# Optimal Disclosure in All-pay Auctions with Interdependent Valuations*

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**Abstract**

We study all-pay auctions with one-sided private information and interdependent valuations. To sharpen the competition and maximize revenue, the auction organizer can design an information disclosure policy through Bayesian persuasion about the bidder with private information. Depending on the bidders' relative strengths and the degree of valuation dependence, the revenue-maximizing disclosure policies can take the form of partial disclosure, full disclosure, or no disclosure. We also show that relative to the no-disclosure benchmark, optimal information disclosure can sometimes improve allocative efficiency, but will always hurt the bidders' total welfare in the resulting all-pay auction.

**Keywords**: All-Pay Auction, Contest, Information Disclosure, Bayesian Persuasion, Stochastic Valuations. **JEL classification**: C72, D44, D82.

## 1 Introduction

All-pay auctions have been widely used in economics to model competitions or conflicts where all contestants invest scarce resources to compete for some valuable prizes. Examples of such competitions include sports, lobbying, R&D races, political campaigns, college admissions, and job promotions. Often, contestants in such real-world competitions are heterogeneous in terms of valuation and information, which can lead to undesirable outcomes, such as discouraged contestants and overall low effort or bids in competition. In

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this paper, we analyze the role of information disclosure for contest organizers in all-pay auctions with asymmetric information.

Information disclosure is a subtle but nevertheless natural tool for contest organizers to level the playing field and sharpen competition among contestants: An organizer can strategically disclose information about the contestants publicly so as to influence the contestants' beliefs about their opponents, which further affect their incentives and behavior in the contest. Indeed, strategic information disclosure is common in real-world competitions. For example, in job promotions, companies can decide whether to reveal information about external candidates to internal candidates. In the US, policy makers can determine lobbying transparency levels by requiring lobbyists to provide financial information, which can reveal information about lobbyists' private interests. As yet another example, the Federal Election Campaign Act requires political candidates to reveal the sources of campaign contributions and report campaign expenditures, which convey information about the extent of a candidate's financial support (see Denter et al., 2014[3]).

We consider a standard two-bidder all-pay auction with one-sided incomplete information and interdependent valuations. Each bidder observes a signal or type, with bidder $A$'s type being private, while bidder $B$'s type being public, and a bidder's valuation of the prize is a linear combination of both bidders' types, which subsumes independent value and common value as polar special cases.

Information disclosure in the all-pay auction is modelled as a disclosure policy that the organizer publicly commits to before bidding, which reveals bidder $A$'s type through Bayesian persuasion as in Kamenica and Gentzkow (2011)[7].[1] Formally, a disclosure policy is a signalling device, which draws random signals conditionally on bidder $A$'s realized type. Upon observing the realized public signal from the signalling device, bidder $B$ updates his belief about $A$'s type, and the bidders then compete in bidding. An optimal disclosure policy elicits the highest total expected (equilibrium) bids from the bidders among all possible disclosure policies.

We first analyze information disclosure in a setting where bidder $A$'s type is binary. Given that our setting presents valuation asymmetry, information asymmetry, and valuation interdependence, the optimal disclosure policy can feature full disclosure, partial disclosure, or no disclosure, depending on the relative strengths and valuation dependence between the bidders. The intuition is that for fixed valuation dependence, the organizer

[1]For example, under full information disclosure, bidder 1's ability becomes common knowledge, while under no information disclosure, bidder $B$'s belief on bidder $A$'s type is the prior belief. The formulation in Kamenica and Gentzkow (2011)[7] enables us to go beyond the above two extreme disclosure policies and consider a more general set of disclosure policies, as we will see.

tries to intensify bidding by revealing a strong bidder $A$'s type, hiding a weak bidder $A$'s type, or partially disclosing bidder $A$'s type to make bidder $B$ believe that bidder $A$ has the same ex ante expected valuation *as much as possible* when bidder $A$'s valuation can be on either side of bidder $B$'s valuation.[2] On the other hand, everything else fixed, when the degree of valuation dependence increases, the bidders' valuations converge, making asymmetric information the dominant impediment to competition. As a result, the optimal disclosure policy tends to transition from partial disclosure to full disclosure, as the bidders' valuations become more interdependent.

We then investigate the effects of optimal information disclosure on allocative efficiency, as well as the bidders' welfare, in the resulting all-pay auction. We find that optimal information disclosure has opposite welfare effects on the bidders. In addition, somewhat surprisingly, optimal information disclosure can sometimes improve allocative efficiency, but the benefits from improved allocative efficiency are only captured by the organizer. Our results here offer some useful implications for regulating information disclosure by organizers in competitions similar to our all-pay auction setting.

Finally, we analyze optimal information disclosure in a general setting with arbitrary finite types. While information disclosure is naturally much more complex in the general setting, we find that as in the binary-type setting, it is optimal for the organizer to construct public (posterior) signals by pooling *at most* two types of bidder $A$ together. Such a somewhat surprising result greatly simplifies the search for optimal disclosure policies in the general setting, and many important intuitions and features of optimal information disclosure in the binary-type setting are also carried over to the general-type setting.

**Literature Review.** Our paper is related and contributes to the contest literature that examines the effects of information disclosure, where most of the existing results focus on comparisons between full disclosure and no disclosure.[3] Morath and Münster (2008)[12] and Fu et al. (2014)[5] consider all-pay auctions with independent and private values and find that fully concealing bidders' types renders a higher expected revenue than fully revealing the information. Lu et al. (2018)[9] investigates deterministic public disclosure policies, which are contingent on bidders' realized types, in an all-pay auction, and find that no disclosure generates the highest expected revenue, a result that is consistent with those in Morath and Münster (2008)[12] and Fu et al. (2014)[5].

[2] By saying bidder $A$ has the same ex ante expected valuation as bidder $B$, we mean that the expected relative valuation of the bidders, as defined in (1) in Section 2, is 1.

[3] There is also a literature on disclosing the number of bidders in contests with stochastic entry (e.g., Chen et al., 2017[2]; Fu et al., 2011[4]; Lim and Matros, 2009[8]).

Unlike the above papers, we identify optimal disclosure policies among the entire (rather than a limited) set of disclosure policies. In this sense, our paper is more closely related to Zhang and Zhou (2016)[15], which identifies optimal disclosure policies among all possible policies, but in a Tullock contest. Given the difference in the discriminatory power of the two contest formats, our optimal disclosure rules are drastically different from that in Zhang and Zhou (2016)[15]. We will relate our results to Zhang and Zhou (2016)[15] in more detail in Section 3.3.3.

Finally, from the perspective of interdependent valuations, our paper is also related to studies on information structures in common-value first price auctions (Milgrom and Weber (1982)[11]) and information disclosure in common-value contests (Wärneryd (2003)[14]). Though our (all-pay auction) setting is more stylized, we consider more general disclosure policies as well as the whole spectrum of valuation interdependence. As a result, we are able to extend and complement the insights in Milgrom and Weber (1982)[11] and Wärneryd (2003)[14] in various directions, which again will be detailed in Section 3.3.3.

## 2 The Model

Consider an all-pay auction with two risk-neutral bidders, $A$ and $B$, and one indivisible prize. Prior to bidding, bidder $A$ observes a private type $t_A$ about the value of the prize, $t_A \in \Omega := \{\tau_0, \tau_1, \dots, \tau_N\}$ where $\Omega$ is a finite set with $0 < \tau_0 < \cdots < \tau_N$, $N \geq 1$, while bidder $B$ observes a type that is publicly known and normalized to $t_B = 1$.[4] Let $\Delta^N = \{\mathbf{p} \in \mathbb{R}^{N+1} | p_k \geq 0, \sum_{k=0}^{N} p_k = 1\}$ be the standard $N$-simplex in $\mathbb{R}^{N+1}$ and $int(\Delta^N)$ be the interior of $\Delta^N$. The commonly known prior distribution of $t_A$ is denoted as $\mathbf{p}^0 = \{p_0^0, \dots, p_N^0\}$ and $\mathbf{p}^0 \in int(\Delta^N)$. After observing their respective types, the bidders submit non-negative bids simultaneously and independently. The highest bid wins the prize, but both bidders pay their respective bids.

**Interdependent Valuations.** Denote a bid profile as $(b_i, b_{-i})$, with $b_i, b_{-i} \in \mathbb{R}_+$ and $i \in \{A, B\}$. Given $(b_i, b_{-i})$, it is common knowledge that bidder $i$'s ex post utility is:

$$u^i(b_i, b_{-i}) = \begin{cases} V^i(t_i, t_{-i}) - b_i & \text{if } b_i > b_{-i} \\ -b_i & \text{if } b_i < b_{-i} \\ \frac{V^i(t_i, t_{-i})}{2} - b_i & \text{if } b_i = b_{-i} \end{cases}.$$

[4] Normalizing bidder $B$'s type simplifies notation and is without loss of generality here, since we do not impose (qualitative) restrictions on the set of bidder $A$'s types, $\Omega = \{\tau_0, \dots, \tau_n\}$.

where $V^i(t_i, t_{-i}) := t_i + \alpha t_{-i}$, with $\alpha \in [0, 1]$. Here, the interdependence parameter $\alpha$ measures the degree to which bidder $i$'s valuation is interdependent with bidder $-i$'s type. This is a familiar interdependence specification (see, e.g., Jehiel and Moldovanu (2001)[6]) and $\alpha$ can be interpreted as, for example, the strength of informational or allocative externalities associated with the all-pay auction setting. Such a specification simplifies equilibrium characterization and more importantly, facilitates a comparative statics analysis in our setting.[5] Under this specification, the bidders have pure independent values when $\alpha = 0$, whereas they have common values when $\alpha = 1$.

To ease notation for equilibrium characterization, given a realized type $\tau_k$, $k = 0, ..., N$, let $V_k^A$ and $V_k^B$ denote, respectively, bidder $A$'s and bidder $B$'s valuations of winning, $v_k$ denote the relative valuation of the bidders, and $\mathbb{E}_{\mathbf{p}}[v]$ the expected relative valuation under distribution $\mathbf{p}$, i.e.,

$$V_k^A \equiv \tau_k + \alpha, V_k^B \equiv 1 + \alpha\tau_k, v_k \equiv V_k^B / V_k^A, \mathbb{E}_{\mathbf{p}}[v] \equiv \textstyle\sum_{k=0}^{N} p_k v_k. \tag{1}$$

Notice that for $\alpha < 1$, $v_k < 1$ (resp., $v_k > 1$) if and only if $\tau_k > 1$ (resp., $\tau_k < 1$), and bidder $A$ has a higher relative valuation *ex ante* if $\mathbb{E}_{\mathbf{p}}[v] < 1$.

**Information Disclosure.** Before the all-pay auction starts, the organizer can choose an information disclosure policy to improve his expected revenue, i.e., he can strategically disclose information about bidder $A$'s type by publicly committing to a disclosure policy before the bidding begins. To allow for a general set of disclosure policies, we model the organizer's information disclosure using Bayesian persuasion (Kamenica and Gentzkow 2011[7]).

To be specific, the organizer can choose a disclosure policy that generates public and probabilistic messages, called signals, conditionally on bidder $A$'s realized type. Formally, a disclosure policy is a pair $(S, \pi)$ consisting of a finite set of signals $S$ and $(N + 1) \cdot |S|$ conditional distributions $\pi(s|t_A)$, $s \in S$. A policy $(S, \pi)$ induces a public posterior belief $\mathbf{p}_s$ for bidder $B$ over bidder $A$'s type for each $s$:

$$\mathbf{p}_s(\tau_k) = p_k = \frac{p_k^0 \pi(s|\tau_k)}{\sum_{j=0}^{N} p_j^0 \pi(s|\tau_j)}. \tag{2}$$

The timing of the interaction is as follows. The organizer first commits to a policy

[5] For equilibrium characterization (with or without information disclosure), it is possible to show that our results do **not** change qualitatively if we consider a general function $V^i(t_i, t_{-i}; \alpha)$ with regularity conditions that $V^i$ is strictly increasing in $t_i$, $t_{-i}$, $\alpha$, and $V^i(t_i, t_{-i}; 0) = t_i$, $V^i(t_i, t_{-i}; 1) = t_i + t_{-i}$.

$(S, \pi)$ publicly. Next bidder $A$'s type $t_A$ is realized according to the prior distribution $\mathbf{p^0}$, which is observed only by bidder $A$. A signal $s \in S$ is then (randomly) generated according to the distribution $\pi(\cdot|t_A)$, which is observed by both bidders. Bidder $B$ forms posterior belief $\mathbf{p}_s$ and then competes with bidder $A$ in bidding. The organizer's objective is to choose an optimal disclosure policy to maximize the total expected revenue from the bidders, through the channel of affecting the (public) posterior of bidder $B$.

While finding an optimal $(S, \pi)$ may come off as unmanageable given seemingly unlimited possibilities, it has been demonstrated in Kamenica and Gentzkow (2011)[7] that public disclosure policies are actually in one-to-one correspondence with distributions of posteriors with supports in the simplex $\Delta^N$ that have expectation $\mathbf{p}^0$. In other words, a policy $(S, \pi)$ corresponds to a convex decomposition

$$\sum_{s \in S} \beta_s \mathbf{p}_s = \mathbf{p}^0 \tag{3}$$

where $\mathbf{p}_s \in \Delta^N$ is, as mentioned, the posterior of bidder $A$'s type conditional on signal $s$ and $\beta_s = \sum_{j=0}^{N} p_j^0 \pi(s|\tau_j)$ is the probability of $s$ with $\sum_{s \in S} \beta_s = 1$. Hence, finding a disclosure policy is equivalent to finding a probability distribution $\tau \in \Delta\left(\Delta^N\right)$ over posteriors, which is parameterized by $\mathbf{p}_s$ and $\beta_s$, and satisfies the *Bayes-plausibility* condition (3). In addition, an application of Carathéodory theorem implies that it suffices to focus on a relatively small set of (*straightforward*) signals, $|S| = |\Omega| = N + 1$. Due to the equivalence between a disclosure policy and a distribution over posteriors, the key element of a disclosure policy is the posteriors $\mathbf{p}_s$ with $\beta_s > 0$ and the signals themselves can be described implicitly—indeed, at least in describing the disclosure policy, one can regard a posterior $\mathbf{p}_s$ as if it were the signal $s$ itself, which is what we will do in the sequel.

# 3 Binary Types

We first consider the setting of binary types, i.e., bidder $A$'s type is drawn from $\{\tau_1, \tau_2\}$, where $0 < \tau_1 < \tau_2$. We call $\tau_1$ the *low* type and $\tau_2$ the *high* type. We first solve for the Bayesian Nash equilibrium of the binary-type setting under *any* distribution over $\{\tau_1, \tau_2\}$ in Section 3.1. Based on the equilibrium analysis, we then characterize the organizer's optimal disclosure policy in Section 3.2. Finally, in Section 3.3, we discuss the issues of allocative efficiency and bidders' welfare under optimal disclosure, as well as relate our results to important studies in the previous literature. Overall, the binary setting not only captures all the essential insights of our optimal disclosure results, but also provides important building blocks for the analysis of optimal disclosure policies in the general non-binary setting, which is presented in Section 4.

### 3.1 Equilibrium Analysis

With finitely many bidders and types, it is a standard result in the all-pay auction literature that there is a unique Bayesian Nash equilibrium in which fierce competition in the perfectly discriminating contest forces both bidders to randomize over the same support $[0, \bar{b}]$, where $\bar{b} \in \left[0, \min\{V_2^A, p_1 V_1^B + p_2 V_2^B\}\right]$ under a public belief distribution $\mathbf{p} = (p_1, p_2)$—here, $V_2^A$ (resp., $p_1 V_1^B + p_2 V_2^B$) is the high-type bidder A's (resp., bidder $B$'s) valuation. In addition, the unique equilibrium is monotonic, with continuous bid distributions and at most one mass point at the zero bid.[6]

Recall our definitions in (1) that $v_k = V_k^B / V_k^A$ is the relative valuation ($k = 1, 2$) and $\mathbb{E}_{\mathbf{p}}[v] = p_1 v_1 + p_2 v_2$ is the expected relative valuation under bidder $B$'s public belief $\mathbf{p}$. Proposition 1 presents the unique equilibrium explicitly:

**Proposition 1 (Unique Equilibrium under Belief p)** *Under bidder B's belief* $\mathbf{p}$*, there is a unique Bayesian Nash equilibrium in the all-pay auction.*

1. $\mathbb{E}_{\mathbf{p}}[v] \leq 1$ : *Type* $\tau_1$ *randomizes uniformly over* $\left[0, p_1 V_1^B\right]$*; type* $\tau_2$ *randomizes uniformly over* $\left[p_1 V_1^B, \sum_{k=1}^2 p_k V_k^B\right]$*; bidder B bids 0 with probability* $1 - \mathbb{E}_{\mathbf{p}}[v]$*, randomizes uniformly over* $\left[0, p_1 V_1^B\right]$ *with probability* $p_1 v_1$*, and randomizes uniformly over* $\left[p_1 V_1^B, \sum_{k=1}^2 p_k V_k^B\right]$ *with probability* $p_2 v_2$*.*

2. $p_2 v_2 < 1 < \mathbb{E}_{\mathbf{p}}[v]$ : *Type* $\tau_1$ *bids* $0$ *with probability* $\left(\mathbb{E}_{\mathbf{p}}[v] - 1\right) / \left(p_1 v_1\right)$*, randomizes uniformly over* $(0, (1 - p_2 v_2) V_1^A]$ *with probability* $(1 - p_2 v_2) / (p_1 v_1)$*; type* $\tau_2$ *randomizes uniformly over* $[(1 - p_2 v_2) V_1^A, (1 - p_2 v_2) V_1^A + p_2 v_2 V_2^A]$*; bidder B randomizes uniformly over* $[0, (1 - p_2 v_2) V_1^A]$ *with probability* $1 - p_2 v_2$*, and randomizes uniformly over* $[(1 - p_2 v_2) V_1^A, (1 - p_2 v_2) V_1^A + p_2 v_2 V_2^A]$ *with probability* $p_2 v_2$*.*

3. $1 \leq p_2 v_2$ *(which requires* $\alpha < 1$*)* : *Type* $\tau_1$ *bids* $0$ *for sure; type* $\tau_2$ *bids* $0$ *with probability* $1 - 1/(p_2 v_2)$*, and randomizes uniformly over* $[0, V_2^A]$ *with probability* $1/(p_2 v_2)$*; bidder B randomizes uniformly over* $[0, V_2^A]$*.*

Proposition 1 shows that the form of the equilibrium hinges on how strong bidder $A$ is relative to bidder $B$ in terms of the expected relative valuation $\mathbb{E}_{\mathbf{p}}[v]$. If $\mathbb{E}_{\mathbf{p}}[v] \leq 1$,

[6] Such equilibrium properties in all-pay auctions are well-known. See, for example, Siegel (2014)[13].

or bidder $A$ is in expectation stronger, bidder $B$, being discouraged, bids 0 with positive probability.[7] If bidder $A$ is in expectation weaker but $A$'s high type $\tau_2$ is strong ($p_2 v_2 < 1 < \mathbb{E}_{\mathbf{p}}[v]$), then bidder $A$'s low type $\tau_1$ is discouraged and bids 0 with positive probability. Finally, if both types of bidder $A$ are weak ($1 \le p_2 v_2$), then bidder $A$'s low type $\tau_1$ bids 0 for sure (i.e., type $\tau_1$ becomes irrelevant and is *implicitly excluded*) and even the high type $\tau_2$ bids 0 with positive probability in equilibrium.

Proposition 1 enables us to derive the bidders' equilibrium bids explicitly:[8]

**Corollary 1** *Denote $EP^i(\mathbf{p})$ as bidder $i$'s expected bid under bidder $B$'s belief $\mathbf{p}$.*

1. $\mathbb{E}_{\mathbf{p}}[v] \le 1$ :

$$EP^A(\mathbf{p}) = \frac{1}{2}p_1^2 V_1^B + p_1 p_2 V_1^B + \frac{1}{2}p_2^2 V_2^B, \;\; EP^B(\mathbf{p}) = \frac{1}{2}p_1^2 V_1^B v_1 + p_1 p_2 V_1^B v_2 + \frac{1}{2}p_2^2 V_2^B v_2.$$

2. $p_2 v_2 < 1 < \mathbb{E}_{\mathbf{p}}[v]$ :

$$\begin{aligned} EP^A(\mathbf{p}) &= \tfrac{(1-p_2 v_2)^2 V_1^A}{2v_1} + (1-p_2 v_2)\, p_2 V_1^A + \tfrac{p_2^2 V_2^A v_2}{2}, \\ EP^B(\mathbf{p}) &= \tfrac{(1-p_2 v_2)^2 V_1^A}{2} + (1-p_2 v_2)\, p_2 V_1^A v_2 + \tfrac{p_2^2 V_2^A v_2^2}{2}. \end{aligned}$$

3. $1 \le p_2 v_2$ :

$$EP^A(\mathbf{p}) = \frac{V_2^A}{2v_2}, EP^B(\mathbf{p}) = \frac{V_2^A}{2}.$$

*In the first two cases, each $EP^i(\mathbf{p})$ is strictly convex in $p_2$.*

The organizer's total expected revenue from the bidders under bidder $B$'s (public) belief $\mathbf{p}$ is hence $ER(\mathbf{p}) \equiv EP^A(\mathbf{p}) + EP^B(\mathbf{p})$, which is also the organizer's objective function in selecting an optimal disclosure policy.

We now illustrate how the equilibrium varies in the interdependence parameter $\alpha$. First consider the special case of $\alpha = 1$, i.e., we have a common-value all-pay auction with one-sided private information. The following corollary immediately follows from Proposition 1 in which the bidders are equally strong in expectation ($\mathbb{E}_{\mathbf{p}}[v] = 1$):

**Corollary 2 (Common Value)** *In the all-pay auction with $\alpha = 1$ and belief $\mathbf{p} \in int(\Delta)$, there is a unique equilibrium where the bidders have an identical ex ante bid distribution,*

[7] Strictly speaking, bidder $B$ bids 0 with positive probability if $\mathbb{E}_{\mathbf{p}}[v] < 1$.

[8] There are multiple but *equivalent* ways in presenting the expected equilibrium bids (i.e., using bidder $A$'s valuations $V_i^A$ or using bidder $B$'s valuations $V_i^B$, $i \in \{1, 2\}$). We choose the presentation of $EP^A(\mathbf{p})$ and $EP^B(\mathbf{p})$ so that the cases in Corollary 1 are consistent with those in Proposition 1.

*each bidder wins with ex ante probability* $1/2$*, and bidder* $A$*'s equilibrium payoff is strictly positive* $(p_1 p_2(\tau_2 - \tau_1))$*, while bidder* $B$*'s equilibrium payoff is* $0$.

It is intuitive that while the bidders have an identical bid distribution and winning probability, bidder $A$'s payoff is strictly positive due to her private information. Corollary 2 is consistent with the equilibrium result (Theorem 1) in Milgrom and Weber (1982)[11] for a first-price auction with common value and one-sided private information.

Next, on the effects of $\alpha$ on the equilibrium, the existence of private information and various cases of the equilibrium prevent one from conducting a clear-cut comparative statics analysis. The following Proposition 2 presents an alternative comparative statics result: *absent private information*, a higher $\alpha$ leads to more fierce competition, and hence a smaller discrepancy between the bidders' equilibrium payoffs:

**Proposition 2 (Interdependence)** *Consider the all-pay auction where both bidders are uninformed about bidder* $A$*'s type. As* $\alpha$ *increases, the difference in the bidders' expected equilibrium payoffs* (*generically*) *strictly decreases and becomes* $0$ *when* $\alpha$ *reaches* $1$.

Proposition 2 is intuitive: as $\alpha$ increases, the valuations of the bidders become closer to each other, which intensifies the competition and transfers more rent from the bidders to the organizer. With complete information, generically, one bidder has a strictly higher valuation than the other and the expected equilibrium payoff of the bidder with a higher valuation is positive and is strictly decreasing in $\alpha$, whereas the equilibrium payoff of the other bidder remains zero. Notice that while private information would certainly complicate the comparative statics, the effect of more intensified competition from a higher $\alpha$ persists, and will impact optimal information disclosure, in the setting with private information.

## 3.2 Optimal Disclosure Policies: Bayesian Persuasion

We now analyze optimal disclosure policies for the organizer. Given the equivalence between disclosure policies and discrete distributions of posteriors satisfying Bayes-plausibility (recall (3) in Section 2), an optimal disclosure policy is a Bayes-plausible distribution of posteriors $(\beta_s, \mathbf{p}_s)$ that maximizes the organizer's expected revenue:

**Definition 1** *A Bayes-plausible disclosure policy* $(\beta_s^*, \mathbf{p}_s^*)$ *is optimal if*

$$(\beta_s^*, \mathbf{p}_s^*) \in \arg\max_{(\beta_s, \mathbf{p}_s)} \sum\nolimits_{s=0}^{N} \beta_s ER(\mathbf{p}_s).$$

With binary types, a disclosure policy for a given prior $\mathbf{p}^0 = (p_1^0, p_2^0)$ corresponds to a pair of posteriors $(\mathbf{p}', \mathbf{p}'')$, where $\mathbf{p}', \mathbf{p}'' \in \triangle$, and a probability $\beta$ such that $\beta \mathbf{p}' + (1-\beta)\,\mathbf{p}'' = \mathbf{p}^0$. As discussed, since $\beta_s^*$ can be recovered from the Bayes-plausibility condition (3), we can ignore the signals in $S$ and focus only on $\mathbf{p}_s^*$ in describing an optimal disclosure policy. Finally, as demonstrated in Kamenica and Gentzkow (2011)[7], the organizer's revenue under an optimal disclosure policy corresponds to the value of the concave closure of $ER(\mathbf{p})$ at the prior $\mathbf{p}^0$.

As a starting point, consider the special case in which the bidders have a common valuation of the prize (i.e., $\alpha = 1$). By Corollary 1, the organizer's expected revenue is strictly convex in the probability of the high type ($p_2$), implying that the concave closure of $ER(\mathbf{p})$ is the linear line connecting the expected revenues at the boundary points. Hence, full disclosure is the unique optimal disclosure. Formally,

**Proposition 3** *In the all-pay auction with $\alpha = 1$, fully disclosing bidder A's private type is the unique optimal disclosure policy.*

Notice that with an identical valuation of the prize regardless of bidder $A$'s type, the competition between the bidders is most intensified under full disclosure, so that both bidders are symmetrically informed. Concealing any information would only provide unnecessary information rent to bidder $A$ and alleviate the competition. The optimality of full disclosure under common value here is consistent with the disclosure results in Wärneryd (2003)[14] for a generalized Tullock lottery contest and Milgrom and Weber (1982)[11] for a standard first-price auction, both featuring common value and one-sided private information (see Section 3.3.3 for more details on this).

When the interdependence of valuations is not perfect (i.e., $\alpha < 1$), two complications arise in characterizing the optimal disclosure policy: *first*, information disclosure has to be more finely tuned to cope with asymmetric strengths of the bidders, which can dampen the competition; *second*, asymmetric valuations also lead to different equilibrium formats (Proposition 1), further complicating the analysis. We now define three signals $\mathbf{p}_s$, so as to explicitly describe optimal disclosure policies for $\alpha < 1$ :[9]

1. $\mathbf{p}^{\tau_i \tau_{-i}}$: $\mathbb{E}_{\mathbf{p}^{\tau_i \tau_{-i}}}[v] = 1$ and $\tau_i > \tau_{-i} > 1$, with beliefs:

$$\Pr(\tau_i | \mathbf{p}^{\tau_i \tau_{-i}}) = \frac{v_{-i} - 1}{v_{-i} - v_i},\ \Pr(\tau_{-i} | \mathbf{p}^{\tau_i \tau_{-i}}) = \frac{1 - v_i}{v_{-i} - v_i}.$$

[9] We adopt somewhat cumbersome notation for the three signals, so that the notation for the binary-type case is consistent with that for the case of general types.

2. $\mathbf{p}^{\tau_i}$: $supp(\mathbf{p}^{\tau_i}) = \{\tau_i\}$, with belief

$$\Pr\left(\tau_i | \mathbf{p}^{\tau_i}\right) = 1.$$

3. $\mathbf{p}^{\tau_i|\tau_{-i}}$: $p_i^{\tau_i|\tau_{-i}} \cdot \tau_i = 1$ and $1 > \tau_i > \tau_{-i}$, with beliefs

$$\Pr\left(\tau_i | \mathbf{p}^{\tau_i|\tau_{-i}}\right) = 1/v_i,\ \Pr\left(\tau_{-i} | \mathbf{p}^{\tau_i|\tau_{-i}}\right) = 1 - 1/v_i.$$

Upon seeing signal $\mathbf{p}^{\tau_2\tau_1}$, bidder $B$ believes that the expected relative valuation is 1. As a result, signal $\mathbf{p}^{\tau_2\tau_1}$ indicates equal strengths of, and intensifies bidding competition between, the bidders. The organizer will hence utilize $\mathbf{p}^{\tau_2\tau_1}$ in the disclosure as much as possible, whenever $\mathbf{p}^{\tau_2\tau_1}$ is credible (i.e., when $\tau_2 > 1 > \tau_1$). Next, upon seeing signal $\mathbf{p}^{\tau_2}$, bidder $B$ believes that $A$ is of type $\tau_2$. Finally, with signal $\mathbf{p}^{\tau_2|\tau_1}$, bidder $B$ believes that bidder $A$ is type $\tau_2$ with probability $\frac{1}{v_2}$. As we will see, $\mathbf{p}^{\tau_2|\tau_1}$ arises in the optimal disclosure only when both types are weak ($1 > \tau_2 > \tau_1$), and the low type $\tau_1$ will bid zero in equilibrium when $\mathbf{p}^{\tau_2|\tau_1}$ is realized.

We now fully characterize the organizer's optimal disclosure policy in three cases. As stated before, we will present the optimal disclosure policies only in terms of the optimal signals $\mathbf{p}_s^*$, written as the support of the distribution $(\beta_s^*, \mathbf{p}_s^*)$.

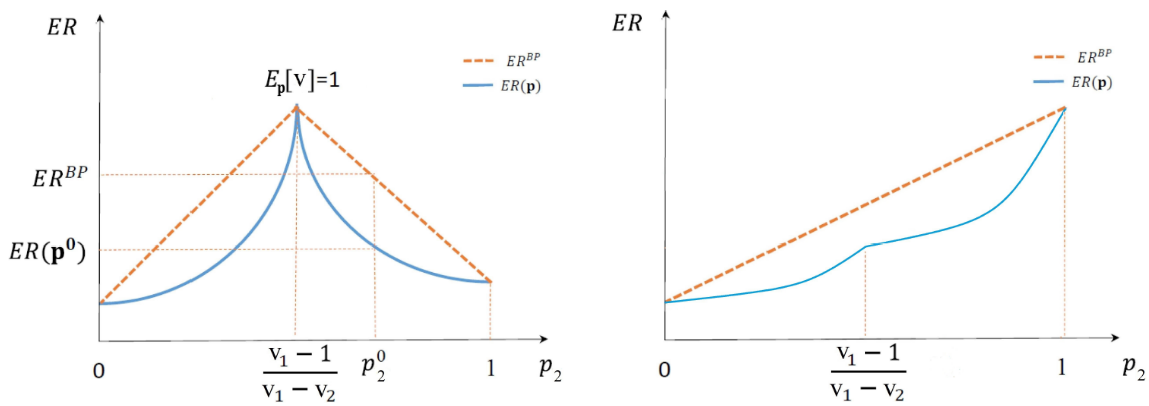
**Proposition 4** *In the all-pay auction, suppose $\tau_1 < 1 < \tau_2$. There is a unique $\hat{\alpha} \in (0,1)$ such that:*

*(i) If $\alpha < \hat{\alpha}$, the optimal disclosure policy $(\beta_s^*, \mathbf{p}_s^*)$ features **partial** disclosure and*

$$supp(\beta_s^*, \mathbf{p}_s^*) = \begin{cases} \{\mathbf{p}^{\tau_2\tau_1}, \mathbf{p}^{\tau_2}\} & \textit{if } \mathbb{E}_{\mathbf{p^0}}[v] < 1 \\ \{\mathbf{p}^{\tau_2\tau_1}\} & \textit{if } \mathbb{E}_{\mathbf{p^0}}[v] = 1 \\ \{\mathbf{p}^{\tau_2\tau_1}, \mathbf{p}^{\tau_1}\} & \textit{if } \mathbb{E}_{\mathbf{p^0}}[v] > 1. \end{cases}$$

*(ii) If $\alpha \geq \hat{\alpha}$, the optimal disclosure policy $(\beta_s^*, \mathbf{p}_s^*)$ features **full** disclosure and $supp(\beta_s^*, \mathbf{p}_s^*) = \{\mathbf{p}^{\tau_1}, \mathbf{p}^{\tau_2}\}$.*

Proposition 4 presents the optimal disclosure policy when $B$'s public type lies in between $A$'s two types. The optimal disclosure rule changes in the interdependence between bidders' valuations. This is shown in Figure 1.[10] First, consider the case in which valuation interdependence is low ($\alpha < \hat{\alpha}$). Generically (i.e., whenever $\mathbb{E}_{\mathbf{p}^0}[v] \neq 1$), it is optimal to partially disclose bidder $A$'s type. If bidder $A$ is ex ante stronger (i.e., $\mathbb{E}_{\mathbf{p}^0}[v] < 1$, or $p_2^0 > \frac{v_1-1}{v_1-v_2}$ in Figure 1), the organizer randomizes between signals $\mathbf{p}^{\tau_2\tau_1}$ and $\mathbf{p}^{\tau_2}$, which effectively hides the low type $\tau_1$ and reveals the high type $\tau_2$ with positive probability. If the bidders are ex ante *exactly* evenly matched (i.e., $\mathbb{E}_{\mathbf{p}^0}[v] = 1$), the competition is already the most intensified at the prior and the organizer reveals nothing. The remaining case of $\mathbb{E}_{\mathbf{p}^0}[v] > 1$, when $p_2^0$ is to the left of $(v_1 - 1) / (v_1 - v_2)$ in Figure 1, is analogous. Such partial disclosure intends to make the bidders as symmetric as possible in expectation. However, compared to full disclosure, the above partial disclosure also incurs the cost of letting bidder $A$ retain some information rent. When valuation interdependence is high ($\alpha \geq \hat{\alpha}$), the bidders are already sufficiently symmetric (and completely so when $\alpha = 1$). As a result, it is not worthwhile to leave any information rent to bidder $A$ and full disclosure is optimal (part (*ii*) of Proposition 4).[11]




Partial Disclosure is Optimal ($\alpha < \hat{\alpha}$) Full Disclosure is Optimal ($\alpha > \hat{\alpha}$)

Figure 1: Optimal Disclosure Policies for Proposition 4 ($\tau_1 < 1 < \tau_2$). Note that $ER(\mathbf{p})$ is the expected revenue under belief $\mathbf{p} = (p_1, p_2)$ and $ER^{BP}$ is that under Bayesian persuasion.

[10]Figure 1 illustrates how much optimal disclosure can improve the organizer's expected revenue. Each solid (blue) curve denotes the equilibrium expected revenue under posterior $\mathbf{p}$, where strict convexity of the expected revenue (to the left and to the right of $(v_1 - 1) / (v_1 - v_2)$) is shown in Corollary 1. When $\alpha < \hat{\alpha}$, the peak is achieved when bidder $B$'s valuation relative to that of $A$ in expectation is 1. Each dashed (orange) line denotes the expected revenue under optimal disclosure.

[11]In the special case of $\alpha = \hat{\alpha}$, the organizer's expected revenues (the blude solid curve) when $p_2 = 0$, $p_2 = 1$, and $p_2 = (v_1 - 1) / (v_1 - v_2)$ lie on the same linear line. As a result, partial disclosure and full disclosure lead to the same expected payoff for the organizer.

**Proposition 5** *In the all-pay auction, suppose* $1 \leq \tau_1 < \tau_2$. *The optimal disclosure policy* $(\beta_s^*, \mathbf{p}_s^*)$ *features* ***full*** *disclosure and supp*$(\beta_s^*, \mathbf{p}_s^*) = \{\mathbf{p}^{\tau_1}, \mathbf{p}^{\tau_2}\}$.

In Proposition 5, both types of bidder $A$ are stronger than bidder $B$. Given such a strong asymmetry, it is optimal for the organizer to leave no information rent, rather than statistically "hide" the high type $\tau_2$ to slightly intensify the competition. As a result, the optimal disclosure policy, illustrated in the left panel of Figure 2, is full disclosure. In particular, Proposition 5 demonstrates that information disclosure, bounded by *Bayes-plausibility* (3), can also be a delicate tool with limited effectiveness whenever there are more powerful forces in dictating the bidders' incentives (e.g., ex ante asymmetry here).

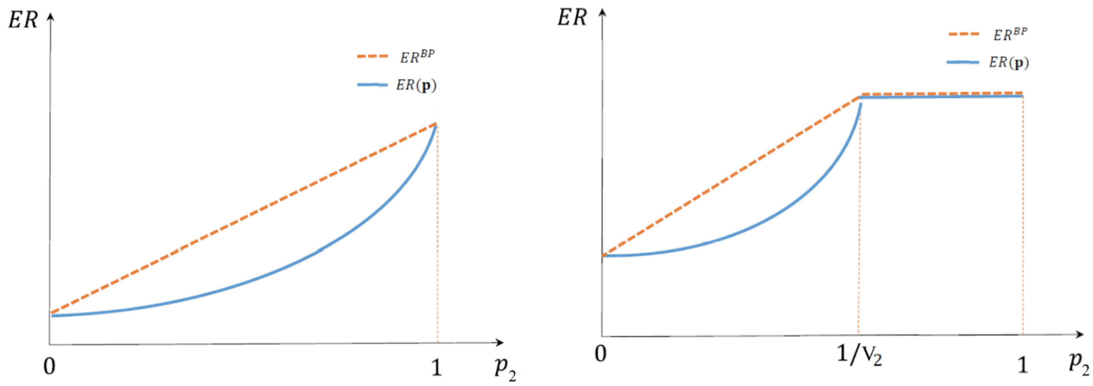
**Proposition 6** *In the all-pay auction, suppose* $\tau_1 < \tau_2 \leq 1$. *The optimal disclosure policy* $(\beta_s^*, \mathbf{p}_s^*)$ *features* ***partial*** *disclosure and*

$$supp(\beta_s^*, \mathbf{p}_s^*) = \begin{cases} \{\mathbf{p}^{\tau_2|\tau_1}, \mathbf{p}^{\tau_1}\} & \text{if } p_2^0 < 1/v_2, \\ \{\mathbf{p}^{\boldsymbol{\tau_2}|\boldsymbol{\tau_1}}, \mathbf{p}^{\tau_2}\} & \text{if } p_2^0 \geq 1/v_2. \end{cases}$$

In Proposition 6, illustrated in the right panel of Figure 2, both types of bidder $A$ are weaker than bidder $B$.[12] Here, the organizer partially discloses bidder $A$'s type when $p_2^0 < 1/v_2$ (i.e., randomizes between $\mathbf{p}^{\tau_2|\tau_1}$ and $\mathbf{p}^{\tau_1}$), as well as when $p_2^0 > 1/v_2$ (i.e., randomizes between $\mathbf{p}^{\tau_2|\tau_1}$ and $\mathbf{p}^{\tau_2}$).[13] Notice that when $p_2^0 \geq 1/v_2$, the weak type $\tau_1$ bids 0 for sure in equilibrium and hence is implicitly excluded in the contest. The organizer's expected revenue is hence constant in posterior $\mathbf{p}$ when $p_2 \geq 1/v_2$ (cf., Case 3 of Corollary 1). Notice that we have asymmetric results in Proposition 5 (full disclosure being optimal) and Proposition 6 (partial or no disclosure being optimal). The reason for such an asymmetry has to do with the cost of leaving information rent to bidder $A$: In both cases of $1 \leq \tau_1 < \tau_2$ and $\tau_1 < \tau_2 \leq 1$, hiding some information will always bring the bidders' expected valuations closer compared to full disclosure, which intensifies competition. However, when both types of bidder $A$ are strong ($1 \leq \tau_1 < \tau_2$), the cost of leaving information rent to $A$ outweighs the benefit from hiding information, while it is exactly the opposite when both types of bidder $A$ are weak ($\tau_1 < \tau_2 \leq 1$).

[12] As Figure 1, Figure 2 shows how much optimal disclosure improves the organizer's expected revenue, where solid (blue) curves represent the equilibrium expected revenue under any posterior $\mathbf{p}$ and dotted (orange) curves denote the organizer's expected revenue under optimal disclosure.

[13] When $p_2^0 \geq 1/v_2$, the organizer's expected payoff from randomizing between $\mathbf{p}^{\tau_2|\tau_1}$ and $\mathbf{p}^{\tau_2}$ is exactly equivalent to that from disclosing nothing (or just disclosing signal $\mathbf{p}^0$). In other words, neither partial disclosure nor no disclosure is strictly optimal when $p_2^0 \geq 1/v_2$.




$1 \leq \tau_1 < \tau_2$ (strong bidder $A$) $\qquad$ $\tau_1 < \tau_2 \leq 1$ (weak bidder $A$)

Figure 2: Optimal Disclosure Policies for Propositions 5 (left) and 6 (right).

In summary, the organizer's optimal disclosure policy in our all-pay auction can be partial disclosure, full disclosure, or no disclosure. And these various optimal disclosure formats can be rationalized by the organizer's single incentive to fine tune information so as to sharpen the competition between the bidders as much as possible, making information disclosure a meaningful and delicate tool for the organizer.

Finally, we discuss how valuation interdependence ($\alpha$) affects optimal disclosure. Such *comparative statics* is most easily illustrated using Figure 1 and Figure 2. First, consider Figure 1 where $\tau_1 < 1 < \tau_2$. In the independent value case ($\alpha = 0$), with or without disclosure, the competition between the bidders is the most fierce when $\mathbb{E}_{\mathbf{p}}[v] = 1$.[14] As $\alpha$ increases, both bidders have higher valuations and the entire expected revenue curve ($ER(\mathbf{p})$) hence shifts upward. However, the two end points of $ER(\mathbf{p})$ increase faster than the interior peak point due to that the benefit of having a posterior of $\mathbb{E}_{\mathbf{p}}[v] = 1$ diminishes as the bidders' valuations also get closer as $\alpha$ increases. This trend continues and the peak point of $ER(\mathbf{p})$ eventually drops below the concave closure of the two end points of $ER(\mathbf{p})$ when $\alpha > \hat{\alpha}$. As a result, everything else fixed, the optimal disclosure policy for the case $\tau_1 < 1 < \tau_2$ transitions from partial disclosure to full disclosure as $\alpha$ increases from 0 to 1. Next, consider the right panel of Figure 2 where $\tau_1 < \tau_2 \leq 1$. As $\alpha$ increases from 0 to 1, other than the entire $ER(\mathbf{p})$ shifting upwards, the critical point of $p_2 = \frac{1}{v_2} = \frac{\alpha+\tau_2}{1+\alpha\tau_2}$ also shifts to the right. At the limit where $\alpha = 1$, both types of bidder $A$ bid 0 with probability 0 (Case 3 of Proposition 1). Here again, the optimal disclosure policy transitions from partial disclosure to full disclosure. Hence, other than the case of a

[14]We have $\mathbb{E}_{\mathbf{p}}[v] = 1$ at the peak point of $ER(\mathbf{p})$ in the left panel of Figure 1, where $\mathbf{p} = (p_1, p_2)$ and $p_2 = \frac{v_1-1}{v_1-v_2}$ (and $p_2 = \frac{\tau_2(1-\tau_1)}{\tau_2-\tau_1}$ when $\alpha = 0$).

strong bidder $A$ ($1 \leq \tau_1 < \tau_2$), the organizer's optimal disclosure policy always transitions from partial disclosure to full disclosure, reflecting the organizer's incentives to reveal "more" information to intensify the bidders' competition as $\alpha$ increases and the bidders' valuations get closer to each other.[15] This is summarized in Corollary 3:

**Corollary 3** *In the all-pay auction, other than the case of* $1 \leq \tau_1 < \tau_2$*, the organizer's optimal disclosure policy always transitions from partial disclosure to full disclosure as* $\alpha$ *increases from* 0 *to* 1*.*

Finally, we present a useful Corollary 4, demonstrating that the organizer's optimal disclosure policy in the common value setting ($\alpha = 1$) differs *qualitatively* from the optimal disclosure policy in the general interdependent value setting ($\alpha < 1$).

**Corollary 4** *In the all-pay auction, full disclosure is an optimal disclosure policy for all parameters* $(\tau_1, \tau_2, \mathbf{p}^0)$ *iff* $\alpha = 1$*, and partial disclosure can be optimal whenever* $\alpha < 1$*.*

Corollary 4 follows immediately from Propositions 3–6. In particular, Proposition 6 and the right panel of Figure 2 jointly imply that for the case of a weak bidder $A$ ($\tau_1 < \tau_2 \leq 1$), the organizer will choose partial disclosure as long as $\alpha < 1$. While simple, Corollary 4 will be of particular importance when we relate our optimal disclosure results with those for the common value setting in the literature (Milgrom and Weber 1982[11], Wärneryd 2003[14]) in Section 3.3.3.

## 3.3 Discussion

In this section, we discuss the effects of optimal information disclosure on allocative efficiency and the bidders' welfare relative to the no-disclosure benchmark of a standard all-pay auction with one-sided private information. Our purpose here is to understand whether allowing the organizer to disclose information is desirable from a normative point of view. We also elaborate in more detail on the relationship between our results with the related literature mentioned previously.

### 3.3.1 Allocative Efficiency

We define the allocative efficiency of an equilibrium outcome in the all-pay auction as the sum of the expected values generated by bidder $A$'s and bidder $B$'s winning. Formally,

[15] It is cumbersome but straightforward to study the explicit transitions of the organizer's optimal policy, which is essentially a binary probability distribution. Such an analysis however does not lead to additional insights other than Corollary 3.

**Definition 2** *The allocative efficiency of an equilibrium outcome under posterior* $\mathbf{p}$ *is*

$$EF(\mathbf{p}) \equiv \textstyle\sum_{k=1}^{2} p_k \Pr\left[\text{bidder } A \text{ wins}|t_A = \tau_k\right] V_k^A + \sum_{k=1}^{2} p_k \Pr\left[\text{bidder } B \text{ wins}|t_A = \tau_k\right] V_k^B.$$

Our analysis in Section 3.2 demonstrates that the organizer generically benefits from optimal disclosure *strictly*, except when bidder $A$ is too weak in expectation (right panel, Figure 2). A natural question is whether such additional revenue is due to improved allocative efficiency from the organizer's optimal information disclosure. The answer to this question is somewhat convoluted. In most cases, optimal information disclosure decreases allocative efficiency relative to the no-disclosure benchmark, implying that in these cases, the organizer's additional revenue comes from her improved ability to extract more surplus from the bidders via optimal information disclosure. There are however limited instances (in our leading case of $\tau_1 < 1 < \tau_2$ where the bidders are roughly matched evenly) where optimal information disclosure can indeed improve allocative efficiency. Proposition 7 below presents such effects formally (here, recall that $\hat{\alpha}$ is defined in Proposition 4):

**Proposition 7** *Optimal disclosure from the organizer decreases allocative efficiency in the all-pay auction relative to the no-disclosure benchmark in all cases, except possibly in the leading case of* $\tau_1 < 1 < \tau_2$*, where*

1. *if* $\alpha > \hat{\alpha}$*, or if* $\alpha < \hat{\alpha}$ *and* $p_2^0 < p_2^{\tau_2\tau_1}$*, then allocative efficiency is lower under optimal disclosure.*

2. *if* $\alpha < \hat{\alpha}$ *and* $p_2^0 > p_2^{\tau_2\tau_1}$*,*

   (a) *when* $\tau_1 \geq \frac{1}{2}$*, allocative efficiency is lower under optimal disclosure;*

   (b) *when* $\tau_1 < \frac{1}{2}$*, there exists* $\tilde{\alpha} \in (0,1)$ *such that allocative efficiency is lower under optimal disclosure if* $\alpha > \tilde{\alpha}$*, higher if* $\alpha < \tilde{\alpha}$*, and the same if* $\alpha = \tilde{\alpha}$*.*[16]

According to Proposition 7, most often the revenue gain from the organizer's optimal disclosure comes from "exploiting" the bidders, except for the restricted scenario (i.e., the leading case $\tau_1 < 1 < \tau_2$) when valuation interdependence is low ($\alpha < \min\{\hat{\alpha}, \tilde{\alpha}\}$), bidder $A$ is ex ante stronger ($p_2^0 > p_2^{\tau_2\tau_1}$), but bidder $A$'s low type is sufficiently weak ($\tau_1 < \frac{1}{2}$).

To see the intuition of Proposition 7, recall that the organizer's optimal disclosure strengthens competition by manipulating the posterior beliefs of bidder $B$. This is typically done by often revealing strong type(s) and hiding weak type(s) of bidder $A$ (relative

[16]Here $\hat{\alpha}$ could be larger than or smaller than $\tilde{\alpha}$, depending on the magnitudes of $\tau_1$ and $\tau_2$.

to that of bidder $B$). In effect, optimal disclosure forces both bidders to bid more aggressively by shifting the probability of winning from an ex ante stronger bidder to the weaker bidder, which is detrimental to allocative efficiency. As a result, the organizer's optimal disclosure usually has a negative effect on allocative efficiency relative to the no-disclosure benchmark, particularly so in the asymmetric cases of $1 \leq \tau_1 < \tau_2$ and $\tau_1 < \tau_2 \leq 1$.

Despite the negative effect, optimal disclosure, somewhat incidentally, can improve allocative efficiency in the restricted scenario mentioned above. Here, in the no-disclosure benchmark (case 1 of Proposition 1 under belief $\mathbf{p}^0$), bidder $B$ bids 0 with positive probability and bidder $A$'s low type $\tau_1$, hiding under the veil of private information, wins with positive probability, leading to inefficiency. Such allocative inefficiency is severe when $\tau_1$ is sufficiently low ($\tau_1 < \frac{1}{2}$) and sufficiently unlikely ($p_2^0 > p_2^{\tau_2\tau_1}$), and valuation interdependence is low ($\alpha < \min\{\hat{\alpha}, \tilde{\alpha}\}$). The organizer's optimal disclosure (left panel of Figure 1) partially reveals type $\tau_1$ and (rightly) shifts winning probability from type $\tau_1$ to bidder $B$, leading to a higher allocative efficiency than the no-disclosure benchmark.[17]

Recently, Lu, Ma, and Wang (2018)[9] analyze information disclosure for an organizer in a symmetric two-bidder all-pay auction with independent private values and binary types. The authors compare revenue and allocative efficiency across three disclosure policies: full disclosure, no disclosure, and partial disclosure.[18] They find that both revenue and efficiency are the lowest under full disclosure and the highest under no disclosure. While their setting and ours differ in several dimensions (interdependence, asymmetric bidders, and general disclosure policy here), there is similarity between the two results in that disclosure is in general bad for allocative efficiency, with similar intuition as well. However, the two sets of results also differ in that disclosure can sometimes improve allocative efficiency in our setting. This difference is mainly caused by that the bidders are asymmetric and our equilibrium under no disclosure can involve non-monotone strategies where a low-value bidder can bid in the same range as a high-value bidder. In a symmetric and independent value setting, however, the equilibrium under no disclosure consists of monotone strategies, and a low type has no chance to win when she meets a high type, resulting in the highest allocative efficiency.

[17] In this restricted scenario, the organizer randomizes between signals $\mathbf{p}^{\tau_2\tau_1}$ and $\mathbf{p}^{\tau_2}$. While only type $\tau_2$ (signal $\mathbf{p}^{\tau_2}$) is revealed with positive probability, bidder $B$ obtains more precise information about bidder $A$'s type and after information disclosure, bidder $B$ bids 0 with probability 0. In addition, since type $\tau_2$ is revealed with positive probability, the negative effect on allocative efficiency from optimal disclosure mentioned previously still exists, as the probability of winning for $\tau_2$, the "player" with the highest valuation, decreases after optimal disclosure.

[18] In Lu, Ma, and Wang (2018)[9] their full disclosure and no disclosure are the same as ours. Their partial disclosure involves publicly disclosing the bidders' values if and only if both values are high (or both values are low).

### 3.3.2 Bidders' Welfare

Our above analysis illustrates that optimal disclosure, though in most cases diminishes allocative efficiency, can sometimes improve allocative efficiency. This leaves open the possibility that optimal information disclosure can benefit both the organizer and the bidders from higher allocative efficiency. We now evaluate various disclosure policies or information structures from the bidders' point of view.

To this end, we consider three simple information structures in our setting, full disclosure, optimal disclosure, and no disclosure, and examine the bidders' welfare under these information structures.[19] The result is summarized in Proposition 8.

**Proposition 8** *In the all-pay auction,*

($i$) *the bidders'* ***total*** *ex ante expected payoff is lower under optimal disclosure than under no disclosure;*

($ii$) *bidder A's (bidder B's) ex ante expected payoff and winning probability are both lower (higher) under optimal disclosure than under no disclosure;*

($iii$) *bidder A's (bidder B's) ex ante expected payoff and winning probability are both lower (higher) under full disclosure than under optimal disclosure.*

Part ($i$) of Proposition 8 shows that allowing the organizer to optimally disclose bidder $A$'s type, while benefiting the organizer, unambiguously hurts the total welfare of the bidders for all parameter specifications. In particular, this implies that though the organizer's optimal disclosure policy can sometimes improve allocative efficiency, such additional benefits are only captured by the organizer.

Parts ($ii$) and ($iii$) compare each bidder's welfare, both in terms of the bidders' expected payoffs and their winning probabilities, across the three information structures. Here, disclosing "more" information about bidder $A$ (from no disclosure to optimal disclosure, and from optimal disclosure to full disclosure) harms the informed bidder $A$ and benefits the uninformed bidder $B$. The intuition is that information disclosure (optimal or full) partially or fully eliminates bidder $A$'s information advantage, which inadvertently enables bidder $B$ to bid more "precisely" against bidder $A$.

There are two implications of Proposition 8 that are worth mentioning explicitly. First, in our all-pay auction, a bidder wins the prize if and only if her (realized) bid is higher

[19]While it is possible to use Bayesian persuasion to identify the optimal disclosure policy from each bidder's point of view, Proposition 8 below will show intuitively that this is not necessary.

than the other bidder. As a result, the change in a bidder's expected payoff is perfectly aligned with the change in the bidder's winning probability in equilibrium across different information structures. The same thing however cannot be said for contests with imperfect discrimination, as shown in Wärneryd (2003)[14]. We will discuss such difference in more detail when we relate our results with the previous literature in the next section.

Second, the limited comparative statics results in parts (*ii*) and (*iii*) are obviously not strict. Under some parameter constellations, the optimal disclosure can coincide with full disclosure or no disclosure. In addition, in some cases, e.g., the case of common value ($\alpha = 1$), each bidder's equilibrium winning probability is identically $\frac{1}{2}$ under any information structure, as established earlier in Corollary 2.

#### 3.3.3 Relation to the Literature

We now relate our results to the previous literature in more detail.

As mentioned, Zhang and Zhou (2016)[15] employs Bayesian persuasion to find that in a Tullock lottery contest with binary types and private values, the optimal disclosure policy is either full disclosure or no disclosure. Our corresponding results for the case of $\alpha = 0$ in Propositions 4-6 differ in that partial disclosure is optimal (in our leading case $\tau_1 < 1 < \tau_2$) unless the bidders' valuations are very asymmetric. This difference is due to that equilibrium outcome and behavior in a lottery contest are less sensitive to information (due to its imperfect discrimination) and optimal disclosure should mainly address the strongest bidder/type's incentives. In a perfectly discriminatory all-pay auction, however, the equilibrium is more sensitive to information, which prompts the organizer to more delicately fine tune information disclosure to guide the bidders' behavior. Taken together, Zhang and Zhou (2016)[15] and our results offer useful snapshots on the effects of the discriminatory power on optimal disclosure in Tullock contests.

Next, as described before, our paper is also related to studies on information structures in common-value first price auctions (Milgrom and Weber (1982)[11]) and in common-value contests (Wärneryd (2003)[14]). Given our more general disclosure policies as well as our general specification of valuation interdependence, we are able to extend the insights in Milgrom and Weber (1982)[11] and Wärneryd (2003)[14] in various directions.

First, on optimal disclosure, both Milgrom and Weber (1982)[11] and Wärneryd (2003)[14], in their respective settings, demonstrate that full disclosure is better than no disclosure for the organizer. Our Proposition 3 shows that for $\alpha = 1$, full disclosure is uniquely optimal among *all disclosure policies* in our all-pay auction, which somewhat sits in between first-price winner-pay auctions and (Tullock) contests. In particular, our result is consistent but is established among a more general disclosure set.[20] More importantly, our Propositions 4-6 also demonstrate that full disclosure being optimal for $\alpha = 1$ is also a *special* result, in that partial/no disclosure can be optimal whenever we (infinitesimally) deviate from the common-value setting ($\alpha < 1$).

Second, on bidders' welfare over various information structures,[21] Proposition 8 demonstrates that the informed bidder $A$ prefers no disclosure to optimal disclosure, and optimal disclosure to full disclosure, while the uninformed bidder $B$ has exactly the opposite preferences. This is consistent with the findings of Milgrom and Weber (1982)[11] (Theorems 4 and 5) for the first-price common-value auction, and the results of Wärneryd (2003)[14] (Section 4) for the lottery contest, but our comparative statics result is obtained in a more general interdependent value setting.[22]

Finally, on bidders' winning probabilities under various information structures, Proposition 8 also shows that a payoff gain (loss) of a bidder from information disclosure is associated with a gain (loss) in probability of winning in our all-pay auction. While our result here is qualitatively identical to that in Milgrom and Weber (1982)[11] and holds for general valuation interdependence, this result differs drastically from that in Wärneryd (2003)[14]. To be specific, Wärneryd (2003)[14] shows that in a contest with common value and one-sided private information, the uninformed bidder wins with strictly higher probability than the informed bidder under no disclosure and both bidders win with equal probability under full disclosure, i.e., disclosing "more" information about the informed bidder increases the informed bidder's winning probability.[23] The comparison of our result here and those in Milgrom and Weber (1982)[11] and Wärneryd (2003)[14] makes it clear that the somewhat surprising result in Wärneryd (2003)[14] is actually due to the noisy winning criterion, rather than the all-pay feature, of a contest. In a Tullock lottery contest, for example, a bidder with a lower bid still has some chance to win the contest, and maximizing expected payoffs is hence less associated with the probability of winning. In particular, the informed bidder will sacrifice winning in low common-value states by submitting low bids, in order to save on expenditure and at the same time, such sacrifice is not as costly due to the noisy winning criterion of the contest.

[20] That full disclosure is strictly better than no disclosure is shown in Corollary of Milgrom and Weber (1982)[11] and in Proposition 5 for lottery contests in Wärneryd (2003)[14]. In addition, though Wä rneryd (2003)[14] only shows that full disclosure is strictly better than no disclosure, it can be shown that full disclosure is actually optimal in his setting as well.

[21] This, for example, can be important for analyzing bidders' incentives to acquire information.

[22] Milgrom and Weber (1982)[11] also consider the cases where the information of the informed bidder is collected either overtly or covertly.

[23] This result is established in Proposition 2 of Wärneryd (2003)[14] for a contest with some general contest successful function, as well as several examples for the Tullock lottery contest.

# 4 Optimal Disclosure for General Types

We now investigate optimal information disclosure for the general setting where bidder $A$'s type is drawn from an arbitrary distribution with finitely many types. Naturally, the analysis of the general setting is much more complex, given the enormous numbers of cases and signals one has to deal with. We will demonstrate, however, that the formats of optimal disclosure here are qualitatively identical to the optimal disclosure formats in the binary-type case. Importantly, one can without loss of generality focus on a set of $N+1$ signals that is approximately similar to the set of signals $\left\{\mathbf{p}^{\tau_i\tau_{-i}}, \mathbf{p}^{\tau_i}, \mathbf{p}^{\tau_i|\tau_{-i}}\right\}$ identified in Section 3.2 to construct optimal disclosure policies for the general setting.

For expositional convenience, divide the set of types into two subsets $\Omega = \mathbf{L} \cup \mathbf{H}$, where $\mathbf{L} := \{\tau_0, \ldots, \tau_m\}$ and $\mathbf{H} := \{\tau_{m+1}, \ldots, \tau_{m+n}\}$, with $\tau_0 < \cdots < \tau_m < 1 < \tau_{m+1} < \cdots < \tau_{m+n}$, $n + m = N$. Next, we consider a special but familiar set of signals:

1′. $\mathbf{P^{HL}} \equiv \{\mathbf{p}^{\tau_h\tau_\ell} | \tau_h \in \mathbf{H} \text{ and } \tau_\ell \in \mathbf{L}\}$;

2′. $\mathbf{P^{H}} \equiv \{\mathbf{p}^{\tau_h} | \tau_h \in \mathbf{H}\}$ and $\mathbf{P^{L}} \equiv \{\mathbf{p}^{\tau_\ell} | \tau_\ell \in \mathbf{L}\}$;

3′. $\mathbf{P^{L|L}} \equiv \{\mathbf{p}^{\tau_\ell|\tau_{\ell'}} | \tau_\ell, \tau_{\ell'} \in \mathbf{L} \text{ and } \tau_\ell > \tau_{\ell'}\}$.

Similar to our previous signals $\left\{\mathbf{p}^{\tau_i\tau_{-i}}, \mathbf{p}^{\tau_i}, \mathbf{p}^{\tau_i|\tau_{-i}}\right\}$, after observing signal $\mathbf{p}^{\tau_h\tau_\ell}$, bidder $B$ believes that bidder $A$'s type is either $\tau_h$ and $\tau_\ell$ with expected relative valuation of 1, and after observing signal $\mathbf{p}^{\tau_i}$, bidder $B$ believes that bidder $A$ is of type $\tau_i$ for sure. A posterior signal $\mathbf{p}^{\tau_\ell|\tau_{\ell'}}$ induces a posterior belief with $\mathbf{p}^{\tau_\ell|\tau_{\ell'}} = 1/v_\ell$ where in the corresponding equilibrium, type $\tau_{\ell'}$ bids 0 for sure, while type $\tau_\ell$ bids 0 with zero probability.[24] To differentiate these signals from those in the binary setting, hereafter we will call the signals $\mathbf{P^{HL}}$, $\mathbf{P^{H}}$, $\mathbf{P^{L}}$, $\mathbf{P^{L|L}}$ *vertices*, and denote $\mathbf{P} \equiv \left[\mathbf{P^{HL}} \cup \mathbf{P^{H}} \cup \mathbf{P^{L}} \cup \mathbf{P^{L|L}}\right]$ as the set of all *vertices*. Finally, for $\mathbf{P}' \subset \mathbf{P}$, we refer to $\Delta(\mathbf{P}')$ as the set of disclosure policies with supporting signals in $\mathbf{P}'$, i.e., a disclosure policy $(\beta_s, \mathbf{p}_s) \in \Delta(\mathbf{P}')$ if $supp(\beta_s, \mathbf{p}_s) \subset \mathbf{P}'$.

By Definition 1, an optimal disclosure policy in the general setting is a probability distribution in $\Delta(\Delta^N)$, parameterized by $(\beta_s, \mathbf{p}_s)$, that satisfies the Bayesian plausibility condition (3) and maximizes the organizer's expected revenue from the all-pay auction. The following Proposition 9 demonstrates that it is without loss of generality to focus on the vertices in set $\mathbf{P}$ defined above, and consequently, the search for an optimal disclosure policy can be reduced to a simpler linear programming problem.

[24] Recall that $v_\ell = V_\ell^B / V_\ell^A$, defined in (1), is the ratio of bidder $B$'s valuation and bidder $A$'s valuation when $A$'s realized type is $\tau_\ell$.

**Proposition 9** *Consider the all-pay auction where A's type is drawn from $\Omega$ according to $\mathbf{p}^0$. A disclosure policy $(\beta_s^*, \mathbf{p}_s^*)$ is optimal if it satisfies the following conditions:*

*(i)* $(\beta_s^*, \mathbf{p}_s^*) \in \Delta(\mathbf{P})$*;*

*(ii)* $(\beta_s^*, \mathbf{p}_s^*)$ *solves the linear programming problem below:*

$$\max_{(\beta_s, \mathbf{p}_s) \in \Delta(\mathbf{P})} \sum_{s \in S} \beta_s ER(\mathbf{p}_s) \ \textit{s.t.} \ \sum_{s \in S} \beta_s \mathbf{p}_s = \mathbf{p}^0.$$

By Proposition 9, optimal disclosure can be achieved by using only vertices in the set $\mathbf{P}$. In particular, notice that every vertex in $\mathbf{P}$ is constructed by pooling no more than two types in $\Omega$. As a result, Proposition 9 greatly simplifies the analysis of optimal disclosure policies, and the formats of the optimal disclosure policies are also similar to those for the binary-type setting, given that we use qualitatively identical signals.

As we show in Lemma 2 (in Appendix), a key step in proving Proposition 9 is to show that for any posterior $\mathbf{p}$ that is not a vertex in $\mathbf{P}$, i.e., $\mathbf{p} \notin \mathbf{P}$, one can always find a probability distribution over some vertices in $\mathbf{P}$ that gives rise to the same posterior beliefs as $\mathbf{p}$, but results in a higher expected revenue than $\mathbf{p}$, using an argument akin to Jensen's inequality.

Proposition 9 seems to resonate with the "exclusion principle" in all-pay auctions (Baye et al., 1993[1]), which states that under some conditions, a contest organizer can increase expected revenue by excluding the strongest contestants so as to level the playing field and strengthen competition among the remaining agents. The organizer chooses a distribution of types, rather than the set of contestants, in our setting with one-sided private information, and Proposition 9 illustrates a similar "exclusion" result when information is endogenous: when disclosing bidder $A$'s type information to $B$, the organizer has incentives to exclude many types in constructing posterior signals so as to avoid excessive information asymmetry, which can dampen the competition. Technically, when more types are included in the posterior signals, bidder $B$ has to play a strategy making all types of bidder $A$ in the posterior signals indifferent in equilibrium, which results in less flexibility for bidder $B$ to bid aggressively, and hence lower both bidders' bids.

It is still cumbersome to present optimal disclosure policies in the general setting, even with the simplification from Proposition 9. Nevertheless, the simplification is indeed helpful in deriving optimal disclosure in the polar common-value and independent value settings. We start with the common-value setting:

**Corollary 5 (Common Valuation Case)** *In the all-pay auction with a general set of types, full disclosure is the unique optimal disclosure policy if $\alpha = 1$, but full disclosure fails to be optimal if $\alpha < 1$ and $|\mathbf{L}| \geq 2$.*

Corollary 5 implies that the results in Proposition 3 and Proposition 6 (and the right panel of Figure 2) for the binary-type setting also hold in the setting with general types.

Next, consider the setting with independent values:

**Corollary 6 (Independent Valuation Case)** *Consider the independent valuation case of the all-pay auction with general types, i.e., $\alpha = 0$.*

[*i*] *Suppose $\mathbb{E}_{\mathbf{p}^0}[v] \leq 1$. Any Bayes-plausible disclosure policy $(\beta_s, \mathbf{p}_s) \in \Delta(\mathbf{P}^{\mathbf{HL}} \cup \mathbf{P}^{\mathbf{H}})$ is an optimal disclosure policy.*

[*ii*] *Suppose $\mathbb{E}_{\mathbf{p}^0}[v] > 1$ with $\sum_{i=1}^{N} p_i^0 v_i = 1$. There exists $\eta \in \{1, ..., m\}$ such that any Bayes-plausible disclosure policy $(\beta_s,\, \mathbf{p}_s) \in \Delta(\{\mathbf{p}^{\tau_h \tau_\ell}\}_{h \geq m+1; \ell=\eta,\ldots,m} \cup \{\mathbf{p}^{\tau_{\ell'}|\tau_0}\}_{\ell'=1,\ldots,\eta})$ is an optimal disclosure policy.*

In case [*i*], bidder $A$ is ex ante stronger. As in the binary-type setting (Proposition 4), optimal disclosure is achieved by concealing weak types (in $\mathbf{L}$) and partially revealing strong types (in $\mathbf{H}$) to bidder $B$. Case [ii] is a representative case in which bidder $A$ is ex ante weaker, i.e., $\mathbb{E}_{\mathbf{p}^0}[v] > 1$.Optimal disclosure is achieved by concealing types in $\mathbf{H}$, and possibly partially revealing types in $\mathbf{L}$ to bidder $B$, exactly the opposite of optimal disclosure in $\mathbb{E}_{\mathbf{p}^0}[v] \leq 1$. The optimal disclosure in case [ii] is similar to that for the corresponding case of $\tau_1 < \tau_2 < 1$ in the binary-type setting (Proposition 6).

In both cases of Corollary 6, the organizer uses a type in $\mathbf{H}$ and a type in $\mathbf{L}$ to construct signals $\mathbf{p}_s$'s such that $\mathbb{E}_{\mathbf{p}_s}[v] = 1$, if possible, so as to perfectly level the playing field. However, in case [*ii*], when there are multiple types in $\mathbf{L}$, in the equilibrium under $\mathbf{p}^0$, only the weakest type $\tau_0$ bids 0, and this will remain true under optimal disclosure. Hence, the best disclosure policy features first pooling less weak types in $\mathbf{L}$ with types in $\mathbf{H}$ to form posterior signals $\mathbf{p}_s$'s with $\mathbb{E}_{\mathbf{p}_s}[v] = 1$ *as much as possible* and then pooling weaker types in $\mathbf{L}$ with type $\tau_0$ to form signals $\mathbf{p}_s$'s in the form of $\mathbf{p}^{\tau_\ell|\tau_0}$. As a result, it "assigns" the probability of bidding 0 completely to $\tau_0$, the residual type, forcing the stronger type to bid more aggressively, which intensifies competition ex ante. We use the following example to further illustrate case [*ii*]:

**Example 1** *Bidder A's set of types is $(\tau_{\ell_0}, \tau_{\ell_1}, \tau_{\ell_2}, \tau_{\ell_3}, \tau_{h_1}, \tau_{h_2}) = (\frac{1}{1.4}, \frac{1}{1.3}, \frac{1}{1.2}, \frac{1}{1.1}, \frac{1}{0.9}, \frac{1}{0.8})$, with prior distribution $\mathbf{p}^0 = (p_{\ell_0}^0, p_{\ell_1}^0, p_{\ell_2}^0, p_{\ell_3}^0, p_{h_1}^0, p_{h_2}^0) = (\frac{1}{10}, \frac{1}{5}, \frac{13}{40}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8})$, and $\alpha = 0$.*

*An optimal disclosure policy for the setting is* $(\beta_s^*, \mathbf{p}_s^*)$ *such that*

$$\{\mathbf{p}_s^*\} = \{\mathbf{p}^{\tau_{\ell_1}|\tau_{\ell_0}}, \mathbf{p}^{\tau_{\ell_2}|\tau_{\ell_0}}, \mathbf{p}^{\tau_{h_2}\tau_{\ell_2}}, \mathbf{p}^{\tau_{h_1}\tau_{\ell_3}}\} \text{ with}$$
$$\{\beta_s^*\} = \left\{\frac{13}{50}, \quad \frac{12}{50}, \quad \frac{1}{4}, \quad \frac{1}{4}\right\}.$$

*Here given* $\mathbb{E}_{\mathbf{p}^0}[v] > 1$*, optimal disclosure involves* (1) *pooling the less weak types* $\tau_{\ell_2}, \tau_{\ell_3}$ *with high types* $\tau_{h_1}, \tau_{h_2}$ *to generate signals* $\mathbf{p}_s \in \{\mathbf{p}^{\tau_{h_2}\tau_{\ell_2}}, \mathbf{p}^{\tau_{h_1}\tau_{\ell_3}}\}$ *such that* $\mathbb{E}_{\mathbf{p}_s}[v] = 1$*, and* (2) *pooling the weaker types* $\tau_{\ell_1}$ *and* $\tau_{\ell_2}$ *with* $\tau_{\ell_0}$ *(who bids 0 for sure under prior* $\mathbf{p}^0$*) to generate signals* $\left\{\mathbf{p}^{\tau_{\ell_1}|\tau_{\ell_0}}, \mathbf{p}^{\tau_{\ell_2}|\tau_{\ell_0}}\right\}$*. Here the "cutoff" type* $\tau_{\ell_2}$ *is used to generate multiple posterior signals* $(\mathbf{p}_s^*)$*.*

*In this example, we can identify another optimal disclosure policy* $(\beta_s^{**}, \mathbf{p}_s^{**})$*:*

$$\{\mathbf{p}_s^{**}\} = \{\mathbf{p}^{\tau_{\ell_1}|\tau_{\ell_0}}, \mathbf{p}^{\tau_{\ell_2}|\tau_{\ell_0}}, \mathbf{p}^{\tau_{h_2}\tau_{\ell_2}}, \mathbf{p}^{\tau_{h_1}\tau_{\ell_2}}, \mathbf{p}^{\tau_{h_2}\tau_{\ell_3}}\} \text{ with}$$
$$\{\beta_s^{**}\} = \left\{\frac{13}{50}, \quad \frac{12}{50}, \quad \frac{1}{8}, \quad \frac{3}{16}, \quad \frac{3}{16}\right\}.$$

*Hence, unlike the binary-type setting, there could be* ***multiple optimal disclosure policies*** *in the setting with general types.*

# 5 Concluding Remarks

In this paper, we have characterized optimal information disclosure for an all-pay auction with one-sided private information and valuation interdependence. We have found that even in such a stylized setting, optimal information design can take various forms such as full disclosure, partial disclosure and no disclosure, depending on the relative (valuation and information) strengths of the bidders and the degree of valuation interdependence. We have also delineated various effects of optimal disclosure on the allocative efficiency and the bidders' welfare in the resulting all-pay auction relative to the traditional no-disclosure benchmark.

Overall, while mathematically obvious given the huge feasible set of information disclosure policies, our analysis shows that information disclosure is indeed a subtle yet versatile instrument in contest design for contest organizers. In particular, our results shed new light on strengths and weaknesses of optimal information disclosure to various parties, which can provide useful implications on how to use and regulate the instrument of information disclosure in practice.

# Appendix. Omitted Proofs.

## Part A: Proofs for the Binary-Type Case.

**Proof of Proposition 4.**

It is straightforward to verify that the equilibrium revenue is strictly convex in $p_2$ over $(0, p_2^{\tau_2\tau_1})$ and over $(p_2^{\tau_2\tau_1}, 1)$. Hence, to find the concave closure of the equilibrium expected revenue we only need to consider the two boundary points, $\mathbf{p}^{\tau_1}$ and $\mathbf{p}^{\tau_2}$, and the critical point $\mathbf{p}^{\tau_2\tau_1}$. Let $\hat{\alpha} \equiv \frac{[(4\tau_2+13)\tau_1^2+(6\tau_2+24)\tau_1+\tau_2^2+4\tau_2+12]^{1/2}+\tau_1-\tau_2-2}{2\tau_2\tau_1+6\tau_1+2}$.[25] When $\alpha > \hat{\alpha}$, the linear curve that connects the expected revenues at the two boundary points is strictly above the expected revenue under distribution $\mathbf{p}^{\tau_2\tau_1}$, or

$$\alpha > \hat{\alpha} \Leftrightarrow ER\left(\mathbf{p}^{\tau_2\tau_1}\right) > p_2^{\tau_2\tau_1} ER\left(\mathbf{p}^{\tau_2}\right) + p_1^{\tau_2\tau_1} ER\left(\mathbf{p}^{\tau_1}\right),$$

and thus full disclosure is optimal.

When $\alpha = \hat{\alpha}$, the expected revenue under distribution $\mathbf{p}^{\tau_2\tau_1}$ is along the linear curve that connects the expected revenues at the two boundary points and thus any Bayes-plausible disclosure policy that randomizes over $\{\mathbf{p}^{\tau_2}, \mathbf{p}^{\tau_1}, \mathbf{p}^{\tau_2\tau_1}\}$ is optimal.

When $\alpha < \hat{\alpha}$, the expected revenue under distribution $\mathbf{p}^{\tau_2\tau_1}$ is strictly above the linear curve that connects the expected revenues at the two boundary points. Thus, if $p_2^0 < p_2^{\tau_2\tau_1}$, the optimal disclosure policy is the one that is Bayes-plausible and randomizes over $\{\mathbf{p}^{\tau_1}, \mathbf{p}^{\tau_2\tau_1}\}$; if $p_2^0 > p_2^{\tau_2\tau_1}$, it is the one that is Bayes-plausible and randomizes over $\{\mathbf{p}^{\tau_2}, \mathbf{p}^{\tau_2\tau_1}\}$; if $p_2^0 = p_2^{\tau_2\tau_1}$, it is exactly $\{\mathbf{p}^{\tau_2\tau_1}\}$. □

**Proof of Proposition 5.**

The equilibrium expected revenue is strictly convex in $p_2$ over $(0, 1)$. Hence, the optimal disclosure, randomizing over $\{\mathbf{p}^{\tau_1}, \mathbf{p}^{\tau_2}\}$, fully discloses $A$'s type. □

**Proof of Proposition 6.**

The equilibrium expected revenue is strictly increasing and strictly convex in $p_2$ over $\left(0, p_2^{\tau_2|\tau_1}\right)$ and constant over $\left[p_2^{\tau_2|\tau_1}, 1\right]$. Hence, the concave closure of the equilibrium expected revenue is the linear curve that connects the expected revenues at $\mathbf{p}^{\tau_1}$ and $\mathbf{p}^{\tau_2|\tau_1}$ and the one that connects expected revenues at $\mathbf{p}^{\tau_2|\tau_1}$ and $\mathbf{p}^{\tau_2}$. □

[25] $\hat{\alpha}$ is a solution to $\alpha^2\tau_1\tau_2 + \alpha\tau_2 + 3\alpha^2\tau_1 - \alpha\tau_1 - \tau_1 + \alpha^2 + 2\alpha - 2 = 0$. It can be shown that $\hat{\alpha}$ is strictly increasing in $\tau_1$ and strictly decreasing in $\tau_2$.

**Proof of Proposition 7.**
The equilibrium expected values generated by bidder $A$'s and bidder $B$'s winning, $EF^A$ and $EF^B$, are respectively:

$$EF^A(\mathbf{p}) = \begin{cases} p_1\left(1 - \frac{p_1 v_1}{2} - p_2 v_2\right) V_1^A + p_2\left(1 - \frac{p_2 v_2}{2}\right) V_2^A & \text{for } \mathbb{E}_{\mathbf{p}}[v] \leq 1 \\ \frac{(1-p_2 v_2)^2}{2v_1} V_1^A + p_2\left(1 - \frac{p_2 v_2}{2}\right) V_2^A & \text{for } p_2 v_2 < 1 < \mathbb{E}_{\mathbf{p}}[v] \\ \frac{V_2^A}{2v_2} & \text{for } 1 \leq p_2 v_2; \end{cases}$$

$$EF^B(\mathbf{p}) = \begin{cases} p_1 v_1 \frac{p_1 V_1^B}{2} + p_2 v_2\left(p_1 V_1^B + \frac{p_2 V_2^B}{2}\right) & \text{for } \mathbb{E}_{\mathbf{p}}[v] \leq 1 \\ (1 - p_2 v_2) p_1\left(1 - \frac{1-p_2 v_2}{2p_1 v_1}\right) V_1^B + p_2 v_2\left(p_1 V_1^B + \frac{p_2 V_2^B}{2}\right) & \text{for } p_2 v_2 < 1 < \mathbb{E}_{\mathbf{p}}[v] \\ p_1 V_1^B + p_2\left(1 - \frac{1}{2p_2 v_2}\right) V_2^B & \text{for } 1 \leq p_2 v_2. \end{cases}$$

Let $\mu(\alpha) \equiv \alpha\tau_1\tau_2 + \alpha^2\tau_2 - \alpha^2\tau_1 + 2\tau_1 + \alpha^2 + \alpha - 1$ and define the allocative efficiency as $EF(\mathbf{p}) \equiv EF^A(\mathbf{p}) + EF^B(\mathbf{p})$. We have:

$$\frac{d^2 EF(\mathbf{p})}{dp_2^2} = \begin{cases} -\frac{(1-\alpha)(\tau_2-\tau_1)}{(\tau_1+\alpha)(\tau_2+\alpha)}\mu(\alpha) & \text{for } \mathbb{E}_{\mathbf{p}}[v] \leq 1 \\ -\frac{(1-\alpha)(\tau_2-\tau_1)(1+\alpha\tau_2)(\alpha\tau_2\tau_1+\tau_2+\tau_1+\alpha^2+\alpha-1)}{(1+\alpha\tau_1)(\tau_2+\alpha)^2} & \text{for } p_2 v_2 < 1 < \mathbb{E}_{\mathbf{p}}[v] \\ 0 & \text{for } 1 \leq p_2 v_2. \end{cases}$$

Also notice that if $\tau_1 > \frac{1}{2}$, $\mu(\alpha) > 0$ for all $\alpha \in [0,1]$; if $\tau_1 \leq \frac{1}{2}$, $\mu(\alpha) > 0$ for $\alpha > \tilde{\alpha} \equiv \frac{\left[\tau_1^2\tau_2^2+(4-6\tau_1)\tau_2+8\tau_1^2-12\tau_1+5\right]^{1/2}-\tau_1\tau_2-1}{2(\tau_2-\tau_1+1)}$, $\mu(\alpha) < 0$ for $\alpha < \tilde{\alpha}$, and $\mu(\alpha) = 0$ for $\alpha = \tilde{\alpha}$.[26]

First, consider the cases of $\tau_1 < 1 < \tau_2$ and $\tau_1 < \tau_2 \leq 1$. In each of the equilibrium cases, $EF(\mathbf{p})$ is concave in $p_2$ over $(0,1)$. By Jensen's inequality, allocative efficiency is lower under optimal disclosure than under no disclosure.[27]

Next, consider our leading case of $\tau_1 < 1 < \tau_2$.

**Case 1.** $\alpha > \hat{\alpha}$. The optimal policy is full disclosure. Let $G(p_2^0) \equiv p_1^0 EF(\mathbf{p}^{\tau_1}) + p_2^0 EF(\mathbf{p}^{\tau_2}) - EF(\mathbf{p}^0)$, which is the efficiency difference between optimal disclosure and no disclosure under the prior $\mathbf{p}^0$. We now show that $G(p_2^0) < 0$.

For $p_2^0 \geq p_2^{\tau_2\tau_1}$ (or $\mathbb{E}_{\mathbf{p}^0}[v] \leq 1$):

$$G(p_2^0) = \frac{\left[-(\tau_2-\tau_1)(\alpha\tau_1+1)\mu(\alpha)p_2^0 - (1-\alpha)^2(1-\tau_1)^3(\tau_2+\alpha)\right](1-\alpha)(1-p_2^0)}{2(\tau_1+\alpha)(\tau_2+\alpha)(\alpha\tau_1+1)},$$

[26] Since $\mu(\alpha)$ is quadratic, and $\mu(1) > 0$, $\mu(0) \leq 0$ (for $\tau_1 \leq 1/2$), $\tilde{\alpha}$ is the unique solution to $\mu(\alpha) = 0$.

[27] Specifically, when $1 \leq \tau_1 < \tau_2$, $EF(\mathbf{p})$ is strictly concave in $p_2$ over $(0,1)$; when $\tau_1 < \tau_2 \leq 1$, $EF(\mathbf{p})$ is strictly concave in $p_2$ over $(0, p_2^{\tau_2|\tau_1})$ and linear over $[p_2^{\tau_2|\tau_1}, 1)$.

which is strictly negative: For $\mu(\alpha) \geq 0$, $G(p_2^0) < 0$ is immediate; for $\mu(\alpha) < 0$, we have:

$$\begin{aligned}
& -(\tau_2 - \tau_1)(\alpha\tau_1 + 1)\mu(\alpha) - (1-\alpha)^2(1-\tau_1)^3(\tau_2 + \alpha) \\
& = \left\{ \begin{array}{c} -(\tau_2 - \tau_1)(\alpha^2\tau_1\tau_2 + \alpha\tau_2 + 3\alpha^2\tau_1 - \alpha\tau_1 - \tau_1 + \alpha^2 + 2\alpha - 2) - (1-\alpha)\left[(\alpha\tau_1^2 + 2\tau_1)(\tau_2 - 1) \right. \\ \left. +\tau_1\tau_2(1-\tau_1) + \tau_1(\tau_2 - \tau_1) + (2\alpha\tau_1 + 1)(\tau_2 - 2\tau_1) + (\alpha\tau_2 + 2\alpha\tau_1) + (1-\alpha)\right] \end{array} \right\} \\
& < 0,
\end{aligned}$$

where the inequality is due to (1) $\mu(\alpha) < 0$ implies $2\tau_1 < 1 < \tau_2$ and (2) $\alpha > \hat{\alpha}$ implies $\alpha^2\tau_1\tau_2 + \alpha\tau_2 + 3\alpha^2\tau_1 - \alpha\tau_1 - \tau_1 + \alpha^2 + 2\alpha - 2 > 0$.

For $p_2^0 \leq p_2^{\tau_2\tau_1}$ (or $1 \leq \mathbb{E}_{\mathbf{p}}[v]$):

$$G(p_2^0) = \left[\frac{v_2(1-v_2)V_2^A}{2} + \frac{v_2^2(v_1 - 1)V_1^A}{2v_1}\right](p_2^0)^2 + \left[\frac{(v_2 - 1)V_2^B}{2} + \frac{(v_1 - 1)(1 - 2v_2)V_1^A}{2v_1}\right] p_2^0.$$

Here $G(p_2^0)$ is strictly convex over $(0, p_2^{\tau_2\tau_1})$, equal to 0 when $p_2^0 = 0$, and strictly negative when $p_2^0 = p_2^{\tau_2\tau_1}$. Hence, $G(p_2^0) < 0$ for $p_2^0 \in (0, p_2^{\tau_2\tau_1}]$.

**Case 2.** $\alpha < \hat{\alpha}$.

For $p_2^0 \in (0, p_2^{\tau_2\tau_1})$ (or $1 < \mathbb{E}_{\mathbf{p}^0}[v]$), optimal disclosure features $\{\mathbf{p}^{\tau_2\tau_1}, \mathbf{p}^{\tau_1}\}$, and $EF(\mathbf{p})$ is strictly concave in $p_2$ over $(0, p_2^{\tau_2\tau_1})$. Hence, allocative efficiency is strictly lower under optimal disclosure by Jensen's inequality.

For $p_2^0 \in (p_2^{\tau_2\tau_1}, 1)$, optimal disclosure features $\{\mathbf{p}^{\tau_2\tau_1}, \mathbf{p}^{\tau_2}\}$. $EF(\mathbf{p})$ is strictly concave in $p_2$ if $\mu(\alpha) > 0$, strictly convex if $\mu(\alpha) < 0$, and linear if $\mu(\alpha) = 0$. By Jensen's inequality, allocative efficiency is strictly lower under optimal disclosure than under no disclosure if $\mu(\alpha) > 0$, strictly higher if $\mu(\alpha) < 0$, and is the same if $\mu(\alpha) = 0$. $\square$

**Proof of Proposition 8.**

First, in the equilibrium under posterior $\mathbf{p}$, each bidder's expected payoff is the bidder's value from winning, net of his expected payment:

$$EU^A(\mathbf{p}) = EF^A(\mathbf{p}) - EP^A(\mathbf{p}) \text{ and } EU^B(\mathbf{p}) = EF^B(\mathbf{p}) - EP^B(\mathbf{p}),$$

where $EF^A$ and $EF^B$ are defined in Proposition 7, and $EP^A$ and $EP^B$, defined in Corollary 1, are calculated according to the three equilibrium formats: (i) $\mathbb{E}_{\mathbf{p}}[v] \leq 1$; (ii) $p_2v_2 < 1 < \mathbb{E}_{\mathbf{p}}[v]$; (iii) $1 \leq p_2v_2$.

First, consider bidder $A$'s expected payoff, $EU^A(\mathbf{p})$. We will use concavity of $EU^A(\mathbf{p})$ and Jensen's inequality to evaluate $EU^A(\mathbf{p})$ under different information structures.

For the simple case of $1 \leq \tau_1 < \tau_2$, $EU^A(\mathbf{p})$ is strictly concave in $p_2$ over $(0, 1)$. By Jensen's inequality, bidder $A$'s expected payoff is strictly lower under optimal disclosure, full disclosure in this case, than under no disclosure.

For the case of $\tau_1 < \tau_2 \leq 1$, $EU^A(\mathbf{p})$ is increasing and concave of in $p_2$ over $(0, 1)$. By Jensen's inequality, bidder $A$'s expected payoff under optimal disclosure, partial disclosure in this case, is lower than that under no disclosure but higher than that under full disclosure.

For the case of $\tau_1 < 1 < \tau_2$, $EU^A(\mathbf{p})$ is strictly concave in $p_2$ over each of the two intervals–$(0, p_2^{\tau_2\tau_1})$ and $(p_2^{\tau_2\tau_1}, 1)$–and $EU^A(\mathbf{p}^{\tau_2\tau_1}) - \left[p_1^{\tau_2\tau_1} EU^A(\mathbf{p}^{\tau_1}) + p_2^{\tau_2\tau_1} EU^A(\mathbf{p}^{\tau_2})\right] = \frac{(v_1-1)(1-v_2)(V_2^B - V_1^B)}{(v_1-v_2)^2} > 0$. Hence, one can immediately verify that bidder $A$'s expected payoff under optimal disclosure (partial disclosure if $a < \hat{a}$ and full disclosure if $a > \hat{a}$, except when $\mathbf{p}^0 = \mathbf{p}^{\tau_2\tau_1}$) is lower than that under no disclosure but higher than that under full disclosure.

The analysis for the expected revenue of bidder $B$ is the mirror image of that of bidder $A$ and is thus omitted.

Next, consider the bidders' total expected payoff, $EU^A + EU^B$. For the case in which $\tau_1 < 1 < \tau_2$ and $\alpha > \hat{\alpha}$, the total expected payoff is strictly lower under optimal disclosure, as the allocative efficiency is strictly lower and the expected revenue is strictly higher under optimal disclosure. For all the other cases, the analysis is qualitatively identical to that for the expected payoff of bidder $A$ and can be done analogously.

Finally, consider bidder $A$'s probability of winning, which can be calculated as:

$$EW^A(\mathbf{p}) = \begin{cases} p_1\left(1 - \frac{p_1 v_1}{2} - p_2 v_2\right) + p_2\left(1 - \frac{p_2 v_2}{2}\right) & \text{for } \mathbb{E}_{\mathbf{p}}[v] \leq 1 \\ \frac{(1-p_2 v_2)^2}{2 v_1} + p_2\left(1 - \frac{p_2 v_2}{2}\right) & \text{for } p_2 v_2 < 1 < \mathbb{E}_{\mathbf{p}}[v] \\ \frac{1}{2 v_2} & \text{for } 1 \leq p_2 v_2 \end{cases}$$

One can verify that similar to $EU^A(\mathbf{p})$, $EW^A(\mathbf{p})$ is strictly concave in $p_2$ in the first two cases and linear in the third case. We hence have similar comparative statics results for $EW^A(\mathbf{p})$ as $EU^A(\mathbf{p})$ over the three information structures. $\square$

## Part B: Equilibrium Analysis for the General-Type Setting.

Consider the following restricted set of posteriors

$$\mathbf{\Gamma} \equiv \left\{ \mathbf{p} \in \Delta^N \middle| \sum_{i=1}^{N} p_i v_i \leq 1 \text{ if } p_0 = 0, \sum_{i=1}^{N} p_i v_i = 1 \text{ if } p_0 > 0 \right\}.$$

Take a distribution $\mathbf{p} \in \mathbf{\Gamma}$. When $p_0 = 0$, $\mathbb{E}_{\mathbf{p}}[v] \leq 1$. When $p_0 > 0$, $\mathbb{E}_{\mathbf{p}}[v] > 1$, and bidder $A$ bids 0 with probability $p_0$ in equilibrium. Hence, $\tau_0$ is a residual type, which collects bidder $A$'s probability of bidding 0. The following reasons motivate us to focus on posterior distributions in the set $\mathbf{\Gamma}$:

- To fully analyze the set of posteriors $\mathbf{p} \in \Delta^N$ (with $N \geq 1$), it is equivalent to analyzing the set of posteriors $\tilde{\mathbf{p}} \in \Delta^N$ with $\tilde{p}_0 = 0$ (and with $N \geq 2$).
- For any $\tilde{\mathbf{p}} \in \Delta^N$ with $\tilde{p}_0 = 0$, we can always find a $\mathbf{p} \in \mathbf{\Gamma}$ such that the equilibrium bids of the two bidders remain the same across the two public belief posteriors.

Proposition 10 below presents the equilibrium for the setting with general types. Following that, Claim 1 illustrates how focusing on set $\mathbf{\Gamma}$ simplifies our analysis.

**Proposition 10 ($(N+1)$-Type Equilibrium)** *Under posterior $\mathbf{p} \in \mathbf{\Gamma}$, there is a unique equilibrium with the following equilibrium strategies:*

- *Type $\tau_0$, if $p_0 > 0$, bids 0. Type $\tau_k$, $k = 1, ..., N$, randomizes uniformly over $\left[\sum_{i=1}^{k-1} p_i V_i^B, \sum_{i=1}^{k} p_i V_i^B\right]$. Bidder $B$ bids 0 with probability $1 - \sum_{i=1}^{N} p_i v_i$ and randomizes uniformly over $\left[\sum_{i=1}^{k-1} p_i V_i^B, \sum_{i=1}^{k} p_i V_i^B\right]$ with probability $p_k v_k$ for $k = 1, ..., N$.*

*The expected payments of the bidders are:*

- $EP^A(\mathbf{p}) = \frac{1}{2} \sum_{k=1}^{N} p_k \cdot \left(\sum_{i=1}^{k-1} p_i V_i^B + \sum_{i=1}^{k} p_i V_i^B\right)$;
- $EP^B(\mathbf{p}) = \frac{1}{2} \sum_{k=1}^{N} p_k v_k \cdot \left(\sum_{i=1}^{k-1} p_i V_i^B + \sum_{i=1}^{k} p_i V_i^B\right)$.

**Proof of Propositions 1 and 10.**

It is straightforward to verify that for each of the three cases, all the players are playing best responses in the prescribed strategy profile. The details are hence omitted. Uniqueness of equilibrium is standard. See, e.g., Proposition 1 of Siegel (2014)[13], and strict monotonicity (condition M) holds in our setting. $\square$

**Claim 1** *For any posterior $\tilde{\mathbf{p}} \in \Delta^N$ with $\tilde{p}_0 = 0$, there exists a $\mathbf{p} \in \mathbf{\Gamma}$ such that each bidder's equilibrium bid distribution remains the same across the posteriors $\tilde{\mathbf{p}}$ and $\mathbf{p}$.*

**Proof of Claim 1.**

Take a $\tilde{\mathbf{p}} \in \Delta^N$ with $\tilde{p}_0 = 0$. There are two cases. First, suppose $\sum_{i=1}^N \tilde{p}_i v_i \leq 1$. Then, $\tilde{\mathbf{p}} \in \mathbf{\Gamma}$. Second, suppose $\sum_{i=1}^N \tilde{p}_i v_i > 1$. We can find a $k \geq 2$ and $\lambda \in [0,1]$ such that $\lambda \tilde{p}_{k-1} v_{k-1} + \sum_{i=k}^N \tilde{p}_i v_i = 1$. Then, we can construct a $\mathbf{p} \in \mathbf{\Gamma}$, where $p_0 = (1-\lambda)\tilde{p}_{k-1} + \sum_{i=0}^{k-2} \tilde{p}_i$, $p_i = \tilde{p}_i$ for $i = k, ..., N$, and $p_i = 0$ for $i = 1, ..., k-2$ if $k > 2$. We can verify that bidders' equilibrium bid distributions under posterior $\tilde{\mathbf{p}}$ are the same as those under posterior $\mathbf{p}$. $\square$

## Part C: Proofs for Optimal Disclosure in the General-Type Setting.

We derive the main result for optimal disclosure policies in three steps. First, we show that if a posterior $\mathbf{p}$ is a reduced lottery of a lottery over two vertices $\mathbf{p}'$ and $\mathbf{p}''$, then replacing $\mathbf{p}$ with $\mathbf{p}'$ and $\mathbf{p}''$ will yield a higher expected revenue (Lemma 1). Recall that $ER(\mathbf{p})$ is defined as the expected revenue under posterior $\mathbf{p}$. Let $\mathbf{p} = \epsilon\mathbf{p}' + (1-\epsilon)\mathbf{p}''$ for some $\epsilon \in (0,1)$. We need to show that $ER(\mathbf{p})$ is convex in $\epsilon$, i.e., the second derivative $L(\mathbf{p}', \mathbf{p}'') \equiv \frac{\partial^2}{\partial \epsilon^2}\left[ER\left(\epsilon\mathbf{p}' + (1-\epsilon)\mathbf{p}''\right)\right]$ is positive. In Lemma 1, we use subscripts on $L$ to differentiate four explicit cases.

Second, we use the result of Lemma 1 and show that for any posterior $\mathbf{p}$ that is not a vertex in $\mathbf{P}$, we can always replace it with vertices in $\mathbf{P}$ whose reduced lottery remains $\mathbf{p}$, but will yield a higher expected revenue (Lemma 2). Thus, it is not optimal to use posteriors that are not vertices in $\mathbf{P}$.

Finally, using Lemma 2 together with the results for binary types, we derive optimal disclosure policies (Proposition 9).

**Lemma 1** *Let us define four functions below.*

*(i)* *Take $\mathbf{p}^{\tau_\ell|\tau_0}, \mathbf{p}^{\tau_{\ell'}|\tau_0} \in \mathbf{P}^{\mathbf{L}|\mathbf{L}}$, where $\tau_\ell > \tau_{\ell'}$. Let*

$$L_1(\mathbf{p}^{\tau_\ell|\tau_0}, \mathbf{p}^{\tau_{\ell'}|\tau_0}) := 2\left[ER(\mathbf{p}^{\tau_\ell|\tau_0}) + ER(\mathbf{p}^{\tau_{\ell'}|\tau_0}) - p_\ell^{\tau_\ell|\tau_0}(1+v_\ell)p_{\ell'}^{\tau_{\ell'}|\tau_0}V_{\ell'}^B\right].$$

*(ii)* *Take $\mathbf{p}^{\tau_{h'}\tau_{\ell'}} \in \mathbf{P}^{\mathbf{HL}}$ and $\mathbf{p}^{\tau_h} \in \mathbf{P}^{\mathbf{H}}$, where $\tau_h \geq \tau_h' > 1 > \tau_{\ell'}$. Let*

$$L_2\left(\mathbf{p}^{\tau_{h'}\tau_{\ell'}}, \mathbf{p}^{\tau_h}\right) := 2\left[ER(\mathbf{p}^{\tau_{h'}\tau_{\ell'}}) + ER(\mathbf{p}^{\tau_h}) - \left(p_{\ell'}^{\tau_{h'}\tau_{\ell'}}V_{\ell'}^B + p_{h'}^{\tau_{h'}\tau_{\ell'}}V_{h'}^B\right)(1+v_h)\right].$$

(iii) *Take* $\mathbf{p}^{\tau_h\tau_\ell} \in \mathbf{P^{HL}}$ *and* $\mathbf{p}^{\tau_{\ell'}|\tau_0} \in \mathbf{P^{L|L}}$, *where* $\tau_\ell > \tau_{\ell'}$. *Let*

$$L_3(\mathbf{p}^{\tau_h\tau_\ell}, \mathbf{p}^{\tau_{\ell'}|\tau_0}) := 2\left[ER(\mathbf{p}^{\tau_h\tau_\ell}) + ER(\mathbf{p}^{\tau_{\ell'}|\tau_0}) - 2p_{\ell'}^{\tau_{\ell'}|\tau_0}V_{\ell'}^B\right].$$

(iv) *Take* $\mathbf{p}^{\tau_h\tau_\ell}, \mathbf{p}^{\tau_{h'}\tau_{\ell'}} \in \mathbf{P^{HL}}$, *where* $\tau_\ell \geq \tau_{\ell'}$ *and* $\mathbf{p}^{\tau_h\tau_\ell} \neq \mathbf{p}^{\tau_{h'}\tau_{\ell'}}$. *Let*

$$\begin{aligned} L_4\left(\mathbf{p}^{\tau_h\tau_\ell}, \mathbf{p}^{\tau_{h'}\tau_{\ell'}}\right) =& L_4\left(\mathbf{p}^{\tau_{h'}\tau_{\ell'}}, \mathbf{p}^{\tau_h\tau_\ell}\right) \\ :=& 2\left[ER\left(\mathbf{p}^{\tau_h\tau_\ell}\right) + ER\left(\mathbf{p}^{\tau_{h'}\tau_{\ell'}}\right) - 2p_{\ell'}^{\tau_{h'}\tau_{\ell'}}V_{\ell'}^B - p_{h'}^{\tau_{h'}\tau_{\ell'}}(1+v_{h'})p_\ell^{\tau_h\tau_\ell}V_\ell^B - \gamma\right], \\ & \textit{where} \quad \gamma = \begin{cases} p_{h'}^{\tau_{h'}\tau_{\ell'}}(1+v_{h'})p_h^{\tau_h\tau_\ell}V_h^B, & \textit{if } h' \geq h \\ p_h^{\tau_h\tau_\ell}(1+v_h)p_{h'}^{\tau_{h'}\tau_{\ell'}}V_{h'}^B, & \textit{if } h' < h. \end{cases} \end{aligned}$$

*Then,* $L_1, L_2, L_3, L_4 > 0$.

**Proof of Lemma 1.**

The proof involves tedious but straightforward calculations, and is hence relegated to an online supplementary appendix. □

**Lemma 2** *Take a* $\mathbf{p} \in \mathbf{\Gamma}$. *There exists a* $(\beta_s, \mathbf{p}_s) \in \Delta(\mathbf{P^{HL}} \cup \mathbf{P^H} \cup \mathbf{P^{L|L}})$ *with* $\sum_{s\in S}\beta_s\mathbf{p}_s = \mathbf{p}$ *such that*

$$\sum_{s\in S}\beta_s ER(\mathbf{p}_s) \geq ER(\mathbf{p})$$

*with equality holds iff* $\mathbf{p} \in \mathbf{P^{HL}} \cup \mathbf{P^H} \cup \mathbf{P^{L|L}}$.

**Proof of Lemma 2.**

For expositional convenience, denote $\beta^{\tau_h\tau_\ell}$, $\beta^{\tau_\ell|\tau_{\ell'}}$, $\beta^{\tau_h}$, and $\beta^{\tau_\ell}$ as the probabilities that bidder $B$ receives signals $\mathbf{p}^{\tau_h\tau_\ell}$, $\mathbf{p}^{\tau_\ell|\tau_{\ell'}}$, $\mathbf{p}^{\tau_h}$, and $\mathbf{p}^{\tau_\ell}$, respectively.

[i]. First, take a take a $\mathbf{p} \in \mathbf{\Gamma}$ with $p_0 = 0$. We can find a disclosure policy $(\beta_s, \mathbf{p}_s) \in \Delta(\mathbf{P^{HL}} \cup \mathbf{P^H})$ with a $\eta \in \{1, ..., n\}$ such that

$$\mathbf{p} = \sum_{\ell=1}^{m}\sum_{h=m+1}^{m+\eta}\beta^{\tau_h\tau_\ell}\mathbf{p}^{\tau_h\tau_\ell} + \sum_{h=m+\eta}^{n}\beta^{\tau_h}\mathbf{p}^{\tau_h} \text{ with } \sum_{\ell=1}^{m}\sum_{h=m+1}^{m+\eta}\beta^{\tau_h\tau_\ell} + \sum_{h=m+\eta}^{m+n}\beta^{\tau_h} = 1.$$

Then, under the posterior $\mathbf{p}$,

$$p_\ell = \sum_{h=m+1}^{m+\eta} \beta^{\tau_h\tau_\ell} p_\ell^{\tau_h\tau_\ell} \text{ for } \ell = 1, ..., m; \tag{4}$$

$$p_h = \begin{cases} \sum_{\ell=1}^m \beta^{\tau_h\tau_\ell} p_h^{\tau_h\tau_\ell} & \text{for } h = m+1, ..., m+\eta-1 \\ \sum_{\ell=1}^m \beta^{\tau_h\tau_\ell} p_h^{\tau_h\tau_\ell} + \beta^{\tau_h} p_h^{\tau_h} & \text{for } h = m+\eta \\ \beta^{\tau_h} p_{\tau_h}^{\tau_h} & \text{for } h = m+\eta+1, ..., m+n. \end{cases} \tag{5}$$

Then, substituting (4) and (5) into the expression of the expected revenue under posterior $\mathbf{p}$ (as given in Proposition 10), it can be verified that:

$$\begin{aligned} ER(\mathbf{p}) =& \sum_{\ell=1}^{m} \sum_{h=m+1}^{m+\eta} \beta^{\tau_h\tau_\ell} ER\left(\mathbf{p}^{\tau_h\tau_\ell}\right) - \frac{1}{4} \sum_{\substack{(\ell,h) \\ m+1\le h\le m+\eta \\ 1\le \ell\le m}} \sum_{\substack{(\ell',h')\neq(\ell,h) \\ m+1\le h'\le m+\eta \\ 1\le \ell'\le m}} \beta^{\tau_h\tau_\ell}\beta^{\tau_{h'}\tau_{\ell'}} L_4\left(\mathbf{p}^{\tau_h\tau_\ell}, \mathbf{p}^{\tau_{h'}\tau_{\ell'}}\right) \\ &+ \sum_{h=m+\eta}^{m+n} \beta^{\tau_h} ER(\mathbf{p}^{\tau_h}) - \frac{1}{2} \sum_{\substack{(\ell',h') \\ m+1\le h'\le m+\eta \\ 1\le \ell\le m}} \sum_{h=m+\eta}^{m+n} \beta^{\tau_{h'}\tau_{\ell'}} \beta^{\tau_h} L_2(\mathbf{p}^{\tau_{h'}\tau_{\ell'}}, \mathbf{p}^{\tau_h}) \\ \le& \sum_{\ell=1}^{m} \sum_{h=m+1}^{m+\eta} \beta^{\tau_h\tau_\ell} ER\left(\mathbf{p}^{\tau_h\tau_\ell}\right) + \sum_{h=m+\eta}^{m+n} \beta^{\tau_h} ER(\mathbf{p}^{\tau_h}), \end{aligned}$$

with equality holds if and only if $\mathbf{p} \in \mathbf{P^{HL}} \cup \mathbf{p^{H}}$.

[ii]. Second, take a $\mathbf{p} \in \mathbf{\Gamma}$ with $p_0 > 0$. We can find a disclosure policy $(\beta_s, \mathbf{p}_s) \in \Delta(\mathbf{P^{HL}} \cup \mathbf{P^{L|L}})$ with a $\eta \in \{1, ..., m\}$ such that

$$\mathbf{p} = \sum_{\ell=1}^{\eta} \beta^{\tau_\ell|\tau_0} \mathbf{p}^{\tau_\ell|\tau_0} + \sum_{\ell=\eta}^{m} \sum_{h=m+1}^{m+n} \beta^{\tau_h\tau_\ell} \mathbf{p}^{\tau_h\tau_\ell} \text{ with } \sum_{\ell=1}^{\eta} \beta^{\tau_\ell|\tau_0} + \sum_{\ell=\eta}^{m} \sum_{h=m+1}^{m+n} \beta^{\tau_h\tau_\ell} = 1.$$

Then, under posterior $\mathbf{p}$,

$$p_\ell = \begin{cases} \beta^{\tau_\ell|\tau_0} p_\ell^{\tau_\ell|\tau_0} & \text{for } \ell = 1, ..., \eta-1 \\ \beta^{\tau_\eta|\tau_0} p_\eta^{\tau_\eta|\tau_0} + \sum_{h=m+1}^{m+n} \beta^{\tau_h\tau_\eta} p_\eta^{\tau_h\tau_\eta} & \text{for } \ell = \eta \\ \sum_{h=m+1}^{m+n} \beta^{\tau_h\tau_\ell} p_\ell^{\tau_h\tau_\ell} & \text{for } \ell = \eta+1, ..., m; \end{cases} \tag{6}$$

$$p_h = \sum_{\ell=\eta}^{m} \beta^{\tau_h\tau_\ell} p_h^{\tau_h\tau_\ell} \text{ for } h = m+1, ..., m+n. \tag{7}$$

Then, substituting (6) and (7) into the expression of the expected revenue under posterior $\mathbf{p}$, it can be verified that:

$$
\begin{aligned}
ER(\mathbf{p}) = & \sum_{\ell=1}^{\eta} \beta^{\tau_\ell|\tau_0} ER(\mathbf{p}^{\tau_\ell|\tau_0}) + \sum_{\ell=\eta}^{m} \sum_{h=m+1}^{m+n} \beta^{\tau_h\tau_\ell} ER\left(\mathbf{p}^{\tau_h\tau_\ell}\right) \\
& - \frac{1}{2} \sum_{\ell=2}^{\eta} \sum_{\ell'=1}^{\ell-1} \beta^{\tau_\ell|\tau_0} \beta^{\tau_{\ell'}|\tau_0} L_1\left(\mathbf{p}^{\tau_\ell|\tau_0}, \mathbf{p}^{\tau_{\ell'}|\tau_0}\right) - \frac{1}{2} \sum_{\ell=\eta}^{m} \sum_{h=m+1}^{m+n} \sum_{\ell'=1}^{\eta} \beta^{\tau_h\tau_\ell} \beta^{\tau_{\ell'}|\tau_0} L_3\left(\mathbf{p}^{\tau_h\tau_\ell}, \mathbf{p}^{\tau_{\ell'}|\tau_0}\right) \\
& - \frac{1}{4} \sum_{\substack{(\ell,h) \\ m+1\le h\le m+n \\ \eta\le\ell\le m}} \sum_{\substack{(\ell',h')\neq(\ell,h) \\ m+1\le h'\le m+n \\ \eta\le\ell\le m}} \beta^{\tau_h\tau_\ell} \beta^{\tau_{h'}\tau_{\ell'}} L_4\left(\mathbf{p}^{\tau_h\tau_\ell}, \mathbf{p}^{\tau_{h'}\tau_{\ell'}}\right) \\
\le & \sum_{\ell=1}^{\eta} \beta^{\tau_\ell|\tau_0} ER(\mathbf{p}^{\tau_\ell|\tau_0}) + \sum_{h=m+1}^{m+n} \sum_{\ell=\eta}^{m} \beta^{\tau_h\tau_\ell} ER\left(\mathbf{p}^{\tau_h\tau_\ell}\right)
\end{aligned}
$$

with equality holds if and only if $\mathbf{p} \in \mathbf{P}^{\mathbf{HL}} \cup \mathbf{p}^{\mathbf{L}|\mathbf{L}}$. □

Lemma 2 demonstrates that it is never strictly better to pool more than two types together.[28] Case $[i]$ in the above proof shows that for any signal $\mathbf{p}$ where $\mathbb{E}_{\mathbf{p}}[v] \le 1$, it is better to replace it with signals in the forms of $\mathbf{p}^{\tau_h\tau_\ell}$ and $\mathbf{p}^{\tau_h}$. Case $[ii]$ shows that for any signal $\mathbf{p}$ where $\mathbb{E}_{\mathbf{p}}[v] > 1$, it is better to replace it with signals in the forms of $\mathbf{p}^{\tau_h\tau_\ell}$ and $\mathbf{p}^{\tau_{\ell'}|\tau_0}$. Technically, the idea behind it is similar to the Jensen's inequality.

**Remark 1** *Note that in case $[ii]$, in the equilibrium under posterior $\mathbf{p}$, type $\tau_0$ bids $0$ with probability $1$ and no other type of bidder A nor bidder B bids $0$ with positive probability. Under our constructed policy $(\beta_s, \mathbf{p}_s) \in \Delta(\mathbf{P}^{\mathbf{HL}} \cup \mathbf{P}^{\mathbf{L}|\mathbf{L}})$, it is also that type $\tau_0$ bids $0$ with probability $1$ and no other type of bidder A nor bidder B bids $0$ with positive probability.*

**Proof of Proposition 9.**

First, as mentioned before, to analyze the optimal disclosure policies for a given prior $\mathbf{p}^0 \in int(\Delta^N)$, it is equivalent to analyzing the optimal disclosure policies for prior $\tilde{\mathbf{p}}^0$ with $\tilde{p}^0_0 = 0$ and $\tilde{p}^0_i > 0$ for $i = 1, ..., N$ (and with $N \ge 2$). Second, following Claim 1, for any posterior $\tilde{\mathbf{p}} \notin \mathbf{P}$ with $\tilde{p}_0 = 0$, we can find a $\mathbf{p} \in \mathbf{\Gamma}$ (if $\mathbb{E}_{\tilde{\mathbf{p}}}[v] > 1$, we need to replace the weakest types (who bid zero in equilibrium) with type $\tau_0$) such that bidder's bid distributions remain the same across the two posteriors. Third, following Lemma 2, we can replace $\mathbf{p}$ with vertices whose reduced lottery remains $\mathbf{p}$, and they will yield a strictly higher expected revenue (if $\mathbb{E}_{\tilde{\mathbf{p}}}[v] > 1$, we need to replace $\tau_0$ back by their original types

[28]In fact, it is never optimal to do so, unless all types except for the highest one are non-active (bid 0 with probability 1).

to the vertices, we can restore the Bayes-plausibility condition). Thus, such a posterior $\tilde{\mathbf{p}}$ will not be used in any optimal disclosure policy. Hence, any solution to the linear programming problem over disclosure policies supported by vertices in $\mathbf{P}$ is an optimal disclosure policy. $\square$

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