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Bridging Bargaining Theory with the Regulation of a Natural Monopoly

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Abstract. In this paper, we integrate the bargaining theory with the problem of regulating a natural monopoly under symmetric information or asymmetric information with complete ignorance. We prove that the unregulated payoffs under symmetric information and the optimally regulated payoffs under asymmetric information define a pair of bargaining sets which are dual to (reflections of) each other. Thanks to this duality, the bargaining solution under asymmetric information can be obtained from the solution under symmetric information by permuting the implied payoffs of the monopolist and consumers provided that the bargaining rule satisfies anonymity and homogeneity. We also show that under symmetric (asymmetric) information the bargaining payoffs (permuted payoffs) obtained under the Egalitarian, Nash, and Kalai-Smorodinsky rules are equivalent to the Cournot-Nash payoffs of unregulated symmetric oligopolies, involving two, three, and four firms, respectively. Moreover, we characterize two bargaining rules using, in addition to (weak or strong) Pareto optimality, several new axioms that depend only on the essentials of the regulation problem.

Keywords: Monopoly regulation; Cournot oligopoly; cooperative bargaining.

JEL codes: C71; C78; D62.

1 Introduction

Regulation of a natural monopolist has been extensively studied under both symmetric and asymmetric information about production costs. Under symmetric information, the regulatory solution proposed by Dupuit (1844, 1952) and Hotelling (1938) suggests that the price of the product should be set at the marginal cost and the monopolist should be given a lump-sum subsidy to cover its fixed cost. If the regulatory objective

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makes an equitable compromise between the welfares of consumers and the monopolist, the solution under symmetric information can be optimal under asymmetric information as well, and it can be implemented, as shown by Loeb and Magat (1979), using a simple incentive scheme that delegates the output decision to the monopolist, which is also entitled –through a subsidy– to the whole economic surplus. Even though this scheme successfully induces the monopoly to choose an output level that would maximize the economic surplus, it also gives rise to an utter inequity because the surplus of consumers after the subsidy payment to the monopolist becomes zero. Because of this inequity, Loeb and Magat's (1979) delegation scheme cannot be optimal in cases where the regulatory objective favors consumers more than the monopolist. The regulatory solution in such cases, as proposed by Baron and Myerson (1982), is (outcome equivalent to) an incentive-compatible direct-revelation mechanism that asks the monopolist to report its cost and that gives no incentive to lie. This mechanism entitles the monopolist to an information rent which can be optimally limited, but not eliminated, by the regulator to maximize the expected social welfare. The information rent depends on many factors, involving the cost parameter of the monopolist, the size of the industry demand, the beliefs of the regulator, and the weight of the monopolist's welfare in the regulatory objective. Depending upon these factors, the monopolist's rent can be very small or high, rendering either the monopolist or consumers, or even both, extremely upset about their regulatory payoffs. Even worse, the regulation may endanger the very existence of the industry. Under symmetric information, the monopolist –if regulated according to the first-best regulatory solution– might become reluctant to operate as it would earn no economic profit. The same reluctance might display itself, yet less strongly, also under asymmetric information if the monopolist does not find the information rent (which is ensured by the regulator to be always nonnegative) sufficiently attractive.

The possibility that regulation might lead the monopolist to abandon the industry is a serious problem to face especially when the regulated product is a socially desirable good that can stimulate economic growth or that may result in new inventions and innovations. In practice, a well-known solution to this problem has been the wide use of patents, allowing inventors (potential monopolists) to obtain rents from their research and development activities. Another solution has been to leave a monopoly (a successful inventor) unregulated when there is reasonable evidence that the monopolist self-regulates its price and activities to avoid regulation.[1] Under the patent solution, the monopolist is entitled to the whole monopoly profit for a pre-determined period, after which it earns the regulated profit. Under the second solution, the monopolist escapes external regulation and earns its operating profit as long as it continues to self-regulate its price and output appropriately. Consequently, under both solutions the lifetime earning of the monopolist becomes a positive fraction of the lifetime total surplus generated in the market.

In this paper, we propose a third solution to ensure the sustainability of the mo-

[1]See Erfle and McMillan (1990) and Glazer and McMillan (1992).

nopolistic industry. We basically consider the possibility where the monopolist and consumers can cooperatively bargain over the set of payoffs attainable under different regulatory objectives. The theory for cooperative bargaining was first introduced by Nash (1950), who formally defined a bargaining problem, consisting of a bargaining set and a disagreement point, in an abstract theoretical setting and proposed a solution –named after him as the Nash bargaining rule– along with a characterization result using several axioms. Nash (1950), and many economists who used his model to propose and/or axiomatize several other solutions, only dealt with symmetric (complete) information. The extension of Nash's (1950) bargaining model to the case of asymmetric (incomplete) information is due to Harsanyi and Selten (1972), followed by Myerson (1979, 1984). Harsanyi and Selten (1972) considered a generalization of the Nash bargaining rule for two-person problems where the bargaining set is obtained by the strict equilibria of a particular choice mechanism under incomplete information, whereas Myerson (1979, 1984) considered problems where the bargaining set may be much larger than the one considered by Harsanyi and Selten (1972) as it contains payoff vectors generated by choice mechanisms satisfying Bayesian incentive-compatibility.[2]

In our paper, we construct the regulatory bargaining problem under symmetric information using the model of Nash (1950) and several bargaining rules developed after (or anonymously before) him. More specifically, we construct the bargaining set under symmetric information as the convex and comprehensive hull of the utility possibility frontier of the 'unregulated' industry, since the regulatory solution (the marginal cost pricing rule) assumed in the literature induces in the utility plane only a single point, which is inconsistent with the sustainability of the industry and also leaves no room for bargaining. On the other hand, we construct the bargaining set under asymmetric information as the convex and comprehensive hull of the utility possibility frontier induced by the regulatory mechanism of Baron and Myerson (1982) for an admissible set of social welfare functions. For both symmetric and asymmetric information cases, we assume that the monopolist is not allowed to operate if the monopolist and consumers fail to agree in bargaining. Therefore, in both cases we choose the disagreement point in bargaining such that consumers' payoff is always zero. On the other hand, we assume that the monopolist's disagreement payoff is equal to zero if its fixed cost is non-sunk and equal to the negative of its fixed cost if it is sunk.

At this point, we should state that our bargaining model under asymmetric information deviates, in an important aspect, from the approach in Myerson (1979, 1984), where agents bargain over the set of incentive-compatible choice mechanisms or

[2] Myerson (1979) and Myerson (1984) mainly differ with respect to the bargaining rules they study. Myerson (1979) shows that the Harsanyi and Selten's (1972) generalization of the Nash bargaining rule is applicable, under a modification for n-person games, to the new bargaining set he constructs using the incentive-compatibility condition, whereas Myerson (1984) shows that the solution proposed by Harsanyi and Selten (1972) violates a probability-invariance axiom that depends only on the decision-theoretically significant structures of the bargaining problem and proposes alternative solutions that satisfy this axiom along with several other plausible axioms.

the implied payoff allocations. In our paper, we assume that the agents (consumers and the monopolist) have already selected the general form of the incentive-compatible mechanism to be used, namely the optimal incentive-compatible regulatory mechanism proposed by Myerson (1982), but have not decided yet on the social welfare function that induces the particulars of this mechanism. Thus, agents in our model do not bargain over the set of admissible mechanisms as in Myerson (1979, 1984), they just bargain using the model of Nash (1950) over the set of social welfare functions, or more precisely over the set of implied expected payoff vectors, given a particular mechanism defined for each possible social welfare function. In this sense, the bargaining model we use under asymmetric information is a hybrid one that integrates the regulatory model of BM (1982) with the bargaining model of Nash (1950).

Given the bargaining problems we have described above, we prove that the bargaining sets under symmetric information and asymmetric information (with complete ignorance) are dual to (reflections of) each other with respect to the 45-degree line passing through the origin (the common disagreement point) when the fixed cost of production is zero. Thanks to this duality, the bargaining solution under asymmetric information can be obtained from the solution under symmetric information by permuting the implied payoffs of the monopolist and consumers provided that the bargaining rule satisfies the axioms of anonymity and homogeneity.[3] We also show that under symmetric (asymmetric) information the bargaining payoffs (permuted payoffs) obtained under the Egalitarian, Nash, and Kalai-Smorodinsky rules are equivalent to the Cournot-Nash payoffs of unregulated symmetric oligopolies, involving two, three, and four firms, respectively. Moreover, we characterize two bargaining rules using, in addition to (weak or strong) Pareto optimality, several new axioms that depend only on the essentials of the regulation problem. Basically, we define these axioms only with reference to the bargaining sets observed under symmetric and asymmetric information. By doing this, we aim to exploit all the information present in the economic environment at hand, as suggested earlier by Roemer (1988). Indeed, his suggestions are consistent with a more general view than implied by our approach in this paper as his suggestions also include the direct use of a mechanism theory on an economic exchange environment instead of the use of a bargaining problem involving a utility possibility frontier (bargaining set) along with a disagreement point. Since the dimension of our economic problem is minimal (due to the singleness of the product/firm to be regulated) and since we already obtain the bargaining set and the disagreement point for our problem (and propose our characterization axioms) using all available economic information (in reference to the regulatory solution of a mechanism design approach by BM), we avoid the general criticism of Roemer (1988) to the classical

[3] The duality concept we introduce for bargaining solutions, and also for bargaining sets, is only concerned with permuting utility allocations and should not be confused with the duality idea used by Chun and Peters (1991) to relate the lexigographic equal-loss solution they propose to the lexiographic Egalitarian solution (Lensberg, 1982; Chun and Peters, 1988) by switching the roles of the ideal point and the disagreement point.

bargaining theory on the ground that under many distributive mechanisms it is impossible to distinctly isolate two economic environments with distinct utility allocations if these environments correspond to the same bargaining problem (involving the same bargaining set and the disagreement point).

The rest of the paper is organized as follows: In Section 2 we present some basic structures and in Section 3 we report our results. Finally, we conclude in Section 4.

2 Basic Structures

Consider a monopolistic market where a single homogeneous good is produced. The inverse demand curve is given by

$$P(q) = a - q, \tag{1}$$

where $q \geq 0$ is the quantity of good demanded and a is a positive real number. We denote by $V(q)$ the gross surplus obtained from q units of production, i.e., $V(q) = \int_0^q P(x)dx$. The cost function of the monopolist is given by

$$C(q, \theta) = z + \theta q, \tag{2}$$

where $q \geq 0$ is the quantity of good supplied, $z \geq 0$ is the fixed cost, and $\theta \in [0, a)$ is the constant marginal cost of production. Assuming $\theta < a$, we ensure that the monopolist always finds it profitable to operate when there is no regulation. Let $\delta \in \{0, 1\}$ be an exogenous constant identifying whether the fixed cost is sunk ($\delta = 1$) or non-sunk ($\delta = 0$).

Let the list $\mathcal{M}^s = (a, z, \delta, \theta)$, along with the assumptions made on them, represent an admissible monopoly market under symmetric information. Given such a list, if the monopolist produces and sells q units of the good, its profit becomes

$$\pi(q, \theta) = P(q)q - \theta q - \delta z = (a - \theta)q - q^2 - z, \tag{3}$$

whereas consumers' surplus becomes

$$CS(q) = V(q) - P(q)q = \frac{q^2}{2}. \tag{4}$$

Below we will consider the regulation of the monopolist in two separate cases depending upon whether the information about the marginal cost is symmetric or asymmetric.

2.1 Regulation under Symmetric Information

Let us first consider the case where the monopolist and consumers have the same (symmetric) information about the marginal cost parameter θ. Notice from equations (3) and (4) that each value of q leads to a distinct welfare distribution in the industry. In the absence of any bargaining between consumers and the firm, the regulator's problem is to choose an output (q) to maximize the social welfare which equals to the sum of the consumers' surplus and a fraction of the firm's profit given by

$$SW(q,\theta) = CS(q,\theta) + \alpha\pi(q,\theta), \tag{5}$$

subject to the individual rationality condition for the firm ensuring $\pi(q,\theta) \geq -\delta z$. The parameter α in the above equation is a known constant that lies in the interval $[0,1]$. The solution to this problem requires, as was proposed by Dupuit (1844, 1952) and Hotelling (1938) for a similar problem, the use of the marginal cost pricing rule (the first-best rule) $p^f(\theta) = \theta$, and correspondingly the output rule $q^f(\theta) = a - \theta$, along with a fixed subsidy $s^f = (1-\delta)z$ which ensures that the firm is paid its fixed cost if this cost is non-sunk and paid nothing otherwise. Notice that this regulatory policy is independent of the welfare weight α, always entitles the firm to the smallest acceptable profit $\pi^f(\theta) = -\delta z$ and consumers to the whole net economic surplus $V(q^f(\theta)) - \theta q^f(\theta) - (1-\delta)z$.

2.2 Regulation Under Asymmetric Information

We now turn to consider the asymmetric information case where the true value of the marginal cost θ is privately known by the monopolist before any regulatory action takes place. Now, the regulator only knows that the parameter θ lies in some interval $[\theta_0, \theta_1)$ with $\theta_1 \geq \theta_0 \geq 0$ and distributed with some probability density function $f(.)$ representing her prior beliefs. We have excluded θ_1 from the admitted values of θ, unlike the model of BM (1982), to make the cost parameter consistent with the possibility that $\theta_0 = 0$ and $\theta_1 = a$, or equivalently consistent with the assumption of $\theta \in [0,a)$ that we have made after equation (2).

The goal of the regulator is to maximize the expected value of the social welfare under her prior beliefs. The solution to this problem is proposed by Baron and Myerson (BM) (1982) who by the Revelation Principle (Dasgupta, Hammond and Maskin, 1979; Myerson, 1979; Harris and Townsend, 1981) restrict themselves, with no loss of generality, to direct mechanisms that ask the firm to report a value for its marginal cost parameter and that give the firm no incentive lie. These mechanisms consist of policy functions $\langle p(.), q(.), r(.), s(.)\rangle$ such that when $\hat{\theta}$ is the firm's report for its private cost parameter, $p(\hat{\theta})$ and $q(\hat{\theta})$ are respectively the price and quantity that satisfy the inverse demand equation $p(\hat{\theta}) = P(q(\hat{\theta}))$, $r(\hat{\theta})$ is the probability that the firm is allowed to produce, and $s(\hat{\theta})$ is the subsidy paid by consumers to the firm. Given the cost report $\hat{\theta}$, the firm with the cost parameter θ obtains the profit $\pi(\hat{\theta},\theta) = p(\hat{\theta})q(\hat{\theta}) - z - \theta q(\hat{\theta}) + s(\hat{\theta})$. A mechanism $\langle p(.), q(.), r(.), s(.)\rangle$ is feasible if it satisfies the incentive-compatibility condition that $\pi(\theta,\theta) \geq \pi(\theta) \equiv \pi(\hat{\theta},\theta)$ for all $\theta, \hat{\theta} \in [\theta_0, \theta_1)$ and the individual rationality condition that $\pi(\theta) \geq -\delta z$ for all $\theta \in [\theta_0, \theta_1)$. (The last condition in BM (1982) is $\pi(\theta) \geq 0$ since they consider the case of $\delta = 0$.) BM (1982) shows that a mechanism $\langle p(.), q(.), r(.), s(.)\rangle$ satisfies the incentive-compatibility condition if $q(.)$ is non-increasing over $[\theta_0, \theta_1)$ and the profit of the firm is equal to

$$\pi(\theta) = \int_{\theta}^{\theta_1} q(x)r(x)dx + \pi(\theta_1). \tag{6}$$

The problem of the regulator is to find a feasible mechanism $\langle p(.), q(.), r(.), s(.)\rangle$ that maximizes the expected social welfare

$$\int_{\theta_0}^{\theta_1} SW(q(\theta), \theta)f(\theta)d\theta = \int_{\theta_0}^{\theta_1} \Big(CS(q(\theta), \theta) - s(\theta) + \alpha\pi(\theta)\Big) f(\theta)d\theta. \tag{7}$$

BM (1982) shows that there exists a unique solution to this problem. To simplify this solution for our results, we shall assume that $F(\theta)/f(\theta)$ is nondecreasing over $[\theta_0, \theta_1)$. Under this assumption which is known as the monotone (inverse) hazard rate property, the optimal regulatory mechanism proposed by BM (1982) suggests setting $\pi(\theta_1) = -\delta z$ and announcing $\langle p^{BM}(.), q^{BM}(.), r^{BM}(.), s^{BM}(.)\rangle$ that satisfy for all $\theta \in [\theta_0, \theta_1)$

$$p^{BM}(\theta) = \theta + (1-\alpha)\frac{F(\theta)}{f(\theta)} \tag{8}$$

$$q^{BM}(\theta) = P^{-1}\left(p^{BM}(\theta)\right) \tag{9}$$

$$r^{BM}(\theta) = \begin{cases} 1 & \text{if } V(q^{BM}(\theta)) - p^{BM}(\theta)q^{BM}(\theta) \geq (1-(1-\alpha)\delta)z \\ 0 & \text{otherwise} \end{cases} \tag{10}$$

$$\begin{aligned} s^{BM}(\theta) &= (1-\delta)z + \theta q^{BM}(\theta) - p^{BM}(\theta)q^{BM}(\theta) \\ &\quad + \int_{\theta}^{\theta_1} q^{BM}(x)r^{BM}(x)dx. \end{aligned} \tag{11}$$

Notice that the firm earns the profit $\int_{\theta}^{\theta_1} q^{BM}(x)r^{BM}(x)dx - \delta z$ when it truthfully reports its marginal cost parameter θ. This profit is the firm's informational rent due to its private information and it is smaller when the interval $[\theta_0, \theta_1)$ is narrower and (given a fixed density function f) the regulator is more certain about the true value of θ. As a matter of fact, the regulator becomes completely informed when the interval $[\theta_0, \theta_1)$ shrinks to $\{\theta\}$, the singleton set containing the true cost parameter. In this case, the informational rent of the firm becomes zero. The informational rent of the firm may also vary with the beliefs of the regulator since the policy functions $q^{BM}(.)$ and $r^{BM}(.)$ affecting this rent always depend on the term $F(\theta)/f(\theta)$, unless $\alpha = 1$.[4]

[4]The dependence on the regulatory mechanism on the prior beliefs of the regulator may lead to a moral hazard problem conjectured by many economists –including Crew and Kleindorfer (1986), Vogelsang (1988), Koray and Sertel (1990)– and formally studied by Koray and Saglam (1999, 2005).

In this study, we shall consider a completely ignorant regulator (endowed with minimal amount of prior information). Thus, we assume that $[\theta_0, \theta_1) = [0, a)$ and that the regulator's prior belief is a maximum entropy probability distribution, namely the uniform distribution f such that $f(\theta) = 1/a$ for all $\theta \in [0, a)$. Under these assumptions, the regulated price and output in equations (8) and (9) respectively become

$$p^{BM}(\theta) = (2 - \alpha)\theta \tag{12}$$

and

$$q^{BM}(\theta) = a - (2 - \alpha)\theta, \tag{13}$$

whereas the probability of regulation in equation (10) becomes

$$r^{BM}(\theta) = \begin{cases} 1 & \text{if } \theta \leq \theta^* \\ 0 & \text{otherwise} \end{cases} \tag{14}$$

with

$$\theta^* = \frac{a - [2(1 - (1 - \alpha)\delta)z]^{1/2}}{2 - \alpha}. \tag{15}$$

Consequently, the expected consumer and producer welfares become

$$\begin{aligned} CW^e &= \int_0^{\theta^*} \left(CS(q^{BM}(\theta), \theta) - s^{BM}(\theta) \right) f(\theta) d\theta \\ &= \frac{a\theta^*}{2} - (\theta^*)^2 + \frac{(4 - \alpha^2)(\theta^*)^3}{6a} - \frac{(1 - \delta)z\theta^*}{a} \end{aligned} \tag{16}$$

and

$$\begin{aligned} PW^e &= \int_0^{\theta^*} \pi(\theta) f(\theta) d\theta \\ &= \frac{(\theta^*)^2}{2} - \frac{(2 - \alpha)(\theta^*)^3}{3a} - \frac{\delta z \theta^*}{a}, \end{aligned} \tag{17}$$

respectively. In Figure 1, we plot the (ex-post) welfares of consumers and the firm (at a particular θ value) obtained under the modified version of the BM (1982) mechanism described above. Notice that the green-colored curve is the adjusted inverse demand function which shows the regulated output $q^{BM}(p)$ at each possible price p. This curve is below the inverse demand function since the regulated price $p^{BM}(\theta)$ is always above the marginal cost of production θ unless α is equal to 1. The green-colored area is the profit, or the information rent, of the firm, whereas the pink-colored area is the tax the firm must pay to consumers out of the operating profit $(p^{BM}(\theta) - \theta)q^{BM}(\theta)$ to be able to earn its information rent $\pi(\theta)$. The light-blue-colored area is the consumer surplus and lastly the black-colored area is the deadweight loss arising under the asymmetric information when $\alpha \neq 1$.

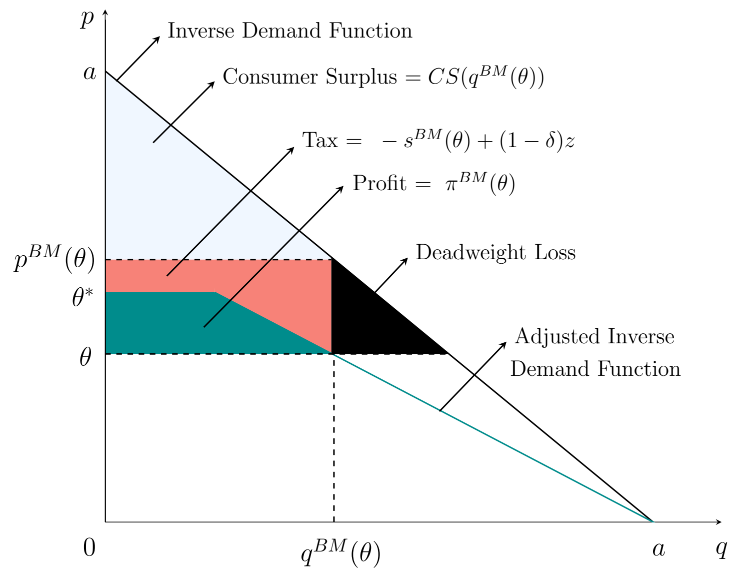

Figure 1. The Welfare Distribution under the Modified Regulatory Mechanism of BM (1982)

2.3 Preliminaries on Cooperative Bargaining

According to Nash (1950), a bargaining problem in two-agent settings is a pair (S, d), where $S \subset \mathbb{R}^2$ is the bargaining set that consists of von Neumann-Morgenstern utility allocations from which the two parties are allowed to cooperatively choose, and $d \in S$ is the disagreement point that denotes the utility allocation the agents would have to face if they should fail to agree on any point in S. It is assumed that S is convex and compact and there exists $s \in S$ such that $s \geq d$. Also, S is d-comprehensive (allowing free disposal of utility); i.e., for all $x, y \in \mathbb{R}^2_+$, $x \in S$ and $x \geq y \geq d$ only if $y \in S$. Let Σ^2 denote the set of all two-person bargaining problems satisfying the above assumptions.

A bargaining rule F which the two parties are assumed to agree upon before the bargaining takes place chooses a point in S, denoted by $F(S, d)$, potentially taking the disagreement point d into consideration. The bargaining rules we shall consider in this paper respect Pareto optimality in the weak or strong sense. For the sake of formality, we define the following. For any $S \subset \mathbb{R}^2$, the set of weakly Pareto optimal allocations in S is $WPO(S) = \{x \in S : y > x \text{ implies } y \notin S\}$ and likewise the set of Pareto optimal allocations in S is $PO(S) = \{x \in S : y \geq x \text{ implies } y \notin S\}$.

In Section 3, we shall investigate the implications of using several bargaining rules, namely the Egalitarian (E), Nash (N), Kalai-Smorodinsky (KS), ρ-Proportional (P^ρ), and β-weighted Utilitarian (U^β) rules. For any bargaining rule $F \in \{E, N, KS,$

$P^\rho, U^\beta\}$, let u_c^F and u_m^F respectively denote the agreement utility of consumers and the monopolist; i.e., $u_c^F = F_1(S,d)$ and $u_m^F = F_2(S,d)$.

The most well-known bargaining rule is the Nash rule introduced by Nash (1950). This rule proposes, for a given two-person problem $(S,d) \in \Sigma^2$, the solution

$$N(S,d) = \text{argmax}_{x \in S}\,(x_1 - d_1)(x_2 - d_2), \tag{18}$$

at which the product of players' net utility gains from agreement is maximized.

Another well-known bargaining rule is the Kalai-Smorodinsky rule, which was first proposed by Raiffa (1953) for two-person games and axiomatized by Kalai and Smorodinsky (1975). For a problem $(S,d) \in \Sigma^2$, this rule finds the allocation

$$KS(S,d) = \max\left\{x \in S : \frac{x_2 - d_2}{x_1 - d_1} = \frac{a_2(S,d) - d_2}{a_1(S,d) - d_1}\right\}, \tag{19}$$

where for each $i = 1,2$, $a_i(S,d) = \max\{x_i : x \in S \text{ and } x_{-i} = d_{-i}\}$ denotes the ideal utility of agent i in S given d. Accordingly, the point $a(S,d) = (a_1(S,d), a_2(S,d))$ is called the ideal point for (S,d). (Notice that the point $a(S,d)$ is outside the bargaining set S unless it is rectangular.) The Kalai-Smorodinsky rule selects the maximum point of the bargaining set S on the line segment connecting the disagreement point d and the ideal point $a(S,d)$.

A third well-known bargaining rule is the Egalitarian rule that was recommended by Rawls (1972) and a generalization of it, known as the class of proportional solutions, was proposed and axiomatized by Kalai (1977). Formally this rule selects for a given bargaining problem $(S,d) \in \Sigma^2$ the solution

$$E(S,d) = \max\{x \in S : x_1 - d_1 = x_2 - d_2\}, \tag{20}$$

at which the utility of the worst-off individual is maximized.

The Egalitarian rule is a member of a class of bargaining rules called Proportional rules. Formally, given any $\rho \in \mathbb{R}_{++}$, a bargaining rule for two-person bargaining problems is called ρ-Proportional (Kalai, 1977) and denoted by P^ρ if for each bargaining problem $(S,d) \in \Sigma^2$

$$P^\rho(S,d) = d + \lambda(S,d)(\rho,1) \text{ and } \lambda(S,d) = \max\{t : d + t(\rho,1) \in S\}. \tag{21}$$

The ρ-Proportional solution suggests the maximum point of the bargaining set S on the line passing through the disagreement point d and the point $(\rho,1)$. Notice that P^ρ coincides with the Egalitarian rule when $\rho = 1$.

Our last bargaining rule of interest is the β-weighted Utilitarian rule. The idea of a utilitarian rule in bargaining is anonymous and can be traced to a few centuries before. On the other hand, the idea of a β-weighted utilitarian rule in regulation can be attributed to Myerson (1982), who introduced this rule as a measure of the social welfare function in regulating a monopolist with unknown costs. Formally, on the space of two-person bargaining problems a bargaining rule is called β-weighted Utilitarian and denoted by U^β if there exists a positive real number β such that for each $(S, d) \in \Sigma^2$

$$U^\beta(S, d) = \{x \in S : x = \operatorname{argmax}_{y \in S} \ (y_1 - d_1) + \beta(y_2 - d_2)\}. \tag{22}$$

In addition to the above bargaining rules, we shall also consider an extreme kind of rule, called the Dictatorial rule, to interpret corner utility allocations in the bargaining set. Formally, a bargaining rule is called Dictatorial for agent i and denoted by D^i if for each $(S, d) \in \Sigma^2$

$$D^i(S, d) = \max\{x \in S : x_i \geq d_i \ \text{ and } \ x_j = d_j \text{ for } j \neq i\}. \tag{23}$$

This solution chooses the maximal point of the bargaining set with the maximal coordinate for agent i. Before moving to our results, we present further preliminaries that will relate the outcomes of the bargaining rules described in (18)-(22) to the welfare distributions in Cournot oligopolies.

2.4 Preliminaries on Cournot Oligopolies

Consider an oligopolistic market that involves n firms that produce a single homogenous good under the same demand function (1) and the same cost function (2) as faced by our monopolist. Let the output profile $q = (q_1, \ldots, q_n)$ list the quantity of these n firms and let $Q = \sum_i^n q_i$ denote the industry output. Also define for any i the profile q_{-i} such that $q = (q_i, q_{-i})$. Given an output profile q, the profit of firm i can be written as

$$\pi_i(q) = P(Q)q_i - \theta q_i - z = (a - \theta)q_i - q_i Q - z. \tag{24}$$

An output profile q^* is called a Cournot-Nash equilibrium if $\pi_i(q_i^*, q_{-i}^*) \geq \pi_i(q, q_{-i}^*)$ for all $i = 1, 2, \ldots, n$ and $q \geq 0$. It is well known that this equilibrium (calculated by simultaneously solving n first-order-necessary conditions) arises when each firm i produces the quantity $q_i^*(n) = (a - \theta)/(n + 1)$. Consequently, the industry output and the product price become

$$Q^*(n) = \frac{n}{n+1}(a - \theta) \tag{25}$$

and

$$p^*(n) = a - \frac{n}{n+1}(a - \theta) \tag{26}$$

respectively, and the industry profit and the consumer surplus become

$$PS^*(n) = n\pi_1^* = \frac{n(a - \theta)^2}{(n+1)^2} - nz \tag{27}$$

and

$$CS^*(n) = \frac{(Q^*)^2}{2} = \frac{n^2(a-\theta)^2}{2(n+1)^2} \tag{28}$$

respectively. It should be clear from above that in the Cournot-Nash equilibrium of an oligopolistic market, the price and industry profit are decreasing, whereas the industry output and consumer surplus are increasing, in the number of firms (n). Moreover, if n approaches infinity, the industry output Q^* approaches to the socially optimal level $a-\theta$, the price to the marginal cost θ, the industry profit to zero, and the consumer surplus to the maximal social surplus $(a-\theta)^2/2$.

3 Results

We shall first investigate the solutions of some well-known bargaining rules in determining the regulatory objective and the induced payoffs under symmetric and asymmetric information, and next we shall present an axiomatic approach to characterize bargaining rules using a series of new axioms that depend on the essentials of the regulatory problem.

3.1 Symmetric Information

We shall first construct the bargaining problem, under symmetric information, for the monopoly market we described in equations (1)-(4). We denote this problem by (S^s, d^s), where the superscript stands for symmetric information. We assume that the monopolist is not allowed to produce if bargaining with consumers fails. In such a case, its utility (profit) becomes $-\delta z$ and consumers' utility becomes zero. So, we set the disagreement point as $d^s = (d_1^s, d_2^s) = (0, -\delta z)$.

Let u_m and u_c denote the utilities of the monopolist and consumers. We can construct the bargaining set S using (3) and (4). These two equations constrain the utilities of the bargaining parties to the equation

$$u_m = 2\sqrt{\nu u_c} - 2u_c - z \tag{29}$$

where $\nu \equiv (a-\theta)^2/2$ denotes the economic surplus under marginal cost pricing rule. We shall relax the above constraint as we allow both the monopolist and consumers to freely dispose of their utilities; i.e. if the allocation (u_m, u_c) is achievable, any allocation $(x, y) \in \mathbb{R}$ with $d_1^s \leq x \leq u_m$ and $d_2^s \leq y \leq u_c$ will be achievable, too. This implies that the bargaining set S^s will be (comprehensive and) given by

$$S^s = \left\{ \begin{array}{ll} (u_c, u_m) : u_c \in [0, \nu]; \quad u_m \geq -z; & \\ \qquad u_m = \nu/2 - z & \text{if } u_c < \nu/4; \\ \qquad u_m \leq 2\sqrt{\nu u_c} - 2u_c - z & \text{if } u_c \geq \nu/4 \end{array} \right\}. \tag{30}$$

In Figure 2, we plot the bargaining problem (S^s, d^s) along with a solution for the case $\delta = 1$ implying $d^s = (0, -z)$. Notice that S^s is compact, convex, and comprehensive as in the bargaining model of Nash (1950). The green curve at the frontier of S^s plots the set of weakly Pareto optimal points, $WPO(S)$. On the other hand, the set of Pareto optimal points, $PO(S^s)$, coincides with the downward sloping part of $WPO(S)$, over which u_c varies between $\nu/4$ and ν.

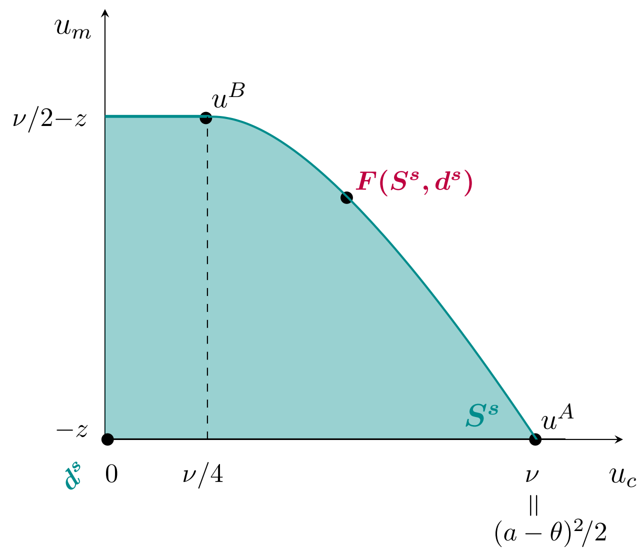

Figure 2. The Bargaining Problem (S, d) under Symmetric Information and Sunk Fixed Cost

Two extreme points (utility allocations) on $PO(S^s)$ needs highlighting. One of them is the allocation u^A which arises if the monopolist is optimally regulated to produce the socially optimal (first best) level of output $q^f = a - \theta$. In this case, the monopolist incurs a loss equal to its fixed cost z while the whole economic surplus $\nu = (a - \theta)^2/2$, which is the maximal surplus that can ever be attained, goes to consumers. The other allocation point is u^B at which the monopolist produces its optimal output level $q^m = (a - \theta)/2$ and earns the highest level of unregulated profit, $\nu/2 - z$, that can be obtained under linear and non-discriminatory prices. Consequently, the consumer surplus becomes $\nu/4$. Obviously, both points u^A and u^B can be obtained as the outcome of some dictatorial rule. Recalling that in Figure 2 consumers are indexed by $i = 1$ and the monopolist by $i = 2$; we should note $D^1(S^s, d^s) = u^A$ and $D^2(S^s, d^s) = u^B$. That is, the welfare distribution generated by the optimal quantity chosen by an unregulated monopolist is equivalent to the utility allocation obtained whenever the monopolist can act as a dictator in the bargaining game (S^s, d^s). Similarly, the welfare distribution generated by the socially optimal quantity chosen by a benevolent regulator is equivalent to the utility allocation obtained whenever consumers as a whole can act as a dictator in the associated bargaining game. Below, we consider non-dictatorial bargaining rules and rank them, for both consumers and the monopolist, with respect to their implied solutions on (S^s, d^s).

Proposition 1. (Egalitarian, Nash and Kalai-Smorodinsky Rules) *Given any market $\mathcal{M}^s$ with $\delta = 1$ and the induced bargaining problem (S^s, d^s),*

(i) the Egalitarian rule yields the utilities $u_c^E = (4/9)\nu$ and $u_m^E = (4/9)\nu - z$;

(ii) the Nash rule yields the utilities $u_c^N = (9/16)\nu$ and $u_m^N = (6/16)\nu - z$;

(iii) the Kalai-Smorodinsky rule yields the utilities $u_c^{KS} = (16/25)\nu$ and $u_m^{KS} = (8/25)\nu - z$.

(The proof of Proposition 1 and several other results are relegated to the Appendix.) Figure 3 illustrates the results in Proposition 1. One can easily calculate that the solutions of the Egalitarian, Nash, and Kalai-Smorodinsky rules require the monopolist's output to be $2q^f/3$, $3q^f/4$, and $4q^f/5$ units respectively, where $q^f = a - \theta$ is the first-best output level induced by the marginal-cost pricing.

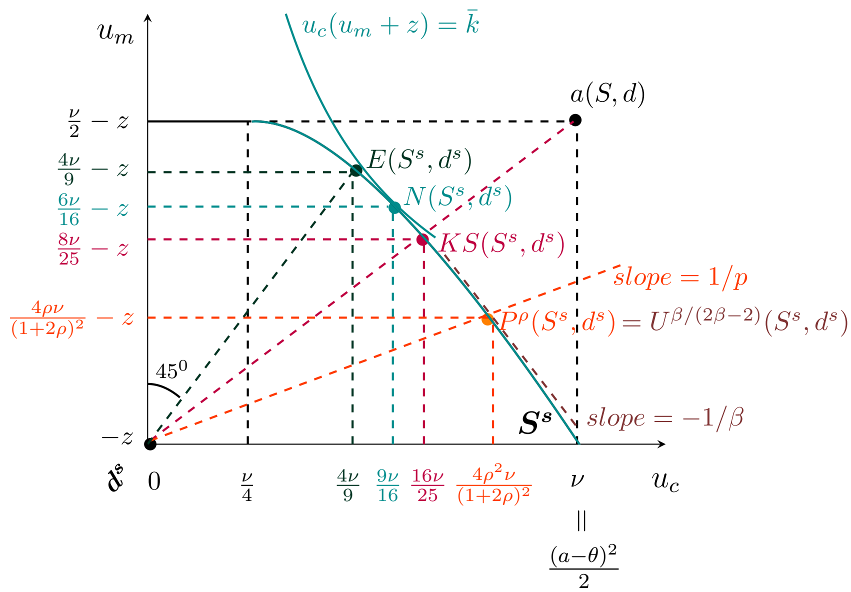

Figure 3. The Solutions of Several Bargaining Rules under Symmetric Information and Sunk Fixed Cost

Also, notice in Figure 3 that $u_c^E < u_c^N < u_c^{KS}$; i.e., under symmetric information consumers prefer to bargain using the Kalai-Smorodinsky rule instead of the other two rules, while they prefer the Nash rule to the Egalitarian rule. Since the solutions to all three rules are found to lie on $PO(S^s)$, i.e., the downward-sloping part of the weak Pareto frontier, the preference ordering of the firm is in the opposite direction to that of consumers.[5] The firm strictly prefers the Egalitarian rule to the other two rules, while it prefers the Nash rule to the Kalai-Smorodinsky rule. The three bargaining rules have also different implications on total welfare. Given the bargaining problem (S^s, d^s) and a bargaining rule F, let $SW^F(S^s, d^s)$ denote the social welfare (total economic surplus) achieved from regulation with bargaining. Naturally, we define it as the sum of the agreement utilities of consumers and the monopolist, i.e. $SW^F(S^s, d^s) = F_1(S^s, d^s) + F_2(S^s, d^s)$. This definition directly induces a preference ordering for the society over different bargaining rules. In fact, from Proposition 1 one can calculate that $SW^E(S^s, d^s) = (8/9)\nu - z$, $SW^N(S^s, d^s) = (15/16)\nu - z$, and $SW^{KS}(S^s, d^s) = (24/25)\nu - z$. So, we find that the society as a whole prefers, as consumers do, the Kalai-Smorodinsky rule to the Nash rule, and the Nash rule to the Egalitarian rule. Interestingly, we find that on the problem (S^s, d^s), the Nash rule is more egalitarian and less utilitarian than the Kalai-Smorodinsky rule, while both of these rules are more utilitarian than the Egalitarian rule.

The social welfare function we have defined above is analogous to a weighted-utilitarian bargaining rule that weighs the utility of consumers and the monopolist equally. It may be of interest to find out what such a bargaining rule would choose on (S^s, d^s). The answer is trivial due to the geometry of the bargaining problem S: the social welfare (the equally weighted total utility) is always maximized at a corner solution where $u_c = \nu$. Below, we show that interior solutions are possible for the β-weighted Utilitarian rule described by (22).

Proposition 2. (β-Weighted Utilitarian Rule) *Given any market $\mathcal{M}^s$ with $\delta = 1$ and the induced bargaining problem (S^s, d^s), the β-weighted Utilitarian rule yields the utilities*

$$u_c^\beta = \begin{cases} \nu & \text{if } \beta \in [0,1] \\ \frac{\beta^2}{(2\beta-1)^2}\nu & \text{if } \beta > 1 \end{cases} \qquad \text{and} \qquad u_m^\beta = \begin{cases} -z & \text{if } \beta \in [0,1] \\ \frac{2(\beta^2-\beta)}{(2\beta-1)^2}\nu - z & \text{if } \beta > 1. \end{cases} \tag{31}$$

The above result implies that $U^\beta(S^s, d^s) = D^1(S^s, d^s) = u^A$ if $\beta \in [0, 1]$ and $\lim_{\beta\to\infty} U^\beta(S^s, d^s) = D^2(S^s, d^s) = u^B$. That is, a weighted Utilitarian rule that weighs the utility of the monopolist not higher than the utility of consumers is welfare-equivalent to a Dictatorial rule under which consumers act as a dictator. In contrast, a weighted Utilitarian rule that weighs the utility of the monopolist infinitely higher than the utility of consumers is welfare-equivalent to a Dictatorial rule where the dictator

[5] In general, for any bargaining problem (S^s, d^s) the solutions of the Nash and Kalai-Smorodinsky rules must always lie on $PO(S^s)$, whereas the solution of the Egalitarian rule may lie on $PO(S^s)$ or $WPO(S^s) \setminus PO(S^s)$ depending upon the geometry of the problem.

is the monopolist. We should also notice that the outcome of a β-weighted Utilitarian rule is always on $PO(S^s)$. Hence, for any point $x = (x_1, x_2) \in PO(S^s)$ one can always find $\beta^x \geq 1$ such that $U^{\beta^x}(S^s, d^s) = x$. In fact, one can easily calculate from (31) this weight as

$$\beta^x = \sqrt{x_1/\nu}/(2\sqrt{x_1/\nu} - 1). \tag{32}$$

The above observation, along with Propositions 1 and 2, implies the following result.

Corollary 1. *Given any market $\mathcal{M}^s$ with $\delta = 1$, the Egalitarian, Nash, and Kalai-Smorodinsky rules are outcome-equivalent on (S^s, d^s) to the β-weighted Utilitarian rules with weights $\beta = 2$, $\beta = 3/2$, and $\beta = 4/3$, respectively.*

We should emphasize that neither the Egalitarian, the Nash, nor the Kalai-Smorodinsky rule is a β-weighted Utilitarian rule. Corollary 1 merely states that their solutions coincide with that of a weighted Utilitarian rule on the bargaining problem (S^s, d^s). An equivalence result in the same spirit can be obtained with reference to proportional rules after we observe the following.

Proposition 3. (ρ-Proportional Rule) *Given any market $\mathcal{M}^s$ with $\delta = 1$ and the induced bargaining problem (S^s, d^s), the ρ-Proportional bargaining rule yields the utilities*

$$u_c^\rho = \begin{cases} \rho(\nu/2) & \text{if } \rho \in (0, 1/2) \\ \frac{4\rho^2}{(1+2\rho)^2}\nu & \text{if } \rho \geq 1/2, \end{cases} \quad \text{and} \quad u_m^\rho = \begin{cases} (\nu/2) - z & \text{if } \rho \in (0, 1/2) \\ \frac{4\rho}{(1+2\rho)^2}\nu - z & \text{if } \rho \geq 1/2. \end{cases} \tag{33}$$

It is clear that $P^\rho(S^s, d^s) = D^1(S^s, d^s) = u^A$ if $\rho = 1/2$ and $\lim_{\rho\to\infty} P^\rho(S^s, d^s) = D^2(S^s, d^s) = u^B$. We should also notice that unlike the β-weighted Utilitarian rule, the ρ-Proportional rule can yield *weakly* Pareto optimal allocations, as well; as a matter of fact, its outcome lies on $WPO(S^s) \setminus PO(S^s)$ if $\rho \in (0, 1/2)$ and lies on $PO(S^s)$ if $\rho \geq 1/2$. Moreover, for any point $x = (x_1, x_2) \in WPO(S^s)$ one can find a proportionality factor $\rho(x) \geq 1$ such that $P^{\rho(x)}(S^s, d^s) = x$. In fact, one can easily calculate from (33) this factor as

$$\rho(x) = x_1/(x_2 + z). \tag{34}$$

The Egalitarian rule is a ρ-Proportional rule with $\rho = 1$. On the other hand, the Nash and Kalai-Smorodinsky rules are independent of ρ, i.e., they can never coincide with any ρ-Proportional rule. However, each of these two rules is outcome-equivalent on (S^s, d^s) to some ρ-Proportional rule, as suggested by equation (34) along with Propositions 1 and 3.

Corollary 2. *Given any market $\mathcal{M}^s$ with $\delta = 1$ the ρ-Proportional rule is outcome-equivalent on (S^s, d^s) to the Nash rule if and only if $\rho = 3/2$ and to the Kalai-Smorodinsky rule if and only if $\rho = 2$.*

A similar equivalence result can be stated between β-weighted Utilitarian and ρ-Proportional rules.

Corollary 3. *Given any market $\mathcal{M}^s$ with $\delta = 1$, the β-weighted Utilitarian rule is outcome-equivalent on (S^s, d^s) to a ρ-Proportional rule if and only if $\beta > 1$ and $\rho = \beta/(2\beta - 2)$.*

Interestingly, each of the Egalitarian, Nash, and Kalai-Smorodinsky rules yields to consumers an amount of utility that can always be obtained in the equilibrium of some unregulated symmetric Cournot oligopoly.

Proposition 4. (Relatedness to Cournot Outcomes) *Given any market $\mathcal{M}^s$ with $\delta = 1$, consumers' utility $F_1(S^s, d^s)$ allocated by a rule F on the bargaining problem (S^s, d^s) is equal to the consumer surplus obtained in an unregulated symmetric Cournot oligopoly involving*

(i) *two firms (duopoly) if F is the Egalitarian rule,*

(ii) *three firms if F is the Nash rule,*

(iii) *four firms if F is the Kalai-Smorodinsky rule,*

(iv) *$\beta/(\beta-1)$ firms if F is the β-weighted Utilitarian rule U^β, provided that $\beta/(\beta-1)$ is a positive integer, and*

(v) *2ρ symmetric firms if F is the ρ-Proportional rule, provided that 2ρ is an integer.*

To better understand the results (i)-(iii) in Proposition 4, we should first notice from (27), (28), and (29) that for any oligopolistic industry with $n \in \{2, \dots, \infty\}$ firms each of which has a fixed cost of z/n, the Cournot equilibrium welfares of consumers and the industry would be given by $(CS^*(n), PS^*(n)) = \left(\frac{n^2}{(n+1)^2}\eta, \frac{2n}{(n+1)^2}\eta - z\right)$ and lie on the Pareto frontier, $PO(S^s)$, of the bargaining set S^s. Given $d^s = (0, -z)$, we can calculate the net welfare ratio $(PS^*(n) - d_2^s)/(CS^*(n) - d_1^s)$ as $2/n$, which is equal to 1 only if $n = 2$. We also know that the Egalitarian solution $E(S^s)$ in S^s lies in $WPO(S^S)$ and $(E_1(S^s) - d_1^s)/(E_2(S^s) - d_2^s) = 1$. Thus, we must have $E(S^s) = (CS^*(2), PS^*(2))$, establishing part (i). We should also note that for the bargaining set S^s, the slope of the ray on which the Kalai-Smorodinsky solution, $KS(S^s)$, lies must be equal to $(a_2(S^s, d^s) - d_2^s)/(a_1(S^s, d^s) - d_1^s) = (\eta/2 - z - (-z))/(\eta - 0) = 1/2$. Moreover, we should observe that $(PS^*(n) - d_2^s)/(CS^*(n) - d_1^s) = 2/n$ can be equal to this ratio only if $n = 4$.

Since $KS(S^s)$ always lies on $PO(S^s)$, we should have $KS(S^s) = (CS^*(4), PS^*(4))$, establishing part (iii). For part (ii), we should notice that the product of utility gains in the Cournot oligopoly is $(PS^*(n) - d_2^s)(CS^*(n) - d_1^s) = \frac{2n^3}{(n+1)^4}$. One can easily show that this is maximized at $n = 3$. Coincidentally, the product of the utility gains $(u^m - d_2^s)(u_c - d_1^s)$ attains its maximum (the Nash solution) over the bargaining problem (S^s, d^s) when $(u_m, u_c) = PS^*(3), CS^*(3)$, establishing part (ii). Interestingly, the allocations that are not generated by any Cournot oligopoly (including an integer number of firms) turn out to be irrelevant in calculating the Nash solution.

Given Corollaries 2 and 3, parts (i)-(iv) of Proposition 4 can all be obtained from part (v). Moreover, when the fixed cost of production is zero, Proposition 4 can be extended to make similar utility comparisons for the monopolist, as well. Now, we shall investigate how the results we have obtained so far may differ when z is sunk or non-sunk ($\delta = 1$ or $\delta = 0$). Let $(S^{s,1}, d^{s,1})$ $(S^{s,0}, d^{s,0})$ denote the bargaining problems for the cases $\delta = 1$ and $\delta = 0$, respectively. Clearly, $S^{s,1} = S^{s,0}$; the bargaining set is not affected by the nature of the fixed cost. On the other hand, $d^{s,1} = (0, -z) \neq (0, 0) = d^{s,0}$, unless $z = 0$. Compared to the problem $(S^{s,1}, d^{s,1})$, the only difference in $(S^{s,0}, d^{s,0})$ is then the rise in the disagreement utility of the monopolist by z. Our task is to estimate the effect of this rise on the agreement utilities of the monopolist and consumers under the previously studied bargaining rules.

An axiom by Livne (1989) called the Strong Disagreement Point Monotonicity (SDPM) requires that given a bargaining problem (S, d) where the bargaining set S is not rectangular, i.e., $a(S, d) \notin S$, if agent i's disagreement utility increases while that of the other agent remains the same, then agent i becomes strictly better off. (A weak inequality-version of this axiom was earlier introduced by Thomson, 1987). This axiom is known to be satisfied by the Kalai-Smorodinsky solution but unfortunately not by the Nash rule, the Egalitarian rule, Proportional rules, and weighted Utilitarian rules. However, these negative results which are obtained under no domain restrictions are not necessarily applicable to our problem at hand because we are not interested in all possible two-person bargaining problems. Even though the bargaining set S^s under symmetric information is consistent with infinitely many markets involving the set of admissible parameters (a, z, δ, θ), the shape of S^s is always the same as drawn in Figures 2 and 3. That is, we are confined to a class of two-person bargaining problems with measure zero. So, it is likely that under some of the well-known bargaining rules for which the SDPM axiom fails, the agreement utility of the monopolist can still be an increasing function of its disagreement utility. On a deeper reflection, this cannot be true, however, for the case of weighted Utilitarian rules in equation (22) since the outcomes of these rules are independent of the disagreement point. That is, $U^\beta(S^{s,1}, d^{s,1}) = U^\beta(S^{s,0}, d^{s,0})$ for all $\beta \geq 0$, since $S^{s,1} = S^{s,0}$. Below, we deal with the other bargaining rules of interest.

Proposition 5. (The Welfare Effect of Sunkness) *Consider any market $\mathcal{M}^s$ and the induced bargaining problems $(S^{s,0}, d^{s,0})$ and $(S^{s,1}, d^{s,1})$ corresponding to $\delta = 0$ and*

$\delta = 1$ *respectively. Under any bargaining rule* $F \in \{E, N, KS, P^\rho\}$ *with* $\rho \geq 1/2$*, the utility of consumers (the firm) is always lower (higher) if* $\delta = 0$*; i.e.* $F_1(S^{s,0}, d^{s,0}) < F_1(S^{s,1}, d^{s,1})$ *and* $F_2(S^{s,0}, d^{s,0}) > F_2(S^{s,1}, d^{s,1})$*.*

Proof. Recall that $S^{s,0} = S^{s,1}$, $d^{s,0} = (0,0)$, and $d^{s,1} = (0, -z)$. For each $\delta \in \{0,1\}$ and $F \in \{E, N, KS, P^\rho\}$, let $u_c^{F,\delta} = F_1(S^{s,\delta}, d^{s,\delta})$ and $u_m^{F,\delta} = F_2(S^{s,\delta}, d^{s,\delta})$.

(i) $F = E$. Equation (20) implies

$$u_m^{E,0} = u_c^{E,0}. \tag{35}$$

From (30) it follows that $u_c^{E,0} \geq \nu/4$, for otherwise (35) would be violated. Therefore,

$$u_m^{E,0} = (a-\theta)(2u_c^{E,0})^{1/2} - 2u_c^{E,0} - z. \tag{36}$$

Solving (35) and (36) together yields

$$E_1(S^{s,0}, d^{s,0}) \equiv u_c^{E,0}(z) = (\sqrt{\nu} + \sqrt{\nu - 3z})^2/9. \tag{37}$$

Notice that $u_c^{E,0}(z)$ is decreasing in z and $u_c^{E,0}(0) = 4\nu/9$. By Proposition 1-(i), $u_c^{E,0}(0) = u_c^{E,1}(z) \equiv u_c^E$ for all z. Since $z > 0$, we have $E_1(S^{s,0}, d^{s,0}) < E_1(S^{s,1}, d^{s,1})$. Also, since $E(s, 1, d^{s,1}) \in PO(S^{s,1})$ by Proposition 1-(i), we have $E_2(S^{s,0}, d^{s,0}) > E_2(S^{s,1}, d^{s,1})$.

(ii) $F = N$. It follows from (18) that an allocation $(u_c, u_m) \in S$ is equal to $N(S, d)$ only if $(u_c, u_m) \in P(S)$. Therefore, $u_c \geq \nu/4$ and

$$u_m = (a-\theta)\sqrt{2u_c} - 2u_c - z. \tag{38}$$

Using $d^{s,0} = (0,0)$ and (38), we can rewrite the Nash product $u_c u_m$ as $u_c[(a-\theta)\sqrt{2u_c} - 2u_c - z]$. Differentiating it with respect to u_c and equating it to zero we get the solution

$$N_1(S^{s,0}, d^{s,0}) \equiv \bar{u}_c^{N,0}(z) = (\sqrt{\nu} + \sqrt{\nu - 16z/9})^2/(64/9). \tag{39}$$

Notice that $u_c^{N,0}(z)$ is decreasing in z and $u_c^{N,0}(0) = 9\nu/16$. By Proposition 1-(ii), $u_c^{N,0}(0) = u_c^{N,1}(z) \equiv u_c^N$ for all z. Since $z > 0$, we have $N_1(S^{s,0}, d^{s,0}) < N_1(S^{s,1}, d^{s,1})$. Also since $N(s, 1, d^{s,1}) \in PO(S^{s,1})$ by Proposition 1-(ii), we have $N_2(S^{s,0}, d^{s,0}) > N_2(S^{s,1}, d^{s,1})$.

(iii) $F = KS$. By Livne (1989), we know that the solution KS satisfies the SDPM axiom. Since $a(S^{s,1}, d^{s,1}) \notin S^{s,1}$, this axiom implies $KS_2(S^{s,0}, d^{s,0}) > KS_2(S^{s,1}, d^{s,1})$. Also since $KS(S^{s,1}, d^{s,1}) \in PO(S^{s,1})$ by Proposition 1-(iii), we have $KS_1(S^{s,0}, d^{s,0}) < KS_1(S^{s,1}, d^{s,1})$. As a matter of fact, we can directly calculate $u_c^{KS,0} = KS_1(S^{s,0}, d^{s,0})$ and $u_m^{KS,0} = KS_1(S^{s,0}, d^{s,0})$. First notice that $a_2(S^{s,0}, d^{s,0}) = \nu/2 - z$. We can find $a_1(S^{s,0}, d^{s,0})$ by inserting $u_m = 0$ into (29) and solving for $u_c = a_1(S^{s,0}, d^{s,0})$, which is equal to $(\sqrt{\nu}+\sqrt{\nu-2z})^2/4$. So, $u_m^{KS,0}/u_c^{KS,0}$ must be equal to $a_2(S^{s,0},d^{s,0})/a_1(S^{s,0},d^{s,0})$ $=(\nu/2-z)(\sqrt{\nu}+\sqrt{\nu-2z})^2/4$ which we denote by $\gamma(z)$. Inserting $u_m^{KS,0}=\gamma(z)u_c^{KS,0}$ into $PO(S^{s,0})$ yields

$$\gamma(z)u_c^{KS,0}=(a-\theta)\sqrt{2u_c^{KS,0}}-2u_c^{KS,0}-z, \tag{40}$$

implying

$$u_c^{KS,0}=\frac{\left(\sqrt{\nu}+\sqrt{\nu-[2+\gamma(z)]z}\right)^2}{[2+\gamma(z)]^2}. \tag{41}$$

(iv) $F=P^\rho$ where $\rho\geq 1/2$. Given $d^{s,0}=(0,0)$, (21) implies

$$u_m^{\rho,0}=u_c^{\rho,0}/\rho. \tag{42}$$

We also have

$$u_m^{\rho,0}=(a-\theta)\sqrt{2u_c^{\rho,0}}-2u_c^{\rho,0}-z. \tag{43}$$

Solving (42) and (43) together, we obtain

$$P_1^\rho(S^{s,0},d^{s,0})\equiv u_c^{\rho,0}(z)=\left(\sqrt{\nu}+\sqrt{\nu-(2+1/\rho)z}\right)^2/(2+1/\rho)^2. \tag{44}$$

Notice that $u_c^{\rho,0}(z)$ is decreasing in z and $u_c^{\rho,0}(0)=4\rho^2\nu/(1+2\rho)^2$. By Proposition 3, $u_c^{\rho,0}(0)=u_c^{\rho,1}(z)\equiv u_c^\rho$ for all z. Since $z>0$, we have $P_1^\rho(S^{s,0},d^{s,0})<P_1^\rho(S^{s,1},d^{s,1})$. Also since $\rho\geq 1/2$, we have $P^\rho(S^{s,1},d^{s,1})\in PO(S^{s,1})$ by Proposition 3, and therefore $P_2^\rho(S^{s,0},d^{s,0})>P_2^\rho(S^{s,1},d^{s,1})$. ∎

The above result shows that the nature of the fixed cost of production affects the welfare of both the monopolist and consumers under the studied well-known bargaining rules, except for the β-Utilitarian rule. The monopolist is always better off when its fixed cost is non-sunk than it is sunk. The reason is that the disagreement utility obtained from bargaining is unilaterally higher for the monopolist when its fixed cost is non-sunk. This unilateral increase in the disagreement utility (by the size of the fixed cost) simply leads to a higher bargaining utility under any proportional rule (with sufficiently small slope), including the Egalitarian rule, since the ray along which the proportional solution is calculated shifts up (by an amount just equal to the size of the fixed cost) and intersects the Pareto frontier of the bargaining set at a superior allocation for the monopolist. For the Nash bargaining rule, the effect of a unilateral increase in the disagreement utility in favor of the monopolist is a change of the Nash product from $u_c(u_m+z)$ to u_cu_m. As the Nash solution is obtained at the interior solution of the Pareto frontier of the bargaining set $PO(S^s)$, this change in the objective function pushes the optimal solution for u_m at the expense of u_c. Finally, for the Kalai-Smorodinsky solution, the aforementioned change in the disagreement utility of the monopolist tilts up the slope of the ray connecting the disagreement point to the ideal point of the bargaining set S^s, leading to a rise in the bargaining utility of the monopolist. Since under all bargaining rules in Proposition 5 the solution always lies on the Pareto frontier of the bargaining set, an improvement in the monopolist's utility corresponds to deterioration in the utility of consumers inevitably. The proof of Proposition 5 also implies that the preference orderings of the monopolist and consumers over the bargaining outcomes of the Egalitarian, Nash, and Kalai-Smorodinsky rules are independent of the value of δ. In other words, the nature of the fixed cost affects the cardinal but not ordinal utilities assigned under these bargaining rules.

Finally, we shall investigate whether Proposition 4 linking the regulation outcomes to Cournot-Nash equilibrium outcomes for some unregulated symmetric oligopolies is also valid when the fixed cost is non-sunk. For convenience, we let $S^{s,\delta}(\mathcal{M}^s)$ denote the bargaining set induced by the monopoly market $\mathcal{M}^s$. Also, we define $\mathcal{M}^s_{-z}$ such that $\mathcal{M}^s = (\mathcal{M}^s_{-z}, z)$. Notice that $S^s(\mathcal{M}^s)$, which is always given by (30), varies with z, even though it is independent of its nature.

Corollary 4. (Unrelatedness to Cournot Outcomes When $\boldsymbol{\delta = 0}$) *Consider any market $\mathcal{M}^s$ with $z > 0$ and $\delta = 0$ and the induced bargaining problem $(S^{s,0}(\mathcal{M}^s), d^{s,0})$. If F is a bargaining rule in the set $\{E, N, KS,\ P^\rho\}$ with $\rho \geq 1/2$, then there exists no integer $n \geq 2$ such that $F_1(S^{s,0}(\mathcal{M}^s), d^{s,0}) = CS^*(n)$ for all $z > 0$.*

Proof. Consider any admissible monopoly market $\mathcal{M}^s$ with $z > 0$ and $\delta = 0$, and the induced bargaining problem $(S^{s,0}(\mathcal{M}^s), d^{s,0})$. First consider any bargaining rule $F \in \{E, N, P^\rho\}$ where $\rho \geq 1/2$. Let $F_1(S^{s,0}(\mathcal{M}^s), d^{s,0}) = u_c^{F,0}(z)$. The proof of Proposition 5 shows $u_c^{F,0}(z)$ is decreasing for all $z > 0$. Therefore, if there exists an integer $n \geq 2$ such that $F_1(S^{s,0}(\mathcal{M}^s), d^{s,0}) = CS^*(n)$ then for any $z' \neq z$ it is true that $F_1(S^{s,0}(\mathcal{M}'), d^{s,0}) \neq CS^*(n)$ if $M' = (\mathcal{M}^s_{-z}, z')$.

Now consider $F = KS$. By Proposition 5, $KS(S^{s,0}(\mathcal{M}^s), d^{s,0}) \neq KS(S^{s,1}(\mathcal{M}^s), d^{s,1})$. Also, it is true that $\lim_{z\to 0} KS(S^{s,0}(\mathcal{M}^s),\ d^{s,0})) = \lim_{z\to 0} KS(S^{s,1}(\mathcal{M}^s),\ d^{s,1}))$. By the continuity of KS, there exists $z' \neq z$ such that $KS(S^{s,0}(\mathcal{M}'), d^{s,0}) \neq KS(S^{s,0}(\mathcal{M}^s), d^{s,0})$ if $M' = (\mathcal{M}^s_{-z}, z')$. So, if there exists an integer $n \geq 2$ such that $KS(S^{s,0}(\mathcal{M}^s), d^{s,0}) = CS^*(n)$, then we have $KS(S^{s,0}(\mathcal{M}'), d^{s,0}) \neq CS^*(n)$. ■

Corollary 4 implies that the result in Proposition 4 that relates the Cournot welfares to the regulatory bargaining utilities is not obtained when the fixed cost is non-sunk. The reason is that the size (or nature) of the fixed cost does not affect consumers' surplus in unregulated Cournot oligopolies whereas it negatively affects consumers' bargaining utilities under well known rules when the fixed cost is non-sunk, as shown in the proof of Proposition 5.

3.2 Asymmetric Information

Now we shall turn to consider the case where the information about the marginal cost of production is asymmetric. As we have seen in Section 2.2, the optimal regulation of the monopolist under asymmetric information requires, without loss of any generality, the use of the incentive-compatible mechanism of BM (1982). Despite this, one may wonder why we should stick to the BM (1982) model of regulation to construct the bargaining set under asymmetric information. In particular, one may ask whether it is possible to obtain a bargaining set with a higher frontier by using a social objective function different from the one used by BM (1982), which is simply the weighted sum of the expected welfares of consumers and the monopolist. The answer is 'no' because the social welfare function in the model of BM (1982) is general enough and always selects a Pareto efficient payoff vector within the set of vectors that are incentive efficient. If instead there were a payoff vector that Pareto dominated the one selected by the BM (1982) mechanism, then it could be possible to increase the expected social welfare in the BM (1982) model by switching to this superior payoff vector. But, this cannot be possible since we already know that the regulatory solution of BM (1982) is ex-ante optimal. Therefore, there is no loss of generality in restricting ourselves to the BM model of regulation to construct the bargaining set. Since this model is valid for, any weighted utilitarian social welfare function where the firm's relative welfare weight α is not higher than one, our first task in this section will be to determine the value of the α parameter using some well-known bargaining rules. But, we have to first introduce some definitions.

Consider any market $\mathcal{M}^s = \langle a, z, \delta, \theta \rangle$ under symmetric information. We say that the list $\mathcal{M}^a = \langle a, z, \delta, \alpha \rangle$ with $\alpha \in [0, 1]$ denotes the extension of $\mathcal{M}^s$ to asymmetric information. Notice that in the list $\mathcal{M}^a$ we suppress the values $\theta_0 = 0$, $\theta_1 = a$, and the belief function $f(\theta) = 1/a$ defined for all $\theta \in [0, a)$, as they are all fixed and known when a particular list for $\mathcal{M}^s$, or correspondingly a list for $\mathcal{M}^a$, is given. Given any market $\mathcal{M}^a$, we denote the expected consumer and producer welfares calculated under the BM (1982) mechanism by $u_c = CW^e$ and $u_m = PW^e$ respectively. Also we denote the bargaining set and the disagreement point associated with the market $\mathcal{M}^a$ respectively by S^a and d^a, where a stands for the asymmetry of information.

Equation (15) implies that under the special assumption where either $z = 0$ or both $\alpha = 0$ and $\delta = 1$, the threshold value θ^* above which the firm is not allowed to operate becomes equal to $a/(2 - \alpha)$, simplifying the expected consumer and producer welfares substantially and making our theoretical analysis tractable. Apparently, the case involving $\alpha = 0$ would not have any merit to study, for in that case the Pareto optimal frontier of the bargaining set would simply reduce to a singleton set, involving the unique allocation with the expected utilities $u_c = a^2/12$ and $u_m = a^2/24 - z$ respectively. In order to ensure that each value of $\alpha \in [0, 1]$ will correspond to some utility allocation in the bargaining set when we equate θ^* to a, we shall assume $z = 0$ (the case of no fixed cost) in the rest of our results. Under this assumption the expected welfare of consumers and the firm respectively become

$$CW^e = \frac{2(1-\alpha)a^2}{6(2-\alpha)^2} \tag{45}$$

and

$$PW^e = \frac{a^2}{6(2-\alpha)^2}. \tag{46}$$

As in the case of symmetric information, we shall assume that the monopolist is not allowed to produce if bargaining with consumers fails. In such a case, the firm obtains zero profit (since $z = 0$) and consumers obtain zero surplus, implying that both of them should expect zero utility in case of disagreement in bargaining. Thus, the disagreement point for bargaining will be $d^a = (0, 0)$. Using this point and equations (45) and (46), we can construct the bargaining set S^a as follows:

$$S^a = \left\{ (u_c, u_m) \in \mathbb{R}^2 : u_c = \tfrac{2(1-\alpha)\eta}{(2-\alpha)^2},\ 0 \leq u_m \leq \tfrac{\eta}{(2-\alpha)^2},\ \ \alpha \in [0,1] \right\} \tag{47}$$

Above, $\eta = a^2/6$ is the maximal expected economic surplus, which arises when $\alpha = 1$ making the regulated price $p^{BM}(\theta)$ equal to the marginal cost θ at each value it takes in the interval $[0, a)$. Notice that $\eta = \mathbb{E}[\nu|\theta] = \int_0^a \nu f(\theta) d\theta$, i.e., η is indeed the expected value of $\nu \equiv (a-\theta)^2/2$, the ex-post economic surplus at the marginal cost level θ. The allocations on the Pareto frontier of S^a change with the welfare weight parameter α. When this parameter is varied over $[0, 1]$, not only the welfare distribution, (u_c, u_m), but also the total welfare, $u_c + u_m$, change. Thus, choosing an allocation from the set $PO(S^a)$ through a bargaining rule simply amounts to choosing a value for α from the interval $[0, 1]$.

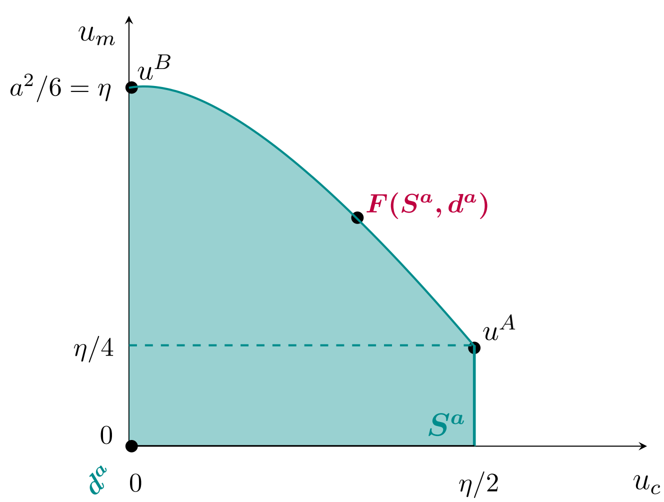

Figure 4. Bargaining Problem (S^a, d^a) under Asymmetric Information

In Figure 4, we plot the bargaining problem (S^a, d^a). Two extreme points on $PO(S^a)$ are marked as u^A and u_B. The allocation point u^A arises in the BM (1982) model when the welfare weight α is equal to 0. In that case, the regulated price becomes $p^{BM}(\theta) = \theta + F(\theta)/f(\theta) = 2\theta$ for any $\theta \in [0, a)$, yielding the maximal expected utility to consumers and the minimal expected utility to the firm on $PO(S^a)$. The allocation point u^B is obtained in the BM (1982) model if $\alpha = 1$. In such a case, the firm is entitled to the maximal expected economic surplus $\eta = a^2/6$, which is obtained under the marginal-cost pricing rule implying $p^{BM}(\theta) = \theta$ for any $\theta \in [0, a)$. Obviously, given the problem (S^a, d^a), allocation points u^A and u^B may arise under dictatorial bargaining rules D^1 and D^2 respectively, where the dictator is the union of consumers under the former rule and the monopolist under the latter rule.

Below, we shall prove that the allocations in the bargaining set S^a are dual to (reflections of) those in the bargaining set S^s with respect to the 45-degree line passing through d^s. We can even feel this visually by comparing Figures 2 and 4, or just their extreme points. The ideal allocation from the viewpoint of consumers, the point $u_A = (\nu, 0) = ((a - \theta)^2/2, 0)$ in Figure 2 is obtained at the marginal-cost pricing in the case of symmetric information. So, for the regulator (and consumers) the expected value of u_A is simply itself. However, for an outside arbitrator who is not informed about the value of θ and want to compare the surpluses in Figures 2 and 4, the expected value of u_A in Figure 2 would be $(\mathbb{E}[\nu|\theta], 0) = (a^2/6, 0)$ or simply $(\eta, 0)$, provided that this arbitrator believes, like the regulator in our model, that θ is distributed uniformly on the support $[0, a)$. Similarly, this arbitrator could calculate the expected value of u_B in Figure 2 as $(0, a^2/12) = (0, \eta/2)$ when $z = 0$. Given the disagreement point $d^s = (0, 0)$ when $z = 0$, and its expected value $\mathbb{E}[d^s|\theta] = (0, 0)$, the maximal utility gain consumers might expect through bargaining would be equal to η according to our arbitrator. And this value is twice as large as that she believes might be obtained by the firm $\eta/2$. Notice that in the case of asymmetric information, the regulator readily plays the role of the outside arbitrator we have introduced above. Comparing the expected utilities we calculated above for Figure 2 to those in Figure 4, the maximal utility gain for the firm, $u^B_m - d^a_m = \eta$, and consumers, $u^A_c - d^a_c = \eta/2$ have a ratio of 2:1 in Figure 4, in contrast to the ratio of 1:2 calculated for Figure 2. The presence of asymmetric information seems to flip the ratio of maximal gains from bargaining. In fact, we can prove that this transformation is more general: the whole bargaining set S^a is a reflection of S^s with respect to the 45-degree line when $z = 0$.

Let us first make the following preparations. To compare, in expected utility terms, the Pareto frontier of the bargaining set S^s we used under symmetric information to the set S^a we shall use under asymmetric information, we can simply calculate the expected value of each point in S^s to get a comparison set $\mathbb{E}[S^s|\theta]$. Formally,

$$\mathbb{E}[S^s|\theta] = \{(x_1, x_2) \in \mathbb{R}^2 : \exists\, (s_1, s_2) \in S^s \quad s.t. \quad x_i = \mathbb{E}[s_i|\theta],\ i = 1, 2.\} \tag{48}$$

Notice that $\mathbb{E}[S^s|\theta]$ is convex and comprehensive because S^s is so. Also, $\mathbb{E}[d^s|\theta] = (0, 0) \in \mathbb{E}[S^s|\theta]$. Since θ is uniformly distributed over $[0, a)$ and since each point (s_1, s_2)

in S^s can be equivalently written, when $z = 0$, as $(h_1\nu, h_2\nu/2)$ for some $h_1, h_2 \in [0,1]$, the expected set $\mathbb{E}[S^s|\theta]$ can be simply obtained from S^s by mapping each point in $(h_1\nu, h_2\nu/2) \in S^s$ to the point $((\eta/\nu)s_1, (\eta/\nu)s_2)$, where $\eta = \mathbb{E}[\nu|\theta]$ as previously noted. Above, the non-constant coefficient η/ν is always positive since $\nu = (a-\theta)^2/2$ is positive for all $\theta \in [0, a)$. Similarly, we can obtain the equation that characterizes $PO(\mathbb{E}[S^s|\theta])$ by applying the expectation operator on the equation (29) to yield u_m and u_c, in expected utilities, satisfying

$$u_m = 2\sqrt{\eta u_c} - 2u_c, \tag{49}$$

for any $u_c \geq \eta/4$. Below, we state the observed relationship between S^s and $\mathbb{E}[S^s|\theta]$ as a lemma to which we shall refer in our results.

Lemma 1. *Given any market $\mathcal{M}^s$ with $z = 0$ and the induced bargaining problem (S^s, d^s), we have $(x, y) \in S^s$ if and only if $(hx, hy) \in \mathbb{E}[S^s|\theta]$ where $h = \eta/\nu$.*

Now, we can state a result showing that the bargaining set $\mathbb{E}[S^s|\theta]$ is dual to (a reflection of) S^a with respect to the 45-degree line passing through the point d^a. We shall use this result to prove our next proposition.

Lemma 2. (Duality of $\mathbb{E}[S^s|\theta]$ and S^a) *Consider any market $\mathcal{M}^s$ with $z = 0$ under symmetric information and its extension $\mathcal{M}^a$ to asymmetric information. Also consider the induced bargaining problems (S^s, d^s) and (S^a, d^a). We have $(x, y) \in \mathbb{E}[S^s|\theta]$ if and only if $(y, x) \in S^a$.*

Proof. First take any (x, y) in $PO(\mathbb{E}[S^s|\theta])$. By (49), $y = 2\sqrt{\eta x} - 2x - z$ where $x \in [\eta/4, \eta]$. We have to show that the point $(2\sqrt{\eta x} - 2x, x)$ is in $PO(S^a)$. If this is true then by equation (47), x must be equal to $\eta/(2-\alpha)^2$ for some $\alpha \in [0, 1]$. Clearly, this holds for $\alpha = 2 - \sqrt{\eta/x}$. Notice that $x \in [\eta/4, \eta]$ implies $\alpha \in [0, 1]$, so x is an admissible utility for player 2 (the firm) in $PO(S^a)$. Also, notice that $\alpha = 2 - \sqrt{\eta/x}$ implies $2(1-\alpha) = 2\sqrt{\eta/x} - 2$. Multiplying this by $\eta/(2-\alpha)^2 = x$, we obtain $2(1-\alpha)\eta/(2-\alpha)^2 = 2\sqrt{\eta x} - 2x$. Since we know that the point $(2(1-\alpha)\eta/(2-\alpha)^2, \eta/(2-\alpha)^2)$ is inside $PO(S^a)$ for any $\alpha \in [0, 1]$, we have proved that $(2\sqrt{\eta x} - 2x, x) \in PO(S^a)$.

Now take any point $(x, y) \in PO(S^a)$. We have to show that $(y, x) \in PO(\mathbb{E}[S^s|\theta])$. Since $(x, y) \in PO(S^a)$, there exists $\alpha \in [0, 1]$ such that $x = 2(1-\alpha)\eta/(2-\alpha)^2$ and $y = \eta/(2-\alpha)^2$. The point (y, x) is in $PO(\mathbb{E}[S^s|\theta])$ only if it satisfies (49). Thus, we must have $x - z = 2\sqrt{\eta y} - 2y$. We can easily check that $y = \eta/(2-\alpha)^2$ and $x = 2(1-\alpha)\eta/(2-\alpha)^2$ satisfy the above equation. Moreover, since $\alpha \in [0, 1]$, $y \in [\eta/4, \eta]$ and $x \in [0, \eta/2]$. Therefore $(y, x) \in PO(\mathbb{E}[S^s|\theta])$.

So far we have proved that $(x, y) \in PO(\mathbb{E}[S^s|\theta])$ if and only if $(y, x) \in PO(S^a)$. Now, pick any $(x', y') \in \mathbb{E}[S^s|\theta] \setminus PO(\mathbb{E}[S^s|\theta])$. We can find some $(x, y) \in PO(\mathbb{E}[S^s|\theta])$ such that $(x', y') \leq (x, y)$ meaning $x' \leq x$ and $y' \leq y$. Let $A(x, y) = [0, x] \times [0, y]$. Clearly, $(x', y') \in A(x, y)$. Also, $A(x, y) \subset \mathbb{E}[S^s|\theta]$, by equations (30) and (48), since

$\mathbb{E}[S^s|\theta]$ is convex and comprehensive. Given the definition of $A(x,y)$, we know that $(y',x') \in A(y,x)$. From the first paragraph of this proof, it follows that $(y,x) \in PO(S^a)$. Then, $A(y,x) \subset S^a$, by equation (47), since S^a is convex and comprehensive. This implies $(y',x') \in S^a$.

Finally, pick any $(x',y') \in S^a \setminus PO(S^a)$. We can find some $(x,y) \in PO(S^a)$ such that $(x',y') \leq (x,y)$ meaning $x' \leq x$ and $y' \leq y$. Let $A(x,y) = [0,x] \times [0,y]$. Clearly, $(x',y') \in A(x,y)$. Also, $A(x,y) \subset S^a$, by equation (47), as S^a is convex and comprehensive. Given the definition of $A(x,y)$, we know that $(y',x') \in A(y/2,x)$. From the second paragraph of this proof, it follows that $(y,x) \in PO(\mathbb{E}[S^s|\theta])$. Then, $A(y,x) \subset \mathbb{E}[S^s|\theta]$, by equation (30) and (48), since $\mathbb{E}[S^s|\theta]$ is convex and comprehensive. This implies $(y',x') \in \mathbb{E}[S^s|\theta]$. ■

Lemma 2 implies that the bargaining set S^a under asymmetric information and the expected bargaining set $\mathbb{E}[S^s|\theta]$ under symmetric information are merely reflections of each other with respect to the 45-degree line, when $z = 0$. This result also implies the following remark.

Remark 1. If an allocation $s \in S^a$ is dual, when $z = 0$, to an allocation $s' \in \mathbb{E}[S^s|\theta]$, then the expected deadweight losses at s and s' are the same; i.e., $\eta - s_1 - s_2 = \eta - s'_1 - s'_2$.

Interestingly, the expected deadweight loss of amount $\eta/4$ that arises under the unregulated monopoly allocation $(\eta/4, \eta/2)$ can also be observed under the optimal regulatory allocation $(\eta/2, \eta/4)$ under asymmetric information if the welfare parameter α is zero, the best value from the viewpoint of consumers. Lemma 1 and Lemma 2 together lead to the following result.

Proposition 6. **(Duality of S^s and S^a)** *Consider any market $\mathcal{M}^s$ with $z = 0$ under symmetric information and its extension $\mathcal{M}^a$ to asymmetric information. Also consider the induced bargaining problems (S^s, d^s) and (S^a, d^a). We have $(x,y) \in S^s$ if and only if $(hy, hx) \in S^a$ where $h = \eta/\nu$.*

Proof. First take any $(x,y) \in S^s$. By Lemma 1, $(hx, hy) \in \mathbb{E}[S^s|\theta]$ where $h = \eta/\nu$. Then, Lemma 2 implies $(hy, hx) \in S^a$. Now, take any $(y,x) \in \mathbb{R}^2$ such that $(hy, hx) \in S^a$. Then, Lemma 2 implies $(hx, hy) \in \mathbb{E}[S^s|\theta]$ and Lemma 1 implies $(x,y) \in S^s$. ■

Proposition 6 can easily be extended to the case with $z > 0$, where we would have $(x,y) \in S^s$ if and only if $(h(y+z), hx - z/2) \in S^a$. When $z = 0$ as in Proposition 6, the bargaining sets $(\eta/\nu)S^s$ and S^a differ only in the identity of the bargainers. In bargaining theory, an axiom, called anonymity, ensures that the identity of the bargainers does not affect the solution, and an axiom, called homogeneity, ensures that scaling all utilities in the bargaining set by the same positive constant does not affect the solution. Now, we shall formally state these axioms. Let φ denote the permutation on the set of agents $N = \{1, 2\}$ with $\varphi(1) = 2$ and $\varphi(2) = 1$. A bargaining rule F satisfies *anonymity* if $F(\varphi S, \varphi d) = \varphi F(S, d)$ for every problem $(S, d) \in \Sigma^2$. On the other hand, a bargaining rule F satisfies *homogeneity* if $F(rS, rd) = rF(S, d)$ for every problem $(S, d) \in \Sigma^2$ and $r \in \mathbb{R}_{++}$.

Proposition 7. (Duality of Solutions on $(\boldsymbol{S^s}, \boldsymbol{d^s})$ and $(\boldsymbol{S^a}, \boldsymbol{d^a})$) *Consider any market $\mathcal{M}^s$ with $z = 0$ under symmetric information and its extension $\mathcal{M}^a$ to asymmetric information. Also consider the induced bargaining problems (S^s, d^s) and (S^a, d^a). If a bargaining rule F satisfies anonymity and homogeneity, then $F_1(S^a, d^a) = hF_2(S^s, d^s)$ and $F_2(S^a, d^a) = hF_1(S^s, d^s)$ where $h = \eta/\nu$.*

Proof. Let F be any bargaining solution that satisfies anonymity and homogeneity. Let $S' = \mathbb{E}[S^s|\theta]$ and $d' = \mathbb{E}[d^s|\theta]$. Then, $S' = hS^s$ by Lemma 1 where $h = \eta/nu$. We also have $d' = hd^s = (0, 0)$, since $z = 0$. Then, $F(S', d') = hF(S^s, d^s)$ by homogeneity. Now, consider the permutation φ on the set of agents $N = \{1, 2\}$ with $\varphi(1) = 2$ and $\varphi(2) = 1$. By Lemma 2, $S' = \varphi S^a$. So, $F(S', d') = \varphi F(S^a, d^a)$ by anonymity. Since $F(S', d') = hF(S^s, d^s)$, it follows that $hF(S^s, d^s) = \varphi F(S^a, d^a)$, implying $F_1(S^a, d^a) = hF_2(S^s, d^s)$ and $F_2(S^a, d^a) = hF_1(S^s, d^s)$. ■

When $z > 0$, the above result can be restated with the help of an additional axiom, known as translation invariance. Formally, a bargaining rule F satisfies translation invariance if $F(S, d) = F(S + k, d + k)$ for every problem (S, d) and every $k \in R^2$. Along the lines in the proof of Proposition 7, one can easily check that a bargaining rule F satisfies anonymity, homogeneity, and translation invariance only if $F_1(S^a, d^a) = h(F_2(S^s, d^s) + z)$ and $F_2(S^a, d^a) = hF_1(S^s, d^s) - z/2$ where $h = \eta/\nu$. Notice that the Egalitarian, Nash, and Kalai-Smorodinsky rules satisfy both anonymity and homogeneity (as well as translation invariance). To find the solutions to these rules under asymmetric information when $z = 0$, it is sufficient, by Proposition 7, to first multiply the utilities under symmetric information by η/ν and then interchange the utilities of consumers and the firm.

Proposition 8. *Given any market $\mathcal{M}^a$ with $z = 0$ and the induced bargaining problem (S^a, d^a),*

(i) the Egalitarian rule yields the utilities $u_c^E = (4/9)\eta$ and $u_m^E = (4/9)\eta - z/2$.

(ii) the Nash rule yields the utilities $u_c^N = (6/16)\eta$ and $u_m^N = (9/16)\eta - z/2$.

(iii) the Kalai-Smorodinsky rule yields the utilities $u_c^{KS} = (8/25)\eta$ and $u_m^{KS} = (16/25)\eta - z/2$.

Figure 5 illustrates the results in Proposition 8. Because of the duality result in Proposition 7, we see that the presence of asymmetric information changes the preference orderings of consumers and the firm over the Egalitarian, Nash, and Kalai-Smorodinsky rules. Under asymmetric information, consumers prefer the most to bargain using the Egalitarian rule, while they prefer the Nash rule to the Kalai-Smorodinsky rule. The preference ordering of the firm is in the opposite direction, as expected. Since the asymmetry of information has only changed the distribution of the induced total welfare but not its expected size under the studied rules, the preference ordering of the society as a whole remains the same. The society still prefers, now just like the firm, the Kalai-Smorodinsky rule to the Nash rule, and the Nash rule to the Egalitarian rule. As in the case of symmetric information, we find that the Nash rule is more egalitarian and less utilitarian than the Kalai-Smorodinsky rule, while both of these rules are more utilitarian than the Egalitarian rule.

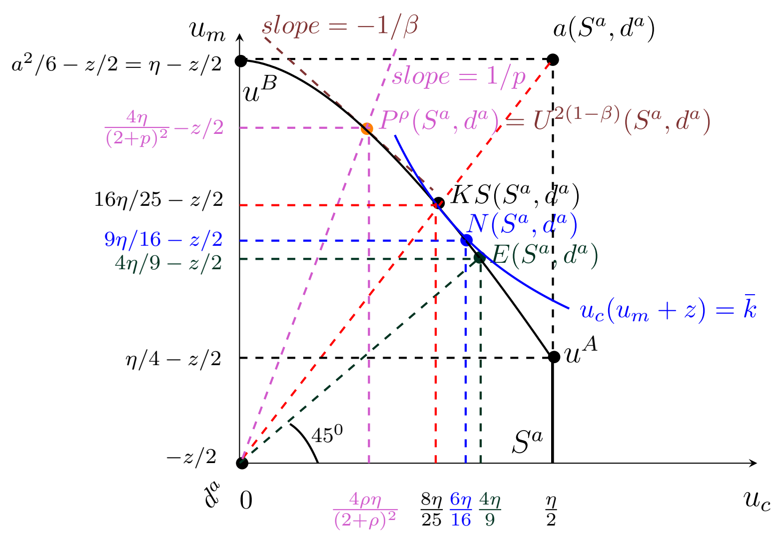

Figure 5. The Solutions of Several Bargaining Rules under Asymmetric Information and Sunk Fixed Cost

The firm is not primarily interested in the expected (ex-ante) welfare as it can calculate its actual (ex-post) welfare using its private information. The solution of the regulatory bargaining is important for the firm to the extent that it determines the α parameter that enters into the output equation $q^{BM}(\theta) = a - (2-\alpha)\theta$, thereby affecting the information rent $\pi(\theta) = \int_0^{\theta^*} q^{BM}(x)dx$. Equating the firm's expected regulatory profit $\eta/(2-\alpha)^2$ in (46), obtained from the BM (1982) mechanism, to its bargaining utilities implied by Proposition 8 we obtain that the Egalitarian, Nash, and Kalai-Smorodinsky rules choose the value of α as $1/2, 2/3$, and $3/4$ respectively. Since both $\theta^* = a/(2-\alpha)$ and $q^{BM}(\theta)$ are increasing in α, we know that the actual (ex-post) regulatory profit, $\pi(\theta)$, of the monopolist is increasing too. So, for any θ, the firm prefers the Kalai-Smorodinsky rule to the Nash rule, and the Nash rule to the Egalitarian rule, even when it is concerned with its ex-post welfare only.

The β-weighted Utilitarian rule does not satisfy the axiom of anonymity unless $\beta = 1$, even though it always satisfies homogeneity. Therefore, we cannot appeal to the duality result in Proposition 7 to find the solution of β-weighted Utilitarian rule under asymmetric information. Below, we make a direct calculation.

Proposition 9. *Given any market $\mathcal{M}^a$ with $z = 0$ and the induced bargaining problem (S^a, d^a), the β-weighted Utilitarian rule yields the utilities*

$$u_c^\beta = \begin{cases} \frac{2(1-\beta)\eta}{(2-\beta)^2} & \text{if } \beta \in [0,1] \\ 0 & \text{if } \beta > 1 \end{cases} \qquad \text{and} \qquad u_m^\beta = \begin{cases} \frac{\eta}{(2-\beta)^2} & \text{if } \beta \in [0,1] \\ \eta & \text{if } \beta > 1. \end{cases} \tag{50}$$

The above result says that if $\beta \in [0,1]$, the β-weighted Utilitarian rule provides each bargaining party with an expected utility payoff that is equal to what could be obtained under the regulatory mechanism of BM (1982) when the welfare weight α were equal to β. After all, this is not surprising since the social welfare function of BM (1982) is simply a β-weighted Utilitarian rule with β restricted to the interval $[0,1]$. On the other hand, if $\beta > 1$ (and also if $\beta = 1$), the β-weighted Utilitarian rule would assign all the expected economic surplus to the firm. If we contrast these results to those in Proposition 2, we see that the welfare distributions are different since the bargaining sets under symmetric and asymmetric information are different. However, the monotonous effects of β on the welfares of two parties remain unchanged, as expected. Propositions 8 and 9 also imply the following.

Corollary 5. *Given any market $\mathcal{M}^a$ with $z = 0$ and the induced bargaining problem (S^a, d^a), the solution of the β-weighted Utilitarian rule coincides with the solution of (i) the Egalitarian rule if and only if $\beta = 1/2$, (ii) the Nash rule if and only if $\beta = 2/3$, and (iii) Kalai-Smorodinsky rule if and only if $\beta = 3/4$.*

The values of β in Corollary 5 that equate the solution of the β-weighted Utilitarian rule to the solutions of the Egalitarian, Nash, or Kalai-Smorodinsky rules under asymmetric information are just the inverse of the respective weights calculated in Corollary 1 under symmetric information. We now turn to consider the ρ-Proportional rule. This rule does not satisfy the axiom of anonymity unless $\rho = 1$ (the Egalitarian solution), even though it always satisfies homogeneity. Therefore, like in the case of β-weighted Utilitarian rule, Proposition 7 will be of no help to find the solution of ρ-Proportional rule under asymmetric information.

Proposition 10. *Given any market $\mathcal{M}^a$ with $z = 0$ and the induced bargaining* *problem* (S^a, d^a)*, the* ρ*-Proportional rule yields the utilities*

$$u_c^\rho = \begin{cases} \frac{4\rho}{(2+\rho)^2}\eta & \text{if } \rho \in (0,2] \\ \frac{1}{2}\eta & \text{if } \rho > 2 \end{cases} \qquad \text{and} \qquad u_m^\rho = \begin{cases} \frac{4}{(2+\rho)^2}\eta & \text{if } \rho \in (0,2] \\ \frac{1}{2\rho}\eta & \text{if } \rho > 2. \end{cases} \tag{51}$$

It is clear that $P^\rho(S^a, d^a) = D^1(S^a, d^a) = u^A$ if $\rho \geq 2$ and $\lim_{\rho \to 0} P^\rho(S^a, d^a) = D^2(S^a, d^a) = u^B$. As in the case of symmetric information, the ρ-Proportional rule can yield *weakly* Pareto optimal allocations, as well: its outcome lies on $WPO(S^a) \backslash PO(S^a)$ if $\rho > 2$ and lies on $PO(S^a)$ if $\rho \in (0,2]$. We also have the following observation.

Corollary 6. *Given any market* $\mathcal{M}^a$ *with* $z = 0$ *and the induced bargaining problem* (S^a, d^a)*, the solution of the* ρ*-Proportional rule coincides with the solution of (i) the Nash rule if and only if* $\rho = 2/3$ *and (ii) the Kalai-Smorodinsky rule if and only if* $\rho = 1/2$*.*

We should also observe that Proposition 10 and Corollary 6 can be obtained from Proposition 3 and Corollary 2 respectively, by interchanging the utilities of the two bargaining parties with each other after replacing the parameter ρ in those earlier results with $1/\rho$ and the maximal economic surplus ν with its expected value η. The same relationship also holds between the results in Proposition 9 and Corollary 5 and those in Proposition 2 and Corollary 1. Using these relationships, Corollary 3 can be restated under asymmetric information as follows.

Corollary 7. *Given any market* $\mathcal{M}^a$ *with* $z = 0$ *and the induced bargaining problem* (S^a, d^a)*, the solutions of* β*-weighted Utilitarian rule and the* ρ*-Proportional rule coincide if and only if* $\beta \in [0, 1)$ *and* $\rho = 2(1 - \beta)$*.*

Interestingly, even under asymmetric information, the solution of each bargaining rule we study can be related to the equilibrium distribution of some unregulated symmetric Cournot oligopoly. However, there is an important difference with respect to the result (Proposition 4) obtained under symmetric information. Each bargaining party is now entitled to what its rival could obtain under no regulation.

Proposition 11. (Relatedness to Permuted Cournot Welfares) *Given any market* $\mathcal{M}^a$ *with* $z = 0$ *and the induced bargaining problem* (S^a, d^a)*, the utility of the firm (consumers) under the bargaining rule* F *is equal to the expected consumer surplus (industry profits) obtained in an unregulated symmetric Cournot oligopoly involving*

(i) *two firms (duopoly) if* F *is the Egalitarian rule,*

(ii) *three firms if* F *is the Nash rule,*

(iii) *four firms if F is the Kalai-Smorodinsky rule,*

(iv) $1/(1-\beta)$ *firms if F is the β-weighted Utilitarian rule U^{β}, provided that* $1/(1-\beta)$ *is an integer, and*

(v) $2/p$ *symmetric firms if F is the ρ-Proportional rule, provided that $2/p$ is an integer.*

Proof. When the regulator has uniformly distributed beliefs on $[0, a)$, it follows from equation (28) that the expected value of the consumer surplus, $\mathbb{E}[CS^*(n)|\theta]$, is $n^2\eta/(n+1)^2$. Also, notice that $F_2(S^a, d^a)$ takes the value $4\eta/9$ when $F = E$, $9\eta/16$ when $F = N$, and $16\eta/25$ when $F = KS$. So, parts (i), (ii), and (iii) follow from equating these values to $n^2\eta/(n+1)^2$ and solving for n.

Part (iv). Notice that if $\beta \geq 1$, then $F_2(S^a, d^a) = u_m^{\beta}$ can be equal to $\mathbb{E}[CS^*(n)|\theta]$ only if $n = \infty$. So, let $\beta \in [0, 1)$. Equating u_m^{β} to $\mathbb{E}[CS^*(n)|\theta]$ implies that $1/(2-\beta)^2 = n^2/(n+1)^2$, implying $n = 1/(1-\beta)$. Since n must be a positive integer not less than 2, the above equality can hold only if $1/(1-\beta)$ is a positive integer.

Part (v). Notice that if $\rho > 2$, then $F_2(S^a, d^a) = u_m^{\rho} = \eta/(2\rho)$ from (51). Then, $u_m^{\rho} < \eta/4$ for all $\rho > 2$. Equating u_c^{ρ} to $\mathbb{E}[CS^*(n)|\theta]$ implies $(n/(n+1))^2\eta < \eta/4$, further implying $n/(n+1) < 1/2$ which cannot hold for any positive integer n. So, let $\rho \in (0, 2]$. Then from (51) it follows that $u_m^{\rho} = 4\eta/(2+p)^2$. Equating this to $\mathbb{E}[CS^*(n)|\theta]$ implies $4/(2+p)^2 = n^2/(1+n)^2$, further implying $n = 2/p$. Since n is an integer, this equality holds only if $2/p$ is an integer. ■

Notice that Proposition 11 can be also obtained from our previous results. The fact that the Egalitarian, Nash, and Kalai-Smorondisky rules satisfy homogeneity and anonymity along with the duality result in Proposition 7 ensures that the result in Proposition 4 establishing "Relatedness to Cournot Outcomes" obtained under symmetric information can be true only for permuted welfares under symmetric information. However, to prove the last two parts of Proposition 11, the duality of bargaining solutions (Proposition 7) is, in general, helpless since the axiom of anonymity is not satisfied by the β-weighted Utilitarian rule unless $\beta = 1$ or by the ρ-Proportional rule unless $\rho = 1$. On the other hand, the duality of bargaining sets in Proposition 6 guarantees that the β-weighted Utilitarian rule under symmetric information and the $1/\beta$-weighted Utilitarian rule under asymmetric information lead to allocations that are permutations of each other. A similar observation is true for the ρ-Proportional rule under symmetric information and $1/\rho$-Proportional rule under asymmetric information. Using these observations, we can trivially obtain parts (iv) and (v) in Proposition 11 from the same parts in Proposition 4, respectively.

3.3 An Axiomatic Approach

The regulatory bargaining problem of consumers and the firm we discussed in the previous parts of this section requires them, in the first place, to select and fix a bargaining rule. The bargaining rules that we have studied are all known to be uniquely characterized by some axioms. For example, in two-agent settings with disagreement point normalized to zero, the Egalitarian rule can be uniquely characterized by Weak Pareto Optimality, Symmetry, and Strong Monotonicity axioms (Kalai, 1977), the Nash rule by Weak Pareto Optimality, Scale Invariance, and Independence of Irrelevant Alternatives axioms (Nash, 1950), and the Kalai-Smorodinsky rule by Weak Pareto Optimality, Scale Invariance, and Individual Monotonicity axioms (Kalai-Smorodinsky, 1975). While some of these axioms, since they were presented, have been considered by common consensus to be plausible and perhaps even desirable, some others have been welcome with some controversies.

If the regulated firm and consumers are content to use any particular bargaining rule, regardless of whether it is perceived by any third party controversial or not, then they can determine the regulatory objective, or specifically the welfare weight parameter α, and the corresponding equilibrium allocations using the approach we outlined in Sections 3.1 and 3.2. However, we should be aware of the fact that the bargaining rules we have studied as well as the other rules known in the literature were all proposed for abstract settings in which any bargaining set, satisfying minimal assumptions such as convexity and comprehensiveness, is admissible. In addition to this abstraction, most of the known bargaining rules pay little, if any, attention to the geometry of the bargaining set or the inherent bargaining powers of agents therein. That is to say a bargaining rule that can be perceived to have a desirable set of axioms may, on the other hand, be impartial to a great part of the information available in the particular bargaining problem at hand. Below, we shall investigate whether/how we can improve the decisions of consumers and the firm in selecting a bargaining rule, by focusing on the geometry of the bargaining sets S^a and $S^e \equiv \mathbb{E}[S^s|\theta]$ and the very essentials of the regulation problem. Let $z = 0$ to ensure by Lemma 2 that S^a and S^e are reflections of each other with respect to the 45-degree line. We need one last bit of definition before we present our axioms and results.

Let $\mathcal{B}(S^a)$ denote the set of bilaterally beneficial allocations in S^a with respect to the expected utility allocation $(\eta/4, \eta/2)$ implied by the unregulated monopoly solution. Formally, we have

$$\mathcal{B}(S^a) = \{(u_c, u_m) \in PO(S^a) : u_c \geq \eta/4 \text{ and } u_m \geq \eta/2\}. \tag{52}$$

Equations (47) and (52) together imply that a welfare weight α leads to a bilaterally beneficial allocation only if $\alpha \leq 2(\sqrt{2}-1) \sim 0.83$ ensuring $u_c \geq \eta/4$ and $\alpha \geq 2-\sqrt{2} \sim$

0.59 ensuring $u_m \geq \eta/2$. Thus, we have

$$\mathcal{B}(S^a) = \Big\{(u_c, u_m):\ u_c = \frac{2(1-\alpha)\eta}{(2-\alpha)^2}, \\ u_m = \frac{\eta}{(2-\alpha)^2},\ \text{and } \alpha \in \Big[2-\sqrt{2}, 2\sqrt{2}-2\Big]\Big\}. \quad (53)$$

Using equation (53), one can calculate that of all the social welfare functions (or of all α values) that are admissible in the BM (1982) model of regulation, only 24% (=%83-%59) of them ensure that both the firm and consumers become better off in ex-ante utilities when the firm is regulated than when it is not. Getting rid of α, equation (53) can alternatively be written as follows:

$$\mathcal{B}(S^a) = \Big\{(u_c, u_m) \in S^a : u_c \in [\eta/4, (\sqrt{2}-1)\eta] \text{ and } u_m \in [\eta/2, \eta/(4-2\sqrt{2})^2]\Big\}. \quad (54)$$

Now we are ready to present a series of axioms.

Pareto Optimality (PO) $F(S^a, d^a) \in PO(S^a)$.

Axiom **PO** and its weak version **WPO**, which requires $F(S^a, d^a) \in WPO(S)$, are standard in axiomatic bargaining. Since $PO(S^a)$ contains all possible outcomes of the BM (1982) mechanism, corresponding to all values of α in $[0, 1]$, it is natural to demand a bargaining solution to satisfy **PO** as the minimum requirement.

Weak Pareto Improvements (WPI) There exists $x \in WPO(S^e)$ such that $F_i(S^a, d^a) \geq x_i$ for each $i = 1, 2$.

WPI requires that the bargaining allocation under asymmetric information is weakly superior to some point in the weak Pareto frontier of the expected bargaining set under symmetric information.

Rationalizable Maximal Earnings (RME) $F_i(S^a, d^a) \leq a_i(S^e, d^e)$ for each $i = 1, 2$.

RME requires that no bargaining party obtains an expected utility under asymmetric information that it could not be expected to obtain (according to the beliefs of the regulator) under symmetric information at any price in the unregulated industry. More specifically, it requires $F_1(S^a, d^a) \leq \eta$ and $F_2(S^a, d^a) \leq \eta/2$, meaning that under asymmetric information consumers cannot expect to obtain from bargaining more than they could get at the optimally regulated price under symmetric information, and the firm cannot expect, before it learns its private information, to obtain from bargaining more than it could earn at the unregulated monopoly price.

Respect for Bilateral Benefits (RBB) $F(S^a, d^a) \in \mathcal{B}(S^a)$.

RBB requires the bargaining solution to lie inside the set of bilaterally beneficial utilities for consumers and the firm. Let $Core(\mathcal{M}^a) \equiv PO(\mathcal{B}(S^a))$ denote the set of core allocations for the market M^a. Notice that **PO** and **RBB** together imply $F(S^a, d^a) \in Core(\mathcal{M}^a)$. Moreover, **RBB** implies **WPI**; but the converse is not true.

No Informational Benefits for Firm (NIBF) There exists $x \in WPO(S^e) \cap WPO(S^a)$ such that $F_2(S^a, d^a) \leq x_2$.

NIBF simply requires that the firm (agent 2) should not expect to benefit from its private information in the bargaining process. In more detail, this axiom states that the agent's bargaining utility under any rule F can never exceed what it could achieve both under symmetric and asymmetric information under a weakly Pareto optimal rule. Notice that the unique point in $WPO(S^e) \cap WPO(S^a)$ is the Egalitarian solution $E(S^a, d^a)$. Therefore, **NIBF** is satisfied by a bargaining rule F only if $F_2(S^a, d^a) \leq E_2(S^a, d^a) = 4\eta/9$. Also notice that **NIBF** implies **RME**; but the converse is not true.

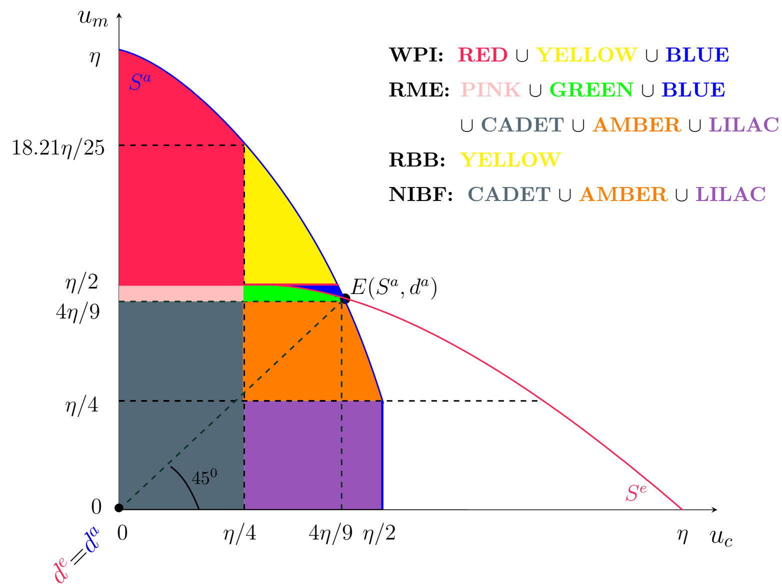

Figure 6. Regions of S^a in Terms of Axioms of WPI, RME, RBB, and **NIBF**.

Figure 6 illustrates various regions of the bargaining set S^a in relation to the axioms **WPI**, **RME**, **RBB**, and **NIBF**. Below, we present several characterizations using these axioms. Since **PO** is our minimum requirement for a bargaining rule, we shall first consider the effect of adding each of the other four axioms one by one to **PO**.

Remark 2. *A bargaining rule satisfies* *(i)* ***PO*** *and* ***WPI*** *if and only if it yields an outcome on* $PO(S^a)$ *obtained with* $\alpha \in [0.5, 1]$,

(ii) ***PO*** *and* ***RME*** *if and only if it yields an outcome on* $PO(S^a)$ *obtained with* $\alpha \in [0, 2-\sqrt{2}] \sim [0, 0.59]$,

(iii) ***PO*** *and* ***RBB*** *if and only if it yields an outcome on* $PO(S^a)$ *obtained with* $\alpha \in [2-\sqrt{2}, 2\sqrt{2}-2] \sim [0.59, 0.83]$,

(iv) ***PO*** *and* ***NIBF*** *if and only if it yields an outcome on* $PO(S^a)$ *obtained with* $\alpha \in [0, 0.5]$.

We obtain the results in Remark 2 by calculating the implication of each axiom on α using (47). In parts (i) and (iii), we can actually relax **PO** by **WPO**. We should also notice that each of the axioms **WPI**, **RME**, **RBB**, and **NIBF** leads to a different characterization when combined with **PO**. However, none of these four axioms is strong enough, when coupled with **PO**, to identify a unique point on $PO(S^e)$ or correspondingly in the unit interval of α values. So, we should consider triples of axioms.

Remark 3. *A bargaining rule satisfies* ***WPO****,* ***WPI****, and* ***RME*** *if and only if it yields an outcome on* $PO(S^a)$ *obtained with* $\alpha \in [1/2, 2-\sqrt{2}] \sim [0.50, 0.59]$.

Altogether, **WPO**, **WPI**, and **RME** considerably narrow down the set of Pareto efficient allocations that a bargaining rule may select on S^a. However, there is still an uncountable number of possibilities. So, we should appeal to our last two axioms, **RBB** and **NIBF**. But, we have to consider each of them in isolation, since no list of axioms that contain both **RBB** and **NIBF** can lead to any characterization.

Remark 4. *There exists no bargaining rule that satisfies both* ***RBB*** *and* ***NIBF****.*

Since **RBB** is a strengthening of **WPI**, we replace **WPI** in Remark 3 with **RBB** to obtain the following characterization.

Proposition 12. *A bargaining rule satisfies* ***WPO****,* ***RME****, and* ***RBB*** *if and only if it is outcome equivalent on* (S^a, d^a) *to a solution that yields the outcome* $((\sqrt{2}-1)\eta, \eta/2-z)$ *obtained with* $\alpha = 2-\sqrt{2} \sim 0.59$.

Recalling that **NIBF** is a strengthening of **RME**, we now replace **RME** in Remark 3 with **NIBF**.

Proposition 13. *A bargaining rule satisfies* ***WPO****,* ***WPI****, and* ***NIBF*** *if and only if it is outcome equivalent on* (S^a, d^a) *to the Egalitarian rule that yields the outcome*

$(4\eta/9, 4/9)$ *obtained with* $\alpha = 1/2$.

We skip the proofs of Propositions 12 and 13 as they directly follow from the implications of **WPO**, **WPI**, **RME**, **RBB**, and **NIBF** on the set S^a, as illustrated in Figure 6. Using (45) and (46), we can check that the total expected welfare under the modified BM (1982) model is $(3-2\alpha)\eta/(2-\alpha)^2$, which is increasing in α. Therefore, the total expected welfare is higher under the bargaining rule characterized in Proposition 12 (implying a solution with $\alpha = 0.59$) than under the Egalitarian rule (implying a solution with $\alpha = 0.50$). Despite this fact, the use of the Egalitarian rule instead of a bargaining rule inducing $\alpha = 0.59$ can be rationalized if the society as a whole prefers the pair of axioms **WPI** and **NIBF** to the pair of axioms **RME** and **RBB**.

4 Conclusion

In this paper, we have attempted to integrate the bargaining theory with the problem of regulating a natural monopoly under symmetric information or asymmetric information with complete ignorance. In the literature, the optimal regulatory solutions proposed under symmetric or asymmetric information are known to be consistent with social welfare functions of linear forms where the weight of consumers' welfare is higher than (or equal to) the weight of the monopolist's welfare. This creates under symmetric information an utter inequity where the monopolist earns zero profit and all economic surplus goes to consumers, rendering the industry unsustainable. Under asymmetric information, a similar situation may arise, in real practices, if the informational rent that must be offered to the monopolist according to the optimal incentive-compatible solution of BM (1982) is not sufficiently high from the viewpoint of the monopolist and/or not sufficiently low from the viewpoint of consumers. This information rent depends on, among several other factors, the relative weight of consumers' welfare in the social objective function. The regulation literature is currently undermining the possibility that even under the well-designed (incentive-compatible and individually rational) regulatory programs offered in the literature, the regulatory solutions may be practically far from being accepted by consumers or the monopolist if the social objective function is not carefully identified by the regulator. The bargaining approach we have offered in this paper helps consumers and the monopolist (or a benevolent regulator acting on behalf of them) make this identification in a natural and theoretically appealing way.

Using this new approach, we have examined the implications of some well-known bargaining rules on the regulatory social objective and the induced welfare distribution under symmetric information as well as under asymmetric information using a duality result. Also, we have related the payoff allocations under some well-known bargaining rules (both under symmetric and asymmetric information) to the Cournot-Nash equilibrium allocations of some unregulated symmetric oligopolies. This result is particularly important under symmetric information since it provides a benchmark to answer how one can restore the aforementioned sustainability of a natural monopoly. For example, in situations the society or the regulator would find it unnecessary or inappropriate to regulate a non-collusive duopoly, they could be certain to achieve the welfare distribution thereof by simply letting consumers and the monopolist bargain over the set of possible payoff allocations using the Egalitarian rule. Our paper also presents an axiomatic approach to identify, under asymmetric information, the bargaining rules that satisfy, in addition to the weak or strong Pareto optimality, some new axioms which only use the relevant information in the regulatory environment. Interestingly, one of our characterization results reveals that the Egalitarian rule can be desirable for the society to be used in negotiations under asymmetric information, as well.

We should consider our work as an initial step to bridge the bargaining theory with the economics of oligopolistic regulation. The bargaining approach we used can be argued to be welfarist as it admits only (expected) utility information, and therefore may be subject to the general criticism faced by welfarism on the ground of its inadequacy for distributive justice (See, for example, Sen, 1979). For higher dimensional problems of interest that involve many firms with many products (possibly with externalities), one may leave our welfarist approach, by applying mechanism theory, following the suggestion of Roemer (1988), directly on the essentials of the regulatory environments, involving a profile of resource/quantity/price choices and utility functions defined on these choices.

Future research may also extend our work in several directions. First, one can relax the assumption of complete ignorance, or the uniformly distributed prior beliefs, to investigate how the bargaining payoffs of consumers could be affected in case consumers or the regulator acquire, and/or the monopolist intentionally reveals or signals, some bit of information about the monopolist's private cost parameter before the regulatory action takes place. This extension would allow one to obtain characterization results in terms of less restrictive axioms. Currently, some of our axioms seem to be more like conditions as they only use the information captured by two (generic) bargaining sets corresponding to symmetric information and asymmetric information. Introducing different levels of ambiguity about the regulator's incomplete information about the monopolist's private cost parameter, it is possible to obtain under asymmetric information uncountably many (essentially different) bargaining sets. In fact, even without the help of such an ambiguity, our axioms can be made richer by changing the demand parameter a and the fixed cost z (which are kept constant in our work) and thereby changing the bargaining sets both under symmetric and asymmetric information.

Second, our model, which assumes linear cost and demand functions for the clarity and simplicity of the new ideas we have introduced, can be extended to allow for non-linear forms of demand and/or cost. While the regulatory theory of BM (1982) also assumes an (affinely) linear cost function (along with a downward sloping demand curve), it is easily applicable to industries with non-linear cost functions. Likewise, the regulatory bargaining approach we introduce in this paper is independent of the cost and demand structure of the industry and can be used as long as the optimal regulatory outcome can be explicitly determined in expected utilities. However, one can show that some of our results, namely the duality of bargaining sets under asymmetric and symmetric information and the relation between the outcomes of well-known bargaining rules and Cournot outcomes may cease to exist under alternative industry settings, for example when the cost function of the monopolist is changed to a quadratic form while all other elements of our model are kept the same. Future research can study whether these particular results can be obtained under any combinations of non-linear cost and/or demand functions and/or non-uniform beliefs.

Finally, one can incorporate into our model the possibility of pre-donation, an idea first introduced to the bargaining literature by Sertel (1992), who showed that in a general two-agent setting the Nash bargaining rule can be manipulated if one of the agents unilaterally commits to pre-donate (transfer) a portion of his/her utility to the other agent. Using this idea, one can profitably study whether any party, consumers or the monopolist, can increase under any well-known bargaining rule its ex-ante and/or ex-post welfare (or even the social welfare) if it pre-donates to the other party a certain fraction of its would-be payoffs.

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Appendix

Proof of Proposition 1. We consider three parts separately.

(i). It follows from (20) that $(u_c^E, u_m^E) \in WPO(S^s)$. Also, $d^s = (0, -z)$ and (20) together imply

$$u_m^E + z = u_c^E. \tag{55}$$

It follows from (30) that $u_c^E > (a-\theta)^2/8$, for otherwise (55) would be violated. Therefore,

$$u_m^E = (a-\theta)\sqrt{2u_c^E} - 2u_c^E - z. \tag{56}$$

Solving (55) and (56) together yields $u_c^E = (2/9)(a-\theta)^2 = 4\nu/9$ and $u_m^E = 4\nu/9 - z$.

(ii). It follows from (18) that an allocation $(u_c, u_m) \in S^s$ can be equal to $N(S^s, d^s)$ only if $(u_c, u_m) \in P(S^s)$. Therefore, $u_c \geq (a-\theta)^2/8$ and

$$u_m = (a-\theta)\sqrt{2u_c} - 2u_c - z. \tag{57}$$

Using (57) we can rewrite the Nash product $u_c(u_m + z)$ as $u_c[(a-\theta)\sqrt{2u_c} - 2u_c]$. Differentiating it with respect to u_c and equating to zero we obtain $u_c^N = (9/32)(a-\theta)^2 = 9\nu/16$, and using (57) we obtain $u_m^N = 6\nu/16 - z$.

(iii). Notice that (19), (30), and $d^s = (0, -z)$ imply

$$KS(S^s, d^s) = \max\left\{(u_c, u_m) \in S^s : \frac{u_m + z}{u_c} = \frac{(a-\theta)^2/4}{(a-\theta)^2/2} = \frac{1}{2}\right\}. \tag{58}$$

It follows from (58) that an allocation $(u_c, u_m) \in S^s$ can be equal to $KS(S^s, d^s)$ only if $(u_c, u_m) \in P(S^s)$. Therefore, $u_c \geq (a-\theta)^2/8$ and

$$u_m = (a-\theta)\sqrt{2u_c} - 2u_c - z. \tag{59}$$

From (58), we also know that

$$u_m = (u_c/2) - z. \tag{60}$$

Equating the right-hand-sides of (59) and (60) we obtain $u_c^K = (8/25)(a-\theta)^2 = 16\nu/25$, and using (60) we obtain $u_m^N = 8\nu/25 - z$, completing the proof. ■

Proof of Proposition 2. Pick any $\beta \in \mathbb{R}_+$. Given the problem (S, d), (22) and (29) imply that the solution $U^\beta(S^s, d^s)$ must be maximizing

$$u_c + \beta(u_m + z) = u_c + \beta\left((a-\theta)2^{1/2}u_c^{1/2} - 2u_c\right).$$

If $\beta \in [0,1]$, then we have a corner solution at the lower extreme of $PO(S^s)$, with $u_c^\beta = \nu$ and $u_m^\beta = -z$. If $\beta > 1$, the solution is in the interior of $PO(S^s)$ and satisfies the first-order condition

$$1 + \frac{\beta}{2}(a-\theta)\sqrt{2}\,(u_c^\beta)^{-1/2} - 2\beta = 0,$$

implying

$$u_c^\beta = \frac{\beta^2}{(2\beta-1)^2}\nu.$$

Inserting this into (29) yields

$$u_m^\beta = \frac{2(\beta^2-\beta)}{(2\beta-1)^2}\nu - z,$$

completing the proof. ∎

Proof of Corollary 1. Directly follows from Propositions 1 and 2. ∎

Proof of Proposition 3. Pick any $\rho \in \mathbb{R}_{++}$. Notice that (21) and $d^s = (0, -z)$ imply

$$u_m^\rho + z = u_c^\rho / p. \tag{61}$$

If $\rho \in (0, 1/2)$, then the solution is on $WPO(S^s) \setminus PO(S^s)$, with $u_c^\rho = \rho\nu/2$ and $u_m^\beta = \nu/2 - z$. On the other hand, if $\rho \geq 1/2$, the solution is on $PO(S^s)$. So, inserting (61) into (29), we can calculate $u_c^\rho = 4\rho^2\nu/(1+2\rho)^2$ and $u_m^\rho = 4\rho\nu/(1+2\rho)^2 - z$. ∎

Proof of Corollary 2. Directly follows from Propositions 1 and 3. ∎

Proof of Corollary 3. Notice from Propositions 2 and 3 that if $\rho \in (0, 1/2)$, then there exists no β that ensures the equality of the solutions U^ρ and U^β on (S^s, d^s). So, let $\rho \geq 1/2$. In that case, $u_c^\rho = 4\rho^2\nu/(1+2\rho)^2$ from Proposition 3. If $\beta \in [0,1]$, then $u_c^\beta = \nu$. We can have $u_c^\rho = u_c^\beta$ only if $\rho = \infty$. So, let $\beta > 1$, implying $u_c^\beta = \beta^2\nu/(2\beta-1)^2$. Then, one can easily show that $u_c^\rho = u_c^\beta$ can hold only if $\rho = \beta/(2\beta-2)$. ∎

Proof of Proposition 4. Parts (i), (ii), and (iii) follow from equating $CS^*(n)$ in (28) to $u_c^E = 4\nu/9$, $u_c^N = 9\nu/16$, and $u_c^{KS} = 16\nu/25$, respectively.

Part (iv). Notice that if $\beta \leq 1$, then u_c^β can be equal to $CS^*(n)$ only if $n = \infty$. So, let $\beta > 1$. Equating u_c^β to $CS^*(n)$ in (28) implies that $\beta^2/(2\beta-1)^2 = n^2/(n+1)^2$, implying $n = \beta/(\beta - 1)$. Since n must be a positive integer not less than 2, the above equality can hold only if $\beta/(\beta - 1)$ is a positive integer.

Part (v). Notice that if $\rho \in (0, 1/2)$, then $u_c^\rho = \rho\nu/2$ from (33). Then, $u_c^\rho < \nu/4$ for all $\rho \in (0, 1/2)$. Equating u_c^ρ to $CS^*(n)$ in (28) implies $(n/(n+1))^2\nu < \nu/4$, further implying $n/(n+1) < 1/2$ which cannot hold for any positive integer n. So, let $\rho > 1/2$. Then, from (33), it follows that $u_c^\rho = 4\rho^2\nu/(1 + 2\rho)^2$. Equating this to $CS^*(n)$ in (28) implies $4\rho^2/(1 + 2\rho)^2 = n^2/(1 + n)^2$, further implying $n = 2\rho$. Since n is an integer, this equality holds only if 2ρ is an integer. ■

Proof of Proposition 8. Since the Egalitarian, Nash, and Kalai-Smorodinsky rules satisfy anonymity and homogeneity axioms, the proof directly follows from Propositions 1 and 7 when $z = 0$. ■

Proof of Proposition 9. Pick any $\beta \in \mathbb{R}_+$. Given the problem (S^a, d^a), (22) and (47) imply that the solution $U^\beta(S^a, d^a)$ must be maximizing

$$u_c + \beta u_m = [2(1 - \alpha) + \beta]/(2 - \alpha)^2.$$

If $\beta \geq 1$, then we have a corner solution at the higher extreme of $PO(S^a)$, with $u_c^\beta = 0$ and $u_m^\beta = \eta$. If $\beta \in (0, 1)$, the solution is in the interior of $PO(S^a)$ and satisfies the first-order condition with respect to α

$$-2(2 - \alpha)^2 + 2[2(1 - \alpha) + \beta](2 - \alpha) = 0,$$

implying $\alpha = \beta$, hence the optimal solutions $u_c^\beta = 2(1 - \beta)\eta/(2 - \beta)^2$ and $u_m^\beta = \eta/(2 - \beta)^2$, completing the proof. ■

Proof of Corollary 5. Directly follows from Propositions 8 and 9. ■

Proof of Proposition 10. Pick any $\rho \in \mathbb{R}_{++}$. Notice that equation (21) and $d^a = (0, 0)$ imply

$$u_m^\rho = u_c^\rho/p. \tag{62}$$

If $\rho > 2$, then $(u_c^\rho, u_m^\rho) \in WPO(S^a) \setminus PO(S^a)$, hence $u_c^\rho = \eta/2$ and $u_m^\rho = \eta/(2\rho)$. If $\rho \in (0, 2]$, the solution is in $PO(S^a)$. So, $u_c^\rho = 2(1-\alpha)\eta/(2-\alpha)^2$ and $u_m^\rho = \eta/(2-\alpha)^2$ for some α, implying $1 = 2(1 - \alpha)/\rho$ or $\alpha = 1 - \rho/2$. It follows that $u_c^\rho = 4\rho\eta/(2 + \rho)^2$ and $u_m^\rho = 4\eta/(2 + \rho)^2$, completing the proof. ■

Proof of Corollary 6. Directly follows from Propositions 8 and 10. ■

Proof of Corollary 7. Notice from Propositions 9 and 10 that if $\rho > 2$, then there exists no β that ensures the equality of the solutions P^ρ and U^β on (S^a, d^a). So, let $\rho \in (0, 2]$. In that case, $u_c^\rho = 4\rho\eta/(2 + \rho)^2$ from Proposition 10. Proposition 9 implies that if $\beta \geq 1$, then $u_c^\beta = 0$. In that case, we can have $u_c^\rho = u_c^\beta$ only if $\rho = 0$, which is not admissible. So, let $\beta \in [0, 1)$, implying $u_c^\beta = 2(1-\beta)\eta/(2-\beta)^2$. Then, one can easily show that $u_c^\rho = u_c^\beta$ can hold only if $\beta \in [0, 1)$ and $\rho = 2(1-\beta)$. $\blacksquare$