Persuading a Manipulative Agent

Yihang Zhou*

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Abstract

In a dynamic Bayesian persuasion game, a sender is seeking approval from a series of receivers before a deadline. I assume that only the receiver can verifiably disclose the current experiment to the next receiver, while the sender cannot. In this case, by deciding to hide the information or not, a receiver manipulates the information used by the following receiver. This manipulation power makes delay possible in equilibrium when receivers are naive. In this naive case, if actions are binary, manipulation power can benefit a receiver while weakly hurting the sender. But with sophisticated receivers, there is no incentive to delay.

Keywords: Bayesian persuasion, Information manipulation, Dynamic games

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*Department of Economics, University of Texas at Austin, yhzhou@utexas.edu

1 Introduction

In a dynamic Bayesian persuasion game with one sender and a series of receivers, if only the receiver can disclose the current experiment to the future receiver, the current receiver has manipulation power on the information used by the next receiver. Though he is not allowed to fabricate evidence, he can choose not to disclose the current information to his successor. The next receiver will know the experiment performed in front of him, but for an experiment in the history, which is performed to a previous receiver, he will only know it when the previous receiver discloses it to him. The sender cannot provide the past information to the current receiver. I want to study whether this manipulation power benefits receivers and how it affects the sender.

As an example, when making policies related to low-carbon energy, policymakers may turn to participatory governance and work closely with firms (Mah and Hills, 2014; Liu, 2019). The policymaker may seek suggestions and evidence from firms and firms can design an information structure to make their suggestions approved, which fits the model of Bayesian persuasion. But the conflict between policymakers and firms is a critical problem in China (Liu and Liu, 2016; Liu, 2019), which leads to the distrust between two sides. Then the government employees have to rely on their predecessors for past information and only take the firms' experiment performed in front of them into consideration. Moreover, when a government employee leaves his position (because of a term limit or position rotation), he may still have some suggestions from firms waiting for a decision. He can make a decision by himself, or delay the decision to his successor. And he is aware that, out of distrust, his successor will require firms to perform an experiment in front of him, and without his disclosure, his successor has no access to today's information.

To describe the situation above, I consider a 2-period dynamic information design model. There are one long-lived sender and 2 short-lived receivers. Receivers have identical preferences. In period 1, the sender chooses an experiment and then the receiver takes an action. The receiver can choose *yes* or *no* to conclude the game or choose to delay the decision to the next receiver. With *yes* or *no* chosen, the game ends and payoffs are realized. Upon delaying, the game proceeds into the next period. Meanwhile, the receiver decides on verifiably disclosing the current experiment and outcome or not, and his payoff depends on the action by the future receiver. With the verification, the next receiver will know today's experiment and outcome. On the contrary, the following receiver has no access to the current information without disclosure, since receivers do not trust the sender. I firstly assume that receivers are naive in the sense that they will assume no experiment was conducted in the past period if the past receiver did not disclose it. So a naive receiver will make a decision based on the verified experiment in the history and the current experiment which is performed in front of him. This naive case can also be considered as that interpreting past information without disclosure is too costly or the receiver has a tight schedule and thus has no time to do something like this. I will visit the sophisticated case where receivers do Bayesian inference seeing no disclosure in Section 4.2. As for the sender, he makes his decision based on the whole history. Finally, in the second period, the actions available to the receiver are just *yes* and *no*. Notice that though in the period 1, the receiver has 4 possible actions (*yes*, *no*, *hide*, *reveal*), there are only two actions that conclude the game.

In the baseline case where receivers have no manipulation power and always disclose the information, the game is simply a repetition of the same Bayesian persuasion and the result will be the same as a static persuasion—there is no delay and the game ends at the first period. But with the manipulation power, delay is possible. Intuitively, by not disclosing, the future receiver will have a different belief from the current receiver. Since in the next period the sender is persuading someone with a different belief from the current receiver, the experiment used may be more informative in the eyes of the current receiver, which can make delay a better choice than acting immediately.

Compared to the static case, the receiver is harder to persuade since he has extra choices. As a result, the sender either provides a more informative experiment to persuade the receiver to act immediately or turns to a less informative experiment and lets the receiver delay.

Which of these two choices is optimal depends on the parameters—the discount factor and the priors of period 1. For 2-period cases, a larger discount factor will make delay more likely[1] since it makes the future payoff more attractive and thus makes the receiver harder to persuade. But if extended to more than 2 periods, the effect of a larger discount factor on delay is ambiguous. When we have $N > 2$ periods, in the periods before $N - 1$, a larger discount factor not only makes delay less costly, it will also make delay more likely in the future. But compared to the case where the sender chooses a more informative experiment to induce immediate actions, the receiver will have a lower payoff when the sender chooses a less informative experiment and lets the receiver delay. So, a higher discount factor also has the effect of making the future payoff less attractive and making the receiver easier to persuade. Since we have two effects in the opposite directions, the total effect will be ambiguous.

The receiver has no value of persuasion from a binary-action Bayesian persuasion game with a state-independent sender[2]. But in my model, if receivers are naive, when the sender chooses a more informative experiment to induce immediate actions, the first receiver is better off compared to the baseline and thus has a positive value of persuasion. When the sender chooses a less informative experiment that induces delay, the value of persuasion to the first receiver is still zero.

The organization of this paper is as follows. Section 2 provides the set-up for the model. Section 3 illustrates the main results of this paper. Section 4 introduces some extensions of the model including multiple-period and multiple-state cases, and sophisticated receivers. Finally, Section 5 is the conclusion remarks.

Related Literature.—There are papers discussing static information manipulation. Fischer and Verrecchia (2000) studied the incentive of the company manager to manipulate the information to the capital market. Goldman and Slezak (2006) looked at how to design a contract to stop the manager from manipulating information and to make him focus on pro-

[1]It is in the sense that more priors can lead to an optimal experiment inducing delay

[2]According to the definition in Kamenica and Gentzkow (2011), it means the receiver's expected payoff from the optimal experiment chosen by the sender is equal to the expected payoff without any experiment.

ductive work. Edmond (2013) discussed how politicians manipulate the information source to affect the citizens' decisions.

But I am studying how information manipulation over time affects the effect of the information provided. This information manipulation can happen when someone is leaving his position and can explain the behavior change at term limits studied by Lopez (2003); Carey, Niemi and Powell (2009); Klein and Sakurai (2015).

Bénabou and Tirole (2002) also studied information manipulation over time. In their model, an agent with time-inconsistent preferences can hide bad news from the future self who is deciding to take a task or not. And his incentive to manipulate information comes from the interest conflict between different selves. In contrast, I assume that all receivers share the same preferences. Another difference is that in my model, instead of a decision problem, there is a game in each period between a sender and a receiver. In my setting, I find that even with identical preferences, the manipulation can still be valuable when receivers are naive. As for sophisticated receivers, they have no incentive to delay and manipulate the information.

This paper also contributes to the literature on Bayesian persuasion started by Kamenica and Gentzkow (2011). It extends Bayesian persuasion to a dynamic circumstance by allowing the receiver to delay and manipulate the information for the following receiver. And the incentive to delay in my model is the manipulation power, which is different from current dynamic persuasion papers with other focuses, such as Ely (2017); Honryo (2018); Orlov, Skrzypacz and Zryumov (2020); Smolin (2020); Bizzotto, Rüdiger and Vigier (2021).

Moreover, though I assume that the sender and the receiver share the same prior in the very first place, priors in the second period can be different between the sender and the receiver endogenously due to information manipulation. This heterogeneous priors problem has been studied by Alonso and Câmara (2016). Also because of information manipulation, the update of the receivers' priors may not be Bayesian across periods, which relates my paper to previous works on persuading a non-Bayesian agent like Levy, de Barreda and

Razin (2018), de Clippel and Zhang (2019) and Galperti (2019).

2 Model

2.1 Timing

I study a 2-period dynamic information design model. There are 3 players: one long-lived sender and 2 short-lived receivers. The unobserved payoff-relevant states of the world are $\Omega = \{g, b\}$. At period t, Receiver t has a prior $p_t = Pr(\omega = g)$ and the sender has a prior p_t^s. Though I assume that in period 1, $p_1 = p_1^s$, I distinguish players' priors since they can be different in period 2 due to information manipulation.

In each period, the sender chooses an experiment: $\pi_t = (\pi_t(.|\omega))_{\omega\in\Omega} \in \bigtimes_{\omega\in\Omega} \Delta(S) = \Pi$, where S is the signal space. I assume that the sender can choose any experiment he likes, as long as outcomes of experiments in different periods are independent conditional on the state (i.e. $P(s_1, s_2|\omega) = \pi_1(s_1|\omega) \cdot \pi_2(s_2|\omega)$). It is saying that the receiver always requires the sender to perform a new experiment in front of him, instead of bringing him some outcomes from the past experiment, since Receiver 2 only believes in the disclosure from the previous receiver.

In period 1, observing the outcome of the experiment s_1, beliefs are updated from p_1 and p_1^s to q_1 and q_1^s. Then Receiver 1 chooses an action $a_1 \in A = \{yes,\ no,\ hide,\ reveal\}$. If *yes* or *no* is chosen, the game ends and payoffs are realized.

On the other hand, if *hide* or *reveal* is chosen, the game proceeds into the next period and the current receiver's payoff is decided by the action chosen by the future receiver. By revealing, Receiver 1 verifiably discloses the experiment and the outcome in period 1 to the future receiver. And by hiding, no outcome or experiment is disclosed. In period 2, the receiver must decide and only *yes* and *no* can be chosen.

This model can be extended to more than two periods, see Section 4.1.

2.2 Belief updating

Receiver 1's hiding behavior can affect players' belief updating across periods. The sender can observe all experiments, outcomes and Receiver 1's hiding behavior in the history, so his prior in period 2 is equal to his posterior in period 1 ($p_2^s = q_1^s$).

I firstly look at a naive receiver who will assume that no experiment was conducted in the past period if the previous receiver did not disclose the information. He observes only the revealed experiment and outcome in period 1 and will ignore period 1 without the disclosure from Receiver 1, so he may have a different updating from the sender. In other words, he will only believe a previous experiment when it has a verification from a previous receiver and will assume that the experiment is uninformative without the verification. As a result, without disclosed information from the last receiver, Receiver 2's prior will stick to the prior in period 1 ($p_2 = p_1$). And with verifiable disclosure, the receiver's belief is updated in the same way as the sender ($p_2 = q_1$). This naive case is corresponding to the situation where interpreting past information without a disclosure is too costly. As for a sophisticated receiver who will do Bayesian inference seeing no disclosure from the past receiver, I will deal with this case in Section 4.2.

2.3 Players' payoffs

When the game ends (*yes* or *no* is chosen), the current receiver gets a payoff depending on the state and the action: $u : \{yes, no\} \times \Omega \to \mathbb{R}$, which is summarized in Table 1. The previous receiver will have a discounted payoff by the discount factor δ. I assume that two receivers have the same preference u, but different receivers may have different expected payoffs since they may have different beliefs because of information manipulation. I also assume that receivers want to match the good state (g) with *yes* and the bad state (b) with *no*. A successful match benefits receivers but a failed match generates zero payoff, i.e. $a, b > 0$. It can be understood as a policymaker seeking to approve a good project but turn down a bad project. Furthermore, with this preference, receivers' payoffs are always

	yes	*no*
good	a	0
bad	0	b

Table 1: Receivers' payoff

non-negative and thus delay is costly to them due to discount.

As for the sender, I assume a pure persuasion here, which means that the sender's payoff at the end of the game is assumed to be state-independent: $v : \{yes, no\} \to \mathbb{R}$. Without loss of generality, I assume that the sender always prefers *yes*, and normalize the payoff from *no* to 0 and *yes* to 1, i.e. $v(yes) = 1$, $v(no) = 0$.

2.4 Strategies and equilibria

Suppose that we are in period 2,[3] the sender observes the whole history (π_1, s_1). But Receiver 2 only considers the history he believes: $(\pi_1, s_1) \times (\pi_2, s_2)$ if $a_1 = reveal$, and (π_2, s_2) if $a_1 = hide$. In other words, only the experiment verifiably revealed and the experiment performed in front of him are taken into consideration.

The sender's strategy maps the whole history into an experiment π_t, and Receiver t's strategy maps the history in his consideration into A for $t = 1$, and into $\{yes, no\}$ for $t = 2$.

The equilibrium concept in this paper is perfect Bayesian equilibrium: players maximize their expected payoffs given other players' strategies and the beliefs generated by Bayesian rule if possible. Moreover, as tie-breakers, I assume that receivers will choose the action preferred by the sender if they are indifferent between two actions and the sender will choose the least informative experiment when indifferent. The first tie-breaker can ensure the upper hemicontinuity of the sender's utility function and thus the existence of Nash equilibria. The second tie-breaker rules out the multiplicity caused by different experiments inducing the same outcome and makes the equilibrium a limit result of games with costly experiments—where the more informative experiment is more costly—as the cost goes to zero.

[3]This would happen only if the game does not end at period 1, i.e. $a_1 \in \{hide,\ reveal\}$

3 Main Results

3.1 Equilibrium results

Before I introduce the results, I firstly define an important concept following Kamenica and Gentzkow (2011), *value of persuasion*, which describes what a player can get from the persuasion game.

Definition 1 *The value of persuasion to a player is his highest expected payoff in an equilibrium minus his expected payoff without any experiment.*

Based on this definition, I introduce the following useful lemma.

Lemma 1 *In static pure Bayesian persuasion with binary actions, the value of persuasion is zero to the receiver.*

Lemma 1 can be easily got from the results in Kamenica and Gentzkow (2011). This lemma is not only used in the proofs, but also brings a comparison between the model and the benchmark—a receiver can have a positive value of persuasion in my model even with only binary concluding actions.

If the initial prior p_1 is making a receiver choose *yes* rather than *no*, the result is straightforward.

Proposition 1 *If $p_1 \geqslant \frac{b}{a+b}$, there is a unique equilibrium where the game ends in period 1.*

To see this, consider the sender choosing an uninformative experiment in period 1. In this case, Receiver 1 will have the same belief as Receiver 2 if he delays. So Receiver 1's delaying payoff is exactly Receiver 2's expected payoff from the experiment in period 2 with a discount. But in period 2, the situation is the same as a static Bayesian persuasion game and thus the equilibrium gives Receiver 2 no value of persuasion, according to Lemma 1. As a result, Receiver 1's payoff from delay without discount is the same as acting immediately.

So, Receiver 1 will take action immediately. Notice that the action preferred by Receiver 1 at $p_1 \geqslant \frac{b}{a+b}$ is *yes*, which is also the preferred action by the sender, so the sender is able to get his highest possible payoff by choosing an uninformative experiment and will not deviate from that. The equilibrium here is that the sender chooses an uninformative experiment and Receiver 1 chooses *yes* immediately. Trivially, this result can be extended to multiple-period cases.

If the initial prior p_1 is inducing *no* rather than *yes*, the result is more interesting.

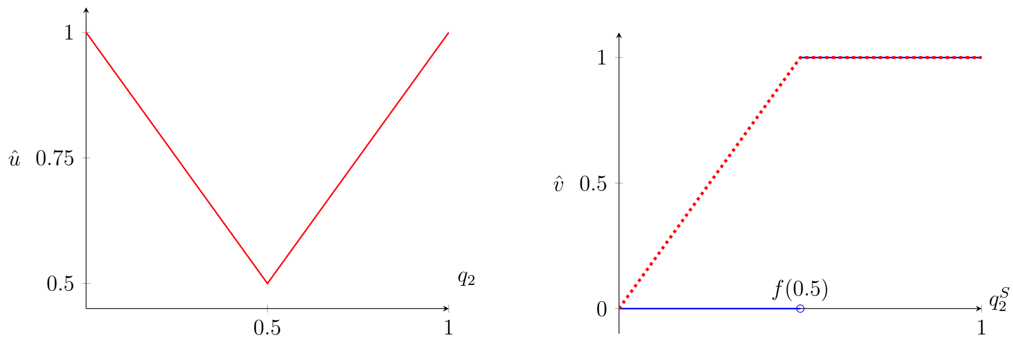
Proposition 2 *For $p_1 < \frac{b}{a+b}$, there exists $\bar{\delta}$ such that:*
If $\delta \geqslant \bar{\delta}$, for cutoff points $0 < \alpha_1(\delta) \leqslant \beta_1(\delta) < \frac{b}{a+b}$, the unique optimal experiment in period 1 induces only immediate actions when $p_1 \notin [\alpha_1(\delta), \beta_1(\delta)]$, and may induce delay when $p_1 \in [\alpha_1(\delta), \beta_1(\delta)]$;

We can solve for the equilibrium by backward induction. In period 2, the situation is exactly the same as a static Bayesian persuasion game since there is no choice of delay for Receiver 2. The payoff functions in period 2 are plotted in Figure 1,[4] and we can easily get the optimal experiment in period 2 by the concave closure method as in Kamenica and Gentzkow (2011). The function $f(.)$ in the graph is the relationship between posteriors: $q_2^s = f(q_2)$. Two posteriors can be different due to the difference between two priors, though posteriors are generated by the same experiment. $f(.)$ is a strictly increasing function according to Alonso and Câmara (2016).

The strategy in period 1 is different from the static case since Receiver 1 can delay—we need to compare Receiver 1's payoffs from delay and immediate actions. Firstly we observe that *reveal* will not be chosen by Receiver 1.

Lemma 2 *The action reveal is always suboptimal.*

[4]Figures in this section is plotted with parameter values $a = b = 1$ and serves the purpose of illustration.



(a) Receiver

(b) Sender

Figure 1: Period 2

If *reveal* is chosen, Receiver 1 and 2 share the same belief, so Receiver 1's payoff is the same as Receiver 2, which is the same as acting immediately due to Lemma 1. Then the discount factor makes *reveal* suboptimal.

As a result, to solve for Receiver 1's strategy, we only need to keep track of action *yes*, *no* and *hide*. The payoffs from *yes* and *no* are easy. If *hide* is chosen, Receiver 1 and 2 can have different expected payoffs from the experiment in period 2 due to different priors. The sender in period 2 will choose a posterior split between 0 and $\frac{b}{a+b}$ for Receiver 2, whose prior is $p_2 = p_1 < \frac{b}{a+b}$ due to hiding. If Receiver 1's belief q_1 is larger than Receiver 2's belief p_2, then the experiment induces a posterior split between 0 and $q > \frac{b}{a+b}$ for Receiver 1. It is a more informative experiment than in the eyes of Receiver 2. Similarly, with $q_1 < p_1$, the experiment in Receiver 1's eyes is less informative than Receiver 2.

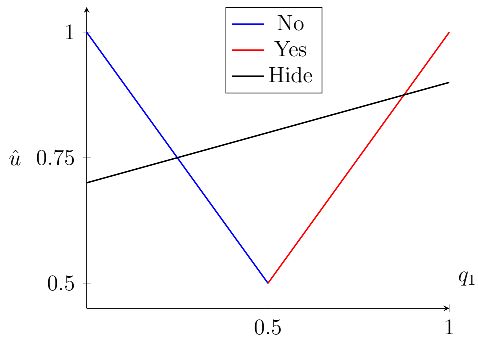
I summarize Receiver 1's strategy under certain p_1 and q_1 in the following lemma.

Lemma 3 *For discount factor $0 < \delta < 1$ and initial belief $p_1 < \frac{b}{a+b}$, there exists $0 < \hat{\delta}(p_1) < 1$:*

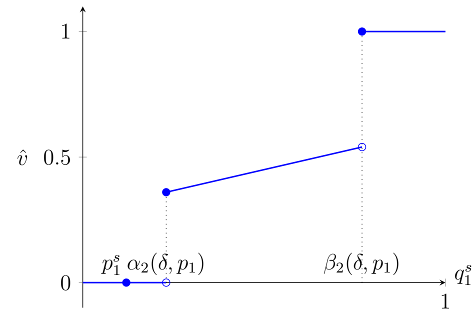
(1) If $\delta > \hat{\delta}(p_1)$, there exist cutoff points $p_1 < \alpha_2(\delta, p_1) < \beta_2(\delta, p_1) < 1$ such that Receiver 1 chooses yes immediately with $q_1 \geqslant \beta_2(\delta, p_1)$; chooses no immediately with $q_1 < \alpha_1(\delta, p_1)$; chooses hide with $\alpha_2(\delta, p_1) \leqslant q_1 < \beta_2(\delta, p_1)$.

(2) If $\delta \leqslant \hat{\delta}(p_1)$, Receiver 1 always acts immediately.

We firstly look at the case with $\delta > \hat{\delta}(p_1)$. Given Receiver 1's strategy, we have the payoff



(a) Receiver



(b) Sender

Figure 2: Period 1

function for the sender and period 1 can be summarized in Figure 2.

From Figure 2 (b), it is easy to see that since we have $p_1^s = p_1 < \alpha_2(\delta, p_1)$, the optimal experiment in period 1 is either a posterior split between 0 and $\alpha_2(\delta, p_1)$ or between 0 and $\beta_2(\delta, p_1)$ in the eyes of the sender and Receiver 1.

I denote the posterior split between 0 and $\alpha_2(\delta, p_1)$ as the *delaying experiment*, since Receiver 1 chooses *hide* at $\alpha_2(\delta, p_1)$, and denote the split between 0 and $\beta_2(\delta, p_1)$ as the *immediate experiment*, since Receiver 1 takes action immediately at both posteriors. Which one is the optimal experiment will depend on the values of the initial priors and the discount factor, which is summarized in Proposition 2.

When $\delta \leqslant \hat{\delta}(p_1)$, there are only immediate actions and the sender just chooses the same experiment as in a static case.

As δ goes up, we will have smaller $\alpha_2(\delta, p_1)$ and larger $\beta_2(\delta, p_1)$, which make Receiver 1 more likely to delay. And this leads to a lower $\alpha_1(\delta)$ and a higher $\beta_1(\delta)$—more priors make the delaying experiment the optimal one. When δ is small enough, no prior can make the delaying experiment optimal.

The intuition is that higher δ makes the cost of delay lower, so that delay is more attractive. With a more attractive option other than immediate actions, Receiver 1 is harder to persuade. As a result, the sender will need a more informative experiment to persuade Receiver 1, which reduces the payoff from the immediate experiment and makes the sender

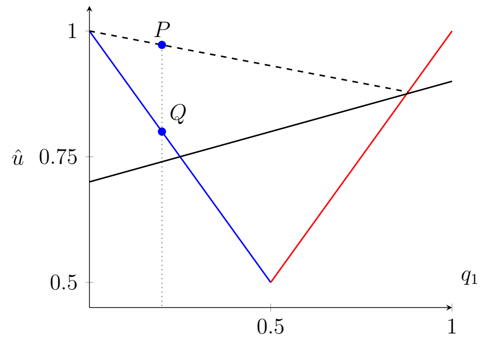


Figure 3: Receiver 1

tend to use the delaying experiment.

3.2 Welfare

For the case with $p_1 \geqslant \frac{b}{a+b}$, in the equilibrium there are an uninformative experiment from the sender and an immediate *yes* from Receiver 1, which is the same as in a static Bayesian persuasion game. So the welfare is also the same.

For the case with $p_1 < \frac{b}{a+b}$, the welfare results are different between the delaying experiment and the immediate experiment.

When the immediate experiment is optimal, Receiver 1 gets a more informative experiment than the static optimal experiment and takes action immediately, so he is better off than the static case, where he has zero value of persuasion. We can easily see this from Figure 3, in which Receiver 1's expected payoff from the immediate experiment is represented by point P.

The delaying experiment is less informative than the static optimal experiment, and Receiver 1's expected payoff from it is represented by the point Q in Figure 3, which is the same as the expected payoff without any experiment. So when the delaying experiment is optimal, Receiver 1 has zero value of persuasion as in the static case.

But for the sender, he is either forced to provide a more informative experiment or is forced to tolerate Receiver 1's delay, and thus is always weakly worse off.

The above is the welfare with both the manipulation power and naive receivers. If we remove the manipulation power, which means that a receiver always reveals the information, the game is simply the repetition of the identical Bayesian persuasions. Then there is no incentive to delay and the game always ends at period 1, leading to the same welfare result as the static case. And in Section 4.2, we will see that with sophisticated receivers who will do Bayesian inference instead of simply assuming no information when they do not observe the disclosure from the past receiver, delay is never an equilibrium choice either.

Considering our application of policymakers and firms, delay only happens when policymakers are able to manipulate the information for naive successors. And they can benefit themselves by this trick at a cost on firms. This trick also threatens the efficiency since it makes delay possible.

3.3 Comparison with benchmark

I compare the model to the benchmark where the receiver has no manipulation power and always discloses the information to see the effect of manipulation power.

In this benchmark, since there is no information manipulation, the game is just the repetition of the same Bayesian persuasions. In the last period, there is no choice of delay and thus players will have the same strategies as in a static persuasion. Recall our Lemma 1, since it is a pure persuasion with binary actions, Receiver 2 gets zero value of persuasion from the sender's optimal experiment. So for Receiver 1, who has the same belief as Receiver 2 due to no manipulation, his payoff from delay is the same as acting immediately at any posterior. And the discount factor further excludes delay from Receiver 1's equilibrium choice. Then backward induction tells us that, even with more than 2 periods, no receiver will delay and the game will end at the very first period, generating the same result as a static persuasion.

Compared with the benchmark, manipulation power brings the incentive for Receiver 1 to delay. With manipulation power and naive receivers, Receiver 1 can have the incentive to delay and hide the information from the following receiver to force the sender to provide a more informative experiment in the next period. Now for the greater good, the receiver may want to delay and manipulate the information to make the future receiver grow.

As discussed, there are two possible situations. In one situation, the sender chooses a less informative experiment in the first period than in the benchmark and Receiver 1 may delay in seek of more information. In the other situation, the sender chooses a more informative experiment than in the benchmark to induce an immediate action in period 1. In the first situation, Receiver 1 gets some extra information in the following period at the cost of less information in period 1, which ends up with the same welfare as the benchmark. When the manipulation power has made the receiver too hard to persuade, he can only ensure the same welfare as no manipulation power.

In the second situation, Receiver 1 gets a more informative experiment than in the benchmark directly in period 1 and thus is better off than the benchmark. When manipulation power increases the difficulty to persuade not too much, Receiver 1 can benefit from it. But for the sender, in either case he is weakly worse off, since in the benchmark he always gets his static optimal payoff in the first period, which is not happening in my model.

4 Extensions

4.1 Multiple periods

The model can be extended to any finite-period case and is still solvable by backward induction. Like the 2-period case, receivers still have incentive to delay and may get positive value of persuasion, though the conditions for different actions and experiments are much more complicated.

By induction, we can get the results for the case with more than 2 periods. The candidate optimal experiments can also be divided into *delaying experiments* and *immediate experiments*, depending on whether delay is possible. But now the cutoff points for the receiver's strategy will be different.

Firstly, *reveal* is not always suboptimal. It is possible that the next receiver will get an immediate experiment without hiding. When this happens, the next receiver gets a positive value of persuasion in the next period. And the current receiver, who shares the same expected payoff with the next receiver, can have a higher payoff than acting immediately.

Furthermore, *yes* can be firstly better than *hide* and then worse than it as q_t goes up. This is because that the optimal experiment in the next period will change from a delaying experiment to an immediate experiment at a point, which causes a jump in the receiver's payoff. I provide an example of the complicated strategy of the receiver in Appendix A2.

And how the discount factor δ affects the choice between the delaying experiment and the immediate experiment in multiple-period cases is also different from the 2-period case, where a higher δ makes the delaying experiment more likely to happen. Here in the multiple-period case, the effect of a higher δ is ambiguous. One effect is that a higher δ makes delay cost lower, which thus makes delay more attractive and makes the receiver harder to persuade. So this effect requires a more informative experiment to persuade the receiver and thus makes the delaying experiment more likely to be optimal. On the other hand, another effect is that a higher δ makes the delaying experiment more likely in the future. But from Section 3.2 we know that the receiver gets a higher payoff from the immediate experiment than from the delaying experiment, so this effect is reducing the payoff from delay and thus is making the receiver easier to persuade. In a 2-period case, we only have the former effect so the overall effect of δ on the delaying experiment is clear. But in multiple-period cases, we have two effects of opposite directions and thus an ambiguous overall effect. An example illustrating this ambiguous effect can be found in Appendix A2.

4.2 Sophisticated receivers

In this section, I look at sophisticated receivers, who will do Bayesian inference seeing no information from a past period, given other players' strategies. With sophisticated receivers, even in the case of multiple periods and multiple states, the current receiver cannot gain informational advantage among following receivers, which removes the incentive for him to delay.

Proposition 3 *With sophisticated receivers, delay never happens in an equilibrium.*

Given an experiment, the current receiver may see different signals while choosing his action. If he hides at only one signal, then following receivers will know for sure what the signal is seeing no information, which makes delay worse than an immediate action. If he hides at multiple signals, the next receiver's belief will be the average of the posteriors induced by hidden signals. Among those posteriors, delay makes the current receiver better off at some posteriors but worse off at others. Since there are always some signals where the current receiver prefers to deviate to an immediate action, hiding at multiple signals cannot be an equilibrium choice either. In conclusion, hiding at only one signal has no effect and hiding at multiple signals always includes points where an immediate action is better.

As we can see, only among naive receivers delay is possible. If the shirking problems we observe from legislators at their term limits are caused by information hiding, there must be a large cost in inferring missing messages or the schedule is too tight for that, since informational advantage from hiding can only be gained from naive receivers.

4.3 Multiple states

Similar results as Proposition 1 and 2 hold in the model with more than two states, as long as receivers' utility function satisfies no-dominant-action requirement. Let $a^*(q)$ be the action preferred by receivers among *yes* and *no* given belief q, we have the following proposition.

Proposition 4 *When* $\Omega = \{\omega_1, \omega_2, ..., \omega_M\}$ *and* $p_t \in \Delta(\Omega)$, *if receivers are naive:*
(1) If $a^*(p_1)$ *is preferred by the sender, there is a unique equilibrium where the game ends in the first period.*

(2) If $a^*(p_1)$ is not preferred by the sender, there is a unique equilibrium. Whether there is delay in equilibrium depends on the values of δ and $p_1 = p_1^s$.

The result (1) is still straightforward, the sender can guarantee a sender-preferred action immediately by providing an uninformative experiment if the initial belief has already ensured the action preferred by the sender from Receiver 1. If the sender has to provide an informative experiment in period 1 in pursuit of a higher payoff, he needs to trade off between more informative experiments that induce immediate actions and less informative experiments that induce delay, which is similar to the binary-state case. More complicated qualities of the project just lead to a more complicated representation of the cutoff points.

The result in Proposition 3 also holds unchanged with more than two states. The Bayesian inference done by the next receiver will remove the information advantage from manipulation.

Proposition 5 *When $\Omega = \{\omega_1, \omega_2, ..., \omega_M\}$ and $p_t \in \Delta(\Omega)$, if receivers are sophisticated, there is no delay in an equilibrium.*

5 Conclusion

I study a model of a sender seeking to persuade a series of manipulative receivers to take an action before a deadline. The optimal action for receivers depends on the state while the sender has a state-independent preference. The receiver can either choose the action by himself or leave the decision to his successor and take the chance to hide his information. If receivers are naive in the sense that they will assume no experiment was conducted in a past period when the previous receiver hides the information, delay with hiding can be better than immediate actions for the receiver and thus delay is possible in an equilibrium. And with naive receivers, the receiver may get a positive value of persuasion in the equilibrium, in contrast with the benchmark case without manipulation power where the receiver always gets zero value of persuasion from the sender's optimal experiment. Not like receivers, the sender is always weakly worse off from receivers' ability to delay and hide the information.

As for sophisticated receivers who do Bayesian inference seeing no information from a past period given other players' strategies, they are not able to gain informational advantage by hiding and thus have no incentive to delay. So the equilibrium in this case is the same as the static persuasion.

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A Mathematical Appendix

A.1 2-period case

A.1.1 Proof for Proposition 1

In period 2, the game is a static Bayesian persuasion, so by the result in Kamenica and Gentzkow (2011), if $p_2 \geqslant \frac{b}{a+b}$, the sender chooses the uninformative experiment; if $p_2 < \frac{b}{a+b}$, the optimal experiment is the posterior split of 0 and $\frac{b}{a+b}$ for Receiver 2.

Back to period 1, Receiver 1 will choose *yes* immediately with posterior $q_1 \geqslant \frac{b}{a+b}$, since the sender will choose uninformative experiment in period 2 if he delays, no matter whether he hides the information or not. With $q_1 < \frac{b}{a+b}$, Receiver 1 will choose *no* immediately. This is because that if he delays, he prefers to reveal the information, then his payoff from delay without discount is the same as acting immediately because of Lemma 1 and the fact that he has the same belief as Receiver 2. As a result, the discount factor ensures that choosing *no* immediately is the only optimal action. Knowing Receiver 1's strategy, we know that the sender is indifferent among posterior splits among $q_1 \in [\frac{b}{a+b}, 1]$ and will not choose a split involving $q_1 < \frac{b}{a+b}$. Finally, according to our tie-breaker, the sender chooses uninformative experiment in period 1 and Receiver 1 chooses *yes* immediately, which is the unique equilibrium here.

A.1.2 Proof for Lemma 2

If Receiver 1 chooses to reveal the information, his belief will be the same as Receiver 2 and thus he will have the same expected payoff as Receiver 2 from delay. Easily, his expected payoff from *reveal* without discount is $a \cdot q_1$ if $q_1 \geqslant \frac{b}{a+b}$, and $a \cdot q_2 \cdot \frac{q_1}{q_2} + b \cdot (1 - \frac{q_1}{q_2}) = b \cdot (1 - q_1)$ if $q_1 < \frac{b}{a+b}$. This payoff without discount is exactly the same as Receiver 1's payoff from taking action immediately, so the discount factor δ makes *reveal* suboptimal.

A.1.3 Proof for Lemma 3

We firstly calculate Receiver 1's payoff from *hide* when the prior and the posterior are p_1 and q_1.

According to Alonso and Câmara (2016), if two players with different priors u_1 and u_2 see the same experiment, their posteriors have the following relationship:

$$u_1^p = \frac{u_2^p \frac{u_1}{u_2}}{u_2^p \frac{u_1}{u_2} + (1-u_2^p)\frac{1-u_1}{1-u_2}}$$

Given hiding, when facing the experiment in period 2, Receiver 1 has belief q_1 but Receiver 2 has belief p_1. Notice that the sender's optimal experiment in period 2 is the split of 0 and $q_2 = \frac{b}{a+b}$ for Receiver 2, so in Receiver 1's eyes, the split is 0 and $q_2' = \frac{q_2\frac{q_1}{p_1}}{q_2\frac{q_1}{p_1}+(1-q_2)\frac{1-q_1}{1-p_1}} > q_2$. Moreover, the split in Receiver 1's eyes is still Bayesian plausible, so the probability for q_2' is $\frac{q_2\frac{q_1}{p_1}+(1-q_2)\frac{1-q_1}{1-p_1}}{\frac{q_2}{p_1}}$. Since Receiver 2 chooses *no* at 0 and *yes* at q_2, Receiver 1 gets $\frac{a\cdot q_2\frac{q_1}{p_1}+b\cdot(1-q_2)\frac{1-q_1}{1-p_1}}{q_2\frac{q_1}{p_1}+(1-q_2)\frac{1-q_1}{1-p_1}}$ by probability $\frac{q_2\frac{q_1}{p_1}+(1-q_2)\frac{1-q_1}{1-p_1}}{\frac{q_2}{p_1}}$, and gets b by probability $1-\frac{q_2\frac{q_1}{p_1}+(1-q_2)\frac{1-q_1}{1-p_1}}{\frac{q_2}{p_1}}$. So, Receiver 1's expected payoff from delay with hiding is $b+\frac{(a-b)\cdot q_2\frac{q_1}{p_1}-b\cdot(1-q_2)\frac{1-q_1}{1-p_1}}{\frac{q_2}{p_1}} = b+(a-b)q_1 - a\frac{p_1}{1-p_1}(1-q_1)$:

$$\mathbf{E}u(hide) = \delta\cdot(b+(a-b)q_1 - a\frac{p_1}{1-p_1}(1-q_1))$$

As for acting immediately, Receiver 1's expected payoff is:

$$\mathbf{E}u(act) = \begin{cases} b\cdot(1-q_1) & ,q_1 < \dfrac{b}{a+b} \\ a\cdot q_1 & ,q_1 \geqslant \dfrac{b}{a+b} \end{cases}$$

Comparing them, we can get that when $\delta > \frac{ab}{2ab-\frac{p_1}{1-p_1}a^2}$, there are $\alpha_2(\delta,p_1) = \frac{b(1-\delta)-p_1(b(1-\delta)-\delta a)}{\delta a+b(1-\delta)-b(1-\delta)p_1}$ and $\beta_2(\delta,p_1) = \frac{b\delta-\delta(a+b)p_1}{a(1-\delta)+b\delta-(a+b\delta)p_1}$, where $p_1 < \alpha_2(\delta,p_1) < \beta_2(\delta,p_1) < 1$, such that Receiver 1 chooses *yes* with $q_1 \geqslant \beta_2(\delta,p_1)$; chooses *no* with $q_1 < \alpha_2(\delta,p_1)$; chooses *hide* in between.

Otherwise, Receiver 1 chooses *yes* with $q_1 \geqslant \frac{b}{a+b}$ and *no* with $q_1 < \frac{b}{a+b}$.

A.1.4 Proof for Proposition 2

Knowing Receiver 1's strategy from Lemma 3, we can figure out the sender's optimal experiment in period 1.

When $\delta \leqslant \frac{ab}{2ab - \frac{p_1}{1-p_1}a^2}$, Receiver 1 chooses *yes* with $q_1 \geqslant \frac{b}{a+b}$ and *no* with $q_1 < \frac{b}{a+b}$. Since $p_1 < \frac{b}{a+b}$, the optimal experiment is clearly the split between 0 and $\frac{b}{a+b}$ in Receiver 1's eyes and this experiment induces only immediate actions.

When $\delta > \frac{ab}{2ab - \frac{p_1}{1-p_1}a^2}$, we have two cutoff points in Receiver 1's strategy $\alpha_2(\delta, p_1)$, $\beta(\delta, p_1)$. Since we have $p_1 < \alpha_2(\delta, p_1) < \beta_2(\delta, p_1)$, the optimal experiment is either the split between 0 and $\alpha_2(\delta, p_1)$ or between 0 and $\beta_2(\delta, p_1)$, where the former one induces *hide* at $\alpha_2(\delta, p_1)$ and the latter one only induces immediate actions. So, they are called the *delaying experiment* and the *immediate experiment* respectively. For the sender, his expected payoff from the immediate experiment is $\frac{p_1}{\beta_2(\delta, p_1)}$. As for the delaying experiment, at 0 he gets 0, at $\alpha_2(\delta, p_1)$ the game proceeds into the next period and in period 2 he faces with an experiment making Receiver 2 whose prior is p_1 believe the split is 0 and $\frac{b}{a+b}$, so his expected payoff from the delaying experiment is $\delta \cdot (p_1 + \frac{a}{b}\frac{p_1}{1-p_1}\frac{1-\alpha_2(\delta,p_1)}{\alpha_2(\delta,p_1)}p_1)$.

So the delaying experiment is the optimal one if and only if:[5]

$$\frac{a}{b} \cdot \frac{\delta^2 a p_1}{b(1-\delta) - p_1(b(1-\delta) - \delta a)} - \frac{a(1-\delta)(1-p_1)}{b\delta - \delta \cdot p_1(a+b)} \geqslant 1 - \delta$$

$$\Leftrightarrow r_1 p_1^2 + r_2 p_1 + r_3 \geqslant 0 \tag{1}$$

Where:

$$r_1 = -\frac{\delta^3 a^2 (a+b)}{b} - a(1-\delta)(b(1-\delta) - \delta a) \\ -\delta(1-\delta)(b(1-\delta) - \delta a)(a+b)$$

$$r_2 = \delta^3 a^2 + a(1-\delta)(b(1-\delta) - \delta a) + ab(1-\delta)^2 \\ + b\delta(1-\delta)^2(a+b) + (1-\delta)\delta b(b(1-\delta) - \delta a)$$

[5]When indifferent between two experiments, the tie-breaker makes the sender choose the delaying experiment, which is less informative.

$$r_3 = -ab(1-\delta)^2 - b^2(1-\delta)^2\delta$$

Notice that for $p_1 = 0$ and $p_1 = \frac{b}{a+b}$, LHS of (A1) is always negative for $\delta \in (0,1)$. Moreover, when $\delta = 1$, we have $r_1 < 0$ and that the solution to (A1) is $p_1 \in [0, \frac{b}{a+b}]$. So, when δ is close enough to 1, the solution to (A1) would be $p_1 \in [\alpha_1(\delta), \beta_1(\delta)]$, where $0 < \alpha_1(\delta) \leqslant \beta_1(\delta) < \frac{b}{a+b}$.

A.2 Multiple-period case

The multiple-period case can be solved by the same method as in the 2-period case, with further backward induction.

Firstly consider the case where $p_1 \geqslant \frac{b}{a+b}$. According to the proof for Proposition 1, backward induction gives us that the game ends at the first period.

As for the case with $p_1 < \frac{b}{a+b}$, period N and period $N-1$ act in the same way as in the 2-period case.

The situation in period $t < N-1$ is more complicated. Firstly, if $p_t \geqslant \frac{b}{a+b}$, as proved in Lemma 2, the game ends at period t with an uninformative experiment. When $p_t < \frac{b}{a+b}$, we need to compare the payoffs from *yes*, *no*, *hide* and *reveal*.

yes and *no* always give us $a \cdot q_t$ and $b \cdot (1-q_1)$ respectively.

Similar to the 2-period case, if *reveal* is chosen, Receiver t and Receiver $t+1$ share the same belief and the same expected payoff from an experiment. If *hide* is chosen, Receiver t and $t+1$ can have a different expected payoff due to different belief.

The calculation method is the same as in a 2-period case, but now the payoff in the next period involves immediate experiments and delaying experiments. According to Section 2.2, there is a jump in receivers' payoff when shifting from a delaying experiment to an immediate experiment, and immediate experiments provide positive value of persuasion. This will make the conditions of different optimal actions different from the 2-period case and more complicated. Now no hiding may be better than hiding due to different types of

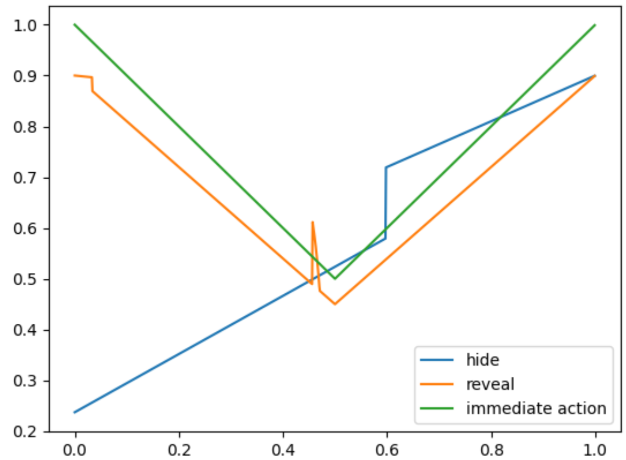


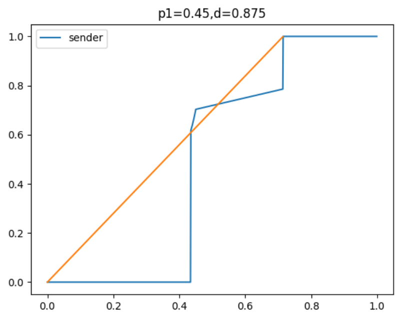
Figure 4: Receiver 1 in a 3-period example

optimal experiments in the next period. And the jump in the receiver's payoff can lead to back and forth.

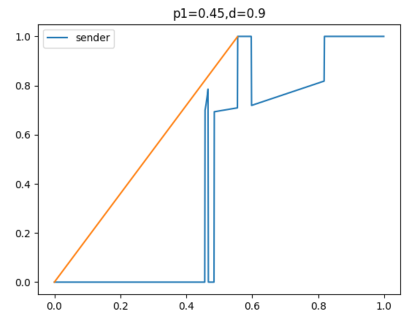
As an example, look at Figure 4, which summarizes Receiver 1's payoffs from different actions in a 3-period case with $a = b = 1$, $p_1 = p_1^s = 0.45$ and $\delta = 0.9$. In this example, Receiver 1's optimal action is $no \rightarrow reveal \rightarrow no \rightarrow hide \rightarrow yes \rightarrow hide \rightarrow yes$, as q_1 goes up.

Such complicated choices of strategies of receivers naturally lead to different criteria in choosing the optimal experiment. And the effect of the discount factor on choosing the optimal experiment is ambiguous as discussed in the main text.

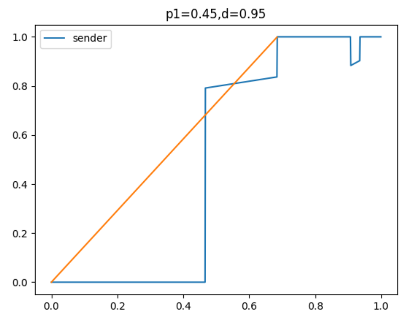
For example, in a 3-period case with $a = b = 1$ and $p_1 = p_1^s = 0.45$, $\delta = 0.875$ or 0.95 makes the optimal experiment a delaying experiment, but $\delta = 0.9$, which is in between, makes the optimal experiment an immediate experiment. See Figure 5 describing the sender's payoff in these three cases. The orange lines in (a) and (c) suggest that the immediate experiment is not the optimal one. And the orange line in (b) suggests that the sender will choose the immediate experiment.



(a) $\delta = 0.8$



(b) $\delta = 0.82$



(c) $\delta = 0.9$

Figure 5: Ambiguous effect of δ

A.3 Sophisticated receivers

A.3.1 Proof for Proposition 3

I will directly prove this proposition under the case with M states and N periods, i.e. $\Omega = \{\omega_1, \omega_2, ..., \omega_M\}$.

By backward induction, we start from period N, which is simply a static persuasion.

As for period $N-1$, I will show that under any experiment, Receiver $N-1$ will not delay in an equilibrium.

Suppose Receiver $N-1$'s possible posteriors are $\mu_1, \mu_2, ..., \mu_G$ with probabilities $P_1, P_2, ..., P_G$.

If Receiver $N-1$ delays without hiding or delays and hides at only one outcome, Receiver N knows exactly what the signal is and has the same belief as Receiver $N-1$'s posterior. Now Receiver $N-1$ has the same expected payoff as Receiver N, but according to Lemma 1, Receiver N has no value of persuasion from the equilibrium and thus has the same expected payoff as no information. As a result, Receiver $N-1$'s payoff from delay is the same as acting immediately, and then the discount factor makes the deviation to an immediate action profitable. So this situation cannot happen in an equilibrium.

If Receiver $N-1$ delays at multiple outcomes, say $\mu_1, \mu_2, ..., \mu_I$ $(I \leqslant G)$. Now Receiver N does not know the signal but the sender knows the signal, so the problem becomes the Bayesian persuasion with an informed sender. Because there is only two possible actions and the sender has access to the fully revealing experiment, the sender's ex-ante optimal experiment is interim optimal. As a result, we can have the sender choose the optimal experiment as if he is not informed. Since now the sender pools the same experiment for all signals hidden, Receiver N cannot know what the signal is from the experiment, and thus his belief seeing no signal is the weighted average $\bar{\mu} = \frac{\sum_{i=1}^{g} P_i \mu_i}{\sum_{i=1}^{g} P_i}$.

If $\bar{\mu}$ can directly induce *yes*, which is preferred by the sender, the sender chooses an uninformative experiment in period N, so clearly deviating to an immediate action can benefit Receiver $N-1$.

If $\bar{\mu}$ induces *no*, the sender chooses an experiment inducing belief q'_1 and q'_2 from prior $\bar{\mu}$ with probabilities P'_1 and P'_2, since there are only two available actions. I also denote the actions induced by q'_1 and q'_2 as a'_1 and a'_2 respectively. According to Alonso and Câmara (2016), Receiver $N-1$ who has a prior μ_i ($i \leqslant I$) in period N will regard posterior q'_j as q'_{ij}, where $q'_{ij}(\omega_0) = \frac{q'_j(\omega_0)\frac{\mu_i(\omega_0)}{\bar{\mu}(\omega_0)}}{\sum_{\omega\in\Omega} q'_j(\omega)\frac{\mu_i(\omega)}{\bar{\mu}(\omega)}}$, and regards P'_j as $P'_{ij} = P'_j \sum_{\omega\in\Omega} q'_j(\omega)\frac{\mu_i(\omega)}{\bar{\mu}(\omega)}$. So the payoff from delay for Receiver $N-1$ with posterior μ_i is $\delta u^i = \delta \sum_{j=1}^{2} P'_{ij} \sum_{\omega\in\Omega} u(a'_j,\omega) q'_{ij}(\omega)$.

Now we have:

$$\begin{aligned}
\frac{\sum_{i=1}^{I} P_i u^i}{\sum_{i=1}^{I} P_i} &= \frac{1}{\sum_{i=1}^{I} P_i} \sum_{j=1}^{2}\sum_{i=1}^{I} P_i P'_{ij} \sum_{\omega\in\Omega} u(a'_j,\omega) q'_{ij}(\omega) \\
&= \frac{1}{\sum_{i=1}^{I} P_i} \sum_{j=1}^{2}\sum_{i=1}^{I} P_i (P'_j \sum_{\omega\in\Omega} q'_j(\omega)\frac{\mu_i(\omega)}{\bar{\mu}(\omega)}) \sum_{\omega\in\Omega} u(a'_j,\omega) q'_{ij}(\omega) \\
&= \sum_{j=1}^{2} P'_j \sum_{i=1}^{I} (\sum_{\omega\in\Omega} q'_j(\omega)\frac{\mu_i(\omega)}{\bar{\mu}(\omega)} \frac{P_i}{\sum_{k=1}^{I} P_k}) \sum_{\omega\in\Omega} u(a'_j,\omega) q'_{ij}(\omega) \\
&= \sum_{j=1}^{2} P'_j \sum_{\omega\in\Omega} u(a'_j,\omega) \sum_{i=1}^{I} (\sum_{\omega\in\Omega} q'_j(\omega)\frac{\mu_i(\omega)}{\bar{\mu}(\omega)} \frac{P_i}{\sum_{k=1}^{I} P_k}) q'_{ij}(\omega) \\
&= \sum_{j=1}^{2} P'_j \sum_{\omega\in\Omega} u(a'_j,\omega) \sum_{i=1}^{I} (\sum_{\omega\in\Omega} q'_j(\omega)\frac{\mu_i(\omega)}{\bar{\mu}(\omega)} \frac{P_i}{\sum_{k=1}^{I} P_k}) \frac{q'_j(\omega)\frac{\mu_i(\omega)}{\bar{\mu}(\omega)}}{\sum_{\omega'\in\Omega} q'_j(\omega')\frac{\mu_i(\omega')}{\bar{\mu}(\omega')}} \\
&= \sum_{j=1}^{2} P'_j \sum_{\omega\in\Omega} u(a'_j,\omega) q'_j(\omega)
\end{aligned}$$

Notice that $\sum_{j=1}^{2} P'_j \sum_{\omega\in\Omega} u(a'_j,\omega) q'_j(\omega)$ is the expected payoff of Receiver N from the optimal experiment of the sender, so we have:

$$\frac{\sum_{i=1}^{I} P_i u^i}{\sum_{i=1}^{I} P_i} = \sum_{j=1}^{2} P'_j \sum_{\omega\in\Omega} u(a'_j,\omega) q'_j(\omega) := \hat{u}(\bar{\mu})$$

Since period N is a static pure persuasion with binary actions, we have $\hat{u}(\bar{\mu}) = \max_a \sum_{\omega\in\Omega} u(a,\omega)\bar{\mu}(\omega)$, which means that $\hat{u}$ is convex. As a result, we have:

$$\frac{\sum_{i=1}^{I} P_i u^i}{\sum_{i=1}^{I} P_i} \leqslant \sum_{i=1}^{I} \frac{P_i \max_a \sum_{\omega\in\Omega} u(a,\omega)\mu_i(\omega)}{\sum_{k=1}^{I} P_k} \tag{2}$$

From (A2), we can conclude that:

$$\exists\ i \leqslant I, u^i \leqslant \max_a \sum_{\omega \in \Omega} u(a, \omega)\mu_i(\omega) \Rightarrow \delta u^i < \max_a \sum_{\omega \in \Omega} u(a, \omega)\mu_i(\omega)$$

So, there must exist one hidden posterior where Receiver $N-1$ can benefit from deviating to an immediate action. And thus hiding at multiple posteriors cannot happen in an equilibrium either.

Then we can conclude that in an equilibrium, there is no delay in period $N-1$.

And we can construct an equilibrium in period $N-1$ where the sender chooses the static optimal experiment and Receiver $N-1$ always takes action immediately. For an static optimal experiment, there must be a posterior inducing the action preferred by the sender, the off-path belief of Receiver N seeing delay with hiding is assigned to this posterior[6]. If Receiver $N-1$ deviates to delay and hide the signal, since the off-path belief of Receiver N is assigned to the posterior inducing the action preferred by the sender, in period N the sender chooses an uninformative experiment and thus this deviation will not be profitable. If Receiver $N-1$ deviates to delay without hiding, by the fact that there is no value of persuasion in the next period, this deviation is not profitable either. As for the sender, if he deviates to another experiment, at any posterior of that experiment, Receiver $N-1$'s delay with hiding will be worse than an immediate action since the off-path belief makes the sender choose an uninformative experiment in the next period, and his delay without hiding is worse than an immediate action as well due to no value of persuasion in the next period. As a result, Receiver $N-1$ will choose an immediate action seeing any outcome of this deviation experiment, and thus this deviation will not give the sender a higher payoff than static optimal. Also notice that since there will be no delay in an equilibrium, only the static optimal experiment can be the equilibrium choice of the sender in period $N-1$.

So we now have that the strategies in period $N-1$ will be the same as in period N, then

[6]Since delay without hiding verifiably reveals the information to Receiver N, that off-path belief of Receiver N is already decided.

further backward induction tells us that there is no delay in an equilibrium.

A.4 Multiple states

A.4.1 Proof for Proposition 4

(1) Firstly let us consider the case where $a^*(p_1)$ is preferred by the sender.

In period 2, players' strategies are just the same as in a static persuasion.

In period 1, when Receiver 1 delays and hides the information, Receiver 2 will has a prior $p_2 = p_1$. Notice that $a^*(p_1)$ is preferred by the sender, so the sender will choose an uninformative experiment in period 2, which makes delay with hiding worse than taking action immediately. When Receiver 1 delays without hiding the information, he has the same belief as Receiver 2 and thus the same expected payoff as Receiver 2. According to Lemma 1, Receiver 2's expected payoff from persuasion is the same as the expected payoff without any information, so Receiver 1's payoff from delay without hiding is also worse than taking action immediately.

So we can see Receiver 1 will choose an immediate action, and thus the sender is indifferent between splits among posteriors that can induce *yes*. Then our tie-breaker will make the sender choose an uninformative experiment.

(2) Then suppose $a^*(p_1)$ is not preferred by the sender.

Again, in period 2, players' strategies are the same as in a static persuasion.

In period 1, if Receiver 1 has a posterior q_1 and chooses *yes* or *no*, his expected payoffs are $u^y(q_1) = \sum_{\omega\in\Omega} u(yes,\omega)q_1(\omega)$ and $u^n(q_1) = \sum_{\omega\in\Omega} u(no,\omega)q_1(\omega)$ respectively.

If Receiver 1 chooses *reveal*, he has the same belief and the same expected payoff as Receiver 2, and thus his payoff will be
$\delta \max_{a\in\{yes,no\}} \sum_{\omega\in\Omega} u(a,\omega)q_1(\omega)$ according to Lemma 1. Clearly, delay without hiding will not be an equilibrium choice of Receiver 1, since there is always an immediate action better than it.

If Receiver 1 chooses to delay with hiding, Receiver 2 has a prior $p_2 = p_1$. Since $a^*(p_1)$

is not preferred by the sender, so the sender will choose an informative experiment. Since there are only 2 actions in period 2, the optimal experiment can be the split of q^n and q^y with probabilities P^n and P^y respectively in the eyes of Receiver 2, where $a^*(q^n) = no$ and $a^*(q^y) = yes$.[7] Moreover, by Bayesian plausibility, we have $P^n q^n + P^y q^y = p_1$. In Receiver 1's eyes, the split is $q_1^n(\omega_0) = \frac{q^n(\omega_0)\frac{q_1(\omega_0)}{p_1(\omega_0)}}{\sum_{\omega\in\Omega} q^n(\omega)\frac{q_1(\omega)}{p_1(\omega)}}$ and $q_1^y(\omega_0) = \frac{q^y(\omega_0)\frac{q_1(\omega_0)}{p_1(\omega_0)}}{\sum_{\omega\in\Omega} q^y(\omega)\frac{q_1(\omega)}{p_1(\omega)}}$ with probabilities $P_1^n = P^n \sum_{\omega\in\Omega} q^n(\omega)\frac{q_1(\omega)}{p_1(\omega)}$ and $P_1^y = P^y \sum_{\omega\in\Omega} q^y(\omega)\frac{q_1(\omega)}{p_1(\omega)}$. Then Receiver 1's payoff from delay with hiding is:

$$\begin{aligned}\delta u^h(q_1) =& \delta(P_1^n \sum_{\omega\in\Omega} u(no,\omega) q_1^n(\omega) + P_1^y \sum_{\omega\in\Omega} u(yes,\omega) q_1^y(\omega))\\ =& \delta(P^n \sum_{\omega\in\Omega} u(no,\omega) q^n(\omega)\frac{q_1(\omega)}{p_1(\omega)} + P^y \sum_{\omega\in\Omega} u(yes,\omega) q^y(\omega)\frac{q_1(\omega)}{p_1(\omega)})\end{aligned}$$

Which action Receiver 1 will choose at q_1 depends on $u^n(q_1)$, $u^y(q_1)$ and $\delta u^h(q_1)$, and the tie-breaker makes Receiver 1 choose only one action.

Notice that at $q_1 = p_1$, $u^h(q_1) = u^n(q_1) > u^y(q_1)$.

If q^y has put some weight on states where *yes* is strictly preferred. Then we can pick a q_1 which only puts positive weight on those states, and thus have $u^n(q_1) < u^h(q_1) \leqslant u^y(q_1)$. So, when δ is large, there exist cutoff points where *yes* and *hide* are indifferent and cutoff points where *no* and *hide* are indifferent. And of course, there are cutoff points where *yes* and *no* are indifferent. It is easy to see that the optimal experiment by the sender will include one of these three kinds of cutoff points, so the optimal experiment can either induce delay or not[8], depending on parameters (δ, p_1 and receivers' preference u). Also, the tie-breaker makes the set of optimal experiments a singleton.

If q^y only put positive weight on 'safe' states where *yes* and *no* are indifferent, for any q_1, we will have $u^h(q_1) = u^n(q_1)$. Then $\delta < 1$ makes *hide* suboptimal.

[7] Since it is an optimal experiment, *yes* and *no* are indifferent at belief q^y, otherwise we can shift two posteriors a little to make P^y larger while q^y still inducing *yes*.

[8] If the optimal experiment includes a cutoff point where *no* and *hide* are indifferent, then delay is possible. Otherwise, there is no delay in equilibrium.

A.4.2 Proof for Proposition 5

See the proof for Proposition 3.