Abstract

Recently, the food delivery platforms such as Uber Eat, Doordash, and Grubhub are becoming more and more popular. In this paper, we investigate competition in the market for food delivery platforms. The paper models this competition in a modified Hotelling setting. We characterize the symmetric equilibrium market structure (either local monopolistic or oligopolistic) and prices. Additionally, we find that the degree of the differentiation affects whether an equilibrium exists. Specifically, if the transportation cost of the Hotelling model is relatively small or large, then a symmetric pure strategy Nash equilibrium exists, whereas for transportation in an intermediate region, a pure strategy Nash equilibrium does not exist.

Keywords: Two-sided Market, Network Externalities

JEL Codes: D43, L11, L13, L87
1 Introduction

With the rapid development of online platform technologies, firms in more industries have adopted these technologies to organize transactions among buyers and sellers. One example is the food delivery market. Morgan Stanley’s research shows that the size of food delivery market was valued at over $350 billion in 2020. On one side of the market, food delivery platforms give customers access to large networks of restaurants. On the other side, platforms give restaurants a greater ability to connect with a larger number of customers. The degree to which restaurants and consumers benefit from these platforms, depends on the prices set by platforms for the two sides of the market. In the food delivery industry, the platforms create network-based two-sided market structure to connect restaurants and consumers. We have witnessed that the prices of these services have been soaring during the past few years. According to a report from The Mercury News, the commission on the seller side ranges from 13.5% to 40% and the service fee on the buyer side is around 15%. Comparing to some other two-sided market industry such as taxi service and credit card, the platforms in the food delivery market really charge a high price. This paper will mainly focus on how will the platform’s price competition in the food delivery industry.

To examine pricing in two-sided markets, Rochet and Tirole (2003) characterize platform optimal prices for each side of a two-sided market in a modified Hotelling model. Their model is innovative in that it characterizes the prices set by platforms for the two sides of the market. These prices depend in part on network externalities created by each side, with the possibility that one side the market subsidizes the other. Armstrong (2006) also uses a modified Hotelling model, in this case in which the “transportation cost” measures the degree of differentiation on the consumer side of the market between platforms. This specification could generalize to both sides of the market. However, we believe that in the food delivery market, only consumers have preferences for platform attributes beyond the number of market participants on a platform. Since the Rochet and Tirole (2003) and Armstrong (2006) seminal papers, numerous papers have modified the Hotelling modeling
to investigate two-sided markets.

Table 1: Literature of Multi-sided Markets Using Hotelling Specification

<table>
<thead>
<tr>
<th>Year</th>
<th>Author</th>
<th>Topic or Industry</th>
</tr>
</thead>
<tbody>
<tr>
<td>2006</td>
<td>Ulrich Kaiser, Julian Wright</td>
<td>Magazine industry</td>
</tr>
<tr>
<td>2007</td>
<td>Andrea Amelio, Bruno Jullien</td>
<td>Bundle pricing strategy</td>
</tr>
<tr>
<td>2012</td>
<td>Nicholas Economides, Joacim Tåg</td>
<td>Network neutrality regulation of the Internet</td>
</tr>
<tr>
<td>2015</td>
<td>Andrei Hagiu, Julian Wright</td>
<td>Tradeoff between multisided market and traditional alternatives.</td>
</tr>
<tr>
<td>2017</td>
<td>Shiyi Wang, Huimiao Chen, Desheng Wu</td>
<td>Taxi-hailing market</td>
</tr>
<tr>
<td>2018</td>
<td>Z.Feng, T.Liu, V.V.Mazalov, J.Zheng</td>
<td>Pricing strategy with heterogeneous agents and limited market size</td>
</tr>
<tr>
<td>2018</td>
<td>Paul Belleflamme, Martin Peitz</td>
<td>Multihoming behavior</td>
</tr>
</tbody>
</table>

Table 1 lists papers that use Armstrong’s modified-Hotelling model of two-sided markets to investigate different industries. However, this model cannot directly applied to the food delivery market because platform membership fees are central to Armstrong’s analysis and transaction volumes per buyer and seller are not included in the Armstrong model. In contrast, food delivery platforms set transaction fees for both restaurants and consumers. These per-transaction fees are the primary, if not the only, revenue sources for the platforms. Additionally, the number of transactions per consumer might be increasing in the number of restaurants and the number of transactions per restaurant might be increasing in the number of consumers as well. Our model incorporates transactions and platforms set per-transaction fees. Furthermore, in our model, consumer preferences per transaction varies
across consumers. For example, when buyers use the platform to order takeout meals, every time they use the platform, they receive a net benefit.

This paper differs from the existing literature in the following ways. First, this paper has a new model for the food delivery market based on Armstrong(2006) and Rochet and Tirole(2003)'s model. This model give us the possibility to analyze the network externalities from sellers to buyers. Second, under our special model, we define the pricing region of competition in the duopoly market. Lastly, this paper discuss the existence of the pure strategy Nash-equilibrium at different level of transportation cost for buyers.

We begin our analysis with a monopoly model, demonstrating a similar result to the Rochet and Tirole(2003) monopoly result. Next, in an environment with two platforms, we examine conditions under which the platforms in equilibrium compete for buyers and conditions in which the platforms do not. We find that, when the degree of platform differentiation is relatively small, resulting in platform competition, or large, resulting in local monopolies, then a symmetric pure strategy Nash equilibrium exists, whereas for the differentiation in an intermediate region, a pure strategy Nash equilibrium does not exist.

2 Model

Consider a food delivery market with two groups: buyers (B) and seller (S). Both groups can choose to interact with the other group through a platform or interact directly. If a buyer chooses to use the platform, her benefit is \( b^B \). If a seller chooses to use the platform, his benefit is \( b^S \). We assume that each buyer’s benefit \( b^B \) is idiosyncratic and with a unit mass of buyer, \( b^B \) is uniformly distributed from \([0, \alpha]\). We also assume that the seller’s benefit \( b^S \) is idiosyncratic and with a unit mass of seller, \( b^S \) is uniformly distributed from \([0, \beta]\). The benefits here are in percentage. For example, for every order, the buyer or seller will enjoy \( b^k \times 100\% \) from that order. On the side of buyers, the benefit to order food delivery are heterogeneous. The reasons could be the different locations to the restaurants or the time
cost for cooking at home. On the side of sellers, the benefit to the restaurant can be different based on the different type of restaurant. For example, the fast food restaurants prefer food delivery more than the fine dinning restaurant.

Assume that $p^S$ and $p^B$ are the proportional priced charged by the platform to the sellers and buyers. This proportional price on the buyer’s side is the service fee on the meal (a certain percentage). On the seller’s side the proportional price is the percentage commission charged to the restaurants. The net benefit per dollar transaction for a participant in group $k$ is $b^k - p^k$. According to Rochet and Tirole (2003)’s specification, this net benefit increase as the number of participants increase on the other side of the market. That is, we can write the utility function for the buyers and sellers in a food delivery market under Rochet and Tirole (2003)’s specification:

$$u^B = (b^B - p^B)n^S,$$

$$u^S = (b^S - p^S)n^B.$$

where $n^k$ denote the proportion of participants in $k$’s group ($k \in \{ B, S \}$). According to Rochet and Tirole (2003), $n^k$ is the demand of platform usage of side $k$. We can also interpret $n^k$ as the probability of group $k$ use the platform as well. For example, the utility of a buyer from ordering the delivery depends on the probability of the seller accepting the order. In this case, $u^k$ represents the expected utility. For instance, when the probability for a seller taking an order increase, the utility level of a buyer increase as well.

Now, let us add Hotelling specification to this model. There are two reasons to use the Hotelling specification: Firstly, the Hotelling specification helps us analysis the platform competition. When the participants have preference between platforms, the Hotelling specification allow us to use the distance to characterise the competition behavior rather than derive the cumulative distribution function to the random variables which measure the agents’ heterogeneity. Secondly, the transportation cost in the Hotelling model tells us the level differentiation between platforms. The level of this differentiation parameter can affect
the equilibrium in the market. According to Armstrong(2006), we can use the Hotelling specification to represent the benefit random variables in our model: $b^B = \alpha - t^B x^B$ and $b^S = \beta - t^S x^S$, where $x$ is the location of the participants from the origin 0. $\alpha$ and $\beta$ are the upper bonds of the marginal benefit for buyer an the marginal revenue for the seller. Here we assume $\alpha$ and $\beta$ are constant exogenous parameter. At consumer side, we assume that the two platforms are located at 0 and 1 on the line respectively, the location of the consumer to each platform are: $x^B_1 = x^B$, $x^B_2 = 1 - x^B$ and each buyer selects one platform for a duopoly market. When the market is monopoly, we assume the platform is located at 0. For the sellers, since different restaurants have different products and different marginal cost. The transportation cost $t^S$ represents the heterogeneity among the sellers.

3 A Platform Monopoly

In this section, we analysis an example of food delivery market in a monopoly environment. The framework of monopoly market helps us to understand how the two groups of agents interact with each other. We can write the utility function in a monopoly environment with the Hotelling specification as:

$$
\begin{align*}
    u^B &= (\alpha - t^B x^B - p^B)n^S, \\
    u^S &= (\beta - t^S x^S - p^S)n^B.
\end{align*}
$$

Here, $n^S$ and $n^B$ are the demand of the platform usage. Under the monopoly market structure, the demand of the platform usage is the proportion of the buyer’s and seller’s who have positive utility:

$$
n^k \equiv Pr(u^k > 0), \ k = B, S.
$$

According to the utility function from equation system (1), we can solve the demand function
in the monopoly market as:

\[ n^{Bm} = \frac{\alpha - p^B}{t^B}, \]
\[ n^{Sm} = \frac{\beta - p^S}{t^S}. \]  

(2)

Now we introduce the price elasticities to the demand function:

\[ \eta^B = -\frac{p^B}{n^B} \frac{\partial n^B}{\partial p^B} = \frac{p^B}{\alpha - p^B}, \quad \eta^S = -\frac{p^S}{n^S} \frac{\partial n^S}{\partial p^S} = \frac{p^S}{\beta - p^S}. \]

(3)

A monopoly platform solves the maximization problem:

\[ \max_{p^S, p^B} \pi^m = (p^S + p^B - c)n^S n^B. \]

(4)

The first order condition to the monopoly platform’s log profit maximization problem are:

\[ \frac{\partial ( \log \pi )}{\partial p^B} = \frac{1}{p^B + p^S - c} - \frac{1}{\alpha - p^B} = 0, \]
\[ \frac{\partial ( \log \pi )}{\partial p^S} = \frac{1}{p^B + p^S - c} - \frac{1}{\beta - p^S} = 0. \]

(5)

By plugging in the elasticities from (3) into the the first order conditions in (5), we can find that:

\[ \frac{p^B}{\eta^B} = \frac{p^S}{\eta^S} = p^S + p^B - c. \]

we can define the total price \( p = p^S + p^B \) and total elasticity \( \eta = \eta^S + \eta^B \), the level of the price can be defined as the Lerner formula:

\[ \frac{p - c}{p} = \frac{1}{\eta}. \]

By solving the system (5), we can conclude the following proposition of food delivery market in monopoly market structure:
Proposition 1  In a food delivery market with a monopoly platform, the optimal prices are:

\[ p_{B}^{*} = \alpha - \frac{\alpha + \beta - c}{3} \]

\[ p_{S}^{*} = \beta - \frac{\alpha + \beta - c}{3} \]  

(6)

the total price \( p = p_{B} + p_{S} \), is given by the standard Lerner formula for the total elasticity: \( \eta = \eta_{B} + \eta_{S} \):

\[ \frac{p - c}{p} = \frac{1}{\eta} \]  

(7)

and the optimal prices also satisfy:

\[ \frac{p_{B}}{\eta_{B}} = \frac{p_{S}}{\eta_{S}} \]  

(8)

4  Platform Competition

4.1  Buyers’ Behavior

Under duopoly competition, we assume that the two platform are located at 0 and 1 on a line. The platforms choose the price to compete for the buyers. The utility functions of a buyer at position \( x \) to buy the meal form the two platforms are:

\[ u_{B1} = (\alpha - xB_{1}t_{B} - p_{B1})n_{1}^{S} \]

\[ u_{B2} = (\alpha - (1 - xB)t_{B} - p_{B2})n_{2}^{S} \]  

(9)

From the utility function, a buyer choose to use platform i if and only if: \( u_{i} > max\{0, u_{j}^{B}\} \).

That is, on the one hand, a buyer choose to join one platform requires the positive utility. On the other hand, the buyer also compare through the platforms to see if they can get a better deal or which food delivery App is better for them. We define \( D_{i}^{B} \) as the demand that

1Notice that, the buyer and seller’s price are depending on \( \alpha \) and \( \beta \), which are their maximum willingness to pay. Therefore \( \alpha + \beta - c \) is the highest possible profit per transaction for the platform. We can interpret the optimal price as the difference between the maximum willingness to pay and one third of the highest possible profit for the platform.
the buyers are indifference with joining platform i or not (0 utility). And $d_i^B$ is the demand that the buyers are indifference with platform 1 and platform 2. We can now define the demand function:

$$n_i^B \equiv \min\{d_i^B, D_i^B\}.$$  \hspace{1cm} (10)

When $D_i^B > d_i^B$, which means all the consumer who prefer platform i rather than platform j, will have positive utility.

Figure 1: Competition

In this case, the market is covered, all the consumer join in the market and choose one platform to buy the meal. In this case, the market is under competition for the buyers. The demand of platform usage on the buyer’s side is $d_i^B$. Here, if a buyer choose platform i, the buyer’s utility on that platform is also positive. When the buyer have the preference on the platform, the number of the sellers who joined in that platform also affect the buyer’s choice. Therefore, when the buyer’s market is under competition, there is a network externality from seller to the buyer. However, this externality does not affect the buyer’s market, when the buyer’s market is not covered:

Figure 2: Local Monopoly

$$d_1^B$$

$$d_2^B = 1 - d_1^B$$
The figure above gives an example of the local monopoly buyer’s market which is not covered. When $D_i^B < d_i^B$, the proportion of the consumers join in platform $i$ is $D_i^B$. In this case, a buyer choose platform $i$ if and only if they have positive utility on that platform. However, a buyer might not join in the market even though they have a preference on the platforms due to the negative utility. In this case, if a buyer choose one platform, they have a strong brand loyalty on that platform. By applying the indifference condition we can solve $D_i^B$ and $d_i^B$ from the utility functions:

\[
D_1^B = \frac{\alpha - p_1^B}{t^B},
\]

\[
D_2^B = \frac{\alpha - p_2^B}{t^B},
\]

\[
d_1^B = \frac{n_2^S(p_2^B + t^B - \alpha) + n_1^S(\alpha - p_1^B)}{t^B(n_1^S + n_2^S)},
\]

\[
d_2^B = \frac{n_1^S(p_1^B + t^B - \alpha) + n_2^S(\alpha - p_2^B)}{t^B(n_1^S + n_2^S)}.
\]

By solving the inequality: $d_i^B < D_i^B$, we can find that

\[
0 < t^B < 2\alpha - p_1^B - p_2^B,
\]

$t^B$ is the transportation cost which measures the level of differentiation between two platforms. Here, when the two platform have similar design, the market is under the competition. Conversely, the platforms are local monopoly for the buyers, when the platforms are different to each other:

\[
t^B > 2\alpha - p_1^B - p_2^B.
\]

For a given market, the transportation cost and marginal benefit parameter are exogenous. The competition in the buyer’s market depends on the pricing strategies by the two platforms. The grey area in figure 4 is the set of the allocation under the competition of buyer’s market.
From the figure above, we can see that the maximum price for a platform charge to the buyers to compete with other platform is $2\alpha - t^B$. Therefore, if platform $i$ set the price $p^B_i$ smaller than $2\alpha - t^B$, its competitor $j$ is able to compete the price less than $2\alpha - t^B - p^B_i$, or charge a higher price to stay local monopoly.

### 4.2 Sellers’ Behavior

The seller’s behaviors differ from consumer’s of following: first, the sellers can sell simultaneously on both platforms while the consumers only pick one platform to order food. Secondly, there are no brand loyalty for the sellers and the sellers only care about the profit. Therefore the platforms are not located on one line for the sellers. The decision for a seller to join one platform is independent of the other. That is, the sellers join the platform if they can make positive profit:

$$u^S_1 = (\alpha - x^S t^S - p^S_1)n^B n^D,$$
$$u^S_2 = (\alpha - x^S t^S - p^S_2)n^B n^D.$$

(12)
we can solve the demand of the sellers on each platform will be:

\[
   n_i^S = \frac{\beta - p_i^S}{t^S}, \forall i \in \{1, 2\}.
\]  

(13)

Here, the demand of seller on platform \(i\) only depends on the price charge by the platform \(i\). The price change on the platform \(j\), no matter on buyers or sellers, do not affect the demand of seller on platform \(i\).

4.3 Demand elasticity

By substituting the demand from the seller’s side in to buyer’s side, we can solve the demand of buyers:

\[
   d_i^B(p_i^B, p_i^S) = \frac{p_i^S(p_i^B + t^B - \alpha + p_i^S\alpha - (p_2^B + t^B)\beta + p_1^B(\beta - p_1^S))}{t^B(p_1^S + p_2^S - 2\beta)},
\]

\[
   d_2^B(p_i^B, p_i^S) = \frac{p_1^Bp_1^S - p_2^Bp_2^S + p_1^S t^B + \alpha(p_2^S - p_1^S) - \beta(p_1^B - p_2^B + t^B)}{t^B(p_1^S + p_2^S - 2\beta)},
\]

(14)

\[\text{where } i = 1, 2.\]

The demand function of buyers for each platforms are the function of the price of both buyers and sellers of both platforms. Therefore, a change in price on any side of any platform can change the buyers demand. In order to analysis the direction of the network externality. We define the elasticity as:  

\[
   \eta_{i}^{Bk} = \frac{\partial d_i^B}{\partial p_i^B} \frac{p_i^k}{d_i^B}
\]

to represent the effect from changing price on market
$j$ side $k$ on market $i$’s demand of buyer. We can show that:

\[ \eta_{i1}^{BB} = \left( \frac{\beta - p_i^S}{\tau B (p_i^S + p_2^S - 2 \beta)} \right) \frac{p_i^B}{d_i^B} < 0, \]

\[ \eta_{i2}^{BB} = \left( \frac{p_2^S - \beta}{\tau B (p_i^S + p_2^S - 2 \beta)} \right) \frac{p_2^B}{d_i^B} > 0, \]

\[ \eta_{i1}^{BS} = \left( \frac{- (p_i^B + p_2^B + \tau B - 2 \alpha)(p_2^S - \beta)}{\tau B (p_i^S + p_2^S - 2 \beta)^2} \right) \frac{p_i^S}{d_i^B} < 0 \]

\[ \eta_{i2}^{BS} = \left( \frac{(p_i^B + p_2^B + \tau B - 2 \alpha)(p_2^S - \beta)}{\tau B (p_i^S + p_2^S - 2 \beta)^2} \right) \frac{p_2^S}{d_i^B} > 0. \]

The elasticity of platform 2 can be derive symmetrically. we can conclude that, the increasing of the price of one platform negatively affect its own demand of buyer and positively affect its competitor’s demand.

### 4.4 Competition in Market For Buyers

We first look at the case that the market is covered. In this case, the platform set the price on the buyer and seller’s side that can maximize their profit:

\[ \pi_i = (p_i^B + p_i^S - c)n_i^S n_i^B. \]

Here, $n_i^S$ and $n_i^B$ is the demand function on seller and buyer’s side respectively, which are the functions of the prices charged on both side. When the platform set the price, it changes the demand function. On the seller’s side, the restaurant join in the market if they can make positive profit. Therefore, the price change on the buyer’s side will not change the strategy on the seller’s side. Therefore, in order to analysis the competition, we can fix the price on the seller’s side.

The competition happens in the buyer’s side. The best response functions on the buyer’s side have two part. First, when the prices allocations are in the competition.

\[ ^2 \text{See figure 3} \]
the best response function by the first order condition of:

\[ \pi_i = (p_i^B + p_i^S - c) n_i^S d_i^B . \]

We assume that, at equilibrium, the platforms match the price on the seller’s side: \( p_1^S = p_2^S = p^S \) (according to the symmetry), the best response function under competition on the buyer’s side is:

\[ Br_i(p_j^B) = \frac{1}{2} (p_j^B - p^S + c + t^B). \]  \( (17) \)

On the buyer’s side, the Nash equilibrium for the competition can be achieved when the best response function crosses. Since the price change on buyers does not affect the demand on seller according to the demand function on (13). Assume the price on seller is exogenous, we can plot the following graphs:

Figure 4: No Nash Equilibrium
The two cases above are not the equilibrium in the market, when \( p^S > c + t^B \) the best response function do not cross. This happens when, the equilibrium seller’s prices are too high: \( p^S > c + t^B \). When \( p^S = c + t^B \), the profit is not maximized. This case happens that the buyer’s price are zero and the seller’s price equals to the sum of marginal cost and transportation cost. We can easily prove that there is always a incentive for the plat form to change their pricing strategy to achieve a equilibrium\(^3\). The requirements for the equilibrium in this market are: first, the best response function crosses in the buyer’s side. Second, the seller’s price maximize the profit. Therefore, figure 5 is the case that the best response function at buyer’s side crosses but the profit is not maximized. In order to have a equilibrium and profit maximization in the market we have:

\(^3\)See the proof in Appendix A
By solving the maximization problem of (16) simultaneously, the optimal prices at equilibrium for the competition are:

\[ p_B^1 = p_B^2 = 2c + 3t_B + 2\alpha - 2\beta, \]
\[ p_S^1 = p_S^2 = 2c + t_B - 2\alpha + 2\beta. \]

Notice that the platform competition must in the competition area. We know that, at equilibrium, \( p_B^1 = p_B^2 \). Therefore, in order to have this equilibrium exist, we need two conditions. First, \( p_B^B > \alpha - \frac{t_B}{2} \). This condition promises the allocation is in the competition area. Second, \( p_S^S < c + t_B \). The price on seller’s side cannot be too high, which might cause the Nash equilibrium not exist. We can solve for the transportation cost, we can conclude that:

**Lemma 1** The Nash equilibrium for the platform competition exists when the transportation
cost $t^B$ satisfy:

$$\frac{2(\beta - \alpha - c)}{3} < t^B < \frac{2}{5}(\alpha + \beta - c), \beta - \alpha > c,$$

or

$$0 < t^B < \frac{2}{5}(\alpha + \beta - c), \beta - \alpha \leq c.$$

We can conclude that if the transportation cost $t^B$ is small, then the market is covered and the Nash equilibrium exist. We can now rewrite the optimal price in (18) as:

$$p_1^{Bc} = p_2^{Bc} = \alpha + \frac{3}{4}t^B - \frac{\alpha + \beta - c}{2},$$

$$p_1^{Sc} = p_2^{Sc} = \beta + \frac{1}{4}t^B - \frac{\alpha + \beta - c}{2}.$$  

and the total price:

$$p_i^{Bc} + p_i^{Sc} = t^B + c$$

The total price in the competition gives a form of classic Hotelling solution. When the differentiation on the buyer’s side is zero, then the total price is under a Bertrand competition structure.

4.5 Local Monopoly

In this section we evaluate the Nash equilibrium when the market is not covered. In this case, notice that $D_i^B < d_i^B$. Hence, the total demand can be small than 1 given that the market is not covered $D_1^B + D_2^B < 1$. Under this case, the change in price on one player will not affect its competitor. The price structure for each platform can be same as monopoly, since each platform are performing as a local monopoly. The level of the price for each platform

\[\text{See figure 2}\]
will be

\[
p_{1}^{Bm} = p_{2}^{Bm} = \alpha - \frac{\alpha + \beta - c}{3},
\]

\[
p_{1}^{Sm} = p_{2}^{Sm} = \beta - \frac{\alpha + \beta - c}{3}.
\]

(20)

According to the figure below, we can solve the range for the transportation cost that a local monopoly equilibrium exist: \( p_{1}^{Bm} = p_{2}^{Bm} > 2\alpha - t^{B} \).

Figure 7: Local Monopoly Equilibrium

Therefore, we can conclude that in a food delivery market, when the transportation cost is large enough:

\[ t^{B} > \frac{\alpha + \beta - c}{3} + \alpha. \]

(21)

we have a equilibrium with the price structure as same as monopoly market.

4.6 Comparing Competition with Local Monopoly

In this section, we compare the local monopoly price to the competition price. We evaluate 4 cases for different level of transportation cost. First we can solve the range of the transportation cost that makes the local monopoly price level higher than the competition price.
level:

\[ t^B < \frac{2}{9}(\alpha + \beta - c). \]  

Proposition 2 In a duopoly food delivery market, we can conclude that:

1. When the transportation cost is small: \( t^B < \frac{2}{9}(\alpha + \beta - c) \), the two platform will compete for the buyers with the Nash equilibrium at:

\[
p_{1c}^{Bc} = p_{2c}^{Bc} = \alpha + \frac{3}{4} t^B - \frac{\alpha + \beta - c}{2},
\]

\[
p_{1c}^{Sc} = p_{2c}^{Sc} = \beta + \frac{1}{4} t^B - \frac{\alpha + \beta - c}{2}.
\]  

2. When the transportation cost is in an intermediate range: \( \frac{2}{9}(\alpha + \beta - c) < t^B < \frac{4\alpha + \beta - c}{3} \), there will be no pure strategy Nash equilibrium.

3. When the transportation cost is large: \( t^B > \frac{4\alpha + \beta - c}{3} \), there is a local monopoly Nash equilibrium at:

\[
p_{1m}^{Bm} = p_{2m}^{Bm} = \alpha - \frac{\alpha + \beta - c}{3},
\]

\[
p_{1m}^{Sm} = p_{2m}^{Sm} = \beta - \frac{\alpha + \beta - c}{3}.
\]  

Based on the propositions and lemma from the previous sections, we can conclude 5 cases:
First, we consider the transportation cost is very small at $t^B < \frac{2}{3}(\alpha + \beta - c)$. In this case, we only have the competition equilibrium exist. If price is at the local monopoly level then $D_1^B + D_2^B > 1$. For this case, the local monopoly equilibrium cannot be achieved. In this market, two platforms are very similar, the price competition is strong, this could make the competition equilibrium price level lower than the monopoly.

Figure 9: Case II. $\frac{2}{3}(\alpha + \beta - c) < t^B < \frac{2}{3}(\alpha + \beta - c)$
Secondly, we have a higher level of transportation cost to allow some degree of differentiation between two platforms. In this case, the price level of competition is higher than the local monopoly price level. This can also cause the local monopoly equilibrium not exist due to the same reason as case I. In this market the, we still have the competition effect. However, the substitution effect between two platform is comparatively lower than case I since the level of transportation cost is higher. This allows the platform to achieve a higher level equilibrium on the buyer’s price than the monopoly.

Figure 10: **Case III.** \( \frac{2(\alpha + \beta - c)}{5} < t^B < \frac{2(\alpha + \beta - c)}{3} \)

Thirdly, we evaluate a special level that both Nash equilibrium cannot be achieved. Since \( t^B > \frac{2}{5}(\alpha + \beta - c) \), the duopolistic equilibrium is not in the competition area. When \( t^B < \frac{2}{3}(\alpha + \beta - c) \), the local monopoly cannot be achieved, because the market is covered and \( D_1^B + D_2^B > 1 \). Hence, in this region, there is no pure strategy Nash Equilibrium.
Next we want to see that if we allow the local monopoly not in the competition area but the price level has intersection with the competition. Under this case the competition price level will be higher than the monopoly price. However, this condition contradict to Lemma 1. Hence, the competition equilibrium does not exist. However, in this case the local monopoly is also not exist\footnote{See the proof in Appendix B}. According to the figure above, we can see that, at this price level, the local monopoly price hits the competition area. This allows its competitor to lower the price and bring the market in to competition. There will always exist an incentive for the players to change their strategies.
The last case is the local monopoly equilibrium. When the transportation cost is large, the market cannot be covered. The only Nash equilibrium in this case is the local monopoly equilibrium.

4.7 Discussion of Transportation Cost

The transportation cost on the buyer’s side \( t^B \) measures the level of differentiation between platforms. The platforms are differentiate in the following ways: the horizontal differentiation, for example, the buyers have preference on the design of the App such as color, basic function, restaurant classification, and payment choice. The platforms can also differentiate from each other vertically through the building the brand loyalty. The previous experience is one important factor. The buyers prefer to the platform that they had the positive previous shopping experience. Most of the food delivery App have the convenience function design, such as: quick order, payment method memory and favourite restaurant recommendation.

From the previous analysis, we know that when the transportation cost is at the intermediate level, there is no equilibrium. We can think about the transportation cost as the matching value. When the matching value is high, the buyers are strictly prefer one platform...
to the other. Each platforms only have their faithful consumers. In this case, we have the local monopoly and the matching between consumers and platforms are strong and stable. That is, even though one platform lower the price to zero, the market is still not covered and the platform cannot ”steal” any consumer from its competitor for any pricing strategy. When the matching value is very low, the platforms have some similarity. The pricing strategy affect their competitor’s behavior. The loyal consumers compare through the platforms and their behavior is based on the price and their taste on platform differentiation. The matching between buyers and platform can be always broken by a lower price from another platform. The equilibrium in the competition happens when the platform are very similar. In this case, the matching is unstable and the consumer do not stick on one platform, they always compare through the platforms to find the best deal. That is, the platforms become more homogeneous to the buyers. Therefore a pure-strategy Nash-equilibrium exist. When the transportation cost is at intermediate level, there is some brand loyalty for some consumers, however, the equilibrium can always be broken by a ”Better Deal”. A different pricing strategy on either buyer or seller’s strategy can always make the equilibrium not be able to achieved. Therefore, even though the platform can attract some loyal consumers, the matching is always unstable.

5 Conclusion and future work

In this paper, we investigate price competition in the food delivery market. With a monopolistic platform, we employ the Lerner index to characterize the equilibrium prices in terms of the price elasticities of the buyer and seller sides of the market. In the duopolistic platform competition, we introduce a new method to analyze equilibrium prices set by the platforms. Specifically, we characterize the scenarios in which the platforms are local monopolies and the complementary scenarios in which the platforms compete for buyers. As the degree of differentiation between the platforms increases, the market transitions from a competitive
environment to local monopolies. We also find that a pure-strategy Nash equilibrium exists when the differentiation is either small or large, whereas an equilibrium does not exist when the degree of differentiation is in an intermediate range.

The future work of this research is to add the third side, namely drivers, of market for food delivery. In our current model, we assume that supply of drivers is perfectly price elastic, resulting in a constant marginal cost to the platforms of food delivery. However, if the supply of drivers is not perfectly price elastic, then the platforms have power in setting driver wages. Our goal of the future research is to find whether the shift from a two-sided market (in which the driver supply curve is perfect price elastic) to a three-sided market changes the type of competition (local monopolistic or oligopolistic) and the equilibrium prices set by the platforms, paying particular attention to the prices paid by the platforms to the drivers.
Appendix A.

Proof.

In this proof we want to show that the pricing allocation: $p_1^B = p_2^B = 0$, $p_1^S = p_2^S = c + t^B$ is not an Nash Equilibrium. In order to simply the notation we need following definitions.

Definition 1 Define the set of an price profile in a duopoly market as:

$$\{p_1^B, p_2^B, p_1^S, p_2^S | p_i^k \in [0, 1], i = 1, 2, k = \{B, S\}\}.$$ 

Definition 2 The profit for platform $i$ with price profile $\{p_1^B, p_2^B, p_1^S, p_2^S\}$ is:

$$\pi_i(p_1^B, p_2^B, p_1^S, p_2^S).$$

We can first set the price profile as: $\{0, 0, c + t^B, c + t^B\}$. Then, we will have:

$$\pi_2(0, 0, c + t^B, c + t^B) = \frac{\beta - (c + t^B)}{2t^S}.$$ 

Now consider another profile as $\{0, p_{1c}^B, c + t^B, p_{2c}^S\}$, where $p_{1c}^B = \frac{2c + 3t^B + 2\alpha - 2\beta}{4}$ and $p_{2c}^S = \frac{2c + t^B - 2\alpha + 2\beta}{4}$ are the optimal prices in the competition. The profit for platform 2 is:

$$\pi_2(0, p_{1c}^B, c + t^B, p_{2c}^S) = \frac{(2c + t^B) - 2(\alpha + \beta))(4c^2 - 13t^B + 4(\alpha + \beta)^2 - 8c(t^B - \alpha + \beta) + 8t^B(\alpha + \beta)}{16t^S(6c + 5t^B - 2(\alpha + 3\beta))}.$$ 

Notice that, the price for seller is $c + t^B$ on at least on side of the market. In order to have the market exist, we must have: $\beta - (c + t^B) > 0$. By taking difference of profit between 2 profits, we can have:

$$\pi_2(0, p_{1c}^B, c + t^B, p_{2c}^S) - \pi_2(0, 0, c + t^B, c + t^B) = \frac{(2c + 3t^B + 2\alpha - 2\beta)^2(2c + t^B - 2(\alpha + \beta))}{16t^S(6c + 5t^B - 2(\alpha + 3\beta))} > 0.$$ 

Therefore, $\{0, 0, c + t^B, c + t^B\}$ can not be a pure strategy Nash Equilibrium. ■
Appendix B.

Proof.

In this proof we want to show that the local monopoly is not an pure strategy Nash when:

\[ \frac{2(\alpha + \beta - c)}{3} < t^B < \frac{4\alpha + \beta - c}{3} \].

We assume that the platform 2 is current using local monopoly pricing strategy. When the platform 2’s price is fixed at the level of local monopoly, the change price of platform 1 will not affect the demand of platform 2. The figure below shows that, by lower the price level, platform 1 can steal the buyer’s from the platform 2 since its pricing strategy are fixed.

![Figure 13](image)

Here, we will evaluate two scenario:

1. Platforms are exactly covering the market.

Under this scenario, the platform 1 will adjust the price to cover the market. That is, \( d_1^B = 1 - D_2^B \). We can solve for the buyer’s price \( p_1^B = \frac{1}{3}(4\alpha + \beta - c - 3t^B) \). By solving the profit maximization problem we can find the optimal seller’s price: \( p_1^S = \frac{1}{6}(4c + 3t^B - 4\alpha + 2\beta) \). The optimal profit that the platform 1 cover the market exactly is:

\[
\frac{(c + 3t^B - \alpha - \beta)(4c + 3t^B - 4(\alpha + \beta))^2}{108t^Bt^S}
\]

Taking difference with the local monopoly profit, we will have:

\[
\frac{(5c + 3t^B - 5(\alpha + \beta))(2c + 3t^B - 2(\alpha + \beta))^2}{108t^Bt^S}
\]
If this difference is positive, we can show that the platform has the incentive to deviate from the local monopoly pricing strategy. We can show that when \( t^B > \frac{5(\alpha + \beta - c)}{3} \), the Nash equilibrium does not exist. We also know that: \( t^B < \frac{4\alpha + \beta - c}{3} \). Then if \( \frac{5(\alpha + \beta - c)}{3} > \frac{4\alpha + \beta - c}{3} \), the local monopoly Nash equilibrium will not exist. By solving this inequality, we need \( c < \frac{1}{4}(\alpha + 4\beta) \).

2. Platform 1 reduce the buyer’s price to 0.

Figure 14

Under this scenario, the optimal demand for platform 1 is \( D_1^B = \frac{\alpha}{t^B} \). According to the figure above, when \( \alpha < t^B \), the maximum demand for the platform 1 will not be higher be: \( \text{Min}\{\frac{\alpha}{t^B}, 1\} \). Assume \( \alpha < t^B \), the difference of optimal profit when \( p_1^B = 0 \) to the local monopoly profit is:

\[
\frac{(4c - \alpha - 4\beta)(c + 2\alpha - \beta)^2}{108t^B t^S}
\]

When this difference is positive we need: \( c > \frac{1}{4}(\alpha + 4\beta) \). We can conclude that, for any value of marginal cost \( c \). We always find an incentive for player 1 to deviate from local monopoly when: \( \frac{2(\alpha + \beta - c)}{3} < t^B < \frac{4\alpha + \beta - c}{3} \). Therefore, the Nash equilibrium dose not exist. ■
Appendix C.

Table 2: Tables of notation in this paper

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>Baseline line marginal benefit for buyers</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Baseline line marginal benefit for sellers</td>
</tr>
<tr>
<td>$t$</td>
<td>transportation cost</td>
</tr>
<tr>
<td>$x$</td>
<td>hotelling location</td>
</tr>
<tr>
<td>$p^B, p^S$</td>
<td>price on each side of the market</td>
</tr>
<tr>
<td>$u^S, u^B$</td>
<td>utility of each side of the market</td>
</tr>
<tr>
<td>$n^B, n^S$</td>
<td>number of buyers and sellers</td>
</tr>
<tr>
<td>$D^B_i$</td>
<td>the set of consumers who prefer platform i to the outside option</td>
</tr>
<tr>
<td>$d^B_i$</td>
<td>the set of consumers who prefer platform i to platform i'</td>
</tr>
<tr>
<td>$Q$</td>
<td>transaction volume</td>
</tr>
</tbody>
</table>
References


