the seemingly contradictory empirical evidence.

I first establish a one-stage signaling model as the benchmark. In the benchmark scenario, the firm can be either a high type or a low type. The high type firm has a positive NPV project and the low type firm has a negative NPV project. The firm can raise capital from investors by issuing either debt or equity. Investors, on the other hand, do not know the firm's type but can observe the firm's choice of securities and update their beliefs. Suppose both types of firms pool at issuing debt, then if the firm turns out to be a low type firm, investors are protected by the standard debt contract feature and can capture all the available proceed. However, when both types of firms pool at issuing equity, investors only capture part of the proceed if they encounter a low type firm. Thus, the low type firm always has an incentive to issue equity if it can pool with the high type firm at a price above the investment cost. Under this logic, an equity issuance becomes a bad signal of a firm's quality. If investors update their beliefs, then the equilibrium solution in the benchmark model is that both types of firms pool at issuing debt.

In the one-stage model, I introduce an equilibrium refinement concept, mimicking test, to accompany the Intuitive Criterion (Cho and Kreps, 1987) when restricting the set of equilibria. Specifically, since the strategy and signal spaces are infinite in my model, applying many existing refinement concepts cannot perfectly rule out corner solutions or solutions that are unreasonable. The mimicking test does not rely on restricting off-equilibrium beliefs. Instead, it considers that in a separating equilibrium, whether the high type firm can find a sequence of actions that are sufficient to

[1] At best, the literature agrees that the evidence on testing the pecking order theory is inconclusive (Leary and Roberts, 2010).

make the low type firm does not want to mimic the strategy of the high type firm.

I then extend the benchmark model to have two stages. In each stage the firm faces an investment project and can choose debt or equity financing. In this two-stage model, the high type firm can first choose to issue equity at a sufficiently low price, e.g., slightly below the investment cost, at stage one to deter the low type firm from issuing equity. When investors are convinced that such behavior can only come from a high type firm, this high type firm can then issue safe debt at the second stage. This alternative equilibrium solution can only be supported in a two-stage model since only the existence of the second project allows the high type firm to have the option of keep lowering the price at stage one. Thus, although issuing equity is a bad signal in the benchmark model, it is possible that issuing severely underpriced equity can become a good signal in a two-stage model.

In order to see how the assumption on a firm's type affects the equilibrium outcome, I re-examine the same research question by assuming the type is distributed continuously on an interval. In the continuous-type setting, I also use a one-stage model as the benchmark for a two-stage model. The results show that if there is only one investment project, the equilibrium outcome under the continuous-type setting resembles its two-type counterpart, and all firms pool at issuing debt. However, when there are two stages, the analysis under the two-type setting is no longer applicable to the continuous-type model. Instead, there are two possibilities in equilibrium. The first possibility is that the lower spectrum of good firms (i.e., firms with positive NPV projects) issue severely underpriced equity to signal their types, whereas the higher spectrum of good firms pool with bad firms (i.e., firms with negative NPV projects) and issue debt. The second possibility is that all good firms issue severely underpriced equity to separate themselves from bad firms.

The comparison between the two-type setting and the continuous-type setting shows that both specifications have their strengths and weaknesses. For instance, the two-type setting is generally simpler and convenient to work with. This simplicity allows us to deduce major inferences quite efficiently. On the other hand, the continuous-type setting requires more computation but can provide us with finer details and predictions. For example, other than the prediction on the debt and equity preference, we can also use the equilibrium outcome under the continuous-type setting to explain the equity issuance announcement effect. Yet this detail is absent under the two-type setting.

Following the literature originated from the costly state verification model (e.g., Townsend, 1979), I also study a variant of the previous signaling model by allowing investors to costly verify a firm's type before deciding whether to participate in the investment activity. I find that when firms are facing more capable investors, good firms are better off no matter which security they decide to issue. Specifically, good firms can offer a lower interest rate or a higher equity price while still provide enough incentives for investors to participate in the issuance. Since the true type of the firm can be revealed either through costly verification (when the cost is low) or through signaling (when the cost is high), the region that can support the pooling at debt equilibrium becomes much smaller comparing with the previous signaling model.

Overall, my findings suggest that even if debt is preferred to equity in a one-stage model, this preference may no longer be true in a two-stage model. In most scenarios, whether a firm prefers debt or equity depends not only on its profitability type and growth opportunities, but also on the market's perception as well as the cost for investors to verify the type. It is very likely that a firm has incentive to violate the pecking order precisely at the time that the adverse selection problem is most severe. Thus, the dynamic element of the asymmetric information casts serious doubt on interpretations of traditional empirical tests of the pecking order theory. Especially, evidence on firms violating the financing hierarchy perhaps only has a limited power on differentiating adverse selection from other leverage determinants. Hopefully, future research can further shed light on this issue.

This paper joins the voluminous development on the capital structure literature by studying the dynamic feature of asymmetric information. Since Akerlof (1970), many studies have investigated how information travels among economic agents, and how the uneven distribution of information has led to deviations from the Modigliani and Miller (1958) world. Earlier studies that use the multi-stage setting mainly focus on one type of security. For instance, Welch (1989) studies the IPO and SEO underpricing patterns when there exists an imitation cost for low quality firms. Chemmanur (1993) shows that IPOs are underpriced when insiders need to provide incentives for outsiders to produce information. Lucas and McDonald (1990) extend Myers and Majluf (1984) to an infinite horizon and predict the stock price behavior around the time of equity issues.[2] On the other hand, the literature that models pecking order

[2]More recent contributions like Daley and Green (2012) model the process that firms have information that is gradually released by news, and investors/buyers can choose to wait. Bond and

violations typically does not involve multiple issues. For example, Nachman and Noe (1994) derive the necessary and sufficient conditions for debt to be optimal rather than equity. Fulghieri and Lukin (2001) discuss how information sensitivity of different securities can make a firm's security choice endogenous.[3] The model in this paper incorporates both aforementioned features in describing corporate financing activities.

To model how security issuances can reveal firms' information to investors, I first focus on the signaling channel (e.g., Spence, 1973; Cho and Kreps, 1987), and then combine both the impact of signaling as well as costly state verification. Prior signaling games in the capital structure literature have explored the possibility that different debt levels can separate good firms from bad firms (Ross, 1977), insiders sometimes imperfectly observe a firm's cash flow (Noe, 1988), and firms underprice in the IPO market to signal their types (Allen and Faulhaber, 1989).[4,5] Subsequently to these early developments, the security design literature builds models that incorporate both the signaling effect and the impact of the information sensitivity of securities. For instance, DeMarzo and Duffie (1999) study how designers trade off between the retention cost and the liquidity cost when including cash flows in the security design. Biais and Mariotti (2005) analyze the issuance problem in the presence of both adverse selection and market power.[6] My model resorts to signaling as well, yet does not consider a possible mix of securities like the security design literature. Instead, I still use the standard debt and equity contracts and focus on how the debt-equity preference can change when firms need to interact with uninformed investors over multiple projects.

Aside from the information friction, other market frictions also contribute to the determinants of real world financing patterns. Numerous alternative explanations

Zhong (2016) provide a unified framework in analyzing SEOs and repurchases.

[3]For other possibilities see, for instance, Fulghieri, Garcia, and Hackbarth (2020), who investigate the scenario in which assets in place and growth options have different exposure to information asymmetry. Bolton and Dewatripont (2005) also summarize some possible scenarios for pecking order violations (e.g., p. 112-120).

[4]In Allen and Faulhaber (1989) good firms have certain possibilities to become bad firms, whereas in my model the type does not change across time.

[5]Other earlier papers that include the signaling effect can be found in, for example, Leland and Pyle (1977), in which the fraction of equity retained by the entrepreneur signals project quality, and Harris and Raviv (1985), who use signaling to explain the puzzle related to convertible debt calls.

[6]For other earlier studies on the security design problem see, e.g., Boot and Thakor (1993), Nachman and Noe (1994). A model of security design with pooling and tranching is studied in DeMarzo (2005).

have emerged to describe the discrepancy between Myers and Majluf (1984) and empirical stylized facts. For instance, issuing equity can alleviate the debt overhang problem (Myers, 1977), reduce the bankruptcy cost (Myers, 1984; Bradley, Jarrell, and Kim, 1984), mitigate the agency cost of debt (Jensen and Meckling, 1976), enlarge a firm's investor base (Merton, 1987). Baker and Wurgler (2002) propose that firms issue equity to time the market. Dittmar and Thakor (2007) develop a model by considering the belief alignment between managers and investors.[7] A dynamic pecking order can also arise when one combines information with other frictions. For example, Bolton and Freixas (2000) and Hennessy, Livdan, and Miranda (2010) combine information with the liquidation cost. Faure-Grimaud and Gromb (2004) build a model in which insiders' effort can change the firm value.[8] While these explanations provide valuable insights in describing the observed financing behavior, the model in the present paper explains the pecking order violation directly based on the adverse selection problem caused by information asymmetry.

My model also contributes to the literature on infinite signaling games. Many well known formulations of Bayesian equilibrium and refinement concepts exist for standard finite games (e.g., Selten, 1975; Kreps and Wilson, 1982; Fudenberg and Tirole, 1991), yet similar developments on infinite games encounter varies difficulties. For instance, the mixed-strategy may not be well defined (e.g., Myerson and Reny, 2020). On the other hand, although literature provides cases with a continuum of types, analyses often rely on certain continuity feature of the players' utility functions (e.g., Mailath, 1987). My paper shows a special case that the infinite game is solvable and provides implications when the sender of the signal has two distinct classes of infinite choices.

Technically, my paper provides a formal illustration of the difference between the two most commonly used model settings in games of incomplete information: two-type versus continuous-type. As shown in Tirole (2006), the two-type setting can be a powerful tool in drawing inferences, and researchers can benefit from the direct applicability of many existing solution concepts developed over the finite type space (e.g., Fudenberg and Tirole, 1991). My analysis indicates that sometimes this technical detail can be irrelevant to the main conclusion, and at other times researchers may have

[7] Another example for heterogeneous priors is Boot and Thakor (2011), who develop a dynamic pecking order model by considering the agency problem between a firm's initial owners and managers.

[8] Another example see Inderst and Mueller (2006), in which the lenders' preference on debt or equity depends on the outcome of their project screening.

to trade off between simplicity and generality when selecting the most appropriate model setup.

Overall, my paper contributes to the long-lasting debate of capital structure theories (for surveys on the empirical evidence see, e.g., Frank and Goyal, 2008; Graham and Leary, 2011), the progress on information economics, and the ongoing vivid development on leverage dynamics (e.g., DeMarzo and He, 2021). As stated in Fama and French (2005), "...both the tradeoff model and the pecking order model have serious problems... Perhaps it is best to regard the two models as stable mates, with each having elements of truth that help explain some aspects of financing decisions." (p. 580-581). To resolve the inconsistency between the empirical evidence and the tradeoff theory, Hennessy and Whited (2005) develop a dynamic tradeoff model by endogenizing several factors like investment and leverage choices.[9] Admati et al. (2018) study leverage dynamics when firms cannot commit to future funding choices. Perhaps complementary to their approach, my model builds on the traditional pecking order theory and extends the setting to incorporate dynamic debt and equity issuing activities.[10]

The rest of the paper proceeds as follows: Section 2 introduces a dynamic model under a two-type setting. Section 3 studies the same dynamic model under a continuous-type setting. Section 4 presents a variant model in which investors can costly verify a firm's type. Section 5 relates the model's implications with the empirical research. Section 6 discusses alternative equilibrium refinements. Section 7 concludes. Appendix A provides a simple extension. All proofs are embedded.

2 A Two-type Model

Consider a firm living in a two-stage economy, with stages are occasionally referred to as today ($t = 1$) and the future ($t = 2$). At each stage the firm faces an investment

[9] A review of related development on dynamic models and structural estimation can be found in Strebulaev and Whited (2012).

[10] To clarify a potential confusion with the terminology, the term "dynamic" in my paper borrows from the game theory literature that studies incomplete information games. Thus, the "dynamic" part of my model implies that the information is updated dynamically. On the other hand, the same term "dynamic" can also be found in the literature that relies on the continuous-time models (Brownian motion). For instance, dynamic capital structure models with changing debt levels can be found in Goldstein, Ju, and Leland (2001), DeMarzo and He (2021). Dynamic real option signaling models that consider corporate decisions as signals can be found in Grenadier and Malenko (2011), Morellec and Schürhoff (2011).

project that requires external financing. These projects are identical and require I amount of investment to yield R_θ at the end of time ($t = 3$). The return of the project can be either high or low, $R_\theta \in \{R_L, R_H\}$, with $R_L < I < R_H$. Thus, the high type firm has a positive NPV project, and the low type firm has a negative NPV project. To ease notation, I use type $\theta \in \{L, H\}$ whenever there is no confusion.

The firm has no financial slack and no intermediate return is available. There are no taxes, transaction cost or time discount. I further assume that there is no managerial distortion, and throughout this paper the manager and the firm are used interchangeably. The firm can choose either equity or debt financing at $t = 1$ and $t = 2$. The timeline is illustrated in Figure 1.

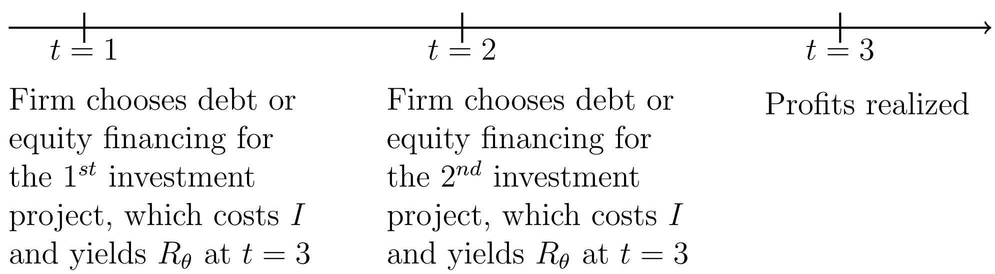


FIGURE 1: Timeline of events. This figure shows the timeline of events for a firm with a profitability type R_θ. There are two identical investment projects that arrive sequentially at $t = 1$ and $t = 2$. The firm can choose between equity and debt to finance each project. Profits are realized at $t = 3$.

In the following sections, I first analyze the incentive for every party in a one-stage model, then study the changes of these incentives in a two-stage setting.

2.1 One project

Suppose only the first project exists. To simplify the discussion, the ownership of the project is represented by one share prior to $t = 1$. The manager knows the exact realization of R_θ. If the manager chooses to issue equity, he decides the price p and the portion α of the project's share to be sold. Thus, the amount of equity issued needs to cover the investment cost $\alpha p = I$.[11] The total return for investors is $(\frac{I}{p}R_\theta - I)$, and the payoff for the firm is $\mathscr{L}_e^F(\theta) = (1 - \frac{I}{p})R_\theta$.

[11] In this paper I only consider the case that the manager raises the required amount of capital. For a different approach, Hart and Moore (1998) study the optimal debt contract in a model that allows the entrepreneur to raise more than needed funds for the investment project.

If the manager chooses to issue debt (or bond), he decides the interest rate r. Thus, the debt contract specifies that the firm will borrow I at $t = 1$ and return $I(1+r)$ at $t = 3$ if $R_\theta \geq I(1+r)$. Otherwise the firm becomes insolvent and pays all its available return R_θ to investors. The total return for investors is $\min[R_\theta - I, Ir]$. The payoff for the firm is $\mathscr{L}_d^F(\theta) = \max[0, R_\theta - I(1+r)]$.

Denote the available action of the firm as $a_F \in \{p, r\}$. Presumably, the manager can also decide not to issue securities. However, not issuing security is weakly dominated by debt issuance, as we can see from the payoff of the firm that $\mathscr{L}_d^F(\theta) \geq 0$. Thus, we can concentrate our discussion on the case of issuance.[12]

Investors have a sufficient amount of wealth to invest but are uninformed about the firm's type. Their prior belief $\mu(\theta)$ about the return distribution is that the probability of θ being a high type is μ, and being a low type is $1-\mu$, i.e., $\mu(H) = \mu$ and $\mu(L) = 1-\mu$. Denote the average of R_θ as $\bar{R}$. Assume investors' prior belief satisfies $\bar{R} = \mu R_H + (1-\mu)R_L > I$. Since all the investors are identical, they can be considered to act collectively as one party. Denote the available action for investors as $a_U \in \{P, N\}$, in which P denotes participate in investment, and N denotes not participate.

The firm moves first to determine which security to issue, and then investors can update their belief based on the firm's choice. Denote $\mu(\theta|a_F)$ as the posterior belief. Thus, we can summarize the expressions of the payoffs for the firm and investors as the following:

$$\mathscr{L}^F(\theta, a_F, a_U) = \begin{cases} \mathscr{L}_e^F(\theta) = (1 - \frac{I}{p})R_\theta & \text{if } a_F = p, a_U = P \\ \mathscr{L}_d^F(\theta) = \max[0, R_\theta - I(1+r)] & \text{if } a_F = r, a_U = P \\ 0 & \text{if } a_U = N \end{cases} \tag{1}$$

$$\mathscr{L}^U(a_F, a_U) = \begin{cases} \mathscr{L}_e^U = \sum_\theta \mu(\theta|p)(\frac{I}{p}R_\theta - I) & \text{if } a_F = p, a_U = P \\ \mathscr{L}_d^U = \sum_\theta \mu(\theta|r)\min[R_\theta - I, Ir] & \text{if } a_F = r, a_U = P \\ 0 & \text{if } a_U = N \end{cases} \tag{2}$$

[12] For example, if the equilibrium is pooling, then the low type firm has no incentive to separate itself by choosing not to issue security. If the equilibrium is separating, then regardless of the issuance decision of the low type firm, investors will react optimally by not investing. Nonetheless, Noe (1988) studies a model that incorporate all three options: issuing debt, equity, and not issuing. In his model, having the option of not issuing security is necessary since the firm already has an asset that generates positive payoff. Yet this setting is absent in the current paper.

Since I focus on pure-strategy equilibria, it is not necessary to specify a probability distribution over actions. Thus, I slightly abuse the notation and continue to use a_F^* and a_U^* to denote strategies for the players. Following the prior literature on signaling games (e.g., Noe, 1988; Welch, 1989), as well as various equilibrium definitions (e.g., Kreps and Wilson, 1982; Fudenberg and Tirole, 1991), I define a perfect Bayesian equilibrium (PBE) as the following:

Definition 1. *A perfect Bayesian equilibrium of the security issuance game is a strategy profile (a_F^*, a_U^*) and posterior beliefs $\mu(\theta|a_F)$ such that:*

(i) $\forall\theta$, $a_F^*(\theta) \in \arg\max_{a_F} \mathscr{L}^F(\theta, a_F, a_U^*)$,

(ii) $\forall a_F$, $a_U^*(a_F) \in \arg\max_{a_U} \mathscr{L}^U(a_F, a_U)$

(iii) Whenever a_F is an on-the-equilibrium action, the posterior belief is given by

$$\mu(\theta|a_F) = \begin{cases} 0 & \text{if } a_F^*(\theta) \neq a_F \\ \frac{\mu(\theta)}{\sum_{\{\theta'|a_F^*(\theta')=a_F\}} \mu(\theta')} & \text{otherwise} \end{cases}$$

and the posterior belief can be any probability distribution if a_F is an off-the-equilibrium action.

Conditions (i) and (ii) in the above definition are the perfection conditions, meaning that the firm maximizes its payoff given investors playing their best responses, and investors act optimally to the firm's action given their posterior beliefs. Condition (iii) means that beliefs of investors update according to Bayes' rule whenever possible.

Figure 2 provides an illustration of the game described above. Strictly speaking, this figure is not correct in the sense that there are infinitely many choices for each security, e.g., many possible prices and interest rates. Nevertheless, we may still find such an illustration helpful when relating the current model to some classical signaling games (e.g., the signaling game in Cho and Kreps, 1987). Since games with incomplete information often face the issue of infinitely many equilibria, and many of those equilibria may not provide meaningful implications, I first introduce these equilibria and then discuss two refinements on the equilibrium selection.[13]

[13]Roughly speaking, there are two types of signaling game settings. The first one is more similar to Spence (1973), in which the cost of signaling is a function of the continuously distributed signals. In this type of setting, the focus of the equilibrium solution is how to separate types within the same class of choices (e.g., education level). Models with this type of setting can be seen in, for

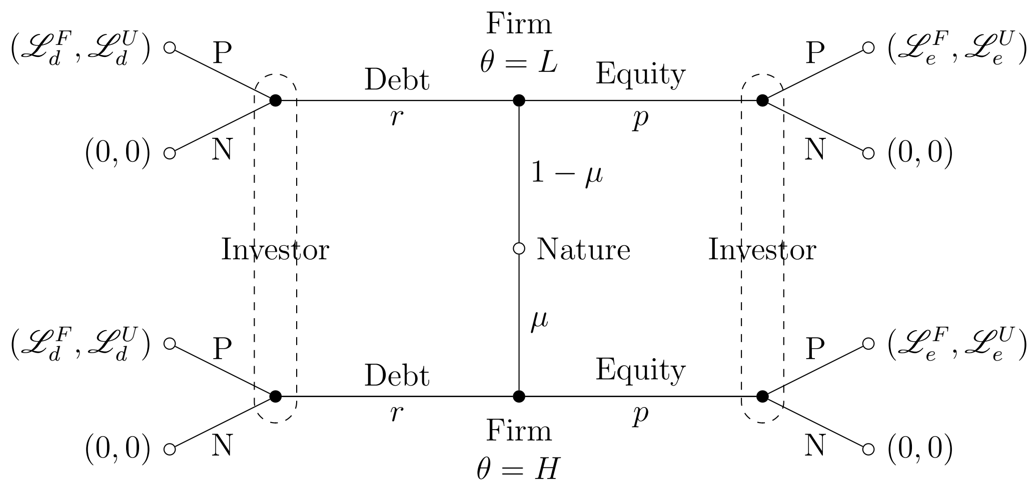


FIGURE 2: Decision Tree. The game is played by the firm and investors. The firm moves first and select a price p or interest rate r. Upon observing what has been offered by the firm, investors move to choose whether to participate (P) or not (N).

Equilibrium 1. ***(Pooling at debt)*** *The following strategy-belief combination constitutes an equilibrium.*

Firm's choice: $a_F^*(\theta) = r^* = \frac{1-\mu}{\mu}\frac{I-R_L}{I}$, $\forall\theta \in \{L, H\}$.

Investors' choice: $a_U^*(r^*) = P$.

Investors' belief along the equilibrium path: $\mu(H|r^*) = \mu$, $\mu(L|r^*) = 1-\mu$.

Investors' belief off the equilibrium path: $\forall a_F \notin \{r^*\}$, $\mu(H|a_F) \leq \mu_0$, $\mu(L|a_F) \geq 1-\mu_0$, *with* $\mu_0 = \frac{I-R_L}{R_H-R_L}$. *Investors' best response to out-of-equilibrium message is not to invest,* $a_U^*(a_F) = N$.

Payoffs for the firm and investors are: $\mathscr{L}_d^F(H) = R_H - I(1+r^*)$, $\mathscr{L}_d^F(L) = 0$, $\mathscr{L}_d^U = 0$.

Proof. The expression of r^* comes from the following:

$$\begin{aligned}
\mathscr{L}_d^U &= \sum_\theta \mu(\theta|r)\min[R_\theta - I, Ir] \\
&= \min[\mu(R_H - I) + (1-\mu)(R_L - I), \mu Ir + (1-\mu)(R_L - I)]
\end{aligned}$$

instance, Ross (1977), DeMarzo and Duffie (1999). The second type of setting is more similar to the beer-quiche example in Cho and Kreps (1987), in which there is a fixed cost for selecting the less preferred choice. In this second type of setting, the focus of a separating equilibrium (if exists) is often a separation in between different classes (e.g., beer or quiche). A model using this type of setting is Welch (1989). In my paper, the setting is more in line with the second type of signaling games, and the focus is the firm's choice between debt and equity.

Investors participate whenever $\mathscr{L}_d^U \geq 0$. Since $\mu(R_H - I) + (1-\mu)(R_L - I) = \bar{R} - I > 0$, then the binding constraint $\mathscr{L}_d^U = \mu I r^* + (1-\mu)(R_L - I) = 0$ gives the expression for r^*.[14]

The expression of μ_0 comes from letting $\mu_0 R_H + (1-\mu_0) R_L = I$. As a result, whenever $\mu(H|a_F) < \mu_0$, investors believe the average firm has a negative NPV project and choose not to invest. □

Equilibrium 2. ***(Pooling at equity)*** *The following strategy-belief combination constitutes an equilibrium.*

Firm's choice: $a_F^*(\theta) = p^* = I$, $\forall \theta \in \{L, H\}$.

Investors' choice: $a_U^*(p^*) = P$.

Investors' belief along the equilibrium path: $\mu(H|p^*) = \mu$, $\mu(L|p^*) = 1 - \mu$.

Investors' belief off the equilibrium path: $\forall a_F \notin \{p^*\}$, $\mu(H|a_F) \leq \mu_0$, $\mu(L|a_F) \geq 1 - \mu_0$, *with* $\mu_0 = \frac{I - R_L}{R_H - R_L}$. *Investors' best response to out-of-equilibrium message is not to invest,* $a_U^*(a_F) = N$.

Payoffs for the firm and investors are: $\mathscr{L}_e^F(H) = 0$, $\mathscr{L}_e^F(L) = 0$, $\mathscr{L}_d^U = \mu(R_H - I) + (1-\mu)(R_L - I) = \bar{R} - I$.

As shown above, the game can have a few pooling equilibria. An immediate candidate of refining pooling equilibria is the Intuitive Criterion developed by Cho and Kreps (1987). In the current model, the Intuitive Criterion can rule out all pooling at equity equilibria if $a_F^*(\theta) = p > I$. The argument goes as follows. In case of $p > I$, the low type firm is strictly better off by choosing equity since $\mathscr{L}_e^F(L) > 0 = \mathscr{L}_d^F(L)$. Thus, it is not reasonable for the low type firm to send the message of debt. However, the high type firm can surely send the message of debt by knowing the payoff of the low type firm. Investors, on the other hand, will correctly interpret the message of debt as it can only come from the high type firm. With this logic, a pooling at equity equilibrium breaks down whenever $p > I$.

Here we can see that although the pooling at debt equilibrium seems more reasonable, and the Cho-Kreps Intuitive Criterion is fairly efficient in reducing the set of equilibria, there is still one pooling at equity equilibrium that can survive this test. Specifically, if both firms choose $p^* = I$, then issuing equity is no longer a strictly

[14]Note that if investors choose not to invest, the payoff is also zero. However, not to invest is not a strategy that can survive reasonable "trembles" from the firm. For instance, if the firm choose $r^* + \varepsilon$, then $\mathscr{L}_d^U > 0$ and $a_U^*(r^*) = P$.

better strategy for the low type firm. In this scenario, the high type firm cannot credibly send the off-equilibrium message.

While the above Equilibrium 2 seems unfavorable, the game also has a few separating equilibria that might look unreasonable.

Equilibrium 3. ***(Separating, H chooses equity and L chooses debt)*** *The following strategy-belief combination constitutes an equilibrium.*

Firm's choice: $a_F^*(H) = p^* = I$, $a_F^*(L) = r$.

Investors' choice: $a_U^*(p^*) = P$, $a_U^*(r) = N$.

Investors' belief along the equilibrium path: $\mu(H|p^*) = 1$, $\mu(L|r) = 1$.

Investors' belief off the equilibrium path: $\forall a_F \notin \{p^*, r\}$, $\mu(H|a_F) \leq \mu_0$, $\mu(L|a_F) \geq 1 - \mu_0$, *with* $\mu_0 = \frac{I-R_L}{R_H-R_L}$. *Investors' best response to out-of-equilibrium message is not to invest,* $a_U^*(a_F) = N$.

Payoffs for the firm and investors are: $\mathscr{L}_e^F(H) = 0$, $\mathscr{L}_d^F(L) = 0$, $\mathscr{L}_e^U = R_H - I$, $\mathscr{L}_d^U = 0$.

Proof. Observe that the high type firm cannot set any price $p > I$, for otherwise the low type can be better off by deviating. □

Equilibrium 4. ***(Separating, H chooses debt and L chooses equity)*** *The following strategy-belief combination constitutes an equilibrium.*

Firm's choice: $a_F^*(H) = r' = 0$, $a_F^*(L) = p$.

Investors' choice: $a_U^*(r') = P$, $a_U^*(p) = N$.

Investors' belief along the equilibrium path: $\mu(H|r') = 1$, $\mu(L|p) = 1$.

Investors' belief off the equilibrium path: $\forall a_F \notin \{p, r'\}$, $\mu(H|a_F) \leq \mu_0$, $\mu(L|a_F) \geq 1 - \mu_0$, *with* $\mu_0 = \frac{I-R_L}{R_H-R_L}$. *Investors' best response to out-of-equilibrium message is not to invest,* $a_U^*(a_F) = N$.

Payoffs for the firm and investors are: $\mathscr{L}_d^F(H) = R_H - I$, $\mathscr{L}_e^F(L) = 0$, $\mathscr{L}^U = 0$.

Equilibrium 5. ***(Separating with different interest rates)*** *The following strategy-belief combination constitutes an equilibrium.*

Firm's choice: $a_F^*(H) = r_1 = 0$, $a_F^*(L) = r_2 > 0$.

Investors' choice: $a_U^*(r_1) = P$, $a_U^*(r_2) = N$.

Investors' belief along the equilibrium path: $\mu(H|r_1) = 1$, $\mu(L|r_2) = 1$.

Investors' belief off the equilibrium path: $\forall a_F \notin \{r_1, r_2\}$, $\mu(H|a_F) \leq \mu_0$, $\mu(L|a_F) \geq 1 - \mu_0$, *with* $\mu_0 = \frac{I-R_L}{R_H-R_L}$. *Investors' best response to out-of-equilibrium message is* *not to invest,* $a^*_U(a_F) = N$*.*

Payoffs for the firm and investors are: $\mathscr{L}^F_d(H) = R_H - I$*,* $\mathscr{L}^F_d(L) = 0$*,* $\mathscr{L}^U = 0$*.*

Among all the possible separating equilibria, only separating with different equity prices is naturally ruled out. For instance, suppose the high type firm chooses price p_1, and the low type firm chooses price p_2. If $p_1 > p_2$, then the low type will strictly benefit from increasing the price. In other words, since investors do not invest in the low type firm whenever it separates itself from the high type firm, it is in the best interest of the low type firm to mimic the high type firm's behavior. However, without further refinement, the game does not generate a clear prediction on how firms behave.

The reason that the above separating equilibria may not seem reasonable is that we are not very sure why a priori the low type firm would have any incentive to pursue such option. For instance, in Equilibrium 4, the low type firm is equally better off by switching to debt financing. Here the concern is that if the low type firm separates itself early, then such behavior essentially prevents all possible future issuances. If a successful issuance is accompanied by any private benefit, or if there is a possibility for the low type firm to switch to a high type firm, then it is in the best interest of the low type firm to try to blend in. The fact that the previous refinement cannot filter out these separating equilibria motivates the use of another refinement, which is described as follows.

Consider the low type firm adopting the following strategy "mimic high type firm's behavior whenever possible". It can be seen that such strategy is a weakly dominating strategy. Thus, I introduce a mimicking test as defined in the following:

Definition 2. *(Mimicking Test) For strategy profiles* $(a^*_F(L), a^*_F(H), a^*_U)$ *in which* $a^*_F(L) \neq a^*_F(H)$*, if* $\nexists$ *a sequence of actions* $a_F(H) \to a^*_F(H)$ *with* $\mathscr{L}^F(H) \geq 0$ *and* $a_F(H)$ *belongs to the same class of security as* $a^*_F(H)$*, such that letting* $a_F(L) = a_F(H)$ *leads to* $\mathscr{L}^F(L, a_F(L), a_F(H)) > \mathscr{L}^F(L, a_F(H), a_F(H))$*,* $\forall a_F(H)$*, then the equilibrium outcome is said to fail the mimicking test.*

The above definition is saying that for any separating equilibrium, if the high type firm cannot find a sequence of actions that the low type firm does not find it profitable to mimic, then such equilibrium is unintuitive and thus should be refined away.[15] We

[15] A minor concern of Definition 2 is that whether it is still necessary to specify the belief of investors. As shown in the above separating equilibria as well as the later Equilibrium 7, constructing beliefs that can support the sequence of actions is relatively straight forward.

can see that after this refinement, Equilibrium 3, Equilibrium 4, and Equilibrium 5 are no longer valid.[16]

If we then revisit the two pooling equilibria, we can see that the high type firm strictly prefers Equilibrium 1, since the payoffs for firms in Equilibrium 1 Pareto dominate those in Equilibrium 2. Thus, a high type firm will choose to issue debt at interest rate r^* when knowing the low type firm will mimic its behavior. Under this logic, Equilibrium 1 becomes the only payoff dominant equilibrium that can survive both the Intuitive Criterion and the mimicking test. I continue to use the combination of these two refinements in the remaining paper when finding the payoff dominant equilibrium solution. Below Proposition 1 summarizes this outcome.

Proposition 1. *(Two-type model) If there is only one project, both the high type firm and the low type firm issue debt at interest rate r^*.*

Finally, it is possible for extensions of the one-stage model to incorporate richer details. In Appendix A I discuss a simple extension of incorporating a randomized initial wealth. In this special case, the extended model nonetheless can be reduced to the basic structure presented in this section. Although changing the benchmark model can be interesting, those extensions deviate from the main theme in the paper. I intend to reserve such variations to future research.

To conclude this section, we can see that in a one-stage model, one can obtain a similar pecking order theory prediction as in Myers (1984), Myers and Majluf (1984), i.e., debt is preferred to equity. However, as I will show in the next section, this preference may no longer be true in a two-stage model.

2.2 Two projects

Consider the two projects as shown in Figure 1. If at $t = 1$ both types of firms pool at issuing debt as in Equilibrium 1, then no information is revealed before $t = 2$ and investors possess the same prior belief when facing the second issuance. Thus, the

[16]Naturally, there are other equilibrium refinement concepts, like the idea of Divinity in Banks and Sobel (1987). For instance, if we look at Equilibrium 2, then it says that the low type firm is more likely to deviate than high type firm (since $\mu_0 < \mu$). While this tendency might be true if the off-equilibrium message is $p > I$, it might be the opposite if the off-equilibrium message is r. Thus, some of these off-equilibrium messages might not be reasonable under alternative refinement concepts. In this paper I introduce a different refinement concept because the mimicking test is quite efficient in restricting the set of equilibria, and can avoid many of the complications that may arise when dealing with out-of-equilibrium beliefs.

issuance game at $t = 2$ repeats itself exactly the same way as in the one-stage model described in the previous section. Under this scenario, we know from Proposition 1 that both firms issue debt. Below Equilibrium 6 summarizes the possible outcome that firms pool at issuing debt twice.

Equilibrium 6. ***(Pooling at debt twice)*** *The following strategy-belief combination appears at both $t = 1$ and $t = 2$.*

Firm's choice: $a_F^*(\theta) = r^* = \frac{1-\mu}{\mu}\frac{I-R_L}{I}$, $\forall\theta \in \{L, H\}$.

Investors' choice: $a_U^*(r^*) = P$.

Investors' belief along the equilibrium path: $\mu(H|r^*) = \mu$, $\mu(L|r^*) = 1 - \mu$.

Investors' belief off the equilibrium path: $\forall a_F \notin \{r^*\}$, $\mu(H|a_F) \leq \mu_0$, $\mu(L|a_F) \geq 1 - \mu_0$, with $\mu_0 = \frac{I-R_L}{R_H-R_L}$. Investors' best response to out-of-equilibrium message is not to invest, $a_U^*(a_F) = N$.

Total payoffs for the firm and investors at $t = 3$ are: $\mathscr{L}_{dd}^F(H) = 2R_H - 2I(1+r^*)$, $\mathscr{L}_{dd}^F(L) = 0$, $\mathscr{L}_{dd}^U = 0$.

On the other hand, when firms have two projects, the above Equilibrium 6 might not be the most desirable outcome for the high type firm. Imagine that if the high type firm can manage to separate itself at $t = 1$, it can raise capital from the same pool of investors at a fair price at $t = 2$. Specifically, when the type is revealed to investors, the high type firm can finance the second project by issuing safe debt and capture the entire proceed $(R_H - I)$. Since issuing debt as in Equilibrium 1 cannot effectively separate the high type from the low type, then the only possible choice for the high type firm is to issue equity at a price that is sufficiently unattractive to the low type firm. Below Equilibrium 7 summarizes this possibility.[17]

Equilibrium 7. ***(Separating, High type issues equity)***

(i) The following strategy-belief combination appears at $t = 1$.

Firm's choice: $a_F^*(H) = p^* - \varepsilon = I - \varepsilon$, $a_F^*(L) = r$.

Investors' choice: $a_U^*(p^* - \varepsilon) = P$, $a_U^*(r) = N$.

Investors' belief along the equilibrium path: $\mu(H|p^* - \varepsilon) = 1$, $\mu(L|r) = 1$.

Investors' belief off the equilibrium path: $\forall a_F \notin \{p^* - \varepsilon, r\}$, $\mu(H|a_F) \leq \mu_0$, $\mu(L|a_F) \geq 1 - \mu_0$, with $\mu_0 = \frac{I-R_L}{R_H-R_L}$. Investors' best response to out-of-equilibrium message is not to invest, $a_U^*(a_F) = N$.

[17] In this equilibrium we can also directly let $\varepsilon \to 0$ and write p^* as the equilibrium outcome, yet the current specification might look more natural at first sight.

(ii) The following strategy appears at $t = 2$.

The high type firm issues safe debt ($r = 0$) and investors participate.

Investors do not participate in any security issued by the low type firm.

(iii) Total payoffs for the firm and investors at $t = 3$ are: $\mathscr{L}^F_{ed}(H) = (1 - \frac{I}{I-\varepsilon})R_H + R_H - I$, $\mathscr{L}^F_{dd}(L) = 0$, $\mathscr{L}^U_{ed} = R_H \frac{I}{I-\varepsilon} - I$.

Proof. Observe that there exists a sequence of ε such that if the low type firm mimics the high type firm, it always gets a negative payoff $\mathscr{L}^F_{ed}(L) = (1 - \frac{I}{I-\varepsilon})R_L + \min[0, R_L - I] < 0$. □

The above equilibrium shows that the high type firm has to issue severely underpriced equity to discourage the low type firm from mimicking. The issuing price for the first project is not just lower than the true type (R_H), but also lower than the average price ($\bar{R}$). In fact, the high type firm has to transfer the entire proceed of the first project to investors (when issues at $p^* = I$).

Naturally, a few interpretation concerns might arise in Equilibrium 7. Specifically, although the cost of ε does not happen in equilibrium, passing the mimicking test requires that there exist such possibilities. For instance, there can be additional cost associated with issuing equity, e.g., fees paid to financial intermediaries. Another way to interpret ε is that the firm can pre-sell a small fraction of its second project to investors.[18]

Second, the outcome that the manager has to sell the entire first project to investors may not look desirable, but practically, if investors don't have a perfect knowledge of the manager's wealth level, or if investors cannot correctly interpret the signal of having skin in the game as in Appendix A, then the manager can also act like investors and secretly purchase the project's shares at price p^*. In this way, effectively only part of the project is sold to outsiders.

Finally, Equilibrium 7 can serve as a good benchmark for a comparison between some model variations. For instance, in Section 4, I consider investors can choose to costly verify a firm's type, and the equilibrium equity price can be well above p^*.

Comparing Equilibrium 7 with the previous Equilibrium 3 we can see that the high type firm does not have any incentive to take the loss of ε if there is only one project,

[18] The logic here shares some similarities with the argument of burning money. When the high type firm has the option of keep lowering the price, in equilibrium it does not have to do so. However, security underpricing is still different than the original burning money idea in the sense that the firm incurs a real loss in the issuance process, yet burning money emphasizes the outcome of not burning (Fudenberg and Tirole, 1991, p. 460-463).

as this action gives a negative total payoff (e.g., revising p^* to $p^* - \varepsilon$ in Equilibrium 3). However, when the high type firm can benefit from the second project, the previous unreasonable Equilibrium 3 might become reasonable as in Equilibrium 7. On the other hand, revisiting the previous Equilibrium 4 can show us why debt financing cannot separate the high type firm from the low type firm. Here we can see that no matter what choices a high type firm makes, it cannot force the payoff of the low type firm to become negative when deviating from the equilibrium. Thus, mimicking the high type firm is always "free" for a low type firm under debt financing, but can be "costly" under equity financing.

Another observation with Equilibrium 7 is that the combination of equity and debt issuance at different stages is necessary to support the equilibrium. Issuing equity twice cannot successfully deter the mimicking attempt of the low type firm. This outcome is consistent with empirical evidence of leverage rebalancing behaviors. For instance, Alti (2006) shows that firms usually issue debt to increase leverage after IPOs.

With the result of Equilibrium 6 and Equilibrium 7, we can obtain the following Proposition 2 by comparing the payoffs of the high type firm.

Proposition 2. *(Two-type model) In the presence of two projects, when* $\mu \in (\mu^*, 1)$*, the high type firm and the low type firm pool at issuing debt at* $t = 1$*. When* $\mu \in (\mu_0, \mu^*)$*, the high type firm issues equity at* $t = 1$ *to separate itself from the low type firm. Where* $\mu_0 = \frac{I - R_L}{R_H - R_L}$*, and* $\mu^* = \frac{2I - 2R_L}{R_H + I - 2R_L}$.

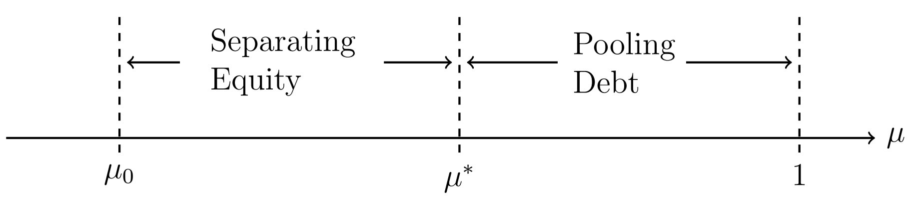


FIGURE 3: An illustration of Proposition 2.

Proof. (1) μ_0 is obtained from the model setting, in which $\bar{R} = \mu R_H + (1 - \mu) R_L > I$. (2) μ^* is obtained by letting $\mathscr{L}^F_{ed}(H) = \mathscr{L}^F_{dd}(H) \Rightarrow (1 - \frac{I}{I - \varepsilon}) R_H + R_H - I = 2R_H - 2I(1 + r^*)$. Given $r^* = \frac{1-\mu}{\mu} \frac{I - R_L}{I}$, we have $\mu^* \to \frac{2I - 2R_L}{R_H + I - 2R_L}$ when $\varepsilon \to 0$. $\square$

Proposition 2 shows that the high type firm has better incentive to pursue the separating equilibrium when the prior belief on the probability of a firm being a high type firm is low. This result is intuitive in the sense that if investors don't have a strong prior belief, they demand a high interest rate when firms are issuing debt. Thus the potential benefit of a high type firm revealing itself is higher. On the other hand, when investors believe most firms are high type firms (μ is very large), then a high type firm prefers debt financing, since issuing equity would have imposed a significant amount of loss.

Comparing Proposition 1 and Proposition 2 we can see that in a two-stage model, debt financing may not be the most preferred choice, and the high type firm has incentive to violate the pecking order when issuing equity can reveal its type to investors. From the society's point of view, Equilibrium 7 is better than Equilibrium 6 because no negative NPV projects can receive funding. Thus, allowing the firm to have repeated interactions with investors can potentially improve the overall welfare and investment efficiency.

3 A Continuous-type Model

In corporate finance, much of the existing inference can be made by assuming that firms have two types, e.g., in various models in Tirole (2006). As shown in the previous section, we can also deduce the main prediction of this paper by employing such a setting. However, although this two-type setting can be sufficient to show the difference between the one-stage and two-stage models, such simplification on the distribution of firms might fail to capture predictions on some other dimensions. Therefore, in this section I re-examine the same research question by assuming that the type is distributed on an interval. Specifically, $R_\theta \in [R_L, R_H]$. Same as before, I use type θ and R_θ interchangeably whenever there is no confusion.

3.1 One project

Denote investors' prior belief about the return distribution as $f(\theta)$, which is the probability density function. To simplify the solution concept, I assume that $f(\theta)$ is continuously distributed on $[R_L, R_H]$, with $R_L \geq 0$ and $f(\theta) > 0$, $\forall R_\theta \in [R_L, R_H]$. Denote the expectation of R_θ as $\bar{R}$. Similar to the previous two-type model, I assume that investors have prior belief that the average firm has a positive NPV project, i.e., $I \in (R_L, \bar{R})$.

The firm moves first with an action $a_F \in \{p, r\}$, and then investors can choose whether to participate or not, $a_U \in \{P, N\}$. Denote $f(\theta|a_F)$ the posterior belief. The expressions of payoffs for the firm and investors are summarized as follows:

$$\mathscr{L}^F(\theta, a_F, a_U) = \begin{cases} \mathscr{L}_e^F(\theta) = (1 - \frac{I}{p})R_\theta & \text{if } a_F = p, a_U = P \\ \mathscr{L}_d^F(\theta) = \max[0, R_\theta - I(1+r)] & \text{if } a_F = r, a_U = P \\ 0 & \text{if } a_U = N \end{cases} \tag{3}$$

$$\mathscr{L}^U(a_F, a_U) = \begin{cases} \mathscr{L}_e^U = \int_{R_L}^{R_H} f(\theta|p)(\frac{I}{p}R_\theta - I)dR_\theta & \text{if } a_F = p, a_U = P \\ \mathscr{L}_d^U = \int_{R_L}^{R_H} f(\theta|r)\min[R_\theta - I, Ir]dR_\theta & \text{if } a_F = r, a_U = P \\ 0 & \text{if } a_U = N \end{cases} \tag{4}$$

I also revise the definition of perfect Bayesian equilibrium:

Definition 3. *A perfect Bayesian equilibrium in the continuous-type model of the security issuance game is a strategy profile* (a_F^*, a_U^*) *and posterior beliefs* $f(\theta|a_F)$ *such that:*

(i) $\forall\theta$, $a_F^*(\theta) \in \arg\max_{a_F} \mathscr{L}^F(\theta, a_F, a_U^*)$,

(ii) $\forall a_F$, $a_U^*(a_F) \in \arg\max_{a_U} \mathscr{L}^U(a_F, a_U)$

(iii) Whenever a_F *is an on-the-equilibrium action, the posterior belief is given by*

$$f(\theta|a_F) = \begin{cases} 0 & \textit{if } a_F^*(\theta) \neq a_F \\ \frac{f(\theta)}{\int_{\{\theta'|a_F^*(\theta')=a_F\}} f(\theta')dR_{\theta'}} & \textit{otherwise} \end{cases}$$

and the posterior belief can be any probability distribution $g(\theta)$*, such that* $\int_{R_L}^{R_H} g(\theta)dR_\theta = 1$*, if* a_F *is an off-the-equilibrium action.*

Similar as in the previous section, conditions (i) and (ii) in the above definition are the perfection conditions, and condition (iii) means that beliefs of investors update according to Bayes' rule whenever possible.

To tackle the problem associated with infinite type sets, observe that loosely speaking, we can still classify firms into two general categories: those that have positive NPV projects (good firms, $R_\theta > I$), and those with negative NPV projects (bad firms, $R_\theta < I$). For firms with negative NPV projects, they are strictly better off if they can pool with some good firms and issue equity at a price above $p^* =$

I. Thus, the previous strategy "mimicking good firms whenever possible" is still weakly dominating its alternatives. In order to focus our attention on the most meaningful equilibrium, I will continue to use the Cho-Kreps Intuitive Criterion and the mimicking test as refinements.

If firms only issue security once, then the below pooling at debt equilibrium is still the most attractive equilibrium to good firms. Define r^{**} to be the smallest positive value to satisfy[19]

$$\mathscr{L}_d^U\big|_{r^{**}} = \int_{R_L}^{R_H} f(\theta)\min[R_\theta - I, Ir^{**}]dR_\theta = 0$$

then the pooling equilibrium can be summarized as the following:

Equilibrium 8. ***(Pooling at debt)*** *The following strategy-belief combination constitutes an equilibrium.*

Firm's choice: $a_F^*(\theta) = r^{**}$, $\forall R_\theta \in [R_L, R_H]$.

Investors' choice: $a_U^*(r^{**}) = P$.

Investors' belief along the equilibrium path: $f(\theta|r^{**}) = f(\theta)$.

Investors' belief off the equilibrium path: $\forall a_F \notin \{r^{**}\}$, $f(\theta|a_F)$ *can be any distribution* $g(\theta)$, *such that* $\int_{R_L}^{R_H} g(\theta)R_\theta dR_\theta \leq I$. *Investors' best response to out-of-equilibrium messages is not to invest,* $a_U^*(a_F) = N$.

Payoffs for the firm and investors are: $\mathscr{L}_d^F(\theta) = \max[0, R_\theta - I(1+r^{**})]$, $\mathscr{L}_d^U = 0$.

Proposition 3 provides a similar summary as Proposition 1. Comparing Equilibrium 8 with Equilibrium 1, we can see that in a one-stage model, the equilibrium outcomes under the continuous-type and the two-type settings resemble each other. However, as I will show in the next section, the inference drawn from different settings can differ significantly in a two-stage model.

Proposition 3. *(Continuous-type model) If there is only one project, all firms issue debt at interest rate* r^{**}.

3.2 Two projects

When the firm's type is distributed on an interval, the previous analysis in the two-type setting cannot be applied directly. For instance, to discourage bad firms from

[19]Since the solution of this equation may not be unique.

issuing equity, the only choice for the good firm is to set price at $p^* - \varepsilon$.[20] However, some very good firms, e.g., firms with $R_\theta \to R_H$, may not find such a strategy profitable. As a result, let us first look at different incentives for different types of firms.

Consider the type distribution illustrated as in Figure 4. If all firms with positive NPV projects ($R_\theta > I$) issue equity at price $p^* - \varepsilon$, then these good firms can collectively convince investors that they have good projects, since bad firms never issue equity at this price. After investors update their belief, good firms can issue safe debt at the second stage and investors will accept their offer. On the other hand, all firms can also pool at issuing debt twice and receive $\max[0, 2R_\theta - 2I(I + r^{**})]$.

If we look at firms with type $R_\theta \in [I, I(1 + r^{**})]$ as shown in the shaded area in Figure 4, these firms have positive NPV projects. However, they can not receive any positive payoff under the pooling at debt equilibrium due to the existence of bad firms. If these good firms issue equity instead, they can surely get a positive payoff from the second project ($R_\theta - I$) if investors are convinced that they are good firms. Thus, firms with type $R_\theta \in [I, I(1 + r^{**})]$ strictly prefer to issue underpriced equity in the first stage. Under this logic, pooling at debt twice cannot become an equilibrium in the two-stage model. When some good firms issue equity, they drive up the equilibrium interest rate (r goes above r^{**}), and further incentivize more good firms to issue equity.

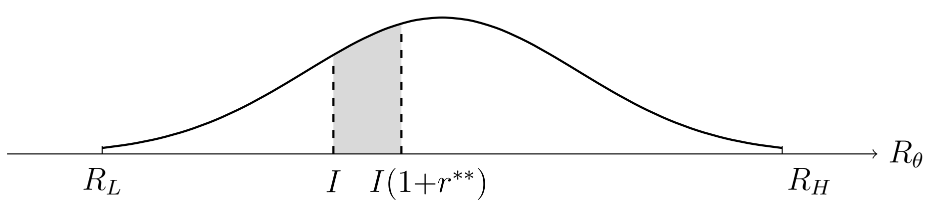


FIGURE 4: An illustration of the firm's type.

With the above discussion, we can see that in contrast with the two-type model, we no longer need to consider the comparison between the pooling at debt twice equilibrium and the separating equilibrium under the continuous-type setting. Instead, the question becomes whether all good firms have incentives to issue equity.

There are two possibilities. The first possibility is part of all good firms issue equity, as shown in Figure 5. In this figure R^* is denoted as the cutoff point, with

[20] Here the actual equilibrium is p^* as shown in Section 2, and the sequence of actions is $p^* - \varepsilon$. In the main text I sometimes denote $p^* - \varepsilon$ since the trade-off is often clearer with this notation.

firms issue debt at interest rate r^{***} when $R_\theta > R^*$ or $R_\theta < I$, and issue equity at price $p^* - \varepsilon$ when $I < R_\theta < R^*$. Before defining R^* and r^{***}, I first summarize this partial separating equilibrium as follows:

Equilibrium 9. ***(Partial Separating)***

(i) The following strategy-belief combination appears at $t = 1$.

Firm's choice: $a_F^*(\theta) = \begin{cases} p^* - \varepsilon & \text{if } R_\theta \in [I, R^*] \\ r^{***} & \text{if } R_\theta \in [R_L, I) \cup (R^*, R_H] \end{cases}$

Investors' choice: $a_U^*(p^* - \varepsilon) = P$, $a_U^*(r^{***}) = P$.

Investors' belief along the equilibrium path:

(a) If observe $p^* - \varepsilon$ then the firm has a positive NPV project.

$$f(\theta|p^* - \varepsilon) = \begin{cases} \frac{f(\theta)}{\int_I^{R^*} f(\theta')dR_{\theta'}} & \text{if } R_\theta \in [I, R^*] \\ 0 & \text{if } R_\theta \in [R_L, I) \cup (R^*, R_H] \end{cases}$$

(b) If observe r^{***} then the firm either has a negative NPV project, or is a very good firm.

$$f(\theta|r^{***}) = \begin{cases} 0 & \text{if } R_\theta \in [I, R^*] \\ \frac{f(\theta)}{\int_{R_L}^I f(\theta')dR_{\theta'} + \int_{R^*}^{R_H} f(\theta')dR_{\theta'}} & \text{if } R_\theta \in [R_L, I) \cup (R^*, R_H] \end{cases}$$

Investors' belief off the equilibrium path: $\forall a_F \notin \{p^* - \varepsilon, r^{***}\}$, $f(\theta|a_F)$ can be any distribution $g(\theta)$, such that $\int_{R_L}^{R_H} g(\theta) R_\theta dR_\theta \leq I$. Investors' best response to out-of-equilibrium messages is not investing, $a_U^*(a_F) = N$.

(ii) The following strategy appears at $t = 2$.

Firms that issue equity at $t = 1$ issue safe debt ($r = 0$) and investors participate. Firms that issue debt at $t = 1$ continue to issue debt at the same interest rate, investors participate. If firms play other strategies, investors do not participate.

(iii) Let $\varepsilon \to 0$, total payoffs for the firm and investors at $t = 3$ are:[21]

$$\mathscr{L}^F = \begin{cases} R_\theta - I & \text{if } R_\theta \in [I, R^*] \\ 2R_\theta - 2I(1 + r^{***}) & \text{if } R_\theta \in (R^*, R_H] \\ 0 & \text{if } R_\theta \in [R_L, I) \end{cases}, \quad \mathscr{L}^U = \int_I^{R^*} f(\theta)(R_\theta - I)dR_\theta.$$

The second possibility is that all good firms issue equity, as shown in Figure 6. In this case we obtain a separating equilibrium as summarized in the below Equilibrium 10.[22]

[21] Here $\mathscr{L}^U$ is the ex ante expected payoff.

[22] Here separating only means that good firms are separated from bad firms. Yet investors still don't know the type for sure. Thus, the equilibrium is more or less a quasi-separating equilibrium.

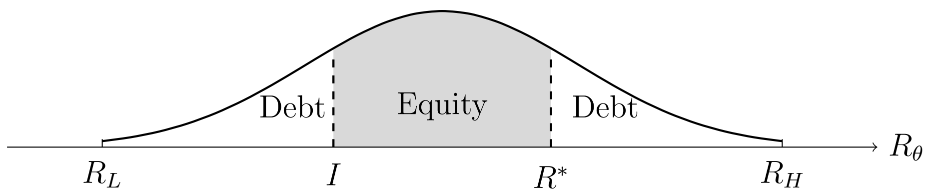


FIGURE 5: An illustration of Equilibrium 9.

Equilibrium 10. ***(Separating)***

(i) The following strategy-belief combination appears at $t = 1$.

Firm's choice: $a_F^*(\theta) = \begin{cases} p^* - \varepsilon & \text{if } R_\theta \in [I, R_H] \\ r & \text{if } R_\theta \in [R_L, I) \end{cases}$

Investors' choice: $a_U^*(p^* - \varepsilon) = P$, $a_U^*(r) = N$.

Investors' belief along the equilibrium path:

(a) If observe $p^* - \varepsilon$ *then the firm has a positive NPV project.*

$$f(\theta|p^* - \varepsilon) = \begin{cases} \frac{f(\theta)}{\int_I^{R_H} f(\theta')dR_{\theta'}} & \text{if } R_\theta \in [I, R_H] \\ 0 & \text{if } R_\theta \in [R_L, I) \end{cases}$$

(b) If observe r *then the firm has a negative NPV project.*

$$f(\theta|r) = \begin{cases} 0 & \text{if } R_\theta \in [I, R_H] \\ \frac{f(\theta)}{\int_{R_L}^{I} f(\theta')dR_{\theta'}} & \text{if } R_\theta \in [R_L, I) \end{cases}$$

Investors' belief off the equilibrium path: $\forall a_F \notin \{p^* - \varepsilon, r\}$, $f(\theta|a_F)$ *can be any distribution* $g(\theta)$, *such that* $\int_{R_L}^{R_H} g(\theta)R_\theta dR_\theta \leq I$. *Investors' best response to out-of-equilibrium messages is not to invest,* $a_U^*(a_F) = N$.

(ii) The following strategy appears at $t = 2$.

Firms that issue equity at $t = 1$ *issue safe debt* $(r = 0)$ *and investors participate. If firms issue debt at* $t = 1$, *investors do not participate.*

(iii) Let $\varepsilon \to 0$, *total payoffs for the firm and investors at* $t = 3$ *are:*[23]

$$\mathscr{L}^F = \begin{cases} R_\theta - I & \text{if } R_\theta \in [I, R_H] \\ 0 & \text{if } R_\theta \in [R_L, I) \end{cases}, \text{ and } \mathscr{L}^U = \int_I^{R_H} f(\theta)(R_\theta - I)dR_\theta.$$

[23] Here $\mathscr{L}^U$ is the ex ante expected payoff.

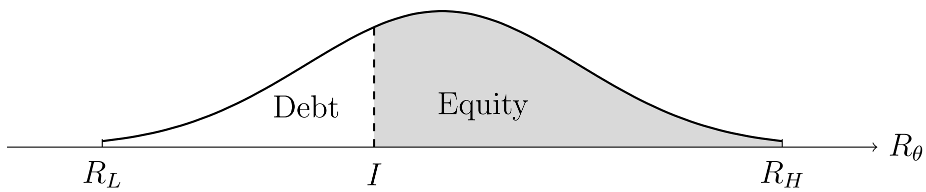


FIGURE 6: An illustration of Equilibrium 10.

Among the above two possibilities, only one of them can show up as the final solution. Intuitively, we can imagine that whether we obtain Equilibrium 9 or Equilibrium 10 depends on the distribution of R_θ and the amount of required investment I. For instance, if I is very large, then many firms have negative NPV projects. In this scenario, it is likely that investors require a very high interest rate. As a result, the potential benefit of issuing equity becomes larger to good firms and we are more likely to observe Equilibrium 10. Below Proposition 4 summarizes the equilibrium solution.

Proposition 4. *(Continuous-type model) In the presence of two projects, define R^* and r^{***} as the joint solution of the following two equations:*

$$R^* = I(1 + 2r^{***}) \tag{5}$$

$$Ir^{***} \int_{R^*}^{R_H} f(\theta) dR_\theta = \int_{R_L}^{I} f(\theta)(I - R_\theta) dR_\theta \tag{6}$$

Further, define I^* as the maximum I such that the solutions of R^* and r^{***} exist.[24] *Then we can find at least one distribution, such that there exists $I^* \in (R_L, \bar{R})$ and the following assertions hold:*

(i) If $I \in (R_L, I^*]$, then we obtain Equilibrium 9, and part of all good firms separate themselves by issuing equity.

(ii) If $I \in (I^*, \bar{R})$, then we obtain Equilibrium 10, and all firms with positive NPV projects issue equity.

Proof. (a) If we obtain Equilibrium 9, then type $R_\theta = R^*$ should be indifferent between issuing debt and equity. From payoffs of firms, we can set $R^* - I = 2R^* - 2I(1 + r^{***})$ when $\varepsilon \to 0$. This gives us Equation (5).

[24] In case of multiple solutions, choose the smallest positive value of r^{***}.

(b) Observe the participation constraint of investors:

$$\begin{aligned}\mathscr{L}_d^U\big|_{r^{***}} &= \int_{R_L}^{R_H} f(\theta|r^{***})\min[R_\theta - I, Ir^{***}]dR_\theta \\ &= \int_{R_L}^{I} f(\theta|r^{***})(R_\theta - I)dR_\theta + \int_{R^*}^{R_H} f(\theta|r^{***})Ir^{***}dR_\theta = 0\end{aligned}$$

Given posterior beliefs as in Equilibrium 9 we can obtain Equation (6).

(c) The example below proves the rest of this proposition. □

Example. Consider a uniform distribution $R_\theta \sim \text{Unif}[0, 2R]$, $f(\theta) = \frac{1}{2R}$. Here $\bar{R} = R$. To ease the notation I will let $\varepsilon \to 0$. Let us first consider Equilibrium 9. In this example, Equation (5) and Equation (6) can be calculated as follows:

$$R^* = I(1 + 2r^{***}) \tag{7}$$

$$r^{***}(2R - R^*) = \frac{1}{2}I \tag{8}$$

The above Equation (7) and Equation (8) have solutions whenever $I \leq I^* = \frac{2}{3}R$. Thus, if we select the smallest positive value for r^{***}, we can find that

$$\begin{aligned}R^* &= \frac{1}{2}[2R + I - \sqrt{(2R - 3I)(2R + I)}] \\ r^{***} &= \frac{1}{4I}[2R - I - \sqrt{(2R - 3I)(2R + I)}]\end{aligned}$$

A few detailed examples are listed below.

(a) Suppose $I = \frac{2}{3}R$, then $R^* = \frac{4}{3}R$ and $r^{***} = \frac{1}{2}$. The equilibrium looks like Figure 7.

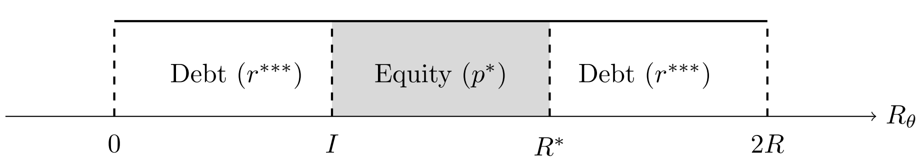


FIGURE 7: An illustration of Equilibrium in Example (a).

(b) Suppose $I = \frac{1}{2}R$, then $R^* = \frac{5-\sqrt{5}}{4}R \approx 0.691R$, and $r^{***} = \frac{3-\sqrt{5}}{4} \approx 0.191$. The equilibrium looks like Figure 8.

(c) Observe from Equation (7) and Equation (8) that whenever I increases, we have both r^{***} and R^* increase. Thus, whenever $I > I^*$, there does not exist a solution

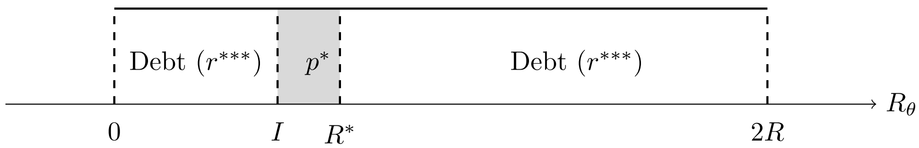


FIGURE 8: An illustration of Equilibrium in Example (b).

that can satisfy both equations.[25] Here we achieve the separating Equilibrium 10 as shown in Figure 9.

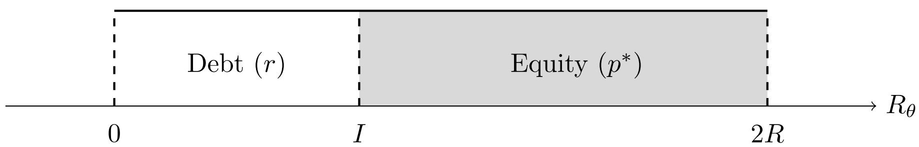


FIGURE 9: An illustration of Equilibrium in Example (c).

To summarize this example, we can see that $I^* = \frac{2}{3}R$, and

(i) If $I \in (0, I^*]$, we obtain Equilibrium 9. Investors' beliefs update according to: (i.1) If observe p^*, then $R_\theta \sim \text{Unif}[I, R^*]$, with $f(\theta|p^*) = \frac{1}{R^*-I}$. (i.2) If observe r^{***}, then $R_\theta \sim \text{Unif}[0, I) \cup [R^*, 2R]$, with $f(\theta|r^{***}) = \frac{1}{I+2R-R^*}$.

(ii) If $I \in (I^*, R)$, we obtain Equilibrium 10. Investors' beliefs update according to: (ii.1) If observe p^*, then $R_\theta \sim \text{Unif}[I, 2R]$, with $f(\theta|p^*) = \frac{1}{2R-I}$. (ii.2) If observe r, then $R_\theta \sim \text{Unif}[0, I)$, with $f(\theta|r) = \frac{1}{I}$. □

To conclude this section, we can see that in a two-stage model, asymmetric information between firms and investors no longer generates a strict preference of debt financing. Instead, at least part of the firms with positive NPV projects prefer to issue underpriced equity. Comparing with the two-type model in Section 2, this section shows that allowing the firm type to have a continuous distribution can dramatically enrich the model predictions. For example, the pattern shown in the above example can also be used to explain the equity issuance announcement effect. If we consider part (b) of this example, then announcing to issue equity will revise the investors' belief to "these firms have an average type around $0.6R$", which is smaller than the prior belief that the average type is R.

[25] To see this, suppose $I = R - \varepsilon \to R$, then in this example we can find the equilibrium interest rate in a one-stage model $r^{**} \to 1$. Thus, according to the argument at the beginning of this section, firms with $R_\theta \in (I, 2I)$ have enough incentive to switch to equity financing. Under this scenario, the remaining portion of the good firms, which is very small (e.g., $R_\theta \in [2R - 2\varepsilon, 2R]$), will be forced into choosing equity as well.

4 When investors can verify the true type

Parallel to the development of signaling games, the literature also explores the possibility that some investors can verify a firm's type. These investors can either be informed investors that costlessly know the true type (e.g., Kyle, 1985; Rock, 1986), special investors that can verify the type at a cost (e.g., Townsend, 1979; Chemmanur, 1993; Fulghieri and Lukin, 2001), or financial intermediaries like investment banks (Baron, 1982; Benveniste and Spindt, 1989; Chemmanur and Fulghieri, 1994). Following these prior developments, in this section I examine a variant of my previous model by considering investors being able to verify the firm's type.

Formally, suppose now uninformed investors can spend a cost c to verify the true type of the firm before deciding whether to participate or not. Denote the action of costly verification as C, and not verify as D. Figure 10 shows the sequence of moves.

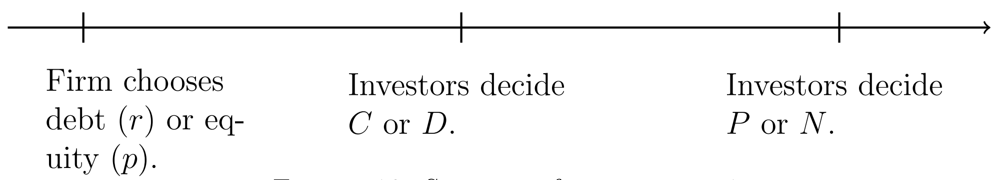


FIGURE 10: Sequence of moves at $t = 1$.

Thus, the augmented action for investors can be represented by $a_U \in \{CP, CN, DP, DN\}$. Clearly, investors don't have any incentive to finance firms with negative NPV projects once the type is verified, and the choice of $\{P, N\}$ depends on the outcome of verification. On the other hand, if investors decide not to verify the type (D), then the game reduces to our previous signaling game. From now on I focus on $a_U \in \{C, D\}$ whenever there is no confusion.

4.1 Two-type model

Let us first consider the case that a firm can be either a high type firm or a low type firm as in Section 2. For the low type firm, it mimics the high type firm whenever possible. For the high type firm, we can restrict its action space to $a_F \in \{r^*, \hat{r}, p^*, \hat{p}\}$, where r^* and p^* are defined in Section 2, and $\hat{r}$ and $\hat{p}$ are defined to be values that investors can ex ante break even if choosing $a_U = C$. Specifically, $\hat{r} = \frac{c}{\mu I}$ and

$\hat{p} = \frac{\mu I}{\mu I + c} R_H$.[26]

Apparently, the equilibrium also depends on the range of the cost c. Let us first focus on the range of $c < \mu(R_H - I)$. Under this scenario, the high type firm does not default under $\hat{r}$ (since $R_H - I > I\hat{r} = \frac{c}{\mu}$), and we also have $\hat{p} > p^*$.

To simplify the discussion, I first separate the choice between debt and equity. Suppose we restrict the high type firm's choice to debt financing. Then the following Lemma 1 summarizes the optimal choice for the firm and investors.

Lemma 1. *If we restrict $a_F(H) \in \{r^*, \hat{r}\}$ at $t = 1$, then*
(a) When $c \in [0, (1-\mu)(I - R_L)]$, $a_F^* = \hat{r}$, and $a_U^* = C$. Information of a firm's type is revealed at $t = 1$ by costly verification. Payoffs for firms are $\mathscr{L}_{dd}^F(H, \hat{r}, C) = 2R_H - 2I - I\hat{r}$, $\mathscr{L}_{dd}^F(L, \hat{r}, C) = 0$.
(b) When $c \in ((1-\mu)(I - R_L), \mu(R_H - I))$, $a_F^* = r^*$, and $a_U^* = D$. No information of type is revealed. Payoffs for firms are $\mathscr{L}_{dd}^F(H, r^*, D) = 2R_H - 2I(1 + r^*)$, $\mathscr{L}_{dd}^F(L, r^*, D) = 0$.

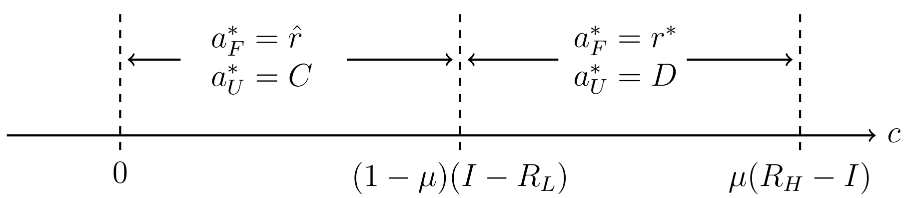


FIGURE 11: An illustration of Lemma 1.

Proof. Since the low type firm mimics the high type firm whenever possible, we only need to consider the incentive for the high type firm. If the high type firm chooses $a_F = \hat{r}$, then

$$\begin{aligned} \mathscr{L}^U(\hat{r}, C) \geq \mathscr{L}^U(\hat{r}, D) \quad &\Leftrightarrow \quad \mu I\hat{r} - c \geq (1-\mu)(R_L - I) + \mu I \hat{r} \\ &\Leftrightarrow \quad c \leq (1-\mu)(I - R_L) \end{aligned}$$

Given expressions of $\hat{r} = \frac{c}{\mu I}$ and $r^* = \frac{(1-\mu)(I-R_L)}{\mu I}$, we also have $\hat{r} \leq r^*$. Thus the firm can choose its preferred interest rate $\hat{r}$ and investors' best response is to verify the true type. Since the type is verified at $t = 1$, the high type firm can issue safe debt at $t = 2$.

[26] If $a_F = \hat{r}$, and $a_U = C$, then $\mathscr{L}^U = \mu I \hat{r} - c = 0$, which gives us the expression for $\hat{r}$. If $a_F = \hat{p}$, and $a_U = C$, then $\mathscr{L}^U = \mu(R_H \frac{I}{\hat{p}} - I) - c = 0$, which gives us the expression for $\hat{p}$.

Thus the total payoff for two projects is $\mathscr{L}^F_{dd}(H, \hat{r}, C) = R_H - I(1 + \hat{r}) + R_H - I = 2R_H - 2I - I\hat{r}$.

If $c > (1 - \mu)(I - R_L)$, then investors' best response to $\hat{r}$ is to choose $a_U = D$. If the firm chooses r^*, then $\mathscr{L}^U(r^*, C) = \mu I r^* - c < 0 = \mathscr{L}^U(r^*, D)$, investors' best response is to choose $a_U = D$. Since investors decide not to verify the true type, the firm will choose r^*, as we have $r^* < \hat{r}$. Here no information is revealed and the security issuance game at $t = 2$ repeats itself as in $t = 1$. □

Similarly, we can also restrict the high type firm's choice to equity financing, and the following Lemma 2 summarizes the optimal choice for the firm and investors.

Lemma 2. *If we restrict $a_F(H) \in \{p^*, \hat{p}\}$ at $t = 1$, then*
(a) When $c \in [0, \mu(\frac{I}{\bar{R}} R_H - I)]$, $a^*_F = \hat{p}$, and $a^*_U = C$. Information of type is revealed at $t = 1$ by costly verification. Payoffs for firms are $\mathscr{L}^F_{ed}(H, \hat{p}, C) = 2R_H - I - \frac{I}{\hat{p}} R_H$, $\mathscr{L}^F_{ed}(L, \hat{p}, C) = 0$.
(b) When $c \in (\mu(\frac{I}{\bar{R}} R_H - I), \mu(R_H - I))$, $a^*_F(H) = p^*$, $a^*_F(L) = r$, and $a^*_U = D$. Information of type is revealed through signaling. Payoffs for firms are $\mathscr{L}^F_{ed}(H, p^*, D) = R_H - I$, $\mathscr{L}^F_{ed}(L, r, D) = 0$.

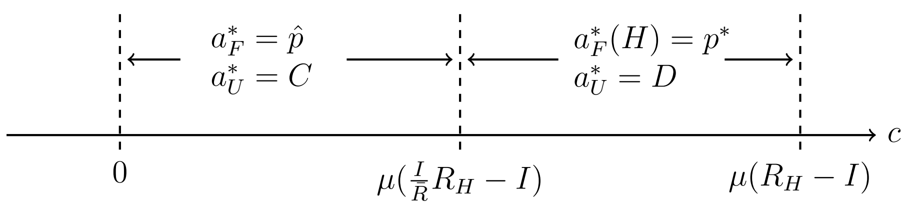


FIGURE 12: An illustration of Lemma 2.

Proof. Again we only need to consider the incentive for the high type firm, since the low type firm mimics the high type firm whenever possible. Note that $\hat{p} > p^*$ whenever $c < \mu(R_H - I)$. Thus the firm strictly prefers $\hat{p}$ whenever possible. If the high type firm chooses $a_F = \hat{p}$, then

$$\begin{aligned}
\mathscr{L}^U(\hat{p}, C) \geq \mathscr{L}^U(\hat{p}, D) \quad &\Leftrightarrow \quad 0 \geq \mu(R_H \frac{I}{\hat{p}} - I) + (1 - \mu)(R_L \frac{I}{\hat{p}} - I) \\
&\Leftrightarrow \quad \hat{p} \geq \bar{R} \\
&\Leftrightarrow \quad c \leq \mu(\frac{I}{\bar{R}} R_H - I)
\end{aligned}$$

Thus the firm can choose its preferred price $\hat{p}$ and investors' best response is to verify the true type.[27] Since the type is verified at $t = 1$, the high type firm can issue safe debt at $t = 2$. Thus the total payoff for two projects is $\mathscr{L}^F_{ed}(H, \hat{p}, C) = (1 - \frac{I}{\hat{p}})R_H + R_H - I$.

If $c > \mu(\frac{I}{R}R_H - I)$, investors' best response to $\hat{p}$ is to choose $a_U = D$. From the analysis in Section 2 we know that any pooling equilibria at $p > p^*$ cannot pass the Cho-Kreps Intuitive Criterion. Thus the firm can only choose p^*, and information of type is revealed at $t = 1$ through signaling. □

A few observations from the above Lemma 1 and Lemma 2 are: First, the high type firm is better off when facing more capable investors. Specifically, when investors can collect information at a smaller cost to verify the type, the high type firm can select a lower interest rate ($\hat{r}$ as in Lemma 1 part a) or a higher stock price ($\hat{p}$ as in Lemma 2 part a) to obtain a larger payoff.

Second, if μ is very high and investors have a strong belief that the firm is more likely to be a high type firm, then there is less incentive for investors to verify the type. For instance, in Lemma 1, the cutoff point $(1 - \mu)(I - R_L)$ is smaller whenever μ is higher.[28]

Third, from the expression of $\hat{p} = \frac{\mu I}{\mu I + c} R_H < R_H$ we know that the high type firm still has to issue underpriced equity whenever $c > 0$. However, since $\hat{p} > p^*$, the magnitude of this underpricing is much smaller comparing with the model in Section 2.

Fourth, investors stay uninformed only if the firm issues debt and the cost of verification is very high (or the prior belief sufficiently strong, i.e., μ is very high). In other words, debt financing at r^* is the only strategy that reveals no information. When the firm issues equity, investors become informed either through signaling or through costly verification. When the firm issues debt, investors can also become informed through costly verification.

Lastly, we can see that although in this modified model investors are more capable at selecting the high type firm, they are actually better off in terms of payoff if they can pretend to be unable of verifying the type. For instance, if we look at the model in

[27] Here the price $\hat{p}$ can pass the Cho-Kreps Intuitive Criterion because when investors' best response is to verify the type, the low type firm receives zero payoff when issuing equity. Thus, issuing equity is no longer a strictly better strategy than issuing debt given investors' best responses.

[28] Such relationship is not linear in Lemma 2.

Section 2, then investors can receive positive payoff when the high type firm chooses equity at p^* (Equilibrium 7). Yet in this section the region for a positive payoff becomes much smaller, as shown in Lemma 2. Thus, investors prefer to play the game in Section 2, but the high type firm would prefer to play the game in this section. Since the firm moves first, it naturally has a first-mover advantage. Here the advantage is that the firm can force investors to collect information and verify the type, for otherwise investors would incur a loss.

So far we have only considered the case in which the cost falls into region $c < \mu(R_H - I)$. For the region of $c \geq \mu(R_H - I)$, the choices for the firm revert back to $\{r^*, p^*\}$, as implied by Lemma 1 and Lemma 2. Thus, equilibrium results for $c \geq \mu(R_H - I)$ are similar to those in Proposition 2. For instance, we can consider the result in Proposition 2 is obtained by letting the cost $c \to +\infty$.

With Lemma 1 and Lemma 2 we can compute the high type firm's preference between all four choices $\{r^*, \hat{r}, p^*, \hat{p}\}$. Apparently, the best choice depends on the cost c and the prior belief μ. The following Proposition 5 summarizes the best choice by the high type firm.

Proposition 5. *(Two-type model) In the presence of two projects, and when investors can verify a firm's type at a cost c,*

(I) If $c \in [0, (1-\mu)(I - R_L)]$, then $a_F^*(H) \in \{\hat{r}, \hat{p}\}$.

(II) If $c \in \big((1-\mu)(I - R_L), \min[\mu(\frac{I}{\bar{R}}R_H - I), 2(1-\mu)(I - R_L)]\big]$, then $a_F^*(H) = \hat{p}$.

(III) If $c \in \big(\min[\mu(\frac{I}{\bar{R}}R_H - I), 2(1-\mu)(I - R_L)], +\infty\big)$ and $\mu \in (\mu_0, \mu^*)$, then $a_F^*(H) = p^*$.

(IV) If $c \in \big(\min[\mu(\frac{I}{\bar{R}}R_H - I), 2(1-\mu)(I - R_L)], +\infty\big)$ and $\mu \in (\mu^*, 1)$, then $a_F^*(H) = r^*$.

Proof. Considering Lemma 1 and Lemma 2, first we want to show the following:

$$
\begin{aligned}
& (1-\mu)(I - R_L) < \mu(\frac{I}{\bar{R}}R_H - I), \quad \forall \mu \in (\mu_0, 1) \\
\Leftrightarrow \quad & \mu(\frac{I}{\bar{R}}R_H - I) - (1-\mu)(I - R_L) > 0 \\
\Leftrightarrow \quad & \mu\frac{I}{\bar{R}}R_H - I + (1-\mu)R_L > 0 \\
\Leftrightarrow \quad & I\frac{\mu R_H - \bar{R}}{\bar{R}} + (1-\mu)R_L > 0 \\
\Leftrightarrow \quad & (1-\mu)R_L(1 - \frac{I}{\bar{R}}) > 0
\end{aligned}
$$

The last line follows naturally as $\bar{R} > I$. Thus we can remove the upper constraint on c, and combine Lemma 1 and Lemma 2 into the following Figure 13.

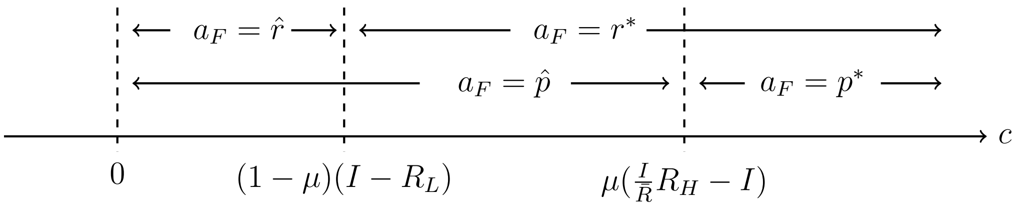


FIGURE 13: Proof of Proposition 5

From Figure 13 we can separate c into the three regions:

(a) If $c \in [0, (1-\mu)(I-R_L)]$, then the high type firm can choose in between $\hat{r}$ and $\hat{p}$. Since $\mathscr{L}^F(\hat{r}) = \mathscr{L}^F(\hat{p})$ can be shown as follows, the high type firm is indifferent between these two options.

$$\mathscr{L}^F(\hat{r}) = 2R_H - 2I - I\hat{r} = 2R_H - 2I - \frac{c}{\mu}$$
$$\mathscr{L}^F(\hat{p}) = 2R_H - I - \frac{I}{\hat{p}}R_H = 2R_H - I - \frac{IR_H}{\mu I R_H}(\mu I + c) = 2R_H - 2I - \frac{c}{\mu}$$

(b) If $c \in \big((1-\mu)(I-R_L), \mu(\frac{I}{R}R_H - I)\big]$, then the high type firm can choose in between r^* and $\hat{p}$. Since

$$\mathscr{L}^F(r^*) = 2R_H - 2I(1+r^*) = 2R_H - 2I - 2\frac{(1-\mu)(I-R_L)}{\mu}$$
$$\mathscr{L}^F(r^*) < \mathscr{L}^F(\hat{p}) \;\Leftrightarrow\; c < 2(1-\mu)(I-R_L)$$

Then if $\mu(\frac{I}{R}R_H - I) < 2(1-\mu)(I-R_L)$, the firm always chooses $\hat{p}$. If $\mu(\frac{I}{R}R_H - I) > 2(1-\mu)(I-R_L)$, then the firm chooses $\hat{p}$ when $c \in \big((1-\mu)(I-R_L), 2(1-\mu)(I-R_L)\big]$, chooses r^* when $c \in \big(2(1-\mu)(I-R_L), \mu(\frac{I}{R}R_H - I)\big]$

(c) If $c \in \big(\mu(\frac{I}{R}R_H - I), +\infty\big)$, then the high type firm can choose in between r^* and p^*. Here the result converges to the case as in Proposition 2. Thus the firm chooses p^* when $\mu \in (\mu_0, \mu^*)$, chooses r^* when $\mu \in (\mu^*, 1)$.

Combining the cases from (a) to (c) we obtain the four regions as shown in this Proposition. □

I use Figure 14 and Figure 15 as two examples to illustrate Proposition 5. Figure 14 shows the specification of $I = 4$, $R_L = 2$, and $R_H = 10$. Figure 15 shows the specification of $I = 4$, $R_L = 3.2$, and $R_H = 10$. Both these two figures draw the four regions summarized by Proposition 5.

To conclude this section, we can see that when investors can verify the firm's type, the region for a pooling equilibrium becomes much smaller. As shown in Proposition 5, among all four regions, only region (IV) supports the pooling at debt equilibrium. Thus, unless the cost of verification is very high and the prior belief is sufficiently strong, the high type firm can always reveal its type to investors. From Figure 14 and Figure 15 we can also see that in this model, there does not exist a strict preference for debt financing. In fact, equity financing can sometimes be the only best option for the high type firm, as shown in regions (II) and (III) in Proposition 5.

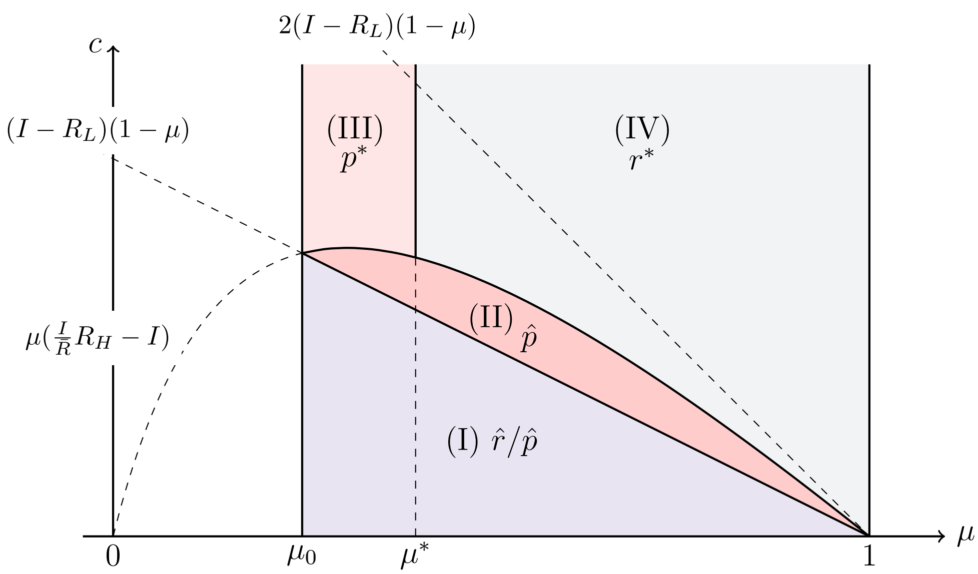


FIGURE 14: An illustration of Proposition 5.

4.2 Continuous-type model

With results from the above two-type model, let us consider the continuous-type specification. As shown in Section 3, the belief updating process is rather complicated. Thus, in this section I focus on debt and equity issuances at $t = 1$ separately. Presumably, the comparison between debt and equity would show a similar pattern as in the two-type model. In addition, since a very high cost c implies that the equilibrium converges to the case in Proposition 4, here I only discuss the case in which $c < \int_{R_L}^{I} f(\theta)(I - R_\theta)dR_\theta$.

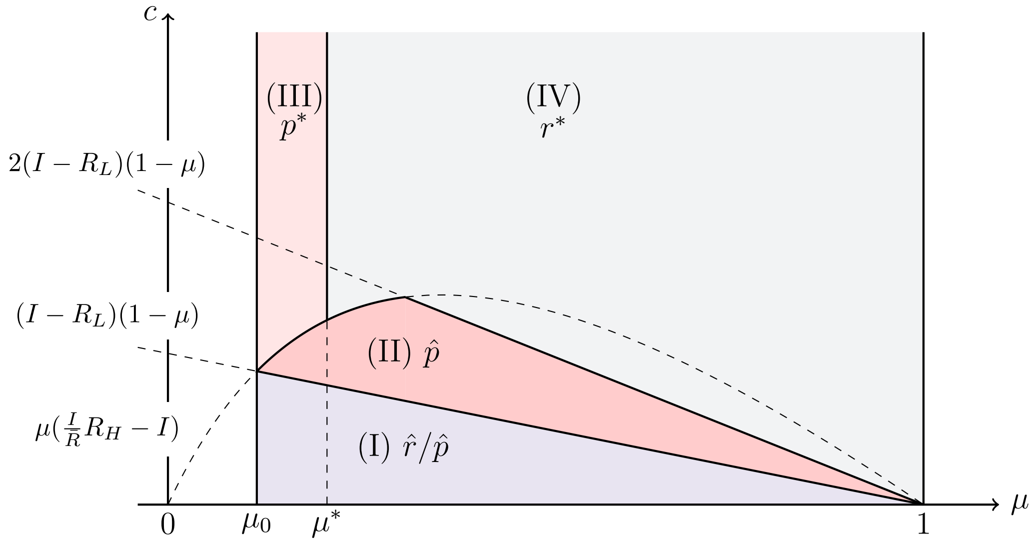


FIGURE 15: Another illustration of Proposition 5.

First, consider all types of firms issue debt at interest rate $r^\dagger$, which is defined as the following:

$$\mathscr{L}^U(r^\dagger, C) = \int_I^{R_H} f(\theta)\min[R_\theta - I, Ir^\dagger]dR_\theta \ - c = 0$$

Then the equilibrium can be summarized as in Equilibrium 11.

Equilibrium 11. ***(Debt only)*** *When $c < \int_{R_L}^{I} f(\theta)(I - R_\theta)dR_\theta$, the following strategy-belief combination constitutes an equilibrium.*

Firm's choice: (a) $a^*_{F,t=1}(\theta) = r^\dagger$, $\forall R_\theta \in [R_L, R_H]$. (b) Good firms issue safe debt at $t = 2$.

Investors choice: $a^*_{U,t=1}(r^\dagger) = C$, and participate only in issuances by good firms.

Investors' belief along the equilibrium path: $f(\theta|r^\dagger) = f(\theta)$.

Payoffs for the firm:

$$\mathscr{L}^F(\theta, r^\dagger, C) = \begin{cases} \max[0, R_\theta - I(1 + r^\dagger)] + R_\theta - I, & \textit{if } R_\theta \in [I, R_H] \\ 0, & \textit{if } R_\theta \in [R_L, I] \end{cases}.$$

Proof. With the definition of $r^\dagger$, we need the following for investors to choose to verify the type:

$$\mathscr{L}^U(r^\dagger, D) < \mathscr{L}^U(r^\dagger, C)$$

$$\Leftrightarrow \int_{R_L}^{I} f(\theta)(R_\theta - I)dR_\theta + \int_{I}^{R_H} f(\theta)\min[R_\theta - I, Ir^\dagger]dR_\theta < 0$$
$$\Leftrightarrow c < \int_{R_L}^{I} f(\theta)(I - R_\theta)dR_\theta$$

If a firm deviates to any $r < r^\dagger$, then investors' best response is to choose $a_U = DN$.[29] Comparing with Figure 4 we can see that, since $r^\dagger < r^{**}$, firms with types $R_\theta \in [I, I(1+r^{**})]$ no longer have incentives to issue underpriced equity at price $p^* - \varepsilon$ to reveal its type. □

The above Equilibrium 11 shows that firms can select an interest rate to incentivize investors to collect information when the cost of doing so is not very high. This prediction is in the same direction as in Lemma 1 in the two-type model.

Second, consider all types of firms issue some kinds of equity. Here we can imagine that since investors can verify the true type, some good firms should be able to issue equity at a price that is larger than p^* but smaller than their actually type R_θ. On the other hand, bad firms will also want to issue at a higher price to take advantage of the asymmetric information. Thus, the equilibrium has to require investors to respond with $a_U = C$ for any price $p > p^*$ in order to pass the Intuitive Criterion. I present one reasonable equilibrium as in Equilibrium 12. However, it is worth mentioning that such equilibrium is not unique. I will briefly explain other possible solutions after this presentation.

Equilibrium 12. ***(Equity only)*** *When $c < \int_{R_L}^{I} f(\theta)(I - R_\theta)dR_\theta$, the following strategy-belief combination constitutes an equilibrium.*

(i) Firm's choice:

$$\text{(a) } a^*_{F,t=1}(\theta) = \begin{cases} p^\dagger = \frac{\eta I}{\eta I + c} R_\theta, & \text{if } R_\theta \in [I + \frac{c}{\eta}, R_H] \\ p^* - \varepsilon, & \text{if } R_\theta \in [I, I + \frac{c}{\eta}] \\ \text{random } p \in [I, \frac{\eta I}{\eta I + c} R_H], & \text{if } R_\theta \in [R_L, I) \end{cases}.$$

(b) Good firms issue safe debt at $t = 2$.

(ii) Investors choice:

(a) $a^*_{U,t=1}(p^* - \varepsilon) = DP$, $a^*_{U,t=1}(p^\dagger/p) = C$ and participate only in issuances by good firms.

(b) $a^*_{U,t=2}(p^* - \varepsilon) = P$, and

[29] Here it is not necessary to specify off-equilibrium beliefs.

$$a^*_{U,t=2}(p^\dagger/p) = \begin{cases} P, & \text{if } R_\theta \in [I+\frac{c}{\eta}, R_H] \text{ and } \mathscr{L}^U_{t=1}(p^\dagger/p, C) \geq 0 \\ N, & \text{if } R_\theta \in [R_L, I] \text{ or } \mathscr{L}^U_{t=1}(p^\dagger/p, C) < 0 \end{cases}$$

(iii) Investors' belief along the equilibrium path:

$$\text{(a) } f(\theta|p^* - \varepsilon) = \begin{cases} \frac{f(\theta)}{\int_I^{I+\frac{c}{\eta}} f(\theta')dR_{\theta'}} & \text{if } R_\theta \in [I, I+\frac{c}{\eta}) \\ 0 & \text{if } R_\theta \in [R_L, I) \cup [I+\frac{c}{\eta}, R_H] \end{cases}$$

(b) If observe $p^\dagger/p$ (since investors cannot distinguish them), then investors believe the firm is of type $R_\theta = \frac{\eta I + c}{\eta I} p^\dagger$ with probability $\eta = \frac{\int_{I+\frac{c}{\eta}}^{R_H} f(\theta)dR_\theta}{1-\int_I^{I+\frac{c}{\eta}} f(\theta)dR_\theta}$, of type $R_\theta \in [R_L, I)$ with $f(\theta|p^\dagger/p) = \frac{f(\theta)}{1-\int_I^{I+\frac{c}{\eta}} f(\theta)dR_\theta}$.

(iv) Payoffs for the firm when $\varepsilon \to 0$:

$$\mathscr{L}^F = \begin{cases} 2R_\theta - 2I - \frac{c}{\eta}, & \text{if } R_\theta \in [I+\frac{c}{\eta}, R_H] \\ R_\theta - I & \text{if } R_\theta \in [I, I+\frac{c}{\eta}) \\ 0, & \text{if } R_\theta \in [R_L, I) \end{cases}.$$

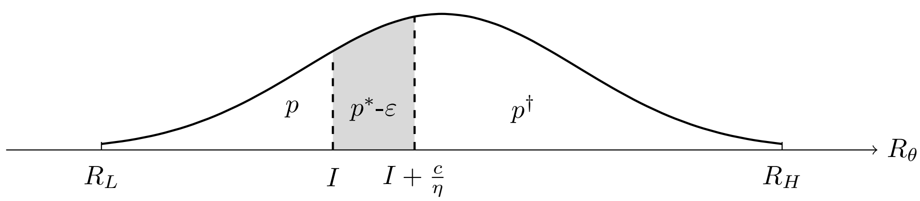


FIGURE 16: An illustration of Equilibrium 12.

Proof. Payoffs for investors at $t = 1$ are

$$\mathscr{L}^U_{t=1}(p^* - \varepsilon) = \int_I^{I+\frac{c}{\eta}} f(\theta|p^* - \varepsilon)(\frac{I}{I-\varepsilon}R_\theta - I)dR_\theta > 0$$

$$\mathscr{L}^U_{t=1}(p^\dagger/p, C) = \eta(\frac{I}{p^\dagger}R_\theta - I) - c = 0 \Leftrightarrow p^\dagger = \frac{\eta I}{\eta I + c}R_\theta$$

$$\mathscr{L}^U_{t=1}(p^\dagger/p, D) = \eta(\frac{I}{p^\dagger}R_\theta - I) + \int_{R_L}^I f(\theta|p^\dagger/p)(\frac{I}{p}R_\theta - I)dR_\theta < 0$$

$$\Leftrightarrow c < \int_{R_L}^I f(\theta|p^\dagger/p)(I - \frac{I}{p}R_\theta)dR_\theta$$

Since $f(\theta|p^\dagger/p) > f(\theta)$, $(I - \frac{I}{p}R_\theta) > (I - R_\theta)$ when $p > I$, then the above last line holds whenever $c < \int_{R_L}^I f(\theta)(I - R_\theta)dR_\theta$.[30] The rest of the proof is described in the

[30]Here we should also specify that $I + \frac{c}{\eta} < R_H \Leftrightarrow c < \eta(R_H - I)$. Yet this restriction is most likely

main context. $\square$

From Equilibrium 12 we can see that some good firms issue underpriced equity at $p^\dagger$. However, these good firms might have incentives to deviate to $p' = R_\theta$, since investors cannot tell the true type without verification and it is still rational for investors to invest after the verification (although investors would lose money ex ante, any price $p \leq R_\theta$ is acceptable ex post). This is why we need investors' strategy at $t = 2$ to be conditional on payoffs at $t = 1$. As shown in the above Equilibrium 12, investors can threaten firms by not participating at $t = 2$ if they lose money at $t = 1$.[31] Such strategy can prevent firms from deviating and is credible since investors earn zero payoffs at $t = 2$ whether they participate or not. Here we can see that the existence of the second stage is crucial for establishing Equilibrium 12, as a one-stage model cannot support $p^\dagger$.

One observation here is that allowing investors' strategy at $t = 1$ to be conditional on the payoffs at $t = 1$ can also be the potential source for alternative equilibrium solutions. For instance, investors can demand firms to further lower the price at $t = 1$, and threaten firms by not participating at $t = 2$ if firms refuse to do so. Firms, on the other hand, might be able to ignore these threats but will have to come up with a different offer at $t = 2$ to provide incentives for investors to participate. This multiplicity of equilibrium problem is most severe in the continuous-type model, since a continuum of type distribution provides more "space" for firms to deviate from underpricing their equity and for investors to imagine the true type. Comparing with the two-type model, we can see that although the two-type model may not be completely immune to the multiple equilibria problem, it can be largely exempt from it.

To conclude this section, we can see that firms are better off when investors are able to verify the type before deciding whether to participate in any investment. From the firm's perspective, whether to use debt or equity financing depends on investors' priors as well as the cost of verification. Typically, there does not exist a strict pecking order of financing. These predictions hold regardless of whether we employ a two-type setting or a continuous-type setting.

to be weaker than $c < \int_{R_L}^{I} f(\theta)(I - R_\theta)dR_\theta$. For instance, if we let $R_H \to +\infty$, then $c < \eta(R_H - I)$ is most likely true. I abbreviate this restriction to simplify the discussion.

[31] $\mathscr{L}^F(p^\dagger) = (1 - \frac{I}{p^\dagger})R_\theta + (R_\theta - I) > (R_\theta - I) = \mathscr{L}^F(p')$

5 Relations with Empirical Research

First, this paper can be related to the IPO and SEO literature. Empirical evidence of underpricing in IPOs and SEOs can be traced back to the early 1970s (e.g., Ibbotson, 1975; Smith, 1977; and Ritter, 1984). Since then, the motivation for issuing underpriced equity has been a puzzle to academics. The results from previous sections provide additional support to the argument that the IPO underpricing and the subsequent SEO announcement effect are due to the asymmetric information problem between firms and investors.[32,33]

Although the IPO and SEO literature can be developed separately without the debt market, many empirical findings suggest that there exists cross impact between debt and equity financing. For instance, Pagano, Panetta, and Zingales (1998) examine a panel of Italian firms and find that IPOs are followed by lower costs of credit. Kisgen (2006) shows that firms near a credit rating change are more likely to issue equity than debt. At the aggregate level, macro evidence (e.g., Erel et al., 2012) also shows that leverage changes according to the fluctuation of the cost of equity and cost of debt. Thus, my paper is also related to the vast majority of empirical capital structure papers that investigate the leverage determinants (e.g., Rajan and Zingales, 1995).

For example, to test the importance of adverse selection, a significant amount of empirical studies have been focusing on how financial deficit predicts leverage dynamics. Starting from Shyam-Sunder and Myers (1999), who argue that the pecking order is a good descriptor, the literature has arrived at different conclusions. For instance, the sample size and time period can lead to a disagreement (Frank and Goyal, 2003), ignoring debt capacity can reduce the testing power (Lemmon and Zender, 2010; Leary and Roberts, 2010), shocks and adjustment costs can make firms deviating from their desired capital structure (Leary and Roberts, 2005), and simulations from structural models (Strebulaev, 2007) or random assignments (Chang and Dasgupta, 2009) can produce testing results largely comparable with real data.

[32] For instance, Rock (1986) considers an information asymmetry model and proposes that issuing equity will suffer from the underpricing problem due to the additional compensation required by uninformed investors. A comprehensive review of IPO motives can be find in Ritter and Welch (2002).

[33] Historically, the SEO announcement return has been used as a proxy for the adverse selection cost (Choe, Masulis, and Nanda, 1993). Alternative determinants for the SEO announcement effect can be seen in, e.g., Kim and Purnanandam (2013).

As mentioned by Graham and Leary (2011), an important and still debatable issue in the capital structure literature is the relative importance of different market frictions, e.g., tax, bankruptcy cost, information asymmetry, and agency conflict. While the past two decades have extensively studied different dynamic versions of the tradeoff theory, tests of a dynamic version of the pecking order theory have encountered various difficulties (e.g., Frank and Goyal, 2008). Although this paper does not aim to empirically differentiate the adverse selection cost versus other frictions, my model suggests that testing the financing hierarchy may not be a reliable method, and perhaps it is better for researchers to find more direct tests of asymmetric information. To the minimum, any tests on the debt-equity preference should consider (or be conditional on) certain intertemporal features.

6 Equilibrium refinements

In this section I discuss alternative equilibrium refinement concepts and how they relate to the Mimicking test proposed in Section 2.

First, there can be two different logics when it comes to introducing an error term (e.g., ϵ) in the decision makers' actions. The first logic follows the contribution of Selten (1975). Specifically, the concept of trembling-hand perfection requires that a player imagines the opponent makes errors by mistake. Proper equilibrium (Myerson, 1978) further revises the concept by requiring players to put less weight on the opponent's actions that are less likely to be made by an error.

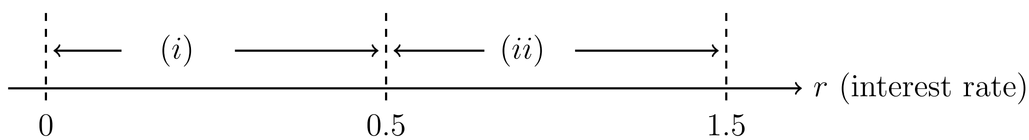
The second type of refinement stresses that all players' choices should be made by a meaningful reasoning. Forward induction (Kohlberg and Mertens, 1986), for instance, specifically focuses on if a node is reached then the receiver of the message is not supposed to consider that such action is done randomly, which makes it in contrast to the first type of refinement logic. I focus my following discussion on two different refinement concepts: the D1 refinement (Cho and Kreps, 1987, Banks and Sobel, 1987), which is now commonly used by many papers on security design (e.g., Nachman and Noe, 1994), and the undefeated equilibria (Mailaith, Okuno-Fujiwara, and Postlewaite, 1993).

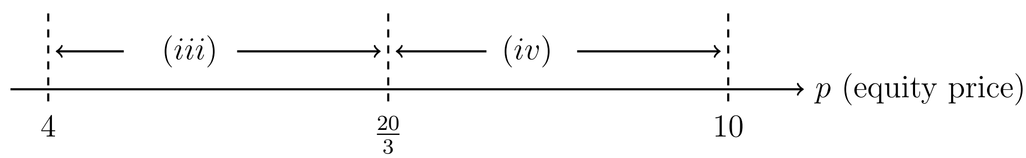
6.1 The D1 refinement

In this section I discuss some problems that arise when applying the D1 refinement to the basic one-stage signaling game in Section 2. w.l.o.g., consider the following example: $I = 4$, $R_L = 2$, $R_H = 10$, and the prior belief $\mu = \frac{1}{2}$.

(1) In Equilibrium 1, we can find that the equilibrium interest rate $r^* = 0.5$, and the payoff for the high type firm is $\mathscr{L}_d^F(H) = 4$. The equilibrium belief requires that whenever investors see an off-equilibrium message, the message is more likely to come from a low type firm, $\mu_0 = 0.25$.

First, if we only consider pure strategies by investors, then one can separate the firm's available strategies into four regions as shown below:





Here the equity price cutoff point is the price that makes the high type firm indifferent between choosing debt or equity. In the four regions, the high type firm is strictly better off in (i) and (iv), whereas the low type firm is strictly better off in (iii) and (iv), and weakly better off in (i) and (ii). Thus, the first difficulty of finding who has more incentive to deviate is that the strictly better off strategies for different types may not be inclusive in each other.

Let us further consider a mixed strategy response by investors. Suppose investors randomize between participate (P) and not participate (N). Then the low type firm's incentive does not change. For the high type firm, the strictly better off regions are reduced because of the mixed strategy response. Thus, it is also likely that under certain conditions, the high type firm no longer has strictly better off deviations. Then one can argue that applying the D1 refinement requires us to specify that whenever investors see an off-equilibrium message, the message comes for sure from the low type firm.

(2) In Equillibrium 4, the high type firm issues debt at zero interest rate, whereas the low type firm issues equity at any price. The equilibrium belief requires that whenever investors see an off-equilibrium message, the message is more likely to come from a low type firm.

In this equilibrium, there's no strategy that can make the high type firm strictly better off. For the low type firm, it can be better off if investors respond with participation (pure or mixed strategy). Thus, the set of high type firm being indifferent is a single point (with fairly priced equity at 10), and this set is certainly a subset of the low type firm's strictly better off set. If applying the D1 refinement, then one can struck out the high type firm for one specific strategy. Here we have two problems. One is that we can at most revise the off-equilibrium beliefs for a small subset of available strategies. The second problem is that even if we revise the belief, the revised belief does not invalidate the equilibrium. Thus, applying D1 refinement here cannot truly refine away Equilibrium 4.

6.2 Undefeated Equilibria

As illustrated above, refinements such as the Intuitive Criterion, D1, or Divinity are built on the idea of investigating the set of types that are more likely to send the off-equilibrium messages. The subsequent literature also points out potential logical inconsistencies that can arise with these refinements. More specifically, the criticism argues that if the sender can reason in the same way as the receiver and further alters his behavior, then the receiver should further updates his belief, which can result in a continuum of belief and action updating process that yields no clear prediction of the equilibrium outcome. On the other hand, applying some refinements can also rule out potential outcomes that are more Pareto efficient.

For example, consider the example used by Mailaith, Okuno-Fujiwara, and Postlewaite (1993) of the Spence's (1973) job market signaling model. The worker derives utility as $u_t(w, e) = w - e/t$, with $t \in \{1, 2\}$ denotes the type, w denotes the wage offered by the firm, and e denotes the education level chosen by the worker. The problem related with applying refinements can arise depending on the prior belief (p) of the worker's type. For instance, the game has a pooling equilibrium in which all workers choose zero education $\tilde{e} = 0$. This equilibrium is ruled out by the Intuitive Criterion when the prior belief (p) is reasonably large. The first issue with inconsistency is that if the worker anticipates that the firm can correctly interpret the off-equilibrium message, then the worker should indeed send the off-equilibrium message. Then if the firm does not observe the off-equilibrium message, the firm should update and consider the pooling outcome being a message send only by the low type worker, which end up upsetting the logic of the initial refinement. The second issue with Pareto efficiency is that when the prior belief (p) is small, the pooling payoffs to workers are higher than the separating equilibrium (Riley outcome, Riley 1979) selected under belief-based refinements. Applying refinements prevent us from studying the possibly more reasonable outcome.

The idea of undefeated equilibria (Mailaith, Okuno-Fujiwara, and Postlewaite, 1993) is to require an equilibrium's beliefs at off-equilibrium messages to be consistent with the on-the-equilibrium beliefs of another equilibrium that Pareto dominates the original equilibrium for these messages. For instance, in the above example, pooling equilibrium with zero education $\tilde{e} = 0$ defeats all other pooling equilibria with $\tilde{e} > 0$ because first, $\tilde{e} = 0$ gives strictly higher payoffs to workers, and second, the later equilibrium requires the firm to believe that $\tilde{e} = 0$ is possibly sent by the low productivity worker, which is not consistent with the beliefs in the $\tilde{e} = 0$ equilibrium.

By construction the concept of undefeated equilibria has a tendency to favor the more Pareto optimal outcome. While this tendency was not an issue in the Spence's signaling model, applying the concept to the model in Section 2 turns out to be quite problematic. If we look at all the five equilibria, i.e., Equilibrium 1 to Equilibrium 5, then Equilibrium 1 can defeat Equilibrium 2 because the equilibrium message r^* gives better payoffs to the issuer and is not sent in Equilibrium 2, the off-equilibrium belief in Equilibrium 2 for r^* is certainly inconsistent with the beliefs in Equilibrium 1. Similarly, Equilibrium 1 can also defeat all separating equilibria in Equilibrium 3 except for the case of $r = r^*$. However, Equilibrium 1 is also defeated by Equilibrium 4 and Equilibrium 5 at the same time. Thus, applying the refinement rules out the most intuitively appealing Equilibrium 1 and leaves us some of the least preferred equilibria such as Equilibrium 4 and Equilibrium 5.

6.3 The Mimicking test

Fundamentally, the game in Section 2 does not satisfy the usual single-crossing property of many signaling games that were studied in the literature. When there are two different classes of signals, i.e., debt and equity, the continuity feature is lost when it comes to the comparison between these two classes. As shown above, we can still apply many existing refinements, but the outcome often turns out to be less desired. I do not claim that one can never find some combinations of existing refinements from the literature that can deliver the outcome. Rather, I believe that it is worthwhile to propose the Mimicking test since it works particularly efficient in the game of this paper.

7 Conclusion

In this paper I develop a two-stage signaling model to describe corporate financing and investment decisions. In each stage the firm faces an investment project that requires outside financing. Equity issuances are accompanied by higher adverse selection costs when there exists information asymmetry between the firm and the security market. Thus, if one stage is viewed separately, the classical pecking order theory holds. However, security issuances are not independent events in a firm's history that has no impact on its future cost of capital. I show that when allowing a firm's behavior to change investors' beliefs, a two-stage model no longer yields a strict preference for debt financing in equilibrium. The firm has incentive to issue underpriced equity if the benefit from the future project can outweigh the current adverse selection cost.

The model in this paper can also be considered as a special application of signaling games with infinite sets of signals and actions. I discuss some problems that arise when applying existing refinements to this security issuance game, and introduce a refinement concept, Mimicking test, that limits the acceptable set of separating equilibria. Technically, a formal comparison between different type settings, i.e., two-type versus continuous-type, highlights their pros and cons. Lastly, I briefly discuss the model's implications to empirical research.

Appendix A Extension

In this section I discuss a simple extension of the one-stage game in Section 2 to incorporate the effect of initial wealth. Suppose with probability λ the firm has an initial wealth of $w_0 < I$, and with probability $1-\lambda$ the firm has an initial wealth of 0. Further, assume λ is unrelated to the prior belief of the type of the projects μ. In other words, suppose whether the firm has initial wealth or not is uncorrelated with the quality of its projects. Effectively, there are four potential types of firms, as shown in the below figure.

	R_L	R_H
w_0	$(1-\mu)\lambda$	$\mu\lambda$
0	$(1-\mu)(1-\lambda)$	$\mu(1-\lambda)$

FIGURE A1: Probability of Types

If the firm has an initial wealth, then the firm can invest w_0 into its project and requires $I - w_0$ outside financing. For example, if the firm issues equity, then the return for the firm can be revised into:

$$\mathscr{L}_e^F(\theta) = (1-\alpha)R_\theta - w_0 = (1 - \frac{I - w_0}{p})R_\theta - w_0$$

If the firm issues debt, then the return for the firm is the following:

$$\mathscr{L}_d^F(\theta) = \max[0, R_\theta - (I - w_0)(1+r)] - w_0$$

Observe that in this extension, type R_L has no incentive to invest its own wealth and issue debt, since $R_L - (I - w_0)(1+r) - w_0 = (R_L - I) + (w_0 - I)r < 0$. Thus, the firm with type (w_0, R_H) can issue fairly priced debt by having skin in the game. As a result, type (w_0, R_H) is surely separated from the remaining three types.

If the remaining three types (w_0, R_L), $(0, R_L)$, and $(0, R_H)$ play the security issuing game, then type (w_0, R_L) is constrained by not investing its own money, for otherwise having skin in the game will reveal its type to investors. As a result, the simple extension here is reduced to the remaining three types (w_0, R_L), $(0, R_L)$, and $(0, R_H)$ playing the one-stage game described in Section 2 with an updated prior belief.

Effectively, the impact of having a randomized initial wealth is to lower investors' prior belief that the firm's project is a high quality project. Potential market failure might arise depending on the parameter λ.

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