QUALITY OVER QUANTITY

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Abstract

We derive the seller's utility maximizing selling mechanism in bilateral trade with interdependent values. Due to the interdependencies in valuations, finding the optimal mechanism is an informed seller problem. It turns out that the optimal mechanism is no longer a take-it-or-leave-it offer for the whole capacity; the seller finds it optimal to decrease the quantity of allocation (or the probability of trade) in order to credibly signal her private information to the buyer.

Keywords: Bilateral Trade, Mechanism Design, Interdependent Values.

JEL: D42, D82.

1. INTRODUCTION

In a standard mechanism design problem it is typically assumed that the mechanism designer (principal) does not possess any payoff-relevant information for the agents (ex-ante contracting). Relaxing this assumption may, however, be essential for many applications as dissolving partnerships, agency contracts, trading with externalities, allocating mineral rights, or other comparable contracting problems in which the principal's choice of a mechanism possibly reveals substantive information to the agents (interim contracting).

One conventional circumstance in which the contract is designed by an informed principal is bilateral trade where the seller, who determines the selling procedure, has private information about the quality of the object which affects the buyer's valuation of the object (market for lemons). In this paper we derive the seller's utility maximizing selling mechanism in such an environment where the players have interdependent values. We show that the optimal mechanism

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is no longer a take-it-or-leave-it offer; the seller finds it optimal to decrease the quantity of allocation (or the probability of trade) in order to credibly signal her type credibly to the buyer (Theorem 1 and Corollary 2). This leads to a non-linear pricing scheme in which the price is increasing in quantity. This behavior can give some rationales for, for instance, joint ownership agreements, part-time employment contracts, or production shortage in the aforementioned applications.

To be downright, we introduce the model next and elaborate the connection of our results to the earlier literature and applications in the last two sections.

2. MODEL

We consider the following (primarily) modest framework. There is a seller, S, who has an object for sale for a buyer, B. The buyer's valuation of the object is given by $v_B(t) = t_B + \alpha_B t_S$ and seller's valuation by $v_S(t) = t_S + \alpha_S t_B$ such that $t_i \in T_i := [\underline{t_i}, \overline{t_i}] \subset \mathbb{R}_+$ is private information of the player i with an interdependence parameter $\alpha_i \in [0, 1)$ for $i \in \{B, S\}$. We assume that $T_S \cap T_B \neq \emptyset$ and denote a type-profile by $t = (t_S, t_B) \in T_S \times T_B =: T$. The seller's belief about the buyer's type, t_B, is given by a distribution function F_B with full support on T_B. The buyer's prior belief about the seller's type, t_S, is given by a distribution F_S with full support on T_S. We assume that t_S and t_B are independently distributed and that F_B is regular, i.e. $\frac{1-F_B(t_B)}{f_B(t_B)}$ is decreasing in t_B.

We assume that $\overline{t_S} \geq \frac{1-\alpha_S}{1-\alpha_B}\overline{t_B}$, which ensures that a seller with the highest type values the object always weakly more than any buyer. Clearly, in a symmetric model where $T_S = T_B$ and $\alpha_S = \alpha_B$ this is satisfied. The use of this assumption is pointed out in the proof of Theorem 1. Implications of relaxing the assumption are discussed in the last section.

The player i's ex-post utility is given by a function $u_i : X \times T \to \mathbb{R}$ for $i \in \{B, S\}$ such that

$$u_B(x;t) = v_B(t)q - p \tag{1}$$

$$u_S(x;t) = p - v_S(t)q, \tag{2}$$

where $x = (q, p)$ and $X := [0, 1] \times \mathbb{R}$ such that q is the probability of sale which belongs to $[0, 1]$, and p is the price of an allocation paid by the buyer to the seller which belongs to $\mathbb{R}$.[1] This is a stylized model of interdependent values with ex-post budget balance.

We have all the ingredients for the Revelation Principle for Bayesian games and we can thus focus on direct revelation mechanisms (see, e.g., Myerson (1981, 1982) or Sugaya and Wolitzky (2021)). That is, an equilibrium of any indirect mechanism with given beliefs corresponds to a truthful equilibrium of a direct revelation mechanism (DRM).

[1]Alternatively q can be interpreted as a quantity of the good with a normalized production capacity (a share of the whole production capacity). We however stick in the conventional language of an allocation probability in the analysis and make alternative interpretations later.

Let the set of all DRMs be $\mathcal{G}$ and $\Gamma = (T, x) \in \mathcal{G}$ be an arbitrary direct mechanism, where $x = (q, p)$ is given by functions $(q, p) : T \to X$.

The timing of the game is as follows. First, valuations $t \in T$ are independently drawn according to F_S and F_B. After learning her type the seller designs a selling mechanism and commits to it. Then the buyer updates her beliefs about the seller's type based on the observed mechanism and decides whether or not to participate the mechanism. If the buyer decides not to participate, the game ends and both players receive their outside option utilities which are normalized to zero. If the buyer participates in the mechanism, the outcomes of the designed game are realized. The timing of the game is similar to that in Myerson (1983) and Maskin and Tirole (1990, 1992). The solution concept is a *Perfect Bayesian Equilibrium* (PBE).[2]

In the next section we give our main results, Theorem 1 and Corollary 1. The seller's utility maximizing mechanisms when the seller's type is unknown and known to the buyer are given by Theorem 1 and Corollary 1, respectively. In Section 4 we give the proof of Theorem 1 which also shows Corollary 1. After that we consider mechanism design by a mediator. The mediator's solution determines the principal's valuation for the full information disclosure or perfect verification of the principal's type to the buyer (Proposition 1). In the last two sections we connect our model to the earlier literature and discuss our findings in a couple of applications.

3. OPTIMAL MECHANISM

The seller's expected utility maximizing mechanism is given by the following theorem. Section 4 is devoted to the proof of this theorem.

THEOREM 1. *The unique seller's utility maximizing mechanism* $\Gamma = (T, x) \in \mathcal{G}$ *is given by the allocation rule*

$$q(t) = \begin{cases} \left(\frac{J(t)}{J(t_B, \underline{t}_S)}\right)^{\lambda}, & \textit{if } J(t) \geq 0 \\ 0, & \textit{otherwise,} \end{cases} \tag{3}$$

where $J(t) = v_B(t) - \frac{1 - F_B(t_B)}{f_B(t_B)} - v_S(t)$ *and* $\lambda := \frac{\alpha_B}{1 - \alpha_B}$, *and by the transfer rule*

$$p(t) = v_B(t)q(t) - \int_{\underline{t_B}}^{t_B} q(s, t_S)ds, \tag{4}$$

for all $t \in T$.

Theorem 1 states that the optimal mechanism is no longer a take-it-or-leave-it offer under even a slightly positive interdependence parameter of the buyer (expect

[2]The seller is fully committed to the designed mechanism and hence almost perfect Bayesian equilibria coincide with weak perfect Bayesian equilibria. We can thus call our solution concept just Perfect Bayesian Equilibrium.

for a seller with the lowest type); the seller makes the mechanism inscrutable by giving up of some of her utility by lowering the allocation probability. This behavior provides that all types of the seller choose the same mechanism and thus the choice of the mechanism does not convey any information from the seller to the buyer. Alternatively we can interpret the mechanism as fully informative signaling: a seller with type t_S chooses a mechanism $(q(t), p(t))$ and reports her type to the buyer. Since the mechanism is undominated and incentive compatible for the seller it perfectly reveals the seller's type. Moreover, the posted mechanism is ex-post incentive compatible and individually rational for the buyer (safe) and hence the buyer chooses the contract truthfully according to her type.

If the seller's type is not payoff-relevant for the buyer — that is, $\alpha_B = 0$, then the mechanism given in Theorem 1 reduces back to a take-it-or-leave-it offer for all $t_S \in T_S$.[3] Clearly, whenever $\alpha_B = \alpha_S = 0$ we end up in the standard solutions of Mussa and Rosen (1978) and Myerson (1981). That is to say, if it is irrelevant for the seller whether the mechanism reveals any information about her type to the buyer then there is no need for (endogenously costly) signaling.[4]

Due to the non-linear allocation probability, also the pricing is non-linear. The equilibrium transfers given in (4) are increasing in q. By assumption $\alpha_B \in [0, 1)$ the exponent in the allocation probability, λ, is nonnegative and the probability of trade, q, is increasing in t_B and decreasing in t_S with strict monotonicities when $\alpha_B > 0$.[5] Moreover, $v_B(t)$ is increasing in t_B. Consequently, the equilibrium pricing given by (4) is also increasing in t_B in the same fashion as q. However, the monotonicity of p with respect to t_S is not guaranteed. This can be easily shown by an example.

EXAMPLE 1. *Consider a symmetric case in which types are uniformly distributed on $[0, 1]$ — that is, $F_i(t_i) = t_i$ for all $t_i \in [0, 1]$ and $i \in \{B, S\}$. Let $\alpha_B = \alpha_S = \frac{1}{2}$ and so $\lambda = 1$. The optimal allocation and transfer rules are $1 - \frac{t_S}{3t_B - 2}$ and*

$$\frac{1}{6}\left[(3t_S + 4)\left(1 - \frac{t_S}{3t_B - 2}\right) - 2t_S \log\left(\frac{t_S}{3t_B - 2}\right)\right], \tag{5}$$

respectively, for all $3t_B - 2 \geq t_S$, and zero otherwise. It is easy to verify that for some values of t_B, say $\frac{9}{10}$, the expression in (5) is increasing for small t_S and decreasing for high t_S.

However, one can show by straightforward calculus that $p(t)/q(t)$ is non-decreasing in t_B and t_S. This is a relevant monotonicity especially when the

[3] Maskin and Tirole (1990) shows that in the quasi-linear case with *private* values the principal neither gains or loses if her type is revealed to the agent before the reporting stage (Proposition 11).

[4] Decreasing the probability of trade is similar kind of signaling to that in Crawford and Sobel (1982) where the cost of signaling is endogenously created in order to have equilibria with partial sorting.

[5] This can be shown by taking the derivatives of $q^*(t) = \left(\frac{J(t)}{J(t_B, \underline{t}_S)}\right)^\lambda$ and $J(t)$ with respect to t_B and t_S and noticing that the $q^*(t)$ and $J(t)$ are both increasing in t_B and decreasing in t_S.

seller is selling a divisible good. Then the equilibrium price per quantity is non-decreasing in marginal costs of production. This interpretation of q is discussed more in the last section.

Unambiguous predictions for q and p by changing α_B cannot be done either. We know that $J(t)$ is increasing in α_B which gives us clear implication of how α_B affects the cutoff types when to allocate at all. This still does not give us the whole effect on q (and so on p);since $\left(\frac{J(t)}{J(t_B,\underline{t}_S)}\right)^\lambda$ is not decreasing in α_B for all $t \in T$. We can only conclude that as α_B goes to zero, the probability of trade converges point-wise to an indicator function $q(t)|_{\alpha_B=0}$ (see Figure 1). That is to say, once $\alpha_B t_S$ gets small, and so the seller's private information is less relevant for the buyer's utility, the closer the optimal mechanism goes to a take-it-or-leave-it-offer.

As for α_S the connection is clear: $J(t)$ and $\left(\frac{J(t)}{J(t_B,\underline{t}_S)}\right)^\lambda$ both are decreasing in t_S and thus q and p are decreasing in α_S.

Figures 1 and 2 illustrate the comparative statics of the optimal mechanism. In Figure 1 the seller's type, t_S, and interdependence parameter, α_S, are fixed and the optimal allocation rule given by Theorem 1 is plotted as a function of the buyer's type, t_B. In Figure 2 the same plot is made by fixing t_B in terms of t_S. In both figures the allocation rule q is depicted with six different interdependence parameters of the buyer: $\alpha_B \in \{0, 0.1, 0.3, 0.5, 0.7, 0.9\}$.

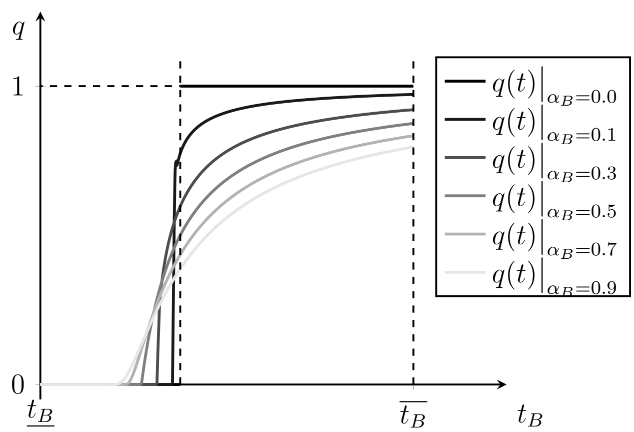

Figure 1: Optimal allocation rule with a fixed $t_S \in T_S$.

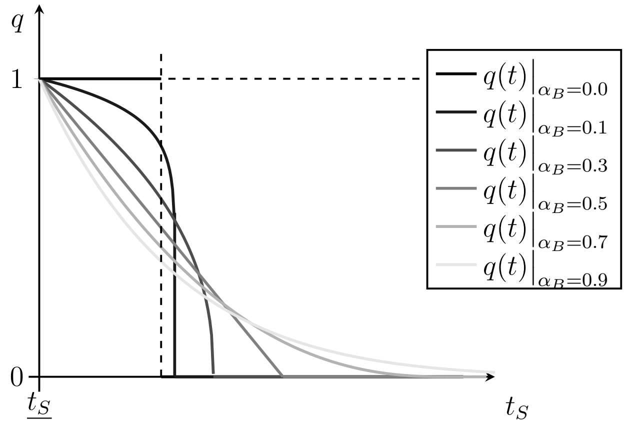

Figure 2: Optimal allocation rule with a fixed $t_B \in T_B$.

If the seller's type is common knowledge for both players, the seller does not need to decrease the allocation probability in order to keep the mechanism inscrutable. In this case both players are better off in equilibrium. The optimal mechanism is given by the following corollary.

COROLLARY 1. *Assume that t_S is common knowledge for both players, i.e. $T_S = \{t_S\}$. The unique seller's utility maximizing mechanism $\Gamma^F = \big(T, x^F\big) \in \mathcal{G}$ is given by the allocation rule*

$$q^F(t) = \begin{cases} 1, & \textit{if } J(t) \geq 0 \\ 0, & \textit{otherwise,} \end{cases} \tag{6}$$

where $J(t) = v_B(t) - \frac{1-F_B(t_B)}{f_B(t_B)} - v_S(t)$, and by the transfer rule $p^F(t) = (t_B^*(t_S) + \alpha_B t_S)q^F(t)$, where $t_B^*(t_S) = \inf\{t_B' \in T_B : J(t_B', t_S) \geq 0\}$.

In other words, when the buyer knows the seller's type, the optimal mechanism is a take-it-or-leave-it offer $t_B^*(t_S) + \alpha_B t_S$. The buyer accepts this offer if her valuation is above $t_B^*(t_S)$ since she values the object at $t_B + \alpha_B t_S$. Since the equilibrium utilities of the seller and the buyer are both increasing in q, both players would be better off if they knew t_S (see the proof of Theorem 1).

Surprisingly, the optimal take-it-or-leave-it offer $t_B^*(t_S) + \alpha_B t_S$ is not always greater than the optimal transfers with an informed seller; the allocation probability in Γ given by Theorem 1 is always weakly smaller than that in Γ^F, but this does not apply to the prices. The seller can increase the price offer due to the buyer's uncertainty about t_S. However, the decrease in q dominates the increase p in terms of the equilibrium utilities of the players and so the seller does not benefit from her private information. These arguments can be easily verified by setting $t_B = 1$ and $t_S = 1/10$ in our example above to get $(q(1, 1/10), p(1, 1/10)) = (9/10, (387 + 20\log(10))/600) \approx (9/10, 0.72)$ and $(q^F(1, 1/10), p^F(1, 1/10)) = (1, 21/30)$.

The optimal mechanism given by Corollary 1 is usually referred to as "full-information" solution in the informed principal literature (ex-post contracting).

As Maskin and Tirole (1990) show, when the seller and the buyer have private values the seller can guarantee herself the same payoff as she would have got if her type was known by the buyer. This can be seen by setting $\alpha_B = 0$ and observing that the optimal mechanisms given by Theorem 1 and Corollary 1 are equivalent. However, as Maskin and Tirole (1992) later propose, this equivalence does no longer hold with interdependent values. The asymmetric information by the seller's side forces the seller to decrease the allocation probability and both players are worse off in comparison with the full-information case; only a seller with the lowest type can use the mechanism given by Corollary 1 even though her type was private information (as pointed out in Jullien and Mariotti (2006), Cai et al. (2007), and Zhao (2018)).

If the buyer's valuation is common knowledge for both players, then the seller still finds it optimal to choose the probability of trade such that it is decreasing in her type. The buyer is left with no surplus since the seller is perfectly informed about the buyer's willingness to pay. This implies that the buyer's individual rationality constraint is binding. Using the binding participation constraint and the seller's incentive compatibility constraint it is possible to solve the seller's utility maximizing mechanism. In this mechanism the seller credibly signals her type to the buyer by giving up some of her equilibrium utility by lowering the allocation probability.

COROLLARY 2. *Assume that t_B is common knowledge for both players, i.e. $T_B = \{t_B\}$. The unique seller's utility maximizing mechanism $\Gamma^{FB} = \left(T, x^{FB}\right) \in \mathcal{G}$ is given by the allocation rule*

$$q^{FB}(t) = \begin{cases} \left(\frac{J^{FB}(t)}{J^{FB}(t_B, \underline{t}_S)}\right)^{\lambda}, & \text{if } J^{FB}(t) \geq 0 \\ 0, & \text{otherwise,} \end{cases} \tag{7}$$

where $J^{FB}(t) = v_B(t) - v_S(t)$ and $\lambda := \frac{\alpha_B}{1-\alpha_B}$, and by the transfer rule

$$p^{FB}(t) = v_S(t)q^{FB}(t) + \int_{t_S}^{\overline{t_S}} q^{FB}(t_B, s)ds, \tag{8}$$

for all $t \in T$.

In comparison with the case where there is asymmetric information both sides we observe the following. The allocation probability is greater in Γ^{FB} than in optimal mechanism Γ which given by Theorem 1. Moreover, Γ^{FB} allocates to a greater share of buyers since $J^{FB}(t) \geq J(t)$ for all $t \in T$. That is to say, asymmetric information from the buyer's side increases ex-ante inefficiency and decreases the equilibrium utilities of both players. However, the aforementioned comparative static properties remains unchanged. In other words, the feature that the optimal mechanism is no longer a take-it-or-leave-it offer arise from the seller's side asymmetric information, not from the buyer's side.[6]

[6] In Myerson (1997) there is a discrete-type example corresponding to our setup which shows that decreasing the quantity offered to the buyer is optimal for the seller.

Before we go the proof of our main result, we give a short example of this setup.

EXAMPLE 2. *Let us simplify further the previous example and assume that $T_B = \{t_B\}$, $F_S(t_S) = t_S$ on $[0, 1]$, $\alpha_S = 0$, and $t_B = \alpha_B = \frac{1}{2}$. Now $q^{FB}(t) = 1 - t_S$ and $p^{FB}(t) = \frac{1}{2}(1 - t_S^2)$ for all $t \in T$. This mechanism yields the buyer zero ex-post utility and the seller gains $\frac{1}{2}(1 - t_S)^2$. A seller with the lowest type 0 gets the full information outcome just by asking the buyer to pay her $\frac{1}{2}$. A seller with the highest type does not trade at all.*

If we assume that the good for sale is divisible, then the equilibrium price per quantity is given by $p^{FB}(t)/q^{FB}(t) = \frac{1}{2}(1 + t_S)$.

4. PROOF OF THEOREM 1

As in Myerson (1983), we call a mechanism *strong solution* iff (if and only if) it is *safe* and *undominated.* Myerson defines the concepts (for multiple agents) as follows:

- A safe mechanism is (interim) incentive compatible and individually rational even if the agents knew the principal's type;
- A mechanism Γ is dominated by mechanism Γ' iff the principal's interim utility in Γ' is greater than or equal to the interim utility of the principal in Γ for all $t_S \in T_S$ such that at least for one $t_S \in T_S$ mechanism Γ' yields strictly greater utility for the principal than mechanism Γ. A mechanism Γ is undominated iff Γ is (interim) incentive compatible and Γ is not dominated by any other (interim) incentive compatible mechanism.

That is, in a bilateral trade model where the seller is the principal and the buyer is the agent, a strong solution is the seller's interim utility maximizing mechanism that is *ex-post* incentive compatible and individually rational for the buyer and *interim* incentive compatible for the seller.[7] Every strong solution allocation forms a PBE.

Myerson (1983) shows that if a strong solution, Γ, exists, it is essentially unique and the principal should always implement it.[8] This is due to the principle of inscrutability: there is no loss of generality in assuming that all types of the principal should choose the same mechanism, and so the actual choice of

[7] In the language of Maskin and Tirole (1992), a mechanism that maximizes the seller's interim utility for each type $t_S \in T_S$ subject to ex-post incentive compatibility and ex-post individual rationality constraints for the buyer and interim incentive compatibility constraint for the seller, is a RSW^* mechanism (Rothschild-Stiglitz-Wilson mechanism, the second definition in Section 8). Maskin and Tirole show that if the players have quasi-linear utilities (as in our case), then the RSW^* allocation cannot be (Pareto) dominated by any other (interim) incentive-compatible mechanism and hence RSW^* is a strong solution.

[8] By *essentially unique* Myerson (1983) means that all strong solutions yield the principal the same interim utility.

mechanism does not reveal any information about the seller's type to the buyer (see Myerson (1983)). The principal should implement a strong solution also in the case she preferred some other incentive compatible mechanism, Γ', since then the agents would update their beliefs about the principal's type based on Γ' (as an evidence that the principal's type must belong to the set preferring Γ' over Γ) which further makes Γ' infeasible once it was selected. In other words, off the equilibrium of a strong solution, the buyer updates her beliefs about the seller's type using Bayes' rule.[9]

Myerson's (1983) first theorem states the following.

LEMMA 1. *(Myerson (1983), Theorem 1) If a strong solution exists, it is the (essentially) unique optimal mechanism for the seller of any type $t_S \in T_S$.*

The proof of Lemma 1 is given in the original paper. For the mathematical convenience, Myerson uses finite type spaces but Lemma 1 remains valid also with infinite type spaces, which is the case in our model. Next we start to derive the strong solution.

Let the buyer's ex-post utility with a given DRM $\Gamma = (T, x)$ be given by the following value function

$$V_B(t) = \max_{t'_B \in T_B} \left(v_B(t) q(t'_B, t_S) - p(t'_B, t_S) \right). \tag{9}$$

By the standard envelope theorem argument (see Milgrom and Segal (2002)) we know that a DRM $\Gamma = (T, x)$ is ex-post incentive compatible for the buyer iff the equilibrium transfers are given by

$$p(t) = v_B(t) q(t) - V_B(\underline{t_B}, t_S) - \int_{\underline{t_B}}^{t_B} q(s, t_S) ds \tag{10}$$

and $q(\cdot, t_S)$ is nondecreasing for all $t_S \in T_S$.

The seller's interim utility be given by the following value function

$$V_S(t_S) = \max_{t'_S \in T_S} \mathbb{E}_{t_B} \left(p(t_B, t'_S) - v_S(t) q(t_B, t'_S) \right). \tag{11}$$

By the envelope theorem for the seller's value function, we know that $V'_S(t_S) = -\mathbb{E}_{t_B} \left(q(t_B, t_S) \right)$ almost everywhere (a.e.) at the truthful equilibrium. Then by the fundamental theorem of calculus the seller's interim equilibrium utility becomes

$$V_S(t_S) = V_S(\overline{t_S}) + \int_{t_S}^{\overline{t_S}} \mathbb{E}_{t_B} \left(q(t_B, s) \right) ds. \tag{12}$$

It is straightforward to show that a DRM $\Gamma = (T, x)$ is interim incentive compatible for the seller iff the seller's interim utility is given by (12), and $\mathbb{E}_{t_B} \left(q(t_B, \cdot) \right)$ is nonincreasing.

[9] See Proposition 7 of Maskin and Tirole (1992) which analyses the connection of a RSW allocation (introduced in footnote 3) to refinements of Cho and Kreps (1987) and Farrell (1985) and Grossman and Perry (1986).

By substituting the transfers (10) into the seller's value function (11), and interchanging the order of integration we observe that the seller's equilibrium utility can be written as

$$V_S(t_S) = \mathbb{E}_{t_B}\left(J(t)q(t) - V_B(\underline{t_B}, t_S)\right), \tag{13}$$

where $J(t) = v_B(t) - \frac{1-F_B(t_B)}{f_B(t_B)} - v_S(t)$.

It is optimal for the seller to set $V_B(\underline{t_B}, t_S) = 0$ for all $t_S \in T_S$ since it weakly increases the equilibrium transfers from any type $t_B \in T_B$ and the probability that the buyer is of type $\underline{t_B}$ is zero. This implies that the ex-post individual rationality constraint for the buyer is satisfied for any $t_B \in T_B$ since $V_B(\cdot, t_S)$ is nondecreasing for all $t_S \in T_S$.

The seller's utility maximizing *safe* mechanism (ex-post incentive compatible for the buyer and interim incentive compatible for the seller) is given by the following maximization problem:

$$\max_{q:T\to[0,1]} \mathbb{E}_{t_B}\left(J(t)q(t)\right) \tag{14}$$

subject to

$$\mathbb{E}_{t_B}\left(J(t)q(t)\right) = V_S(\overline{t_S}) + \int_{t_S}^{\overline{t_S}} \mathbb{E}_{t_B}\left(q(t_B, s)\right) ds \tag{15}$$

$$q(\cdot, t_S) \text{ nondecreasing} \tag{16}$$

$$\mathbb{E}_{t_B}\left(q(t_B, \cdot)\right) \text{ nonincreasing,} \tag{17}$$

for all $t_S \in T_S$. That is, if there exists a function q which solves that optimization problem above, then q and its corresponding transfers (10) are undominated and thus form a *strong solution.*[10]

From this optimization problem we can conclude that whenever t_S is common knowledge, i.e. $T_S = \{t_S\}$, the constraints (15) and (17) are redundant and the optimal mechanism can be solved point-wise from the optimization problem since the solution satisfies the monotonicity constraint (16). This gives us Corollary 1.

Let us next turn our attention to the seller's incentive compatibility constraint (15). By assumption $\overline{t_S} \geq \frac{1-\alpha_S}{1-\alpha_B}\overline{t_B}$ we know that $J(t_B, \overline{t_S}) \leq 0$ for all $t_B \in T_B$ and hence we must have $V_S(\overline{t_S}) = 0$ in equilibrium. This implies that the seller's incentive compatible constraint (15) becomes

$$\mathbb{E}_{t_B}\left(J(t)q(t)\right) = \int_{t_S}^{\overline{t_S}} \mathbb{E}_{t_B}\left(q(t_B, s)\right) ds \tag{18}$$

for any $t_S \in T_S$.

[10] Maskin and Tirole (1992) referred to the solution of this optimization problem as a RSW^* allocation.

Then multiply both sides of (18) by $\mu > 0$ and form the Lagrangian of the seller's problem (omitting the monotonicity constraints (16) and (17)). After rearranging the terms, the Lagrangian can be written as

$$\mathbb{E}_{t_B}\left[(1+\mu)\left[J(t)q(t) - \frac{\mu}{1+\mu}\int_{t_S}^{\overline{t_S}} q(t_B, s)ds\right]\right]. \tag{19}$$

This expression is negative for all $t \notin \{t' \in T : J(t') \geq 0\}$ since $\frac{\mu}{1+\mu} > 0$. Hence, an allocation rule

$$q(t) = \begin{cases} q^*(t), & \text{for } J(t) \geq 0 \\ 0, & \text{otherwise} \end{cases} \tag{20}$$

for some $q^* : T \to [0,1]$, which solves (18) and satisfies monotonicity conditions (16) and (17), dominates all the other safe mechanisms.

Let us then find an allocation rule q given in (20) that solves (18). Denote $t_B^*(t_S) = \inf\{t'_B \in T_B : J(t'_B, t_S) \geq 0\}$ and $t_S^*(t_B) = \sup\{t'_S \in T_S : J(t_B, t'_S) \geq 0\}$. Using this notation the seller's incentive compatibility constraint (18) can be written as

$$\int_{t_B^*(t_S)}^{\overline{t_B}} \left(J(t)q^*(t) - \int_{t_S}^{t_S^*(t_B)} q^*(t_B, s)ds\right) dF_B(t_B) = 0, \tag{21}$$

for all $t_S \in T_S$. Assumption $\overline{t_S} \geq \frac{1-\alpha_S}{1-\alpha_B}\overline{t_B}$ guarantees that $t_B^*(\overline{t_S}) = \overline{t_B}$ and hence incentive compatibility requires that constraint (21) must hold for all $t_B^*(t_S) \in \left[t_B^*(\underline{t_S}), \overline{t_B}\right]$. This further implies that (21) must hold point-wise, i.e. the integrand of (21) must be zero almost everywhere.[11] That is, q^* must solve the following integral equation:

$$J(t)q^*(t) = \int_{t_S}^{t_S^*(t_B)} q^*(t_B, s)ds, \tag{22}$$

for almost all $t \in \{t' \in T : J(t') \geq 0\}$.

The equation in (22) is a linear Volterra integral equation of the second kind (or more generally Fredholm equation) for t_S which has a *unique* continuous solution with given boundary conditions (see, e.g., Kress et al. (1989), Theorem 3.10). It is straightforward to verify that a function

$$q^*(t) = \beta(t_B)J(t)^\lambda, \tag{23}$$

where $\lambda := \frac{\alpha_B}{1-\alpha_B}$, solves (22). That is, in every strong solution an allocation rule q which satisfies incentive compatible must be of form (23). This excludes all non-stochastic mechanisms for all $\alpha_B > 0$.

[11] If a function g is integrable on $[a,b] \subset \mathbb{R}$ and $\int_{[x,b]} g = 0$ for all $x \in [a,b]$, then $g = 0$ almost everywhere on $[a,b]$.

It is optimal for the seller to set $q^*(t)$ as high as possible and so we must have that $q^*(t_B, \underline{t_S}) = 1$ for $t_B \geq t_B^*(\underline{t_S})$ at the optimum. This gives us the boundary condition from which we can solve $\beta(t_B) = \frac{1}{J(t_B,\underline{t_S})^\lambda}$. This solution satisfies monotonicity constraints (16) and (17) for all $\alpha_B \in [0, 1)$.

Consequently, the unique strong solution is given by the physical allocation

$$q(t) = \begin{cases} \left(\frac{J(t)}{J(t_B,\underline{t_S})}\right)^\lambda, & \text{if } J(t) \geq 0 \\ 0, & \text{otherwise,} \end{cases} \tag{24}$$

and its corresponding transfers (10).

Lastly we prove Corollary 2. Assume that t_B is common knowledge for both players — that is, $T_B = \{t_B\}$. Then the equilibrium transfers are given by (12):

$$p(t) = v_S(t)q(t) + V_S(\overline{t_S}) + \int_{t_S}^{\overline{t_S}} q(t_B, s)ds \tag{25}$$

for all $t \in T$. It is straightforward to show that a mechanism $\Gamma = (T, x)$ is incentive compatible for the seller iff the equilibrium transfers satisfy (25) and $q(t_B, \cdot)$ is non-increasing. Consequently, the seller's problem is to design a mechanism $\Gamma \in \mathcal{G}$ to maximize $p(t)-v_S(t)q(t)$ subject to (25), $q(t_B, \cdot)$ non-increasing, and the buyer's *ex-post* individual rationality constraint $v_B(t)q(t) - p(t) \geq 0$ for all $t \in T$. By substituting (25) into the buyer's participation constraint we observe

$$(v_B(t) - v_S(t))\, q(t) \geq V_S(\overline{t_S}) + \int_{t_S}^{\overline{t_S}} q(t_B, s)ds, \tag{26}$$

must hold for all $t \in T$. This implies that we must have $q(t) = 0$ for all $t \in T$ such that $v_B(t) < v_S(t)$. As for $t \in T$ such that $v_B(t) \geq v_S(t)$ it is optimal for the seller to choose mechanism such that (26) holds at equality. The binding individual rationality constraint forms an integral equation similar to (22) which has, by the same reasoning, the unique seller's utility maximizing solution, and the allocation rule becomes

$$q^{FB}(t) = \begin{cases} \left(\frac{J^{FB}(t)}{J^{FB}(t_B,\underline{t_S})}\right)^\lambda, & \text{if } J^{FB}(t) \geq 0 \\ 0, & \text{otherwise,} \end{cases} \tag{27}$$

where $J^{FB}(t) = v_B(t) - v_S(t)$ and $\lambda := \frac{\alpha_B}{1-\alpha_B}$. This result holds without assuming $\overline{t_S} \geq \frac{1-\alpha_S}{1-\alpha_B}\overline{t_B}$ and with $V_S(\overline{t_S}) \geq 0$ which guarantees that (26) is binding.

□

5. TRADING WITH A MEDIATOR

Suppose that there is a trustworthy mediator (a broker) who arranges the trade. Let us assume that the mediator is able to verify the seller's type without any extra costs and always reveals it honestly to the buyer. The mediator must design a mechanism that gives the seller weakly better payoff than the mechanism given in Theorem 1 or otherwise the seller arranges the trade by herself. On the other hand, the greatest expected payoff that the mediator can get from the trade is given by the mechanism given in Corollary 1. Hence, the mediator can make the buyer a take-it-or-leave-it offer $t_B^*(t_S)+\alpha_B t_S$ (see Corollary 1) and give the seller payment $r(t_S)+\alpha_B t_S$ if the buyer accepts the offer, where $r(t_S)$ is determined by the following equation:

$$V_S(t_S)=\int_{t_B^*(t_S)}^{\overline{t_B}} J(t)q^*(t)dF_B(t_B)=\int_{r(t_S)}^{\overline{t_B}} J(t)dF_B(t_B), \tag{28}$$

where q^* is given by Theorem 1.

By receiving $r(t_S)+\alpha_B t_S$ from the mediator the seller yields the same utility as she would have got from settling the trade by herself. The mediator's offer is feasible since $V_S(t_S)$ is strictly decreasing in t_S and hence there exists a unique $r(t_S)\le t_B^*(t_S)$ for each type $t_S\in T_S$.[12] When the buyer receives the offer from the mediator who reliably verifies the seller's type, she accepts the offer if her valuation $t_B+\alpha_B t_S$ is greater than the price $t_B^*(t_S)+\alpha_B t_S$. The mediator hence profits the spread between the price that she is receiving from the buyer and the price that she paying to the seller: $b(t_S)=t_B^*(t_S)-r(t_S)\ge 0$.

Since the mediator can verify the seller's type, the incentive compatibility for the seller can be enforced by giving the seller 0 payment if $t_S'\neq t_S$ and $r(t_S)+\alpha_B t_S$ if $t_S'=t_S$.[13] Based on this reasoning we get the following result.

PROPOSITION 1. *The optimal mechanism designed by a trustworthy mediator who can verify the seller's type $t_S\in T_S$ is a take-it-or-leave-it offer $t_B^*(t_S)+\alpha_B t_S$ to the buyer with a brokerage $b(t_S)=t_B^*(t_S)-r(t_S)\ge 0$ paid by the seller to the mediator, where $t_B^*(t_S)=\inf\{t_B'\in T_B : J(t_B',t_S)\ge 0\}$ and $r(t_S)$ is given by the equation (28).*

[12]This is by the facts $\int_{r(t_S)}^{\overline{t_B}} J(t)dF_B(t_B)=-\alpha_S\mathbb{E}_{t_B}(t_B)-(1-\alpha_B)t_S<0\le V_S(t_S)$, when $r(t_S)=\underline{t_B}$ and $V_S(t_S)\le\int_{r(t_S)}^{\overline{t_B}} J(t)dF_B(t_B)$, when $r(t_S)=t_B^*(t_S)$ with strict inequality for all $t_S>\underline{t_S}$ such that $t_B^*(t_S)<\overline{t_B}$. Moreover, $\int_{r(t_S)}^{\overline{t_B}} J(t)dF_B(t_B)$ is continuous and strictly increasing in $r(t_S)$ whenever $r(t_S)\le t_B^*(t_S)$ and hence there exists a unique $r(t_S)$ that solves (28) and satisfies $r(t_S)\le t_B^*(t_S)$.

[13]Note that the uniqueness of the optimal mechanism given by Theorem 1 implies that a take-it-or-leave-it offer determined by $r(t_S)$ is not incentive compatible for the seller. It is easy to show that this is, indeed, the case by assuming the opposite: if a take-it-or-leave-it mechanism determined by r is a strong solution then (28) holds. In this case r is a homeomorphism (by the reasoning above) and then there exists an inverse of r, $r^{-1}:T_B\to T_S$, and the seller's incentive compatibility constraint (18) with the take-it-or-leave-it offer determined by r becomes $\int_{r(t_S)}^{\overline{t_B}}\left(J(t)-r^{-1}(t_B)+t_S\right)dF_B(t_B)=0$, for all $t_S\in T_S$. Since r maps all values on $\left[t_B^*(\underline{t_S}),\overline{t_B}\right]$, the incentive compatibility constraint must hold point-wise and so r^{-1} must satisfy $r^{-1}(t_B)=J(t)+t_S=(1-\alpha_S)t_B-\frac{1-F_B(t_B)}{f_B(t_B)}+\alpha_B t_S$ for all $t_S\in T_S$. However, if $\alpha_B>0$, then r^{-1} should depend also on t_S which is a contradiction. This implies that there is no strong solution that is a take-it-or-leave-it offer.

It is worth of highlighting that $b(t_S)$ is also the maximum price that the seller is willing to pay from a credible verification of her type. That is, if the seller has the possibility to perfectly disclose her private information via the mediator to the buyer at some cost, then it is profitable for the seller to do so if the cost is less than $b(t_S)$. Alternatively, $b(t_S)$ can be interpreted as the loss that the seller meets due to the asymmetric information by her side as discussed in Section 3.

Assume next that there is no possibility of verification or the mediator does not know the seller's type. In this case, the mediator designs a selling mechanism that maximizes the seller's ex-ante utility subject to interim incentive compatibility constraints for both parties. The mediator constructs a mechanism that is interim individually rational for the buyer and ex-ante budget balanced (without loss of generality by Börgers and Norman (2009)).

Let us make a short-cut based on the well-known steps (see, for instance, Börgers (2015), Myerson and Satterthwaite (1983), or the proof of Theorem 1) and introduce the following result.

PROPOSITION 2. *The seller-optimal mechanism designed by a mediator is given by the allocation rule*

$$q^E(t) = \begin{cases} 1, & \text{if } v_B(t) - \frac{1-F_B(t_B)}{f_B(t_B)} - v_S(t) \geq \frac{\mu}{1+\mu}\frac{F_S(t_S)}{f_S(t_S)} \\ 0, & \text{otherwise,} \end{cases} \tag{29}$$

where $\mu > 0$ is the Lagrange multiplier that solves ex-ante budget balance when the interim transfers of the buyer and the seller are, respectively, given by

$$\mathbb{E}_{t_S}\left[p_B^E(t)\right] = \mathbb{E}_{t_S}\left[v_B(t)q(t) - \int_{\underline{t_B}}^{t_B} q(s, t_S)ds\right] \tag{30}$$

$$\mathbb{E}_{t_B}\left[p_S^E(t)\right] = \mathbb{E}_{t_B}\left[v_S(t)q(t) + \int_{t_S}^{\overline{t_S}} q(t_B, s)ds\right]. \tag{31}$$

From this proposition we can see that the optimal mechanism is a take-it-or-leave-it offer that is greater than in the optimal mechanism when the seller's type is common knowledge (Corollary 1). The optimal mechanism designed by the mediator is not, however, a strong solution. This is due to the fact a seller with a high type accepts the mechanism ex ante, but designs a new mechanism (strong solution) after learning her type at interim level since it yields her better payoff. The following short example illustrates this phenomenon.

EXAMPLE 3. *Suppose that that the types of the players are both independently and uniformly distributed on $[0, 1]$ and $\alpha_S = \alpha_B = \frac{1}{2}$. Let the equilibrium utility of a seller of type $t_S \in T_S$ in a strong solution be denoted by $V_S(t_S)$ (Theorem 1), the utility in a case where t_S is common knowledge by $V_S^F(t_S)$ (Corollary 1), and in the optimal mechanism designed by a mediator by $V_S^E(t_S)$ (Proposition 2).*

After some (tedious) calculations (mimicking the steps in Börgers (2015)), we observe that the Lagrange multiplier that solves the ex-ante budget balance in the mediator's problem is given by $\frac{\mu}{1+\mu} = \frac{\alpha_B}{2} = \frac{1}{4}$.

The seller's interim utilities are given by

$$V_S(t_S) = \frac{1}{12}\left(3t_S^2 - 4t_S + 1 - 2t_S^2\log(t_S)\right) \tag{32}$$

$$V_S^F(t_S) = \frac{1}{12}\left(1 - t_S\right)^2 \tag{33}$$

$$V_S^E(t_S) = \frac{1}{12}\left(\frac{3}{4}t_S^2 - 2t_S + 1\right). \tag{34}$$

We observe that the mechanism designed by the mediator gives the sellers with types (approximately) less than 0.63 *greater utility than the strong solution. However, all the sellers who have type greater than this would deviate to the strong solution.*

6. RELATED LITERATURE

Mechanism design with an informed principal has been first studied by Myerson (1983) whose shoulders we are standing on in this paper. Myerson introduces several pivotal solution concepts for our analysis as safe mechanisms, strong solutions, and inscrutability of the principal. While Myerson analyzes a general model of multiple agents, Maskin and Tirole (1990, 1992) study a single agent setup as in this paper. Maskin and Tirole (1990) assume private values and Maskin and Tirole (1992) common values. That is, our paper belongs to the intersection of Myerson (1983) and Maskin and Tirole (1992) which is clearly elaborated in the last section of Maskin and Tirole (1992). Therefore, our analysis could have been equivalently done using the foundations of Maskin and Tirole (1992) as well (see footnotes 2 and 6).

Mylovanov and Tröger (2012) generalize the approach of Maskin and Tirole (1990) in a private values setup and establish existence of equilibria. Mylovanov and Tröger (2014) derive the equilibrium allocation characterization by imposing additional structure of transferable utility to Mylovanov and Tröger (2012).[14]

In an informed-principal problem the contract (mechanism) is designed after the principal privately learns her type or signal (interim contracts). Natural benchmarks to interim contracts used in the literature are ex-ante and ex-post contracts — that is, contracts designed before the principal learns her type or signal and contracts designed before the agents learn the principal's private information, respectively. The equivalence between the outcomes of these three contracts (ex-ante, interim, and ex-post) is shown in many *private values* environments: first in Maskin and Tirole (1990) with risk-neutral players and transferable utilities and later, e.g., in Tan (1996), Yilankaya (1999), Skreta (2011), and Mylovanov

[14]Utility is transferable if one player can transfer part of its utility to another player without any additional cost.

and Tröger (2014). However, Fleckinger (2007) and Mylovanov and Tröger (2014) show that the equivalence does not generally hold in private-values models.[15] This is also the case in interdependent or common values models as Maskin and Tirole (1992) and we have indicated in this paper (comparing interim and ex-post contracts).

One of the closest papers to ours is Koessler and Skreta (2016). Koessler and Skreta consider a bilateral trade setup in which the buyer's valuation of the object is a function of the buyer's and seller's types. The types are private information of the players, which leads to an informed seller problem similar to our setup. However, the crucial difference to our model is that Koessler and Skreta assume that the value of the object for the seller is type-independent and hence does not affect the seller's incentives. Koessler and Skreta (2016) show that in an ex-ante revenue-maximizing equilibrium, the seller benefits from her private information.[16] This result holds for a larger set of equilibrium allocations than strong solutions, since the ex-ante optimal allocation is not safe in general. Nonetheless, the findings of Koessler and Skreta (2016) are in stark contrast to ours; in our model the seller would always weakly benefit from credible information disclosure.

Cella (2008) studies mechanism selection by an informed principal and a single agent with correlated types. Cella shows that the principal can extract extra information rent from the agent in comparison with the ex-post contracts. Skreta (2011) considers optimal information disclosure by an informed principal who maximizes her expected revenue after observing a vector of signals correlated with the agents' valuations. Skreta shows that under general allocation environments under the *agents with interdependent values* it is optimal for the principal to disclose no information. This is in stark contrast to our results: under a slight interdependence in *the principal's and the agent's* values it is optimal for the principal to fully disclose her private information. Severinov (2008) provides conditions under which an ex-post efficient solution exists in an informed principal problem under interdependent values (among all players). Balkenborg and Makris (2015) studies undominated mechanisms designed by an informed principal who has common values with the agent.[17]

Jullien and Mariotti (2006) consider a second-price auction with an informed seller who announces an informative reserve price in advance. Jullien and Mariotti characterize the equilibria of the game (see Cai et al. (2007) and Tsuchihashi (2020) for reserve price signaling). Zhao (2018) studies optimal auction design by an informed seller and observes that reserve price signaling is optimal for the seller (the connection to our results is elaborated in the next section). Both, Jullien and Mariotti (2006) and Zhao (2018), find that the optimal reserve price is higher than

[15]Mylovanov and Tröger (2012, 2014) provide the comprehensive survey of the informed principal literature with private values.

[16] In the language of Myerson (1983), the set of ex-ante optimal allocations coincides with the set of *core* mechanisms.

[17]Informed-principal problems in moral-hazard environments are studied, for instance, by Beaudry (1994), Jost (1996), Benabou and Tirole (2003), Kaya (2010), and Wagner et al. (2015) among many others.

that in the full-information case where the seller's type is common knowledge.

There is an extensive literature on auctions and mechanism design with interdependent values by an *uninformed* seller. The revenue rankings of auctions with interdependent values between the buyers are pioneered by Milgrom and Weber (1982). Crémer and McLean (1985, 1988) show that the seller is capable to extract the full surplus from the buyers if either the valuations of the buyers are interdependent (1985) or the signals of the buyers' valuations are correlated (1988).[18] This mechanism is usually referred as the generalized Vickrey-Clarke-Groves mechanism, which is later studied, for instance, by Ausubel (1999), Dasgupta and Maskin (2000), and Perry and Reny (2002). Their focus is on efficient design under interdependent values as in Holmström and Myerson (1983), Jehiel and Moldovanu (2001), Fieseler et al. (2003), Mezzetti (2004), and Li (2017). The revenue maximizing mechanism with interdependent values are studied, e.g., in Myerson (1981) and Roughgarden and Talgam-Cohen (2016). For bargaining with interdependent values one can see, for instance, Deneckere and Liang (2006) and Fuchs and Skrzypacz (2013). The crucial feature in these models typically is that the uninformed party makes all the offers.

7. DISCUSSION

We have shown that in bilateral trade with interdependent values, the seller-optimal mechanism is no longer a take-it-or-leave-it offer; the seller finds it optimal to decrease the probability of trade in order to credibly signal her type.

Decreasing the probability of trade is, however, a somewhat counter-intuitive and ambiguous result. It may explain why do we sometimes observe trading situations in which a seller seems to act like she does not want to sell the good after all; the seller would rather not to sell the good than to give a wrong signal of the true valuation of the object. This kind of behavior fits perhaps to some more rare occasions as art selling or antiques and thus does not give very comprehensive positive theory. Moreover, implementing a randomized allocation significantly suffers from commitment problems. A seller, who had randomized to not to sell the object to a buyer knows that the buyer is still willing to buy the object. In this case committing to not to sell the object is unprofitable for both parties.

Decreased q can be also interpreted as delayed trade. Let a common discount factor for the seller and buyer be $\delta \in (0,1)$ and time continuous. Then let us reformulate the optimal mechanism given by Theorem 1 as $(q(t), p(t)) = \big(\hat{q}(t; T(t))e^{-\delta T(t)}, \hat{p}(t; T(t))e^{-\delta T(t)}\big)$, where $\hat{q}(t;T)e^{-\delta T(t)}$ and $\hat{p}(t;T(t))e^{-\delta T(t)}$ are respectively the discounted allocation probability and price in period $T(t)$ determined by the reports t. By setting $\hat{q}(t;T) = 1$ whenever $q(t)$ is positive and solving $T(t)$ for reported $t \in T$, we get the following interpretation of the optimal

[18] McAfee et al. (1989) assume that the signals are continuously distributed and that the buyers assign the same value to the object. They show that almost all the surplus can be extracted also in this case. Myerson (1981) was the first one to point out that the full surplus extraction may be possible if the signals of the buyers' valuations are correlated.

mechanism. If $J(t) \geq 0$, then the seller *commits* to postpone selling the object for $T(t)$ periods. The price of the object in period $T(t)$ is given by $\hat{p}(t) = p(t)/q(t)$.

Nonetheless, these two interpretation are not the whole story since our model is adjustable for many applications. Next we exemplify how our results generalize to explain several important economic phenomena.

Market for Lemons. In the spirit of Akerlof (1970) consider a market for goods with an unknown quality t_S. There are buyers whose are willing to buy unity quantity of the good at price lower than $v_B(t) = t_B + \alpha_B t_S$. The ex-post valuation of the buyers is a combination of the buyers private information t_B and the quality of the goods t_S, which is private information of a seller. The seller's production costs are given by $v_S(t)q = t_S q$ and the production capacity is limited to unity (by normalization). Based on Theorem 1, only a seller who has the lowest quality, $\underline{t_S}$, supplies the whole capacity and sellers with higher qualities induce production shortage; the supplier can signal the quality to the buyers by not producing the whole capacity. This signaling behavior gives one rationale for deficient supplies of new high quality or luxury goods.[19]

An alternative lemon story in this context can be found from advertising models. Suppose that a monopolist chooses a share q of demand that she is able to capture by investing in advertising. The production costs of quality t_S are given by $v_S(t)q = t_S q$. According to Theorem 1, a high-quality monopolist best signals its type with a low level of demand-enhancing advertising. This behavior predicts a negative relationship between advertising and product quality (see Figure 2).[20]

In both examples low-quality products have higher supply than high-quality products which indicates the similar market outcomes as in Akerlof (1970).

Joint Ownership. In the models of partnership there are two or more parties who initially own shares of the company. The partners have private information about their valuations of the ownership and they want to renegotiate the shares of the company in order to achieve a profitable balance of ownership for all parties (see, for instance, Cramton et al. (1987), Jehiel and Pauzner (2006), and Loertscher and Wasser (2019)). In this context, our model can be interpreted as a special case of a partnership dissolution in which the seller initially owns the whole company and is willing to sell a share q to the buyer at price p. The seller has private information about the profitability of the firm. The buyer's valuation of the full ownership is given by a weighted sum of her own profitability t_B and the seller's private information about the firm's current profitability t_S. In this case the seller wants to signal the buyer that the firm is of a high profitability by offering a partial ownership to the buyer; keeping a share of the company to herself the

[19]For quality signaling via product scarcity see Stock and Balachander (2005). Stock and Balachander show that a high-quality monopoly firm that signals quality by inducing shortage can make more profit than using price alone.

[20]To some extent this prediction is supported by advertising literature. Many of these studies can be found from Bagwell (2007) which provides comprehensive empirical and theoretical survey on advertising and quality literature.

seller credibly reveals the profitability of the company.

This example generalizes to applications in which there is a seller who owns an asset (or unity mass of assets), which profitability is private information of the seller. The seller determines the asset pricing scheme in order to sell the asset(s).

Part-time Employment. Consider a worker (seller) who offers to work a share q of her working hours in a company (buyer). The productivity of the worker is partly known by the firm and partly known by the worker herself. The worker proposes a part-time contract of employment in order to signal that she is competent and profitable in her current position and thus also for the firm. In this way the worker chooses the optimal allocation of her work load: she works share q of her working hours in the new company and share $1-q$ in her current position which yields her $v_S(t) = t_S$. It is profitable for the worker to negotiate such a contract since there are positive externalities in the firm's production (by interdependence $v_B(t) = t_B + \alpha_B t_S$) and so the company is willing to pay a high salary for the worker.

Trading with Externalities. Consider next a trade between two firms, $\{B, S\}$. The firms' profits are negatively interacted (e.g. due to competition) such that firm i's profit is given by $\hat{\pi}_i = r_i - \alpha_i \hat{\pi}_j$, where r_i is firm i's gains from the trade and $\hat{\pi}_j$ is the profit of the rival firm multiplied by an externality parameter α_i. By presuming that $\alpha_i \in [0, 1)$ we can solve the reduced-form profits from the trade: $\pi_i := (1 - \alpha_i \alpha_j)\hat{\pi}_i = r_i - \alpha_i r_j$ for both $i \in \{B, S\}$.[21]

Assume next that firm S is selling goods to firm B. The profits from the trade are given by $r_B(x; t) = qt_B - p$ and $r_S(x; t) = p - t_S q$, where an allocation $x = (q, p) \in X = [0, 1] \times \mathbb{R}$ is given by the quantity $q \in [0, 1]$ and by the price of the goods $p \in \mathbb{R}$. The buyer's valuation of the goods, t_B, and the seller's marginal cost of producing the goods, t_S, are private information of the firms. The ex-post profits of the firms can be written as

$$u_B(x; t) = (t_B + \alpha_B t_S)\, q - (1 + \alpha_B)p =: v_B(t)q - p_B \tag{35}$$

$$u_S(x; t) = (1 + \alpha_S)p - \ (t_S + \alpha_S t_B)\, q =: p_S - v_S(t)q. \tag{36}$$

This is, indeed, the model we considered in this paper except the prices are now $p_i = (1 + \alpha_i)p$ for both $i \in \{B, S\}$. Since the only difference is in the budget balance, Theorem 1 and Corollary 1 are straightforward to generalize into this example. If the buying firm, B, receives negative externalities from the trade, the

[21] This is essentially a model of socially interacted agents. The model can be found from Becker's 1974 seminal article "A Theory of Social Interactions", where he introduces an economic theory under social interactions. The theory incorporates a general treatment of interactions into the theory of consumer demand in order to explain, for instance, intrafamily relations, charity, merit goods, and envy and hatred. That is to say, social interactions between the principal and agent may lead to an informed principal problem. Our results hence shed light on optimal allocation between bilateral trade under social interactions as well. For more general socially interacted utility functions see, e.g., Bergstrom (1999) and Bourlès et al. (2017).

selling firm, S, finds it profitable to decrease the quantity that she is offering to firm B. In this way firm S can efficiently signal its high production costs which is in the interest of firm B. The pricing scheme can be expressed as a function of quantity sold: $P(q(t)) = v_B(t)q(t) - \int_{\underline{t_B}}^{t_B} q(s, t_S)ds$.

Mineral Rights. Lastly, consider a seller who owns the mineral rights to exploit an area for the minerals it harbors. There is a single potential buyer whose valuation of the right is given by its own private productivity t_B and the amount of minerals in the area, t_S, which is private information of the seller. If the buyer cannot verify the amount of minerals in the area in advance to contracting, the seller finds it optimal to signal this information by offering a joint ownership of the minerals (Theorem 1). However, as Corollary 1 shows, it is profitable for both parties to let the buyer acquire information about the amount of minerals, and the seller is even willing to pay the information acquisition if it is relatively inexpensive (Proposition 1).[22]

All the examples given above offer simplistic and shallow standpoint to the applications which are certainly more intrinsic than we have illustrated. Our intention is not to give unequivocal explanation to these phenomena but rather highlight possible implications of an informed principal to some of the well-known frameworks.

In the analysis and examples where both players have private information about their valuations we have assumed that the seller's highest valuation for the object is at least as high as the buyer's highest valuation ex post $\overline{t_S} + \alpha_S\overline{t_B} \geq \overline{t_B} + \alpha_B\overline{t_S}$. This assumption plays a crucial role in our results of two-sided asymmetric information: the seller's incentive compatibility constraint holds point-wise which makes it possible to solve the unique closed-form solution for the seller's utility maximizing problem. Relaxing this assumption makes it possible that the optimal solution may be restored into a take-it-or-leave-it offer. Zhao (2018) shows when $\overline{t_S} < \frac{1-\alpha_S}{1-\alpha_B}\overline{t_B}$ there exists a solution to the optimization problem in (14) such that the seller's interim allocation probability is strictly decreasing in t_S (as long as it is positive) and strictly less than that in the optimal mechanism when the seller's type is common knowledge for all $t_S > \underline{t_S}$ (Zhao (2018), Theorem 1 and Proposition 4). According to Zhao, the optimal way to decrease the interim allocation probability is not to decrease the ex-post allocation probability, as in our case, but to increase the take-it-or-leave-it offer (the reserve price in a multiple-agent model) from the full-information counterpart. That is to say, when the seller's highest type is relatively small, it is optimal for the seller to engage only in price signaling.

[22]On information acquisition in mechanism design and auctions with interdependent values see, for instance, Bergemann and Välimäki (2002, 2005) and Bergemann et al. (2009).

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