Preventive-Service Fraud in Credence Good Markets

Edi Karni *

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Abstract

Fraud in markets for preventive services is persistent and prevasive. Examples include preventive dental care and automotive maintenance intended to prevent problems that, if they materialized, would require costly treatment or repair. The market is modeled as a stochastic dynamic game of incomplete information in which the players are customers and service providers. It is analyzed using the notion of weak perfect Bayesian equilibrium. The services provided are credence because the customers lack the expertise necessary to assess the need for the recommended service both ex ante and ex post. In such markets, fraud is a prevalent equilibrium phenomenon that is somewaht mitigated by customers' loyalty and providers' reputation.

Keywords: Credence-quality good markets; preventive-services fraud; Stochastic game of incomplete information.

*Department of Economics, Johns Hopkins University. email: karni@jhu.edu

1 Introduction

In May 2019 the *Atlantic* ran an article entitled "The Trouble With Dentistry," which exposed the practice of excessive diagnosis and treatment endemic to the dental industry.

> ... dentistry's struggle to embrace scientific inquiry has left dentists with considerable latitude to advise unnecessary procedures—whether intentionally or not. The standard euphemism for this proclivity is overtreatment. Favored procedures, many of which are elaborate and steeply priced, include root canals, the application of crowns and veneers, teeth whitening and filing, deep cleaning, gum grafts, fillings for "microcavities"—incipient lesions that do not require immediate treatment—and superfluous restorations and replacements, such as swapping old metal fillings for modern resin ones.

Included in the article were the following testimonies:

> Trish Walraven, who worked as a dental hygienist for 25 years and now manages a dental-software company with her husband in Texas, recalls many troubling cases: "We would see patients seeking a second opinion, and they had treatment plans telling them they need eight fillings in virgin teeth. We would look at X-rays and say, 'You've got to be kidding me.' It was blatantly overtreatment—drilling into teeth that did not need it whatsoever.

The article also mentions a recent field experiment that provides some clues about its pervasiveness.

> A team of researchers at ETH Zurich, a Swiss university, asked a volunteer patient with three tiny, shallow cavities to visit 180 randomly selected dentists in Zurich. The Swiss Dental Guidelines state that such minor cavities do not require fillings; rather, the dentist should monitor the decay and encourage the patient to brush regularly, which can reverse the damage. Despite this, 50 of the 180 dentists suggested unnecessary treatment. Their recommendations were incongruous: Collectively, the overzealous dentists singled out 13 different teeth for drilling; each advised one to six fillings. Similarly, in an investigation for *Reader's Digest*, the writer William Ecenbarger visited 50 dentists in 28 states in the U.S. and received prescriptions ranging from a single crown to a full-mouth reconstruction, with the price tag starting at about $500 and going up to nearly $30,000.

The practices of excessive diagnosis and overtreatment described in the article are also found in other industries, such as car service and maintenance.[1] Both dental patients and car owners, having routine checkup or undergoing emergency treatment or service, may receive diagnoses that include recommendations for treatments intended to preempt problems looming on the horizon. Customers usually lack the expertise to assess the validity of the assessment and the necessity of the recom-

[1] Beck et. al (2014) report that in an experimental setting, car mechanics are significantly more prone to supplying unnecessary services than student subjects.

mended treatment , both ex ante and ex post. Their only recourse is to accept the recommended treatment or service or seek a second opinion. The service providers' incentive to prescribe overtreatment is the business it generates.

Darby and Karni (1973) were the first to identify the fundamental ingredients of the problem of fraudulent prescription of unnecessary services, in what they dubbed, "markets for credence-quality good and services," – information asymmetry and the bundling of diagnosis and service. Numerous studies confirm the prevalence of fraudulent behavior in such markets.[2]

The nature and extent of prescription of unnecessary services depend on the characteristics of the credence services markets. The modeling and analysis of these markets must therefor be based on the specific features of the market under consideration. In this paper, I model and analyze the dentistry and auto-maintenance industries based on their distinct characteristics.

Following Chiu and Karni (2021), I invoke a game-theoretic approach to analyze the equilibrium behavior in credence-service markets in which multiple service providers compete for servicing customers. An important aspect of the markets being considered is, what Darby and Karni (1973) referred to as "client relationship." To capture this aspect, I assume that service providers have loyal clienteles. The customers' loyalty is manifested by considering their regular providers the default option for routine maintenance or emergency service and they return to this provider for service if they obtain a second opinion that agrees with the first provider's prescription. Ex ante the providers are assumed to be identical in every respect. Ex post,

[2] Dulleck and Kerschbamer (2006) and Chiu and Karni (2021) include surveys of the literature.

however, the providers may, endogenously, cultivate different-size clienteles, which affect their behavior, and at any point in time they may have different queues of customers waiting to be served. Both the size of the clientele and the length of the queues are the providers' private information. Customers have different incomes and, consequently, distinct risk attitudes, knowledge of which is their private information.

The distinct and critical aspect of the model is the asymmetric ability to detect and assess the need for treatment/service that is necessary to prevent problems. The service providers are assume to have the expertise necessary to detect and assess these needs, and the customers do not.

The next section describes the credence service market and the game that depicts the interactions among the market participants. Section 3 includes the equilibrium analysis and discusses its economic implications. Section 4 discusses the sources and implications of providers' reputation. Section 5 discusses of the robustness of the model and identifies possible extensions. Section 6 provides the proofs.

2 The Credence Service Market

2.1 An Overview

Consider a market for credence-quality service populated by finite number, n, of customers and a number, $m < n$, of service providers. An important feature of this market is the "customer-provider relationship." These relationships, built and maintained through repeated interactions, are manifested by the customers' inclination to seek periodic maintenance service (e.g., teeth cleaning, oil change) and, if necessary, emergency service from their regular providers. To capture this feature of the market, I assume that each provider has loyal customers who schedule the routine maintenance service and visit the provider first in case of an emergency. Formally, let $(C_1, ..., C_m)$ be a partition of the set C of customers and $i \in C_j$ indicate that customer i belongs to the clientele of provider j. The value of the customer loyalty to the provider is the expected present value of the future services the customer purchases. Let this value be denoted by $v_i \in [0, \bar{v}]$ and assume that it is common knowledge.[3]

Customers seeking a second opinion schedule an appointment with another provider. If they detect fraudulent behavior on the part of the provider to whose clientele they originally belonged, they switch their loyalty and join the clientele of the provider from whom they sought the second opinion.

Ex ante, all service providers are assumed to be identical in every respect. However, at any time, providers may have different queues of customers for scheduled services. Because the lengths of the queues are the providers' private information, customers perceived all providers other than their own as identical.

The information asymmetry in this market is two-sided. The customers' private information is their risk preferences, which are assumed to be determined by their incomes. The service providers' private information is the length of their queues.

In addition, there is expertise asymmetry. The service providers possess expert knowledge that allows them, upon inspection, to anticipate problems, assess their severity, and determine the necessary preemptive treatments. As a result, after the

[3] See discussion of the implications of relaxing the common knowledge of v_i assumption in Section 4.

inspection, there is information asymmetry. The provider is informed about the potential problems whereas the customer is not.

To simplify the exposition, I assume that there are two states, ω_0 and ω_1.[4] In state ω_0, no imminent problem is detected and no preemptive intervention is required. In state ω_1, a looming problem is detected that requires preemptive intervention to avoid a more costly treatment in the future. Let $\Omega = \{\omega_0, \omega_1\}$ denote the state space.

When a customer arrives at the service facility for a scheduled appointment or emergency service, the provider delivers the required service and observes (i.e., diagnoses) the state. Based on the diagnosis, the provider may prescribe, ω_0, no preemptive treatment or, ω_1, preemptive treatment. Lacking the expertise to form an independent assessment of the state, customers have no way of knowing whether the proposed treatment is necessary. Upon receiving a prescription, the customer may accept it or seek a second opinion. If the customer seeks a second opinion, she receives a second prescription and must decide whether to accept it or return to the first provider. I assume that if the second opinion agrees with the first, loyalty makes the customer return to her regular provider for service.

Providers maintain facilities with installed service capacities. The service market is competitive, so the hourly service price charged by all service providers is identical. Using the service hour as the numeraire, the marginal cost of maintaining a service station is c, regardless of whether it is in use or not. The profit generated by a service hour is $1 - c$ if the station is occupied and $-c$ if it is not. At each point in time, t, the queue (i.e., the number of scheduled maintenance service hours) of

[4] For a discussion of the implications of relaxing this assumption, see Section 4.

provider j is $Q_j(t) \in \left[0, \bar{Q}_j\right]$, where the upper bound is a function of the size of the provider's clientele (i.e., $\bar{Q}_j = \mid C_j \mid \omega_1$). Let G_j denote the cumulative distribution function of Q_j which, in equilibrium, is determined by the probabilities of the states, the providers' prescriptions, the customers' arrival rate and their decisions. Let $\mathcal{G}$ be the set of cumulative distribution functions on $\left[0, \bar{Q}_j\right]$ endowed with the topology of weak convergence.

Customers are assumed to have identical preferences and different incomes, denoted by $y_i \in [0, \bar{y}]$. The income distribution is depicted by a cumulative distribution function, F on $[0, \bar{y}]$, that is differentiable and has full support.

2.2 The Stage Game

The credence service market is modeled as a stochastic game of incomplete information and analyzed using the concept of weak perfect Bayesian equilibrium. The players are the service providers and the customers. A stage game is initiated when a customer arrives at the service facility of her regular provider.

2.2.1 The extensive form stage game

The stage game begins with nature assigning the customer a state, $\omega \in \Omega$.[5] The probabilities of being assigned the states ω_0 and ω_1 are $\mu(\omega_0)$ and $\mu(\omega_1) = (1 - \mu(\omega_0))$, respectively, and are assumed to be common knowledge. At time t, a customer, $i \in C_j$, shows up for a routine maintenance or emergency service, initiating the stage game $\Gamma(\omega, Q_j(t), v_i, i \in C_j)$. The players in this stage game are the customer, i; the

[5] Figure 1 depicts the structure of the stage game in extensive form.

provider, j; and a provider, h, whom the customer selects, at random, if she decides to seek a second opinion. Henceforth, I refer to the provider to whose clientele the customer belongs, as the first provider and to the second opinion provider as the second provider.

The first provider observes the state (i.e., his information sets are $\{(\omega_0, y) \mid y \in [0, \bar{y}]\}$ and $\{(\omega_1, y) \mid y \in [0, \bar{y}]\}$) and chooses a prescription $p_1 : \Omega \rightarrow \Omega$, where $p_1(\omega_k) = \omega_0$, $k \in \{0, 1\}$ means that no preemptive service is recommended and $p_1(\omega_k) = \omega_1$ means that preemptive ω_1 hours of service are recommended. To simplify the exposition, I assume that if the state is ω_1, then the provider must prescribe ω_1. Formally, $p_1(\omega_1) = \omega_1$.[6]

Unable to observe the state, and not knowing the provider's total number of scheduled maintenance service hours, $Q_j(t)$, the customer's initial information set is $H_c = \{(\omega, Q_j(t)) \mid \omega \in \Omega, Q_j(t) \in [0, \bar{Q}_j]\}$. Assume that the distribution of ω and $Q_j(t)$ are independent. Then the customer's prior beliefs that the stage game is $\Gamma(\omega_1, Q_j(t), v_i, i \in C_j)$ are $\{\mu(\omega) G_j(dQ_j(t)) \mid \omega \in \Omega, Q_j(t) \in [0, \bar{Q}_j]\}$. After having received the first provider's prescription, the customer updates her beliefs. Her posterior belief that the stage game is $\Gamma(\omega_1, Q_j(t), v_i, i \in C_j)$ are given by $\{\mu(\omega \mid p_1, v_i) G_j(dQ_j(t) \mid p_1, v_i) \mid p_1 \in \Omega, \omega \in \Omega, Q_j(t) \in [0, \bar{Q}_j], v_i \in [0, \bar{v}]\}$.

Having been prescribed $p_1 \in \Omega$, if the customer seeks a second opinion she must pay a diagnosis fee, $D < \omega_1$.[7] Assume that the search for prescription is with recall - in other words, having received a second prescription, the customer can either accept

[6] This assumption is sustained as an equlibrium outcome. It can also be justified if the customer can sue for malpractice if the state is revealed to be ω_1 and the provider prescribed ω_0.

[7] The assumption is that the fee is smaller than the cost of service. Otherwise, customers will never seek a second opinion, and providers will always prescribe ω_1.

it or return to the first provider and accept his prescription or continue the search seeking a third opinion. As the service price is the same, a customer seeking a second opinion selects a provider at random, with equal probabilities. As an ingredient of the client relationship, I assume that if the second prescription agrees with the first, then the customer accepts the prescription and service of the first provider.

Providers know their clients; when an unfamiliar customer shows up, they infer that she is seeking a second opinion. The second provider implements a prescription policy $p_2 : \Omega \rightarrow \Omega$. Not knowing the prescription that the customer receives on her first visit, the second provider's information sets are $H_h^0 = \{(\omega_0, p_1 = \omega_0), (\omega_0, p_1 = \omega_1)\}$ and $H_h^1 = \{(\omega_1, p_1 = \omega_0), (\omega_1, p_1 = \omega_1)\}$. However, the second provider knows that the customer will accept his prescription if and only if the state is ω_0, the first provider prescribed $p_1(\omega_0) = \omega_1$, and he prescribes $p_2(\omega_0) = \omega_0$. In every other instance, the customer returns to the first provider.

In the final stage, the customer, having obtained two prescriptions, p_1 and p_2, must decide whose prescription to accept and whether to remain loyal to the provider to whose clientele she initially belonged or switch her loyalty to the second provider. *Fraud is said to be committed if the state is* ω_0, *and the first provider prescribes* $p_1(\omega_0) = \omega_1$.

Figure 1

2.2.2 The customers

As we shall see later, truthful description (i.e., $p_2(\omega_k) = \omega_k$, $k \in \{0, 1\}$) is the second provider's dominant strategy. Consequently, the customers have no incentive to seek more than two opinions. With slight abuse of notations, let p_1, $p_2 \in \Omega$ denote the prescriptions of the first and second provider, respectively.

The customers' strategies: The customers' strategies are a mappings $\sigma : [0, \bar{y}] \rightarrow \Sigma_1 \times \Sigma_2$, where, for each income $y \in [0, \bar{y}]$, $\Sigma_1(y) := \{\sigma_1(y) : \Omega \rightarrow \{0, 1\}\}$ and $\Sigma_2(y, p_1) := \{\sigma_2(y, p_1) : \Omega \rightarrow \{0, 1\}\}$. Given $(y, p_1) \in [0, \bar{y}] \times \Omega$, $\sigma_1(y, p_1) = 1$, means that the customer accepts the prescription of the first provider and terminates the search; $\sigma_1(y, p_1) = 0$ means that she seeks a second prescription. Similarly, $\sigma_2(y, p_1, p_2) = 1$ means that the customer, having sought a second opinion, accepts the second provider's prescription, and $\sigma_2(y, p_1, p_2) = 0$ means that she rejects the second provider's prescription and accepts the prescription of the first provider. Let Σ denote the set of customers' strategies.

The customers' beliefs: Given the customer's information set $H_c = \{(\omega, Q_j(t)) \mid \omega \in \Omega, Q_j(t) \in [0, \bar{Q}_j]\}$, her prior beliefs are μ and G_j. Upon obtaining the first prescription, p_1, the customer updates her beliefs about the true state and the probability distribution of the provider's queue by applying Bayes' rule. Given the customer's value, v_i, her posterior beliefs conditional on the first provider' prescription, p_1, are represented by $\mu(\cdot \mid p_1, v_i)$ on Ω and the conditional CDF $G_j(\cdot \mid p_1, \omega_i)$ on $[0, \bar{Q}_j]$.

If the customer decides to seek a second opinion, she selects a provider, h, at random. The provider recognizes that the customer does not belong to his clientele and infers that she visited another provider first and is looking for a second opinion.

This sequence of events happens only if the first provider prescribed ω_1. The second provider observes the state. If the state is ω_1, the second provider must subscribe $p_2(\omega_1) = \omega_1$, in which case the second provider earns the diagnostic fee, D, and the customer, finding no evidence that the first provider overprescribed, returns to him for service. If the state is ω_0, the second provider infers that the first provider prescribed $p_1(\omega_0) = \omega_1$. Hence, if the second provider prescribes $p_2(\omega_0) = \omega_0$, then in addition to collecting the diagnosis fee, the second provider adds the customer to his clientele. Thus, prescribing $p_2(\omega_0) = \omega_0$ is the second provider's dominant strategy. Combining these arguments, we conclude that *the second provider's dominant strategy is to prescribes truthfully; consequently, the second prescription reveals the true state. The customer has no incentive to continue her search.*

The customers' payoffs: Assume that the customer's utility function displays decreasing absolute risk aversion, and let $\mathbb{R}_{++}$ be the range of the Arrow-Pratt measure of absolute risk aversion.[8] Thus, accepting a prescribed service p_1 on her first visit, the utility of a customer whose income is y is $u(y - p_1)$, $p_1 \in \Omega$. Seeking a second opinion and accepting the least costly prescription, the customer's utility is $u(y - D - \min\{p_1, p_2\})$. Clearly, conditional on $p_1 = \omega_0$, the customer always accepts the prescription and stops the search. Conditional on $p_1 = \omega_1$ the customer's expected utility from seeking a second prescription is:

$$\bar{u}(y) := \mu(\omega_1 \mid p_1 = \omega_1, v_i)\, u(y - D - \omega_1) + \mu(\omega_0 \mid p_1 = \omega_1, v_i))u(y - D), \qquad (1)$$

[8]This assumption, which involves some loss of generality, is intended to simplify the exposition using the customer's income to parametrize her risk attitudes. Example of such utility functions are $u(y) = y^{\alpha}$, where $\alpha \in (0, 1]$, and $u(y) = \log y$.

and the customer's utility of accepting the first provider's prescription is $u(y-\omega_1)$.

Let y^* be the solution to the equation $\bar{u}(y) = u(y-\omega_1)$. Then, $\bar{u}(y) \geq u(y-\omega_1)$ if $y \geq y^*$, and $\bar{u}(y) < u(y-\omega_1)$ if $y < y^*$. In the former case, the customer exhibits a low degree of risk aversion and is willing to take the risk of seeking a second opinion. In the latter case, she is sufficiently risk averse that she prefers to avoid taking the risk involved in seeking a second opinion and accepts the prescription.

2.2.3 The providers

Each provider $j = 1, ..., m$, has a set, C_j, of loyal customers, and at any point in time, t, a queue of length $Q_j(t)$ expressed in terms of hours committed to serving customers that have accepted the provider's prescriptions. When a customer shows up at the service station, the provider responds differently depending on whether or not the customer belongs to his clientele.

The providers' strategies: Provider j's *pure prescription strategy* is a mapping $p : \Omega \times [0, \bar{Q}_j] \times [0, \bar{v}] \times \{C_j, C \backslash C_j\} \rightarrow \Omega$, where $p(\omega, Q_j(t), v_i, i \in C_j) = p_1(\omega, Q_j(t), v_i)$ and $p(\omega, Q_j(t), v_i, i \in C/C_j) = p_2(\omega, Q_j(t), v_i)$ denote the provider's prescriptions as a function of the state ω; his queue, $Q_j(t)$; the "customer's value," v_i; and whether or not the customer belongs to his clientele. In particular, $i \in C_j$ implies that j is the first provider and $i \in C/C_j$ implies that the provider was selected at random for a second opinion. The asymmetries between providers is a consequence of their relation to the customer.

The providers' beliefs: The first provider knows the customer's value, v_i, and believes that if he prescribes $p_1 = \omega_0$, the probability that the customer seeks a second opinion is zero and that if he prescribes $p_1 = \omega_1$, the probability that the customer seeks a second opinion is $(1 - F(y^*))$ (i.e., the probability that the customer's income is sufficiently high and, consequently, her aversion to risk sufficiently low that she is willing to bear the risk of seeking a second opinion). Moreover, the first provider anticipates that, should the customer seek a second opinion, the second provider will prescribe truthfully.

The second provider believes that the customer shows up for a second opinion only if she was prescribed $p_1 = \omega_1$ on her first visit.

The providers' payoffs: It is convenient to study the providers' payoffs starting with the second provider. If the customer seeks a second opinion, the new provider, h, recognizes that $i \notin C_h$ and infers that the customer must have received the prescription $p_1 = \omega_1$ on her first visit. Thus, regardless of the state, if $p_2 = \omega_1$, then the second provider's payoff is the diagnosis fee, D. If the state is ω_0, then prescribing $p_2(\omega_0) = \omega_0$, the provider collects the diagnosis fee and, in addition, the customer will join his clientele, which is worth v_i in expected present value. Hence, prescribing truthfully (i.e., $p_2(\omega_0) = \omega_0$ and $p_2(\omega_1) = \omega_1$) is the second provider's dominant strategy and his payoff is $D + \Pr\{p_1(\omega_0, Q_j(t), v_i) = \omega_1\}v_i$.

Consider next the first provider's payoff. At the start of a stage game, $\Gamma(\omega, Q_j(t), v_i, i \in C_j)$, if $\omega = \omega_1$ provider j must prescribe $p_1(\omega_1) = \omega_1$, anticipating that this prescription will be accepted, immediately or after the customer visits a second provider, with probability one. In this case, the length of j's queue at the end of the game is $Q_j(t) + \omega_1$. If $\omega = \omega_0$ and the provider prescribes $p_1(\omega_0) = \omega_0$, then the prescription will be accepted and the length of his queue at the end of the game will be $Q_j(t)$.

If he prescribes $p_1(\omega_0) = \omega_1$ then, with probability $(1 - F(y^*))$ the customer seeks a second opinion, following which the first provider's prescription is rejected, the queue remains $Q_j(t)$, and the customer removes herself from the provider's clientele. With probability $F(y^*)$, the prescription is accepted, the length of the queue at the end of the game is $Q_j(t) + \omega_1$, and the customer is retained.

The arrival of customers is an exogenously given stochastic process. Denote by $\Phi_j(\tau) = C_j \Phi(\tau)/C$ the probability that the elapsed time since the end of the preceding stage game and the arrival of the customer during which no other customer arrives is τ. This probability depends on the common exogenous arrival process, depicted by $\Phi(\tau)$, and the likelihood that the next arriving customer belongs to the clientele of provider j. With positive arrival probability, $\Phi_j(0) = 1$ and Φ_j is a strictly decreasing convex function. I assume that Φ_j has full support on $[0, \infty)$ and is absolutely continuous with respect to the Lebesgue measure. Moreover, Φ_j is specific to the provider, since the arrival rate depends on the size of the provider's clientele. Let $\rho_j(Q, \omega_1) := (1 - \Phi_j(Q)) \mu(\omega_1)$ denote the probability that a new customer arrives in period, Q, in the state ω_1. Then $\rho_j(0, \omega_1) = 0$ and is monotonic increasing and concave in Q (e.g., $\Phi(\tau) = e^{-\beta_j \tau}$ and $\rho_j(Q, \omega_1) = \left(1 - e^{-\beta_j Q}\right) \mu(\omega_1)$).

Consider first a customer $i \in C_j$, arriving at time t. Just before the start of the stage game, $\Gamma(\omega, Q_j(t), v_i, i \in C_j)$, provider j expects to earn cash flow from servicing the customers in his queue, yielding a discounted value

$$w(Q_j(t)) := \int_0^{Q_j(t)} (1 - c) e^{-r\tau} d\tau, \tag{2}$$

where r is the discount rate.

If the state is ω_1, then $p_1 = \omega_1$ and, with probability one, the prescription is accepted and the provider's payoff is

$$w\left(Q_j\left(t\right) + \omega_1 \mid p_1 = \omega_1\right) = w\left(Q_j\left(t\right)\right) + e^{-rQ_j(t)}\left(1 - c\right)\omega_1 + v_i. \tag{3}$$

If the state is ω_0 and the provider prescribes $p_1 = \omega_0$, then the customer accepts the prescription with probability one. With probability $1 - \rho_j\left(Q_j\left(t\right), \omega_1\right)$ no new customer arrives during the period $Q_j\left(t\right)$ in the state ω_1, in which case the provider will be idle for the duration ω_1 incurring a loss of $c\omega_1$. Thus, the provider's payoff is

$$w\left(Q_j\left(t\right) \mid p_1 = \omega_0\right) = w\left(Q_j\left(t\right)\right) + v_i - e^{-rQ_j(t)}\left(1 - \rho_j\left(Q_j\left(t\right), \omega_1\right)\right)c\omega_1. \tag{4}$$

If the state is ω_0 and the provider prescribes $p_1 = \omega_1$, then the customer accepts the prescription with probability $F\left(y^*\right)$. In this case the provider's payoff is

$$w\left(Q_j\left(t\right) + \omega_1 \mid p_1 = \omega_1\right). \tag{5}$$

With probability $1 - F\left(y^*\right)$ the customer seeks a second opinion. Because the second provider's prescription reveals the state, the customer accepts the second prescription and switches her loyalty to the second provider. Consequently, if no new customer arrives during time $Q_j\left(t\right)$ in state ω_1, the first provider's expected payoff of prescribing ω_1 when the true state is ω_0 is

$$V\left(Q_j\left(t\right), v_i \mid p_1 = \omega_1\right) \tag{6}$$

$$= w\left(Q_j(t)\right) + \left[F(y^*)\left(e^{-rQ_j(t)}(1-c)\,\omega_1 + v_1\right) - (1-F(y^*))\,e^{-rQ_j(t)}c\omega_1\right](1-\rho_j(Q_j(t),\omega_1).$$

Hence, the first provider prescribes truthfully (i.e., $p_1(\omega_1, Q_j(t), v_i) = \omega_1$ and $p_1(\omega_0, Q_j(t), v_i) = \omega_0$) if and only if

$$V(Q_j(t), v_i \mid p_1 = \omega_1) \leq w(Q_j(t) \mid p_1 = \omega_0). \tag{7}$$

This condition may be written as

$$\omega_1 e^{-rQ_j(t)} F(y^*)(1-\rho_j(Q_j(t),\omega_1)) - (1-F(y^*)(1-\rho_j(Q_j(t),\omega_1)))v_i \leq 0. \tag{8}$$

Define $\alpha_j(Q(t), y^*) = F(y^*)(1-\rho_j(Q(t),\omega_1))$. Then (8) may be written as:

$$e^{-rQ_j(t)}\omega_1\alpha_j(Q_j(t), y^*) - v_i(1-\alpha_j(Q_j(t), y^*)) \leq 0. \tag{9}$$

But

$$\frac{d\alpha(Q_j, y^*)}{dQ_j} = -F(y^*)(\rho'(Q_j(t),\omega_1) < 0 \tag{10}$$

and $\lim_{Q_j\to\infty}\alpha_j(Q_j, y^*) = 0$. Hence, if $Q_j(t)$ is long enough, then (9) holds with strict inequality. At $Q_j(t) = 0$, $\alpha_j(Q_j(t), y^*) = F(y^*)$. Hence, the left-hand side of (9) is $F(y^*)\omega_1 + (1-F(y^*))v_i$. Thus, if $F(y^*)\omega_1 + (1-F(y^*))v_i \leq 0$ then the first provider will prescribe truthfully. If $F(y^*)\omega_1 + (1-F(y^*))v_i > 0$, then there is a unique $Q_j^*(v_i) \in (0, \bar{Q}_j]$ such that $e^{-rQ_j}\omega_1\alpha_j(Q_j, y^*) - v_i(1-\alpha_j(Q_j, y^*)) = 0$, and for all $Q_j(t) < Q_j^*(v_i)$ the provider finds it profitable to prescribe ω_1 when the true state is ω_0. If $Q_j(t) \geq Q_j^*(v_i)$, then truthful prescription is the provider's best response.

3 Equilibrium Analysis

3.1 Stage game equilibrium

The analysis of the game invokes the concept of weak perfect Bayesian equilibrium. Formally, a *system of beliefs* η in extensive form game Γ_E is a specification of a probability $\eta(x) \in [0,1]$ for each decision node x in Γ_E such that $\Sigma_{x \in H} \eta(x) = 1$, for all informations sets H. A strategy profile ζ in the extensive form game Γ_E is *sequentially rational at the information set H given the system of beliefs η* if player h, who moves at the information set H, maximizes his expected utility given the strategies of the other players. If the strategy profile satisfies this condition for all information sets H, then it is *sequentially rational given the system of beliefs η.*

A profile of strategies and system of beliefs (ζ, η) is a *weak perfect Bayesian equilibrium* (PBE) in the extensive form game Γ_E if (i) the strategy profile ζ is sequentially rational given the system of beliefs η and (ii) the system of beliefs η is derived from the strategy profile η using Bayes' rule whenever possible (i.e., at every information set that is reachable with positive probability, the probability of its decision nodes is updated using Bayes' rule).

I describe next the system of beliefs and the strategy profile of the extensive form stage game $\Gamma(\omega, Q_j(t), i)$.

The Customers

Following her first visit, the customer $i \in C_j$ obtains a prescription p_1 on the basis of which she updates her beliefs about the states using Bayes' rule. Specifically, denote by $p_1(\cdot \mid \omega, Q_j(t), v_i)$ the probability on Ω conditional on the state, the provider's queue, and the customer's loyalty value. Recall that $p_1(\omega_1 \mid \omega_1, Q_j(t), v_i) = 1$, for all $Q_j(t)$ and v_i. Thus, the customer's posterior probability on Ω conditional on the first provider prescribing $p_1 = \omega_1$ is:

$$\mu(\omega_1 \mid p_1 = \omega_1, v_i) = \frac{\mu(\omega_1)}{\mu(\omega_1) + \mu(\omega_0) \int_0^{\bar{Q}_j} p_1(\omega_1 \mid \omega_0, Q_j(t), v_i) dG_j(Q_j(t) \mid p_1 = \omega_1, v_i)}, \tag{11}$$

where

$$G_j(Q_j(t) \mid p_1 = \omega_1, v_i) = \frac{G_j(Q_j(t)) + \int_0^{Q_j(t)} p_1(\omega_1 \mid \omega_0, s, v_i) dG_j(s)}{1 + \int_0^{\bar{Q}_j} p_1(\omega_1 \mid \omega_0, s, v_i) dG_j(s)}. \tag{12}$$

and

$$p_1(\omega_1 \mid \omega_0, Q_j(t), v_i) \in [0, 1]. \tag{13}$$

The customer's posterior probability on Ω conditional on the first provider prescribing $p_1 = \omega_0$ is $p_1(\omega_0 \mid \omega_1, Q_j(t), v_i) = 0$, for all $Q_j(t)$ and v_i. Hence,

$$\mu(\omega_1 \mid p_1 = \omega_0, v_i) = 0. \tag{14}$$

The customers' system of beliefs is $(\mu, G_j, \mu(\cdot \mid p_1, v_i), G_j(\cdot \mid p_1, v_i))$, where μ and G_j are, respectively, the priors on the state space and the supplier's queue, and $\mu(\cdot \mid p_1, v_i)$ and $G_j(\cdot \mid p_1, v_i)$ are the corresponding posteriors.

To delineate the customer's optimal strategy, we need to consider several possibilities. The least risk-averse customer is risk neutral. Such a customer is indifferent between accepting the prescription $p_1 = \omega_1$ and seeking a second opinion if $D = \mu(\omega_0 \mid p_1 = \omega_1, v_i)\,\omega_1$. Hence, if $D \geq \mu(\omega_0 \mid p_1 = \omega_1, v_i)\,\omega_1$, then

$$u(y-\omega_1) > \mu(\omega_0 \mid p_1 = \omega_1, v_i)\, u(y - D) + \mu(\omega_1 \mid p_1 = \omega_1, v_i)\, u(y-\omega_1-D) = \bar{u}(y, v_i) \tag{15}$$

for all y and all risk-averse utility functions. Consequently, regardless of her income, customer i accepts the first provider's prescription. If $D < \mu(\omega_0 \mid p_1 = \omega_1, v_i)\,\omega_1$, then there exist $y^*(v_i)$ such that $u(y^*-\omega_1) = \bar{u}(y^*, v_i)$.[9] In this case, given that the utility function displays decreasing absolute risk aversion customer i's optimal strategy is $\sigma_1^*(y, \omega_0) = 1$, for all y, $\sigma_1^*(y, \omega_1) = 1$ if $y \leq y^*(v_i)$ and $\sigma_1(y, \omega_0) = 0$ if $y > y^*(v_i)$. If the customer decides to seek a second opinion her strategy is $\sigma_2^*(y, \omega_1, \omega_0) = 1$ (i.e., the customer accepts the second provider's prescription) and $\sigma_2^*(y, \omega_1, \omega_1) = 0$ (i.e., loyalty makes the customer return to the first provider).

Definition 1. A *reservation-utility strategy* $\sigma : \mathbb{R}_+ \to \Sigma_1 \times \Sigma_2$, with reservation utility $u(y^*(v_i))$, consists of two mappings: $\sigma_1 : \mathbb{R}_+ \times \Omega \to \{0, 1\}$ and $\sigma_2 : \mathbb{R}_+ \times \Omega^2 \to \{0, 1\}$ and a function $u : \mathbb{R}_+ \to \mathbb{R}_+$ such that:

(i) $\sigma_1(y, \omega_0) = 1$ for all y, $\sigma_1(y, \omega_1) = 1$ if $u(y - \omega_1) > u(y^*(v_i))$ and $\sigma_1(y, \omega_1) = 0$ otherwise.

(ii) $\sigma_2(y, \omega_1, \omega_0) = 1$ and $\sigma_2(y, \omega_1, \omega_1) = 0$, for all $y \in \mathbb{R}_+$.

Proposition 1. *The customer's best-response strategy to the suppliers' pure*

[9] Here I assume, without essential loss of generality, that $\bar{y}$ is large enough that the customer displays a safficiently low degree of risk aversion when her income is close to $\bar{y}$. Otherwise, assume that D is sufficiently small that customers with lower income may still be inclined to bear the risk of seeking a second opinion

prescriptions strategy profile (p_1, p_2) *is a reservation-utility strategy with reservation utility* $u(y^*(v_i))$ *, where the reservation income,* $y^*(v_i)$ *, is the solution to* $u(y - \omega_1) = \bar{u}(y, v_i)$ *.*

The Providers

Given that the customers' incomes are private information, the information sets of the first provider are $H_0 := \{(\omega_0, y) \mid y \in [0, \bar{y}]\}$ and $H_1 := \{(\omega_1, y) \mid y \in [0, \bar{y}]\}$. At each information set, the provider must choose between prescribing ω_0 and ω_1. By assumption, at the information set H_1 the provider must prescribe $p_1 = \omega_1$ with probability one. At the information set H_0, the provider prescribes $p_1 = \omega_0$ if the inequality (8) holds and $p_1 = \omega_1$, otherwise.

Denote by $Q_j^*(y^*, v_i)$ the solution to (8) with equality. Then, if $Q_j(t) \geq Q_j^*(y^*, v_i)$, the provider prescribes $p_1(\omega_0, Q_j(t), v_i) = \omega_0$ and if $Q_j(t) < Q_j^*(y^*, v_i)$, he prescribes $p_1(\omega_0, Q_j(t), v_i) = \omega_1$. Hence, the provider's strategy is fully characterized by the function $Q_j^*(y^*, v_i)$. In particular, $p_1(\omega_1 \mid \omega_0, v_i) = G_j\left(Q_j^*(y^*, v_i)\right)$. Hence, the customer's ex ante expected utility, $\bar{U}(y)$ under the reservation-utility strategy is

$$u(y - \omega_1)\left(\mu(\omega_1) + \mu(\omega_0) G_j\left(Q_j^*(y^*, v_i)\right)\right), \text{ if } y \leq y^*(v_i), \tag{16}$$

and

$$u(y - D) G_j\left(Q_j^*(y^*, v_i)\right) \mu(\omega_0) + u(y - D - \omega_1) \mu(\omega_1), \text{ if } y > y^*(v_i). \tag{17}$$

An immediate implication of the linearity of $\bar{U}$ in $G_j\left(Q_j^*(y^*, v_i)\right)$ is that the customers' ex ante expected utility under the reservation-utility strategy is continuous in the provider's strategy. Formally,

Lemma 1. *The customer's reservation utility,* $u\left(y^{*}\left(v_i\right)\right)$, *is continuous in the provider's prescription* $p_1(\omega_1 \mid \omega_0, Q_j\left(t\right), v_i) = G_j\left(Q_j^{*}\left(y^{*}, v_i\right)\right)$.

From the first provider's viewpoint, the probability that customer i accepts the prescription ω_1 is a continuous (monotonic decreasing) function of $p_1(\omega_1 \mid \omega_0, Q_j\left(t\right), v_i)$.

3.2 The provider's queue

The evolution of a provider's queue is driven by the random arrival of customers seeking maintenance or emergency service, their state, the provider's prescriptions, and customers' responses. To trace the evolution of a provider's queue, let g_j^{*} denote the the distribution of provider j's queue *hypothesized* by a customer $i \in C_j$.

Because the customer does not know the state, ω, or the length, Q_j, of the provider's queue in the preceding stage game or how much time has passed since the preceding stage game ended (i.e., the waiting time t), her *perceived* distribution of the provider's queue is the unconditional expectations of $g_j\left(\hat{Q}_j \mid Q_j, \omega\right)$.

For $\hat{Q}_j > 0$ to obtain, it must be the case that the preceding stage game ended with $Q_j = \hat{Q}_j + t$, where $t \in [0, Q_j)$. Hence, conditional on the preceding stage game starting with $(\omega, Q_j) \in \Omega \times \left[0, \bar{Q}_j\right]$, the probability of $\hat{Q}_j > 0$ is

$$g_j\left(\hat{Q}_j\right) = \int_0^{\infty} \int_{t<Q_j}^{\bar{Q}_j} \left[g_j^{*}\left(\hat{Q}_j \mid Q_j, \omega_0\right) \mu\left(\omega_0\right) + g_j^{*}\left(\hat{Q}_j \mid Q_j, \omega_1\right) \mu\left(\omega_1\right)\right] g_j^{*}\left(Q_j\right) dQ_j d\Phi\left(t\right). \tag{18}$$

For $\hat{Q}_j = 0$ to obtain, it must be the case that $t \geq Q_j$. The probability of this event is

$$g_j(0) = \int_0^{\infty} \int_{Q_j \le t} \left[g_j^*(0 \mid Q_j, \omega_0)\, \mu(\omega_0) + g_j^*(0 \mid Q_j, \omega_1)\, \mu(\omega_1)\right] g_j^*(Q_j)\, dQ_j d\Phi(t)\,. \tag{19}$$

If the received probability agrees with the hypothesize probability, then

$$g_j^*\left(\hat{Q}_j\right) = \int_0^{\infty} \int_0^{\bar{Q}_j} \left[g_j^*\left(\hat{Q}_j \mid Q_j, \omega_0\right) \mu(\omega_0) + g_j^*\left(\hat{Q}_j \mid Q_j, \omega_1\right) \mu(\omega_1)\right] g_j^*(Q_j)\, dQ_j d\Phi(t)\,. \tag{20}$$

Lemma 2. *Let g_j^* be given by(20). Then* (a) *g_j^* has full support $\left[0, \bar{Q}_j\right]$ and an atom at zero and* (b) *g_j^* is absolutely continuous with respect to the Lebesgue measure.*

The customer's unconditional expected CDF of the provider's queue is

$$G_j^*(Q') = g_j^*(0) + \int_0^{Q'} g_j^*(y)\, dy, \text{ for all } Q' \in \left[0, \bar{Q}_j\right].$$

For the new customer to encounter a queue that is shorter or equal to Q', the preceding stage game must have ended with either $Q \le Q'$ or, if $Q \ge \left[Q', \bar{Q}_j\right]$, the time elapsed since the end of the preceding stage game during which no new customer arrives is $\tau \ge Q - Q'$. These events may obtain under three possible scenarios:

(a) The preceding stage game started with (ω_0, Q) and the provider prescribed $p_1 = \omega_0$ (which is accepted) or the provider prescribed $p_1 = \omega_1$, and the customer sought a second opinion and rejected the provider's prescription. The provider prescribes $p_1 = \omega_0$ if $Q \ge Q_j^*(y^*, v)$ and $p_1 = \omega_1$ otherwise, where v denotes the loyalty value of the preceding customer. Let J be the CDF on $[0, \bar{v}]$, the range of possible customer's loyalty values. Assume that J has full support. Then the probability of this scenario is

$$\mu(\omega_0)\left[\int_0^{\bar{v}}\left(1-G_j\left(Q_j^*(y^*,v)\,F(y^*)\right)dJ(v)\right)\int_{Q'}^{\bar{Q}_j}\Phi_j(Q)\,dG_j(Q)+G_j(Q')\right].^{10} \tag{21}$$

(b) The preceding stage game began with (ω_1, Q), in which case the provider prescribes $p_1 = \omega_1$ with probability one and the customer accepts it (immediately or after having sought a second opinion). The probability of this scenario is

$$\mu(\omega_1)\left[\int_{Q'-\omega_1}^{\bar{Q}_j}\Phi_j(Q+\omega_1)\,dG_j(Q)+G_j(Q'-\omega_1)\right]. \tag{22}$$

Note that if $Q' = 0$, then $G_j(Q' - \omega_1) = 0$ and since $Q \geq 0$, $\Phi_j(Q + \omega_1 - Q') = \Phi_j(Q + \omega_1) \geq \Phi_j(\bar{Q}_j + \omega_1)$. Hence, $G_j(0) > \mu(\omega_1)\,\Phi_j(\bar{Q}_j + \omega_1) > 0$.

(c) The preceding stage game began with (ω_0, Q), the provider prescribed $p_1 = \omega_1$, and it is accepted. The probability of this scenario is

$$\mu(\omega_0)\left[\int_0^{\bar{v}}G_j\left(Q_j^*(y^*,v)\right)F(y^*)\,dJ(v)\int_{Q'-\omega_1}^{\bar{Q}_j}\Phi_j(Q+\omega_1)\,dG_j(Q)+G_j(Q'-\omega_1)\right]. \tag{23}$$

Let $\bar{\lambda}(y^*, Q^*(y^*)) = \mu(\omega_0)\int_0^{\bar{v}}(1 - G_j(Q^*(y^*, v))\,F(y^*))\,dJ(v)$. Then the unconditional CDF of the length of the queue upon the arrival of the current customer

[10] To see this, note that, for $Q \geq Q'$, $\Phi(Q - Q') = \int_{Q-Q'}^{\infty}\Phi'(\tau)\,d\tau$ is the probability that the time elapsed since the end of the preceding stage game exceeds $Q - Q'$.

being shorter or equal to Q' is

$$\hat{G}_j(Q') := \bar{\lambda}(y^*, Q^*(y^*)) \left[\int_{Q'}^{\bar{Q}_j} \Phi_j(Q)\, dG_j(Q) + G_j(Q') \right] + \tag{24}$$

$$\left(1 - \bar{\lambda}(y^*, Q^*(y^*))\right) \left[\int_{Q'-\omega_1}^{\bar{Q}_j} \Phi_j(Q + \omega_1)\, dG_j(Q) + G_j(Q' - \omega_1) \right]. \tag{25}$$

Note that $\bar{\lambda}(y^*, Q^*(y^*)) < \mu(\omega_0)$ underscores the fact that the decisions of the customer and the provider shift the weight so that the prescription ω_1 is more likely than the probability of the state ω_1. This bias is a reflection of the provider's fraudulent behavior.

The probability of $Q' = 0$ is the probability of the event "the last stage game ends with a queue Q and the elapsed time since that end of that game during which no new customer arrives is equal or exceeds Q." Formally, let $Q \geq 0$ denote the queue at the end of the last stage game. Then

$$G_j(0 \mid Q) = \Phi_j(Q) \tag{26}$$

and the unconditional probability of $Q' = 0$ is

$$\hat{G}_j(0) := \int_0^{\bar{Q}_j} \Phi_j(Q)\, dG_j(Q) > 0. \tag{27}$$

Since Φ is monotonic decreasing and convex, the longer is Q, the smaller is the probability, $G_j(0 \mid Q)$, that the provider finds himself idle. Consequently, a longer queue reduces the provider's incentive to prescribe unnecessary service.

For every given $(y^*, Q^*(y^*)) \in [0, \bar{y}] \times [0, \bar{Q}_j]$, define a function $\Upsilon(\cdot \mid y^*, Q^*(y^*)) :$ $\mathcal{G} \rightarrow \mathcal{G}$ by $\hat{G}_j = \Upsilon(G_j \mid y^*, Q^*(y^*))$, in (24).

Lemma 3. *For every given $(y^*, Q^*(y^*)) \in [0, \bar{y}] \times [0, \bar{Q}_j]$, $\Upsilon(\cdot \mid y^*, Q^*(y^*))$ has fixed point. Moreover, if G_j^* is a fixed point of Υ then it is continuous in y^* and $Q^*(y^*)$.*

3.3 Equilibrium

Given the state, ω; the value of the customer's loyalty, v; and the customer's reservation-income, $y^*(v)$, the first provider's best response to the customer's reservation utility strategy is characterized by a reservation-queue length $Q^*(y^*, v)$ such that if the provider's actual queue exceeds it, he prescribes truthfully and if it is short of it, he prescribes unnecessary services. Truthful prescription is the second provider's dominant strategy. The customer's best response to the providers' strategies is a reservation-utility strategy characterized by a reservation income, $y^*(v)$, above which she seeks a second opinion whenever the prescription of the first provider is ω_1 and below which the prescription is accepted. Hence, given the value of the customer, v, the equilibrium strategy profile of the stage game is characterized by $(Q^*(y^*, v), p_2^*, y^*(v))$, where $p_2^*(\omega_k) = \omega_k$, $k = 0, 1$, and the corresponding steady-state service providers' queue-length cumulative distribution functions, $G_j^*(Q')$, $j = 1, ..., m$. Let ϑ denote the second provider's beliefs conditional on observing the state. Thus, $\vartheta(\omega_1 \mid \omega_1) = \Pr(p_1(\omega_1) = \omega_1 \mid \omega_1) = 1$ and $\vartheta(\omega_1 \mid \omega_0) = \Pr(p_1(\omega_0) = \omega_1 \mid \omega_0) = 1$.

Theorem 1. *There exist weak perfect Bayesian equilibrium in pure strategies of the stage game $\Gamma(\omega, Q_j, v_i, i \in C_j)$.*

3.4 Behavioral implications

The main behavioral implication of the analysis is that a certain level of fraud (i.e., recommendation of unnecessary preventive treatment or service) is endemic to the competitive equilibrium the markets considered here. The analysis also reveals certain characteristics of the fraudulent behavior. In particular, fraud is perpetrated by provider j against customer i when the length of the provider's queue $Q(t) \leq Q_j^*(y^*, v_i)$. Hence, given the customer's value, v_i and her reservation income, y^*, the probability of fraudulent prescription, is $G_j^*\left(Q_j^*(y^*, v_i)\right)$. *Providers are more likely to commit fraud when their queues are shorter, out of fear of finding themselves idle.*

Recall that $Q_j^*(y^*, v_i)$ is the solution $\omega_1 \alpha(Q_j, y^*) = v_i(1 - \alpha(Q_j, y^*))$ if an interior solution exist and $Q_j^*(y^*, v_i) = \bar{Q}_j$ otherwise. The left-hand side of this equation is the expected marginal benefit of overprescribing preventive service; the right-hand side is the expected marginal loss. Since $\partial \alpha(Q_j, y^*) / \partial Q_j < 0$, $Q_j^*(y^*, v_i)$ is an decreasing function of v_i. In other words, the *likelihood that a provider prescribes unnecessary preemptive services is smaller the higher is the value of keeping the customer's loyalty.* The intuition of this claim is clear. The more valuable the customer, the less inclined the provider is to risk losing her. Consequently, valued customers are less likely to be defrauded. *The client relationship helps mitigate the problem of fraudulent recommendation for preemptive treatment.*

Naturally, client relationships are of limited duration (e.g., the client's expected longevity, moving to a different location, the provider going out of business). Since the shorter the horizon the lower is the value of the customer's loyalty, the analysis suggests that fraud is more likely to be committed when the provider anticipates the client relationship to be of shorter duration. At the extreme, a customer who is *recognized as transient* (e.g., a motorist with out-of-state license plates stopping for mechanical service while on a trip) is much more likely to be warned that preventive (unnecessary) service is recommended to avoid mechanical breakdown down the road. To grasp this observation, note that, since $\omega_1 \alpha \left(Q_j, y^*\right) > 0$, for all Q_j, if the value of the customer $v_i = 0$, which is the case when the customer is transient, as the provider has nothing to lose and something to be gained by recommending unnecessary preventive service. In this case, fraudulent prescription is the provider's dominant strategy.[11] Consequently, in equilibrium, a transient customer learns nothing from the prescription and must decide whether or not to accept the recommended service on the basis of her prior, μ. Seeking a second opinion in this case yields no useful information either as the second provider has no reason to prescribe differently.

Another implication of the analysis is that high-income, less risk-averse customers are more likely to seek a second opinion than low-income, more risk-averse, customers. Consequently, high-income customers are more likely to discover when fraud is committed by the provider to whose clientele they belong and, as a result, are more likely to change their providers than low-income customers.

Finally, consider a new customer arriving on the market and choosing a provider. If the customer is recognized as non-transient newcomer, then the provider behaves as if the customer belongs to his clientele. If the provider does not recognize the newcomer for as a new non-transient customer, he will have to assume that she is seeking a second opinion and prescribes truthfully.

[11] In this case $Q_j^* \left(y^*, v_i\right) = \bar{Q}_j$ and $G^* \left(\bar{Q}_j\right) = 1$.

4 Reputation Dynamics

Building and sustaining a reputation for honesty require that providers implement distinct prescription strategies in equilibrium. In our model, given the same clientele, all providers adopt the same equilibrium prescription strategies. Recognizing that, customers do not have illusions that some providers are inherently more honest that others. Therefore, incorporating a reputation for honesty and analyze its implications for the perpetration of fraud requires that some providers be inherently more reluctant to overprescribe preventive service than others or their position in the market induces them to refrain from overprescribing treatment when other providers do. If the inclination for honest dealing with customers is the provider's private information, then a reputation for honesty is built over time, as customers who seek second opinions notice and spread the word that some providers are less likely to recommend service when it is not needed.

4.1 Authentic honesty

One way to introduce reputation into the model is to relax the assumption that providers are expected-value maximizers (i.e., risk neutral) and to suppose instead that they exhibit varying degrees of risk aversion. In this case, ceteris paribus, the more risk averse the provider, the less he is inclined to recommend unnecessary preventive treatment or service.[12] Formally, let provider j's utility function be denoted by u_j, and denote by $Q_j^*(u_j)$ the provider's queue length below which he would recommend unnecessary preventive treatment.

[12] The ceteris paribus includes the clientele size and composition,and the values of the customer.

Proposition 2: *Ceteris paribus, the equilibrium strategies of providers are such that* $Q_j^*(u_j) < Q_j^*(u_k)$ *if and only if* u_j *displays greater absolute risk aversion than* u_k.

By setting a shorter queue below which the provider prescribes unnecessary preventive measures, the more risk-averse provider is less likely to do so. The intuitive explanation of this observation is that when overprescribing preventive measures, the provider runs the risk of losing the customer. A more risk-averse provider can reduce the risk by implementing a strategy that reduces the likelihood of bearing this risk (i.e., overprescribing when the queue is shorter).

A more risk-averse and, consequently, more honest, provider will retain more of his clientele and attract and retain more second-opinion seekers and increase the size of his clientele. The increased clientele produces a first-order stochastically dominated shift of the distribution of waiting time between the arrival of customers (i.e., shorter waiting time become more likely), which reduces the expected time of being idle. The reduction in the probability of being idle further induces the provider to refrain from overprescribing and, thereby, enhances his reputation. *The feedback loop between the building of a reputation and honest prescribing mitigates the inclination to commit fraud.* Formally, denoting by $Q_j^*(\mid C_j \mid / \mid C \mid)$ the provider's queue length below which he would recommend unnecessary preventive treatment, then we have:

Proposition 3: *Ceteris paribus, the equilibrium strategies of the providers are such that* $Q_j^*(\mid C_j \mid / \mid C \mid) < Q_j^*\left(\mid C_j' \mid / \mid C \mid\right)$ *if and only if* $\mid C_j \mid < \mid C_j' \mid$.

The intuition of this observation is as follows: The larger the share of customers belonging to the clientele of provider j, the more favorable is the distribution of the waiting time between customers' arrivals which, in turn, reduces the probability of the provider being idle for every queue length and, consequently, the optimal queue length below which the provider recommends unnecessary preventive service.

4.2 Spurious honesty

The argument above attributes the source of reputation building to an exogenous factor – namely, inherently diverse risk attitudes. Once started, however, reputation building is enhanced by the increase in the provider's clientele. This analysis suggests that a reputation may be established endogenously even if all providers exhibits the same risk attitudes.

The equilibrium plays of providers and customers generate a random process of reallocation of customers among providers. During this process, providers who prescribe unnecessary services will lose some clients that seek second opinions. This process results in changes in the size and compositions of the providers' clientele. Providers whose clientele grow by adding second-opinion seekers (i.e., less risk-averse clients) experience the shift in the distribution of the waiting time described above, which induces them to prescribe more honestly; providers whose clienteles shrink in the process are induced to subscribe unnecessary services more often. In the long run, the evolution of the clientele and the feedback dynamics may allow customers to identify providers as less inclined to defraud their customers, thus establishing their reputation for being more honest. Thus, *reputation may arise endogenously in equilibrium.*

It is worth underscoring that the building of clientele and reputation are dynamic processes that, once started, enhance each other.

5 Concluding Remarks

The model of this paper delineates a credence-good market with some special characteristics. In particular, it highlights the role of what Darby and Karni (1973) dubbed client relationships, which emerge when providers and customers interact repeatedly. The fear of losing loyal customers deters fraud and mitigates the problem of prescribing unnecessary preventive treatments and services. The analysis underscores the role of reputation in mitigating fraud.

A general conclusion of the analysis is that *the less risk averse customers are and more risk averse the providers are, the less fraud will be perpetrated in the market under study.*

In the interest of preserving tractability, the model of this paper includes some simplifying assumptions. The assumption that the state space is a doubleton has two important implications. If over-treatment or excessive service is prescribed, it can be of only one value, which allows the second provider to determine whether fraud was committed and, consequently, induce him to prescribe truthfully. A richer state space would permit different levels of unnecessary service prescriptions. In this case, the second provider may decide to prescribe unnecessary service, taking the chance that the first provider prescribed an even higher level of service. In this case, the second provider attracts the customer and enjoys the extra income from overprescribing. Awareness of this possibility may induce customers to seek more than two opinions, updating their beliefs according to the prescriptions they elicit. The customers' optimal stopping rule will still be characterized by reservation-utility strategy, and providers will still be more likely to recommend unnecessary services when they face the risk of being idle (i.e., when their queue is short) and the cost of losing a loyal client is smaller. Enriching the state space would thus complicate the analysis without yielding new qualitative conclusions or insights.

Another aspect of the model that deserves further attention is the assumption that the value of future services expected from a client is common knowledge in the stage game. This assumption implies that the expected duration and intensity of the client relationship is common knowledge in the stage game. In practice, this is not the case, the parties must act on their perceptions of the customer's value. The customer may try to impress the provider of her loyalty in order to incentivize him to prescribe honestly. A provider who intends to retire may conceal his intention in order to reduce the customer's suspicion that she is being defrauded. These considerations suggest that another game of signaling and deception may be played. Analysis of this aspect of the client relationship may reveal additional behavioral subtleties but is beyond the scope of this paper. I suspect, however, that these behavioral subtleties are of second-order significance relative to the main conclusions of this paper.

The assumption that customers' incomes are private information is realistic. However, upon repeated interaction, the provider may obtain some information that would allow him to assess, albeit coarsely, the income of the customer and adjust his prescription accordingly. In particular, the provider may be reluctant to recommend unnecessary preventive services if the customer is perceived to be rich, for fear that such customer is likely to seek a second opinion and will be lost. The benefit of being perceived as rich opens up the possibility of another game in which customers signal that they are rich to induce truthful prescription. This extension of the model is worth further study.

I assumed that all customers have the same utility function and that their heterogeneity is induced by their diverse incomes. This assumption, and the choice of a utility function displaying decreasing absolute risk aversion, allows the risk attitudes to be captured by the customers income. A more general approach would introduce heterogeneity by depicting customers' types by their income and risk preferences, two ingredients of choice behavior that jointly determine the optimal search strategy.

6 Proofs

6.1 Proof of Lemma 1

By (1), $u(y^*)$ is continuous in $\mu(\omega_1 \mid p_1 = \omega_1)$. By (11), $\mu(\omega_1 \mid p_1 = \omega_1)$ is continuos in $\bar{p}_1(\omega_1 \mid \omega_0, v)$ which, by linearity, $\bar{p}_1(\omega_1 \mid \omega_0, v)$ is continuos in $p_1(\omega_1 \mid \omega_0, v)$. ▲

6.2 Proof of Lemma 2

(a) To prove that g_j^* has full support note that any point $Q \in (0, \bar{Q}_j]$ can be reach if the preceding stage game ended with $Q + \tau$ and the elapsed time is $\tau \geq 0$. Since Φ has full support, the question boils down to whether any point in $(0, \bar{Q}_j]$ may be reached after a finite sequence of stage games.

If the state is ω_1 then regardless of the state of the provider queue he prescribes

$p_1 = \omega_1$, which is accepted with probability one. Because the elapsed time distribution Φ_j has full support, every $Q = (0, \bar{Q}_j]$ can be reach from $\bar{Q}_j$ if the elapsed time is $\bar{Q}_j - Q$. Since the probability of this event is $\Phi_j\left(\bar{Q}_j - Q\right) = \int_Q^{\bar{Q}_j} d\Phi_j(\tau) > 0$, suffices it to establish that $\bar{Q}_j$ can be reached with positive probability form $Q = 0$.

Consider the event $Q = 0$ and assume that during a period $\Delta < \omega_1$, every customer belonging to the clientele of j arrives in the state ω_1, (i.e.,$\mid C_j \mid$ stage games are initiated during the period Δ). Then, by the end of the period the length of the provider's queue will be $\omega_1 \times \mid C_j \mid = \bar{Q}_j$. The probability of this event is $\left(\left(1 - \Phi_j(\Delta)\right)\mu(\omega_1)\right)^{|C_j|} > 0$.

That prove that $g_j^*(0) > 0$ suffices it to note that

$$g_j^*(0) = \int_0^\infty \int_{Q_j \le t} \left[g_j^*(0 \mid Q_j, \omega_0)\,\mu(\omega_0) + g_j^*(0 \mid Q_j, \omega_1)\,\mu(\omega_1)\right] g_j^*(Q_j)\,dQ_j d\Phi(\tau) >$$

$$\left[g_j^*\left(0 \mid \bar{Q}_j, \omega_0\right)\mu(\omega_0) + g_j^*\left(0 \mid \bar{Q}_j, \omega_1\right)\mu(\omega_1)\right] g_j^*\left(\bar{Q}_j\right) \int_{\bar{Q}_j}^\infty d\Phi(\tau) > 0.$$

(b) Given $Q_j \in \left[0, \bar{Q}_j\right]$, the queue at the start of the next stage game is $\hat{Q}_j - t$, where the transition from Q_j to $\hat{Q}_j$ is the outcome of the state, ω, the provider's prescription and the customer decision, and t is the elapsed time since the end of the preceding stage game. The transition probability is determined by the probability distribution, μ on Ω and the strategies of the provider and the customer. Thus, because the perceived distribution, g_j^*, is arbitrary, the resulting random variable $\hat{Q}_j$ is arbitrarily distributed according to a probability measure $\hat{g}_j$.

Define $\zeta(t) = -t$, $t > 0$ and observe that, since the random variables $\hat{Q}_j$ and $-t$ are stochastically independent, their sum is distributed according to the convolution of $\hat{g}_j$ and $\Phi\left(\zeta^{-1}(-t)\right)$. Thus, by the Fubini-Toneli theorem,

$$\hat{g}_j * \Phi\left(\zeta^{-1}(-t)\right)(\mathcal{B}) = \int_0^\infty \left[\int_0^{\bar{Q}_j} \mathbf{1}_{\mathcal{B}}\left(\hat{Q}_j + \zeta(t)\right) \hat{g}_j(Q_j)\, dQ_j\right] d\Phi(t)$$

for all $\mathcal{B}$ in the Borel sigma algebra on $\left[0, \bar{Q}_j\right]$.

Because the Φ is absolutely continuous with respect to the Lebesgue measure on $\mathbb{R}$, the measure of $Q_j - t$ is absolutely continuous with respect to the Lebesgue measure on $\mathbb{R}$. Hence, it is non-atomic, except at 0.

Suppose that $\lambda(\mathcal{B}) = 0$, where λ denotes that Lebesgue measure on $\mathbb{R}$, and $\hat{g}_j(\mathcal{B}) > 0$. Let $Q_j^0 = \left[0, \bar{Q}_j\right]$ such that $\neg\left(Q_j^0 = Q_j' - t\right), Q_j' \in \mathcal{B}$. Then, $\hat{g}_j\left(Q_j^0\right) > 0$. For every $t \in [0, \infty)$ let $Q_j^0(t) := Q_j^0 + t \in \left[Q_j^0, \bar{Q}_j\right]$. Let $\mathcal{P} = \{P_z \mid z \in \mathbb{N}\}$ be a partition of $\left[Q_j^0, \bar{Q}_j\right]$ such that $P_z = \{Q_j^0(t) \mid \hat{g}_j\left(Q_j^0(t)\right) \in \left[(z+1)^{-1}, z^{-1}\right]\}$. Because Φ is absolutely continuous with respect to the Lebesgue measure on $\mathbb{R}$, $\hat{g}_j\left(Q_j^0\right) > 0$ implies that at least one element of the partition is uncountable. Let such element of $\mathcal{P}$ be denoted $P_{\hat{z}}$, then $\hat{g}_j\left(Q_j^0(t)\right) > (\hat{z}+1)^{-1} > 0$. Pick a countable number of elements of $P_{\hat{z}}$, $\{Q_j^0(t_\ell) \mid \ell \in \mathbb{N}\}$. Then

$$\hat{g}_j\left(\cup_{\ell\in\mathbb{N}} Q_j^0(t_\ell)\right) = \Sigma_{\ell\in\mathbb{N}} \hat{g}_j\left(Q_j^0(t_\ell)\right) \geq \Sigma_{\ell\in\mathbb{N}} (\hat{z}+1)^{-1} = \infty.$$

But $\hat{g}_j$ is bounded, which is a contradiction. Thus, $\hat{g}_j$ is absolutely continuous with respect to the Lebesgue measure on $\left[0, \bar{Q}_j\right]$. ▲

6.3 Proof of Lemma 3

By Prokhorov's theorem, the domain, $\mathcal{G}$, of $\Upsilon\left(\cdot \mid y^{*}, Q^{*}\left(y^{*}\right)\right)$ is compact in the topology of weak convergence and is obviously convex.

Let $\left(G_j^n\right)$ be a sequence in $\mathcal{G}$ that converges to G_j in the topology of weak convergence. Then, for all continuous real-valued functions f on $\left[0, \bar{Q}_j\right]$, $\lim_{n\rightarrow\infty}\int_0^{\bar{Q}_j} f dG_j^n = \int_0^{\bar{Q}_j} f dG_j$. Since Φ is continuous, by (24), for all $Q' \in [0, \bar{Q}_j]$

$$\begin{aligned}
&\lim_{n\rightarrow\infty}\Upsilon\left(G_j^n\left(Q'\right)\right) \\
&:= \bar{\lambda}\left(y^{*}, Q^{*}\left(y^{*}\right)\right)\left[\int_{Q'}^{\bar{Q}_j}\Phi_j\left(Q\right)dG_j^n\left(Q\right) + G_j^n\left(Q'\right)\right] + \left(1-\bar{\lambda}\left(y^{*}, Q^{*}\left(y^{*}\right)\right)\right)\left[\int_{Q'-\omega_1}^{\bar{Q}_j}\Phi_j\left(Q+\omega_1\right)dG_j^n\left(Q\right)\right. \qquad (28) \\
&= \bar{\lambda}\left(y^{*}, Q^{*}\left(y^{*}\right)\right)\left[\int_{Q'}^{\bar{Q}_j}\Phi_j\left(Q\right)dG_j\left(Q\right) + G_j\left(Q'\right)\right] + \left(1-\bar{\lambda}\left(y^{*}, Q^{*}\left(y^{*}\right)\right)\right)\left[\int_{Q'-\omega_1}^{\bar{Q}_j}\Phi_j\left(Q+\omega_1\right)dG_j\left(Q\right) + \right. \qquad (29)
\end{aligned}$$

Thus, $\Upsilon\left(\cdot \mid y^{*}, Q^{*}\left(y^{*}\right)\right)$ is continuous. The conclusion that Υ has fixed point is implied by Brouwer's fixed point theorem. That the fixed point is continuous with respect to follows from the continuity of $\bar{\lambda}\left(y^{*}, Q^{*}\left(y^{*}\right)\right)$ in these variables. ▲

6.4 Proof of Theorem 1

Proof. Given v_i define a mapping $\Psi : [0, \bar{y}] \times \left[0, \bar{Q}\right] \times \mathcal{G} \rightarrow [0, \bar{y}] \times \left[0, \bar{Q}\right] \times \mathcal{G}$ by:

$$\Psi\left(y^{*}\left(v_i\right), Q_j^{*}\left(y^{*}, v_i\right), , G_j^{*}\left(\cdot \mid y^{*}\left(v_i\right), Q_j^{*}\right)\right) = \left(y^{**}\left(v_i\right), Q_j^{**}\left(y^{**}, v_i\right), G_j^{*}\left(\cdot \mid y^{**}\left(v_i\right), Q_j^{*}\left(y^{**}, v_i\right)\right)\right),$$

where $y^{**}(v_i)$ is the customer's reservation income given $\left(Q_j^*(y^*, v_i),\right)$ and $G_j^*\left(\cdot \mid y^*(v_i), Q_j^*\right)$, $Q_j^{**}(y^{**}, v_i)$ is the provider's best response to $y^*(v_i)$ and $G_j^*\left(\cdot \mid y^*(v_i), Q_j^*\right)$, and $G_j^*\left(\cdot \mid y^*(v_i), Q_j^*(y^*, v_i\right.$ $\Upsilon\left(G_j^*\left(\cdot \mid y^*(v_i), Q_j^*\right)\right)$.

Since each of the sets in the domain of Ψ is compact and convex, the product $[0, \infty] \times \left[0, \bar{Q}\right] \times \mathcal{G}$ is compact (in the product topology) and convex.

By (11) $\mu(\cdot \mid p_1, v_i)$ is continuous in G_j^*. Consequently, by (15), y^* is continuous in G_j. Since F is continuous in y, it follows from (8) that $Q_j^*(y, v_i)$ is continuous in y. By Lemma 3, $G^*(\cdot \mid y, Q_j)$ is continuous in y and Q_j. Hence, Ψ is a continuous mapping. By Brouwer's fixed point theorem, it has a fixed point $\left(y^*, Q_j^*(y^*, v_i), G_j^*\left(\cdot \mid y^*, Q_j^*(y^*, v_i)\right)\right)$.

Consider next the system of beliefs $\left(\mu, G_j^*, \mu(\cdot \mid p_1, v_i), G_j^*(\cdot \mid p_1, v_i), F, \vartheta\right)$, where μ is the prior distribution on Ω, and $G_j^*\left(\cdot \mid y^*(v_i), Q_j^*(y^*, v_i)\right)$ is the fixed point of Υ given $y^*(v_i)$ and $Q_j^*(y^*, v_i)$, and $\mu(\cdot \mid p_1, v_i)$ and $G^*\left(\cdot \mid y^*(v_i), Q_j^*(y^*, v_i)\right)$, given, respectively, in (11) and (12), are obtained by the application of Bayes' rule.[13]

Thus, $\left(y^*(v_i), Q_j^*(y^*, v_i), G_j^*\left(\cdot \mid y^*(v_i), Q_j^*(v_i)\right)\right)$ are sequentially rational given the system of beliefs $\left(\mu, G_j^*, \mu(\cdot \mid p_1, v_i), G_j^*(\cdot \mid p_1, v_i), F, \vartheta\right)$, and the system of beliefs is derived from the strategy profile, using Bayes' rule on the equilibrium path. ■

[13] The application of Bayes rule is possible since none of these beliefs is off the equilibrium path.

6.5 Proof of Proposition 2

The utility function u_j exhibits greater degree of absolute risk aversion than u_k if and only if, (by (9)),

$$u_j\left(e^{-rQ}\omega_1\right)\alpha_j\left(Q,y^*\right)+u_j\left(-v_i\right)\left(1-\alpha_j\left(Q,y^*\right)\right)<u_k\left(e^{-rQ_k}\omega_1\right)\alpha_k\left(Q,y^*\right)+u_k\left(-v_i\right)\left(1-\alpha_k\left(Q,y^*\right)\right). \tag{30}$$

for all $Q \geq 0$. Let $Q_k^*\left(u_k\right)$ denote the solution of

$$u_k\left(e^{-rQ_k}\omega_1\right)\alpha_k\left(Q_k,y^*\right)+u_k\left(-v_i\right)\left(1-\alpha_k\left(Q_k,y^*\right)\right)=0. \tag{31}$$

Hence,

$$u_j\left(e^{-rQ_k^*(u_k)}\omega_1\right)\alpha_j\left(Q_k^*\left(u_k\right),y^*\right)+u_j\left(-v_i\right)\left(1-\alpha_j\left(Q_k^*\left(u_k\right),y^*\right)\right)<0. \tag{32}$$

But

$$\frac{d\left[u_j\left(e^{-rQ_j}\omega_1\right)\alpha_j\left(Q,y^*\right)+u_j\left(-v_i\right)\left(1-\alpha_j\left(Q,y^*\right)\right)\right]}{dQ} \tag{33}$$
$$=-re^{-rQ_j}u_j'\left(e^{-rQ_j(t)}\omega_1\right)\alpha_j\left(Q,y^*\right)+\left[u_j\left(e^{-rQ_j}\omega_1\right)-u_k\left(-v_i\right)\right]\frac{d\alpha_j\left(Q,y^*\right)}{dQ}<0,$$

where the inequality is implies by (10) and the fact that $u_j\left(e^{-rQ_j}\omega_1\right)-u_k\left(-v_i\right)>0$. Hence, the solution, $Q_j^*\left(u_j\right)$, of

$$u_j\left(e^{-rQ}\omega_1\right)\alpha_j\left(Q,y^*\right)+u_j\left(-v_i\right)\left(1-\alpha_j\left(Q,y^*\right)\right)=0, \tag{34}$$

satisfies $Q_j^*(u_j) < Q_k^*(u_k)$. Thus, $Q_j^*(u_j) < Q_k^*(u_k)$ if and only if u_j exhibits greater degree of absolute risk aversion than u_k. ■

6.6 Proof of Proposition 3

By definition $\alpha_j(Q, y^*) = F(y^*)(1 - \rho_j(Q, \omega_1))$, where, $\rho_j(Q, \omega_1) := (1 - \mid C_j \mid / \mid C \mid \Phi(Q))\,\mu(\omega_1)$. Hence, $\mid C_j' \mid / \mid C \mid > \mid C_j \mid / \mid C \mid$ if and only if $\alpha_j\left(Q, y^* \mid C_j'\right) > \alpha_j(Q, y^* \mid C_j)$. Let $Q_j^*\left(\mid C_j' \mid / \mid C \mid\right)$ denote the solution of

$$e^{-rQ}\omega_1\alpha_j\left(Q, y^* \mid C_j'\right) - v_i\left(1 - \alpha_j\left(Q, y^* \mid C_j'\right)\right) = 0.$$

Then

$$e^{-rQ}\omega_1\alpha_j\left(Q_j^*(\mid C_j \mid / \mid C \mid), y^* \mid C_j\right) - v_i\left(1 - \alpha_j\left(Q_j^*(\mid C_j \mid / \mid C \mid), y^* \mid C_j\right)\right) < 0.$$

Hence, by (33), the solution, $Q_j^*(\mid C_j \mid / \mid C \mid)$ to

$$e^{-rQ}\omega_1\alpha_j(Q, y^* \mid C_j) - v_i(1 - \alpha_j(Q, y^* \mid C_j)) = 0.$$

satisfies $Q_j^*(\mid C_j \mid / \mid C \mid) > Q_j^*(\mid C_j \mid / \mid C \mid)$ if and only if $\mid C_j' \mid / \mid C \mid > \mid C_j \mid / \mid C \mid$. ■

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