The Feasible Set and Folk Theorems for Infinitely Repeated Games with Switching Costs *

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1 Introduction

Switching costs appear naturally in many real-life scenarios, as changing an action might incur additional costs compared to maintaining it for an extra period. This can occur due to set-up costs [Akerlof and Yellen, 1985a,b], movement costs [Filar and Schultz, 1986], or costs of time of inactivity (such as in scheduling problems, see Yavuz and Jeffcoat [2007]).

The introduction of switching costs to a repeated game changes the game. In particular a player that alternates between two of his actions will lose some payoff due to the cost of switching. This impacts both the worst case payoff he can defend (i.e., his individually rational level), and the payoffs he can receive in equilibrium. The natural questions that follow are which payoffs can a player obtain in a repeated game with switching costs, how they depart from the settings without switching costs and how they change as the switching costs increase. Partial answers were established in the asymptotic cases of small and large switching costs for example in Chakrabarti [1990],Lipman and Wang [2000], Lipman and Wang [2009] or [Burkov and Chaib-Draa, 2014].

In this paper, we study the general effect of switching costs on non-zero-sum scenarios, characterize the set of feasible payoffs, and establish Folk Theorem for different time horizons and payoff accumulation methods (infinitely repeated undiscounted, infinitely discounted and finitely repeated). This allow us to provide comparative static between several levels of costs in each case. This analysis can benefit subsequent studies in the above mentioned fields and

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others which take switching costs into consideration.

2 Results

2.1 Infinitetly repeated game with undiscounted payoffs

This case will serve as a benchmark.First, we study the shape of the set of the equilibrium payoffs in the game. We show that when the switching costs are symmetric, this set is equal to the feasible set of a one-shot auxiliary game, constructed by considering only the first two stages of the repeated game. Our result provides a novel and simple method to calculate the feasible set, since we show that it depends solely on the primary parameters of the game. The symmetry of the switching costs is essential for this result to hold and contains the mostly studied framework of constant switching cost. For assymetric costs, we exhibit a superset.

Second, we obtain a Folk Theorem. The set of SPE payoffs can be characterized as the intersection of the feasible payoffs and the set of individually rational payoffs (in the infinitely repeated game with undiscounted payoff).We construct explicitely Subgame Perfect Equilibria without correlation device due to the special structure of our transition structure. As a consequence of this Folk Theorem, we show that an increase in the switching costs can only have a negative effect on a player.

2.2 Infinitely repeated game with discounted payoffs

For infinitely repeated game with discounted payoffs and discounted case, we have different results depending on the payoffs function and the patience of the players. We established an asymptotic Folk Theorem using Dutta [1995] showing that the set of discounted SPE converges to the undiscounted equilibrium set under the Full dimensionality Assumption. This is in particular the case if all players have some switching cost. Hence asymptotically, a higher cost has a negative effect on a player. On the contrary, we provide an example showing that a player may benefit from a switching cost if players are not patient.

2.3 Finitely repeated game with mean-average payoff

For finitely repeated games, we have different results between patient and impatient players but also depending on the structure of the game. We established an asymptotic Folk Theorem using Marlats [2015] showing that the set of discounted SPE converges to the undiscounted equilibrium set under the Full dimensionality Assumption and a sufficiently rich structure of

the undiscounted SPE payoffs. Hence asymptotically, a higher cost has a negative effect on a player. On the contrary, we provide an example where these conditions are not satisfied and show that a player may benefit from a switching cost in arbitrarly long finitely repeated game.

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