Self-fulfilling debt crises and limits to arbitrage*

Gaston Chaumont
University of Rochester

Grey Gordon
The Federal Reserve Bank of Richmond

Bruno Sultanum
The Federal Reserve Bank of Richmond

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Abstract

Self-fulfilling crises, where investors' expectations of a default cause a default, are a defining feature of sovereign debt markets. However, we show that a self-fulfilling crisis creates an arbitrage opportunity. Suppose that there are two equilibria in an economy: a self-fulfilling crisis where investors do not rollover the debt and default happens and a non-crisis where investors rollover the debt and default does not happen. In the self-fulling crisis, a large investor or coalition of small investors with sufficient resources could bid up the sovereign debt to its non-crisis equilibrium price. This price results in zero profits on the purchase of the new issuance *and* a gain for existing debt holders. We propose an equilibrium refinement based on absence of this arbitrage opportunity and show that (i) there is a unique equilibrium which survives the refinement, and (ii) in the unique equilibrium the sovereign does not default due to rollover risk. Further, we show that the possibility of indeterminacy increases when resource constraints are tight or the country's debt is large relative to investor resources. Last, we extend our model to allow investors to participate in credit-default swap (CDS) markets. Contrary to conventional wisdom, we find the ability to trade in CDS markets can *dampen* indeterminacy by allowing investors to sell CDS protection and use those proceeds to execute the arbitrage trade.

JEL Classification: F34, E44, G12, G15

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*Contact: Gaston Chaumont: gaston.chaumont@rochester.edu, Grey Gordon: greygordon@gmail.com, Bruno Sultanum: bruno@sultanum.com. The views expressed are those of the authors and do not necessarily those of the Federal Reserve Bank of Richmond or the Federal Reserve System. We are grateful to Tim Kehoe, Manuel Amador, Yan Bai, Gabriel Mihalache and seminar participants at the 2021 meeting of the SED, the XXIII UTDT Workshop in International Finance and Economics and the University of Rochester. All errors remain our own.

1 Introduction

Self-fulfilling crises are a defining feature of sovereign debt markets. When investors consider lending to a sovereign, they must take into account what other investors will do. If other investors lend to the sovereign at attractive prices, the sovereign can rollover existing debt and remain solvent. If not, the sovereign may not be able to pay for all the debt that comes due, resulting in default. As a small investor, one's own investment decisions are immaterial and prices hinge entirely on what others do. When everyone thinks in this way, expectations of default can be self-fulfilling: If investors expect default, it is optimal not to lend and the sovereign defaults; if investors expect repayment, it is optimal to lend and the sovereign repays. But is it necessary that all investors behave as if they are small to have self-fulfilling debt crises? In this paper, we show the answer is "yes" when there are no limits to arbitrage and "sometimes" when there are limits.

To establish this, we first consider the case of no limits to arbitrage and allow investors to form arbitrarily small coalitions. In this case, an arbitrage opportunity exists. Specifically, whenever the sunspot would result in default, the coalition of investors can bid up the bond price to the fundamental value, i.e., the price when the sunspot would result in no-default. In doing so, they prevent default and earn zero return in the auction itself while still obtaining a financial gain. In the benchmark, this financial gain is modeled as all investors holding a portion of the predetermined debt stock. However, one can alternatively model the gain by allowing a secondary market where initial bond holdings can be bought at depressed prices by coalition investors. In either case, the result of this arbitrage opportunity is that any expectation-driven-default equilibrium is eliminated when applying a non-arbitrage refinement, which leaves only the repayment equilibrium as the unique outcome.

To execute this arbitrage opportunity, the coalition must be able to bid a sufficient amount in the sovereign debt auction. What matters for the coalition to exploit this opportunity is the aggregate resources of the coalition. As the coalition shrinks in size, the amount of resources required by each investor goes to infinite. Elimination of the indeterminate equilibrium requires that the coalition can buy a sufficiently large amount of bonds. As the size of the coalition gets smaller, or the resources available to individual investors shrinks, indeterminacy is restored.

As an extension of our theoretical results, we consider the role of a specific type of secondary sovereign debt market, that of credit-default swaps (CDS). Contrary to conventional wisdom that CDS markets *amplify* indeterminacy by allowing investors to bet against a sovereign and raise its borrowing costs, we show that CDS markets *dampen* indeterminacy by allowing investors to bet in favor of the sovereign.[1] Specifically, the presence of a CDS market allows a coalition of investors to circumvent their resource limits and further reduce the indeterminacy region using the following strategy. In a sunspot resulting in default, investors can raise revenue by selling CDS, that is, sell claims that pay out when the sovereign defaults. Since the sunspot results in default with probability one, the competitive price of these claims is the risk-free price. Consequently investors can raise arbitrarily large amounts of capital, circumventing their resource constraints, and implement the arbitrage opportunity already stated by bidding up the sovereign's debt price in the auction. This strategy results in repayment, and eliminates the indeterminacy region when using a no-arbitrage refinement.

We also consider deviations from the case in which investors have zero mass. In reality, it is usually the case that a few large banks are the main buyers of sovereign bonds at issuance, instead of a large number of small investors. Large investors could themselves affect the result of an auction without the existence of a coalition of small investors. Thus, we study two extensions of the model in which there are no coalitions of investors but there is a large investor that can purchase newly issued bonds. In the first extension, we allow a large bank to participate in sovereign bond auctions under the same rules for small investors. In the second extension, we allow the government and the large bank to trade bonds bilaterally, as it is usually the case in a syndicated bond issuance. We show that no self-fulfilling default equilibrium survives the no-arbitrage refinement whenever a large investor has enough resources to roll-over government's

[1] Some friction-less models with CDS and bonds can result in a higher bond prices when CDS are banned. Intuitively, allowing investors to sell CDS effectively increases the supply of sovereign-bond-like instruments, depressing the price. Consequently, banning naked CDS can improve prices. Chaumont, Gordon, Sultanum and Tobin (2020) discuss both friction-less and frictional models.

debt.

Our results highlight that financial frictions play a key role in *promoting* indeterminacy. Consequently, the policy recommendations that come from our research are that CDS trade should be unrestricted and that, especially in times of crises, investors costs of funds should be low. We note that the arbitrage opportunities arise off-equilibrium. Therefore, these arbitrage opportunities do not present themselves on the equilibrium path, so the collusive, price-manipulating behavior does not actually happen. Rather, the threat of price-manipulation eliminates the indeterminacy.

Related Literature. The sovereign debt literature, since its conception in Eaton and Gersovitz (1981), has had a focus on determining possible equilibria in sovereign debt markets. In some particular environments it has been shown that there is a unique equilibrium (see Auclert and Rognlie (2016) and Aguiar and Amador (2019)). However, the literature has shown that agents' expectations can lead to multiple equilibria in models of sovereign default.

In their seminal paper, Cole and Kehoe (2000), show that government's inability to commit to repay its debt obligations can lead investors to believe that a default will occur. Default expectations may then result in investors not willing to bid a positive price for newly issued bonds in sovereign debt auctions. The failure to obtain revenues from new debt issues reduces government's ability to repay its debt obligations and may result in a self-fulfilling default decision. This type of multiplicity is present in Chatterjee and Eyigungor (2012), Conesa and Kehoe (2017), Roch and Uhlig (2018), Aguiar et al. (2016), Aguiar and Amador (2020) and a recent extension by Aguiar et al. (2020), among others. The possibility of a self-fulfilling rollover crisis has improved the literature's ability to explain the movements in interest rate spreads and why governments may prefer to issue long-term debt over short-term debt. Recent work by Bocola and Dovis (2019) quantifies the role of rollover risk in driving credit spreads of Italian bonds over German bonds during the period of 2008-2012 and find that on average 13% of the spread is explain by a non-fundamental component that arises due to the possibility of a self-fulfilling debt crisis.

Our contributions to the literature on self-fulfilling debt crises are two-fold. Firstly, we show that self-fulfilling debt crises equilibria do not survive to our non-arbitrage refinement and that financial frictions are at the core of these kind of equilibria. Secondly, we show how CDS contracts can be used to relax financial frictions associated to borrowing constraints and eliminate the possibility of self-fulfilling debt crises.

A strand of the literature following Calvo (1988) shows how the feedback between debt accumulation and interest rates can generate multiple equilibria. Investors expectations about the probability of a default may result in higher or lower interest rates on sovereign debt, and different interest rates may induce different amounts of debt accumulation that validate investors' expectations. In the simplest version of the model, there are two equilibria. A first equilibrium in which interest rates, debt issuances and default risk are low. And, another equilibrium in which a higher expectation of default leads to lower bond prices and the need to issue more debt to finance a given fiscal deficit. Larger debt accumulation is then consistent with more default risk and investors' expectations are validated in equilibrium. This kind of multiplicity is also present in Lorenzoni and Werning (2019) and Ayres et al. (2019). In section 5.1 we should how our non-arbitrage refinement applies to this setup as well.

2 Benchmark model

In this section, we present our benchmark environment, equilibrium definition and characterize three equilibrium regions: the non-default region, the default region, and the sunspot equilibria region.

2.1 Environment

There are two periods, $t = 1, 2$, and one general purpose consumption good per period. The economy is populated by a set $I = [0, 1]$ of lenders with no mass point, and a government. In the beginning of period $t = 1$, the government has an endowment $Y_1 > 0$, and an initial debt burden $B_1 > 0$, which is equally distributed across lenders. There are two random variables. There is a publicly observed sunspot variable, ζ, drawn in period $t = 1$ from a uniform distribution in the interval $[0, 1]$, and government (potential) revenue in period $t = 2$, denoted by Y_2, which follows a distribution F with continuous density f in the support $[\underline{Y}, \bar{Y}] \subset \mathbb{R}_{++}$.

The government can finance spending and repayment of B_1 by issuing new debt $B_2 \in \mathbb{R}_+$ in a Dutch auction that we describe later. After observing the auction price, q, and revenue, qB_2, the government can default on B_1 and the newly-issued debt B_2. In the end of period $t = 1$, the government derives utility from spending. In the beginning of period $t = 2$, the government observes the realization of the revenue Y_2. At this point, the government has again the option to default on B_2, if it has not already done it in the end of period $t = 1$, and it then derives utility from government spending.

When the government defaults, either in the end of period $t = 1$ or beginning of period $t = 2$, the government loses a fraction of the revenue. Let $\delta_t = 1$ denote if the government defaults at time t or earlier and $\delta_t = 0$ otherwise. To capture the loss due to default, we assume that the government keeps a fraction $h(\delta_t)$ of the endowment Y_t, where $0 < h(1) < h(0) = 1$.

Finally, the government derives utility

$$U(G_1, G_2) = \mathbb{E}\left[u(G_1) + \beta u(G_2)\right] \tag{1}$$

from a spending stream $\{G_1, G_2\}$, where $\beta \in (0, 1)$, and the function $u(\cdot)$ is continuously differentiable, strictly concave and $\lim_{x \searrow 0} u'(x) = \infty$. We assume the following:

Assumption 1. $u'(Y_1 - B_1) > \beta \mathbb{E}_{Y_2}\left[u'(Y_2)\right]$.

The above assumption guarantees that the government wants to borrow in period $t = 1$. It is reasonable to presume that some governments are better off saving rather than borrowing. However, we focus on the government ability to finance debt and whether it leads to default, which naturally is not the case for a government that is saving.

Lenders have discount factor $1/R = \beta$ and transferable utility—meaning, the utility of consuming and cost of producing is one in terms of utility. In period $t = 1$, lenders can buy new issuance of government bonds, B_2, in the primary auction. For simplicity, we do not consider a secondary market for previously issued bonds. The addition of a secondary market would complicate the notation without changing any results.

The government issues new bonds in a Dutch auction. It first announces the supply of bonds to be auctioned, B_2^g, and the reservation price q_2^{min}. Then, each lender $i \in I$ submits a bid in the form of a one-step demand function, $bid^i = (\hat{b}_2^i, q_2^i)$, where the demand is $\hat{b}_2^i$ for any price $q_2 \leq q_2^i$, and zero otherwise. The aggregate demand for bonds in the auction is

$$\hat{B}(q_2) = \int_{i \in I} \hat{b}_2^i \mathbb{1}_{\{q_2 \leq q_2^i\}} di. \tag{2}$$

The final price for the auction is

$$q_2^* := \sup_{q_2} \{q_2;\ q_2 \geq q_2^{min} \text{ and } \hat{B}(q_2) \geq B_2^g\}, \tag{3}$$

when the set $\{q_2;\ q_2 \geq q_2^{min} \text{ and } \hat{B}(q_2) \geq B_2^g\}$ is not empty, and q_2^{min} otherwise.

The amount of bonds allocated to each lender i is

$$b_2^i = \begin{cases} 0 & \text{if } \ q_2^i < q_2^* \\ \frac{B_2 - \hat{B}^+(q_2^*)}{\hat{B}(q_2^*) - \hat{B}^+(q_2^*)} \hat{b}_2^i & \text{if } \ q_2^i = q_2^* \text{ and } B_2^g < \hat{B}(q_2^*) \\ \hat{b}_2^i & \text{if } \ \text{either } q_2^i > q_2^*, \text{ or } q_2^i = q_2^* \text{ and } B_2^g \geq \hat{B}(q_2^*) \end{cases}, \tag{4}$$

and the total is

$$B_2^* = \begin{cases} B_2^g & \text{if } \ B_2^g < \hat{B}(q_2^*) \\ \hat{B}(q_2^*) & \text{if } \ B_2^g \geq \hat{B}(q_2^*) \end{cases}, \tag{5}$$

where $\hat{B}^+(q_2^*) = \lim_{q_2 \searrow q^*} \hat{B}(q_2^*)$, and the lender pays $q_2^* b_2^i$. We formulate the auction to resemble the competitive portion of the auction for US Treasuries.[2] However, our main results do not hinge on the details of the auction itself.

Each lender $i \in I$ cannot spend in the auction more than a given endowment $\bar{a} > 0$, which for now we assume to be infinity.[3]

[2]See *www.newyorkfed.org/aboutthefed/fedpoint/fed41.html* for details on US Treasury auctions. Most countries issue bonds in a way that is similar to the US. They have 2 rounds, the 1st round for the competitive bids, and the 2nd round for non-competitive bids. Some smaller countries do not conduct auctions but instead they do syndicated issues to a few banks.

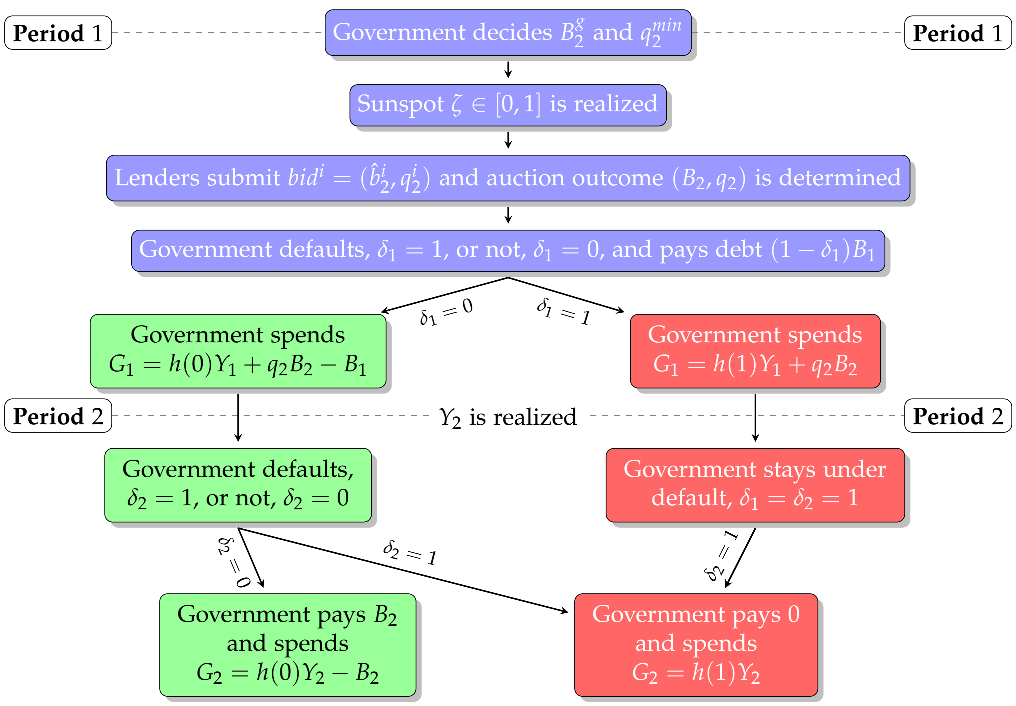
[3]The more general interpretation is that each lender has a funding cost $cost(\cdot)$ as a function of the

Assumption 2. *Lenders have deep pockets—that is, $\bar{a} = \infty$.*

Under assumption 2, lenders have deep pockets. As a result, they can take advantage of any arbitrage opportunity. When $\bar{a}$ is finite, the constraint on spending can impose some limits to arbitrage. The limit to arbitrage case is interesting because it changes the conditions for the refinement we propose. When the constraint binds, the refinement may not eliminate certain equilibria. As we will show, this case provides economic insights on how self-fulfilling debt crisis are generated in equilibrium.

The sequence of actions is depict in Figure 1.

Figure 1: Sequence of actions



2.2 Equilibrium

We study symmetric equilibria, where all lenders chose the same action, and solve the model backwards starting from period $t = 2$.

funding amount a. We assume an extreme version where $cost(a) = 1$ for $a \leq \bar{a}$ and $cost(a) = \infty$ for $a > \bar{a}$. Our results are robust to alternative formulations.

Period $t = 2:$ The government problem in period $t = 2$ is straightforward. The relevant state is (B_2, δ_1, Y_2). If the government defaulted in period $t = 1$, it remains under default in period $t = 2$. In this case, $\delta_2 = 1$ and government spending is $G_2 = h(1)Y_2$.

If the government does not default in period $t = 1$, it has to decide whether to default choosing $\delta_2 = 1$, or not, choosing $\delta_2 = 0$. The government will default if, and only if, the government spending is higher under default. That is, if, and only if

$$h(1)Y_2 > Y_2 - B_2 \quad \Longleftrightarrow \quad B_2 > [1 - h(1)]Y_2,$$

where we assume that the government does not default when it is indifferent between defaulting or not. The value function of the government in period $t = 2$ satisfies

$$V_2(B_2, \delta_1, Y_2) = \begin{cases} u\big(Y_2 - B_2\big) & \text{if } \delta_1 = 0 \text{ and } B_2 \leq [1 - h(1)]Y_2, \text{ and} \\ u\big(h(1)Y_2\big) & \text{otherwise.} \end{cases} \tag{6}$$

Associated with the value function V_2, we have the default policy function

$$\delta_2^*(B_2, \delta_1, Y_2) = \begin{cases} 0 & \text{if } \delta_1 = 0 \text{ and } B_2 \leq [1 - h(1)]Y_2, \text{ and} \\ 1 & \text{otherwise.} \end{cases} \tag{7}$$

Period $t = 1:$ We start analysing period $t = 1$ with the government default decision. The relevant state at this point is the final auction outcome (B_2, q_2); that is, the amount of bond sold and its price per unit in auction. Note that the amount of bonds sold in the auction, B_2, can be smaller than the offered, B_2^g. The government defaults if, and only if, its utility of defaulting is strictly higher than its utility of not defaulting.

Let the government interim utility, $W(B_2, q_2, \delta_1)$, be given by

$$W(B_2, q_2, \delta_1) = u\Big(h(\delta_1)Y_1 + q_2 B_2 - (1 - \delta_1)B_1\Big) + \beta \mathbb{E}\big[V_2(B_2, \delta_1, Y_2)\big]. \tag{8}$$

The government interim utility is a function of the auction outcomes B_2 and q_2, and his default decision δ_1. The expectation is taken over the period $t = 2$ revenue Y_2.

The government chooses to default if, and only if, $W(B_2, q_2, 1) > W(B_2, q_2, 0)$. Therefore, the default policy function in period $t = 1$ is given by

$$\delta_1^*(B_2, q_2) = \begin{cases} 0 & \text{if } W(B_2, q_2, 0) \geq W(B_2, q_2, 1) \\ 1 & \text{if } W(B_2, q_2, 0) < W(B_2, q_2, 1) \end{cases}. \tag{9}$$

Now we turn to lenders bid decision in the auction as a function of the bond supply, B_2^g, reservation price, q_2^{min}, and sunspot realization, ζ. Consider the bid profile $\boldsymbol{bid}(B_2^g, q_2^{min}, \zeta) = \{bid^j(B_2^g, q_2^{min}, \zeta))\}_j$ and associated auction outcomes $B_2^*(B_2^g, q_2^{min}, \zeta)$ and $q_2^*(B_2^g, q_2^{min}, \zeta)$ implied by the auction rules and bid profile. That is, $B_2^*(B_2^g, q_2^{min}, \zeta)$ and $q_2^*(B_2^g, q_2^{min}, \zeta)$ solve equations (3) and (5) given $\boldsymbol{bid}(B_2^g, q_2^{min}, \zeta)$.

At the time of the auction, all lenders know the supply of bonds in the auction, B_2^g, reservation price, q_2^{min}, and sunspot realization, ζ. Lender $i \in I$ takes as given the bid of other lenders in the auction, $\boldsymbol{bid}_{-i}(B_2^g, q_2^{min}, \zeta) = \{bid^j(B_2^g, q_2^{min}, \zeta))\}_{j \neq i}$. Since lender i has zero mass, her bid has no influence on the auction outcome so the quantity sold and price in associated with bid profile $\boldsymbol{bid}_{-\boldsymbol{i}}(B_2^g, q_2^{min}, \zeta)$ are the same for all $i \in [0, 1]$ and coincide with $B_2^*(B_2^g, q_2^{min}, \zeta)$ and $q_2^*(B_2^g, q_2^{min}, \zeta)$. Therefore, lender i is willing to buy any quantity of bonds if its price is no greater than

$$q_2(B_2^g, q_2^{min}, \zeta) = \frac{1}{R} \mathbb{E}_{Y_2} \Big\{ 1 - \delta_2^* \big(B_2^*, \delta_1^*(B_2^*, q_2^*), Y_2 \big) \Big\}, \tag{10}$$

where we omitted the arguments of $B_2^* = B_2^*(B_2^g, q_2^{min}, \zeta)$ and $q_2^* = q_2^*(B_2^g, q_2^{min}, \zeta)$ in equation (10) to keep it short.

Lender i bids

$$bid^i(B_2^g, q_2^{min}, \zeta) = \big(\bar{b}(B_2^g, q_2^{min}, \zeta), q_2(B_2^g, q_2^{min}, \zeta) \big), \tag{11}$$

where $\bar{b}(B_2^g, q_2^{min}, \zeta)$ satisfies $q_2(B_2^g, q_2^{min}, \zeta)\bar{b}(B_2^g, q_2^{min}, \zeta) \leq \bar{a}$. That is, the lender cannot commit to spend in the auction more than it is endowment $\bar{a}$.

Now we turn to the government choice of bond issuance B_2^g. The government anticipates the bond price $q_2^*(B_2^g, q_2^{min}, \zeta)$ and default decision $\delta_1^*\big(B_2, q_2\big)$. Then, it solves

$$(B_2^{g*}, q_2^{min*}) \in \underset{B_2^g, q_2^{min}}{\arg\max} \left\{ \mathbb{E}\Big[W\Big(B_2, q_2^*(B_2^g, q_2^{min}, \zeta), \delta_1^*\big(B_2, q_2^*(B_2^g, q_2^{min}, \zeta)\big)\Big)\Big]\right\}, \tag{12}$$

where the expectation is taken over the sunspot variable ζ. The expected utility of the government in the beginning of period $t = 1$ is

$$V_1 = \mathbb{E}\Big[W\Big(B_2^*, q_2^*(B_2^{g*}, q_2^{min*}, \zeta), \delta_1^*\big(B_2^*, q_2^*(B_2^{g*}, q_2^{min*}, \zeta)\big)\Big)\Big], \tag{13}$$

where the expectation is taken over ζ. Below we define a symmetric equilibrium.

Definition 1. *An equilibrium is a family* $\big\{V_1, V_2, \delta_1^*, \delta_2^*, B_2^{g*}, q_2^{min*}, bid^*, (b_2^{i*})_i, B_2^*, q_2^*\big\}$ *where*

1. *the government period-*2 *value function* $V_2(B_2, \delta_1, Y_2)$ *satisfies equation* (6)*, and the period-*2 *policy function* $\delta_2^*(B_2, \delta_1, Y_2)$ *satisfies equation* (7)*;*
2. *the government period-*1 *utility* V_1 *satisfies equation* (13)*, and the period-*1 *policy functions* $\delta_1^*(B_2, q_2)$ *and* B_2^{g*}, q_2^{min*} *satisfy equations* (7) *and* (12)*;*
3. *the lenders bid function is* $bid^i(B_2^g, q_2^{min}, \zeta) = \big(\bar{b}(B_2^g, q_2^{min}, \zeta), q_2^*(B_2^g, q_2^{min}, \zeta)\big)$*, where* $q_2^*(B_2^g, q_2^{min}, \zeta)\bar{b}(B_2^g, q_2^{min}, \zeta) = \bar{a}$ *if* $q_2^*(B_2^g, q_2^{min}, \zeta) > 0$*, and the period-*2 *bond price function* $q_2^*(\cdot)$ *satisfies equation* (10)*; and*
4. *the equilibrium period-*2 *bond price* $q_2^*(B_2^g, q_2^{min}, \zeta)$ *solves* (3)*, and lenders period-*2 *bond allocation in the auction* $b_2^{i*}(B_2^g, q_2^{min}, \zeta)$ *solves equation* (4) *for all* $i \in I$*.*

It is worth noticing three limitations of the above definition. First, it requires that the allocation is symmetric across lenders, while the same equilibrium prices and aggregate quantities supports a continuum of equilibria since lenders are risky neutral. Second, we do not specify privately mixed strategy equilibrium. Third, as discussed before, we do not have a secondary market for the previously-issued bonds. None of these limitations play a role in generating our results.

2.3 Solution and multiplicity

Equilibrium is characterized by three regions of pairs (Y_1, B_1). There is a non-default region where the unique equilibrium is one where the government does not default in period $t = 1$ with probability one. There is a default region where the unique equilibrium is one where the government defaults in period $t = 1$ with probability one. And, there is a region with multiple equilibria. This last region features sunspot equilibria with positive probability of both default and repayment.

2.3.1 The non-default region

Let us first characterize the non-default region. From equations (6) and (8), we have that the government's gain of defaulting in period $t = 1$ is

$$
\begin{aligned}
&W(B_2, q_2, 1) - W(B_2, q_2, 0) = \\
&\quad u(h(1)Y_1 + q_2 B_2) - u(Y_1 + q_2 B_2 - B_1) + \beta \mathbb{E}\left[V_2(B_2, 1, Y_2) - V_2(B_2, 0, Y_2)\right] = \\
&\quad \underbrace{u\Big(h(1)Y_1 + q_2 B_2\Big) - u\Big(Y_1 + q_2 B_2 - B_1\Big)}_{\text{Current gains due to default}} - \beta \underbrace{\mathbb{E}\big[\max\{u(Y_2 - B_2) - u(h(1)Y_2), 0\}\big]}_{\text{Future losses due to default}}.
\end{aligned}
$$

The government defaults in period $t = 1$ if $W(B_2, q_2, 1) - W(B_2, q_2, 0) > 0$. We can show this inequality does not hold when $[1 - h(1)]Y_1 \geq B_1$—that is, when the period-1 debt is less than the default costs incurred in period $t = 1$. To see this, note that defaulting in period $t = 1$ is never beneficial in terms of period-2 payoff since defaulting in period $t = 1$ the government loses the option value of defaulting in period $t = 2$. This option value is given by the term $\mathbb{E}\big[\max\{u(Y_2 - B_2) - u(h(1)Y_2), 0\}\big]$ and is always positive. If in addition we have that $[1 - h(1)]Y_1 \geq B_1$, then there are also no current gains due to default because $u(h(1)Y_1 + qB_2) - u(Y_1 + q_2 B_2 - B_1) \leq 0$.

When $[1 - h(1)]Y_1 < B_1$, the default decision in period $t = 1$ depends on the difference between current gains from default and future losses. But these gains and losses are functions of the issuance B_2, which is an equilibrium object. In order to characterize the non-default region, consider the problem

$$\max_{B_2 \geq 0} \left\{ u(h(0)Y_1 + q_2^{nd}(B_2)B_2 - B_1) + \beta \mathbb{E}_{Y_2}\left[V_2(B_2, 0, Y_2) \right] \right\} \tag{14}$$
$$\text{subject to } W(B_2, q^{nd}(B_2), 0) \geq W(B_2, q^{nd}(B_2), 1),$$

where $q_2^{nd}(B_2) := \frac{1 - F(B_2/[1-h(1)])}{R}$. For a given issuance choice B_2, $q_2^{nd}(B_2)$ is the price associated with no default in period $t = 1$. This is the highest price the government can get for its debt and induces the highest issuance.

The solution to problem (14), which always exists, induces the highest possible welfare for the government under the assumption that there is no default in period $t = 1$. If the government in this case does not have incentives to default for any price $q_2 \in [0, q^{nd}(B_2)]$, then there is no equilibrium with default in period $t = 1$. Proposition 1 below formalizes this result and characterizes the non-default region as a function only of Y_1 and B_1. Its proof, and all proofs thereafter, are available in the appendix.

Proposition 1. *Let $B_2^{nd}(Y_1, B_1)$ be a solution to problem* (14)*. If the inequality*

$$\underbrace{u(h(1)Y_1) - u(Y_1 - B_1)}_{\text{Max current gains due to default}} \leq \beta \underbrace{\mathbb{E}\left[\max\{u(Y_2 - B_2^{nd}(Y_1, B_1)) - u(h(1)Y_2), 0\} \right]}_{\text{Future losses due to default at issuance } B_2^{nd}(Y_1, B_1)} \tag{15}$$

is satisfied then, in any equilibrium, there is zero probability of default in period $t = 1$. All the equilibria are payoff equivalent and the government utility is the maximum in problem (14)*.*

2.3.2 The default region

In Section 2.3.1, we provided conditions on Y_1 and B_1 such that all equilibria feature no government default in period $t = 1$. Similarly, we now provide conditions such that all equilibria feature government default in period $t = 1$.

Proposition 2. *If the inequality*

$$\sup_{B_2 \geq 0} \left\{ W(B_2, q_2^{nd}(B_2), 0) - W(B_2, q_2^{nd}(B_2), 1) \right\} < 0 \tag{16}$$

is satisfied, then, in any equilibrium, there is probability one of default in period $t = 1$. Moreover, all the equilibria are payoff equivalent and the government utility is $u\big(h(1)Y_1\big) + \beta\mathbb{E}\big[u\big(h(1)Y_2\big)\big]$.

This result is straightforward. If for every B, the government is better off by defaulting when investors pay the non-default price $q_2^{nd}(B)$, then there cannot be an equilibrium with repayment and default occurs with probability one.

2.3.3 The sunspot equilibrium region

In sections 2.3.1 and 2.3.2, inequalities (15) and (16) characterize the non-default and default regions. In these regions, equilibria either feature government repayment, when inequality (15) is satisfied, or default, when inequality (16) is satisfied. In this section, we show that when neither inequalities (15) or (16) are satisfied, there are equilibria with probability one of repayment in period $t = 1$, but also sunspot equilibrium where both repayment and default occurs with strictly positive probability.

Proposition 3. *If inequalities* (15) *and* (16) *are not satisfied, that is, if*

$$u(h(1)Y_1) - u(Y_1 - B_1) > \beta\mathbb{E}\big[\max\{u(Y_2 - B_2^{nd}(Y_1, B_1)) - u(h(1)Y_2), 0\}\big], \tag{17}$$

for any $B_2^{nd}(Y_1, B_1)$ that solves problem (14)*; and*

$$\sup_B \Big\{W(B, q_2^{nd}(B), 0) - W(B, q_2^{nd}(B), 1)\Big\} \geq 0, \tag{18}$$

then there is a non-default equilibrium with repayment in period $t = 1$, and sunspot equilibria featuring strictly positive probability of repayment and default in period $t = 1$. The equilibria are welfare ranked and attaining the maximum with no default in period $t = 1$.

Propositions 1–3 fully characterize all the possible equilibrium outcomes. When the conditions of Propositions 1 or 2 are satisfied, there is either equilibrium with or without default—but not both. Only when the conditions of Proposition 3 are satisfied that we get indeterminacy with sunspot equilibrium.

The sunspot equilibria we obtain, however, relies on limits to arbitrage that are embed into the model. Specifically, it relies on the assumption that investors can only buy a measure zero of bonds. To see this, assume that investors can bid on a positive measure of bonds in the auction and consider a sunspot equilibrium where, for some sunspot realizations, bonds are priced at their fundamental value and the government does not default in period $t = 1$; while for other realizations, all bonds are priced at zero and the government defaults in period $t = 1$. Does this equilibrium present an arbitrage opportunity? The answer is yes.

Suppose we are in a default realization of the sunspot. The lender can bid in the auction to buy all the bonds offered, B_2^{g*}, at the fundamental price of the non-default sunspot realization, $q_2^{nd}(B_2^{g*})$. This bid creates a floor on the auction price. In this case, it is optimal for the government not to default in period $t = 1$, as it would in the non-default sunspot realizations. The lender is indifferent between buying or not buying the bonds in the auction at the fundamental value $q_2^{nd}(B_2^{g*})$ because it is the expected payoff of the bonds. Since the lender owns $B_1 > 0$ and there is not period $t = 1$ default, she is paid its face value, which she would otherwise not get paid. This generates a strictly positive profit and, therefore, it is an arbitrage opportunity. Also note that, if there were a secondary market for period-1 bonds, the lender could at the same time buy more period-1 bonds and increase profits. So adding a secondary market for period-1 bonds would only strengthen our results.

A profitable arbitrage strategy would, in principle, preclude this outcome from being an equilibrium. It does not happen here because each lender can only bid on a measure zero of bonds, and not all the newly issued bonds B_2^{g*} in the auction. By assumption, only a positive measure of lenders can buy a positive measure of bonds. As a result, a lender alone cannot take advantage of the low price of the bonds in the auction to the point of undoing the default as an equilibrium outcome.[4]

[4] We could, alternatively, let investors use a Dirac function when biding in the auction. In this case, the integral over the bids can be strictly positive even if only one (measure zero) bidder has a positive bid.

3 The non-arbitrage refinement

In this section, we study an economy where a fraction of lenders ϵ can collude to take advantage of arbitrage opportunities for as long as to collude is an equilibrium in itself for those lenders. That is, we let lenders coordinate their actions but they are not able to commit to collusive behavior—they play a coalition game.

One should think of $\epsilon > 0$ as being an arbitrarily small number. In particular, we assume that $\epsilon \in (0, \bar{\epsilon})$, where $\epsilon < \bar{\epsilon} := \min \left\{ 1 - \frac{B_2^{nd}(Y_1, B_1)}{\bar{a}}; \frac{1}{2} \right\}$. This restriction guarantees that the purchase of any equilibrium bond issuance B_2^{g*} can be supported using resources not from the coalition so coalition members are not the marginal bidders. This is not a binding constraint when $\bar{a}$ is infinite. However, as we discuss in Section 3.3, this assumption becomes relevant when $\bar{a}$ is finite and resources are scarce.

3.1 The coalition game

Consider a symmetric equilibrium $\mathcal{E} = \left\{ V_1, V_2, \delta_1^*, \delta_2^*, B_2^{g*}, q_2^{min*}, bid^*, (b_2^{i*})_i, B_2^*, q_2^* \right\}$, and fix a subset of lenders $I^\epsilon \subset I$ with measure $\epsilon \in (0, \bar{\epsilon})$. We refer to $\mathcal{E}$ as the original equilibrium, and the set of lenders I^ϵ as the coalition. The original equilibrium $\mathcal{E}$ implies a game for the coalition lenders $i \in I^\epsilon$, where they take as given the bid strategy bid^* and period-1 bond demand $\hat{b}_1$ of the lenders $j \in I \setminus I^\epsilon$ that are not part of the coalition, as well as the government bond supply B_2^{g*} and default policies δ_1^* and δ_2^*.

A strategy profile for a lender $i \in I^\epsilon$ is the following. As before, at the time of the auction lenders know the bond supply B_2^g and the sunspot variable ζ. Lender $i \in I^\epsilon$ takes as given the bid of other lenders in the auction, $\boldsymbol{bid}^{\boldsymbol{\epsilon}}_{\boldsymbol{-i}}(B_2^g, q_2^{min}, \zeta) = \left\{ bid^j(B_2^g, q_2^{min}, \zeta) \right\}_{j \neq i}$, which includes the bid bid_j^* for investors $j \neq i$ that can be part of the coalition or not. Let $B_2^{\epsilon*}(B_2^g, q_2^{min}, \zeta)$ and $q_2^{\epsilon*}(B_2^g, q_2^{min}, \zeta)$ solve equations (3) and (5) given $\boldsymbol{bid}^{\boldsymbol{\epsilon}}_{\boldsymbol{-i}}$. Lender $i \in I^\epsilon$ is willing to buy any quantity of bonds as long as the price is no greater than

$$q_2^\epsilon(B_2^g, q_2^{min}, \zeta) = \frac{1}{R} \mathbb{E}_{Y_2} \left\{ 1 - \delta_2^* \left(B_2^{\epsilon*}, \delta_1^*(B_2^{\epsilon*}, q_2^{\epsilon*}), Y_2 \right) \right\}. \tag{19}$$

We omitted the arguments of $B_2^{\epsilon*} = B_2^{\epsilon*}(B_2^g, q_2^{min}, \zeta)$ and $q_2^{\epsilon*} = q_2^{\epsilon*}(B_2^g, q_2^{min}, \zeta)$ in equation

(19) to keep it short. Lender $i \in I^\epsilon$ bids

$$bid^i(B_2^g, q_2^{min}, \zeta) = \left(\bar{b}^\epsilon(B_2^g, q_2^{min}, \zeta), q_2^\epsilon(B_2^g, q_2^{min}, \zeta)\right), \tag{20}$$

where $\bar{b}^\epsilon(B_2^g, q_2^{min}, \zeta)$ satisfies $q_2^\epsilon(B_2^g, q_2^{min}, \zeta)\bar{b}^\epsilon(B_2^g, q_2^{min}, \zeta) \leq \bar{a}$. As before, the lender cannot commit to spend in the auction more than it is endowment $\bar{a}$.

Now we can define a symmetric equilibrium for the coalition I^ϵ.

Definition 2. *Given an equilibrium $\mathcal{E} = \left\{V_1, V_2, \delta_1^*, \delta_2^*, B_2^{g*}, q_2^{min*}, bid^*, (b_2^{i*})_i, B_2^*, q_2^*\right\}$, a symmetric coalition equilibrium for I^ϵ is a family $\mathcal{E}^\epsilon = \left\{bid^{\epsilon*}, (b_2^{\epsilon i*})_{i\in I^\epsilon}, B_2^{\epsilon*}, q_2^{\epsilon*}\right\}$ satisfying:*

1. *the lenders bid function is $bid^{\epsilon*}(B_2^g, q_2^{min}, \zeta) = \left(\bar{b}^\epsilon(B_2^g, q_2^{min}, \zeta), q_2^\epsilon(B_2^g, q_2^{min}, \zeta)\right)$, where $q_2^\epsilon(B_2^g, q_2^{min}, \zeta)\bar{b}^\epsilon(B_2^g, q_2^{min}, \zeta) \leq \bar{a}$, and the bond price $q_2^{\epsilon*} = q_2^\epsilon$ satisfies equation* (19)*;*
2. *the equilibrium period-*2 *quantity sold and bond price, $B_2^{\epsilon*}(B_2^g, q_2^{min}, \zeta)$ and $q_2^{\epsilon*}(B_2^g, q_2^{min}, \zeta)$, solve equations* (3) *and* (5)*; and*
3. *lenders bond allocation in the auction $b_2^{\epsilon i*}(B_2^g, q_2^{min}, \zeta)$ solves equation* (4) *for all $i \in I^\epsilon$.*

The equilibrium in Definition 2 is very similar to the one given by Definition 7. The difference is that it takes the strategies of the government and all lenders $i \in I \setminus I^\epsilon$ out the coalition as given. However, it is still best response for the lenders in the coalition I^ϵ to take actions consistent with the original equilibrium.

Lemma 1. *Consider an equilibrium $\mathcal{E} = \left\{V_1, V_2, \delta_1^*, \delta_2^*, B_2^{g*}, q_2^{min*}, bid^*, (b_2^{i*})_i, B_2^*, q_2^*\right\}$. Then, $\mathcal{E}^\epsilon = \left\{bid^*, (b_2^{i*})_{i\in I^\epsilon}, B_2^*, q_2^*\right\}$ is an equilibrium for the coalition I^ϵ associated with $\mathcal{E}$.*

3.2 The refinement

It is always an equilibrium for the coalition members to replicate the actions of the original equilibrium, but in some instances they could be better off with a joint deviation. When this is the case, the original equilibrium is not robust in the sense that a coalition could improve their payoff without any additional assumptions such as commitment within the coalition members. We formalize this idea below.

Consider an equilibrium $\mathcal{E}^\epsilon$ for the coalition I^ϵ associated with an equilibrium $\mathcal{E}$.

Lender's $i \in I^\epsilon$ payoff in the equilibrium $\mathcal{E}^\epsilon$ given an auction (B_2^g, q_2^{min}) is

$$v(B_2^g, q_2^{min}; \mathcal{E}, \mathcal{E}^\epsilon) = \mathbb{E}_{\zeta, Y_2}\left\{ \left[1 - \delta_1^*\left(B_2^{\epsilon *}, q_2^{\epsilon *}\right)\right] B_1 + \right. \\ \left. \left[\frac{1 - \delta_2^*\left(B_2^{\epsilon *}, \delta_1^*(B_2^{\epsilon *}, q_2^{\epsilon *}), Y_2\right)}{R} - q_2^{\epsilon *}\right] b_2^{\epsilon i *} \right\}, \tag{21}$$

where we omitted the arguments of $B_2^{\epsilon *} = B_2^*(B_2^g, q_2^{min}, \zeta)$ and $q_2^{\epsilon *} = q_2^*(B_2^g, q_2^{min}, \zeta)$ in equation (21) to keep it short.

Before providing our refinement definition, it is useful to define the payoff in the original equilibrium, which is the same payoff as in the coalition equilibrium where its members replicate the strategy of the original equilibrium, as described in Lemma 1. With slightly abuse of notation, call such payoff $v(B_2^g, q_2^{min}; \mathcal{E})$, which is given by

$$v(B_2^g, q_2^{min}; \mathcal{E}) = \mathbb{E}_{\zeta, Y_2}\left\{ \left[1 - \delta_1^*\left(B_2^*, q_2^*\right)\right] B_1 + \right. \\ \left. \left[\frac{1 - \delta_2^*\left(B_2^*, \delta_1^*(B_2^*, q_2^*), Y_2\right)}{R} - q_2^*\right] b_2^{i *} \right\}, \tag{22}$$

where again we omitted the arguments of $B_2^{\epsilon *} = B_2^*(B_2^g, q_2^{min}, \zeta)$ and $q_2^{\epsilon *} = q_2^*(B_2^g, q_2^{min}, \zeta)$.

We can easily characterize the payoff in equation (22). Under Assumption 2, the constraint $\bar{a}$ does not bind. As a result, in equilibrium the bond price $q_2^*(B_2^g, q_2^{min}, \zeta)$ is given by the expected repayment (one minus the expected default) so it cancels out in equation (22), which we formalize in the following lemma.

Lemma 2. *Let $\mathcal{E} = \left\{V_1, V_2, \delta_1^*, \delta_2^*, B_2^{g*}, q_2^{min*}, bid^*, (b_2^{i*})_i, B_2^*, q_2^*\right\}$ be an equilibrium. Then the equilibrium payoff given in equation* (22) *is $v(B_2^g, q_2^{min}; \mathcal{E}) = \mathbb{E}_\zeta\left[1 - \delta_1^*\left(B_2^*, q_2^*\right)\right] B_1$.*

We now provide our refinement definition.

Definition 3. *Let $\mathcal{E}$ be an equilibrium. We say that $\mathcal{E}$ survives the non-arbitrage refinement if $v(B_2^g, q_2^{min}; \mathcal{E}) \geq v(B_2^g, q_2^{min}; \mathcal{E}, \mathcal{E}^\epsilon)$ for all bond issuance $B_2^g \geq 0$, reservation price $q_2^{min} \geq 0$, and coalition equilibrium $\mathcal{E}^\epsilon$ associated with $\mathcal{E}$.*

An equilibrium survives the non-arbitrage refinement if a coalition does not find it profitable to play an equilibrium different from the original one. It is important to note that such condition has to be satisfied for all bond issuance $B_2^g \geq 0$ and reservation price q_2^{min}—not only for the ones associated with the equilibrium B_2^{g*} and q_2^{min*}. That guarantees that the government belief over outcomes out of the equilibrium path are also consistent with the non-arbitrage refinement.

Consider an equilibrium $\mathcal{E} = \left\{V_1, V_2, \delta_1^*, \delta_2^*, B_2^{g*}, q_2^{min*}, bid^*, (b_2^{i*})_i, B_2^*, q_2^*\right\}$. We call $\mathcal{E}$ a sunspot equilibrium if the probability of government default and repayment in period $t = 1$ are both strictly positive—that is, the probability of period-1 default $\mathbb{E}\left[\delta_1^*\left(B_2^*, q_2^*(B_2^*, \zeta)\right)\right] \in (0, 1)$. Below we provide our main result.

Proposition 4. *If there exists an equilibrium $\mathcal{E} = \left\{V_1, V_2, \delta_1^*, \delta_2^*, B_2^{g*}, q_2^{min*}, bid^*, (b_2^{i*})_i, B_2^*, q_2^*\right\}$ with strictly positive probability of period-1 repayment, $\mathbb{E}_\zeta\left[1 - \delta_1^*\left(B_2^*, q_2^*(B_2^*, \zeta)\right)\right] > 0$, and assumption 2 holds, then the probability of period-1 repayment has to be exactly one for all equilibria that survives the non-arbitrage refinement.*

An interesting corollary of Proposition 4 is that, in the sunspot equilibrium region defined by equations (17) and (18), every equilibrium that survives the non-arbitrage refinement has repayment in period $t = 1$ and are payoff equivalent. That is, Proposition 4 implies only two regions for equilibria that are robust to the non-arbitrage refinement. When condition (16) is satisfied, there is always default in period $t = 1$, lenders payoff is zero and the government payoff is $u(h(1)Y_1) + \beta\mathbb{E}_{Y_2}\left[u(h(1)Y_2)\right]$. When condition (16) is not satisfied, there is no default in period $t = 1$, lenders payoff is B_1 and the government payoff is the maximum of problem (14). Note that there could still be multiple equilibria, but these equilibria are payoff equivalent.

Corollary 1. *No sunspot equilibria survive the non-arbitrage refinement under assumption 2.*

The equilibria that survive the non-arbitrage refinement provide lenders and the government with the highest equilibrium payoff so it is important to understand under what conditions our equilibrium refinement based on non-arbitrage fails.

3.3 Limits to arbitrage

The previous results relied on investors having deep pockets—that is, $\bar{a}$ was infinity—as stated in Assumption 2. This assumption allowed even a small coalition of lenders to take advantage of the arbitrage opportunity emerging from sunspot equilibria. As a result, sunspot equilibria did not survive the non-arbitrage refinement. In what follows, we study properties of the refinement and equlibria when $\bar{a}$ is finite.

Assumption 3. *Lenders have finite resources—that is, $\bar{a} < \infty$.*

Under assumption 3, whether a coalition of lenders can take advantage of an arbitrage opportunity depends on whether it has enough resources. Consider a coalition I^ϵ of size $\epsilon > 0$. In a sunspot equilibrium, given new issuance B_2, the coalition can purchase all issued bonds at the non-default price if $\epsilon\bar{a} \geq q_2^{nd}(B_2)B_2$. In this case, the coalition can undo any sunspot equilibria by buying these bonds and making a profit. Moreover, if for some solution to Problem (14), given by $B_2^{nd}(Y_1, B_1)$, we have that $\epsilon\bar{a} \geq q_2^{nd}\left(B_2^{nd}(Y_1, B_1)\right)B_2^{nd}(Y_1, B_1)$, then it must be the case that the government achieves the utility associated with Problem (14). That is because the government can always issue $B_2^{nd}(Y_1, B_1)$, and any equilibrium that survives the refinement does not have default in period $t = 1$ associated with this issuance level.

Proposition 5. *Assume the conditions of Proposition 3 are satisfied—that is, equations* (17) *and* (18)*. Then no sunspot equilibrium survives the non-arbitrage refinement given in Definition 3 if $\epsilon\bar{a} \geq q_2^{nd}\left(B_2^{nd}(Y_1, B_1)\right)B_2^{nd}(Y_1, B_1)$ for some $B_2^{nd}(Y_1, B_1)$ that solves Problem* (14)*.*

Proposition 5 shows that a coalition is able to undo sunspot equilibria when it has enough resources—that is, when $\epsilon\bar{a} \geq q_2^{nd}\left(B_2^{nd}(Y_1, B_1)\right)B_2^{nd}(Y_1, B_1)$. As discussed before, this is a sufficient condition because it allows the coalition to exploit the arbitrage involving buying the new bonds in the auction at the non-default price. It is not necessary though. The coalition could have enough resources to buy the bonds cheaper, but at a price still high enough to eliminate the incentives of the government to default. There is of course a limit on how much a coalition can do with limited resources.

Proposition 6. *Assume the conditions of Proposition 3 are satisfied—that is, equations* (17) *and* (18)*. If $\epsilon\bar{a}$ is sufficiently small, then there exists sunspot equilibria that survives the non-arbitrage refinement given in Definition 3.*

When the coalition has little resources, meaning $\epsilon\bar{a}$ is close to zero, there is no price that the coalition can support in the auction which would eliminate the incentives of the government to default. As a result, some sunspot equilibria survive.

4 Model extensions

In Section 3, we showed how only fundamental equilibria survived our proposed refinement when limits to arbitrage do no bind. In this section, we provide several extensions of our benchmark model to better understand our results.

4.1 Credit-default swaps

Propositions 5 and 6, combined, show that whether the refinement eliminates sunspot equilibria depends on how much resources are available to the coalition. When the resources available are small, the coalition does not have enough resources to support a bond price that is high enough so the government would not want to default. As a result, the non-arbitrage refinement does not eliminate sunspot equilibria. In this section, we show that when lenders have access to a CDS market, they can use this market to raise resources, which reestablishes our main result that sunspot equilibria do not survive the non-arbitrage refinement we propose under certain parameters.

Lenders trade CDS contracts to protect against loses due to government default in period $t = 1$. We assume that CDSs settlement happens in the same period they are bought immediately after the default decision, so it is not a state variable, and without loss of generality we assume that there is no coupon payments.[5] That is, one unit of CDS bought in the beginning of period $t = 1$, before the government decides to default

[5]For simplicity, we do not consider a CDS market in period $t = 2$. That is inconsequential because the government default decision in the last period is deterministic. For this same reason, settling the CDS in period $t = 1$ or $t = 2$ is also inconsequential.

or not on B_1 and B_2, pays one unit of consumption good in the end of period $t = 1$ if the government defaults, and zero otherwise. CDSs are traded in a competitive market at price $p(B_2^g, q_2^{min}, \zeta)$, which opens immediately before the auction for new bonds.

We also assume that each lender CDS position x must belong to an interval $[\underline{x}, \bar{x}]$, satisfying $\underline{x} < 0 < \bar{x}$. This is a regularity condition that guarantees lenders will not buy/sell infinity amounts of CDS, in which case payoffs would not be well defined.

Lenders take the CDS price, $p = p^*(B_2^g, q_2^{min}, \zeta)$, and the price outcome of the auction for newly issued bonds, $q_2 = q_2^*(B_2^g, q_2^{min}, \zeta)$, as given. A CDS pays one if the government defaults in period $t = 1$, and zero otherwise. Therefore, lenders demand of CDS is a function of the expected default decision of the government.

There is no uncertainty when lenders are deciding their demand for CDS because Y_2 only enters the default decision $\delta_2^*(B_2, \delta_1, Y_2)$ in period $t = 2$, so the expected default decision is just $\delta_1^*(B_2, q_2)$. If p equals $\delta_1^*(B_2, q_2)$, lenders are willing to buy or sell any quantity of CDS $x \in [\underline{x}, \bar{x}]$. If p is lower than $\delta_1^*(B_2, q_2)$, lenders would like to buy infinite amounts of CDSs—or the upper bound $\bar{x}$. If p is higher than $\delta_1^*(B_2, q_2)$, lenders would like to sell infinite amounts of CDSs—or the lower bound $\underline{x}$. As a result, the demand for CDSs, which is a correspondence in this case, is

$$x(B_2, q_2, p) = \begin{cases} \bar{x} & \text{if } \; p < \delta_1^*(B_2, q_2) \\ [\underline{x}, \bar{x}] & \text{if } \; p = \delta_1^*(B_2, q_2) \\ \underline{x} & \text{if } \; p > \delta_1^*(B_2, q_2) \end{cases} . \tag{23}$$

Below we define a symmetric equilibrium for an economy with a CDS market.

Definition 4. *A symmetric equilibrium for an economy with a CDS market is given by a family* $\{V_1, V_2, \delta_1^*, \delta_2^*, B_2^{g*}, q_2^{min*}, bid^*, (b_2^{i*})_i, B_2^*, q_2^*, (x^{i*})_i, p^*\}$ *such that*

1. $\{V_1, V_2, \delta_1^*, \delta_2^*, B_2^{g*}, q_2^{min*}, bid^*, (b_2^{i*})_i, B_2^*, q_2^*\}$ *is a symmetric equilibrium;*
2. $p^*(B_2^g, q_2^{min}, \zeta) = \delta_1^*\left(B_2^*(B_2^g, q_2^{min}, \zeta), q_2^*(B_2^g, q_2^{min}, \zeta)\right)$, *and* $x^{i*} = 0$ *for all lender* $i \in I$.

Similarly, we can define a symmetric equilibrium for the coalition I^ϵ in an economy with a CDS market.

Definition 5. *A symmetric equilibrium for the coalition I^ϵ, given an equilibrium for an economy with a CDS market $\mathcal{E} = \{V_1, V_2, \delta_1^*, \delta_2^*, B_2^{g*}, q_2^{min*}, bid^*, (b_2^{i*})_i, B_2^*, q_2^*, (x^{i*})_i, p^*\}$, is a family $\mathcal{E}^\epsilon = \{bid^{\epsilon*}, (b_2^{\epsilon i*})_{i\in I^\epsilon}, B_2^{\epsilon*}, q_2^{\epsilon*}, x^{\epsilon*}\}$ satisfying:*

1. *the lenders bid function is $bid^{\epsilon*}(B_2^g, q_2^{min}, \zeta) = \left(\bar{b}^\epsilon(B_2^g, q_2^{min}, \zeta), q_2^\epsilon(B_2^g, q_2^{min}, \zeta)\right)$, where $q_2^\epsilon(B_2^g, q_2^{min}, \zeta)\bar{b}^\epsilon(B_2^g, q_2^{min}, \zeta) \leq \bar{a} - p^*(B_2^g, q_2^{min}, \zeta)x^{\epsilon*}(B_2^g, q_2^{min}, \zeta)$, and the bond price $q_2^{\epsilon*} = q_2^\epsilon$ satisfies equation* (19)*;*
2. *the equilibrium period-2 quantity sold and bond price, $B_2^{\epsilon*}(B_2^g, q_2^{min}, \zeta)$ and $q_2^{\epsilon*}(B_2^g, q_2^{min}, \zeta)$, solve equations* (3) *and* (5)*; and*
3. *lenders bond allocation in the auction $b_2^{\epsilon i*}(B_2^g, q_2^{min}, \zeta)$ solves equation* (4) *for all $i \in I^\epsilon$.*
4. *the CDS demand is $x^{\epsilon*}(B_2^g, q_2^{min}, \zeta) = x\left(B_2^{\epsilon*}(B_2^g, q_2^{min}, \zeta), q_2^{\epsilon*}(B_2^g, q_2^{min}, \zeta), p^*(B_2^g, q_2^{min}, \zeta)\right)$ for all $(B_2^g, q_2^{min}, \zeta)$;*

The refinement definition is the same as in Definition 3 and we omit it here. Below we have our main result for this section.

Proposition 7. *Assume the conditions of Proposition 3 are satisfied—that is, equations* (17) *and* (18)*. Then no sunspot equilibrium survives the non-arbitrage refinement given in Definition 3 if $\epsilon[\bar{a} + |\underline{x}|] \geq q_2^{nd}\left(B_2^{nd}(Y_1, B_1)\right)B_2^{nd}(Y_1, B_1)$ for some $B_2^{nd}(Y_1, B_1)$ that solves Problem* (14)*.*

The term $|\underline{x}|$ strengthens the non-arbitrage condition in Proposition 7 relative to Proposition 5, allowing it to eliminate more sunspot equilibria. It is worth noticing that it operates by increasing the investment capacity of investors. This implies that any other financial asset, or regulation, that relax borrowing constraints—that is, increase $\bar{a}$—would have similar effects in terms of eliminating sunspot equilibria.

There is an important assumption embedded in Definition 5 that is used to generate Proposition 7. The CDS price taken as given by lenders in the coalition is p^*, which is the price in the original equilibrium $\mathcal{E}$. One legitimate question is why the lenders out of the coalition do not update their beliefs once they see coalition members trading CDSs, in which case we would have another price $p^{\epsilon*}$ for the CDS.

To state the problem bluntly, the coalition arbitrage away sunspot equilibria by selling CDSs to raise funds and bid for bonds in the auction. But if the lenders out of the coalition see the coalition members selling CDS, why don't they anticipate that the coalition will save the government? And if they do anticipate it, why pay anything for protection? In Section 4.3, we allow for this possibility and obtain similar results.

4.2 Large lender: the role of bank syndicates

We have shown that no sunspot equilibrium survives the non-arbitrage refinement in Definition 3 whenever there is a coalition with enough resources. However, it is often the case that governments can sell bonds to bank syndicates besides selling to lenders in a primary auction. In this section, we modify our environment to allow for such type of bond issuance. We consider a single large financial institution, or bank syndicate, that can buy sovereign bonds directly from the government through a syndicated issuance.

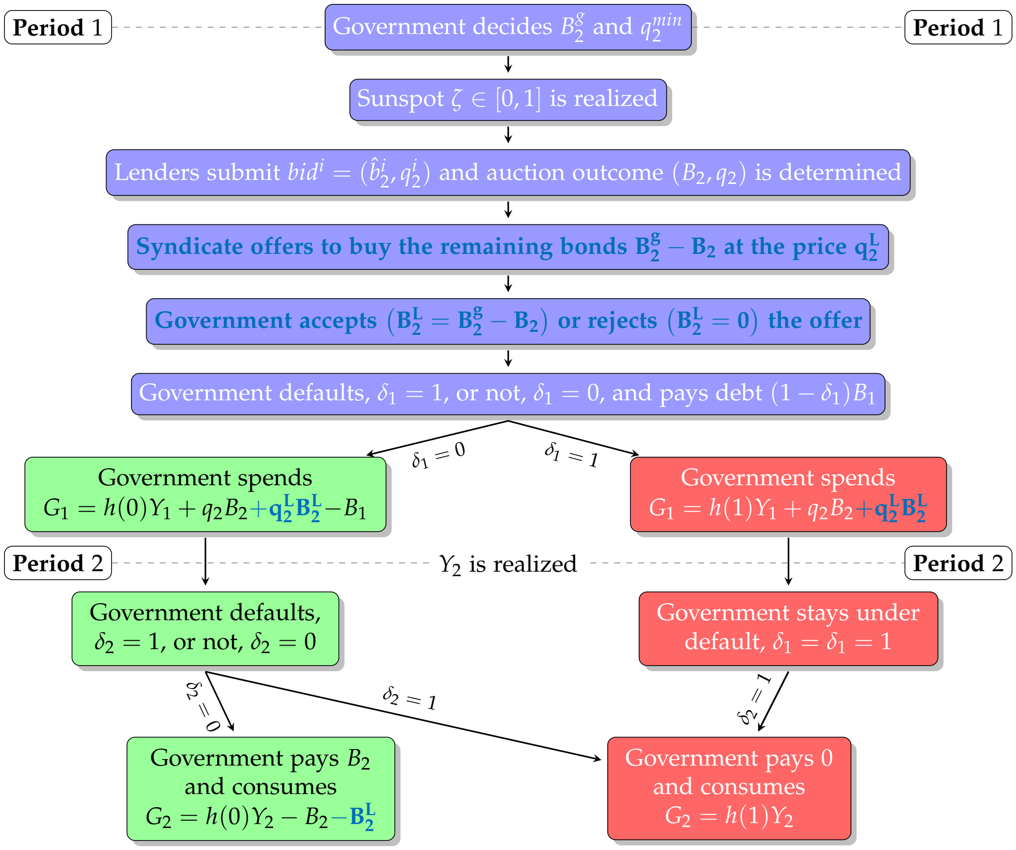
We model the bank syndicate as a lender i_L, with positive mass and available resources $\epsilon\bar{a}$. To keep the total resources of the economy comparable to the previous sections, the remaining lenders have total mass $1-\epsilon$, and each of them has measure zero and wealth equal to $\bar{a}$. We also assume that the syndicate does not hold period 1 bonds.[6] We can also interpret the lender i_L as a coalition of lenders, as before, but that can now commit to maximize the welfare of the coalition so lenders actions do not have to be individually optimal, as in Section 3.

The sequence of actions is depict in Figure 2, where the added actions relative to the benchmark model are highlighted in bold blue. To simplify the analysis, we consider the case in which i_L only acquires bonds through a syndicated loan but does not participate in the competitive auction. We also assume that i_L makes a take-it-or-leave-it offer to buy the remaining bonds not sold in the auction, $B_2^g - B_2$, at an endogeneous price q_2^L—that is, the syndicate chooses the price but the quantity traded is only whatever is left from the auction. Neither assumptions are necessary to generate our main results.

Period $t = 2$ **:** To solve the model, we proceed as before by backward induction. The solution to the government problem in period $t = 2$ is basically the same with the

[6]This assumption can be relaxed in a model with a secondary market for bonds, such as the one we present in Section 4.3. It is function is to guarantee that the syndicate does not enter an negotiation with the government holding period-1 bonds, otherwise the government would have more bargaining in the negotiation which could distort its issuance decisions in equilibrium.

Figure 2: Sequence of actions



difference that now the debt to be paid is $B_2 + B_2^L$. That is, the value function of the government and default policy function in period $t = 2$ are

$$\tilde{V}_2(B_2, B_2^L, \delta_1, Y_2) = V_2(B_2 + B_2^L, \delta_1, Y_2), \quad \text{and} \tag{24}$$

$$\tilde{\delta}_2^*(B_2 + B_2^L, \delta_1, Y_2) = \delta_2^*(B_2 + B_2^L, \delta_1, Y_2), \tag{25}$$

where V_2 and δ_2^* satisfy equations (6) and (7).

Period $t = 1$ **:** In period $t = 1$, the default decision as a function of the outcomes of the auction and syndicate offer is also similar to the solution characterized by equations (8) and (9). The government defaults if, and only if, its utility of defaulting is strictly higher than its utility of not defaulting. Let the government interim utility be given by

$$\tilde{W}(R_1, B_2, B_2^L, \delta_1) = u\Big(h(\delta_1)Y_1 + R_1 - (1-\delta_1)B_1\Big) + \beta \mathbb{E}\left[\tilde{V}_2(B_2, B_2^L, \delta_1, Y_2)\right], \tag{26}$$

for a revenue $R_1 = q_2 B_2 + q_2^L B_2^L$. The government defaults if, and only if, $\tilde{W}(R_1, B_2, B_2^L, 1)$ is strictly bigger than $\tilde{W}(R_1, B_2, B_2^L, 0)$. The period-1 default policy function is given by

$$\tilde{\delta}_1^*(R_1, B_2, B_2^L) = \begin{cases} 0 & \text{if } \tilde{W}(R_1, B_2, B_2^L, 0) \geq \tilde{W}(R_1, B_2, B_2^L, 0) \\ 1 & \text{if } \tilde{W}(R_1, B_2, B_2^L, 0) < \tilde{W}(R_1, B_2, B_2^L, 1) \end{cases}. \tag{27}$$

The syndicate's problem to find the optimal TIOLI offer is given by

$$v(B_2, B_2^L, q_2) = \max_{q_2^L \geq 0} \left\{ \mathbb{E}_{Y_2} \left[\frac{1 - \tilde{\delta}_2^*(B_2 + B_2^L, 0, Y_2)}{R} \right] - q_2^L \right\} B_2^L \tag{28}$$

subject to

$$q_2^L B_2^L \leq \epsilon \bar{a}, \quad \text{and} \tag{29}$$

$$\tilde{W}(q_2 B_2 + q_2^L B_2^L, B_2, B_2^L, 0) \geq \max_{\delta_1 = 0,1} \left\{ \tilde{W}(q_2 B_2, B_2, 0, \delta_1) \right\}. \tag{30}$$

The syndicate faces two constraints when deciding on the optimal price offer q_2^L. Equation (29) states that it cannot lend more than the funds it has, and equation (30) states that it has to make an offer that the government accepts and does not default immediately on it. When $B_2^L = 0$, there may not exist $q_2^L \geq 0$ satisfying constraint (30) since it becomes independent of q_2^L. When $B_2^L > 0$, it is always feasible for the syndicate to offer a sufficient high price which satisfies constraint (30). However, such high price may not be feasible—that is, it does not satisfy constraint (29)—or may not be profitable—that is, it may imply that $v(B_2, B_2^L, q_2) < 0$. If there is no bond price $q_2^L \geq 0$ satisfying constraints (29) and (30), we define $v(B_2, B_2^L, q_2) = -\infty$.

Let $\mathcal{Q}_2^L$ be the argmax set of problem (28). The optimal take-it-or-leave-it offer is

$$q_2^{*L}(B_2, B_2^L, q_2) = \begin{cases} 0 & \text{if } v(B_2, B_2^L, q_2) < 0 \\ \sup \mathcal{Q}_2^L & \text{if } v(B_2, B_2^L, q_2) \geq 0 \end{cases}, \tag{31}$$

and the quantity bought by the syndicate is

$$B_2^{*L}(B_2, B_2^L, q_2) = \begin{cases} 0 & \text{if } \ v(B_2, B_2^L, q_2) < 0 \\ B_2^L & \text{if } \ v(B_2, B_2^L, q_2) \geq 0 \end{cases}. \tag{32}$$

The syndicate may be indifferent between funding the government or not if both lead to zero expected profits. We assume that in this case the syndicate offers the highest optimal price, $\sup \mathcal{Q}_2^L$. This is consistent with our prior assumption that the government does not default when it is indifferent between defaulting or not.

The equilibrium bidding strategy of investors in an auction is still characterized by equations (10) and (11), with the the caveat that now the default policy functions are given by $\tilde{\delta}_1^*$ and $\tilde{\delta}_2^*$ instead of δ_1^* and δ_2^*. At the time of the auction, all lenders know the supply of bonds in the auction, B_2^g, reservation price, q_2^{min}, and sunspot realization, ζ. They also anticipate the response functions of the syndicate and government.

Consider the bid profile $\boldsymbol{bid}(B_2^g, q_2^{min}, \zeta) = \left\{bid^j(B_2^g, q_2^{min}, \zeta)\right)\}_j$ and associated auction outcomes $B_2^*(B_2^g, q_2^{min}, \zeta)$ and $q_2^*(B_2^g, q_2^{min}, \zeta)$ implied by the auction rules and bid profile. That is, $B_2^*(B_2^g, q_2^{min}, \zeta)$ and $q_2^*(B_2^g, q_2^{min}, \zeta)$ solve equations (3) and (5) given $\boldsymbol{bid}(B_2^g, q_2^{min}, \zeta)$. Lender $i \in I$ takes as given the bid of other lenders in the auction, $\boldsymbol{bid}_{-i}(B_2^g, q_2^{min}, \zeta) = \left\{bid^j(B_2^g, q_2^{min}, \zeta)\right)\}_{j \neq i}$, as well as the response functions of the syndicate and government, and is willing to buy bonds if its price is no greater than

$$q_2(B_2^g, q_2^{min}, \zeta) = \frac{\mathbb{E}_{Y_2}\left[1 - \tilde{\delta}_2^*(B_2^* + B_2^{*L}, \tilde{\delta}_1^*(R_1^*, B_2^*, B_2^{*L}), Y_2)\right]}{R}, \tag{33}$$

where $R_1^* = q_2^* B_2^* + q_2^{*L} B_2^{*L}$. We omitted the arguments of $B_2^* = B_2^*(B_2^g, q_2^{min}, \zeta)$, $q_2^* = q_2^*(B_2^g, q_2^{min}, \zeta)$, $B_2^{*L} = B_2^{*L}(B_2^*, B_2^g - B_2^*, q_2^*)$ and $q_2^{*L} = B_2^{*L}(B_2^*, B_2^g - B_2^*, q_2^*)$ to keep it short.

Lender i bids

$$bid^i(B_2^g, q_2^{min}, \zeta) = \left(\bar{b}(B_2^g, q_2^{min}, \zeta), q_2(B_2^g, q_2^{min}, \zeta)\right), \tag{34}$$

where $\bar{b}(B_2^g, q_2^{min}, \zeta)$ satisfies $q_2(B_2^g, q_2^{min}, \zeta)\bar{b}(B_2^g, q_2^{min}, \zeta) \leq \bar{a}$. That is, the lender cannot commit to spend in the auction more than it is endowment $\bar{a}$.

The last step is to solve for the optimal debt issuance of the government, B_2^g, and reservation price, q_2^{min}. The government anticipates the auction bond quantity $B_2^*(B_2^g, q_2^{min}, \zeta)$ and price $q_2^*(B_2^g, q_2^{min}, \zeta)$, the syndicate quantity purchased $B_2^{*L}(B_2^g, q_2^{min}, \zeta)$ and price $q_2^{*L}(B_2^g, q_2^{min}, \zeta)$, and default decision $\tilde{\delta}_1^*(R_1^*, B_2^*, B_2^{*L})$. Then, it solves

$$\tilde{V}_1 = \max_{B_2^g, q_2^{min}} \left\{ \mathbb{E}_\zeta \left[\tilde{W}(R_1^*, B_2^*, B_2^{*L}, \delta_1) \right] \right\}, \tag{35}$$

for a revenue $R_1^* = q_2^* B_2^* + q_2^{*L} B_2^{*L}$, where $B_2^* = B_2^*(B_2^g, q_2^{min}, \zeta)$, $q_2^* = q_2^*(B_2^g, q_2^{min}, \zeta)$, $B_2^{*L} = B_2^{*L}(B_2^*, B_2^g - B_2^*, q_2^*)$, $q_2^{*L} = q_2^{*L}(B_2^*, B_2^g - B_2^*, q_2^*)$, and the expectation is taken over the sunspot variable ζ. Below we define a symmetric equilibrium.

Definition 6. *A family* $\left\{\tilde{V}_1, \tilde{V}_2, \tilde{\delta}_1^*, \tilde{\delta}_2^*, \tilde{B}_2^{g*}, q_2^{min*}, bid^*, (b_2^{i*})_i, B_2^*, q_2^*, v, B_2^{L*}, q_2^{L*}\right\}$ *is an equilibrium if it satisfies the following conditions:*

1. *the government period-*2 *value function* $\tilde{V}_2(B_2, B_2^L, \delta_1, Y_2)$ *satisfies equation* (24), *and the period-*2 *policy function* $\delta_2^*(B_2, \delta_1, Y_2)$ *satisfies equation* (25);
2. *the government period-*1 *value function* $\tilde{V}_1$ *satisfies equation* (35), *where the government bond issuance and reservation price* (B_2^{g*}, q_2^{min*}) *achieves the maximum in this problem, and the period-*1 *policy function* $\tilde{\delta}_1^*(R_1, B_2, B_2^L)$ *satisfies equation* (25);
3. *lender* $bid^i(B_2^g, q_2^{min}, \zeta)$ *satisfies* (34) *for all* i, *where* $q_2^*(B_2^g, q_2^{min}, \zeta)\bar{b}(B_2^g, q_2^{min}, \zeta) = \bar{a}$ *if* $q_2^*(B_2^g, q_2^{min}, \zeta) > 0$, *and the period-*2 *bond price function* $q_2^*(B_2^g, q_2^{min}, \zeta)$ *satisfies* (33);
4. *the equilibrium period-*2 *bond price* $q_2^*(B_2^g, q_2^{min}, \zeta)$ *solves* (3), *and lenders period-*2 *bond allocation in the auction* $b_2^{i*}(B_2^g, q_2^{min}, \zeta)$ *solves equation* (4) *for all* $i \in I$; *and*
5. *the syndicate take-it-or-leave-it offer satisfies* (31), *and quantity purchased satisfies* (32).

The important difference between the equilibrium definition 7 of our benchmark model and the one presented above is that lenders anticipate the offer made by the syndicate. If it is profitable to rescue the government after a failed auction, the syndicate will do it, and lenders know it. As before, a government defaulting for lacking of funding presents an arbitrage opportunity for the syndicate when there is sunspot equilibria. If the government is strictly better not defaulting in the good equilibrium, the syndicate can profit by offering slightly less than the price of the non-default equilibrium.

Proposition 8. *Assume the conditions of Proposition 3 are satisfied—that is, equations* (17) *and* (18)*. Then there exists no sunspot equilibrium if* $\epsilon\bar{a} \geq q_2^{nd}\left(B_2^{nd}(Y_1, B_1)\right)B_2^{nd}(Y_1, B_1)$ *for some* $B_2^{nd}(Y_1, B_1)$ *that solves Problem* (14)*. Reversely, for a given economy there exists small enough* $\epsilon > 0$ *associated with sunspot equilibria.*

There are two interesting observations from the above result worth mentioning. First, the syndicate does not rely on their previous bond holds, ϵB_1, to turn into a profit when rescuing the government after a failed auction due to sunspots. The reason is that the syndicate can always offer to the government a price q_2^L that is lower then it fair value $\mathbb{E}_{Y_2}\left[\frac{1-\tilde{\delta}_2^*(B_2+B_2^L,0,Y_2)}{R}\right]$. That is in contrast with the refinement based on the coalition where they actually do not profit from buying assets at the auction, only from avoiding the default on their B_1 holding. Because there is no competition within the syndicate, they collude on a lower TIOLI price, while the coalition equilibrium has to be individually optimal so they compete to buy bonds.

The second observation is that, while the coalition provides a refinement of equilibria, the extension of the model with a bank syndicate eliminates all sunspot equilibria. The reasoning is similar to the first observation. Since the syndicate collude to take action, there is no chance of miscoordination among the syndicate's banks. The syndicate rescues the government whenever it is profitable to do so, and in a sunspot equilibrium it is always profitable to rescue the government in the default sunspot state.

4.3 Out-of-equilibrium beliefs and credit-default swaps

In Section 4.1, we showed, for the case of CDSs, that other financial instruments can improve financial stability by allowing agents in the coalition to raisefunds and relax their resource constraint. However, we assumed that the trade of CDSs did not change non-coalition lenders' beliefs over the default state. That is, lenders out of the coalition who bought protection did so at the price consistent with default, even though by observing the coalition's lenders selling protection they should have inferred that they would rescue the government from defaulting.

In terms of the refinement theory, strictly speaking this is not a problem. If the non-coalition lenders are the majority of lenders, which we assume to be the case, they price the CDS. When they believe the government is going to default, they price it consistently. But in the case of a sunspot default, the coalition lenders would want to sell protection and save the government under those prices. Then non-coalition lenders should update their initial beliefs, and those beliefs are not robust to the coalition game—resulting in sunspot equilibria being refined away.

However, there are some concerns to apply this results in practice. If coalition lenders know that how non-coalition lenders will update their belief, and how the CDS price will move against them, once they try to sell CDSs, they also know this attempt is pointless. So why would they coordinate to rescue the government in the first place? One possible answer is that lenders may also have other reasons to trade CDSs, like liquidity shocks, and observing some CDSs being sold is not enough information to make lenders update their beliefs. While one could make such argument theoretically consistent by extending the model with a decentralized CDS market and making lenders receive preference shocks, one could still argue that a large enough movement in the market, sufficient to rescue a government that is deeply indebted, would provide a strong enough signal for non-coalition lenders to update their beliefs.

We provide a different answer to this problem. Let us consider an extension to the model in Section 4.2 where the bank syndicate can trade both period 1 bonds and CDSs prior to making an offer to the government. Analogous to Section 4.1, each lender can sell at most $-\underline{x}$, and buy at most $\bar{x}$, units of CDSs. Since the bank syndicate has measure ϵ, we assume that it can sell $-\epsilon\underline{x}$, and and buy $\epsilon\bar{x}$. The syndicate does not hold bonds so we assume that it can short an amount $-\epsilon\underline{b}_1$ of bonds, where $\underline{b}_1 \leq 0$, and can buy at most $\epsilon\bar{a}/q_1$ of bonds, where q_1 is the price of period-1 it pays per unit of bonds.

We assume that after the government bond auction, and prior to the syndicate TIOLI offer to the government, the syndicate makes a TIOLI offer to lenders $\mathcal{X} = (b_1, q_1, x, p)$ to buy b_1 units of the bond at price q_1 and x units of the CDS at price p. The offer has to satisfy $b_1 \in \left[\frac{\epsilon\underline{b}_1}{1-\epsilon}, \frac{\epsilon\bar{a}}{(1-\epsilon)q_1}\right]$ and $x \in \left[\frac{\epsilon\underline{x}}{1-\epsilon}, \frac{\epsilon\bar{x}}{1-\epsilon}\right]$ to be feasible.

The equilibrium definition follow the same principles of Definition 6 and we omit it here to keep the presentation short. One important difference is that lenders now have to update their beliefs as a function of the offer $\mathcal{X} = (b_1, q_1, x, p)$ made by the syndicate. This action should be out of the equilibrium path since the syndicate's offer is anticipated by lenders. However, this update by lenders will discipline what offers the syndicate can make as we will see later.

After observing the offer $\mathcal{X} = (b_1, q_1, x, p)$, lenders can potentially update their beliefs. But note that bonds and their associated CDSs have an inherent relationship; whenever one pays off, the other does not. As a result, one's price is the mirror of the other. That is, let $\mu(B_2, B_2^L, q_2, \xi, \mathcal{X})$ be the probability lenders assign to the government defaulting in period $t = 1$ after observing the auction outcomes and the offer from the syndicate. The price that makes lenders indifferent between buying or selling the bond b_1 is then $q_1 = 1 - \mu(B_2, B_2^L, q_2, \mathcal{X})$. Since the CDS pays out when the period $t = 1$ bond does not, the price that makes lenders indifferent between buying or selling the CDS is $p = \mu(B_2, B_2^L, q_2, \xi, \mathcal{X})$. And what about one unit of each? The answer is $q_1 + p = 1$, which is independent of the belief $\mu(B_2, B_2^L, q_2, \xi, \mathcal{X})$.

The above implies that an offer $\mathcal{X} = (b_1, q_1, x, p)$ with $b_1 = x$ and $q_1 + p = 1$ is always accepted, not matter what is the particular combination of q_1 and p. Note that the syndicate is also indifferent between selling or not a bundle $\mathcal{X} = (b_1, q_1, x, p)$ with $b_1 = x$ and $q_1 + p = 1$. So even if it makes such offer but it ended up not being able to rescue the government, the trading of $\mathcal{X}$ does not impose a risk to the syndicate.

Proposition 9. *Assume the conditions of Proposition 3 are satisfied—that is, equations* (17) *and* (18). *If* $\epsilon[\bar{a} + \min\{|\underline{b}_1|, |\underline{x}|\}] \geq q_2^{nd}(B_2^{nd}(Y_1, B_1))B_2^{nd}(Y_1, B_1)$ *for some* $B_2^{nd}(Y_1, B_1)$ *that solves Problem* (14)*, then there exists no sunspot equilibrium.*

The intuition for Proposition 9 is straightforward from the discussion above. There is no cost for the syndicate to raise funds by offering to a bundle $\mathcal{X} = (b_1, q_1, x, p)$ with $b_1 = x = -\frac{\epsilon \min\{|\underline{b}_1|, |\underline{x}|\}}{1-\epsilon}$ and $q_1 = p = 1/2$. This bundle generates revenue $\epsilon \min\{|\underline{b}_1|, |\underline{x}|\}$, which can then be used to fund the government.

There is a very interesting aspect of this result when compared to the extension with only CDSs, presented in Section 4.1. When the syndicate can only sell CDSs, that is $\underline{b}_1 = 0$, then it will not be able to raise funds by selling CDSs if lenders update their beliefs and think the CDS fair price is zero. Similarly, when the syndicate can only sell bonds, that is $\underline{x} = 0$, then it will not be able to raise funds by selling bonds if lenders keep their beliefs and think the bond fair price is zero. Having both markets simultaneously allows the syndicate to raise funds no matter the beliefs of lenders.

4.4 Long term debt

Our non-arbitrage refinement extends to a model of long term debt. To save space and notation, we formalize the argument in appendix B.1. In this section we briefly describe how we extend the model to consider long term debt, we state the result informally and we provide intuition of why the non-arbitrage refinement works in this extension.

We modify the environment minimally by adding an initial period $t = 0$ and allowing maturity to be longer than one period. We assume that all non-defaulted bonds outstanding at the beginning of period $t = 2$ mature in that period. However, as it is usually assumed in long term debt models, e.g., Chatterjee and Eyigungor (2012), in periods $t = 0, 1$ outstanding bonds mature with probability $\lambda \in (0, 1)$ independently of when they were issued. That is, the duration of bonds outstanding at the beginning of periods $t = 0, 1$ is greater than one period. For simplicity, we assume that if the government defaults on its debt at any period it remains in financial autarky forever.

It is useful to define the price associated with no default in the current period $q_{t+1}^{\lambda,nd}$ in this version of the model. It is given by

$$q_{t+1}^{\lambda,nd}(B_{t+1}) := \begin{cases} \frac{1}{R}\mathbb{E}_{Y_1,\zeta_1}\left[\left(1-\delta_1(B_2^\lambda, q_2^\lambda)\right)\left(\lambda + (1-\lambda)q_2^\lambda\right)\right] & \text{if} \quad t=0, \text{ and} \\ \frac{1-F\left(B_2^\lambda/[1-h(1)]\right)}{R} & \text{if} \quad t=1, \end{cases} \tag{36}$$

where the superscript λ refers to the version of the model with maturity probability given by λ. For $t = 0$, B_2^λ and q_2^λ are determined optimally in equilibrium depending on the state of the economy and all agents decisions but we omit such dependence to simplify notation. For a given issuance choice B_{t+1}^λ, $q_{t+1}^{\lambda,nd}(B_{t+1}^\lambda)$ is the price associated with no default in period t. This is the highest price the government can get for its debt and induces the highest issuance.

Proposition 10. *[Informal] Assume that in some states of the economy conditions for a sunspot equilibrium hold, and both repayment and default can occur with strictly positive probability in any period $t = 0, 1$. Then no sunspot equilibrium survives the non-arbitrage refinement if the coalition has enough resources to: (i) bid the price for the bond consistent with no default in the current period, $q_{t+1}^{\lambda,nd}(B_{t+1})$; and (ii) afford to purchase the total amount of debt optimally chosen by the government when it expects that the price of bonds to be given by $q_{t+1}^{\lambda,nd}(B_{t+1})$.*

As in the case of short term debt, if a coalition of investors have enough resources to purchase enough bonds at the price associated to repayment in the current period, it is profitable for the coalition to eliminate self-fulfilling default equilibria.

Some remarks are in order. Firstly, with long term debt, it is profitable for the coalition to pay $q_1^{\lambda,nd}(B_1)$ and eliminate a self-fulfilling default in period $t = 0$, independently of the likelihood of a self-fulfilling default in $t = 1$. Notice that the price schedule $q_1^{\lambda,nd}(B_1)$ depends on q_2^{λ}, which incorporates the expectation of a self-fulfilling default in period $t = 1$, instead of depending on $q_2^{\lambda,nd}(B_2)$. In other words, in order for the coalition to make a profit from eliminating a self-fulfilling default in the current period, it does not need to have enough resources to eliminate all future self-fulfilling defaults. Secondly, because the non-arbitrage refinement is based on a coalition of investors taking specific actions outside the equilibrium path, self-fulfilling defaults are eliminated without compromising the budget constraint of the coalition of investors in future periods. Thirdly, despite the previous two remarks, the amount of bonds issued by the government in the current period does affect future budget constraints of investors. Therefore, the debt choice affects the probability of future occurrences of self-fulfilling crises, as in previous work in the literature. The key difference is that self-fulling defaults not only arise depending on the amount of sovereign debt outstanding and output, but also depending on the amount of wealth available to the coalition of investors.

5 Discussion on alternative models

Our model builds closely on the type of rollover crisis studied by Cole and Kehoe (2000), but similar arguments can be applied more broadly. Here we informally discuss how similar arguments apply to two alternative frameworks where multiple equilibria arise. We first focus on the type of multiplicity introduced by Calvo (1988). In his framework, multiplicity arises because, for a given budget deficit to be financed, multiple combinations of debt issuance and debt price can cover such deficit and be consistent with some expectation about default risk in equilibrium. We then study the framework proposed by Aguiar et al. (2020) which extends Cole and Kehoe (2000) by introducing uncertainty between the time of the auction and the repayment decision. We leave the formal details and proofs to appendices B.2 and B.3.

5.1 Multiplicity of equilibria à la Calvo (1988)

Calvo (1988) proposes a two periods model where multiple interest rate can arise in equilibrium, depending on investor's (rational) expectations about the probability of a default. As opposed to Cole and Kehoe (2000), a government does not choose to issue a certain amount of debt but, instead, it chooses to raise a certain amount in revenues, for example to cover some amount of fiscal deficit. Thus, a certain deficit d can be covered by a continuum of combinations of debt issuance B' and bond price q such that $qB' = d$. Among all possible combinations of price and quantity, more than one of them can be consistent with a no-arbitrage condition determining the price for some expected default probability, and an optimal default decision for a given value of B' that supports such expected default risk. This type of multiplicity also arises in Lorenzoni and Werning (2019) and Ayres et al. (2019).

Following Ayres et al. (2019), we slightly modify our benchmark model of section 2. As before there are two periods, $t = 1, 2$. At the beginning of period $t = 1$, the country has initial endowment, Y_1, and an initial stock of debt, $B_1 > 0$, with duration larger than one period. To allow for long term debt, we assume that each unit of debt matures with probability $\lambda \in (0, 1)$ at the beginning of $t = 1$, which is different from the model of section 2. As in our benchmark model, all the stock of debt at the beginning of $t = 2$ is assumed to mature in that period.

The most important differences with respect to our benchmark model are the timing of actions in period $t = 1$ and that the government chooses an amount of resources to be collected through debt issuance instead of next period's stock of debt. Within period $t = 1$, the timing of actions is as follows: (i) the government decides to repay maturing debt or default on the whole stock of debt and the sunspot shock ζ is realized; (ii) the government announces the size of the deficit d_1 (in units of good) to be covered with newly issued debt, (iii) each lender $i \in I$ submits a bid in the form of a one-step demand function, $bid^i = (\hat{b}_2^i, q_2^i)$, where the demand is $\hat{b}_2^i$ for any price $q_2 \leq q_2^i$, and zero otherwise; (iv) the government orders the bid from highest to lowest price and issues enough units of the bond to raise d_1 units of goods when investors pay the price of the marginal bid as in section 2.1.

In period $t = 2$, if the country is not in default, $\delta_1 = 0$, the government observes the realization of the endowment $Y_2 \in y^\ell, y^h$, with $y^\ell < y^h$, and stock of debt B_2 outstanding and decides whether to repay or default. Let $p \in (0, 1)$ be the probability that y^ℓ is realized. The default decision in period $t = 2$ is thus characterized by equation (7) as in our benchmark model of section 2.

In period $t = 1$ investors submit their bids after observing the realization of a sunspot shock, ζ, and the announced deficit, d_1. The price submitted by individual investors has to be consistent with the expected default probability and the no-arbitrage condition equating the expected return of the sovereign bond and the return of the risk free asset, R. That is,

$$R = \rho(q; d_1) \equiv \left[1 - \Pr\left(Y_2(1 - h(1)) < \frac{d_1}{q}\right)\right] \frac{1}{q}, \tag{37}$$

where q is the cutoff price in the auction and $\rho(q; d_1)$ represents the expected return to the lender. Given the optimal default policy of period $t = 2$, for each given deficit d_1, the expected return is

$$\rho(q; d_1) = \begin{cases} \frac{1}{q} & \text{if} \quad \frac{1}{q} \leq \frac{y^\ell[1-h(1)]}{d_1} \\ \frac{1-p}{q} & \text{if} \quad \frac{y^\ell[1-h(1)]}{d_1} < \frac{1}{q} \leq \frac{y^h[1-h(1)]}{d_1} \\ 0 & \text{if} \quad \frac{1}{q} > \frac{y^h[1-h(1)]}{d_1} \end{cases}.$$

We can thus define the correspondence relating the deficit d_1 to bond price:

$$\mathcal{Q}(d_1) = \begin{cases} \frac{1}{R} & \text{if} \quad d_1 \leq \frac{y^\ell[1-h(1)]}{R} \\ \frac{1-p}{R} & \text{if} \quad \frac{(1-p)y^\ell[1-h(1)]}{R} < d_1 \leq \frac{(1-p)y^h[1-h(1)]}{R} \\ 0 & \text{if} \quad d_1 > \frac{y^h[1-h(1)]}{R} \end{cases}.$$

It then follows that a necessary condition for multiple equilibrium prices is

$$d_1 \leq \frac{y^\ell[1-h(1)]}{R} < \frac{d_1}{1-p} \leq \frac{y^h[1-h(1)]}{R}.$$

After observing the initial state (Y_1, B_1, ζ), and anticipating the market response, the government chooses whether to repay its debt or default. In addition, in case of choosing to repay, the government decides how many resources to raise issuing new debt d_1. We focus on the values of (Y_1, B_1) such that the government does not default at the beginning of period $t = 1$, as those are the state in which the non-arbitrage refinement is relevant. After repayment, the government chooses d_1 to solve

$$\begin{aligned} d_1^* &= \arg\max u(Y_1 + d_1 - \lambda B_1) + \beta \mathbb{E}_{Y_2} \max\{u(Y_2 - B_2), u(h(1)Y_2)\} \\ &\text{subject to } B_2 = (1-\lambda)B_1 + d_1/q_2(d_1, \zeta). \end{aligned} \tag{38}$$

Definition 7. *An equilibrium is a family* $\{\delta_1^*, \delta_2^*, d_1^*, bid^*, (b_2^{i*})_i, B_2^*, q_2^*\}$ *where*

1. *the government period-*2 *policy function* $\delta_2^*(B_2, 0, Y_2)$ *satisfies equation* (7)*;*
2. *the government period-*1 *policy function* d_1^* *satisfies equation* (38)*;*
3. *the lenders bid function is* $bid^i(d_1, \zeta) = \left(\bar{b}(d_1, \zeta), q_2^*(d_1, \zeta)\right)$*, where* $q_2^*(d_1, \zeta)\bar{b}(d_1, \zeta) = \bar{a}$ *if* $q_2^*(d_1, \zeta) > 0$*, and the period-*2 *bond price function* $q_2^*(\cdot)$ *satisfies equation* (37)*; and*
4. *lenders period-*2 *bond allocation in the auction* $b_2^{i*}(d_1, \zeta)$ *solves equation* (4) *for all* $i \in I$*.*

Consider now the problem of the government assuming that the price schedule is consistent with no default when there are multiple equilibria, given by

$$\max_{d_1 \geq 0} \left\{ u(h(0)Y_1 + d_1 - \lambda B_1) + \beta \mathbb{E}_{Y_2} \left[V_2\big((1-\lambda)B_1 + d_1/q^{nd}(d_1), 0, Y_2\big) \right] \right\} \tag{39}$$

where $q_2^{nd}(d_1) := \frac{1}{R}$ if $d_1 R < y^h[1 - h(1)]$ and $q_2^{nd}(d_1) = 0$ otherwise. For a given deficit d_1, $q_2^{nd}(d_1)$ is the price associated with no default in period $t = 1$. This is the highest price the government can get for its debt and induces the highest issuance.

Proposition 11. *Suppose there is a coalition of investors such that $d_1^{nd} \leq \epsilon \bar{a}$, where d_1^{nd} solves problem* (39). *Then, the equilibrium with low bond price does not survive the non-arbitrage refinement.*

As before, the intuition behind this result is that investors in the coalition have incentives to pay the high price for the bond consistent with no-default because, even though they make zero profit for the newly purchased bonds, they increase the value of the bonds that they carried in their balance sheet from the past, as only a fractions λ of their bonds mature at the beginning of the period $t = 1$.[7]

5.2 Aguiar, Chatterjee, Cole and Stangebye (2021)

Aguiar et al. (2020) extends the Cole and Kehoe (2000) environment to introduce uncertainty between the time of the auction and the repayment. Specifically, they add a shock to the value function of the government in case of default. Such uncertainty generates another type of equilibria where bonds price are depressed, but strictly positive, and default and repayment occur with positive probability depending of the government shock.[8] This is in contrast with Cole and Kehoe (2000) where, at the time of the auction, based on the sunspot realization lenders know whether the government will default or not, which result in either the bond price being zero or being associated with probability one of repayment in the current period.

[7] We setup a model of long-term debt as a way to have investors holding unmatured debt and who could profit from the non-arbitrage refinement. Alternatively, we could have a model in which the government could issue debt in multiple rounds as in Lorenzoni and Werning (2019) and short term debt. In such case, if the low price equilibrium was about to be played, the coalition of investors could purchase bonds at a price that is higher than the low price equilibrium but that forces the government to issue bonds in a next round. Then, it could purchase enough bonds in the following round to eliminate the low price equilibrium and make a profit out of the bonds purchased in a previous round.

[8] Another interpretation of their result is that they establish the existence of a mixed strategy equilibrium. A zero bond price makes the government strictly better of by defaulting, a sufficiently high price makes the government strictly better off by not defaulting, and there is a price in between where the government is indifferent and play a mixed strategy with the exact probabilities that support the price.

Can this new type of equilibrium survive the refinement we propose here or the existence of a bank syndicate? Let us consider this possibility. Aguiar et al. (2020) established conditions for three equilibrium prices: $p_l = 0 < p_m < p_h$. At the price p_l the government defaults for sure, at the price p_m the government defaults with probability $\mu \in (0, 1)$ strictly between 0 and 1, and at the price p_h the government does not default. Note that the conditions are such that the government is strictly better of by defaulting when facing p_l, and strictly better off by not defaulting when facing p_h.

Now suppose the bank syndicate joins the primary auction for bonds when lenders are playing the equilibrium with price p_m. Since the government is strictly better off by not defaulting when facing p_h, and indifferent when facing p_m, the syndicate can submit a bid to buy the whole supply at price $p^* = p_h - \gamma$ for $\gamma \in (0, p_h - p_m)$ and the government will not default. In this case, the price p_h is the fair value of the bonds, and the syndicate's expected profit is $(p_h - p^*)B^g = \gamma B^g > 0$. Assuming the syndicate has enough resources to buy the whole supply of bonds at a price $p^* > p_m$, the syndicate can make strictly positive profit and, as a result, the price p_m is not an equilibrium outcome.

As with the model we presented, this result does not depend on the syndicate holding period $t = 1$ bonds, being able to buy period $t = 1$ bonds in the secondary market or sell CDSs in the secondary market, but such extensions would only strengthen the incentives for the syndicate to submit a higher bid at the auction. The only restriction is whether the coalition or syndicate has enough resources to bid up the auction price.

6 Empirical analysis

Our theory predicts that, controlling for fundamental risk, a larger country is more susceptible to self-fulfilling crises. That is because larger financing needs requires a coalition, or syndicate, to have more resources to rescue the government during a self-fulfilling crisis—making the crisis more likely to occur. This is in contrast with most models, where homotheticity implies that the a country's size should be irrelevant.

In this section, we investigate whether this implication holds empirically. We use two measures for country's size: Debt and GDP. Our measure of sunspot risk is the rolling

10-day standard deviation of the 1-year implied default probability (IDP), which we label Vol IDP. We compute IDP using CDS spreads at a daily frequency. Using a tight window limits the potential for fundamental risk (such as the release of quarterly GDP numbers) to explain a large share of IDP volatility.

Our preferred way of controlling for fundamental risk is to use the IDP level as its frequency is higher than traditional measures like Debt/GDP (daily rather than quarterly) and has considerably more explanatory power than them. Because the theory is about what happens before default, we also restrict the sample to have an IDP of less than 40%. We focus on emerging economies by also requiring the IDP to be greater than 1%. Table 1 lists summary statistics. In the appendix we provide a full description of our data and additional summary statistics by country.

Table 1: Summary statistics

	count	mean	sd	p25	p75	min	max
Daily variables							
CDS midpt.	58412	307.20	368.78	140.25	330.00	100.50	5073.23
IDP	58412	2.96	3.23	1.39	3.25	1.00	39.79
Vol IDP	58412	0.11	0.27	0.03	0.11	0.00	15.90
Quarterly variables							
	count	mean	sd	p25	p75	min	max
Log debt	890	25.52	1.63	24.22	26.51	20.98	30.07
Log GDP	890	26.22	1.57	24.81	27.60	22.95	30.13
Debt/GDP	890	0.64	0.46	0.32	0.84	0.06	2.43
Year	890	2014	3.39	2011	2017	2007	2021

Our benchmark specification is

$$\text{Vol IDP}_{k,q,t} = \alpha_q + \beta \log \text{Debt}_{k,q} + \gamma_1 \text{IDP}_{k,q,t} + \gamma_2 \text{IDP}^2_{k,q,t} + \epsilon_{k,q,t} \tag{40}$$

where k indexes countries, q time at a quarterly frequency, and t time at a daily frequency. This specification controls for fundamental risk by using the implied default probability and time fixed effects. We expect $\beta > 0$ as countries with more debt require more coalition or syndicate resources to execute the arbitrage opportunity.

The result is in model 1 of Table 2. As expected the coefficient is positive, and it is also statistically significant at conventional confidence levels.[9] The magnitude is of modest size, with a one standard deviation increase in log debt implying an increase in volatility equal to 6% of Vol IDP's standard deviation. Consistent with our expectation that the IDP absorbs all the fundamental default risk, including debt/GDP and its square (as is done in model 2) contributes almost no explanatory power, increasing the R^2 by a negligible amount.

Table 2: Volatility controlling for fundamental risk

	(1) Vol IDP	(2) Vol IDP	(3) Vol IDP	(4) Vol IDP
Log debt	0.0102	0.0110		
	(10.88)	(9.67)		
Log GDP			0.0113	0.0111
			(9.68)	(9.95)
IDP	0.0433	0.0437	0.0449	0.0438
	(10.19)	(9.80)	(10.25)	(9.82)
IDP^2	0.000338	0.000320	0.000293	0.000317
	(1.56)	(1.43)	(1.34)	(1.41)
Debt/GDP		-0.0671		-0.0311
		(-3.68)		(-1.83)
Debt/GDP^2		0.0332		0.0224
		(2.85)		(1.95)
Quarter FEs	Yes	Yes	Yes	Yes
Observations	58412	58412	58412	58412
R^2	0.39	0.39	0.39	0.39

Note: Newey-West standards errors with 10 lags have been used; t-stats are in parentheses; all regressions include a non reported constant.

While the amount of debt is the appropriate measure of size viewed through the model lens, a possible worry is it may be correlated with some other omitted variable that is producing $\beta > 0$. To alleviate this concern, we replace debt as a measure of size with GDP in models 3 and 4. In both models, the coefficient is still positive and similar

[9] We use Newey-West standard errors to correct for the autocorrelation in $\epsilon_{k,q,t}$ induced by the rolling window construction of Vol IDP$_{k,q,t}$.

to when using debt. Since more debt else equal would tend to worsen the country's situation while more GDP else equal would improve it, it is reassuring to have the same sign coefficient in both specifications.[10]

Another implication from the model is that we should expect the increased susceptibility to sunspots only when the country is in the self-fulfilling region. In general, identifying whether a country is in the self-fulfilling region is very difficult. However, at very low levels of the IDP, the model would predict the country is not in the crisis region. Consequently, the effect of size on IDP volatility should be smaller at small IDP levels and larger at high IDP levels. To test this, we consider a semi-parametric specification where the response of IDP volatility to size-dependence is allowed to vary by IDP bin. Specifically, we use

$$\text{Vol IDP}_{k,q,t} = \alpha_q + \sum_j \beta_j \mathbf{1}_{[\text{IDP}_{k,q,t} \in [p_j, p_{j+1})]} \log \text{Debt}_{k,q} + \gamma_1 \text{IDP}_{k,q,t} + \gamma_2 \text{IDP}^2_{k,q,t} + \epsilon_{k,q,t} \quad (41)$$

for $\{p_j\}$ the IDP quintiles.

As shown in model 1 of Table 3, the results are consistent with theory. At low levels of the IDP, the elasticity of IDP volatility to size is small and positive while at high IDP levels the elasticity is much larger. At the highest IDP level, a one standard deviation in size implies a volatility increase equal to 43% of Vol IDP's standard deviation. The estimates are also much more statistically significant at the higher IDP quantiles. For brevity, we do not report the estimates using log GDP in place of log debt, but the results are almost identical.

7 Concluding remarks

Self-fulfilling debt crisis has been an appealing explanation for many sovereign defaults events, and with the recent increase in governments' debt around the world, it will certainly stay in the mind of policy makers and academics for years to come. Traditionally,

[10] Most of our variation and statistical power comes from cross-sectional differences in size across countries (such as comparing Argentina and Uruguay). Including country fixed effects, the estimates become statistically insignificant.

Table 3: Level and volatility controlling for fundamental risk nonparametrically

	(1) Vol IDP		(2) Vol IDP	
IDP Q=2	-0.0700	(-1.53)	-0.0824	(-1.71)
IDP Q=3	-0.0584	(-1.27)	-0.0874	(-1.82)
IDP Q=4	-0.111	(-1.99)	-0.0978	(-1.70)
IDP Q=5	-1.459	(-8.59)	-1.328	(-8.69)
IDP Q=1 $\times$ Log debt	0.00330	(2.10)	0.00372	(2.06)
IDP Q=2 $\times$ Log debt	0.00653	(6.82)	0.00738	(6.90)
IDP Q=3 $\times$ Log debt	0.00717	(7.89)	0.00874	(8.40)
IDP Q=4 $\times$ Log debt	0.00998	(7.47)	0.00991	(6.71)
IDP Q=5 $\times$ Log debt	0.0709	(10.41)	0.0660	(10.84)
Debt/GDP			-0.0966	(-4.45)
Debt/GDP^2			0.0572	(3.91)
Quarter FEs	Yes		Yes	
Observations	58412		58412	
R^2	0.17		0.17	

Note: Newey-West standards errors with 10 lags have been used; t-stats are in parentheses; all regressions include a non reported constant.

the recommendation for governments to prevent self-fulfilling debt crisis has been to stay in a *safe* region, where the government is able to rollover its debt regardless of the market conditions for its debt. Going beyond that level would expose the government to default events that are unrelated to government fundamentals.

The above recommendation, however, only reflects the borrower side of the market and misses the actions that can be taken by lenders. As we show, a self-full-filing debt crisis creates an arbitrage opportunity where lenders can profit by rolling over government's debt. The basic principle is simple. During a self-fulfilling crisis, there are unrealized gains from trade. The value of the bonds of a well funded government is higher than what those liabilities are being traded. Therefore, a market participant with enough resources can fund the government and profit by increasing the value of the bonds they hold, or buying more bonds at depressed prices. As we show, the market participant could be a coalition of small lenders, a single large lender or a bank syndicate. We also show, for the particular case of credit-default swaps, that financial markets other than the bond market can provide resources to be channelled to the government—reducing the risk to self-fulling crisis.

The theory we propose opens the literature to several avenues for future research. The time horizon plays an important role in determining the amount of resources needed by the arbitrageur. If the arbitrageur needs to fund the government for several years until the government is clear from a confidence crisis, that may inhibit its ability to fund the government as needed due to limited resources. Another question is on the formation of coalitions. In 2020 we saw investors coalitions, formed around online forums, save firms that were doomed to file for bankruptcy. Can we influence the organization of such coalitions in order to eliminate negative, self-full-filing outcomes? And if a bank syndicate or large investor is ultimately preventing the self-fulfilling crisis, can she use that influence to extract rents from the government like a monopolist.

References

Aguiar, M. and M. Amador (2019). A contraction for sovereign debt models. *Journal of Economic Theory 183*, 842–875.

Aguiar, M. and M. Amador (2020). Self-fulfilling debt dilution: Maturity and multiplicity in debt models. *American Economic Review 110*(9), 2783–2818.

Aguiar, M., S. Chatterjee, H. Cole, and Z. Stangebye (2016). Quantitative models of sovereign debt crises. In *Handbook of Macroeconomics*, Volume 2, pp. 1697–1755. Elsevier.

Aguiar, M., S. Chatterjee, H. L. Cole, and Z. Stangebye (2020). Self-fulfilling debt crises, revisited. *Available at SSRN 3587646*.

Auclert, A. and M. Rognlie (2016). Unique equilibrium in the eaton–gersovitz model of sovereign debt. *Journal of Monetary Economics 84*, 134–146.

Augustin, P., F. Saleh, and H. Xu (2020). Cds returns. *Journal of Economic Dynamics and Control 118*, 103977.

Ayres, J., G. Navarro, J. P. Nicolini, P. Teles, et al. (2019). Self-fulfilling debt crises with long stagnations. Technical report, Federal Reserve Bank of Minneapolis.

Bocola, L. and A. Dovis (2019). Self-fulfilling debt crises: A quantitative analysis. *American Economic Review 109*(12), 4343–77.

Calvo, G. (1988). Servicing the public debt: The role of expectations. *The American Economics Review 78*(4), 647–661.

Chatterjee, S. and B. Eyigungor (2012). Maturity, indebtedness, and default risk. *American Economic Review 102*(6), 2674–99.

Cole, H. L. and T. J. Kehoe (2000). Self-fulfilling debt crises. *The Review of Economic Studies 67*(1), 91–116.

Conesa, J. C. and T. J. Kehoe (2017). Gambling for redemption and self-fulfilling debt crises. *Economic Theory 64*(4), 707–740.

Eaton, J. and M. Gersovitz (1981). Debt with potential repudiation: Theoretical and empirical analysis. *The Review of Economic Studies 48*(2), 289–309.

Lorenzoni, G. and I. Werning (2019). Slow moving debt crises. *American Economic Review 109*(9), 3229–63.

Roch, F. and H. Uhlig (2018). The dynamics of sovereign debt crises and bailouts. *Journal of International Economics 114*, 1–13.

A Appendix: Proofs

Proof of Proposition 1: Assume that equation (15) holds and suppose by the way of contradiction that there is an equilibrium $\left\{V_1, V_2, \delta_1^*, \delta_2^*, B_2^*, bid^*, (b_2^{i*})_i, q_1^*, q_2^*, \hat{b}_1, (b_1^{i*})_i\right\}$ with positive probability of period-1 default. That is, $\delta_1^*(B_2^*, q_2^*(B_2^*, \zeta)) = 1$ for all ζ in some set $\mathcal{A} \subset [0,1]$ with strictly positive measure. As we mentioned before, if $\delta_1^*(B_2^*, q_2^*(B_2^*, \zeta)) = 1$ for some realization of $\zeta \in [0,1]$ then we must have $[1-h(1)]Y_1 < B_1$. Otherwise, there is no default since there are no current gains from doing so.

If $B_2^* \leq B_2^{nd}(Y_1, B_1)$ we have a contradiction. Indeed, from equation (9) we know $q_2^*(B_2^*, \zeta) = 0$ for all $\zeta \in \mathcal{A}$. As a result, for all $\zeta \in \mathcal{A}$,

$$
\begin{aligned}
&W(B_2^*, 0, 1) - W(B_2^*, 0, 0) = \\
&\quad u(h(1)Y_1) - u(Y_1 - B_1) - \beta \mathbb{E}_{Y_2}\big[\max\{u(Y_2 - B_2^*) - u(h(1)Y_2), 0\}\big] \leq \\
&\quad u(h(1)Y_1) - u(Y_1 - B_1) - \beta \mathbb{E}_{Y_2}\big[\max\{u(Y_2 - B_2^{nd}(Y_1, B_1)) - u(h(1)Y_2), 0\}\big] \leq 0,
\end{aligned}
$$

where the first inequality is implied by $B_2^* \leq B_2^{nd}(Y_1, B_1)$ and the second by equation (15). But then $W(B_2^*, 0, 1) \leq W(B_2^*, 0, 0)$ and the government does not default, which contradicts the assumption that $\delta_1^*(B_2^*, q_2^*(B_2^*, \zeta)) = 1$.

Now assume that $B_2^* > B_2^{nd}(Y_1, B_1)$. The equilibrium definition implies that B_2^* and δ_1^* solve the maximization

$$
\max_{B_2, \delta_1} \mathbb{E}_\zeta \Big\{ u(h(\delta_1(B_2, q_2^*(B_2^g, q_2^{min}, \zeta))Y_1 + q_2^*(B_2^g, q_2^{min}, \zeta)B_2 - B_1[1 - \delta_1]) + \beta \mathbb{E}_{Y_2}\left[V_2(B_2, \delta_1, Y_2)\right] \Big\}.
$$

We can consider two possibilities separately. First, consider the case that $q_2^*(B_2^*, \zeta) = 0$ for a measure one of ζ. This would be the case if either $\delta_1^*(B_2^*, q_2^*(B_2^*, \zeta)) = 1$ for a measure one of ζ or $\delta_2^*(B_2^*, 0, Y_2) = 1$ for a measure one of Y_2. In this case, the maximum above is $u(h(1)Y_1) + \beta \mathbb{E}\big[u(h(1)Y_2)\big]$. But equation (15) combined with assumption 1 (which implies that $B_2^{nd}(Y_1, B_1) > 0$ is strictly greater than zero), implies that

$$
\begin{aligned}
u(h(1)Y_1) + \beta \mathbb{E}\big[u(h(1)Y_2)\big] &< u(Y_1 + q_2^{nd}(B_2^{nd}(Y_1, B_1))B_2^{nd}(Y_1, B_1) - B_1) \\
&\quad + \beta \mathbb{E}\big[\max\{u(Y_2 - B_2^{nd}(Y_1, B_1)), u(h(1)Y_2)\}\big].
\end{aligned}
$$

Which contradicts B_2^* and δ_1^* solving the maximization above.

Now consider the case that $q_2^*(B_2^*, \zeta) > 0$ for a positive measure of ζ. That has two implications. First, $W(B, q^{nd}(B^*), 0) \geq W(B, q^{nd}(B^*), 1)$—otherwise, it would be optimal to default in period $t = 1$ at debt B^* and $q_2^*(B_2^*, \zeta) = 0$ for all ζ. Second, $\delta_2^*(B_2^*, 0, Y_2) = 0$ for a positive measure of Y_2—otherwise again $q_2^*(B_2^*, \zeta) = 0$ for all ζ. Given these two facts, we have that

$$
\begin{aligned}
&\mathbb{E}_\zeta\Big\{u(h(\delta_1^*(B_2^*,q_2^*(B_2^*,\zeta)))Y_1+q_2^*(B_2^*,\zeta)B_2^*-B_1[1-\delta_1^*(B_2^*,q_2^*(B_2^*,\zeta))])\\
&\qquad+\beta\mathbb{E}_{Y_2}\left[V_2(B_2,\delta_1^*(B_2^*,q_2^*(B_2^*,\zeta)),Y_2)\right]\Big\}\\
&<\mathbb{E}_\zeta\Big\{u(h(\delta_1^*(B_2^*,q_2^*(B_2^*,\zeta)))Y_1+q_2^*(B_2^*,\zeta)B_2^*-B_1[1-\delta_1^*(B_2^*,q_2^*(B_2^*,\zeta))])\\
&\qquad+\beta\mathbb{E}_{Y_2}\left[V_2(B_2^*,0,Y_2)\right]\Big\}\\
&=\mathbb{E}_\zeta\Big\{u(h(\delta_1^*(B_2^*,q_2^*(B_2^*,\zeta)))Y_1+q_2^*(B_2^*,\zeta)B_2^*-B_1[1-\delta_1^*(B_2^*,q_2^*(B_2^*,\zeta))]\\
&\qquad+\beta\mathbb{E}\big[\max\{u(Y_2-B_2^*),u(h(1)Y_2)\}\big]\Big\}\\
&\le\mathbb{E}_\zeta\Big\{u(h(\delta_1^*(B_2^*,q_2^*(B_2^*,\zeta)))Y_1+q_2^{nd}(B_2^*)B_2^*-B_1[1-\delta_1^*(B_2^*,q_2^*(B_2^*,\zeta))])\\
&\qquad+\beta\mathbb{E}\big[\max\{u(Y_2-B_2^*),u(h(1)Y_2)\}\big]\Big\}\\
&\le u(Y_1+q_2^{nd}(B_2^{nd}(Y_1,B_1))B_2^{nd}(Y_1,B_1)-B_1)+\beta\mathbb{E}\big[\max\{u(Y_2-B_2^{nd}(Y_1,B_1)),u(h(1)Y_2)\}\big].
\end{aligned}
$$

The first inequality above comes from the fact that the government is always better off in period $t=2$ if he gets there without having defaulted in period $t=1$ (this inequality is strict because for a positive measure of Y_2 he is better off by not defaulting); the second inequality comes from $q_2^*(B_2^*,\zeta)\le q_2^{nd}(B_2^*)$, which has to hold since $q_2^{nd}(B_2^*)$ is computed assuming that there is no default in period $t=1$; the third inequality comes from the definition of $B_2^{nd}(Y_1,B_1)$ and $\delta_1^{nd}(Y_1,B_1)$, and from observing that equation (15) implies that $\delta_1^{nd}(Y_1,B_1)$ has to equal zero.

This contradicts the hypothesis that B_2^* and δ_1^* are optimally chosen, solving the maximization stated earlier. This concludes the proof. □

Proof of Proposition 2: Consider an equilibrium $\Big\{V_1,V_2,\delta_1^*,\delta_2^*,B_2^*,bid^*,(b_2^{i*})_i,q_1^*,q_2^*,\hat{b}_1,(b_1^{i*})_i\Big\}$. Note that we must have $q_2^*(B_2^*,\zeta)$ smaller or equal to $q_2^{nd}(B^*)$ for every ζ. But then

$$
\begin{aligned}
&W(B_2^*,q_2(B_2^*,\zeta),0)-W(B_2,q_2(B_2^*,\zeta),1)\\
&=u(Y_1+q_2(B_2^*,\zeta)B_2^*-B_1)-u(h(1)Y_1+q_2(B_2^*,\zeta)B_2^*)+\beta\mathbb{E}\big[V_2(B_2^*,0,Y_2)-V_2(B_2^*,1,Y_2)\big]\\
&\le u(Y_1+q_2^{nd}(B^*)B_2^*-B_1)-u(h(1)Y_1+q_2^{nd}(B^*)B_2^*)+\beta\mathbb{E}\big[V_2(B_2^*,0,Y_2)-V_2(B_2^*,1,Y_2)\big]
\end{aligned}
$$

$$\leq \sup_{B} \Big\{ W(B, q_2^{nd}(B), 0) - W(B, q_2^{nd}(B), 1) \Big\} < 0,$$

where the inequalities come from the concavity of $u(\cdot)$ and equation (16). The above inequality implies that the government best response has to be to default in period $t = 1$. As a result, in any equilibrium, there is probability one of default in period $t = 1$, and the government utility is $u\big(h(1)Y_1\big) + \beta\mathbb{E}\big[u\big(h(1)Y_1\big)\big]$. This concludes the proof. $\square$

Proof of Proposition 3: First let us show that there is a no-default equilibrium. Suppose lenders believe the government will default if, and only if, the issuance choice B_2 satisfy

$$W(B_2, q_2^{nd}(B_2), 0) < W(B_2, q_2^{nd}(B), 1).$$

That is a consistent belief since $W(B_2, q_2^{nd}(B_2), 0) < W(B_2, q_2^{nd}(B_2), 1)$ implies that $W(B_2, 0, 0) < W(B_2, 0, 1)$ due to the concavity of the utility function.

In this case, the optimal choice of the government is to choose $B_2 = B_2^{nd}(Y_1, B_1)$. To see this, first note that such B_2 exists since, by equation (16), the set of B_2 satisfying $W(B, q_2^{nd}(B), 0) \geq W(B, q_2^{nd}(B_2), 1)$ is non-empty. We can also show that this set is compact due to the weak inequality and because it has to be bounded above—if B_2 is large enough, it is always optimal to default on period $t = 1$. Choosing $B_2^* = B_2^{nd}(Y_1, B_1)$ also has to be better than picking a B_2 satisfying $W(B_2, q_2^{nd}(B_2), 0) < W(B_2, q_2^{nd}(B_2), 1)$ and defaulting for sure in period $t = 1$ because

$$W(B_2^*, q_2^{nd}(B_2^*), 0) \geq W(B_2^*, q_2^{nd}(B_2^*), 1) \geq W(B_2^{nd}(Y_1, B_1), 0, 1) = W(B_2, 0, 1)$$

for any B_2 satisfying $W(B_2, q_2^{nd}(B_2), 0) < W(B_2, q_2^{nd}(B_2), 1)$. The last equality comes simply from the fact that, if the issuance bond price is zero and I default, then issuance volume B_2 does not affect the welfare.

Now let us look at the sunspot equilibrium. Consider the problem

$$\max_{B_2} \Big\{ (1 - \bar{\zeta}) W(B_2, q^{nd}(B_2), 0) + \bar{\zeta} W(B_2, q^{nd}(B_2), 1) \Big\}$$
$$\text{subject to } W(B_2, q^{nd}(B_2), 0) \geq W(B_2, q^{nd}(B_2), 1),$$

where $\bar{\zeta} \in (0,1)$. From the solution of the above problem—label it $B_2^{\bar{\zeta}}$—we can construct an equilibrium where the government issues $B_2^{\bar{\zeta}}$ and; if $\zeta \leq \bar{\zeta}$: lenders believe the government will default, the realized bond price q_2 is zero, and the government defaults; and if $\zeta > \bar{\zeta}$: lenders belief the government will not default, the realized bond price $q_2(B_2^{\bar{\zeta}})$ is zero, and the government does not default.

By construction not to default is optimal when $\zeta > \bar{\zeta}$ and bond price is $q^{nd}(B_2^{\bar{\zeta}})$. That is because $B_2^{\bar{\zeta}}$ satisfy the constraint $W(B_2, q^{nd}(B_2), 0) \geq W(B_2, q^{nd}(B_2), 1)$ imposed in the above problem. To see the default case note that, for $\bar{\zeta}$ small enough, we know $B_2^{\bar{\zeta}}$ has to be close to some solution $B_2(Y_1, B_1)$ of problem 14. But then, because inequality (15) is not satisfied, we have that $W(B_2^{\bar{\zeta}}, 0, 0) < W(B_2^{\bar{\zeta}}, 0, 1)$ and it is optimal for the government to default. Since the optimal government decisions are consistent with lenders beliefs

For the last, we need that picking B_2 satisfying the constraint $W(B_2, q^{nd}(B_2), 0) \geq W(B_2, q^{nd}(B_2), 1)$ is better than picking one that doesn't satisfy this constraint. For that to be true we assume lenders believe the government will default otherwise. Then we can justify this as an equilibrium outcome in the same way we did for the non-default equilibrium, which also implies that the government highest utility is the maximum in problem (14) in the non-default equilibrium. This concludes the proof. $\square$

Proof of Lemma 1: This proof is straightforward. When lenders $i \in I^{\epsilon}$ replicate the strategies of the equilibrium $\mathcal{E}$, then conditions 1–2 in the equilibrium definition 2 are restatements of conditions 3–4 in the equilibrium definition 7. Then, since $\mathcal{E}$ is an equilibrium, conditions 3–4 in definition 7 are satisfied, which implies the result. $\square$

Proof of Lemma 2: When $\bar{a}$ does not bind, the conditions imposed in equilibrium definition 7 imply that $q_2^{\epsilon *}(B_2^g, q_2^{min}, \zeta) = \frac{1-\delta_2^*\left(B_2, \delta_1^*(B_2^*, q_2^{\epsilon *}(B_2^*, \zeta)), Y_2\right)}{R}$. By replacing this equality in the payoff equation (21), we obtain the result. $\square$

Proof of Proposition 4 *(Sketch)*: Suppose we are in a default realization of the sunspot. The lender can bid in the auction to buy all the bonds offered, B_2^*, at any price not larger than the fundamental price of the non-default sunspot realization, $q_2^{nd}(B_2^*)$. This bid creates a floor on the auction price. In this case, it is optimal for the government not to default in period $t = 1$ since that was the outcome of the non-default sunspot realizations when the government faces the auction price $q_2^{nd}(B_2^*)$. The lender is indifferent between buying or not buying the bonds, B_2^*, in the auction at the fundamental value $q_2^{nd}(B_2^*)$ because it is the expected payoff of the bonds. The lender is also paid the face value of the period-1 bonds, $B_1 > 0$, which she would otherwise not get paid because the government was going to default. This generates a strictly positive profit—and, therefore, an arbitrage opportunity—that is inconsistent with the equilibrium refinement. $\square$

Proof of Proposition 5 *(Sketch)*: In the subgame associated with issuance $B_2^{nd}(Y_1, B_1)$, it must be the case that the government achieves the utility associated with Problem (14) since there cannot be a default—otherwise the coalition would deviate. As a result, the government can always issue $B_2^{nd}(Y_1, B_1)$, and any equilibrium that survives the refinement does not have default in period $t = 1$ associated with this issuance level. Then, we cannot have issuance B^* with default in period one. $\square$

B Appendix: Extensions and Alternative models

B.1 Long term debt

In this section we extend our analysis to show that the non-arbitrage refinement extends to a model of long term debt. We modify the environment minimally by adding an initial period $t = 0$ and allowing maturity to be longer than one period.

The maturity structure works as follows. All non-defaulted bonds outstanding at the beginning of period $t = 2$ mature in that period. However, in periods $t = 0, 1$ outstanding bonds mature with probability $\lambda \in (0, 1)$ independently of when they were issues, as it is usually assumed in long term debt models, e.g., Chatterjee and Eyigungor (2012). For simplicity, we assume that if the government defaults on its debt at any period it remains in financial autarky until the end of period $t = 2$.

Introducing period $t = 0$ and long-term debt requires some additional adjustments to our model. To differentiate value and policy functions from the ones in our model of section 2, we add a superscript λ to all equilibrium objects of the long term debt model in this section. Let the interim value for the government in period $t = 1$ of equation (8) be re-defined as

$$W_1^\lambda(Y_1, B_1^\lambda, \delta_0^\lambda, B_2^\lambda, q_2^\lambda, \delta_1^\lambda) = u\left(h(\max\{\delta_0^\lambda, \delta_1^\lambda\}) + (1-\delta_0^\lambda)q_2^\lambda[B_2^\lambda - (1-\lambda)B_1^\lambda]\right.$$
$$\left. -(1-\max\{\delta_0^\lambda, \delta_1^\lambda\})\lambda B_1^\lambda\right) + \beta\mathbb{E}[V_2^\lambda(B_2^\lambda, \delta_1^\lambda, Y_2)], \tag{42}$$

where expectations are taken over the realization of Y_2. For given initial values (B_0^λ, Y_0) and assuming that the government is not in default, let the government interim utility at $t = 0$ be given by

$$W_0^\lambda(Y_0, B_0^\lambda, B_1^\lambda, q_1^\lambda, \delta_0^\lambda) = u\left(h(\delta_0^\lambda) + q_2^\lambda[B_1^\lambda - (1-\lambda)B_0^\lambda] - (1-\delta_0^\lambda)\lambda B_0^\lambda\right)$$
$$+ \beta\mathbb{E}[V_1^\lambda(B_1^\lambda, \delta_0^\lambda, Y_1)], \tag{43}$$

where the expectation is taken over Y_1, and

$$V_1^\lambda(B_1^\lambda, \delta_0^\lambda, Y_1) = \begin{cases} \max_{B_2^{\lambda,g}, q_2^{\lambda,min}} \mathbb{E}_{\zeta_1}\left[W_1^\lambda\left(B_1^\lambda, 0, B_2^\lambda, q_2^\lambda, \delta_1^\lambda(B_2^\lambda, q_2^\lambda)\right)\right] & \text{if } \delta_0^\lambda = 0, \text{ and} \\ u\big(h(1)Y_1\big) + \mathbb{E}_{Y_2}\left[u\big(h(1)Y_2\big)\right] & \text{otherwise.} \end{cases} \tag{44}$$

where we remove the dependence of B_2^λ and q_2^λ on $(B_1^\lambda, B_2^{\lambda,g}, q_2^{\lambda,min}, \zeta_1)$ to simplify notation, and the sub-index under the expectation operators represent the source of uncertainty.

Also, in order to characterize the non-default region fot $t = 0, 1$, we need to consider the problem

$$\max_{B_{t+1}\geq 0}\left\{u\left(h(0)Y_t + q_{t+1}^{\lambda,nd}(B_{t+1})[B_{t+1} - (1-\lambda)B_t^\lambda] - \lambda B_t^\lambda\right) + \beta\mathbb{E}_{Y_{t+1}}\left[V_{t+1}^\lambda\big(B_{t+1}, 0, Y_{t+1}\big)\right]\right\} \tag{45}$$

subject to $W_t^\lambda(Y_t, B_t^\lambda, B_{t+1}, q_{t+1}^{\lambda,nd}(B_{t+1}), 0) \geq W_t^\lambda(Y_t, B_t^\lambda, B_{t+1}, q_{t+1}^{\lambda,nd}(B_{t+1}), 1)$, where

$$q_{t+1}^{\lambda,nd}(B_{t+1}) := \begin{cases} \frac{1}{R}\mathbb{E}_{Y_1,\zeta_1}\left[(1-\delta_1(B_2^\lambda,q_2^\lambda))\left(\lambda+(1-\lambda)q_2^\lambda)\right)\right] & \text{if} \quad t=0, \text{ and} \\ \frac{1-F\left(B_2^\lambda/[1-h(1)]\right)}{R} & \text{if} \quad t=1. \end{cases} \tag{46}$$

For $t = 0$, we omit the dependence of B_2^λ and q_2^λ on $(B_1, B_2^{\lambda,g}, q_2^{\lambda,min}, \zeta_1)$ to simplify notation. For a given issuance choice B_{t+1}^λ, $q_{t+1}^{\lambda,nd}(B_{t+1}^\lambda)$ is the price associated with no default in period t. This is the highest price the government can get for its debt and induces the highest issuance.

If $\delta_0^\lambda = 0$, all the results in sections 2 and 3 remain valid for the sub-game starting in period $t = 1$. Let $\mathcal{E} = (\cdot; Y_1, B_1, \delta_0)$ be an equilibrium of the two period model of section 2 for initial values $(Y_1, B_1, \delta_0 = 0)$. Also, let $\mathcal{E}_1^\lambda = (\cdot; Y_1, B_1, \delta_0)$ be an equilibrium of the sub-game starting in period $t = 1$ for the long term debt model of this section, given initial conditions (Y_1, B_1, δ_0) at the beginning of $t = 1$. Let a_1 denote investors' wealth at the beginning of period $t = 1$.

Lemma 3. *Let* $\mathcal{E} = (V_1, V_2, \delta_1^*, \delta_2^*, q_2^{min*}, bid^*, (b_2^{i*})_i, B_2^*, q_2^*; Y_1, B_1, 0)$ *be an equilibrium of the two period model of section 2. Then,* $\mathcal{E}_1^\lambda = (V_1, V_2, \delta_1^*, \delta_2^*, q_2^{min*}, bid^*, (b_2^{i*})_i, B_2^* - (1-\lambda)B*, q_2^*; Y_1, \lambda B_1, 0)$ *is an equivalent equilibrium of the sub-game starting in period* $t = 1$. *Moreover, no equilibrium of the sub-game starting in period 1 satisfies the non-arbitrage refinement if* $\epsilon a_1 > q_2^{nd}(B_2^{nd}(Y_1, B_1) - (1-\lambda)B_1))[B_2^{nd}(Y_1, B_1) - (1-\lambda)B_1)]$ *for some* $[B_2^{nd}(Y_1, B_1) - (1-\lambda)B_1)]$ *that solves Problem* (14).

To study the role of long term debt, we proceed backwards and solve the problems of the government and the investors at $t = 0$. As before, we start analyzing $t = 0$ with the government default decision. The relevant state at this point is the auction outcome $(B_1^\lambda, q_1^\lambda)$, where it is possible that the amount of bonds sold in the auction, B_1^λ, is smaller than the amount offered by the government, $B_1^{\lambda,g}$. Therefore, the default policy function in period $t = 0$ is given by

$$\delta_1^*(B_2, q_2) = \begin{cases} 0 & \text{if} \quad W_0^\lambda(Y_0, B_0^\lambda, B_1^\lambda, q_1^\lambda, 0) \geq W_0^\lambda(Y_0, B_0^\lambda, B_1^\lambda, q_1^\lambda, 1) \\ 1 & \text{if} \quad W_0^\lambda(Y_0, B_0^\lambda, B_1^\lambda, q_1^\lambda, 0) < W_0^\lambda(Y_0, B_0^\lambda, B_1^\lambda, q_1^\lambda, 1) \end{cases} . \tag{47}$$

Anticipating government's default decision, lenders bid decision in the auction as a function of the bond supply, $B_1^{\lambda,g}$, reservation price, $q_1^{\lambda,min}$, and sunspot realization, ζ_0. Consider the bid profile $\boldsymbol{bid}(B_1^{\lambda,g}, q_1^{\lambda,min}, \zeta_0) = \{bid^j(B_1^{\lambda,g}, q_1^{\lambda,min}, \zeta_0))\}_j$ and associated auction outcomes $B_1^{\lambda*}(B_1^{\lambda,g}, q_1^{\lambda,min}, \zeta_0)$ and $q_1^{\lambda*}(B_1^{\lambda,g}, q_1^{\lambda,min}, \zeta_0)$ implied by the auction rules and bid profile.

As before, lender $i \in I$ takes as given the bid of other lenders in the auction, $\boldsymbol{bid}_{-i}(B_1^{\lambda,g}, q_1^{\lambda,min}, \zeta_0) = \{bid^j(B_1^{\lambda,g}, q_1^{\lambda,min}, \zeta_0))\}_{j\neq i}$. Since lender i has zero mass, her bid has no influence on the auction outcome so the quantity sold and price in associated with bid profile $\boldsymbol{bid_{-i}}(B_1^{\lambda,g}, q_1^{\lambda,min}, \zeta_0)$ are the same for all $i \in [0,1]$ and coincide with $B_1^{\lambda*}(B_1^{\lambda,g}, q_1^{\lambda,min}, \zeta_0)$ and $q_1^{\lambda*}(B_1^{\lambda,g}, q_1^{\lambda,min}, \zeta_0)$. Therefore, lender i is willing to buy any quantity of bonds if its price is no greater than

$$q_1^{\lambda}(B_1^{\lambda,g}, q_1^{\lambda,min}, \zeta_0) = \frac{1}{R}\mathbb{E}_{Y_1,\zeta_1}\left\{\left[1 - \delta_1(B_2^{\lambda*}, q_2^{\lambda*}, \delta_0^{\lambda}(B_1^{\lambda*}, q_1^{\lambda*}))\right]\left[\lambda + (1-\lambda)q_2^{\lambda*}\right]\right\}, \quad (48)$$

where we omitted the arguments of $B_1^{\lambda*} = B_1^{\lambda*}(B_1^{\lambda,g}, q_1^{\lambda,min}, \zeta_0)$, $q_1^{\lambda*} = q_1^{\lambda*}(B_1^{\lambda,g}, q_1^{\lambda,min}, \zeta_0)$, $B_2^{\lambda*} = B_2^{\lambda*}(Y_1, B_2^{\lambda,g}, q_2^{\lambda,min}, \zeta_1)$, $q_2^{\lambda*} = q_2^{\lambda*}(Y_1, B_2^{\lambda,g}, q_2^{\lambda,min}, \zeta_1)$ in equation (48) to keep it short.

Let lenders' endowments be all given by a_0. Lender i bids

$$bid^i(B_1^{\lambda,g}, q_1^{\lambda,min}, \zeta_0) = \left(\bar{b}(B_1^{\lambda,g}, q_1^{\lambda,min}, \zeta_0), q_1(B_1^{\lambda,g}, q_1^{\lambda,min}, \zeta_0)\right), \quad (49)$$

where $\bar{b}(B_1^{\lambda,g}, q_1^{\lambda,min}, \zeta_0)$ satisfies $q_1^{\lambda}(B_1^{\lambda,g}, q_1^{\lambda,min}, \zeta_0)\bar{b}(B_1^{\lambda,g}, q_1^{\lambda,min}, \zeta_0) \leq a_0$. That is, the lender cannot commit to spend in the auction more than it is endowment a_0.

Now we turn to the government choice of bond issuance $B_1^{\lambda,g}$. The government anticipates the bond price $q_1^{\lambda*}(B_1^{\lambda,g}, q_1^{\lambda,min}, \zeta_0)$ and default decision $\delta_0^{\lambda*}(B_1^{\lambda}, q_1^{\lambda})$. Then, it solves

$$(B_1^{\lambda,g*}, q_1^{\lambda,min*}) \in \underset{B_1^{\lambda,g}, q_1^{\lambda,min}}{\arg\max}\left\{\mathbb{E}_{\zeta_0}\left[W_0^{\lambda}\left(Y_0, B_0^{\lambda}, B_1^{\lambda}, q_1^{\lambda*}(B_1^{\lambda,g}, q_1^{\lambda,min}, \zeta_0), \delta_0^{\lambda*}\left(B_1^{\lambda}, q_1^{\lambda*}(B_1^{\lambda,g}, q_1^{\lambda,min}, \zeta_0)\right)\right)\right]\right\}. \quad (50)$$

The expected utility of the government in the beginning of period $t = 0$ is

$$V_0^\lambda = \mathbb{E}_{\zeta_0}\left[W_0^\lambda\Big(Y_0, B_0^\lambda, B_1^{\lambda*}, q_1^{\lambda*}(B_1^{\lambda,g*}, q_1^{\lambda,min*}, \zeta_0), \delta_0^{\lambda*}\big(B_1^{\lambda*}, q_1^{\lambda*}(B_1^{\lambda,g*}, q_1^{\lambda,min*}, \zeta_0)\big)\Big)\right]. \quad (51)$$

Definition 8. *An equilibrium is a family* $\Big\{V_0^\lambda, V_1^\lambda, V_2^\lambda, \delta_0^{\lambda*}, \delta_1^{\lambda*}, \delta_2^{\lambda*}, B_1^{\lambda,g*}, q_1^{\lambda,min*}, B_2^{\lambda,g*}, q_2^{\lambda,min*},$ $bid_0^*, bid_1^*, (b_2^{i*})_i, B_1^{\lambda*}, q_1^{\lambda*}, B_2^{\lambda*}, q_2^{\lambda*}\Big\}$ *where*

1. *the family* $\mathcal{E}_1^\lambda = \Big(V_1^\lambda(B_1^\lambda, 0, Y_1^\lambda), V_2^\lambda, \delta_1^{\lambda*}, \delta_2^{\lambda*}., B_2^{\lambda,g*}, q_2^{\lambda,min*}, (b_2^{i*})_i, B_2^{\lambda*} + (1-\lambda)B_1^\lambda, q_2^{\lambda*};$ $Y_1, B_1^\lambda/\lambda, 0\Big)$ *is an equilibrium in the sub-game starting in period* $t = 1$ *according to Definition 7, for all pairs* (Y_1, B_1).
2. *the government period-*0 *utility* V_0^λ *satisfies equation* (51)*, and the period-*0 *policy functions* $\delta_0^{\lambda*}(B_1^\lambda, q_1^\lambda)$ *and* $B_1^{\lambda,g*}$, $q_1^{\lambda,min*}$ *satisfy equations* (47) *and* (50)*;*
3. *the lenders bid function is* $bid^i(B_1^{\lambda,g}, q_1^{\lambda,min}, \zeta_0) = \big(\bar{b}(B_1^{\lambda,g}, q_1^{\lambda,min}, \zeta_0), q_1^{\lambda*}(B_1^{\lambda,g}, q_1^{\lambda,min}, \zeta_0)\big)$*, where* $q_1^{\lambda*}(B_1^{\lambda,g}, q_1^{\lambda,min}, \zeta_0)\bar{b}(B_1^{\lambda,g}, q_1^{\lambda,min}, \zeta_0) = \bar{a}$ *if* $q_1^{\lambda*}(B_1^{\lambda,g}, q_1^{\lambda,min}, \zeta_0) > 0$*, and the period-*1 *bond price function* $q_1^{\lambda*}(\cdot)$ *satisfies equation* (48)*; and*
4. *the equilibrium period-*1 *bond price* $q_1^{\lambda*}(B_1^{\lambda,g}, q_1^{\lambda,min}, \zeta_0)$ *solves* (3) *applied to period-*1 *bond price, and lenders period-*1 *bond allocation in the auction* $b_1^{i*}(B_1^{\lambda,g}, q_1^{\lambda,min}, \zeta_0)$ *solves equation* (4) *applied to period-*11 *bond allocation for all* $i \in I$.

In period $t = 0$, the conditions for the existence of a self-fulfilling debt crises are given by

$$u(h(1)Y_0) - u(Y_0 - \lambda B_0^\lambda) > \beta\mathbb{E}_{Y_1}\left[\max\{V^{1(B_1^{\lambda,nd},0,Y_1) - V_1^\lambda(B_1^{\lambda,nd},1,Y_1),0\}}\right], \quad (52)$$

for any $B_1^{\lambda,nd}(Y_0, B_0)$ that solves problem (45) at $t = 0$; and

$$\sup_B \Big\{W_0^\lambda(B, q_1^{\lambda,nd}(B), 0) - W_0^\lambda(B, q_1^{\lambda,nd}(B), 1)\Big\} \geq 0, \quad (53)$$

where $q_1^{\lambda,nd}$ is defined as in equation (46). REFINEMENT:

Lender's $i \in I^\epsilon$ payoff in the equilibrium $\mathcal{E}^\epsilon$ given an auction (B_1^g, q_1^{min}) is

$$v(B_1^{\lambda,g}, q_1^{\lambda,min}; \mathcal{E}, \mathcal{E}^\epsilon) = \mathbb{E}_{\zeta_0,Y_1,\zeta_1}\Big\{\Big[1 - \delta_0^{\lambda*}(B_1^{\lambda\epsilon*}, q_1^{\lambda\epsilon*})\Big]\lambda B_0^\lambda +$$

$$\left[\frac{1-\delta_1^{\lambda*}\left(Y_1, B_2^{\lambda*}, q_2^{\lambda*}, \delta_0^{\lambda}(B_1^{\lambda*}, q_1^{\lambda*})\right)}{R}\left(\lambda+(1-\lambda)q_2^{\lambda\epsilon*}\right)-q_1^{\epsilon*}\right] b_1^{\epsilon i*}\Bigg\},\tag{54}$$

where we omitted the arguments of $B_1^{\epsilon*} = B_1^*(B_1^g, q_1^{min}, \zeta_0)$ and $q_1^{\epsilon*} = q_1^*(B_1^g, q_1^{min}, \zeta_0)$ in equation (54) to keep it short.

Proposition 12. *If there exists an equilibrium* $\mathcal{E}^\lambda = \Big\{V_0^\lambda, V_1^\lambda, V_2^\lambda, \delta_0^{\lambda*}, \delta_1^{\lambda*}, \delta_2^{\lambda*}, B_1^{\lambda,g*}, q_1^{\lambda,min*}, B_2^{\lambda,g*}, q_2^{\lambda,min*}, bid_0^*, bid_1^*, (b_2^{i*})_i, B_1^{\lambda*}, q_1^{\lambda*}, B_2^{\lambda*}, q_2^{\lambda*}\Big\}$ *with strictly positive probability of period-0 repayment,* $\mathbb{E}_{\zeta_0}\left[1-\delta_0^{\lambda*}(B_1^{\lambda*}, q_1^{\lambda*}(B_1^{\lambda*}, \zeta_0))\right] > 0$, *and assumption 2 holds, then the probability of period-0 repayment has to be exactly one for all equilibria that survive the non-arbitrage refinement.*

TO BE COMPLETED

B.2 Calvo (1988)

B.3 Aguiar et al (2021)

C Appendix: Data

Our raw data runs from 2007 to 2021 and contains observations of daily Sovereign CDS prices joined with quarterly gross central government debt and quarterly NSA annualized nominal GDP in U.S. dollars as available. We obtain the NY end of day CDS bid and ask prices for the 5-Yr USD Senior Sovereign CDS from CMA DataVision via Bloomberg with the CMAN pricing source setting. We directly source quarterly total, short, and long term gross central government debt in U.S. dollars from the joint World Bank and IMF Quarterly Public Sector Debt (QPSD) database. For GDP, we first source quarterly NSA nominal GDP in local currency from the IMF's International Financial Statistics (IFS) before converting to U.S. dollars with IFS's local currency exchange rates (or Euro exchange rate where applicable) and annualizing. For completeness we further enhance the data by imputing total, short, or long term debt when only two of the three variables are provided.

From the raw data we construct the CDS Midpoint as the average of the bid and ask price, and use it to derive the approximate 1-Yr Implied Default Probability (IDP) in percent. We assume a zero recovery rate and continuous coupon payments such that $IDP = 100 * (1 - e^{-\frac{CDSmidpt.}{10000}})$ following Augustin et al. (2020). We also calculate the volatility of the 1-Yr Implied Default Probability (IDP Vol) as the trailing 10-day standard deviation of IDP.

Table 4 reports summary statistics for IDP by country.

Table 4: IDP summary statistics by country

	count	mean	sd		count	mean	sd
Argentina	2033	11.28	8.38	Japan	191	1.20	0.14
Australia	22	1.26	0.21	Kenya	844	4.12	1.13
Austria	303	1.60	0.33	Korea, Rep.	62	1.59	0.18
Belgium	611	1.93	0.66	Latvia	1311	2.43	1.45
Brazil	2853	1.93	0.81	Lithuania	1202	1.99	0.76
Bulgaria	2002	1.93	0.85	Mexico	2268	1.36	0.40
China	315	1.23	0.16	Netherlands	160	1.16	0.09
Colombia	2389	1.48	0.49	New Zealand	45	1.23	0.30
Costa Rica	2888	2.85	1.10	Philippines	1332	1.37	0.30
Croatia	2075	2.82	0.94	Poland	692	1.72	0.52
Cyprus	1832	4.55	3.52	Portugal	2125	3.74	2.81
Czech Republic	308	1.30	0.23	Romania	2216	2.00	0.98
El Salvador	1968	3.70	0.63	Russian Federation	2655	2.01	0.97
Estonia	502	1.62	1.06	Rwanda	1032	3.91	1.36
France	338	1.69	0.37	Slovak Republic	380	2.06	0.62
Germany	61	1.05	0.04	Slovenia	1282	2.19	1.10
Greece	2450	7.89	6.75	Spain	1259	2.57	1.23
Guatemala	1943	2.64	0.84	Sri Lanka	409	7.59	3.69
Hungary	2011	2.58	1.31	Sweden	16	1.24	0.16
Iceland	1676	2.28	1.21	Switzerland	12	1.40	0.25
Indonesia	2346	1.63	0.40	Thailand	3	1.10	0.10
Ireland	1089	3.81	2.34	Turkey	3097	2.55	1.05
Israel	1065	1.41	0.30	United Kingdom	25	1.19	0.19
Italy	2616	1.99	1.01	Uruguay	98	1.48	0.36