An Experimental Investigation of Global Games with Strategic Substitutes*

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Abstract

We experimentally investigate behavior in a global game where actions are strategic substitutes. Following the theoretical foundations of Harrison and Jara-Moroni (2021), we focus on a 3 agent, binary action game where payoffs depend on some underlying value of a state fundamental. For some values of the state, the game predicts multiple equilibria. Furthermore, payoffs are heterogeneous across agents which results in an ordering of agent "types." The global game equilibrium selection results in a unique equilibrium in which agents adopt threshold strategies, with thresholds following the order of types. Our experiment provides some support for the theory. 2/3 of the subjects adopt threshold strategies with few mistakes. While the estimated thresholds deviate from point predictions, the comparative statics still hold. Finally, a majority of outcomes correspond to the global games equilibrium even in regions of multiplicity.

Keywords: Global Games; Strategic Substitutes; Uncertainty; Equilibrium Selection; Experiment

JEL Classification: C72; D82;

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1 Introduction

The theory of global games (Carlsson and van Damme, 1993; Frankel et al., 2003) has been used extensively as a way to refine predictions in strategic settings with multiple equilibrium predictions. The global games principle works by introducing a small amount of noise in the game where agents observe a private, but noisy signal about the payoff fundamental. The theory suggests that, for a small amount of noise, this game with incomplete information always yields a unique Bayesian Nash equilibrium: agents follow cutoff strategies in the sense that they choose a certain action if their private signal about the payoff fundamental is high enough. Theory suggests that this behavior should persist as noise vanishes and we recover the original game, resulting in equilibrium selection in regions of multiplicity. While initially applied to coordination games where actions are strategic complements (see Morris and Shin, 2003 for a survey), the theory has advanced to incorporate more general payoff structures (Hoffmann and Sabarwal, 2019a,b). More recently, Harrison and Jara-Moroni (2015, 2021) have provided a characterization of the global games equilibrium in settings where actions are strategic substitutes.

In this paper, we experimentally test the validity of the global games theory in strategic situations where actions are strategic substitutes. Leveraging the theoretical foundations of Harrison and Jara-Moroni (2021), we consider a simple model in which 3 agents take part in a simultaneous game with binary actions (0 or 1). Action 0 yields a fixed payoff, whereas action 1 yields a payoff that is dependent on a state fundamental drawn from a continuous distribution over a bounded support, and the number of agents who choose action 1. Actions are strategic substitutes in the sense that more agents choosing action 1 lowers the payoff from choosing action 1. Payoffs are heterogeneous across agents resulting in an ordering of agents types. We face the issue of multiplicity of equilibria for intermediate values of the state fundamental under the setting of complete information. Following the global games approach, if we endow the game with a small amount of noise, we obtain a unique equilibrium where agents adopt threshold strategies. Furthermore, these thresholds follow the same payoff ordering in that they are monotonic across agent types. This behavior persists as noise vanishes resulting in a unique equilibrium prediction in regions of multiple equilibrium for the game with complete information.

Our experiment is geared towards testing the comparative static predictions of the model. In our experiment, subjects take part in 75 rounds of repeated play of the 3 agent game. In each round, the computer randomly generates a value of the state from a known distribution. Our treatments vary the signal precision. In the complete information treatment, the value of the state is publicly observed and common knowledge. In the incomplete information treatment, subjects observe a private, noisy signal about the state. In each round, subjects make decisions as *each* of the three agent types in the game while playing against the remaining types.

The data provides some support to the validity of the global games theory. We observe that around 2/3 of the subject population adopts threshold strategies for each type, albeit imperfectly with at most 10 mistakes per type. Even though the estimated thresholds depart from the point predictions of the model, the comparative static predictions still hold. Choices and estimated thresholds are monotonic and follow the ordering induced by the agent types. We also find that a majority of outcomes in the region with multiple equilibria coincide with the global game prediction.

We focus on games with strategic substitutes since several classes of such games have been used in empirical work. One example is the market entry game in which a group of firms have to simultaneously decide on entering a market, the profitability of which is decreasing in the number of entrants. The market entry game has been used extensively in empirical industrial organization to investigate behavior in markets for automobiles (Bresnahan and Reiss, 1990), airlines (Berry, 1992; Ciliberto and Tamer, 2009), construction contractors Bajari et al. (2010) and others.[1]. The issue of multiplicity in such games creates identification and misspecification problems in empirical analysis. One way to get around these issues is to adopt an (oftentimes ad-hoc) equilibrium selection rule and estimate the model under the assumption that the selected equilibrium will be played (see, for example, Bjorn and Vuong, 1984; Jia, 2008). The global games principle rule can potentially act as a theoretically founded equilibrium selection rule that can help empirical analysis.

The global games theory has been extensively studied in the laboratory. Heinemann et al. (2004) are the first to experimentally study global games in a speculative attack model. They look at behavior in both complete and incomplete information treatments and find support for subjects adopting threshold strategies. Helland et al. (2018); Szkup and Trevino (2020) look at the effect of

[1]Berry and Reiss (2007) provides a survey on empirical applications of entry games to field data

varying signal precision on behavior and find a reversal of the comparative statics (relative to the theory). Avoyan (2019) incorporates communication (in binary form) to the setting of Szkup and Trevino (2020). Darai et al. (2017) looks at how public information (measured as the average of private signals) influences coordination in a model of joint investment decisions. In a recent paper Heinemann (2018) shows that the global games theory is a poor predictor of behavior in settings with strategic complementarities and asymmetric payoffs. However, all of these papers focus on settings where actions are strategic complements. To the best of our knowledge, we are the first to experimentally investigate behavior in global games where actions are strategic substitutes.

The rest of this paper is structured as follows. Section 2 presents the theoretical model and predictions. Section 3 presents the experimental design. Section 4 presents the results. Section 5 concludes.

2 Model and Predictions

Our theoretical model follows a similar structure to that of Example 5 in Harrison and Jara-Moroni (2021). There are 3 agents, each choosing between two actions $a_i \in \{0, 1\}$. There is a state of the world θ which is uniformly distributed over the interval $\Theta = [\underline{\theta}, \overline{\theta}]$, the value and the distribution being common knowledge among all agents. The payoffs to each agent i are given as:

$$u_i(a_i, n; \theta) = \begin{cases} \theta - n\delta - c_i & \text{if } a_i = 1 \\ r & \text{if } a_i = 0 \end{cases}$$

where $n = \sum_{j \neq i} a_j \in \{0, 1, 2\}$ is the number of *opponents* choosing action 1, $\delta > 0$ is a penalty term that reduces an agent's payoff from choosing 1 if more agents choose action 1, $c_i > 0$ is an agent specific cost to choosing action 1, and $r > 0$ is an agent's reservation utility.[2]

Denote $\Delta u_i(n; \theta) = u_i(1, n; \theta) - u_i(0, n; \theta) = \theta - n\delta - c_i - r$ to be the marginal benefit to an agent i from choosing the higher action, assuming that exactly n opponents choose action 1. There are a couple of points to note. First, $\Delta u_i(n; \theta)$ is continuous and monotonically increasing in θ — a higher

[2]One can interpret this model as in which a group of 3 firms simultaneously choose to enter a market. The market profitability is captured by the parameter θ. Each additional entrant reduces the payoffs of all entrants by δ and in addition to this, there is a cost of entry c_i.

state *increases* the incentive to choose action 1. Second, $\Delta u_i(n+1;\theta) - \Delta u_i(n;\theta) = -\delta < 0$ for all θ so this is a game of strategic substitutes — a higher number of opponents choosing action 1 *reduces* the incentive to choose action 1.

Following convention in the global games literature, for each agent i, we assume the existence of "indifference points" $\{\theta_i^L, \theta_i^H\}$ which make an agent indifferent between choosing action 1 provided no other opponent or all other opponents have chosen action 1 respectively. Specifically, θ_i^L is the unique solution to $\Delta u_i(0, \theta_i^L) = 0$ and θ_i^H is the unique solution to $\Delta u_i(2, \theta_i^H) = 0$. These indifference points establish the existence of dominance regions. For each agent i, 0 is the strictly dominant strategy for $\theta < \theta_i^L$ and 1 is the strictly dominant strategy for $\theta > \theta_i^H$.[3]

Furthermore, given continuity, monotonicity and strategic substitutability, for each agent, there exists a value θ_i^M that makes the agent indifferent between choosing action 1 or 0 when exactly one opponent chooses action 1. This means that θ_i^M uniquely solves $\Delta u_i(1, \theta_i^M) = 0$. The model parameters yield:

$$\begin{aligned}
\theta_i^L &= c_i + r \\
\theta_i^M &= c_i + \delta + r \\
\theta_i^H &= c_i + 2\delta + r
\end{aligned}$$

Note that the indifference points are monotonically increasing i.e. $\theta_i^L < \theta_i^M < \theta_i^H$ for all agents i.

Finally, we incorporate heterogeneity in agent payoffs by assuming $0 < c_1 < c_2 < c_3$. This payoff asymmetry induces an ordering of players based on their incentive to choose action 1. More specifically, $\Delta u_1(n;\theta) > \Delta u_2(n;\theta) > \Delta u_3(n;\theta)$ for all n and all θ. Thus for any given θ and n, agent 1 has the highest incentive to choose action 1 and agent 3 has the lowest incentive to choose action 1. This implies that the indifference points above also follow an order: $\theta_1^L < \theta_2^L < \theta_3^L$; $\theta_1^M < \theta_2^M < \theta_3^M$; and $\theta_1^H < \theta_2^H < \theta_3^H$. We also assume $c_3 - c_1 < \delta$ which implies that $\theta_3^L < \theta_1^M$ and $\theta_3^M < \theta_3^H$.

We focus on pure strategy equilibria where a strategy for agent i, denoted $s_i(\theta) \in \{0, 1\}$, maps the state to an action. Denote a strategy profile to be a tuple $s(\theta) = (s_1(\theta), s_2(\theta), s_3(\theta))$. Figure 1 (adapted from Harrison and Jara-Moroni) below traces out choices and equilibrium predictions

[3] For dominance regions to exist, it must be the case that $\underline{\theta} < \min_i\{\underline{\theta}_i\}$ and $\overline{\theta} > \max_i\{\overline{\theta}_i\}$

conditional on the value of the state θ. Note that we have regions where the model yields multiple

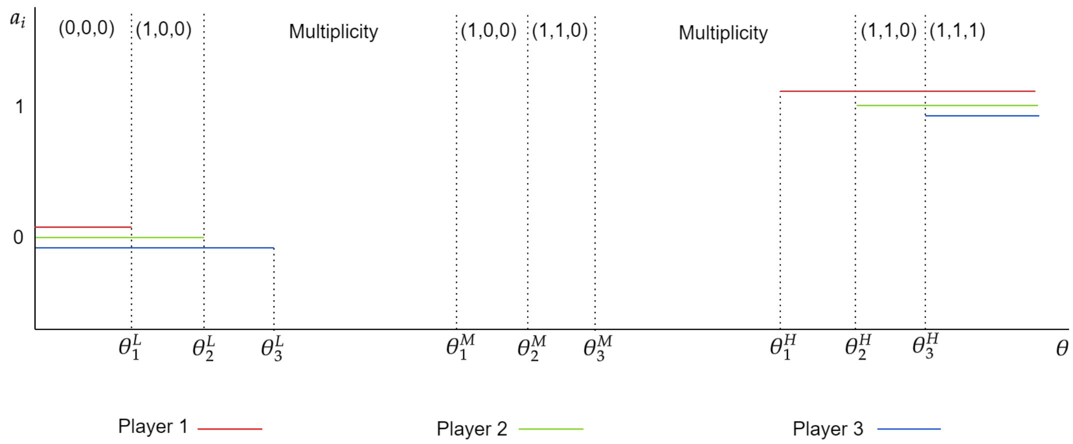


Figure 1: Equilibrium Prediction

equilibria. Specifically, the model yields multiple predictions in regions of $\theta_2^L < \theta < \theta_1^M$ and $\theta_3^M < \theta < \theta_2^H$. Table 1 below characterizes the pure strategy equilibria in these regions. A common characteristic of the equilibria in these specific regions is that they all agree on the number of agents choosing action 1, yet are agnostic about which agent chooses action 1.

Region	Equilibrium Prediction
$\theta_2^L < \theta < \theta_3^L$	(1,0,0) or (0,1,0)
$\theta_3^L < \theta < \theta_1^M$	(1,0,0) or (0,1,0) or (0,0,1)
$\theta_3^M < \theta < \theta_1^H$	(1,1,0) or (1,0,1) or (0,1,1)
$\theta_1^H < \theta < \theta_2^H$	(1,1,0) or (1,0,1)

Table 1: Multiplicity of Equilibria

Harrison and Jara-Moroni (2021) show that we can use the Global Games approach as an equilibrium selection mechanism for this game with strategic substitutes. The Global Games approach works on the principle that one can recover a unique equilibrium prediction in a similar game where, instead of observing the true value of the state, each agent observes a noisy signal about the state. In the equilibrium for this game with *incomplete* information, agents adopt threshold strategies in which they choose action 1 if the value of their private signal exceeds some threshold, and choose action 0 otherwise. Equilibrium selection in the *complete* information game is achieved in the limit when the noise in the *incomplete* information game goes to 0. We formalize this below.

2.1 The Global Games Equilibrium Selection Principle

Consider the incomplete information game where, instead of observing the value of θ, each agent observes a private signal $\theta_i = \theta + \sigma\varepsilon_i$ where ε is an i.i.d random variable drawn from a uniform distribution over $[-0.5, 0.5]$ and $\sigma > 0$ is a scale parameter. Note that the signals $\theta_i \in \tilde{\Theta} \equiv [\underline{\theta} - 0.5\sigma, \overline{\theta} + 0.5\sigma]$. We again focus on pure strategies $s_i : \tilde{\Theta} \to \{0, 1\}$ where s_i prescribes, for each signal θ_i, a pure action 0 or 1. The following proposition shows that there is a unique Bayes-Nash Equilibrium of this game with incomplete information, in which agents follow threshold strategies.

Proposition 1. *For $\sigma > 0$ sufficiently small, the unique BNE of the game with incomplete information s^σ takes the form:*

$$s_i^\sigma(\theta_i) = \begin{cases} 1 \text{ } \textit{if } \theta_i > \theta_i^\sigma \\ 0 \text{ } \textit{if } \theta_i < \theta_i^\sigma \end{cases}$$

where $\theta_1^\sigma = \theta_1^L \equiv c_1 + r$, $\theta_2^\sigma = \theta_2^M \equiv c_2 + r + \delta$ and $\theta_3^\sigma = \theta_3^H \equiv c_3 + r + 2\delta$.

Proof. It is easy to see that the model satisfies assumptions of *Strategic Substitutability, Continuity, Monotonicity, Indifference Points* and *Payoff Asymmetry* (Assumptions A1–A5) of Harrison and Jara-Moroni (2021). Let $\sigma < \overline{\sigma}$ where $\overline{\sigma} = \frac{1}{2}(\min\{c_2 - c_1, c_3 - c_2\} - \alpha)$ for $\alpha > 0$ really small.[4]

Define the strategy s^σ as follows: For each agent i, $s_i^\sigma(\theta_i) = \begin{cases} 1 \text{ if } \theta_i > \theta_i^\sigma \\ 0 \text{ if } \theta_i < \theta_i^\sigma \end{cases}$ where θ_i^σ solve, for

[4]This is a direct application of Lemma 2 (specifically Equation 10) and the upper bound for the noise threshold for Proposition 1 in Harrison and Jara-Moroni (2021). In particular, we need $\alpha < \min_{j>i}\{c_j - c_i\}$.

each $i \in \{1, 2, 3\}$

$$\int_{\theta_1^\sigma - 0.5\sigma}^{\theta_1^\sigma + 0.5\sigma} (\theta - c_1 - r) dP_{\sigma,1}(\theta|\theta_1) = 0$$
$$\int_{\theta_2^\sigma - 0.5\sigma}^{\theta_2^\sigma + 0.5\sigma} (\theta - c_2 - \delta - r) dP_{\sigma,2}(\theta|\theta_2) = 0$$
$$\int_{\theta_3^\sigma - 0.5\sigma}^{\theta_3^\sigma + 0.5\sigma} (\theta - c_3 - 2\delta - r) dP_{\sigma,3}(\theta|\theta_3) = 0$$

Note that, since ε_i and θ are uniformly distributed, for each agent i, the posterior distribution of $\theta|\theta_i$ is uniformly distributed over $[\theta_i - 0.5\sigma, \theta_i + 0.5\sigma]$.[5] Then for each agent i, $dP_{\sigma,i}(\theta|\theta_i) = \frac{1}{\sigma}$ and therefore:

$$\theta_1^\sigma = c_1 + r;\ \theta_2^\sigma = c_2 + r + \delta;\ \theta_3^\sigma = c_3 + r + 2\delta;$$

An application of Lemma 4 in Harrison and Jara-Moroni (2021) shows that s^σ is indeed a Bayes-Nash equilibrium of the game with incomplete information. Uniqueness follows from the application of Proposition 1 in the same paper. ■

Given uniqueness of equilibrium in the game with incomplete information, we can now use the Global Games equilibrium selection principle. The result is recorded below:

Proposition 2. *As $\sigma \longrightarrow 0$, the unique BNE involves switching strategies of the following form:*

$$s_i^*(\theta_i) = \begin{cases} 1 \text{ if } \theta_i > \theta_i^* \\ 0 \text{ if } \theta_i < \theta_i^* \end{cases}$$

where $\theta_1^* = c_1 + r$, $\theta_2^* = c_2 + r + \delta$ and $\theta_3^* = c_3 + r + 2\delta$.

Proof. This is a direct application of Theorem 1 in Harrison and Jara-Moroni (2021). ■

Figure 2 below illustrates the Global Games equilibrium selection principle. There are a few key points to note. First, agents follow threshold strategies in equilibrium for both the complete

[5] We assume that the cutoffs $\theta_i^\sigma \in (\underline{\theta} + 0.5\sigma, \overline{\theta} - 0.5\sigma)$ for all i. This holds true, so long as $\underline{\theta} < c_1 + r - 0.5\sigma$ and $\overline{\theta} > c_3 + r + 2\delta + 0.5\sigma$.

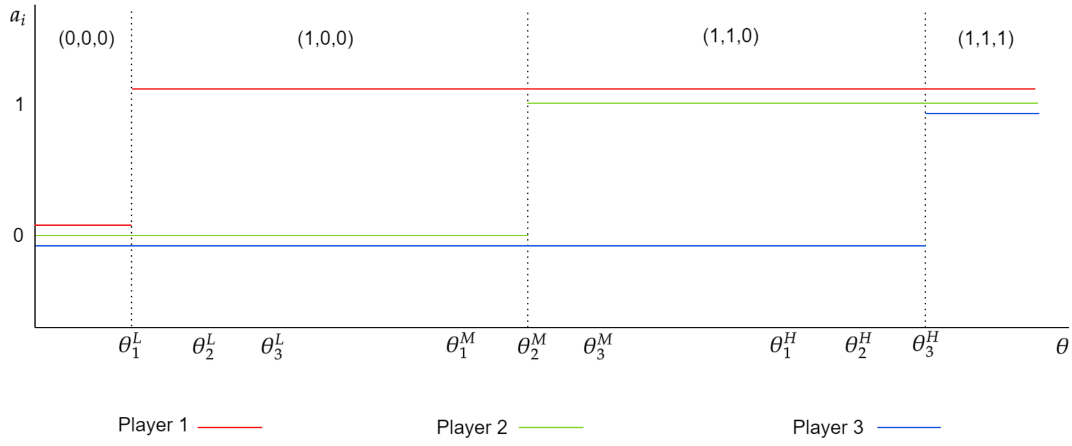


Figure 2: Equilibrium Selection

and incomplete information games. Furthermore, these threshold strategies are identical for both games. This result is driven by the distribution assumptions used in the model and linearity of payoffs which implies that the thresholds in the incomplete information game are independent of the noise parameter σ.[6]

Second, the theoretical thresholds follow the same ordering as the agents, i.e. $\theta_1^* < \theta_2^* < \theta_3^*$. These thresholds are precisely the indifference points for each agent. Agent 1's threshold is the value that makes them indifferent between choosing action 1 or 0 when no other opponent chooses action 1. Agent 2's threshold is the value that makes them indifferent between choosing action 1 or 0 when exactly one other opponent chooses 1. Finally, agent 3's threshold is the value that makes them indifferent between choosing action 1 or 0 when both agents 1 and 2 choose action 1. As a consequence, only agent 1 chooses action 1 in the region $\theta_1^L < \theta < \theta_2^M$ and only agents 1 and 2 choose action 1 in the region $\theta_2^M < \theta < \theta_3^H$

[6]The result of Harrison and Jara-Moroni (2021) is more general and does not require any distributional assumptions, so long as the supports of θ and ε are bounded and the distributions are continuous. For any given distributional assumptions:

$$\theta_1^\sigma \in [\theta_1^L - 0.5\sigma, \theta_1^L + 0.5\sigma]$$
$$\theta_2^\sigma \in [\theta_2^M - 0.5\sigma, \theta_2^M + 0.5\sigma]$$
$$\theta_3^\sigma \in [\theta_3^H - 0.5\sigma, \theta_3^H + 0.5\sigma]$$

3 Experimental Design and Hypotheses

The theoretical results from Section 2 yield the following two streams of comparative statics which drive our experimental design:

1) Comparing behavior between the complete and incomplete information settings, and;

2) Comparing behavior between different types of agents (captured by c_i).

We considered two treatments, one of complete information and one of incomplete information. A subject took part in only one treatment. The procedures for both treatments were nearly identical, save for the fact that in the complete information, the true value of θ was common knowledge amongst all subjects, whereas in the incomplete information treatment, subjects only received a private noisy signal that contained information about the value of θ. Each session consisted of 75 independent and identical rounds. In each round, subjects were randomly re-assigned into groups of 3 following which, depending on the treatment, subjects took part in either the complete or incomplete version of the game.[7] The parameters used for the experiment are given in Table 2 below.

Parameter	$\underline{\theta}$	$\overline{\theta}$	δ	r	c_1	c_2	c_3	σ
Value	100	350	75	100	25	50	75	10 (Incomplete Information)

Table 2: Parameters

The timeline for each round proceeded as follows. First the computer randomly drew a value of θ from a uniform distribution over the interval [100, 350] with values being reported up to 2 decimal places. We generated entire sequences of 75 state values prior to each session. Many sequences were generated. Given the large range of the state space, there was always a possibility to a sequence to contain large gaps in the state draws. To minimize this, we opted to select entire sequences which had few gaps and had a couple of state values each in the intervals (120, 130), (220, 230) and (320, 330) in the last 60 rounds.

Following the draw of θ, subjects were then shown a private signal containing information about θ. In complete information treatments, their private signal was in fact the true value of θ. In the

[7]We incorporate random re-matching each period to minimize the possibility of repeated play behavior usually seen in experiments with fixed matching and a large number of periods.

incomplete information treatments, the private signal was a noisy signal generated as $\theta_i = \theta + \sigma\varepsilon_i$ where ε_i was (independently) uniformly distributed over [-0.5,0.5] and $\sigma = 10$.[8] After observing the value of their signal, each subject in a group then took part in *three* simultaneous, binary action games in which, for each game, they had to choose between options A (action 1 in the model) and B (action 0 in the model). The payoff structure, shown in Table 3, for each game was similar, except for the fact that we re-assigned the types for each game. In particular, for game 1, $c_i = 25$, $c_j = 50$, $c_k = 75$; for game 2, $c_i = 50$, $c_j = 75$, $c_k = 25$ and for game 3, $c_i = 75$, $c_j = 25$, $c_k = 50$. Essentially, each subject played 3 games, one for $c_1 = 25$, one for $c_2 = 50$ and one for $c = 75$ with each game being played against the other two types. This design allowed us to conduct a within subjects analysis when comparing decisions across types.

		Number of Players Choosing A		
		1	2	3
Your Choice	A	$\theta - c_i$	$\theta - c_i - 75$	$\theta - c_i - 150$
	B	100		
Player 2's Choice	A	$\theta - c_j$	$\theta - c_j - 75$	$\theta - c_j - 150$
	B	100		
Player 3's Choice	A	$\theta - c_k$	$\theta - c_k - 75$	$\theta - c_k - 150$
	B	100		

Table 3: Payoff Structure

Note that, for each subject, there were two possible ways to assign group members to Players 2 and 3. This results in 6 possible combinations per round. For feedback and payment purposes, we randomly selected one possible assignment for each subject and selected the appropriate decisions for that assignment. The assignments were selected to ensure consistency of payoffs across all subjects within a group. At the end of the round, subjects received feedback on;

1) The value of θ and their private signal,

[8]Given conditions for uniqueness, any value of $\sigma < 12.5$ would suffice.

2) Their decisions for each game,

3) The number of players choosing A for each game and;

4) Their payoffs for the round.

At the end of the experiment, subjects were paid for the outcome of each of the 3 games in one randomly selected round.

3.1 Theoretical Predictions and Hypotheses

The indifference points implied by the parameters used for the experiment are shown in Table 4. Mapping these points to the model shows that, in the game with complete information, we have multiple equilibrium predictions for $150 < \theta < 200$ and for $250 < \theta < 300$ which amounts to 40% of the state space. Propositions 1 and 2 suggest that subjects should follow threshold strategies.

c_i	θ_i^L	θ_i^M	θ_i^H	θ_i^*
25	125	200	275	125
50	150	225	300	225
75	175	250	325	325

Table 4: Indifference Points and Equilibrium Thresholds

The corresponding thresholds are shown in the last column of Table 4. The equilibrium selection principle yields unique equilibria, given θ, for the complete information treatment which are shown in Table 5.

Range of θ	Equilibrium
$100 < \theta < 125$	(0,0,0)
$125 < \theta < 225$	(1,0,0)
$225 < \theta < 325$	(1,1,0)
$325 < \theta < 350$	(1,1,1)

Table 5: Global Games Prediction

Our first set of hypotheses deals with aggregate behavior which leverage the comparative statics of the predictions. The predictions, while sharp, suggest that an agent's choice is weakly increasing in the observed signal. Furthermore, the incentive to choose action A actually decreases with type c_i. As such, we expect aggregate behavior to follow these monotone comparative statics. This hypothesis is recorded below.

Hypothesis 1 (Aggregate behavior)**.** *Expect, on average, frequency of action A to increase with* θ *and to decrease with* c_i

Our second set of hypotheses deals with individual subject behavior. The theory provides very sharp point predictions about the strategies undertaken by subjects under both treatments. As such, we expect subjects to follow some kind of threshold strategy. Furthermore, we expect that these thresholds are increasing in c_i. This hypothesis is recorded below.

Hypothesis 2 (Individual behavior)**.** *Expect subjects to adopt threshold strategies in line with theoretical predictions.*

Our last set of hypotheses compares behavior in both complete and incomplete information treatments. The theoretical predictions show that agents follow threshold strategies in both treatments. Furthermore, the thresholds are identical in both treatments. As such, when comparing treatments, we should expect to observe very similar, if not identical, behavior. This hypothesis is summarized below.

Hypothesis 3 (Complete and Incomplete Information)**.** *Expect, on average, similar behavior across complete and incomplete information*

3.2 Procedures

Sessions were conducted at the Ohio State Experimental Economics Laboratory in the months of January through March of 2022. Subjects were students at the Ohio State University and were recruited using OSU's online recruitment system ORSEE (Greiner, 2015). Subjects were provided with paper copies of the instructions which are reproduced in Appendix A. Instructions were read out loud at the beginning of the experiment and subjects were allowed to ask clarification questions.[9] Instructions were also programmed into the software, so after the oral description, subjects were

[9]At the end of the experiment, subjects were asked to indicate if they were able to understand the structure of the experiment. All but two subjects responded in the affirmative.

able to read the instructions again at their own pace prior to starting the experiment. Subjects were paid for the outcome of one randomly determined round at the end of the session, at the rate of 25 points to $1. Earnings averaged around $19 for sessions lasting around 1 hour and 30 minutes. The experiment was programmed using oTree (Chen et al., 2016).

3.3 Data Description

We conducted 3 sessions for each treatment. Each session had at least 12 participants. We collected data on 42 subjects for the complete information treatment and 45 subjects for the incomplete information treatment. We opted to select 3 sequences of state draws following the criteria outlined above. Each sequence was used for one complete information treatment and one incomplete information treatment. This choice allows us to compare decisions across treatments since the sequences are matched.

We focus on behavior after 15 rounds of play. There are two reasons for this. First, the criteria for sequence selection focuses on having minimal gaps and at least a few state values around the theoretical thresholds for the last 60 rounds. Second, we wanted to provide subjects with enough experience to get familiar with the experiment. In a post experiment survey, we asked subjects if there were any sources of confusion for them in understanding the experiment. Several subjects responded that they were better able to understand the experiment after a few rounds of play.

4 Results

We first provide a broad overview of the results in relation to the hypotheses outlined in Section 3.1. We first look at aggregate behavior, then turn towards individual behavior and finally compare behavior across complete and incomplete information treatments. We find that

1. Aggregate behavior follows the comparative static predictions of the model
2. Only 2/3rd of the subject population follows some form of threshold strategy. The estimated thresholds for these subjects are significantly different from the theoretical predictions for $c = 25$ and $c = 75$.

3. There is a significant difference in behavior across treatments. For $c = 25$, frequencies of subjects choosing A are higher (for low signals) under complete information, a consequence of the fact that estimated thresholds for $c = 25$ are *lower* in the complete information treatment.

We touch upon these results in detail in the following subsections.

4.1 Aggregate Behavior

We first focus on Hypothesis 1 which looks at aggregate behavior. Figure 3 below shows the empirical frequencies along with standard error bars of subjects choosing option A across realized values of the state θ in the complete information treatment and the private signal θ_i in the incomplete information treatment for all types c_i. We observe similar comparative statics across both treatments.

Firstly, behavior is close to predictions in the dominance regions. For $\theta < 125$, we observe at maximum 12% of observations across all types in which subjects choose A which is dominated. Conversely, for high realizations $\theta > 325$, we have at least 85% subjects choosing the dominant action A across all types.

Secondly, we observe that frequencies are (weakly) increasing in the realized values for all c_i under both treatments. Furthermore, we observe significant jumps in frequencies around the theoretical thresholds. For $c = 25$ frequencies average around 12-15% for signals below 125 and jump to nearly 70% for complete information and 60% for incomplete information for signals between 125 and 150. For $c = 50$, frequencies average around 30% for signals between 200 and 225 and jump to 70% for complete information and 60% under incomplete information for signals between 225 and 250. Finally, for $c = 75$, frequencies average around 50% to 60% for signals between 300 and 325 and jump to 87% under complete information, and 90% under incomplete information for signals above 325.

Lastly, we observe that frequencies of choosing A follow a monotone ordering in c_i. Specifically, conditioning on signal realizations, frequencies for $c = 25$ are higher than those for $c = 50$ which themselves are higher than those for $c = 75$. These results are summarized below.

Result 1. *Aggregate behavior follows in line with predicted comparative statics. Frequencies of choosing A are (weakly) increasing in the value of the signal and decreasing in the type c_i. Furthermore, we observe*

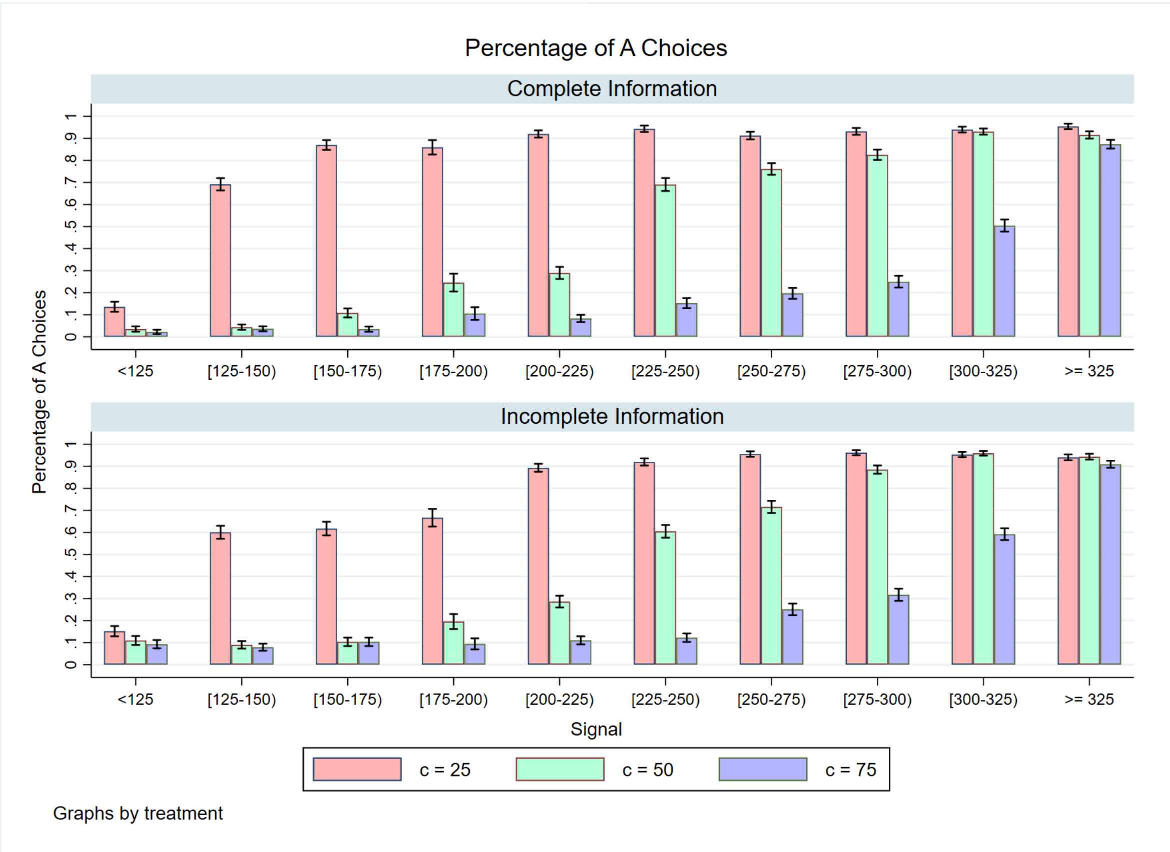


Figure 3: Frequency of choosing A

significant jumps in frequencies around the predicted thresholds for each type.

4.1.1 On Global Games Equilibrium Selection

While the aggregate frequencies are in line with the comparative static predictions, it remains to be seen how the global games prediction performs with respect to other equilibria. Table 6 below shows the empirical frequencies of the global games equilibrium for the complete information treatment and compares it to other pure strategy equilibria and non-equilibrium outcomes. We find

Range of θ	Prediction	Global Games	Other Equilibrium	Non-Equilibrium
$100 < \theta < 125$	(0,0,0)	81.1%	N.A	18.9%
$125 < \theta < 150$	(1,0,0)	63.0%	N.A	37.0%
$150 < \theta < 175$	(1,0,0)	74.0%	0.9%	25.1%
$175 < \theta < 200$	(1,0,0)	57.9%	3.5%	38.6%
$200 < \theta < 225$	(1,0,0)	62.3%	N.A	37.7%
$225 < \theta < 250$	(1,1,0)	55.0%	N.A	45.0%
$250 < \theta < 275$	(1,1,0)	58.3%	8.3%	33.3%
$275 < \theta < 300$	(1,1,0)	57.6%	4.9%	37.5%
$300 < \theta < 325$	(1,1,0)	42.9%	N.A	57.1%
$325 < \theta < 350$	(1,1,1)	76.5%	N.A	23.5%

Table 6: Frequencies of Outcomes

that a majority of outcomes correspond to the global games equilibrium prediction. In regions of dominance ($\theta < 125$ and $\theta > 325$), the global games prediction accounts for at least 76.5% of the outcomes. In regions with a unique equilibrium, we find that play conforms to the prediction for 42.9% of the observations ($300 < \theta < 325$) to 63% of the observations ($125 < \theta < 150$). We also find that, in regions of multiplicity ($150 < \theta < 200$ and $250 < \theta < 300$), the global games prediction corresponds to 57.6% to 74% of the observations whereas only a handful of observations (at most 8.3%) correspond to other pure strategy equilibrium. Thus, our results provide some evidence suggesting that the global games principle is a valid equilibrium selection mechanism, especially in regions with multiple equilibria.

Result 2. *A majority of outcomes in the complete information treatment are in line with the global games* *equilibrium prediction.*

4.2 Individual Behavior

In this section, we focus on Hypothesis 2 and look at behavior at the individual subject level. We first focus on whether or not subjects adopt threshold strategies in behavior. We adopt the following approach to classify whether or not an individual subject follows a threshold strategy. We first follow the convention in the global games literature and fit a logistic regression to each individual subject's data. Specifically, for each subject, we regress the subject's choice (either A or B) on their observed signal. The CDF of the logistic distribution, for each subject i, can be expressed as:

$$\Pr(A) = \frac{1}{1 + \exp(a_i + b_i \theta_i)}$$

where θ_i is the signal observed by the subject. Using the estimated parameters, we then "back out" the estimated threshold as signal at which a subject is exactly indifferent between choosing A and B (i.e. $\Pr(A) = 0.5$). The estimated threshold can then be stated as $\tilde{\theta}_i = -\frac{a_i}{b_i}$.[10]

We observe a lot of heterogeneity in subject behavior. In particular, very few subjects follow clear threshold strategies with a clear switching point. However, we do observe some regularity in behavior across a majority of observations that indicate that subjects follow some monotone strategy with some degree of noise.[11] To capture this noise, we estimate the number of "mistakes" a subject makes in their choices. Specifically, we classify an observation as a mistake if a subject chooses action A if the signal is below her estimated threshold. Likewise, an observation is also considered a mistake if a subject chooses action B if the signal is above her estimated threshold. We consider an individual subject to follow a threshold strategy if she makes *at most* 10 mistakes for each type c_i. Figure 4 below shows the distribution of the number of mistakes for each type c_i.

We find that a majority of subjects make at most 10 mistakes. For $c = 25$, $c = 50$ and $c = 75$, we have 75 (86%), 68 (78%) and 65 (75%) of subjects making making at most 10 mistakes respectively. Furthermore, 58 subjects (67%) make at most 10 mistakes for each type c_i, with 26 (62%) in the

[10]For subjects who follow a clear threshold strategy, we estimate their threshold as the average between the highest signal for which they choose B and the lowest signal for which they choose A.

[11]Appendix B provides some examples of subject behavior.

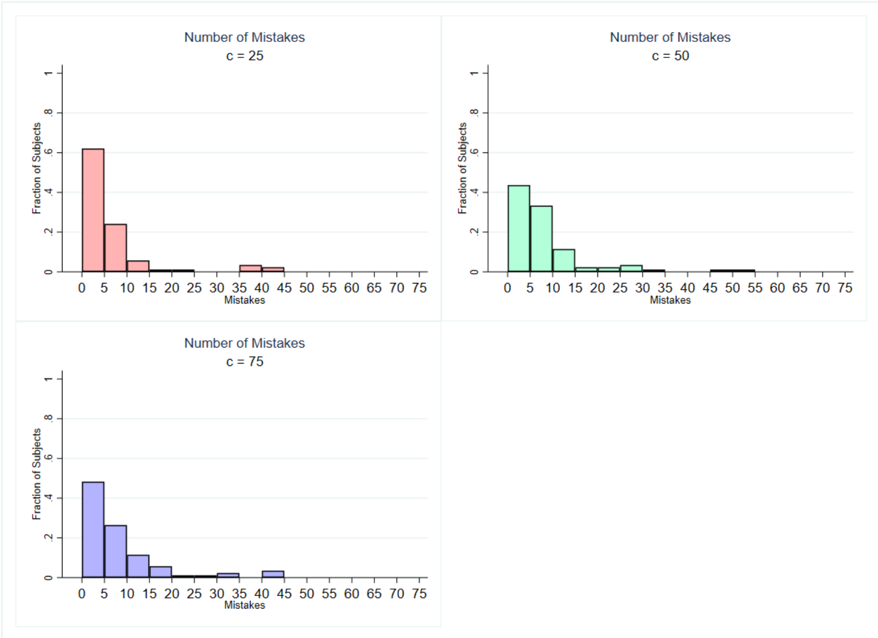


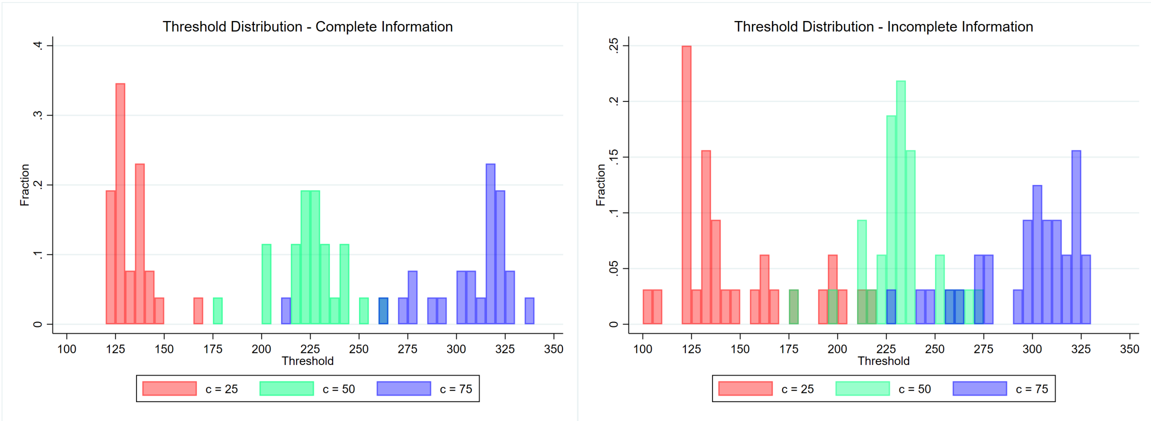
Figure 4: Distribution of mistakes in Subject behavior

complete information treatment and 32 (71%) in the incomplete information treatment. This result is summarized below.

Result 3. *2/3 of the subjects follow a threshold strategy with at most 10 mistakes.*

For the remainder of this subsection, we focus on only these subjects. Figure 5 below shows the distribution of estimated thresholds for each type under both complete and incomplete information. There is a clear separation in the distribution of thresholds across types for both treatments with the degree of separation being more pronounced in the complete information treatment. Table 7 provides a quantitative comparison of the thresholds with respect to the theoretical predictions. We find that the average estimated threshold for $c = 25$ is significantly higher than the theoretical prediction for both complete and incomplete information (two-sided $t-$tests, $p < 0.05$). Furthermore, we observe that the average for $c = 75$ is significantly lower than the theoretical predictions (two-sided $t-$tests, $p < 0.05$). In contrast to the above, we do not observe any statistically significant difference in the estimated thresholds for $c = 50$ from the theoretical prediction.

We also find that the estimated thresholds are in line with the comparative static predictions



(a) Complete Information (b) Incomplete Information

Figure 5: Estimated Thresholds

	c=25	c=50	c=75
Prediction	125	225	325
Mean (Complete)	132.9*	225.9	304.4*
	(1.93)	(3.34)	(5.20)
Mean (Incomplete)	148.0*	231.8	297.1*
	(5.66)	(3.41)	(4.74)
Predicted Difference	100	100	
Difference (Complete)	93.0*	78.5*	
	(3.22)	(4.68)	
Difference (Incomplete)	83.8*	65.3*	
	(4.90)	(5.19)	

Notes: Estimates are mean values of estimated thresholds. Standard errors in parentheses. * indicates estimates are different from prediction at the 5% significance level.

Table 7: Threshold Estimates

for both treatments. The average estimated threshold for $c = 25$ is significantly lower than that for $c = 50$, which again is significantly lower than that for $c = 75$. However, the average distance between the thresholds is significantly lower under the predicted value for both treatments. These results are summarized below

Result 4. *Estimated thresholds follow comparative static predictions. Thresholds are significantly higher than predicted for* $c = 25$*, but lower for* $c = 75$*.*

4.3 Comparison between Complete and Incomplete Information

We finally turn towards Hypothesis 3, comparing behavior between Complete and Incomplete Information treatments. Recall that under Hypothesis 3, we expect behavior to be fairly similar across both treatments for all types.

We first take a look at aggregate behavior. Figure 6 below compares the average frequencies of subjects choosing A between complete and incomplete information across all types. When comparing behavior for $c = 25$, we find significant differences across treatments for low signal realizations. In particular, for signals between 125 and 200, we observe higher average frequencies under complete information. Behavior becomes statistically indistinguishable, however, for high signal realizations. For $c = 50$, we find that frequencies are higher under incomplete information for $\theta < 150$, higher under complete information for $225 < \theta < 275$ and higher under incomplete information for $275 < \theta < 300$. Finally, for $c = 75$, we observe higher average frequencies under the incomplete information for both low ($\theta < 175$) and high ($\theta > 275$) signal realizations. These results are summarized below

Result 5. *For low signals, frequencies for choosing A are higher under the complete information for* $c = 25$*, but higher under the incomplete information for* $c = 50$ *and* $c = 75$*. For high signals, behavior is similar in both treatments for* $c = 25$*, but frequencies for choosing A are higher under the incomplete information for* $c = 75$*.*

We now turn towards analyzing individual thresholds. Recall that 26 subjects in the complete information treatment and 32 subjects in the incomplete information treatment followed a monotone threshold strategy. Figure 7 below compares the distributions of these estimated thresholds across

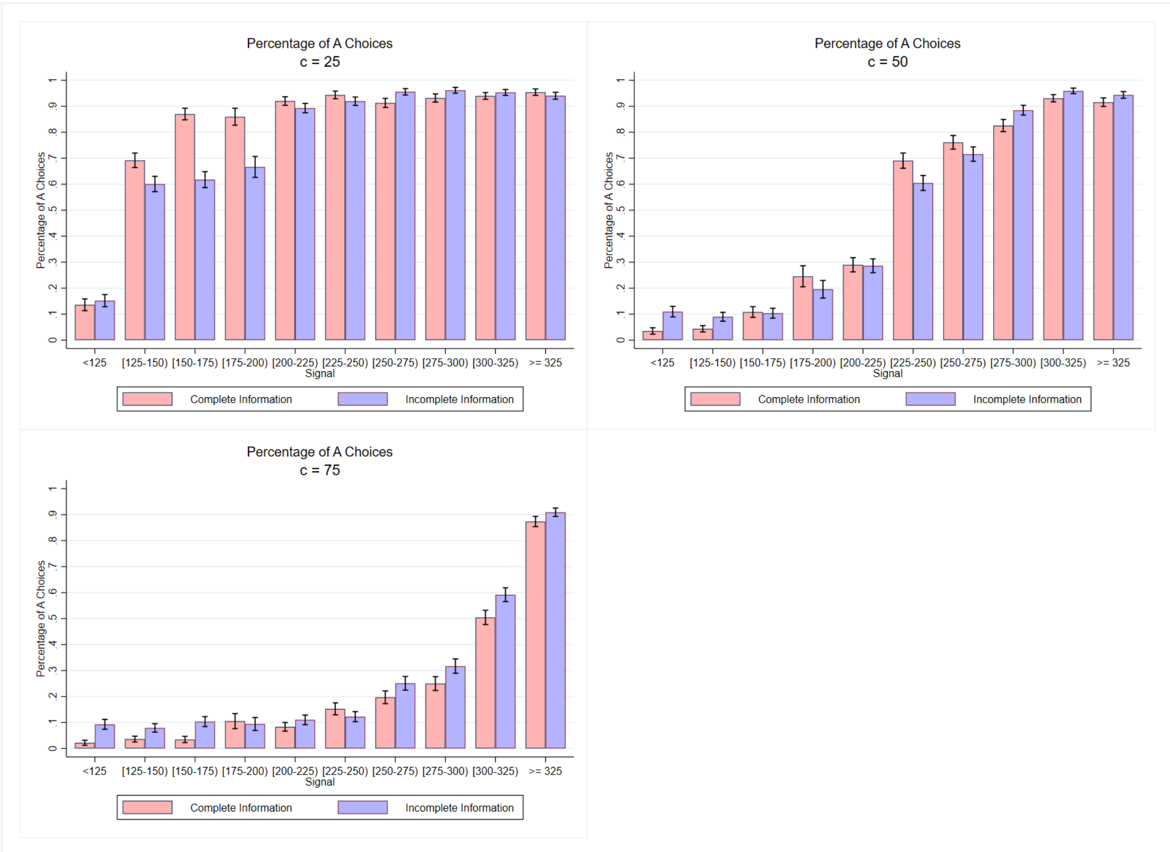


Figure 6: Comparison of Frequencies (Complete and Incomplete Information)

all types. While we do see some more variance in the thresholds under incomplete information treatments, the difference in distributions is only statistically significant for $c = 25$ (Kolmogorov-Smirnov $p < 0.1$). We cannot reject the null hypothesis of no difference in distributions for $c = 50$ and $c = 75$ (Kolmogorov-Smirnov $p > 0.1$). This result is summarized below

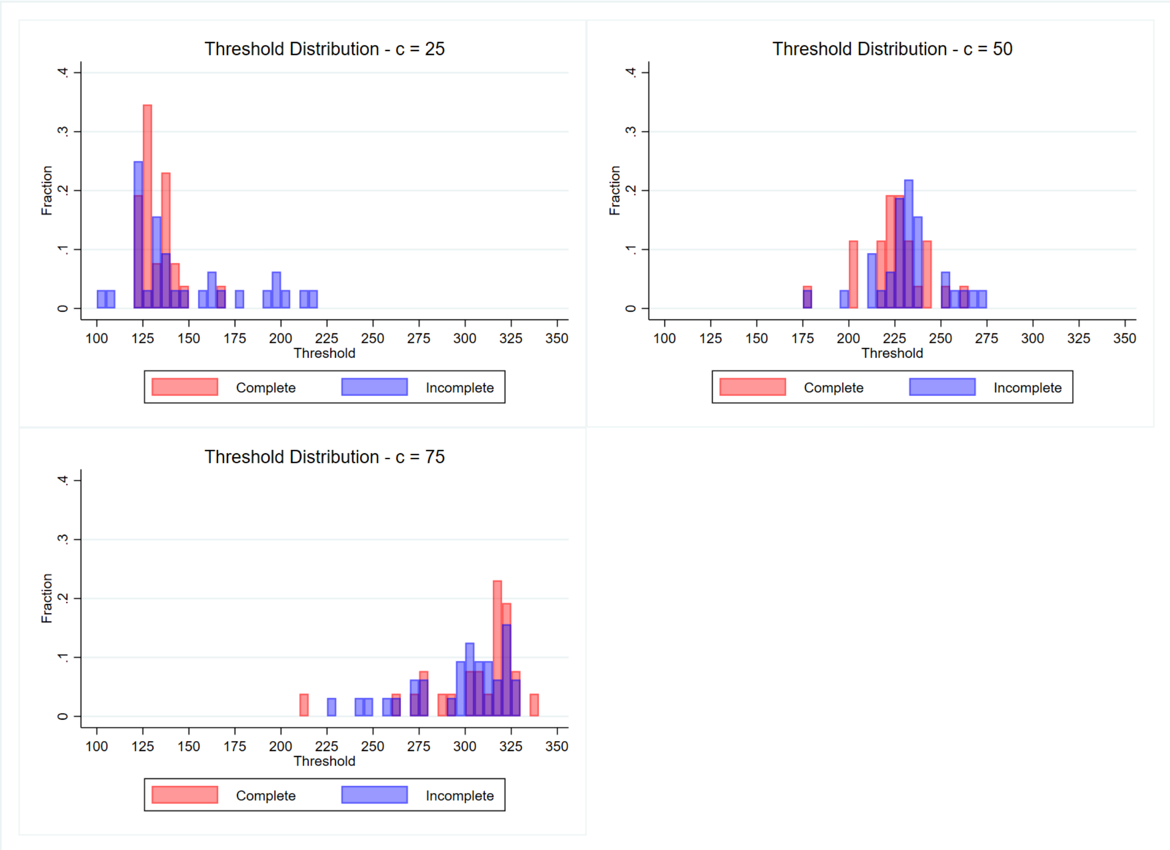


Figure 7: Comparison of Thresholds (Complete and Incomplete Information)

Result 6. *Estimated thresholds for subjects following a threshold strategy are similar across treatments for* $c = 50$ *and* $c = 75$*. Estimated thresholds for* $c = 25$ *are significantly lower under the complete information treatment.*

5 Conclusions

In this paper we experimentally investigate behavior in a global game where actions are strategic substitutes. In our experiment, subjects take part in a 3 agent, binary action game where payoffs are heterogeneous across agents and depend on some underlying state fundamental. Depending on the value of the state fundamental, the game with complete information permits regions with multiple equilibria. We test the global games equilibrium selection result of Harrison and Jara-Moroni (2021) by having subjects take part the game with complete *and* incomplete information. Our experimental results provide some support to the theoretical predictions. 66% of the subjects adopt threshold strategies, as outlined by the theory, with at most 10 mistakes. While the estimated thresholds depart from theoretical predictions, the comparative static predictions still hold. Furthermore, a majority of outcomes in regions of multiplicity in the complete information game correspond to the unique equilibrium selected under the global games principle.

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Appendix A Instructions for the Experiment

We reproduce the instructions for the incomplete information treatment below. The instructions were largely the same for the complete information, save for the removal of information regarding the private signal. For the instructions, we use the letter X to denote the value of the state θ.

Instructions

Welcome and thank you for taking part in this Economics Experiment. This experiment will last for around 1.5 hours. If you read the instructions carefully, you can earn a considerable amount of money depending on your decisions, the decisions of others and chance. Your earnings will be paid out to you in cash after the experiment.

Please turn off your cell phones for the duration of the experiment. Only use the software provided to you on your devices. Failure to comply with these rules will result in dismissal from this experiment and as a result, you will not be paid any earnings you may have otherwise received. If you have any questions, please raise your hands.

Overview

This experiment consists of 75 rounds. At the beginning of each round, you will be randomly matched into a group with 2 other participants. You will be re-matched randomly into different groups every round and you will never know the identity of your group members in any round.

In each round you will make 3 decisions with your group members. You will earn points in each round for each decision. Your payment for this experiment will depend on the points you earn **in one randomly selected round.** Your payment will be converted to dollars at the rate of

$1 = 25 points

You will also earn a show-up fee of $5 in addition to your payment from the experiment.

Next

Instructions

Rounds

At the beginning of each round, the computer will randomly generate a number X. Each period, the computer will generate a new value for X from a **uniform distribution ranging from 100.00 to 350.00.** This means that any number between 100.00 and 350.00, in increments of 0.01, is equally likely to be generated. The value of X will be drawn independently for each round, meaning that each draw is unaffected by the draws in earlier or later periods. **Note that this number will be the same for all decisions and group members for a given round.**

Private Signal: Instead of observing the actual value of X, you will observe the value of a *private signal*. This signal will serve as a hint for the value of X and is constructed as follows.

Your private signal = X + your private Y

In each round, the computer will randomly generate a private number Y from a **uniform distribution ranging from -5.00 to 5.00.** This means that any number between -5.00 and 5.00, in increments of 0.01, is equally likely to be generated. This number will be drawn independently of X and for each round, meaning that the draw of Y is not affected by the true value of X or by the value of your Y in any other round. This number Y is private, meaning that the computer will draw a number for each of the group members and your number Y is drawn independently of the numbers drawn for the other group members. **You will not see the value of Y or X; you will only see your private signal.**

You will not know the values of your group members' private signals.

Each round has 3 decisions. For each decision, you must choose between two options: **Option A and Option B.** You will make your decisions at the same time as your group members. Your earnings for each decision will depend on:

1. Your choice,
2. The number of participants in your group choosing option A and,
3. The random number X generated at the start of the round.

On your screen, you will see the value of your private signal (X + Y) as well as the payoff tables for each decision. You will then, for each decision, click the choice (either A or B). To submit your choices, click the Next button.

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Instructions

Payoffs

The following tables summarize your, and your group members' earnings for each decision and combination of choices.

Decision 1

		Number of Players Choosing A		
		1	2	3
Your Choice	A	X - 25	X - 100	X - 175
	B	100		
Player 2's Choice	A	X - 50	X - 125	X - 200
	B	100		
Player 3's Choice	A	X - 75	X - 150	X - 225
	B	100		

Decision 2

		Number of Players Choosing A		
		1	2	3
Your Choice	A	X - 50	X - 125	X - 200
	B	100		
Player 2's Choice	A	X - 75	X - 150	X - 225
	B	100		
Player 3's Choice	A	X - 25	X - 100	X - 175
	B	100		

Decision 3

		Number of Players Choosing A		
		1	2	3
Your Choice	A	X - 75	X - 150	X - 225
	B	100		
Player 2's Choice	A	X - 25	X - 100	X - 175
	B	100		
Player 3's Choice	A	X - 50	X - 125	X - 200
	B	100		

Each cell represents the payoff to the relevant person for each decision and choice scenario. If you choose A and are the only person choosing A in your group for that decision, your payoff for that decision is X-25 (decision 1), X-50 (decision 2) or X-75 (decision 3). Each additional group member choosing A *reduces your payoff by 75 points if you choose A*. Choosing B always gives you a payoff of 100 points for any decision.

How the choices of other group members for each decision are selected is explained as follows:

For each round, the computer will randomly assign one of your group members as Player 2, and the remaining group member as Player 3. The relevant choices to determine the number of people choosing A for each decision are given below:

	Payoff for Decision 1	**Payoff for Decision 2**	**Payoff for Decision 3**
Your Choice	Decision 1	Decision 2	Decision 3
Player 2's Choice	Decision 2	Decision 3	Decision 1
Player 3's Choice	Decision 3	Decision 1	Decision 2

In words, to compute your payoffs for Decision 1, we look at your choice in Decision 1, Player 2's choice in Decision 2 and Player 3's choice in Decision 3. To compute your payoffs for Decision 2, we look at your choice in Decision 2, Player 2's choice in Decision 3 and Player 3's choice in Decision 1. Finally, to compute your payoffs for Decision 3, we look at your choice in Decision 3, Player 2's choice in Decision 1 and Player 3's choice in Decision 2.

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Instructions

Feedback

Once everyone has submitted their choices, you will be shown the outcome for the round. On this screen, you will see

1. The value of X,
2. Your private signal (X + Y),
3. The payoff tables for each decision,
4. Your choices for each decision,
5. The number of people who chose A in your group and,
6. Your payoffs for each decision and the total payoff for the round.

At the end of the 75 rounds, the computer will randomly select one round for payment. Your screen will show the following information for that round:

1. The round chosen for payment,
2. The value of X for that round,
3. Your private signal (X + Y),
4. Your choices for each decision,
5. The number of people who chose A in your group for each decision and,
6. Your payoffs (and final payment) for that round.

At the end of the experiment, we will ask you some survey questions.

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Instructions

Summary

To summarize:

1. The experiment consists of 75 rounds.
2. You will be randomly re-assigned into a group of 3 participants at the beginning of every round.
3. At the beginning of each round, the computer will randomly generate a number X between 100.00 and 350.00 (each number is equally likely).
4. Instead of observing the value of X, you will be shown a *private signal* X+Y, where Y is a private number randomly generated between -5.00 and 5.00 (any number in between is equally likely).
5. You and your group members will take part in 3 decisions. Each decision involves you choosing between Option A and Option B
6. Your payoff for each decision depends on your choice, the number of people choosing A and the value of X.
7. After 75 rounds are completed, the computer will select one round at random for payment.
8. Payment will be at the rate of $1=25 points. You will also earn a $5 show-up fee.

Please click the next button to continue on to the experiment.

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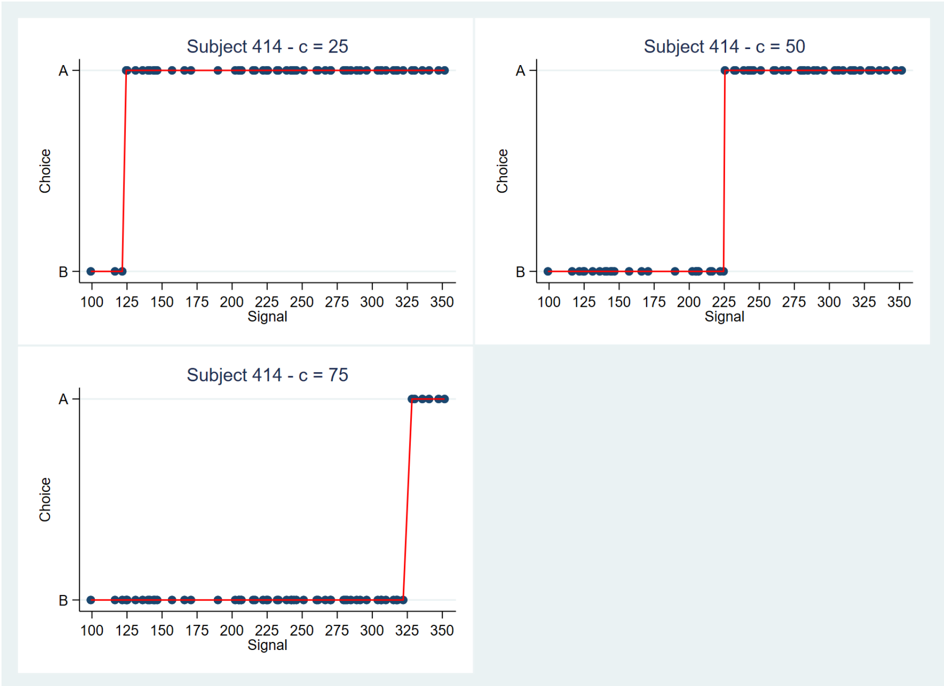
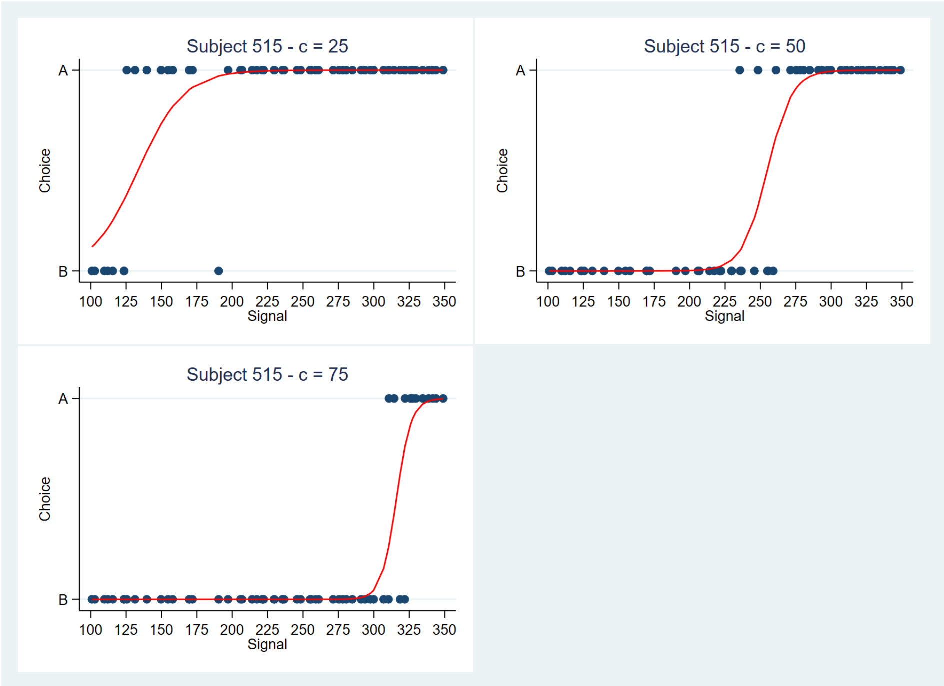
Appendix B Example Subject Behavior

The figures below provide some example subjects whose behavior is in line with some threshold strategy with varying degrees of noise

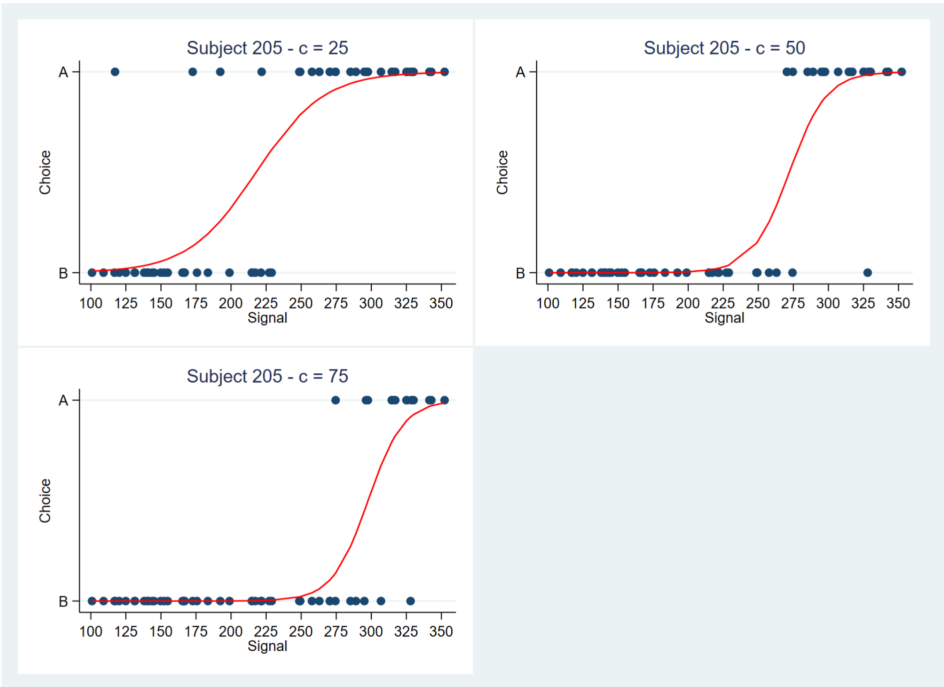


(a) No Noise

Figure 8: Examples of Subject Behavior

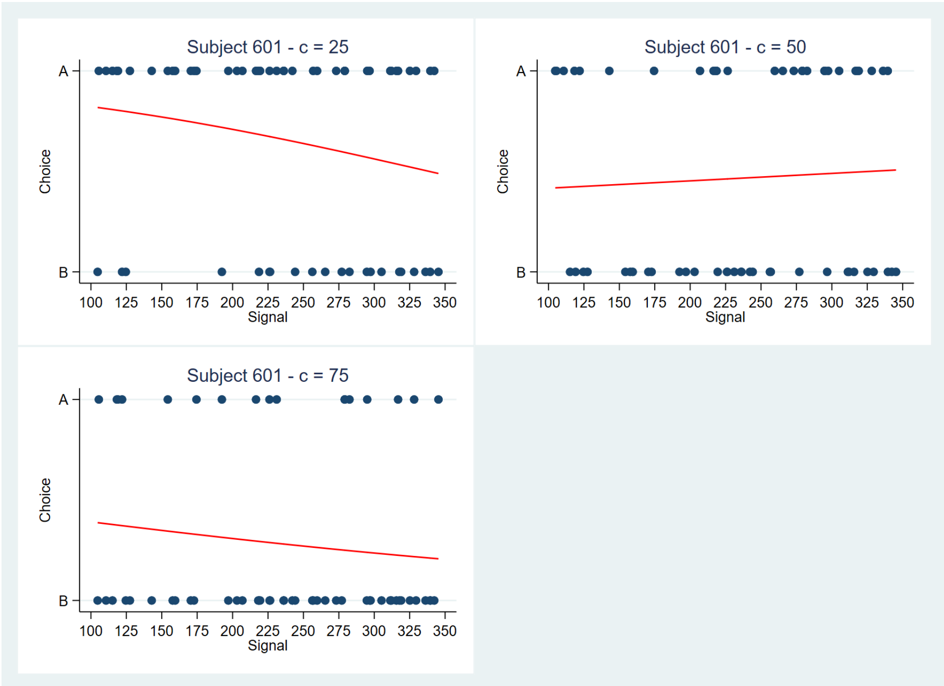


(b) Small Noise



(c) Moderate Noise

Figure 8: Examples of Subject Behavior



(d) Pure Noise

Figure 8: Examples of Subject Behavior