Market Structure and Adverse Selection

Dakang Huang and Christopher Sandmann

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Abstract

We consider an insurance economy plagued by adverse selection where a planner pre-assigns roles to prospective sellers. This choice determines which sellers a buyer can jointly trade with. To date, only two polar market structures have been explored. Under exclusive competition as in Rothschild and Stiglitz (1976), each buyer can trade with at most one seller. Under nonexclusive competition as in Attar, Mariotti and Salanié (2011,2014,2021,2022), buyers can trade with arbitrarily many sellers. While the choice of market structure matters, the welfare comparison is ambiguous: Exclusive competition gives rise to separation, low prices for low risk types yet frequently involves rationing. Nonexclusive competition forces low risk types to pool with high risk types and thereby pay higher prices, but does not involve rationing. In this paper we propose an intermediate market structure—partial exclusive competition —whereby each seller belongs to one of two subgroups; buyers can trade with at most one seller from each subgroup. We show that in every equilibrium one subgroup of sellers proposes pooling contracts, and there always exist equilibria under which separation arises for the other subgroup. This ensures that the low risk agent's welfare is greater than under nonexclusive competition.

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Contents

1 Introduction (preliminary version)

Since Akerlof (1970) first showed that markets can unravel in the presence of adverse selection, a large literature has investigated the role of information asymmetries on market outcomes. Under adverse selection, those most eager to trade are also the most costly to serve. In the market for lemons, this may lead the uninformed side of the market to refrain from trading altogether. The subsequent analysis by Rotschild and Stiglitz (1976) and Attar Marrioti and Salanié (2011,2014,2021,2022) revealed that the extent to which competitive markets unravel very much depends on the market structure in place. Under exclusive competition analyzed by Rotschild and Stiglitz, buyers can trade with at most one seller (e.g., car insurance). An equilibrium need not exist; if it does, it involves rationing of low risk buyers. More recently, Attar, Mariotti and Salanié analyze the market structure where buyers can purchase multiple contracts from different sellers (e.g. annuities.). They show that an equilibrium involves pooling at fair prices that depend on the proportion of different risk in the economy.

Nonexclusive and exclusive competition are polar cases of a larger family of market structures by which we refer to any trading rule that specifies which sellers a buyer can jointly trade with. The interpretation we give is that a benevolent planner pre-assigns roles to prospective sellers. Neither nonexclusive nor exclusive competition is fully convincing from a normative point of view: Rationing may be extreme under exclusive competition; restrictive trading rules prevent Pareto gains which could arise due to additional pooling contracts. Meanwhile nonexclusive competition offers (too) great (a) scope for market-destabilizing deviations.

In this paper we analyze a middle ground market structure—partial exclusive competition —that seeks to retain the greater flexibility from nonexclusive competition while limiting the scope for destabilizing deviations. Under "1+1" partial exclusive competition each seller belongs to one of two subgroups so that buyers can trade with at most one seller from each subgroup. While our analysis is strictly theoretical, it is worth mentioning that in this instance practice is ahead of theory. In the publicly regulated health insurance market in France, buyers purchase one basic insurance contract from one seller, and complement it with another contract "Mutuelle" from another seller. This naturally gives rise to two groups of contracts (basic and complementary). A buyer can exclusively purchase one contract within each group, but nonexclusively combine contracts

In the 2013 UK annuity markets, five million buyers owned six millions annuities between the two groups.

How do equilibria under "1+1" partial exclusive competition compare? We show that in any equilibrium at least one of the two subgroups of sellers must propose pooling contracts: distinct buyer types purchase the same contract from sellers in that group at the average price. This stands in contrast to equilibrium allocations under exclusive competition that must be separating. We also show that there always exist equilibria in which the other subgroup proposes separating contracts. This stands in contrast to equilibrium allocations under nonexclusive competition—known as the Jaynes-Hellwig-Glosten (JHG) allocation —which can never involve contracts priced below the pooling price. From a welfare point of view, this finding gives credence to the idea that partial exclusive competition is indeed a middle ground between fully exclusive and nonexclusive competition. If the high type's preferred allocation were JHG, and the low type's preferred allocation was RS, then the high type prefers any allocation which arises as an equilibrium under *partial exclusivity* over RS, and the low type prefers said allocation over JHG.

As an extension, we consider the alternative market structure where the planner increases the number of sellers in one group that the buyer can jointly trade with. We show that when the planner changes from "1+1" to "$\lambda + 1$" partial exclusive structure (a buyer can now trade with at most λ sellers in subgroup one), the available set of equilibria shrinks. Moreover, the available equilibria converges to the JHG allocation as λ becomes large. On an intuitive level, a greater λ leads to more nonexclusive competition which enhances the set of possible deviations and thereby destroys more possible equilibria.

We also find that if $\lambda_1 + \lambda_2$ is big enough, then the $\lambda_1 + \lambda_2$ partial exclusive market structure (a buyer can now trade with at most λ_1 sellers in subgroup one and λ_2 sellers in subgroup two) is equivalent to $(\lambda_1 + \lambda_2 - 1) + 1$ partial exclusive structure. This feature allow us simplify the search for other partial exclusive market structure.

This paper is organized as follows. Section 2 introduces the model and proposes the notion of a market structure. Section 3 illustrates the examples of exclusive and nonexclusive competition as two (polar) market structures . Section 4 presents our main results: the market can have different equilibria that have never been studied before. Section 5 presents as an extension the limiting case as the scope of nonexclusive competition increases. Section 6 considers the robustness of equilibria when we allow for competition in menus.

2 The Model: An Insurance Economy

We study competitive pricing under adverse selection in the insurance economy. Buyers purchase coverage in exchange for an insurance premium. The basic setting of the model is consistent with the setting of nonexclusive environment as described by Attar, Mariotti and Salanié (2021,2022). However, we will use the this setting to study a unified framework which not only contains exclusive and nonexclusive competition , but also some other market structures.

2.1 Buyers and Sellers

Buyers The model can be interpreted as if there was a continuum of buyers, but for descriptive purposes, it is best to think of a single consumer of unknown types. There are two buyers, indexed by their type $1, 2$. Types are buyers' private information; and the commonly known proportion is m_1 to type 1 and $m_2 \equiv 1 - m_1$ to type 2. A buyer type i holds preferences over aggregate trades; if she trades with sellers $A \subseteq \{1, ..., K\}$ her aggregate trades are henceforth given by $Q = \sum_{k \in A} q^k$ and $T = \sum_{k \in A} t^k$. Preferences over aggregate trades are represented by a utility $U_i(Q, T)$ which is quasi-concave, increasing in its first, and decreasing in its second argument, twice continuously differentiable, and satisfy the Inada's condition. All buyers have as a common outside option the null trade so that $U_i(0, 0) = 0$. Finally, buyer types are ordered in the sense that higher types have a greater propensity to consume.

Sellers and Contracts A finite number of ex-ante identical sellers $\{1, ..., K\}$ competes in quantities and tariffs so that each seller proposes a single contract (q^k, t^k), involving a non-negative quantity and price. A contract (q, t) between a seller and a buyer covers a fiexd fraction $q \geq 0$ of the loss for a premium t, with unit price of $\frac{t}{q}$. Sellers are perfectly substitutable in that the cost of providing aggregate quantity q to some buyer type i does not depend on the identity nor quantity provided by individual sellers. If a seller trade a contract (q, t) with a buyer of type i, then the seller earns an expected profit $t - c_i q$, where c_i is the marginal cost of serving buyer type $i \in \{1, 2\}$. We assume there is adverse selection. This means that those types more eager to trade are also the types that are more costly to serve, which is characterized by the assumptions as follows:

Assumption 1 (single-crossing)**.** *For any two* (Q', T'), (Q'', T'') *such that* $Q'' \geq$

Q',

$$U_1(Q'',T'') \geq U_1(Q',T') \quad \Rightarrow \quad U_2(Q'',T'') > U_2(Q',T')$$

.

Throughout it will be convenient to consider utility functions which are continuously differentiable. Thus define the buyer type i's marginal rate of substitution of coverage for premia as

$$\tau_i(Q,T) = -\frac{\partial_1 U_i(Q,T)}{\partial_2 U_i(Q,T)}$$

which is also the slope of buyer type i's indifference curve.

Lemma 1. *Under single-crossing, for any quantity-tariff pair* $(Q,T), \tau_2(Q,T) > \tau_1(Q,T)$.

Assumption 2 (Common Value)**.** *Type 2 is more costly for sellers:* $c_2 > c_1$.

Finally, it will be useful to also introduce the average cost $c \equiv c_1 m_1 + c_2 m_2$. Clearly, $c_1 < c < c_2$.

Remark: We use this basic setting of economy to capture the key elements of insurance market, however, it can be used in more general settings. As in Attar, Mariotti and Salanié (2022), based on the aggregate trades (Q,T), buyer type i's utility function can be represented by:

$$U_i(Q,T) \equiv \int v_i \left(W_0 - (1-Q)l - T\right) f_i(l)\boldsymbol{l}(\mathrm{d}l) \tag{1}$$

where W_0 is buyers' initial wealth, and $\boldsymbol{l}$ is a fixed loss measure with different risk according to a density f_i , v_i the utility index of a consumer of type i buyers. So $U_i(Q,T)$ is the expected utility of buyers' final wealth. Then the serving cost of type i is defined by $c_i \equiv \int l f_i(l)\boldsymbol{l}(\mathrm{d}l)$. If $\boldsymbol{l}$ is the counting measure on $\{0, L\}$, and $c_i \equiv f_i(L)L$, with $f_2(L) > f_1(L)$. It is actually the economy of Rothschild and Stiglitz (1976), where type 2 is the more risky type.

2.2 Market Structures and Timeline

The main innovation of our model is the definition of market structures. A market structure is a trading rule which describes the set of sellers with whom a buyer can jointly trade. From the viewpoint of the planner, she could stipulate the jointly trade rule to the market, which not related to the quantity-tariff pair , just the numbers of sellers that one buyer could trade with.

Definition 1. *A market structure $\mathcal{M}$ is a (non-empty) collection of subsets of sellers with whom a buyer can jointly trade: $\mathcal{M} \subseteq \mathcal{P}(\{1, ..., K\})$.*

Here $\mathcal{P}(\{1, ..., K\})$ denotes the power set, i.e., the set of all subsets of $\{1, ..., K\}$. Taking a market structure perspective means that a benevolent planner or a platform could choose $\mathcal{M}$ before sellers make their offers.

The Simultaneously Game We consider a competitive-screening game in which firms compete by posting a single contract, the benevolent planner could stipulate the jointly trade rule before the market. The game unfolds as follows:

- Stage 0: The planner chooses a market structure $\mathcal{M} \in \mathcal{P}(\{1, ..., K\})$.
- Stage 1: Each seller k proposes a contract $(q^k, t^k) \in \mathbb{R}_+^2$.
- Stage 2: After privately learning her type i, each buyer chooses some $M \in \mathcal{M}$, trades with all sellers $k \in M$ and derives utility $U_i\big(\sum_{k \in M} q^k, \sum_{k \in M} t^k\big)$.

The stage 0 describes how can the planner affect market only through a simple trading rule. It is not strong intervention which related to stipulate the detail of different contracts market, it is just very week intervention, only stipulate the maximal sellers they could jointly trade. So in stage 0, there is no restriction on the quantity-tariff pair, no instruction "suggest" that what contracts should the sellers propose, only the total trade numbers with sellers. We also assume that the trading rule does not discriminate between individual sellers. We do not award individual sellers with quasi-monopoly powers, nor award profitable trading opportunities to some sellers but not to others.

Stage 1 and stage 2 are main game the sellers and buyers play in the market. In the stage 1, all the sellers simultaneously propose a single contract to the market, which means that a seller cannot react to the contracts posted by other sellers and also a seller cannot propose a contract contingent on the other sellers' contracts.

The choice of market structure can describe different features in the simultaneously game, introducing stage 0 allow us include different environments of insurance market. We would like to share three different environments in the insurance market, which based on exclusive, nonexclusive and partial exclusive environment.

In the car insurance market, one buyer with private information is restricted to purchase at most one seller's insurance in the market, that is to say , the contracts between sellers are exclusive to each other, thus we call it exclusive environment. By choosing the market structure $\mathcal{M} = \big\{\emptyset, \{1\}, \{2\}, ..., \{K\}\big\}$ in stage 0, it means that a buyer can only trade with at most one seller, which consistent with the exclusive environment.

In the annuity market, one buyer can purchase multiple contracts with different sellers, there is no restriction on the numbers of contracts they could trade, we call it nonexclusive environment. By choosing the market structure $\mathcal{M} = \mathcal{P}(\{1, ..., K\})$ in stage 0, we can also study this kind of environment in the same model.

In the health market, like the "Mutuell" in France, one buyer can purchase at most one basic insurance from a seller and can purchase at most one complementary insurance from a seller. It is like there are two groups of contacts (basic and complementary), a buyer can exclusive purchase one contract within the group, but can nonexclusive combine the two groups. We call it partial exclusive environment. By choosing the market structure $\mathcal{M} = \big\{\emptyset, \{1\}, \{2, \}, ..., \{K_1\}\big\} \times \big\{\emptyset, \{K_1+1\}, \{K_1+2\}, ..., \{K\}\big\}$, we can also study this partial exclusive environment in our model.

Example 1. **(i)** *Exclusive competition:* $\mathcal{M} = \mathcal{M}_E := \big\{\emptyset, \{1\}, \{2\}, ..., \{K\}\big\}$

(ii) *Nonexclusive competition:* $\mathcal{M} = \mathcal{M}_N := \mathcal{P}(\{1, ..., K\})$

(iii) *Partial exclusive competition:* $\mathcal{M} = \mathcal{M}_1^1 := \big\{\emptyset, \{1\}, \{2, \}, ..., \{K_1\}\big\} \times \big\{\emptyset, \{K_1 + 1\}, \{K_1 + 2\}, ..., \{K\}\big\}$,

As the examples above, by choosing different structure in stage 0, we can study various environments in our model. In the real world , if there is no restriction on the trade of insurance market , the nature of the markets is nonexclusive, due to one seller can not monitor the trade between the buyers and other sellers. Thus, if there is no restriction on the trade, it is consistent with the nonexclusive environment of Attar, Mariotti and Salanié (2022). However, like what happens in car insurance, there exists the restriction or rule that one buyer can only purchase at most one contract in the market, like in Rothschild and Stiglitz (1976), the trade between buyers and sellers is restricted to exclusive. In here, we can find that exclusive trade comes from the restriction of some rule or regulator, this is why we include stage 0 as a part of our model. By choosing $\mathcal{M} = \mathcal{M}_E$, our model includes the exclusive environment which studied by Rothschild and Stiglitz (1976), Wilson (1977), Hellwig (1977) and so on. By choosing $\mathcal{M} = \mathcal{M}_N$, our model includes the nonexclusive environment which studied by Jaynes (1978), Hellwig (1988), Attar, Mariotti and Salanié (2011,2014,2021,2022) and so on. However, when we assume that the planner can choose the market structure, we can study more than what the literature has studied before. Like the example in health insurance, the planner can divide sellers into two subgroups, and then stipulate that buyers can only purchase at most one contacts of subgroup inside, but can purchase contracts in both subgroups. This new structure include some exclusivity in each subgroup, but also exists nonexclusive between subgroups. This kind of structure never studied by other literature, but it happens in the life, our model of stage 0 allow us can study this kind of structure, and it is also possible to study many other partial exclusive environment with different structures.

2.3 Equilibrium

Our equilibrium concept is perfect Bayesian equilibrium in pure strategies. After the planner has chosen the market structure $\mathcal{M}$ in stage 0, sellers propose the contracts that maximize their expected profit; and buyers trade with the subset of sellers $M \in \mathcal{M}$ that maximize their utility.

Definition 2. *Given the market structure $\mathcal{M}$, an equilibrium is a tuple $(\mathcal{C}, S)$ where $\mathcal{C} = \{(q^k, t^k)\}_{k\in\{1,\dots,K\}}$ is the set of trades offered by the sellers and $S = (S_i)_{i\in\{1,2\}}$ is the buyers' strategy profile $S_i : \mathbb{R}_+^{2K} \to \mathcal{M}$. Buyers' strategy profile S must satisfy buyer optimality*

$$S_i(\mathcal{C}') \in \arg\max_{M\in\mathcal{M}} U_i\Big(\sum_{k\in M} q^k, \sum_{k\in M} t^k\Big) \quad \forall \mathcal{C}' \in \mathbb{R}_+^{2K};$$

the trades offered must satisfy seller optimality

$$(q^k, t^k) \in \arg\max_{(q'^k, t'^k)\in\mathbb{R}_+^2} \sum_{i\in\{1,2\}} \big[t^k - c_i\, q^k\big] m_i\, \mathbf{1}\Big\{k \in S_i\big(\{(q^\ell, t^\ell), (q'^k, t'^k)\}_{\ell\neq k}\big)\Big\}.$$

In a word, the buyers' and the sellers' choices are optimal given the market structure and other participants behaviour. Throughout the analysis, we will focus on perfect Bayesian equilibria for a given market structure, hereafter simply equilibria. Then if in an equilibrium there exists at least one seller that does not trade, then we say this seller is inactive, and we call this equilibrium a free-entry equilibrium.

3 Exclusive and Nonexclusive Competition

In this section, we discuss two polar market structures: nonexclusive and totally exclusive competition. Under exclusive competition, the restriction on buyers is strong: a buyer can only trade with at most one seller in the market.

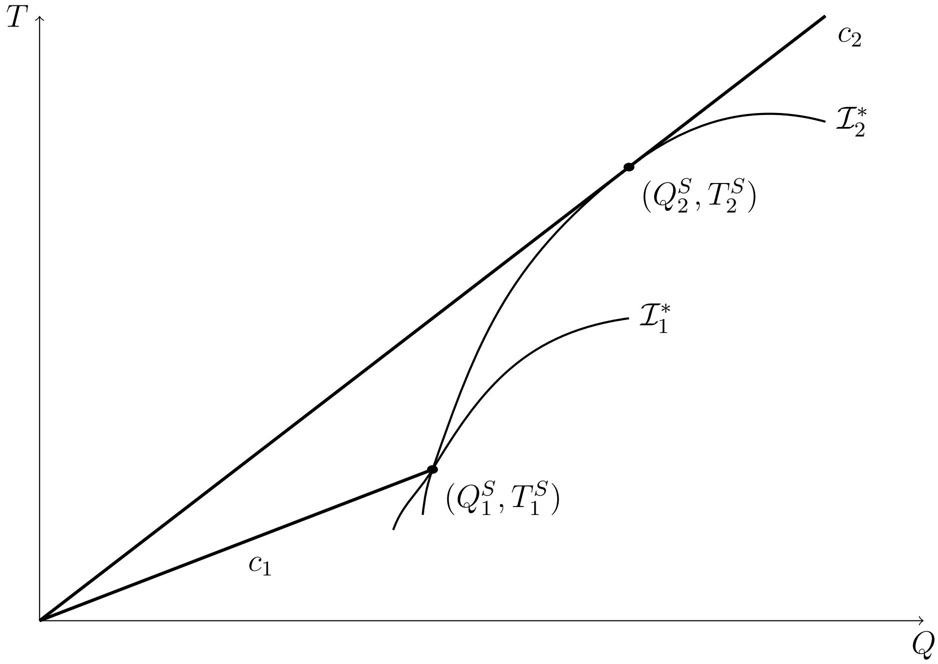

Figure 1: RS allocation

It is equivalent to setting $\mathcal{M} = \mathcal{M}_E := \{\emptyset, \{1\}, \{2\}, ..., \{K\}\}$ in the stage 0. Under nonexclusive competition, there is no restriction on how many sellers a buyer can trade with. Thus, it is equivalent to setting $\mathcal{M} = \mathcal{M}_N = \mathcal{P}(\{1, ..., K\})$ in stage 0. These are two polar structures among all the structures in our model, and the results of these two polar cases have been studied in the literature.

3.1 Exclusive competition

In this subsection, we describe the contractual outcomes that have been referred to as the Rothschild-Stiglitz (RS) allocation in the insurance literature (see Rothschild and Stiglitz 1976). They solve the following optimization problem:

Definition 3 (Rotschild-Stiglitz (RS))**.** *The RS allocation is the separating allocation (Q_1^S, T_1^S) and (Q_2^S, T_2^S) where*

$$Q_2^S = \arg\max_{Q_2 \geq 0} U_2(Q_2, c_2 Q_2), \quad T_2^S = c_2 Q_2^S$$
$$Q_1^S = \arg\max_{Q_1 \geq 0} U_1(Q_1, c_1 Q_1), \quad T_1^S = c_1 Q_2^S$$
$$\textit{subject to } U_2(Q_2^S, T_2^S) \geq U_2(Q_1^S, T_1^S).$$

Lemma 2 (Equilibrium in exclusive structure.)**.** *Given that $\mathcal{M} = \mathcal{M}_E := \{\emptyset, \{1\}, \{2\}, ..., \{K\}\}$ in stage 0, the unique equilibrium candidate in the simultaneous competition game is Rotschild-Stiglitz allocation. When the proportion* *of type 1 is small enough, the equilibrium exists.*

The market structure $\mathcal{M}_E$ replicates the results of Rotschild and Stiglitz (1976). In equilibrium, type 1 and type 2 purchase different separating contracts. Type 2 obtains full insurance with the unit price equal to the type 2's serving cost c_2; type 1 achieves partial insurance with unit price equal to c_1. If the equilibrium exists in this game, then this RS allocation is second-best efficient. (If $\tau_1(Q_1^S, T_1^S) \neq c_1$, then $U_2(Q_2^S, T_2^S) = U_2(Q_1^S, T_1^S)$)

3.2 Nonexclusive Competition

In this subsection, we define the unique equilibrium candidate in the nonexclusive environment. By choosing $\mathcal{M} = \mathcal{M}_N$ in stage 0, we can replicate this environment in our model. Firstly, we describe the results that have been referred to as the Jaynes-Hellwig-Glosten (JHG) allocation in the insurance literature (see Jaynes (1978),Hellwig (1988), Glosten (1994), Attar, Mariotti and Salanié (2011,2014,2020,2021). They solve the following optimization problem:

Definition 4 (Jaynes-Hellwig-Glosten (JHG))**.** *The JHG allocation is the partially pooling allocation* (Q_1^P, T_1^P) *and* (Q_2^P, T_2^P) *where*

$$Q_1^P = \arg\max_{Q_1 \geq 0} U_1(Q_1, cQ_1), \tag{2}$$

$$T_1^P = cQ_1^P \tag{3}$$

$$Q_2^P - Q_1^P = \arg\max_{q \geq 0} U_2(Q_1^P + q, T_1^P + c_2 q), \tag{4}$$

$$T_2^P - T_1^P = c_2(Q_2^P - Q_1^P). \tag{5}$$

Based on the definition above , we can find that in JHG allocation, there are two layers of active unit price contracts: basic layer with unit price of c, which serve both type 1 and type 2; and an optional layer with unit price of c_2, which is only purchased by type 2. For both layers, the coverage-premium pair is actually fairly priced given the buyers who purchase it. The quantity is also an optimal choice for different types of buyers given the unit price proposed by sellers.

According to Attar, Mariotti and Salanié (2020,2021), the JHG allocation is the unique equilibrium candidate in the nonexclusive environment. Indeed, under free entry, it is easy to see that JHG is the unique equilibrium candidate. Assume that $(Q_1^{P'}, T_1^{P'}), (Q_2^{P'}, T_2^{P'})$ are aggregate equilibrium trades in this nonexclusive economy. Firstly, we would have that $T_2^{P'} - T_1^{P'} = c_2(Q_2^{P'} - T_2^{P'})$ and $Q_2^{P'} - Q_1^{P'} = \arg\max_{q \geq 0} U_2(Q_1^{P'} + q, T_1^{P'} + c_2 q)$; otherwise, any unit price lower than c_2 can only have negative profit for the sellers who propose this optional

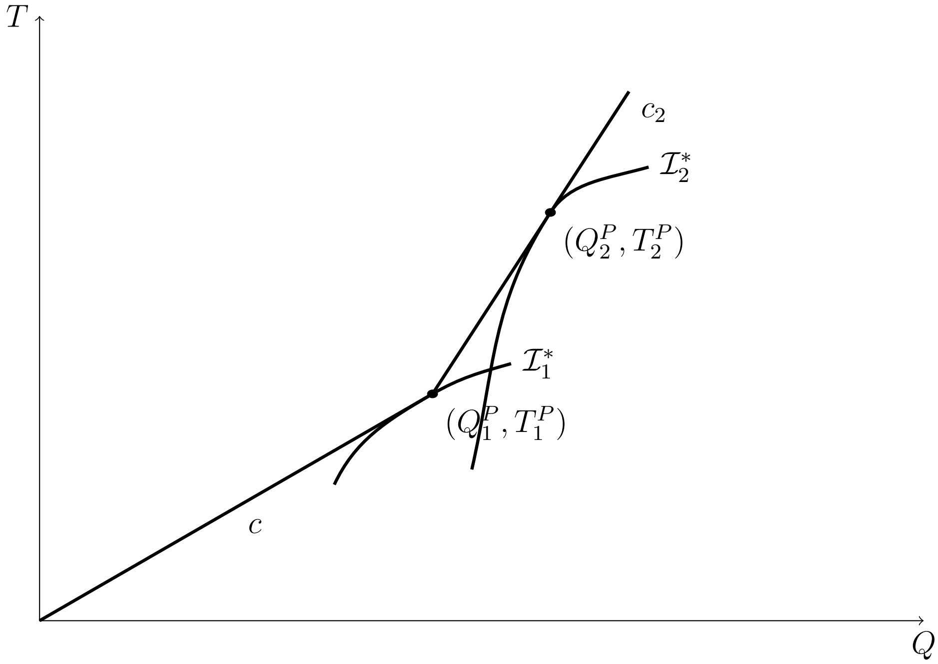

Figure 2: JHG alloaction

contract, and for the other allocations which have unit price no lower than c_2 and not satisfy the optimal quantity, an inactive seller can propose a contract with a unit price slightly higher than c_2 can attract type 2 and achieve positive profit. Then by the same logic, for the basic layer , due to budget balance, we should have that $T_1^{P'} \geq cQ_1^{P'}$. Then if $(Q_1^{P'}, T_1^{P'})$ can not propose the same utility level as (Q_1^P, T_1^P), an inactive seller could propose a contract with unit price slightly higher than c which attracts type 1 for sure, even also attracts type 2, it is also profitable. Thus, under free entry, JHG allocation is the unique candidate equilibrium. And in our model, due to that the sellers in stage 1 can only propose one contract, due to there is no monopoly power in equilibrium, so at least two sellers would propose the same contract, which means we always have inactive seller in the market. Thus, JHG allocation is the unique equilibrium candidate in our model with structure of nonexclusive.

Definition 5 (Large Pooling)**.** *In JHG allocation , We say* (Q_1^P, T_1^P) *and* (Q_2^P, T_2^P) *satisfy the large pooling condition if*

$$U_2(Q_2^P, T_2^P) \geq U_2(2Q_1^P, 2T_1^P)$$
$$Q_1^P > Q_2^P - Q_1^P$$

Lemma 3 (Equilibrium in nonexclusive structure.)**.** *Given that* $\mathcal{M} = \mathcal{M}_N := \mathcal{P}(\{1, ..., K\})$ *in stage 0, the unique equilibrium candidate in the simultaneous* *competition game is JHG allocation. If the "Large Pooling" holds, JHG allocation could be the equilibrium in nonexclusive environment.*

By choosing $\mathcal{M} = \mathcal{M}_N$ in stage 0, we replicate the results in nonexclusive environment with our model. In the equilibrium , both type 1 and type 2 would purchase a same level of basic aggregate quantity-tariff with unit price of c, then type 2 would purchase an additional layer of quantity-tariff pairs with unit price of c_2. The incentive compatible condition is this allocation is not binding due to that the inactive seller could pivot on the type 1's contract.

When RS allocation is an equilibrium in exclusive environment, then comparing to JHG allocation, RS allocation is better for type 1 , worse for type 2. When RS allocation is not an equilibrium in exclusive environment, then JHG allocation is better both for type 1 and type 2 buyers.

4 A New Structure: "1+1" Partial Exclusive Competition

As what we mentioned, in the heath insurance market, a buyer can purchase at most one basic contract and at most one optional contract in the market. We can find some exclusive feature in each subgroups: the basic contract is exclusive with other basic contract. We can also find some nonexclusive feature between two subgroups: basic and optional contracts are nonexclusive to each other. Thus, we say this environment has partial exclusivity. We capture the feature of this new environment in real life , we started by choosing that $\mathcal{M} = \mathcal{M}_1^1 := \big\{\emptyset, \{1\}, \{2,\}, ..., \{K_1\}\big\} \times \big\{\emptyset, \{K_1+1\}, \{K_1+2\}, ..., \{K\}\big\}$, this structure contains very little nonexclusivity. To easily understand this structure, we can also define the "1+1" partial exclusive structure as follow:

Definition 6. *Consider a partition of $\mathcal{K} = \{1, ..., K\}$ into two disjoint sets A_1 and A_2. Then "1+1" partial exclusive structure is*

$$\mathcal{M}_1^1 = \big\{\{k, l\} : k \in A_1 \cup \{\emptyset\}, \ell \in A_2 \cup \{\emptyset\}\big\}.$$

In other words: any buyer can trade with none, one or any two sellers which do not belong to the same subgroup. The buyer can purchase at most one contract in set A_1 and at most one contract in set A_2 (We assume that in each subgroup, there are at least 4 sellers in each group). Then we can recall that the timing of the game is as follow:

Timing of the game

- Stage 0: The planner chooses the market structure $\mathcal{M}_1^1$.
- Stage 1: Each seller k proposes a contract $(q^k, t^k) \in \mathbb{R}_+^2$.
- Stage 2: After privately learning her type i, each buyer chooses some $M \in \mathcal{M}$, trades with all sellers $k \in M$ and derives utility $U\big(\sum_{k\in M} q^k, \sum_{k\in M} t^k\big)$.

4.1 The possible equilibrium form

Under exclusive competition, the allocation consists of two separating contracts. Under nonexclusive competition, the allocation consists of one pooling contract and one additional contract. Unlike under exclusive or nonexclusive competition, "1+1" partial exclusive competition involves two distinct subgroups A_1 and A_2. We study the possible contracts that can arise in each subgroup:

Definition 7. *"pooling" means that both buyer types purchase the same contract in one subgroup, whereas "separating" means that distinct buyer types purchase different contracts in one subgroup.*

Since there are only two subgroups in the "1+1" partial exclusivity setting, equilibrium trading must take one of two forms.

Under (1)"Pooling+Pooling" type 1 and type 2 buyers purchase the same contracts in both groups. Under (2) "Pooling+Separating" type 1 and type 2 purchase the same contracts in one group and purchase different contracts in the other subgroup. Under (3) "Separating+Separating" type 1 and type 2 buyers purchase different contracts in both subgroups. To simplify our analysis, we can firstly prove a basic proposition which can help us rule out one possible form of equilibrium.

Proposition 1. *No equilibrium is of the form "Separating+Separating" in the structure of "1+1" partial exclusivity.*

To prove the proposition above, we firstly study the feature of the unit price in equilibrium. We will discuss the possible situation that for different types of buyers purchase the contracts, then by using the assumption of free-entry, and combine with the argument of Bertrand competition, we can solve the possible unit price in different cases. (In there, we use the premise that there is at least one inactive seller in the equilibrium, we will prove in later that there always exist inactive sellers in equilibrium to propose latent contract.)

Lemma 4 (Fair Unit Pricing)**.** *Under free-entry equilibrium, if an active contract is purchased by type 2 buyers only, the unit price should be c_2; If purchased* *by both types, the unit price should be c; If purchased by type 1 buyers only , then the unit price should locate in* $[c_1, c]$.

The sketch of fair unit pricing: The budget balanced condition of sellers could give us the lower bound unit price of each situation , due to that the unit price of an insurance should at least equal to the serving cost. Assume in equilibrium, there is an active contract (q^k, t^k) purchased by buyers. Then if only type 2 buyers purchase this contract , then unit price should be at least c_2; If both types purchase it ,the unit price should be at least c; If only type 1 purchases it , the unit price should be at least c_1.

Then we can study the upper bound of unit price of the contracts by using the Bertrand competition argument in different situations. (1)If only type 2 buyer purchases (q^k, t^k), and $t^k > c_2 q^k$, then an inactive seller can propose $(q^k, t^k - \epsilon)$ with ϵ small enough such that $t^k - c_2 q^k - \epsilon > 0$ to attract type 2 buyers, no matter type 1 buys or not,it is a profitable deviation, contradiction. Thus, if a contract only sold to type 2 in equilibrium, then the unit price is c_2. (2)If both type 2 and type 1 purchase the (q^k, t^k), and $t^k > cq^k$, in this case, an inactive seller can propose $(q^k, t^k - \epsilon)$ with ϵ small enough such that $t^k - cq^k - \epsilon > 0$ to attract both types and it is profitable, contradiction. Thus , the pooling active contracts in equilibrium should have unit price equal to c. (3) By the same logic, the upper bound of unit price that only sold by type 1 is also c. In here, we can not have the unit price is c_1, due to any lower price may also attracts type 2 buyers.

After we studied the feature of the unit price in equilibrium in the fair unit price lemma, we can prove the proposition 1 by using the idea of pivoting and profitable deviation from the inactive sellers. Which as follows:

Proof of Proposition 1. According to the structure in stage 0, we have two subgroups which are A_1 and A_2 . If there is an equilibrium implemented with the structure of "separating + separating" form, then from lemma 1, the active separating contract for type 2 buyers should have unit price with c_2. "separating+separating " means that type 2 would purchase two contracts with unit price equal c_2 in both subgroup A_1 and A_2. Let us define the equilibrium allocation is (Q_1^*, T_1^*) for type 1 buyers, and (Q_2^*, T_2^*) for type 2 buyers.Then we have that:

$$T_2^* = c_2 Q_2^*$$

Then we look at type 1 buyers' allocation, under "Inada's condition", type

1 buyers purchase at least one contract in the "1+1" partial exclusive structure game. WLOG, we assume that the active trade (q_1, t_1) for type 1 happens in A_1, then by lemma 1, we have that $t_1 \leq cq_1$. In this case, an inactive seller m in subgroup A_2 could propose a contract $(q_m, t_m) = (Q_2^* - q_1, T_2^* - t_1 - \epsilon)$ with ϵ very small, this contract attracts type 2 for sure, if it does not attract type 1 , the profit would be

$$f_2(T_2^* - t_1 - \epsilon - c_2(Q_2^* - q_1)) = f_2(c_2 q_1 - t_1 - \epsilon) > 0$$

If it also attracts type 1, then the profit would be

$$T_2^* - t_1 - \epsilon - c(Q_2^* - q_1) = T_2^* - cQ_2^* + cq_1 - t_1 - \epsilon > 0$$

Thus, in both situation the inactive seller can achieve positive profit by pivoting on type 1 buyers' contract. Thus, it is impossible that the equilibrium could have a structure with "separating + separating".

4.2 "Pooling + Pooling" and "Pooling + Separating"

According to the proposition in last subsection, we can conclude that the possible equilibrium form in the "1+1" partial exclusive structure can only be "pooling+separating" and "pooling+pooling". For the possible form of "pooling+pooling", which means that type 1 and type 2 buyers purchase the same contracts in both subgroups A_1 and A_2. It follows from the "fair unit price" lemma, the allocation in "pooling+pooling" should have the unit price of c. Then by using the same trick in nonexclusive structure, if we define (Q_1^*, T_1^*) and (Q_2^*, T_2^*) as the equilibrium allocation, we can get that the possible equilibrium with the form of "pooling+pooling" should have that $(Q_1^*, T_1^*) = (Q_2^*, T_2^*) = (Q_1^p, T_1^P)$, where (Q_1^p, T_1^P) is the type 1's quantity-tariff pair in JHG allocation. For any other pair (Q, T) with $t = cQ$, an inactive seller could propose a contract with $(Q_1^p, T_1^P + \epsilon)$, it attracts both type buyers and achieve positive profit. Thus, for the form of "pooling+pooling", the only possible equilibrium is (Q_1^p, T_1^P) for both types.

In the next, we will focus on the possible equilibrium with the form of "pooling + separating". WLOG, assume the subgroup A_1 provides the active pooling contracts in the equilibrium, while subgroup A_2 provides the active separating contracts. Assume that in the equilibrium, the allocation is (Q_1^*, T_1^*) for type 1 buyers, and (Q_2^*, T_2^*) for type 2 buyers. According to the lemma of "fair unit price", the pooling contract (Q^*, T^*) in subgroup A_1 and the separating contract

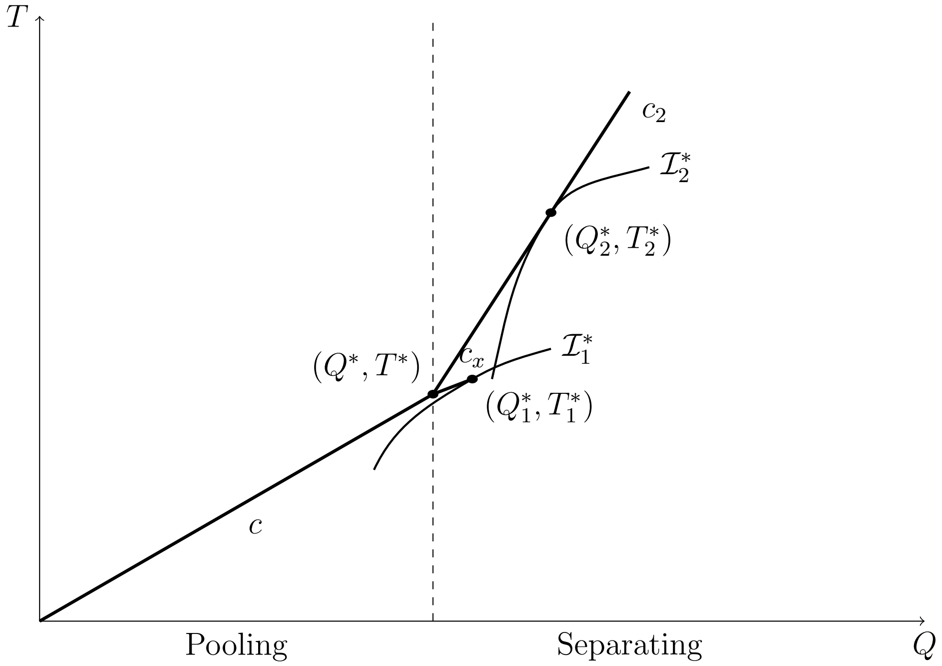

Figure 3: Pooling+Separating

in subgroup A_2 should satisfy:

$$T^* = c \times Q^*$$
$$T_2^* - T^* = c_2 \times (Q_2^* - Q^*)$$

Then by solving these two equations , we can have the relationship between the pooling contracts and the allocation of type 2 buyers, which is that $(Q^*, T^*) = (\frac{c_2 Q_2^* - T_2^*}{c_2 - c}, c\frac{c_2 Q_2^* - T_2^*}{c_2 - c})$, which gives the pooling active contracts provided by subgroup A_1. Then we can use the "fair unit price " lemma to get the relationship between type 1's allocation and the pooling contract.

Lemma 5. *In the structure of "Pooling + Separating" equilibrium, "fair unit price" lemma implies that* $c_2(Q_1^* - Q^*) \leq T_1^* - T^* \leq c(Q_1^* - Q^*)$.

In this lemma, it contains two cases of possible "separating" in subgroup A_2, the first case is that if type 1 will not purchase any contract (so this separating is that type 1 purchase null contract while type 2 purchase a positive quantity contract), then this inequality is binding. The second case is that type 1 also purchase a positive quantity contract, then the unit price of this contract should between $[c_1, c]$.

4.3 Necessary conditions with allocation

This subsection provides the necessary conditions that an allocation of equilibrium should have. That is to say, for a given (Q_1^*, T_1^*) and (Q_2^*, T_2^*), we can rule out this allocation as an equilibrium if it is not satisfy the condition in this subsection. It start with the basic requirement of incentive compatible condition, then by using the possible a pooling deviation, we can have our fair marginal pricing condition, after that, considering the pivoting based on type 1's separating contracts, we can solve the no pivoting condition.

Proposition 2. ***Incentive compatibility buyers:*** *The aggregate trades* $(Q_i^*, T_i^*), i = 1, 2$ *satisfy incentive compatibility:*

$$U_x(Q_x^*, T_x^*) \geq U_x(Q_y^*, T_y^*) \quad \textit{for all } x, y \in \{1, 2\}.$$

This necessary condition is the basic requirements of that one allocation could be an equilibrium, otherwise , type 2 and type 1 can mimic each other. Then we will see the second condition that an equilibrium should have.

Proposition 3. ***Fair Marginal purchasing:*** *In the allocation of equilibrium, the marginal utility of type i buyers should equal the tail expectation serving cost from type i,* $\tau_1(Q_1^*, T_1^*) = c$, *and* $T_1^* \leq cQ_1^*$, *where* $c = f_1c_1 + f_2c_2$. *and* $\tau_2(Q_2^*, T_2^*) = c_2$ *if* $(Q_1^*, T_1^*) \neq (Q_2^*, T_2^*)$.

The "Fair marginal purchasing" is saying that, in the equilibrium, type 1 buyers would at least have a utility no less that $max\{U_1(Q, cQ) : Q \geq 0\}$, and in the same time, the marginal substitution of of quantity for tariff for type 1 should equal to c which is the serving cost of both types, and the marginal substitution of of quantity for tariff for type 2 is c_2 in the form of *pooling + separating*. Otherwise, in the form of "pooling+pooling", type 2 and type 1 will have the same aggregate quantity and tariff pair.

Proof. (1) By the same argument in nonexclusive environment, we will have the $T_1^* \leq cQ_1^*$. Then We need to prove that $\tau_1(Q_1^*, T_1^*) = c$. By contrast, if $\tau_1(Q_1^*, T_1^*) \neq c$, an inactive seller in subgroup A_1 can deviate by issuing a contract with $(Q^* + \delta_1, T^* + \epsilon_1)$ with δ_1 and ϵ_1 chosen so that:

$$\tau_1(Q_1^*, T_1^*)\delta_1 > \epsilon_1 > c\delta_1$$

When δ_1 and ϵ_1 are small enough, "$\tau_1(Q_1^*, T_1^*)\delta_1 > \epsilon_1$" implies that the new contract is attractive for type 1 buyers. (If δ_1 is positive, it is the case of undersupply; If δ_1 is negative , it is the case of the oversupply). Case 1, if $(Q^*+\delta_1, T^*+\epsilon_1)$ also attracts type 2 buyers, then the seller could achieve a positive profit with $\epsilon_1 - c\delta_1 > 0$ with second inequality ; Case 2, if $(Q^*+\delta_1, T^*+\epsilon_1)$ doesn't attract type 2 buyers, then the deviation achieve the profit with $f_1 \times (T^* - c_1 Q^* + \epsilon_1 - c_1\delta_1) > 0$, still get positive profit with deviation.

(2) Then we show that $\tau_2(Q_2^*, T_2^*) = c_2$ when $(Q_1^*, T_1^*) \neq (Q_2^*, T_2^*)$. Otherwise, an inactive seller in subgroup A_2 can deviate by proposing a contract $(Q_2^* - Q^* + \delta_2, T_2^* - T^* + \epsilon_2)$ with δ_2 and ϵ_2 chosen so that:

$$\tau_2(Q_2^*, T_2^*)\delta_2 > \epsilon_2 > c_2\delta_2$$

When δ_2 and ϵ_2 are small enough, "$\tau_2(Q_2^*, T_2^*)\delta_2 > \epsilon_2$" implies that the new contract is attractive for type 2 buyers. Case 1, if this contract doesn't attract type 1 buyers, then the profit for deviator is $f_2(\epsilon_2 - c_2\delta_2) > 0$; Case 2, if this contract also attract type 1 buyers, then the profit is

$$\begin{aligned}&f_2(\epsilon_2 - c_2\delta_2) + f_1(T_2^* - T^* + \epsilon_2 - c_1(Q_2^* - Q^* + \delta_2))\\ =&\epsilon_2 - c_2\delta_2 + f_1(c_2 - c_1)(Q_2^* - Q^* + \delta_2) > 0\end{aligned}$$

Thus, if $\tau_2(Q_2^*, T_2^*) \neq c_2$, there exists profitable deviation, contradiction.

Proposition 4. ***No Pivoting Condition(NPC):***
No inactive seller in subgroup A_1 can make a profit by combining the separating contract designed for type 1 in subgroup A_2 and attract type 2 buyers. Which is that:

$$T_2^* - T_1^* + \frac{c(c_2Q_2^* - T_2^*)}{c_2 - c_1} \leq c_2(Q_2^* - Q_1^* + \frac{(c_2Q_2^* - T_2^*)}{c_2 - c_1}) \tag{6}$$

In the "1+1" partial exclusive structure, the exclusivity of subgroup A_2 provide the possibility that type 1 and type 2 buyers choose different contracts inside the subgroup, which is impossible in nonexclusive environment because an inactive seller can propose a contract pivoting on type 1's lower unit price contract and then attracts type 2 buyers. However, due to the trade between A_1 and A_2 is nonexclusive, thus, we need to consider that the pivoting comes from the sellers in group A_1, one seller in A_1 could propose one pooling contract together with additional pivoting contract to attract type 2 buyers, if the profit of pivoting contract great than the loss of pooling contract to type 2, then this deviation is profitable, then this allocation is not an equilibrium. In this NPC condition, we provide a condition that there is no pivoting is profitable. This

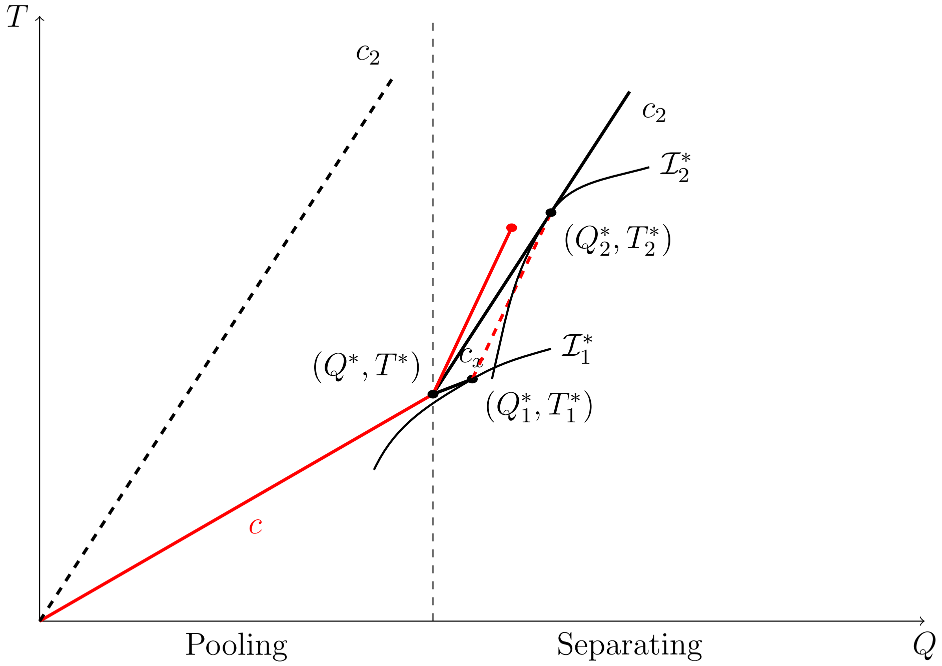

Figure 4: Pivoting is not profitable

condition also tell us one allocation could be an equilibrium only if that the pooling contract is big enough.

Proof. If the an equilibrium with the structure of "pooling+separating" , by using the fair unit price lemma , the active pooling contract in subgroup A_1 should be $(Q^*, T^*) = (\frac{c_2 Q_2^* - T_2^*}{c_2 - c}, c\frac{c_2 Q_2^* - T_2^*}{c_2 - c})$. Then the inequality in NPC condition is equivalent to say that

$$T_2^* - T_1^* + T^* \leq c_2(Q_2^* - Q_1^* + Q^*)$$

Notice that $(Q_1^* - Q^*, T_1^* - T^*)$ is the active separating contract in subgroup A_2 that serving for type 1 buyers, and $(Q_2^* - Q^*, T_2^* - T^*)$ is the active separating contract in subgroup A_2 that serving for type 2 buyers.

Next, we show that inequality above should be hold in equilibrium. Otherwise, we will have that :

$$T_2^* - (T_1^* - T^*) > c_2(Q_2^* - (Q_1^* - Q^*))$$

Then an inactive seller in subgroup A_1 could propose a contract $(Q_2^* - (Q_1^* - Q^*), T_2^* - (T_1^* - T^*) - \epsilon)$ with ϵ small enough such that:

$$T_2^* - (T_1^* - T^*) - \epsilon > c_2(Q_2^* - (Q_1^* - Q^*))$$

The contract $(Q_2^* - (Q_1^* - Q^*), T_2^* - (T_1^* - T^*) - \epsilon)$ in subgroup A_1 is attractive for type 2 buyers, due to type 2 buyers can combine this contract with the separating contract $(Q_1^* - Q^*, T_1^* - T^*)$ in subgroup A_2, then achieve the aggregate contract with $(Q_2^*, T_2^* - \epsilon)$. If the new contract only attract type 2 buyers, then the seller could get profit with $f_2(T_2^* - (T_1^* - T^*) - \epsilon - c_2(Q_2^* - (Q_1^* - Q^*))) > 0$; If the new contract also attract type 1 buyers , then the seller could get profit with $T_2^* - (T_1^* - T^*) - \epsilon - c(Q_2^* - (Q_1^* - Q^*)) > T_2^* - (T_1^* - T^*) - \epsilon - c_2(Q_2^* - (Q_1^* - Q^*)) > 0$, again positive profit. Thus, in equilibrium with structure of "pooling + separating", the condition of NPC inequality should be hold to prevent the pivoting deviation.

4.4 Off-equilibrium path: latent contracts are necessary

In the equilibrium, there are pooling contracts or some separating contracts traded in the market. On the equilibrium path, if there is any equilibrium allocation $(Q_1^*, T_1^*), (Q_2^*, T_2^*)$ for type 1 and type 2 buyers. Then $(Q_1^*, T_1^*), (Q_2^*, T_2^*)$ should satisfy all the necessary conditions as above. However, there are other some other contracts may be proposed by some sellers without being traded on the equilibrium path, but they are also necessary to sustain the equilibrium. As what we mentioned, we will focus on the the form of "pooling + separating " , and then extend the results also to the "pooling + pooling" form.

On the equilibrium path, We assume sellers in subgroup A_1 provide the on-equilibrium path pooling contracts , while sellers in subgroup A_2 provide the on-equilibrium path separating contracts. According to "fair unit price " lemma, we can have the on-equilibrium path pooling contract in subgroup A_1 are $(Q^*, T^*) = (\frac{c_2 Q_2^* - T_2^*}{c_2 - c}, c\frac{c_2 Q_2^* - T_2^*}{c_2 - c})$, and on-equilibrium path separating contracts in subgroup A_2 are the $(q_2^*, t_2^*) = (Q_2^* - Q^*, T_2^* - T^*)$,and $(q_1^*, t_1^*) = (Q_1^* - Q^*, T_2^* - T^*)$.

In the allocations with cross-subsidy between type 1 and type 2 buyers, there is a very important deviation may destroy the equilibrium, this deviation could design to attract type 1 buyers only with lower unit price but then has less serving cost. For example, one possible cream-skimming deviation is that one seller in A_1 proposes the a contract $(Q^* - \epsilon, T^* - c_x\epsilon)$, with c_x in (c, c_2) and ϵ very small. If there is no other kinds of contract in the market, this deviation will attracts only type 1 buyers and achieve positive profit. Thus, we would like to find some method to block this kind of deviations, a useful way is to make the deviations also attract type 2 as well, by combining with some latent contract.

Followed with Attar , Mariotti and Salanié (2021), we will start dealing with the large cream-skimming deviations.It is a deviation designed for attracting type 1 only, even if she doesn't trade with other contracts. We call them **large cream-skimming deviations** which proposes a large quantity-tariff (Q', T') pair and gives a better utility than $U_1(Q_1^*, T_1^*)$ and attract type 1. To deal with this kind of deviation, we would like to find a latent contract to block the possible large cream-skimming deviations:

Definition 8. *A contract (q^ℓ, t^ℓ) blocks large cream-skimming deviations if:*

$$\text{for each } (q,t), U_1(q,t) \geq U_1(Q_1^*, T_1^*) \text{ implies } U_2(q+q^\ell.t+t^\ell) \geq U_2(Q_2^*, T_2^*) \quad (7)$$

We would like to find a single latent that could block ll large cream-skimming deviations. By using the similar argument in Attar, Mariotti and Salanié (2022), we can have that the only possible candidate (q^ℓ, t^ℓ) which can block all large cream-skimming deviations is the contract which defined as below:

$$\begin{aligned} & U_2\left(Q_1^*+q^\ell, T_1^*+t^\ell\right)=U_2\left(Q_2^*, T_2^*\right) \\ & \tau_2\left(Q_1^*+q^\ell, T_1^*+t^\ell\right)=c \end{aligned} \quad (8)$$

The latent contract described by (13) is the unique candidate of the condition (12) , to make sure that this (q^ℓ, t^ℓ) could satisfy that block large cream-skimming deviation, we need an assumption on the utility function which related to the Gaussian curvature $\kappa_i\left(Q_0, T_0\right)$ of every type i 's indifference curve at any aggregate trade (Q_0, T_0). Denoting by $T=\mathcal{I}_i\left(Q, U_i\left(Q_0, T_0\right)\right)$ the functional expression of this indifference curve, we have

$$\kappa_i\left(Q_0, T_0\right) \equiv \frac{1}{\left\|\nabla U_i\right\|^3}\left|\begin{array}{cc} -\nabla^2 U_i & \nabla U_i \\ -\nabla U_i^{\top} & 0 \end{array}\right|\left(Q_0, T_0\right)=-\frac{\frac{\partial^2 \mathcal{I}_i}{\partial Q^2}\left(Q_0, U_i\left(Q_0, T_0\right)\right)}{\left\{1+\left[\frac{\partial \mathcal{I}_i}{\partial Q}\left(Q_0, U_i\left(Q_0, T_0\right)\right)\right]^2\right\}^{\frac{3}{2}}}$$

Assumption 3. ***Flatter Curvatures (2-order single-crossing):***

For each $i, \kappa_i>0$. Moreover, one of the following statements holds:

(i) For all Q_1, Q_2, T_1, T_2, if $\tau_1\left(Q_1, T_1\right)=\tau_2\left(Q_2, T_2\right)$, then $\kappa_1\left(Q_1, T_1\right)>\kappa_2\left(Q_2, T_2\right)$.

(ii) For all Q_1, Q_2, T_1, T_2, if $\tau_1\left(Q_1, T_1\right)=\tau_2\left(Q_2, T_2\right)$, then $\kappa_1\left(Q_1, T_1\right)=\kappa_2\left(Q_2, T_2\right)$.

The Flatter Curvatures is saying that the indifference curve of type 2 is more flatter than indifference curve of type 1 when the marginal substitution of quantity for tariff is the same. Or we can say that any contracts attracts type 1 , by combining with latent contract (q^ℓ, t^ℓ), it is also attractive for type 2 buyers.

Lemma 6. *If buyers' preferences satisfy Assumption Flatter Curvatures, then the contract (q^ℓ, t^ℓ) in one subgroup can block large cream-skimming deviations in the other subgroup.*

4.5 The Equilibrium Characterization with "pooling+separating "Form

In this subsection, we will show that for any $(Q_1^*, T_1^*), (Q_2^*, T_2^*)$ which satisfy all the necessary conditions and also the fair unit price lemma, then this allocation can be implemented by some equilibrium in the structure of "1+1" partial exclusive setting. In this subsection, we will focus on the "pooling + separating " form, and in the next subsection, we will show the construction of "pooling + pooling "situation.

4.5.1 The equilibrium contracts

To get the equilibrium, Firstly , the market needs some sellers who propose the pooling contracts and also some sellers who propose the additional separating contracts for two different types. Then, to block the possible cream-skimming deviation, some latent contracts are also needed.

The Pooling Contracts. As what mentioned before , the equilibrium structure will consist of "pooling + separating " , WLOG, let the sellers in subgroup A_1 provide the active pooling contracts. Assuming that the pooling contract is (Q^*, T^*), then according to fair unit price lemma, we could have that :

$$(Q^*, T^*) = (\frac{c_2 Q_2^* - T_2^*}{c_2 - c}, c\frac{c_2 Q_2^* - T_2^*}{c_2 - c}) \tag{9}$$

To avoid the market power of sellers, there are at least two sellers in subgroup A_1 propose the contract of (Q^*, T^*), both type 1 and type 2 buyers would purchase the contract in equilibrium, and the unit price of this pooling contract is c which is equal the serving cost for both types. Due to the restriction of exclusive inside the subgroup, type 1 and type 2 can only purchase at most one pooling contract in this setting.

The Separating Contracts. In the subgroup A_2, type 1 and type 2 purchase different quantity for the coverage. To get the aggregate allocation of type 2, there are at least two sellers in this subgroup propose the type 2's separating contract:

$$(q_2^*, t_2^*) = (Q_2^* - Q^*, T_2^* - T^*)$$

To implement the type 1's aggregate quantity-tariff pair, there are at least two sellers in subgroup A_2 propose the type 1's separating contract:

$$(q_1^*, t_1^*) = (Q_1^* - Q^*, T_2^* - T^*)$$

The unit price of (q_2^*, t_2^*) will equal to c_2 and unit price of (q_1^*, t_1^*) will locate between c_1 and c. To achieve the aggregate allocation, type 1 buyers purchase one pooling contract (Q^*, T^*) in subgroup A_1 and purchase one separating contract (q_1^*, t_1^*) in subgroup A_2, type 1 could get (Q_1^*, T_1^*) in total. Then, for type 2 , she can purchase one pooling contract in subgroup A_1 and additional buying the contract (q_2^*, t_2^*) in subgroup A_2, which enables type 2 to reach her aggregate trade (Q_2^*, T_2^*).

As we mentioned before, the possible cream-skimming deviation could destroy the equilibrium, to sustain the equilibrium, we also need some latent contracts which not active in the equilibrium but could block the cream-skimming deviation.

Blocking Large Cream-Skimming Deviations. As what we discussed before, the (q^ℓ, t^ℓ) could block large cream-skimming deviation, so for each subgroup A_1 and A_2, there exists at least one seller proposes the latent contract (q^ℓ, t^ℓ). With assumption Flatter Curvatures, these contracts can block the cream-skimming deviations in any group.

Blocking Small Cream-Skimming Deviations. If a deviation happens in subgroup A_1, type 1 buyers can combine contracts (q_1^*, t_1^*) issued in subgroup A_2, then a deviation could attract type 1 with a small amount of coverage. To block such deviation in subgroup A_1, we require that there exists at least one seller in subgroup A_2 proposes the latent contract:

$$(q_2^\ell, t_2^\ell) \equiv (q^\ell, t^\ell) + (q_1^*, t_1^*) \tag{10}$$

If a deviation happens in subgroup A_2, type 1 buyers can combine contracts (Q^*, T^*) issued in subgroup A_1, then a deviation could attract type 1 with a small amount of coverage. Thus, to block such a deviation in subgroup A_1, we require that there exists at least one seller in subgroup A_1 proposes the latent contract:

$$(q_1^\ell, t_1^\ell) \equiv (q^\ell, t^\ell) + (Q^*, T^*) \tag{11}$$

According to the Flatter Curvature, if a small cream-skimming deviation (q, t) with $t/q < c$ in A_1 combine with the contract (q_1^*, t_1^*) in A_2 is attractive for type 1, then it is also attractive for type 2 by combining (q, t) in A_1 and (q_2^ℓ, t_2^ℓ) in A_2. Then with the same logic, if a small cream-skimming deviation (q, t) with $t/q < c$ in A_2 combine with the contract (Q^*, T^*) is attractive for type 1, then it is also attractive for type 2 by combining (q, t) in A_2 and (q_1^ℓ, t_1^ℓ) in A_1.

Then we need to consider the case that if type 1 can combine the latent contract in one subgroup and attracted by a cream-skimming deviation in the other group. Let n_ℓ be the smallest integral number satisfies $n_\ell q^\ell > Q_2^*$, then for each subgroup $g, g \in \{1, 2\}$, we define the latent contracts with $k \leq n_\ell$:

$$(q_{gk}^\ell, t_{gk}^\ell) \equiv k \times (q_\ell, t_\ell) \tag{12}$$

If a deviation (q, t) happens in A_1, and type 1 can combine (q, t) and $(q_{2(k-1)}^\ell, t_{g(k-1)}^\ell)$ to have better utility, then the latent contract $(q_{2k}^\ell, t_{2k}^\ell)$ can block this deviation. And due to $n_\ell q^\ell > Q_2^*$, if any (q, t) that combine with $n^\ell \times (q^\ell, t^\ell)$ attracts type 1 buyers, it will reach a aggregate quantity larger than Q_2^*, by single-crossing, it also attracts type 2 buyers. The same logic , we can have the deviation (q, t) happens in A_1 and type 1 can combine (q, t) with $(q_{1(k-1)}^\ell, t_{1(k-1)}^\ell)$ are blocked.

Next , we consider the case a deviation (q, t) in subgroup A_1 , such that if type 1 can combine the latent contract (q_2^ℓ, t_2^ℓ) and (q, t) to get better utility, We then define some latent contracts with $k \leq n^\ell$ can block steps by steps:

$$(q_{2k}^{\ell'}, t_{2k}^{\ell'}) = (q_1^*, t_1^*) + k \times (q^\ell, t^\ell) \tag{13}$$

Then , if there is some seller propose some (q, t) in A_2, and type 1 can combine (q, t) and (q_1^ℓ, t_1^ℓ) to get better utility, notice that , if $q > q_1^*$, the combination of $q + q_1^\ell > Q_2^*$, by single-crossing, this deviation should attract type 2 also. If $q < q_1^*$, (q, t) attracts type 1 only if $t/q < c_1$, it is not profitable.

4.5.2 The Implementation of equilibrium

Theorem 1. *Under Flatter Curvature assumption , if $(Q_1^*, T_1^*), (Q_2^*, T_2^*)$ satisfy the Incentive compatible,fair unit price lemma, $\tau_1(Q_1^*, T_1^*) = c$,$\tau_2(Q_2^*, T_2^*) = c_2$ and "no pivoting condition", then this allocation can be implemented by "Pooling+Separating" allocation with "1+1" partial exclusive structure.*

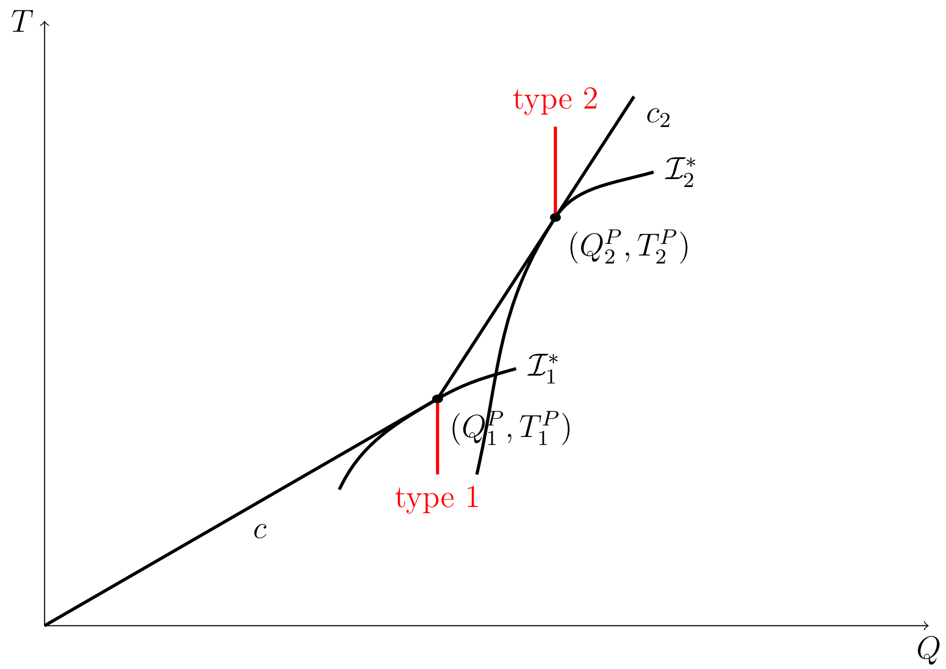

Figure 5: Equilibria compared with JHG in CARA

This theorem provide us a criterion to check whether an allocation can be implemented by an equilibrium of "pooling + separating" form in the "1+1" partial exclusive structure.

According to the criterion, we can find that in some equilibria, it is possible that the sellers who propose type 1 buyers' separating contracts in group A_2 could get positive profit. That is to say , the sellers in A_2 propose separating contract to type 1 with unit price $c_x > c_1$ can get positive profit. This is an interesting result because sellers get positive profit in a competitive market. The reason of the profitable sellers consist of three parts: (1)The exclusivity inside of subgroup A_2 gives the possibility of separating contracts to type 1 buyers. (2) The latent contracts ensure that any lower unit price contracts will attracts also type 2 buyers. (3) Type 2 buyers' preference can make no other sellers want to propose same contracts, for example, if only two sellers propose type 1's separating contract, type 2 purchases type 2's contracts, otherwise , purchase type 1's contract and one latent contract.

If we focus on the zero profit equilibria, then the equilibria allocation smooth changing for both types, and it actually gives the efficient allocation frontier in the "1+1" partial exclusive structure. Among all the equilibria ,JHG allocation is the most attractive for type 2 buyers but gives the worst utility to type 1 buyers. The allocation which let the NPC condition binding gives type 1 buyers best utility but worst utility for type 2 buyers.

4.6 The Equilibrium Characterization with "pooling+pooling "Form

As we discussed before, if there is an equilibrium with the form of "pooling+pooling" , then by the fair unit price lemma, and the possible pooling deviation, we can get that the unique possible equilibrium with this form is that $(Q_1^*, T_1^*) = (Q_2^*, T_2^*) = (Q_1^P, T_1^P)$.

In this allocation, we can find that type 1 purchase the optimal quantity given the unit price c. However, by single-crossing condition, type 2 still doesn't purchase enough quantity given the unit price of c. To ensure that the this kind of allocation is an equilibrium, we need that no sellers can propose a contract profitable attracts type 2: $U_2(Q_1^P, T_1^P) \geq max\{U_2(\frac{Q_1^P}{2} + Q, \frac{T_1^P}{2} + c_2 Q) : Q \geq 0\}$. Then by similar construction of "pooling+separating" form , we can have our implementation on "pooling + pooling " form.

Theorem 2. *Under Flatter Curvature assumption , if* $(Q_1^*, T_1^*) = (Q_2^*, T_2^*) = (Q_1^P, T_1^P)$ *and* $U_2(Q_1^P, T_1^P) \geq max\{U_2(\frac{Q_1^P}{2} + Q, \frac{T_1^P}{2} + c_2 Q) : Q \geq 0\}$*, then this allocation can be implemented by "Pooling+Pooling" allocation with "1+1" partial exclusive structure.*

In the equilibrium, there are two kinds of contracts, active contracts in both subgroups mainly consist of pooling contracts, and inactive contracts are the latent contracts which can block the cream-skimming deviations. The sketch of construction is as follows:

The pooling contract. There are at least two sellers in both subgroups A_1 and A_2 propose the pooling contract with $\frac{1}{2}(Q_1^P, T_1^p)$. In equilibrium , both type of buyers will choose one $\frac{1}{2}(Q_1^P, T_1^p)$ contracts in each subgroup. Thus , the average serving cost is same as the unit price c, and due to at least two sellers propose same contracts , there is no monopoly power for these active sellers.

The Latent contract. Take $(Q_1^*, T_1^*) = (Q_2^*, T_2^*) = (Q_1^P, T_1^P)$, using the same definition of the latent contract (q^ℓ, t^ℓ) that block the large cream-skimming deviation.Then we use the same argument as "pooling+separating", in each subgroup, we have some sellers propose (1) (q^ℓ, t^ℓ) to block the possible large cream-skimming deviation. (2) $(q^\ell, t^\ell) + \frac{1}{2}(Q_1^P, T_1^P)$ to block the cream-skimming deviation which based on the pooling contract. (3) latent contracts $k*(q^\ell, t^\ell)$ to block the latent contract $(k-1)*(q^\ell, t^\ell)$. (4) latent cotnracts $k*(q^\ell, t^\ell) + \frac{1}{2}(Q_1^P, T_1^P)$ to block the latent contract $(k-1)*(q^\ell, t^\ell) + \frac{1}{2}(Q_1^P, T_1^P)$.

5 Other Market Structures

This section will study two kinds of market structures, and we can find some similarity between different structures.

5.1 "$\lambda + 1$" partial exclusive competition

In the last section, we studied the "1+1" partial exclusive structure, and we got a sort of allocations that could be sustained in that structure, let the $\mathcal{E}_1^1$ be the set of all the equilibria in this structure. In this subsection, we study some similar structures we call that "$\lambda + 1$" partial exclusive structure $\mathcal{M}_\lambda^1$, which means the sellers are divided into two subgroups, in one subgroup, buyers can at most trade with λ sellers while in the other subgroup, buyers can at most trade with one seller. We find that the set of equilibria in structure $\mathcal{M}_\lambda^1$, defined by $\mathcal{E}_\lambda^1$, included by the $\mathcal{E}_1^1$, and when the λ increases, the set of equilibria will converge to JHG allocation.

Actually, in the "$\lambda + 1$" partial exclusive structure, use the same logic in last section. We will have that the possible equilibrium form are "pooling+pooling" and "pooling+separating", the first form is contains a unique equilibrium, for the second form, the necessary condition of Fair unit price lemma, incentive compatible and Fair Marginal purchasing would be same as in the "$1 + 1$" partial exclusive structure. However, for the no pivoting condition, $\lambda + 1$ structure has more strict condition.

Let us recall the necessary condition of No pivoting condition, in the form of "pooling+separating", firstly , we have that the allocation is (Q_1^*, T_1^*) and (Q_2^*, T_2^*), then the pooling aggregate contract in subgroup A_1 is $(Q^*, T^*) = (\frac{c_2 Q_2^* - T_2^*}{c_2 - c}, c\frac{c_2 Q_2^* - T_2^*}{c_2 - c})$, and the active separating contracts in A_2 is $(q_1^*, t_1^*) = (Q_1^* - Q^*, T_1^* - T^*)$ and $(q_2^*, t_2^*) = (Q_2^* - Q^*, T_2^* - T^*)$. Given the separating contract (q_1^*, t_1^*) trade with type 1 in subgroup A_2, one inactive seller in subgroup A_1 could pivot on this contract and try to attract type 2 buyers. To attract type 2 , the seller should also propose the basic pooling contract as the opportunity cost : In total, the seller could have profit on pivoting with $T_2^* - T_1^* - c_2(Q_2^* - Q_1^*)$, and the maximal possible cost with the pooling contract is $\frac{1}{\lambda}(c_2 Q^* - T^*)$. To make this pivoting in subgroup A_1 is not a profitable deviation, we need that the profit is less than the cost for providing pooling contracts. which is

$$T_2^* - T_1^* - c_2(Q_2^* - Q_1^*) \leq \frac{1}{\lambda}(c_2 Q^* - T^*) \tag{14}$$

From the necessary condition above, we can find that: (1) When λ increasing , the inequality becomes more strict, this structure contains less equilibria. (2) If λ is big enough, then the inequality becomes that $T_2^* - T_1^* - c_2(Q_2^* - Q_1^*) \leq 0$, combined with fair unit price lemma, it implies the unique equilibrium is JHG allocation.

Theorem 3 (Convergence of equilibria)**.** *Denote $\mathcal{E}_\lambda^1$ be the set of of equilibria in the market structure $\mathcal{M}_\lambda^1$. For two different market structure $\mathcal{M}_{\lambda 1}^1$ and $\mathcal{M}_{\lambda 2}^1$, if $\lambda 2 > \lambda 1$, then $\mathcal{E}_{\lambda 2}^1 \subseteq \mathcal{E}_{\lambda 1}^1$. When λ close to infinite, $\mathcal{E}_\lambda^1$ converge to JHG allocation.*

5.2 "$\lambda_1 + \lambda_2$" partial exclusive competition(TBD)

In the structure of "$\lambda + 1$" partial exclusive structure,trade at most λ in subgroup A_1 gives A_1 some nonexclusivity, but in subgroup A_2, it still keeps the restriction that a buyer can only purchase exclusively with other sellers inside of A_2. With this kind of structure, we can see the effect by only controlling one subgroup's degree of nonexclusivity, we found that with increasing of nonexclusivity in one subgroup, the equilibria set convergent to JHG allocation. In this subsection, we would like to check what will happen if there exist some more nonexclusivity in both subgroups.

Lemma: In any equilibrium, type 1 could trade at most one separating contract.

The intuition of the lemma is that if type 1 trade at least 2 separating contracts in the market, then a seller can pivot on type 1's separating contracts

Then according to this lemma, the possible candidate of equilibrium can only be divided into two cases:

Case (1) If type 1 trade with no separating contract, then the only candidate allocation trade for type 1 is (Q_1^P, T_1^P), while type 2's allocation trade could be (Q_2^P, T_2^P) or (Q_1^P, T_1^P).

Case (2) If type 1 trade with one separating contract, assume it was in subgroup A_2, then in equilibrium, buyers purchase $\lambda_2 - 1$ numbers of pooling contracts and one separating contract, otherwise, the inactive seller can pivot on type 1's separating contract.

Theorem 4. *Equivalence structures Denote $\mathcal{E}_{\lambda_1}^{\lambda_2}$ be the set of of equilibria in the market structure $\mathcal{M}_{\lambda_1}^{\lambda_2}$. Denote $\mathcal{E}_{\lambda_1+\lambda_2-1}^{1}$ be the set of of equilibria in the market structure $\mathcal{M}_{\lambda_1+\lambda_2-1}^{1}$. Then $\mathcal{E}_{\lambda_1}^{\lambda_2} \subseteq \mathcal{E}_{\lambda_1+\lambda_2-1}^{1}$, and when $\lambda_1 + \lambda_2$ is big enough, $\mathcal{E}_{\lambda_1}^{\lambda_2} = \mathcal{E}_{\lambda_1+\lambda_2-1}^{1}$.*

6 Robustness: Menu Game in Partial Exclusive Competition

In our model , we assume that the sellers in the market can only propose a single contract to the market, it simplify our analysis and show the results in a straightforward way. In this section, we will also show the results that if the sellers could propose menu contracts, that is to say,a seller propose a set of contracts and a buyer can only choose one of contract from the menu of this seller. In the setting of regulation games proposed by Attar, Mariotti and Salanié (2021b),we show that the results in our model doesn't change. When there is no regulation, we show that the unique equilibrium in the menu game is JHG allocation.

6.1 Menu games with regulation

The timeline of regulation game is as follow:

- Stage 0: The planner chooses the market structure $\mathcal{M}_1^1$.
- Stage 1: Every firm k posts a compact menu of contracts C^k that contains at least the no-trade contract $(0, 0)$.
- Stage 2: After privately learning her type i, each buyer chooses some $M \in \mathcal{M}_1^1$, trades with all sellers $k \in M$ and chooses a contract in $\mathcal{C}^k$, then derives utility $U\big(\sum_{k\in M} q^k, \sum_{k\in M} t^k\big)$.
- Stage 3: If a firm overall earns a nonnegative profit but incurs a loss on a contract it actually trades, then its profit is confiscated and the firm is fined.

There are two different setting with the regulation game compared with our original model: (1) Firstly, the sellers in the market could propose more than one contract, but a buyer can only choose at most one contract from the menu. The menu of the sellers allow they play a competitive-screening game. (2) Secondly, stage 3 provides an additional monitoring on sellers, it make sure that there is no cross-subsidies between different contracts.

In this regulation game , it is easy to get that the necessary condition of incentive-compatible, fair marginal purchasing, no pivoting condition should be also satisfied by using the possible deviation assumption. Moreover, the fair unit price lemma should be also satisfied, due to there is no cross-subsidies from other contracts, we can actually treat that all the contracts in the menu should earn nonegative profit, then serving type 2 should have unit price of c_2 and serving for both types should have unit price of c due to the Bertrand competition and budget-balanced. And then in the equilibrium , no contract should aim at type 2, because any contract attract type 2 only will achieve negative profit. Then the deviation to have positive profit only related to type 1's contracts, however, the latent contract in the construction of section 4 can block any deviation attracts type 1.

Theorem 5 (Equilibria in regulation game). *If $(Q_1^*, T_1^*), (Q_2^*, T_2^*)$ can be one of equilibrium in the single contract game with "1+1" partial exclusive structure, then it is also the equilibrium of the regulation game with menu offer sellers.*

6.2 Menu games without regulation

In a totally nonexclusive structure and sellers could propose menu contracts ,according to Attar,Mariotti and Salanié (2014), if we drop out the stage 3 which regulate the cross-subsidizing between different contracts, JHG allocation is not a equilibrium. In our partial exclusive setting , we allow some exclusive and also same nonexclusive in the market structure, we show that in the menu game, "1+1" partial exclusive structure has unique equilibrium candidate and it could be the equilibrium when the utility function satisfy some assumptions. The timeline of the competitive menu game is as follows:

- Stage 0: The planner chooses the market structure $\mathcal{M}_1^1$.
- Stage 1: Each seller k can propose a compact menu of contracts $\mathcal{C}^k$ to the market.
- Stage 2: After privately learning her type i, each buyer chooses some $M \in \mathcal{M}_1^1$, trades with all sellers $k \in M$ and chooses a contract in $\mathcal{C}^k$, then derives utility $U\big(\sum\limits_{k \in M} q^k, \sum\limits_{k \in M} t^k\big)$.

Remark: In stage 2, a seller k can propose a menu consist of finite number of contracts or a menu consist of a tariff with quantity.

Similar with the method of single contract game , we will focus on the equilibrium form of “pooling+separating”. (Actually, “pooling+pooling” and “separating+separating” can be ruled out by the argument of double deviation.) Before find the necessary condition , we will show the fair unit price lemma in the menu games.

Lemma 7 (Fair unit price in menu)**.** *Under free-entry equilibrium, in each subgroup, if an active contract is purchased by both types, the unit price of this contract should equal c; if an active contract is purchased by type 2 only, the unit price should lower than* c_2*; if an active contract is purchased by type 1 only, the unit price should between* $[c_1, c]$*.*

This The fair unit price lemma in menu is very similar with the lemma in single contract game, however, if one contract of purchased by type 2, the unit price should no larger than c_2, but it could smaller than c_2 because of the possible cross-subsidy from other contract traded with type 1. Then due to sellers can not make profit on type 2, the unit price for the contract trade with type 1 should be larger than c_1. Then the upper bound of each kinds of trade can be deduced by Bertrand competition argument, thus the fair unit price lemma in menu.

6.2.1 Motivating example: “1+1” partial exclusive case

The unique equilibrium candidate: JHG allocation

In the game with single contract , we got many equilibria with the structure of “pooling + separating”, as what we did, if (Q_1^*, T_1^*) and (Q_2^*, T_2^*) are the equilibrium allocation with “pooling + separating ” form. Assume the sellers in subgroup A_1 propose the active pooling contract (Q^*, T^*), then the sellers in A_2 propose the active separating contracts $(q_1^*, t_1^*) = (Q_1^* - Q^*, T_1^* - T^*)$ and $(q_2^*, t_2^*) = (Q_2^* - Q^*, T_2^* - T^*)$. Then by fair unit price lemma in menu , we have that $T^* = cQ^*$, and $t_2^* \leq c_2 q_2^*$. and $c_1 q_1^* \leq t_1^* \leq c q_1^*$.

And then the two necessary condition we got in single-contract setting should also hold in that sellers can propose menu of contracts setting , which as below:

Proposition 5. ***Incentive compatibility buyers:*** *The aggregate trades* $(Q_i^*, T_i^*), i = 1, 2$ *satisfy incentive compatibility:*

$$U_x(Q_x^*, T_x^*) \geq U_x(Q_y^*, T_y^*) \quad \textit{for all } x, y \in \{1, 2\}.$$

Proposition 6. ***Fair Marginal purchasing:*** *In the allocation of equilibrium, the marginal utility of type i buyers should equal the tail expectation serving cost* *from type i,* $\tau_1(Q_1^*, T_1^*) = c$ *where* $c = f_1c_1 + f_2c_2$ *, and then* $\tau_2(Q_2^*, T_2^*) = c_2$

Then we will discuss what would be the possible equilibrium cases in multiple menus game, actually, in the form of "pooling +separating" we can have more necessary conditions which can be concluded by the argument of double deviation by a seller with menu contracts. For example, in the subgroup A_2, the separating contracts should satisfy some conditions as follow:

Proposition 7. *In the equilibrium ,* $t_1^* = cq_1^*$ *and* $t_2^* - t_1^* = c_2(q_2^* - q_1^*)$*.*

To prove this proposition, we can divide the situation of separating contract into two case, in the first case, if the active separating contract proposed by different sellers, we that the type 1's contract will be the null contract; in the second case, if the active separating contract proposed by one seller, we show that the difference between two contracts is also fair priced.

Lemma 8. *If the separating contracts provided by two different sellers in* A_2*, then* $q_1^* = t_1^* = 0$ *and* $t_2^* = c_2q_2^*$*.*

The main idea of this lemma is that , if there is one seller only serving for type 1 with some unit price lower than c_2 , then an inactive seller in A_1 could propose a double deviation, one contract pivot on type 1's separating contract to attract type 2, and the other contract propose a lower price attracts type 1. Then this double deviation will have positive profit if type 1's separating contract is not null contract.

Proof. Firstly, if the two separating contracts provided by two different sellers, the contract (q_2^*, t_2^*) should get zero-profit according to lemma 1 and budget balance of seller, thus, we get that $t_2^* = c_2q_2^*$. Then we claim for the contract of (q_1^*, t_1^*), according to lemma 1, we have that $t_1^* \leq cq_1^*$, then we claim that it is impossible to trade with $q_1^* > 0$ in the equilibrium.

Otherwise, assume (q_1^*, t_1^*) active traded by type 1 in equilibrium with $q_1^* > 0$. Consider an inactive seller k in subgroup A_1, and propose a menu with contracts $\mathcal{C}^k = \{c_1^k, c_2^k\}$, where $c_1^k = (Q^*, T^* - \epsilon_1)$ and $c_2^k = (Q_2^* - q_1^*, T_2^* - t_1^* - \epsilon_2)$ for some numbers of ϵ_1 and ϵ_2, it is easy to get that $c_1^k + (q_1^*, t_1^*) = (Q_1^*, t_1^* - \epsilon_1)$ and $c_2^k + (q_1^*, t_1^*) = (Q_2^*, T_2^* - \epsilon_2)$, the aggregate trade for type 1 and type 2 are better than (Q_1^*, T_1^*) and (Q_2^*, T_2^*). Then if ϵ_1 is small enough , then we can always find a small ϵ_2 such that type 2 prefer c_2^k than c_1^k. Then the profit for seller k would be

$$f_1(T^* - \epsilon_1 - c_1Q^*) + f_2(T_2^* - t_1^* - \epsilon_2 - c_2(Q_2^* - q_1^*)) \tag{15}$$

we know that $(Q_2^*, T_2^*) = (Q^* + q_2^*, T^* + t_2^*)$ and we know that $t_2^* = c_2q_2^*$. Let

ϵ_1 and ϵ_2 close to 0, then by (1), the profit for seller k would be

$$[T^* - (f_1c_1 + f_2c_2)Q^*] + (t_2^* - c_2q_2^*) + (cq_1^* - t_1^*) + (c_2 - c)q_1^* \tag{16}$$

In (2), we know that $T^* - cQ^* = 0, t_2^* - c_2q_2^* = 0$ and $cq_1^* - t_1^* \geq 0$ and $(c_2 - c)q_1^* > 0$. That is to say, the seller k in group A_1 can get strict positive profit by proposing $\mathcal{C}^k = \{c_1^k, c_2^k\}$, contradiction.

Lemma 9. *If the separating contracts provided by same seller in A_2, then $t_1^* = cq_1^*$ and $t_2^* - t_1^* = c_2(q_2^* - q_1^*)$*

Proof. In "pooling + separating " form of allocation, we first show that if the a seller trade (q_1^*, t_1^*) and (q_2^*, t_2^*) in equilibrium path, then $t_2^* - t_1^* = c_2(q_2^* - q_1^*)$.

Step (1): $t_2^* - t_1^* \geq c_2(q_2^* - q_1^*)$, otherwise, if $t_2^* - t_1^* < c_2(q_2^* - q_1^*)$, then the profit of trade with both types for this seller would be :

$$\begin{aligned} &f_1(t_1^* - c_1q_1^*) + f_2(t_2^* - c_2q_2^*) \\ =&[t_1^* - cq_1^*] + f_2[t_2^* - t_1^* - c_2(q_2^* - q_1^*)] \end{aligned}$$

The first term $t_1^* - cq_1^* \leq 0$ according to lemma 1, if the second term $t_2^* - t_1^* - c_2(q_2^* - q_1^*) < 0$, then the profit of this seller is negative, contradiction.

Step (2): $t_2^* - t_1^* \leq c_2(q_2^* - q_1^*)$, otherwise, assume that $t_2^* - t_1^* > c_2(q_2^* - q_1^*)$. Consider an inactive seller k in subgroup A_1, and propose a menu with contracts $\mathcal{C}^k = \{c_1^k, c_2^k\}$, where $c_1^k = (Q^*, T^* - \epsilon_1)$ and $c_2^k = (Q_2^* - q_1^*, T_2^* - t_1^* - \epsilon_2)$ for some numbers of ϵ_1 and ϵ_2, it is easy to get that $c_1^k + (q_1^*, t_1^*) = (Q_1^*, t_1^* - \epsilon_1)$ and $c_2^k + (q_1^*, t_1^*) = (Q_2^*, T_2^* - \epsilon_2)$, the aggregate trade for type 1 and type 2 are better than (Q_1^*, T_1^*) and (Q_2^*, T_2^*). Then if ϵ_1 is small enough , then we can always find a small ϵ_2 such that type 2 prefer c_2^k than c_1^k. The profit of this deviation would be:

$$f_1(T^* - \epsilon_1 - c_1Q^*) + f_2(T_2^* - t_1^* - \epsilon_2 - c_2(Q_2^* - q_1^*))$$

When ϵ_1 and ϵ_2 close to 0, the profit for seller k will be

$$\begin{aligned} &f_1(T^* - c_1Q^*) + f_2(T_2^* - t_1^* - c_2(Q_2^* - q_1^*)) \\ =&[T^* - (f_1c_1 + f_2c_2)Q^*] + f_2[t_2^* - t_1^* - c_2(q_2^* - q_1^*)] \end{aligned}$$

We know that $T^* - (f_1c_1 + f_2c_2)Q^* = 0$, if $t_2^* - t_1^* - c_2(q_2^* - q_1^*) > 0$ then the seller k get strict positive profit by proposing $\mathcal{C}^k$, contradiction again. Thus, $t_2^* - t_1^* \leq c_2(q_2^* - q_1^*)$.

According to step (1) and (2), we have that $t_2^* - t_1^* = c_2(q_2^* - q_1^*)$. Then according to step (1), the profit of the seller who provides the contracts (q_1^*, t_1^*) and (q_2^*, t_2^*) is :

$$[t_1^* - cq_1^*] + f_2[t_2^* - t_1^* - c_2(q_2^* - q_1^*)] \tag{17}$$

Due to $t_2^* - t_1^* = c_2(q_2^* - q_1^*)$, then budget balance requires that $t_1^* - cq_1^* \geq 0$, and according to lemma 1, we have $t_1^* - cq_1^* \leq 0$, thus , we have that $t_1^* = cq_1^*$.

From the lemma above, we know that no matter the separating contract provided by one seller or different sellers, we have that : $T_2^* - T_1^* = t_2^* - t_1^* = c_2(q_2^* - q_1^*) = c_2(Q_2^* - Q_1^*)$. Then by using fair unit price lemma, we have the the pooling contracts in A_1 have unit price of c, combined with $t_1^* = cq_1^*$, we can have that $T_1^* = cQ_1^*$. Then according to the necessary condition of Fair purchasing buyers, we have that $\tau_1(Q_1^*, T_1^*) = c$, and then $\tau_2(Q_2^*, T_2^*) = c_2$. Thus, the only equilibrium candidate is actually JHG allocation.

Theorem 6. *Under the setting of sellers could propose menu, and in the "1+1" partial exclusive structure, the unique equilibrium candidate is JHG allocation:* (Q_1^*, T_1^*) *and* (Q_2^*, T_2^*) *defined as below:*

$$\begin{aligned}
Q_1^* &= argmax\{U_1(Q, cQ) : Q \geq 0\} \\
T_1^* &= cQ_1^* \\
Q_2^* - Q_1^* &= argmax\{U_2(Q_1^* + Q, T_1^* + c_2Q) : Q \geq 0\} \\
T_2^* - T_1^* &= c_2(Q_2^* - Q_1^*)
\end{aligned}$$

Existence of equilibrium

In the allocation of form "pooling + separating ", we know that the only possible equilibrium candidate with "pooling+ separating " structure is JHG allocation (Q_1^*, T_1^*) and (Q_2^*, T_2^*) . In this part, we would like to construct the possible implementation of the equilibrium.

An equilibrium menus

The menus in subgroup A_1 support the JHG allocation.

We wanted that the sellers in subgroup A_1 could propose at least the pooling contract (Q_1^*, T_1^*), due to the sellers can propose a menu of contracts or a tariff specifies on different quantity, we hope that the sellers who propose this kind of pooling could also help the market block the cream-skimming deviation in A_2, then we can construct at least two sellers in A_1 provides the tariff with

$$T(Q) = 1_{\{Q \leq Q_1^*\}} cQ + 1_{\{Q > Q_1^*\}} [cQ_1^* + c_2(Q - Q_1^*)] \tag{18}$$

Actually, this kind of tariff is entry-proof to the sellers of A_2, and due to buyers can at most purchase from one seller in A_1, so the sellers who propose this kind of tariff in A_1 will not affect the possible profit of others. Let $\mathcal{C}_1^*$ be the menu of this tariff in subgroup A_1.

The menus in A_2 with additional separating contracts. The sellers in A_2 was designed to provide the additional contracts $(Q_2^* - Q_1^*, T_2^* - T_1^*)$, to block some potential deviation , we construct at least two sellers in A_2 propose the tariff with

$$T(Q) = c_2 Q \tag{19}$$

Let $\mathcal{C}_2^*$ be the menu of this tariff in subgroup A_1.

The latent contract in A_2 to block the large cream-skimming deviation in A_1 As we discussed before, to avoid the large cream-skimming in A_1, we need that there is at least one seller in subgroup A_2 propose (q^ℓ, t^ℓ) defined by (9). With the assumption Flatter Curvature, this kind of contract in A_2 can block the cream-skimming deviation in A_1.

The latent contract in A_2 to block the small cream-skimming deviation in A_1

There are two possible small deviation in A_1: (1) the first is propose some (q, t) in A_1 and attracts type 1 if she can combine (q, t) with latent contact (q^ell, t^ell) to get better utility: in this case, Let n_ℓ be the smallest integral number satisfies $n_\ell q^\ell > Q_2^*$, we just need to construct the seller who propose menu with n_ℓ latent contract also has the contracts as :

$$(q_k^\ell, t_k^\ell) \equiv k \times (q_\ell, t_\ell) \tag{20}$$

. Let $\mathcal{C}_l^1$ be the menu of this tariff in subgroup A_1. (2) The second case is (q, t) in A_1 and attracts type 1 if she can combine (q, t) with additional separating contracts tariff, in this case : we just need construct at least one seller in A_2 propose the tariff with quantity great than q^ℓ as :

$$T(Q + q^\ell) = t^\ell + c_2 Q \tag{21}$$

Let $\mathcal{C}_l^2$ be the menu of this tariff in subgroup A_1.Then all small skimming deviation are blocked by these $\mathcal{C}_l^1$ and $\mathcal{C}_l^2$ of latent contracts.

Theorem 7. *Under assumption of Flatter curvature (ii), JHG allocation is an equilibrium in which there are at least two sellers propose $\mathcal{C}_1^*$ in subgroup A_1, and there are at least two sellers propose $\mathcal{C}_2^*$, $\mathcal{C}_l^1$ and $\mathcal{C}_l^2$ in subgroup A_2.*

To ensure that JHG allocation is an allocation, we need to deal to two main deviations: the first one is the cream-skimming deviation , we could deal with that by using the latent contract which is similar which what we did in theorem 1. The second deviation is double deviation, in which a seller in subgroup A_1 tries to screen type 1 and type 2 by proposing a menu of contracts. Actually, in a totally nonexclusive environment , no equilibrium exists if the type 1 trade with positive contracts(Attar, Mariotti and Salanié (2014)). This is because that one seller can have profitable deviation by posting two contracts, one contract a little bit cheaper than (Q_1^*, T_1^*), the other pivoting on the (Q_1^*, T_1^*). In our "1+1" partial exclusive setting, this kind of double deviation is impossible because of the exclusivity of subgroup A_1, the pivoting contract is exclusive with the (Q_1^*, T_1^*). Thus, the double deviation destroy equilibrium in nonexclusive not worked in "1+1" partial exclusive model. But another possible double deviation is pivoting on latent contract and then attracts type 2 buyers, under assumption Flatter curvature (ii), any pivoting contract based on latent contract to attract type 2 should have unit price lower than c, and by the translate indifference curve, it would also attract type 1, which means that deviation is not profitable.

According to theorem 6, we find that in the setting of "1+1" partial exclusive structure, JHG allocation can be an equilibrium in menu game. Different with Attar,Mariotti and Salanié (2021b), we do not need that the planner monitor the cross-subsidy between different contracts, instead, the planner only need set the trade rule in the stage 0 , and we will get the JHG allocation.

6.2.2 General cases: "$\lambda + 1$"and "$\lambda_1 + \lambda_2$" partial exclusive cases

After the motivating example of "$1 + 1$" partial exclusive structure case, we can along the same trick of proving the uniqueness, get that JHG allocation is the unique possible equilibrium in menu game without regulation in the "$\lambda + 1$" and "$\lambda_1 + \lambda_2$" structure. Then we will construct that JHG allocation can be an equilibrium if some assumption can be sustain in the preference.

Theorem 8. *If the preference of buyers satisfy the perfect translation property (Flatter Curvature (ii)), then in "$\lambda+1$" and "$\lambda_1+\lambda_2$" partial exclusive structure, JHG allocation is the equilibrium and in the menu game.*

The trick of existence is similar to "1 + 1" case: Firstly, we need the sellers in subgroup A_1 propose the entry-proof tariff, which ensures that there is no deviation in subgroup A_2. Then we need the sellers in A_2 propose the complementary contract with unit price c_2, and then the latent contract which can block the cream-deviation in subgroup A_1. Take the "$\lambda + 1$" structure as the example, there exist at least $\lambda + 1$ sellers propose the menu with $T(Q) = 1_{\{Q \leq \frac{Q_1^*}{\lambda}\}} cQ + 1_{\{Q > \frac{Q_1^*}{\lambda}\}} [c\frac{Q_1^*}{\lambda} + c_2(Q - \frac{Q_1^*}{\lambda})]$. It is easy to get that combination of λ sellers in A_1 forms entry-proof tariff, thus, there is no profitable deviation in A_2. And due to the restriction of trade at most "λ" seller in A_1, it is not possible for type 2 trade pooling contract larger than Q_1^*. Thus, the possible profitable deviation only exists in A_1 group sellers.

For the cream-skimming deviation in A_1, the latent contract in A_2 can block it. Then the only possible profitable deviation in A_1 is double deviation which propose a menu with different contracts c_1^k, c_2^k to attract different types, where $c_1^k = (q_1^k, t_1^k), c_2^k = (q_2^k, t_2^k)$.$c_1^k$ attracts type 1 only if the unit price lower than c: (1)If c_2^k combine complementary on latent contract in A_2, then we will have that $t_2^k < cq_2^k$, and by perfect translation of utility function, if c_2^k is preferred by type 2 than c_1^k with combination of latent contract , then type 1 also prefer c_2^k , but the average cost for serving both type is c, negative profit. (2) If there is no type 2's combination with latent contract in this menu, firstly, if a buyer choose one of contract of the deviation, the tariff given by other sellers in this A_1 is $T'(Q) = 1_{\{Q \leq \frac{\lambda-1}{\lambda} Q_1^*\}} cQ + 1_{\{Q > \frac{\lambda-1}{\lambda} Q_1^*\}} [c\frac{\lambda-1}{\lambda} Q_1^* + c_2(Q - \frac{\lambda-1}{\lambda} Q_1^*)]$.

Then we can divide the possible c_1^k in two cases, the first one is that $q_1^k < \frac{1}{\lambda} Q_1^*$, in this case c_2^k attracts type 2 only if the combination of c_2^k can give better utility than (Q_2^*, T_2^*) , but this indicates that the total profit of c_1^k and c_2^k will be negative.As in the figure **small double deviation**, c_2^k can be split as the $\frac{1}{\lambda} Q_1^* + (q_2', t_2')$ with $t_2' < c_2 q_2'$, so it is straightforward to get the total profit is negative.

As in the figure **large double deviation**,the second possible deviation is that $q_1^k > Q_1^*$, there are two cases for the type 2's contracts: c_2^k can have unit price lower than c_1^k or greater than c_1^k. When c_2^k has lower unit price and attracts type 2 ,the convexity of the available tariff suggests that c_2^k also more attractive for type 1, which means c_2^k attracts both type with unit price lower c, which is not profitable. Then c_2^k has larger unit price of c_1^k, by the same trick in small double deviation, the total profit is negative. To conclude, JHG allocation can be an equilibrium in this menu game.

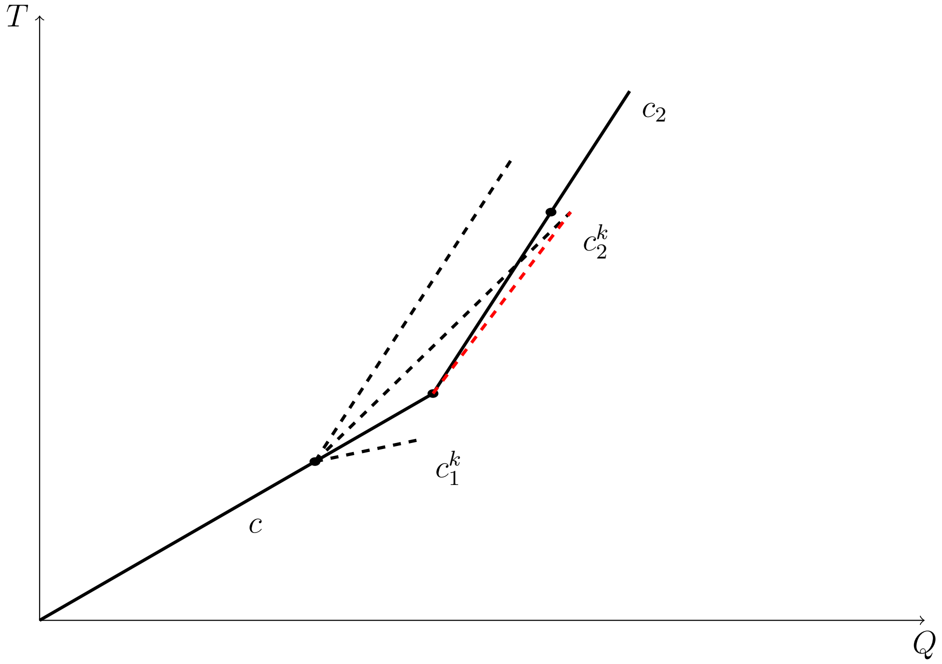

Figure 6: The small double deviation

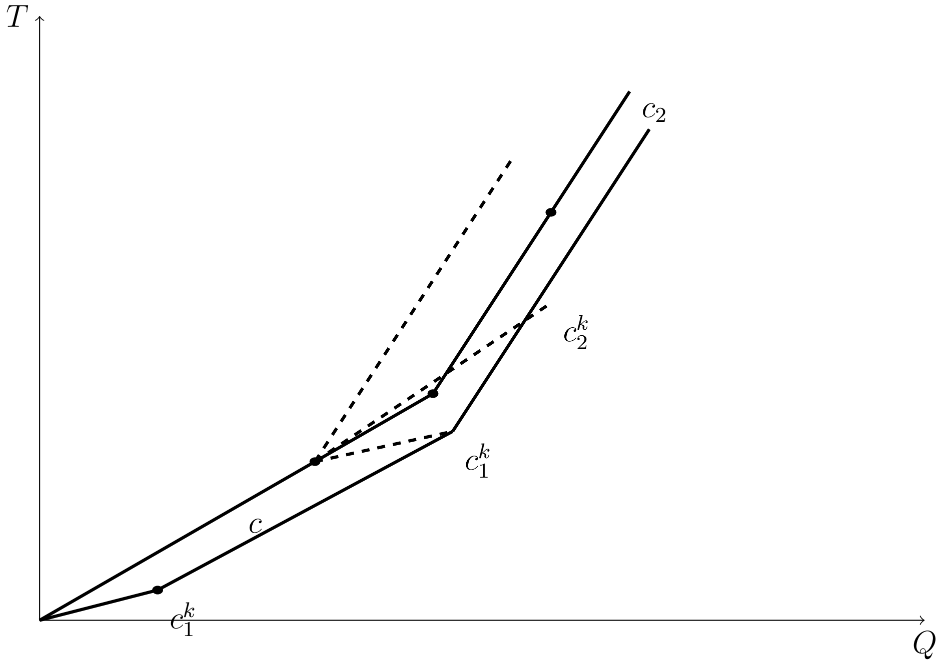

Figure 7: The large double deviation

7 Appendix

7.1 Proof of Theorem 1

The proof consist of 4 steps:

Step 1 We first construct the subgroup's active contracts which can implement $(Q_1^*, T_1^*), (Q_2^*, T_2^*)$, and also the latent contract which can help sustain the equilibrium.

For the subgroup A_1:

1. The pooling contracts: there are at least two sellers propose (Q^*, T^*) defined by (9) in group A_1.
2. The latent contract to block large cream-skimming deviation in group A_2: there is at least one seller in group A_1 propose (q^ℓ, t^ℓ) .
3. The latent contract to block small cream-skimming deviation in group A_2 :
   - There is at least one seller in group A_1 propose (q_1^ℓ, t_1^ℓ) .
   - For each $k \leq n_\ell$, there is at least one seller in A_1 proposes $(q_{1k}^\ell, t_{1k}^\ell)$.
4. Any other sellers in this subgroup propose either one of contract in 1,2,and 3.

For the subgroup A_2:

1. The separating contracts: there are at least two sellers propose $(Q_2^* - Q^*, T_2^* - T^*)$, and at least two sellers propose $(Q_1^* - Q^*, T_1^* - T^*)$ in group A_2.
2. The latent contract to block large cream-skimming deviation in group A_1: there is at least one seller in group A_1 propose (q^ℓ, t^ℓ).
3. The latent contract to block small cream-skimming deviation in group A_1:
   - There is at least one seller in group A_2 propose (q_2^ℓ, t_2^ℓ).
   - For each $k \leq n_\ell$, there is at least one seller in A_2 proposes $(q_{2k}^\ell, t_{2k}^\ell)$.
   - For each $k \leq n_\ell$, there is at least one seller in A_2 proposes $(q_{2k}^{\ell'}, t_{2k}^{\ell'})$.
4. Any other sellers in this subgroup propose either one of contract in 1,2,and 3.

With the contracts like above, type 1 buyers can purchase one pooling contract (Q^*, T^*) in A_1, and purchase one separating contract $(Q_1^* - Q^*, T_1^* - T^*)$ in A_2 to achieve the aggregate trade (Q_1^*, T_1^*). Type 2 buyers can purchase one pooling contract (Q^*, T^*) in A_1, and and purchase one separating contract

$(Q_2^* - Q^*, T_2^* - T^*)$ in A_2 to achieve the aggregate trade (Q_2^*, T_2^*).

Step 2 We next claim that, on the equilibrium path, two types consumers have a best response given the available contracts as step 1's construction.

(1) Consider first type 1 buyers. due to $\tau_1(Q_1^*, T_1^*) = c$, and $T^* = cQ^*$, $\frac{t_1^*}{q_1^*} \in [c_1, c]$, What we can get is that :

$$U_1(Q_1^*, T_1^*) \geq max\{U_1(Q, cQ) : Q \geq 0\} \tag{22}$$

$$U_1(Q_1^*, T_1^*) \geq max\{U_1(q_1^* + Q, t_1^* + cQ) : Q \geq 0\} \tag{23}$$

If there is an profitable deviation for type 1: (1) If the deviation of type 1 does not contain contract (q_1^*, t_1^*), then all other contracts in step 1 have premium rates at least equal to c, then by (16) the utility of this deviation $\leq max\{U_1(Q, cQ) : Q \geq 0\} \leq U_1(Q_1^*, T_1^*)$. (2) If the deviation of type 1 contains contract (q_1^*, t_1^*) in subgroup A_2, then all the contracts in A_1 have premium rates at least equal to c, then by (17) the utility of deviation $\leq max\{U_1(q_1^* + Q, t_1^* + cQ) : Q \geq 0\} \leq U_1(Q_1^*, T_1^*)$. So the optimal utility that type 1 can get is actually $U_1(Q_1^*, T_1^*)$, and choose (Q^*, T^*) in A_1 and (q_1^*, t_1^*) in A_2 can reach this aggregate trade of (Q_1^*, T_1^*).

(2) Consider next type 2 buyers. By trade (Q^*, T^*) in A_1 , and (q_2^*, t_2^*) in A_2, type 2 can reach the aggregate trade (Q_2^*, T_2^*). Next ,we show that type 2 has no profitable deviation given the contracts in step 1.

First, If type 2 purchases pooling in A_1, then by incentive compatible condition , trade (q_1^*, t_1^*) is not a profitable deviation. If the type 2 trade a pooling in A_1 and one latent contract in A_2, the aggregate trade will be on the line or above the line with slope c that supports her upper contour set of (Q_2^*, T_2^*), which has a lower utility than (Q_2^*, T_2^*).

Second,If type 2 purchases the contract (q_1^ℓ, t_1^ℓ) in A_1, then if purchase (q_1^*, t_1^*) in A_2, the aggregate trade will be $(Q_1^* + q^\ell, T_1^* + t^\ell)$,which has the same utility with (Q_2^*, T_2^*). And due to any other contract in A_2 has a premium rate great than c, and $\tau_2(Q_1^* + q^\ell, T_1^* + t^\ell) = c$. Then if type combine it with other contracts (the unit price at least c) in A_2, the combinations will locate above the line with slope c that supports her upper contour set of (Q_2^*, T_2^*).

Third, If type 2 purchases the contract (q^ℓ, t^ℓ) in A_1, by combining with (q_1^*, t_1^*) in A_2, the aggregate trade will be on the line with slope c that supports her upper contour set of (Q_2^*, T_2^*), given all the other contracts in A_2 have premium great than c, then the combination of (q^ℓ, t^ℓ) and them will locate above the line with slope c that supports her upper contour set of (Q_2^*, T_2^*). which gives worse utility than (Q_2^*, T_2^*). Thus, (q^ℓ, t^ℓ) combine any contract (q, t) in A_2 will locate at least the line with slope c that supports her upper contour set of (Q_2^*, T_2^*).

Fourth, If type 2 purchases the contract $(q_{1k}^\ell, t_{1k}^\ell)$, and combine with a contract (q, t) in A_2, the aggregate contracts can be treat $[(q^\ell, t^\ell)+(q, t)]+(q_{1k}^\ell, t_{1k}^\ell)$, the first part will locate at least the line with slope c that supports her upper contour set of (Q_2^*, T_2^*) according to third case, the last part with premium rate great than c, again, it gives worse payoff than (Q_2^*, T_2^*).

Fifth, the last case we need to consider is that type 2 purchase only one active contract in all group: (1) for latent contract only , all the single latent contract locate at least the line with slope c that supports her upper contour set of (Q_2^*, T_2^*);(2) for (q_2^*, t_2^*) or (Q^*, T^*) only, due to

$$U_2(Q_2^*, T_2^*) \geq max\{U_2(Q^* + Q, T^* + c_2 Q) : Q \geq 0\}$$

(Q_2^*, T_2^*) gives a better utility than either one of these two kinds contracts. (3) consider (q_1^*, t_1^*) only, We know that $Q_2^* > Q_1^*$ and $\tau_2(Q_2^*, T_2^*) = c_2$, and $U_2(Q_2^*, T_2^*) \geq U_2(Q^* + q_1^*, cQ^* + t_1^*)$, according to that $U_1(Q^* + q_1^*, cQ^* + t_1^*) \geq U_1(q_1^*, t_1^*)$, by single-crossing, we have $U_2(Q^* + q_1^*, cQ^* + t_1^*) > U_2(q_1^*, t_1^*)$, as a result, type 2 can not be better off by choosing only one contract.

so it is best response for type 2 to choose (Q^*, T^*) in A_1 and (q_2^*, t_2^*) in A_2 to reach (Q_2^*, T_2^*).

Remark: before the next two steps: we claim that no seller could attract both type with positive profit due to the proposition of "Fair Marginal Purchasing". Then what we need to prove is that no seller can find profitable deviation to attract only one type buyers.

Step 3 We then prove that no seller has profitable deviation that only attracts type 2 buyers.

First, we claim that no seller in subgroup A_2 has profitable deviation only attracts type 2 buyers, otherwise, if a seller k can have profitable deviation with (q, t), if type 2 trade with pooling in A_1, according to that $\tau_2(Q_2^*, T_2^*) = c_2$. we have that $U_2(Q_2^*, T_2^*) = max\{U_2(Q^* + Q, T^* + c_2Q) : Q \geq 0\}$, (q, t) is attractive only if $t < c_2q$, not profitable. If type 2 trades with latent contract in A_1, the contract in A_1 will at least locate on the line with slope c that support her upper contour set of (Q_2^*, T_2^*), so (q, t) attracts type 2 only if $t < cq$,not profitable again.

Second, We claim that no seller in subgroup A_1 has profitable deviation only attract type 2 buyers. Assume the deviation contract is (q, t). According to proposition 4, no seller can profitably attract type 2 by combining (q_1^*, t_1^*) in A_2. Then, if type 2 trades with latent contract in A_2, the contract in A_2 will at least locate on the line with slope c that support her upper contour set of (Q_2^*, T_2^*), so (q, t) attracts type 2 only if $t < cq$,not profitable again. Last case, if type 2 trades with (q_2^*, t_2^*) in A_2 and has a better utility than $U_2(Q_2^*, T_2^*)$, then $t < c_2q$ for sure. still not profitable again.

Step 4 We next prove that no seller has profitable deviation that only attract type 1 buyers.

First, according to fair unit price lemma and fair marginal purchasing proposition, we know that $\tau_1(Q_1^*, T_1^*) = c$ and $T^* = cQ^*$, we also know that $c_1q_1 \leq t_1 \leq cq_1$ so we can get that $U_1(Q_1^*, T_1^*) \geq max\{U_1(Q, cQ) : Q \geq 0\}$ and $U_1(Q_1^*, T_1^*) \geq max\{U_1(q_1^* + Q, t_1^* + cQ) : Q \geq 0\}$. If there is a seller propose (q, t) as cream-skimming deviation, then to attract type 1 consumers, it requires that $t \leq cq$. And this contract is profitable only if $t > c_1q$. So any cream-skimming deviation must belong to the cone :

$$X \equiv \{(q, t) : cq \geq t \geq c_1q\} \tag{24}$$

Then, we discuss the cream-skimming deviation in four different cases: Large cream-skimming deviation in A_1, Large cream-skimming deviation in A_2, Small cream-skimming deviation in A_1 (combining with (q_1^*, t_1^*)), Small cream-skimming deviation in A_2 (combining with (Q^*, T^*)).

Case(1): Large cream-skimming deviation in A_1, that is a contract $(q, t) \in X$ proposed by some seller in A_1 such that :

$$U_1(q, t) \geq U_1(Q_1^*, T_1^*) \tag{25}$$

By using the lemma 3 and with the latent contract (q^ℓ, t^ℓ) in A_2, we have that :

$$U_2(q + q^\ell, t + t^\ell) \geq U_2(Q_2^*, T_2^*) \tag{26}$$

So (q,t) in A_1 also attracts type 2 with the latent contract (q^ℓ, t^ℓ)in subgroup A_2. Then we can construct the buyers' best response that both types trade with (q,t). However, if (q,t) trade with both types, the marginal serving cost is c, and $t \leq cq$ means this contract can get at most 0 profit.

Case(2): Large cream-skimming deviation in A_2, this case is very similar with case (1).

Case(3): Small cream-skimming deviation in A_1, in this case,

1. If a small cream-skimming deviation (q,t), type 1 can combine (q,t) and separating contract (q_1^*, t_1^*) in subgroup A_2. That is to say , a contract $(q,t) \in X$ proposed by some seller in A_1 such that :

$$U_1(q + q_1^*, t + t_1^*) \geq U_1(Q_1^*, T_1^*) \tag{27}$$

   Under the assumption Flatter curvature, and with the lemma 3, what we can get from (19) is

$$U_2(q + q_1^* + q^\ell, t + t_1^* + t^\ell) \geq U_2(Q_2^*, T_2^*) \tag{28}$$

   And as what we construct in step 1 , $(q_2^\ell, t_2^\ell) = (q_1^* + q^\ell, t_1^* + t^\ell)$ which is the latent contract in A_2. Thus (q,t) also attracts type 2 in combination with (q_2^ℓ, t_2^ℓ). If both consumers purchase (q,t) , the marginal serving cost would be c, which makes the profit at most 0.

2. If a small cream-skimming deviation is (q,t), and type 1 can combine (q,t) and latent contract $(q_{2k}^\ell, t_{2k}^\ell)$.$(1 \leq k \leq n_\ell - 1)$ in subgroup A_2 to get better utility, then this deviation will be blocked by $(q_{2(k+1)}^\ell, t_{2(k+1)}^\ell)$. and then if type 1 combines (q,t) with $(q_{2n_\ell}^\ell, t_{2n_\ell}^\ell)$, the aggregate quantity is larger than (Q_2^*, T_2^*), so still attract type 2 also. Thus, the (q,t) satisfies this case also attracts type 2 .

3. If a small cream-skimming deviation is (q,t), and type 1 can combine (q,t) and latent contract $(q_{2k}^{\ell'}, t_{2k}^{\ell'})$. By the same trick as last case, any (q,t) attract type 1 also attracts type 2.

Case(4):Small cream-skimming deviation in A_2,in this case,

1. If a cream-skimming deviation $(q,t) \in X$ proposed by some seller in A_2 and type 1 can get better utility by combining (q,t) and (Q^*, T^*) in A_1, such that :

$$U_1(q + Q^*, t + T^*) \geq U_1(Q_1^*, T_1^*) \tag{29}$$

Under the assumption Flatter curvature, this kind of of deviation is also blocked :

$$U_2(q+Q^*+q^\ell, t+T^*+t^\ell) \geq U_2(Q_2^*, T_2^*) \tag{30}$$

And as what we construct in step 1, $(q_1^\ell, t_1^\ell) = (Q^*+q^\ell, T^*+t^\ell)$ which is one of the latent contract in subgroup A_1. Hence (q,t) in A_2 also attracts type 2 in combination with (q_1^ℓ, t_1^ℓ) in A_1.If both consumers purchase (q,t) , the marginal serving cost would be c, which makes the profit at most 0.

2. If a cream-skimming deviation $(q,t) \in X$ proposed by some seller in A_2 and type 1 can get better utility by combining (q,t) and latent contract $(q_{1k}^\ell, t_{1k}^\ell)$.$(1 \leq k \leq n_\ell - 1)$ in A_1, then this deviation will be blocked by $(q_{1(k+1)}^\ell, t_{1(k+1)}^\ell)$. If type 1 can combine (q,t) with $(q_{1n_\ell}^\ell, t_{1n_\ell}^\ell)$, the aggregate quantity is larger than (Q_2^*, T_2^*), so still attract type 2 also. Thus, the (q,t) satisfies this case also attracts type 2 .
3. If a cream-skimming deviation $(q,t) \in X$ proposed by some seller in A_2 and type 1 can get better utility by combining (q,t) and latent contract (q_1^ℓ, t_1^ℓ). If $q > q_1^*$, then the combination of $q + q_1^\ell$ has aggregate quantity larger than Q_2^*, then by single-crossing, it is also attractive for type 2, so it is not profitable given $\frac{t}{q} \leq c$. However, if $q \leq q_1^*$, (q,t) attracts type 1 only if $t/q < c_1$, it is also not profitable.

7.2 Proof of Theorem 7.

Step 1, we construct the equilibrium menus of each group:

For the subgroup A_1:

1. At least two sellers propose the entry-proof tariff with $T(Q) = 1_{\{Q \leq Q_1^*\}} cQ + 1_{\{Q > Q_1^*\}}[cQ_1^* + c_2(Q - Q_1^*)]$.
2. Other sellers could propose the same menu as 1 or null contracts

For the subgroup A_2:

1. At least two sellers propose the separating tariff with: $T(Q) = c_2 Q$
2. At least one sellers propose the menu tariff with: $T(Q+q^\ell) = t^\ell + c_2 Q$ $(Q \geq 0)$
3. At least one sellers propose the menu with contracts : $(q_k^\ell, t_k^\ell) \equiv k \times (q_\ell, t_\ell)$, where $k \leq n_\ell$.
4. Other sellers could propose one of 1,2,3 or null menus.

Then to get the JHG allocation, type 1 and type 2 purchase (Q_1^*, T_1^*) in A_1 and then purchase $(Q_2^* - Q_1^*, T_2^* - T_1^*)$ in subgroup A_2.

Step 2, We show that it is best response to both types:

(1) Consider first for type 1 : we know that all the contracts in two subgroup have the premium rate at least c , so the maximal utility that type 1 can get $U_1^* \leq max\{U_1(Q, cQ) : Q \geq 0\} = U_1(Q_1^*, T_1^*)$, so it is best response for type 1.

(2) Then for type 2 buyers: (1) If combining with any latent contract , given any other menu has unit price at least c, the combination of latent contract and others will locate at least the line with slope c that supports her upper contour set of (Q_2^*, T_2^*), which gives type 2 worse utility. (2) If not combining with latent contract, $\tau_2(Q_2^*, T_2^*) = c_2$ means no additional contracts with unit price of c_2 is attractive for type 2, and the maximal pooling quantity of type 2 can have is Q_1^*, thus the max utility of type 2 $U_2^* \leq max\{U_2(Q_1^* + Q, T_1^* + c_2^*) : Q \geq 0\} = U_2(Q_2^*, T_2^*)$.Thus, it is also best response to type 2 buyers.

Step 3, We then prove that no seller has profitable deviation that only attracts type 2 buyers. According to the construction in step 1, type 2 can always get the utility $U_2(Q_2^*, T_2^*)$ even if any one of sellers deviate to other menus. Thus, given the utility $U_2(Q_2^*, T_2^*)$, a contract (q, t) attracts type 2 should satisfy that $t < c_2 q$, but if only attracts type 2 buyers, it is not profitable deviation.

Step 4, We next prove that no seller has profitable deviation that only attracts type 1 buyers.

Firstly, if one seller deviates to other menu, type 1 can still achieve the utility $U_1(Q_1^*, T_1^*)$ with other sellers with the construction in step 1. Next, if a seller in A_2 try to propose a contract (q, t) to attracts type 1 buyers, according to Attar, Mariotti and Salanié (2021a), due to the tariff in A_1 is entry-proof to the market, any seller in A_2 can at most achieve 0 profit given the menu in A_1.

Then, there is a seller propose (q, t) as cream-skimming deviation in A_1, then to attract type 1 consumers, it requires that $t \leq cq$. And this contract is profitable only if $t > c_1 q$. So any cream-skimming deviation must belong to the cone :

$$X \equiv \{(q, t) : cq \geq t \geq c_1 q\} \tag{31}$$

We can discuss the cream-skimming deviation in two cases: Large cream-skimming deviation in A_1,Small cream-skimming deviation in A_1.

case (1), Large cream-skimming deviation in A_1.that is a contract $(q, t) \in X$

proposed by some seller in A_1 such that :

$$U_1(q,t) > U_1(Q_1^*, T_1^*) \tag{32}$$

By using the lemma 4 and with the latent contract (q^ℓ, t^ℓ) in A_2, we have that :

$$U_2(q+q^\ell, t+t^\ell) > U_2(Q_2^*, T_2^*) \tag{33}$$

So (q,t) in A_1 also attracts type 2 with the latent contract (q^ℓ, t^ℓ)in subgroup A_2. However, if (q,t) trade with both types, the marginal serving cost is c, and $t \leq cq$ means this contract can get at most 0 profit.

Case (2), Small cream-skimming deviation in A_1 .

1. If a small cream-skimming deviation (q,t), type 1 can combine (q,t) and a contract $(Q, c_2 Q)$ for some Q in subgroup A_2. which makes that

$$U_1(q+Q, t+c_2 Q) > U_1(Q_1^*, T_1^*) \tag{34}$$

   Then by using the lemma 2 , (17) also means that

$$U_2(q+Q+q^\ell, t+c_2 Q+t^\ell) > U_2(Q_2^*, T_2^*) \tag{35}$$

   while the contract of $(Q+q^\ell, c_2 Q+t^\ell)$ is one of latent contract as construction in step 1. so this small deviation was blocked by the latent contract in subgroup A_2.

2. If a small cream-skimming deviation is (q,t), and type 1 can combine (q,t) and latent contract (q_k^ℓ, t_k^ℓ).$(1 \leq k \leq n_\ell - 1)$ in subgroup A_2 to get better utility, then this deviation will be blocked by $(q_{(k+1)}^\ell, t_{(k+1)}^\ell)$. and then if type 1 combines (q,t) with $(q_{n_\ell}^\ell, t_{n_\ell}^\ell)$, the aggregate quantity is larger than (Q_2^*, T_2^*), so still attract type 2 also. Thus, the (q,t) satisfies this case also attracts type 2 .

Step 5, In the end, we show that no seller k has profitable deviation that attracts both types with menu $\{c_1^k, c_2^k\}$.

If $c_1^k = c_2^k$, then it is a pooling contracts to market and trying to attract both types with contract (q^k, t^k). We know that $U_1(Q_1^*, T_1^*) = max\{U_1(Q, cQ) : Q \geq 0\}$, then (q^k, t^k) attracts both types only if $t^k < cq^k$, but the serving cost to both type is c, which means the deviation with $t^k < cq^k$ is not profitable.

If $c_1^k \neq c_1^k$, then we can discuss the case as follow:

1. If the seller is in the subgroup A_2, then given the tariff in subgroup A_1 is entry-proof to the market with nonexclusivity , due to A_1 and A_2 are non-exclusive trade, then the maximal profit of seller k can get is 0, so it is not profitable deviation.

2. If the seller is in subgroup A_1, and $c_1^k = (q_1^k, t_1^k), c_2^k = (q_2^k, t_2^k)$ with $q_1^k \leq Q_1^*$. c_1^k attracts type 1 only if $t_1^k < cq_1^k$, then if c_2^k is attractive for type 2 without combination with other contracts, which means c_2^k gives better utility than (Q_2^*, T_2^*), it is easy to conclude that the total profit of c_1^k, c_2^k is negative.If c_2^k attract type 2 with combination of other contract in A_2, then we can discuss the situation in two cases:
   (1) type 2 can combine c_2^k and the contract with unit price c_2 in A_2 to have better utility, then we will have that $q_2^k > Q_1^* \geq q_1^k$, there exist one (q', t') such that $c_2^k = (Q_1^; T_1^*) + (q', t')$ with $t' < c_2 q'$, then the aggregate profit of c_1^k, c_2^k will be negative.
   (2) if type 2 combine c_2^k with a latent contract in A_2, then we have that $t_2^k < cq_2^k$ and type 2 prefers c_2^k than c_1^k, then due to the assumption flatter curvature (ii), by perfect translate with latent contract, we will also have that type 1 also prefers c_2^k than c_1^k, but the serving cost of both types is c which makes the deviation of seller get negative profit again. Thus, there is no profitable deviation in this case.

7.3 Proof of Theorem 8.

The uniqueness is the same as the trick in the "$1+1$" partial exclusive structure, we then show the existence of equilibrium in the cases mentioned.

Step 1, For the structure of "$\lambda+1$", we construct the equilibrium menus of each group:

For the subgroup A_1:

1. At least $\lambda+1$ sellers propose the tariff with $T(Q) = 1_{\{Q \leq \frac{Q_1^*}{\lambda}\}} cQ + 1_{\{Q > \frac{Q_1^*}{\lambda}\}} [c\frac{Q_1^*}{\lambda} + c_2(Q - \frac{Q_1^*}{\lambda})]$.
2. Other sellers could propose the same menu as 1 or null contracts

For the subgroup A_2:

1. At least two sellers propose the separating tariff with: $T(Q) = c_2 Q$
2. At least one sellers propose the menu tariff with: $T(Q+q^\ell) = t^\ell + c_2 Q \ \ (Q \geq 0)$

3. At least one sellers propose the menu with contracts : $(q_k^\ell, t_k^\ell) \equiv k \times (q_\ell, t_\ell)$, where $k \leq n_\ell$.
4. Other sellers could propose one of 1,2,3 or null menus.

Then to get the JHG allocation, type 1 and type 2 purchase (Q_1^*, T_1^*) in A_1 and then purchase $(Q_2^* - Q_1^*, T_2^* - T_1^*)$ in subgroup A_2.

Step 2, We show that it is best response to both types:

(1) Consider first for type 1 : we know that all the contracts in two subgroup have the premium rate at least c , so the maximal utility that type 1 can get $U_1^* \leq max\{U_1(Q, cQ) : Q \geq 0\} = U_1(Q_1^*, T_1^*)$, so it is best response for type 1.

(2) Then for type 2 buyers: (1) If combining with any latent contract , given any other menu has unit price at least c, the combination of latent contract and others will locate at least the line with slope c that supports her upper contour set of (Q_2^*, T_2^*), which gives type 2 worse utility. (2) If not combining with latent contract, $\tau_2(Q_2^*, T_2^*) = c_2$ means no additional contracts with unit price of c_2 is attractive for type 2, and the maximal pooling quantity of type 2 can have is Q_1^*, thus the max utility of type 2 $U_2^* \leq max\{U_2(Q_1^* + Q, T_1^* + c_2^*) : Q \geq 0\} = U_2(Q_2^*, T_2^*)$.Thus, it is also best response to type 2 buyers.

Step 3, We then prove that no seller has profitable deviation that only attracts type 2 buyers. According to the construction in step 1, type 2 can always get the utility $U_2(Q_2^*, T_2^*)$ even if any one of sellers deviate to other menus. Thus, given the utility $U_2(Q_2^*, T_2^*)$, a contract (q, t) attracts type 2 should satisfy that $t < c_2 q$, but if only attracts type 2 buyers, it is not profitable deviation.

Step 4, Cream-skimming deviation: We next prove that no seller has profitable deviation that only attracts type 1 buyers.

Firstly, if one seller deviates to other menu, type 1 can still achieve the utility $U_1(Q_1^*, T_1^*)$ with other sellers with the construction in step 1. Next, if a seller in A_2 try to propose a contract (q, t) to attracts type 1 buyers, according to Attar, Mariotti and Salanié (2021a), due to the tariff in A_1 is entry-proof to the market, any seller in A_2 can at most achieve 0 profit given the menu in A_1.

Then, there is a seller propose (q, t) as cream-skimming deviation in A_1, then to attract type 1 consumers, it requires that $t \leq cq$. And this contract is profitable only if $t > c_1 q$. So any cream-skimming deviation must belong to the cone :

$$X \equiv \{(q,t) : cq \geq t \geq c_1 q\} \tag{36}$$

We can discuss the cream-skimming deviation in two cases: Large cream-skimming deviation in A_1,Small cream-skimming deviation in A_1.

case (1), Large cream-skimming deviation in A_1.that is a contract $(q,t) \in X$ proposed by some seller in A_1 such that :

$$U_1(q,t) > U_1(Q_1^*, T_1^*) \tag{37}$$

By using the lemma 4 and with the latent contract (q^ℓ, t^ℓ) in A_2, we have that :

$$U_2(q + q^\ell, t + t^\ell) > U_2(Q_2^*, T_2^*) \tag{38}$$

So (q,t) in A_1 also attracts type 2 with the latent contract (q^ℓ, t^ℓ)in subgroup A_2. However, if (q,t) trade with both types, the marginal serving cost is c, and $t \leq cq$ means this contract can get at most 0 profit.

Case (2), Small cream-skimming deviation in A_1 .

1. If a small cream-skimming deviation (q,t), type 1 can combine (q,t) and a contract $(Q, c_2 Q)$ for some Q in subgroup A_2. which makes that

$$U_1(q + Q, t + c_2 Q) > U_1(Q_1^*, T_1^*) \tag{39}$$

   Then by using the lemma 2 , (17) also means that

$$U_2(q + Q + q^\ell, t + c_2 Q + t^\ell) > U_2(Q_2^*, T_2^*) \tag{40}$$

   while the contract of $(Q + q^\ell, c_2 Q + t^\ell)$ is one of latent contract as construction in step 1. so this small deviation was blocked by the latent contract in subgroup A_2.

2. If a small cream-skimming deviation is (q,t), and type 1 can combine (q,t) and latent contract (q_k^ℓ, t_k^ℓ).$(1 \leq k \leq n_\ell - 1)$ in subgroup A_2 to get better utility, then this deviation will be blocked by $(q_{(k+1)}^\ell, t_{(k+1)}^\ell)$. and then if type 1 combines (q,t) with $(q_{n_\ell}^\ell, t_{n_\ell}^\ell)$, the aggregate quantity is larger than (Q_2^*, T_2^*), so still attract type 2 also. Thus, the (q,t) satisfies this case also attracts type 2 .

Step 5, In the end, we show that no seller k has profitable deviation that attracts both types with menu $\{c_1^k, c_2^k\}$.

If $c_1^k = c_2^k$, then it is a pooling contracts to market and trying to attract both types with contract (q^k, t^k). We know that $U_1(Q_1^*, T_1^*) = max\{U_1(Q, cQ) : Q \geq 0\}$, then (q^k, t^k) attracts both types only if $t^k < cq^k$, but the serving cost to both type is c, which means the deviation with $t^k < cq^k$ is not profitable.

If $c_1^k \neq c_1^k$, then we can discuss the case as follow:

1. If the seller is in the subgroup A_2, then given the tariff in subgroup A_1 is entry-proof to the market with nonexclusivity , due to A_1 and A_2 are non-exclusive trade, then the maximal profit of seller k can get is 0, so it is not profitable deviation.

   Then we consider the profitable deviation that attracts different types with different contracts within one menu. In most menu game with nonexclusive structure, double-deviation is a usual problem for sustain the equilibrium. However, in the "$\lambda + 1$" structure, we will show that the partial exclusive feature can help us prevent the classical double deviation which as proposed in Attar, Mariotti and Salanie (2014): when the double deviation happens in subgroup A_1, then, there will be two contract c_1^k, c_2^k which aim type 1 and type 2, where $c_1^k = (q_1^k, t_1^k), c_2^k = (q_2^k, t_2^k)$.

   We discuss the situation in two different cases: Case (1), given the deviation c_1^k, c_2^k in A_1, type 2 would not combine with the latent contract in subgroup A_2 ; Case(2) , given the deviation c_1^k, c_2^k in A_1, type 2 would combine with the latent contract in subgroup A_2 .

   In all the situations, we need to be aware of that, if buyer choose the menu from a seller who propose some deviation, then with the restriction of trade at most λ sellers in A_1, the available tariff in this structure is $T^*(Q) = 1_{\{Q \leq \frac{\lambda-1}{\lambda} Q_1^*\}} cQ + 1_{\{Q > \frac{\lambda-1}{\lambda} Q_1^*\}} [c\frac{\lambda-1}{\lambda} Q_1^* + c_2(Q - \frac{\lambda-1}{\lambda} Q_1^*)]$.

   Case (1.1) small contract c_1^k: When $q_1^k \leq \frac{1}{\lambda} Q_1^*$, then type 1 chooses contracts from other $\lambda - 1$ sellers in A_1, and combined with c_1^k. In this case, c_1^k attracts type 1 only if $t_1^k < cq_1^k$, then due to (Q_2^*, T_2^*) is available for type 2, then c_2^k attracts type 2 only if $\frac{\lambda-1}{\lambda}(Q_1^*, T_1^*) + c_2^k$ give better utility than (Q_2^*, T_2^*), however, this kind of c_2^k and c_1^k will result negative profit in total.

Case (1.2) Large contract c_1^k: When $q_1^k > \frac{1}{\lambda}Q_1^*$, then c_1^k attracts type 1 only if $t_1^k < cq_1^k$. Considering the contract c_2^k:

(1) if $q_2^k < q_1^k$, then c_2^k attracts type 2 only if c_2^k has lower unit price and the convex contour tariff combined with c_2^k contains the convex contour tariff combined with c_1^k, but in this case, c_2^k is more attractive than c_1^k for type 1 also, the serving cost of both type is c while we have that the unit price of c_2^k is lower than c, unprofitable.

(2) if $q_2^k \geq q_1^k$, we know that $q_2^k \geq q_1^k > \frac{1}{\lambda}Q_1^*$. Then we can always find a (q_2', t_2') such that $c_2^k = (q_2^k{}', t_2^k{}') + c_1^k$, due to c_2^k is more attractive than c_1^k to type 1, then we know that $t_2' < c_2 q_2^k{}'$, then the profit of this double deviation is $m_2(t_2^k{}' - c_2 q_2^k{}') + t_1^k - cq_1^k < 0$, not profitable.

Case (2) Double deviation based on latent contract in A_2. if type 2 combine c_2^k with a latent contract in A_2, due to it is better than JHG, than we have that $t_2^k < cq_2^k$ and means that type 2 prefers c_2^k than c_1^k,then due to the assumption flatter curvature (ii), by perfect translation with latent contract, we will also have that type 1 also prefers c_2^k than c_1^k, but the serving cost of both types is c which makes the deviation get negative profit again. Thus, there is no profitable deviation in this case. Q.E.D

Then, to prove the existence in $\lambda_1 + \lambda_2$ structure, we just need to change a little bit in step 1 and constructing the equilibrium menus of each group as follows:

For the subgroup A_1:

(a) At least $\lambda_1 + 1$ sellers propose the tariff with $T(Q) = 1_{\{Q \leq \frac{Q_1^*}{\lambda_1}\}} cQ + 1_{\{Q > \frac{Q_1^*}{\lambda_1}\}} [c\frac{Q_1^*}{\lambda_1} + c_2(Q - \frac{Q_1^*}{\lambda_1})]$.

(b) Other sellers could propose the same menu as 1 or null contracts

For the subgroup A_2:

(a) At least two sellers propose the separating tariff with: $T(Q) = c_2 Q$

(b) At least one sellers propose the menu tariff with: $T(Q + q^\ell) = t^\ell + c_2 Q \quad (Q \geq 0)$

(c) At least one sellers propose the menu with contracts : $(q_k^\ell, t_k^\ell) \equiv k \times (q_\ell, t_\ell)$, where $k \leq n_\ell$.

(d) Other sellers could propose one of 1,2,3 or null menus.

Then to get the JHG allocation, type 1 and type 2 purchase (Q_1^*, T_1^*) in A_1 and then purchase $(Q_2^* - Q_1^*, T_2^* - T_1^*)$ in subgroup A_2. Q.E.D