Pareto Gains of Pre-Donation in Monopoly Regulation

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Abstract. The Revelation Principle implies that given any admissible social welfare function, the outcome of Baron and Myerson's (1982) (BM) optimal direct-revelation mechanism under incentive constraints cannot be dominated by any other mechanism in expected utilities. However, since the expected total surplus varies with a change in the social welfare function, Pareto improvements should be possible if the monopolist and consumers can agree, by means of side payments that reveal no additional information to the regulator, on the use of an alternative social welfare function which would generate a lower expected deadweight loss. We check the validity of this intuition by integrating the BM mechanism with an induced cooperative bargaining model where unilateral pre-donation by consumers or the monopolist is allowed. Under this new mechanism monopolist's pre-donation in the *ex-ante* stage always leads to *ex-ante* Pareto improvement while a certain amount of it eliminates the expected deadweight loss. Moreover, if optimally designed in the *interim* stage, the monopolist's pre-donation may also lead under some cost parameters to *interim* (and also *ex-post*) Pareto improvement. Consumers, on the other hand, have no incentive to make a unilateral pre-donation, nor to reverse the optimal pre-donation of the monopolist.

Keywords: Monopoly regulation; cooperative bargaining; pre-donation.

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1 Introduction

A seminal paper by Baron and Myerson (1982) (henceforth, BM) shows that a monopolist with unknown costs can be optimally regulated by a direct-revelation mechanism which cannot be dominated by any other mechanism in terms of the expected welfare distribution. However, since the expected total surplus implied by their mechanism varies with a change in the expected social welfare function, intuition suggests that Pareto improvements may be possible if the monopolist and consumers can agree, by means of side payments, on the use of an alternative social welfare function which would generate a lower expected deadweight loss. In this paper, we check the validity of this intuition by integrating the BM mechanism with an induced cooperative bargaining model that allows unilateral pre-donation by consumers or the monopolist. To explain this integrated mechanism and what it can achieve in more detail, we shall briefly re-introduce below the regulation problem considered by BM along with their solution.

Using the Revelation Principle (Dasgupta, Hammond, and Maskin, 1979; Myerson, 1979; Harris and Townsend, 1981), BM restrict themselves, for monopoly regulation, to direct revelation mechanisms that ask the monopolist to report its unknown cost parameter and that give it no incentive to lie. Using such mechanisms, BM calculate, on behalf of a benevolent and computationally able regulator, the smallest individually-rational subsidy that must be offered by consumers, at each value of the cost parameter, to the monopolist in order to induce it to truthful revelation. The information about this subsidy function allows the regulator to calculate the welfare of the monopolist (operating profit plus the subsidy received) and the welfare of consumers (consumer surplus net of the subsidy) as a function of the monopolist's possible cost reports. BM brings together these two welfare functions, representing the conflicting interests of the two parties, under a generalized social welfare function, an important novelty for the regulation literature at the time. Formally, they define the ex-post social welfare function as the sum of consumer welfare and a fraction of the monopolist's welfare. Given the regulator's incomplete information represented by a commonly known prior belief about the unknown cost parameter, BM assume that the regulator's task is to maximize the expected value of the ex-post social welfare over the set of cost reports ensuring the operation of the monopolist.

The Bayesian approach thus introduced by BM to the regulation literature is indispensable, as it was immediately revealed by their regulatory solution that there can exist no feasible direct-revelation mechanism that can maximize the *ex-post* social welfare function unless this function treats consumers and the monopolist equally. In this restrictive case, the optimal regulatory solution that ensures marginal cost pricing coincides (in terms of the welfare allocation) with the earlier solution of Loeb and Magat (1979) (henceforth, LM) achieved by the use of a delegation mechanism where the monopolist is entitled, through an output-dependent subsidy scheme, to the sole right to enjoy the whole economic surplus at the output it is delegated to choose. This solution by LM, however, cannot be optimal when the social welfare function is of an asymmetric form assigning to the welfare of the monopolist a weight less than one. The reason is that marginal cost pricing would then lead to a suboptimally high level of subsidy, both in the LM and BM model, which would reduce the actual (and expected) social welfare below a level that is inevitable. The solution proposed by BM under these asymmetric forms of welfare functions requires the price of the good to be always above the marginal cost of the monopolist in order to limit the subsidy paid to the monopolist, hence its informational rent (the monopolist's welfare).

In this study, we ask whether we can obtain a regulatory outcome that is Pareto superior to the outcome of BM in terms of the expected or actual welfare distribution. Notice that this question is not necessarily invalidated by the Revelation Principle, which, for our problem, would state that if a socially efficient allocation rule (maximizing a given social welfare function at each cost parameter) can be implemented by an arbitrary mechanism, then the same rule can be implemented by an incentive-compatible direct mechanism. This principle merely implies that once we fix an expected social welfare function in the BM model of regulation where the welfare weights of the monopolist and consumers are pre-determined and do not change during the regulatory process, no mechanism of any form can generate a higher expected social welfare than the direct revelation mechanism of BM. An implication of this result is that the welfare allocations that correspond to different (expected) social welfare functions cannot be Pareto ranked, further implying that the regulator cannot have any meta preference or ranking over the set of possible social welfare functions when she tries to construct such a preference comparing the welfare allocations induced by the BM mechanism. If she had such a meta preference, the optimality of the regulatory mechanism would clearly require the regulator to select the best social welfare function in terms of the induced welfare allocation and announce it as part of the mechanism before the regulatory action takes place. The regulator's lack of a meta preference over the set of social welfare functions should not mean, however, a complete impartialness for her or the society on whose behalf she acts. Under the BM mechanism, the expected total surplus, or the equally weighted sum of the monopolist's and consumers' welfares, does vary with a change in the social welfare function. Therefore, *ex-ante* Pareto improvements should generally be possible if the monopolist and consumers can agree –by means of some constant side payments that will not harm the incentive-compatibility constraints– on the use of an alternative social welfare function that generates a lower deadweight loss than predicted by the BM model.

The desired Pareto improvements over the outcome of the BM mechanism can be achieved only if the superior mechanism we are looking for may yield welfare allocations that are not attainable by the BM mechanism. Aiming to explore such a superior mechanism, we will augment the BM mechanism with some additional elements that will map, with the help of some pre-committed side payments decided upon by consumers or the monopolist, each social welfare function that can be (initially) chosen by the regulator in the BM mechanism to a new social welfare function that will be used in the augmented mechanism. The resulting regulatory mechanism will be incentive-compatible, like the BM mechanism, only if the regulator can perfectly commit not to use any additional information revealed by the augmented mechanism to update her prior beliefs about the monopolist's private cost. We will show that in some informational situations, we do not even need this commitment on the part of the regulator since the augmented mechanism would reveal no additional information to the regulator than she would already observe under the BM mechanism. To understand why the outcome of the augmented mechanism may Pareto dominate the original mechanism, we should note that our results could alternatively be obtained in a setup where the monopolist could effectively bribe (incentivize) consumers to authorize the regulator to value the monopolist's profit more highly than she truly does. The regulator's increased effective concern with the monopolist's profit would lead her to implement smaller output distortions. The more limited distortions would in turn reduce deadweight loss. The increase in total surplus, coupled with *ex- ante* transfer payments from the monopolist to consumers, could admit Pareto gains. Here, we should acknowledge that the very idea that social welfare can be increased when a policy-maker (or a regulator) commits herself to maximize an objective that differs from her true objective was first proposed –to the best of our best knowledge– by Besanko and Spulber (1993) within the context of a horizontal merger in a Betrand duopoly. Our paper shows that this idea can also be extended to the context of monopoly regulation with the help of a bargaining model with pre-donations.

In more detail, we make the aforementioned augmentation or modification to the BM mechanism by adding, before the revelation of the cost information, an initial stage involving a cooperative bargaining game between consumers and the monopolist over the possible regulatory outcomes, hence over the possible social welfare functions, under the possibility of pre-donation. We model this cooperative bargaining game as in Saglam (2021), who shows that the BM model of regulation is isomorphic to a cooperative bargaining problem a la Nash (1950) with appropriate elements. On the other hand, we borrow our insight as to the potential welfare benefits of pre-donation in a Pareto sense from the literature pioneered by Sertel (1992), who shows that in simple bargaining problems (where the bargaining set has a linear frontier) the two-person Nash bargaining rule can be manipulated via pre-donations: the bargaining party with the higher valuation can alter the bargaining set always to its benefit.[1] A more recent work by Akin et al. (2011) in the same direction even shows that in simple

[1]For more in this literature, see Sertel and Orbay (1998), Orbay (2003), Akyol (2008), Akin et al. (2011), among others.

n-person bargaining problems the manipulation of Kalai-Smorodisnky rule through pre-donation may lead to (strong) Pareto improvements. Motivated by these results, we aim to explore whether a modified mechanism bringing the regulatory bargaining idea of Saglam (2021) and the pre-donation idea of Sertel (1992) together may lead to Pareto improvements over the BM mechanism.

Since the utilities in the bargaining setup of Nash (1950), and accordingly in Saglam (2021), are von Neumann-Morgenstern (expected) utilities, any Pareto improvement which may be deduced by only inspecting the effect of pre-donation on bargaining solutions is bound to be an *ex-ante* improvement, defined in expected utilities. However, we will also deal with ex-post improvements. To make both types of improvements meaningful for the monopolist, we will consider two informational stages in our extended regulatory model. The first stage is called the *ex-ante* stage where the monopolist has not learned yet the actual value of its cost parameter and it shares the regulator's beliefs. The second stage is the *interim* stage where the monopolist privately knows the actual value of its cost parameter. Associated with these two stages, our model will have two variants, depending upon whether pre-donations occur in the *ex-ante* stage or the *interim* stage. However, we will retain the assumption from the BM model that information revelation will occur in the *interim* stage. We will also assume that both the monopolist and consumers will be informed by the regulator as to the details of the regulatory mechanism at the beginning of the stage they are allowed to make pre-donation. Given these assumptions, we observe that if consumers should decide whether and how much to pre-donate in the *ex-ante* or *interim* stage (which they can never distinguish from each other based on their own information in the model), they should always consider the maximization of their *ex-ante* payoffs. On the other hand, the monopolist should take into account its *ex-ante* payoff if it makes pre-donation decisions in the *ex-ante* stage and its *interim* or equivalently *ex-post* payoff if it makes these decisions in the *interim* stage.

We show that any amount of pre-donation made by the monopolist in the *ex-ante* stage always leads to *ex-ante* Pareto improvement in the welfare allocation while a certain amount of it eliminates the expected deadweight loss. Moreover, pre-donation in the *ex-ante* stage reveals no information about the monopolist's private costs, hence it creates no commitment problem on the part of the regulator. We also show that the pre-donation of the monopolist, if optimally designed in the *interim* stage, may also lead under some cost parameters to *ex-post* Pareto improvement. Since the optimal pre-donation of the monopolist is not independent of its private cost information in the *interim* stage, the monopolist unintentionally reveals some part of this information regardless whether it chooses to pre-donate or not. However, since the monopolist can always commit to pre-donation functions that will increase the expected utility of consumers and since such increases would be verifiable before the cost revelation occurs, a benevolent regulator may find it beneficial to perfectly commit *ex-ante* not to use the information that would be revealed by pre-donation to update her prior beliefs about the monopolist's private cost information. Such commitment is in the spirit of Loeb and Magat's (1979) analysis in which the regulator can make binding commitments before the monopolist becomes privately informed about its cost. Finally, we show that consumers have no incentive to make a unilateral pre-donation, nor to reverse the optimal pre-donation of the monopolist.

Here, we should note that our work that studies the effects of pre-donation in the regulatory framework of BM (1982) can be extended to other forms of regulation, such as the rate-of-return regulation (ROR-R) and the price-cap regulation (PC-R) that have been frequently used for many decades in the United States, the Europe, and the South America to regulate utilities, especially in industries involving water supply, gas, electricity, telecommunications, and railroad transportation.[2] Under ROR-R, the monopoly is allowed to earn a pre-specified return on the invested capital and recover all operating costs, taxes, and depreciation allowances. Under PC-R, the regulator puts a cap (ceiling) on the product's price, below which the monopoly is allowed to choose any price.[3] We argue that both ROR-R and PC-R can be re-modeled using the bargaining approach in Saglam (2021) to allow one to investigate, as we do in this paper, whether or when the pre-donation of consumers or the monopoly may yield desirable welfare effects. In fact, the idea of re-modeling regulation through the lens of bargaining may not be unexpected in the case of ROR-R, where the reasonable levels of the rate of return and operating costs are determined in public hearings where consumers and the monopoly share their claims and opinions in the presence of the regulator who acts as an arbitrator.

While the idea of bargaining with *pre-donation* in the context of incentive regulation is novel to our paper, *unmediated negotiations* are frequently observed in regulatory practice. For instance, Littlechild (2009) shows how stipulated settlements replaced or supplemented, in the past, the process of litigation in Florida. He reports that during the period 1976-2002, 30 percent of earnings in the telephone, gas, and electricity sectors were settled by stipulations between the public utilities and the Office of Public Counsel (the Consumer Advocate). These settlements or negotiations took place, like pre-donations in our model, without requiring the presence or the signature of the regulatory body (Florida Public Service Commission) and without any public record in many cases. Moreover, the stipulations usually brought rate reductions, much like

[2] See Liston (1993) and Parker and Kirkpatrick (2005) for comprehensive discussions on PC-R and ROR-R in comparison to some other forms of regulation.

[3] Despite their popularity, both forms of regulation are known to lead to economic distortions, among several other problems in implementation. The fact that ROR-R fully subsidizes the operating costs removes the incentive of the monopoly to produce at the minimum cost, whereas the fact that PC-R allows the monopoly to capture any surplus (as long as its price is below the cap) induces the monopoly to lower the quality of its product. See Liston (1993) for more on these issues.

the price decreases under the monopolist-optimal pre-donation in our model.[4]

The rest of the paper is organized as follows. Section 2 introduces the basic structures, Section 3 presents our results, and finally Section 4 concludes.

2 Basic Structures

Consider a monopolist producing a single good under the inverse demand function

$$P(q) = a - q, \tag{1}$$

where $a > 0$. The monopolist is subject to a cost function

$$C(q, \theta) = \theta q \ \text{ if } q > 0 \ \text{ and } \ C(0, \theta) = 0, \tag{2}$$

where $q \geq 0$ denotes the quantity of supply and $\theta \in [0, a)$ denotes the constant marginal cost which is privately known by the monopolist. On the other hand, the support of θ, the demand parameter a as well as the form of the inverse demand and cost functions described are common knowledge.

The monopolist is optimally regulated by a benevolent regulator who believes that the private cost parameter of the monopolist is uniformly distributed on the interval $[0, a)$ according to the probability density function $f(\theta)$ such that $f(\theta) = 1/a$ if $\theta \in [0, a)$ and $f(\theta) = 0$ otherwise. The problem facing the regulator is choosing the optimal price of the good to maximize the expected social welfare under her beliefs. We should notice that the regulatory structure described above simplifies the structure considered by Baron and Myerson (BM) (1982), where the cost function is affinely linear, involving a fixed part as well, whereas the inverse demand function and the regulator's beliefs are not restricted to any specific forms. While we make our simplifications for the sake of clarity and tractability; it will become clear throughout our analysis that our results can be extended to other forms of regulatory structures, as well.

The solution to the regulatory problem we have described above is proposed by BM in their more general structure. According to this solution, the regulator can, with no loss of generality, restrict herself to incentive-compatible direct revelation mechanisms that ask the monopolist to report its parameter θ and that gives the monopolist no incentive for lying. These mechanisms involve functions $\langle p(.), q(.), r(.), s(.)\rangle$ such that when $\tilde{\theta}$ is the cost report of the monopolist, $p(\tilde{\theta})$ and $q(\tilde{\theta})$ become the price and quantity satisfying $p(\tilde{\theta}) = a - q(\tilde{\theta})$, $r(\tilde{\theta})$ becomes the probability that the regulated monopolist is allowed to produce and sell, and $s(\tilde{\theta})$ becomes the expected subsidy paid by consumers to the monopolist to ensure a truthful response.

[4] In fact, Littlechild (2009) shows that the mean value of a rate reduction was eight times larger with a stipulation than without, while the median value was more than 50 times larger.

Given a mechanism $\langle p(.), q(.), r(.), s(.)\rangle$, if the monopolist with the true marginal cost θ submits the cost report $\tilde{\theta}$, it obtains the regulated profit $\pi(\tilde{\theta}, \theta) = [p(\tilde{\theta})q(\tilde{\theta}) - \theta q(\tilde{\theta})]r(\theta) + s(\tilde{\theta})$. This mechanism is called feasible if (i) it is incentive-compatible; i.e. $\pi(\theta) \equiv \pi(\theta, \theta) \geq \pi(\tilde{\theta}, \theta)$ for all $\theta, \tilde{\theta} \in [0, a)$ and (ii) it is individual rational; i.e., $\pi(\theta) \geq 0$ for all $\theta \in [0, a)$. The first condition implies that the function $q(.)$ is non-increasing over $[0, a)$ and

$$\pi(\theta) = \int_0^a q(x)r(x)dx + \pi(a). \tag{3}$$

Given a feasible mechanism $\langle p(.), q(.), r(.), s(.)\rangle$, the actual welfare of consumers can be calculated as

$$CW(\theta) = \left[\int_0^{q(\theta)} (a - x)dx - p(\theta)q(\theta)\right] r(\theta) - s(\theta). \tag{4}$$

Using $s(\theta) = \pi(\theta) - [p(\theta)q(\theta) - \theta q(\theta)]r(\theta)$, the above equation can be simplified as

$$CW(\theta) = \left[\int_0^{q(\theta)} (a - x)dx - \theta q(\theta)\right] r(\theta) - \pi(\theta). \tag{5}$$

The actual social welfare can be defined, as in the BM model, by the equation

$$SW(\theta) = CW(\theta) + \alpha\pi(\theta), \tag{6}$$

where α is a fixed parameter in $[0, 1]$. The problem facing the regulator is to find a feasible mechanism $\langle p(.), q(.), r(.), s(.)\rangle$ that maximizes the expected social welfare

$$\begin{aligned} SW^e &\equiv \int_0^a SW(\theta)f(\theta)d\theta \\ &= \int_0^a \left(\left[\int_0^{q(\theta)} (a - x)dx - \theta q(\theta)\right] r(\theta) - (1 - \alpha)\pi(\theta)\right) f(\theta)d\theta. \end{aligned} \tag{7}$$

We can observe from the above equation along with (3) that any mechanism maximizing SW^e must yield $\pi(a) = 0$. To completely characterize this mechanism, we can modify the optimal mechanism of BM for the special forms of demand, cost, and belief functions in our model. This modification results in the optimal mechanism $\langle p^*(.), q^*(.), r^*(.), s^*(.)\rangle$ satisfying

$$p^*(\theta) = (2 - \alpha)\theta \tag{8}$$

$$q^*(\theta) = a - (2 - \alpha)\theta \tag{9}$$

$$r^*(\theta) = \begin{cases} 1 & \text{if } \theta \leq \theta^* \equiv \dfrac{a}{2-\alpha} \\ 0 & \text{otherwise} \end{cases} \tag{10}$$

$$s^*(\theta) = \int_0^a q^*(x)r^*(x)dx + [\theta q^*(\theta) - p^*(\theta)q^*(\theta)]r^*(\theta) \tag{11}$$

for all $\theta \in [0, a)$. The above mechanism yields to the monopolist the actual welfare given by

$$\begin{aligned} \pi(\theta, \alpha) \equiv \pi(\theta) &= \int_\theta^{\theta^*(\alpha)} q^*(x, \alpha)dx = \\ &= \left(\frac{2-\alpha}{2}\right)\theta^2 - a\theta + \frac{a^2}{2(2-\alpha)}, \end{aligned} \tag{12}$$

if $\theta \in [0, \theta^*(\alpha))$ and $\pi(\theta, \alpha) = 0$ otherwise. On the other hand, the consumer welfare would become

$$\begin{aligned} CW(\theta, \alpha) \equiv CW(\theta) &= \left[\int_0^{q^*(x,\alpha)} (a-x)dx - \theta q^*(x, \alpha)\right] r^*(\theta) - \pi(\theta, \alpha) \\ &= (a-\theta)\left[a - (2-\alpha)\theta\right] - \frac{1}{2}\left[a - (2-\alpha)\theta\right]^2 \\ &\quad - \left(\frac{2-\alpha}{2}\right)\theta^2 + a\theta - \frac{a^2}{2(2-\alpha)} \end{aligned} \tag{13}$$

if $\theta \in [0, \theta^*(\alpha))$ and $CW(\theta, \alpha) = 0$ otherwise. From the viewpoint of consumers and the regulator, the above welfares are unknown before θ is revealed by the monopolist. But, they can calculate the expected values of these welfares as

$$CW^e(\alpha) = \int_0^a CW(\theta, \alpha)r^*(\theta)f(\theta)d\theta = \frac{2(1-\alpha)a^2}{6(2-\alpha)^2} \tag{14}$$

and

$$PW^e(\alpha) = \int_0^a \pi(\theta, \alpha)r^*(\theta)f(\theta)d\theta = \frac{a^2}{6(2-\alpha)^2} \tag{15}$$

respectively. Notice that the pair $(CW^e(\alpha), PW^e(\alpha))$ denotes the expected welfare (utility) distribution generated by the BM mechanism when the monopolist's welfare is weighted by α in the social welfare function. We will denote this pair simply by $W(\alpha)$. Likewise, we will henceforth denote $p^*(\theta), q^*(\theta), r^*(\theta), s^*(\theta)$, and θ^*, by the variables $p^*(\theta, \alpha), q^*(\theta, \alpha)$, $r^*(\theta, \alpha), s^*(\theta, \alpha)$, and $\theta^*(\alpha)$, respectively.

We can now calculate the expected economic surplus, $ES^e(\alpha) \equiv CW^e(\alpha)+PW^e(\alpha)$

$$ES^e(\alpha) = \frac{(3-2\alpha)a^2}{6(2-\alpha)^2}. \tag{16}$$

Notice that $ES^e(\alpha)$ attains its maximum value of $a^2/6$ if $\alpha = 1$, in which case $ES^e(\alpha)$ coincides with $SW^e(\alpha)$. Let V denote this maximal surplus; i.e. $V \equiv a^2/6$. Notice that V is the expected value of the actual surplus $\nu(\theta) \equiv (a-\theta)^2/2$ under the regulator's belief f; i.e., $V = E[\nu(\theta)|f]$.

Given V, we can write for any α the expected economic surplus as $ES^e(\alpha) = (3-2\alpha)V/(2-\alpha)^2$. We can also define, for any value of α, the expected deadweight loss $DW^e(\alpha) \equiv V - ES^e(\alpha)$ and calculate it as

$$DW^e(\alpha) = \frac{(1-\alpha)^2}{(2-\alpha)^2}V. \tag{17}$$

We should notice that the distribution of expected welfare, $(CW^e(\alpha), PW^e(\alpha))$, as well as the expected deadweight loss, $DW^e(\alpha)$, varies with the parameter α. In particular, we can observe that the triplet $(CW^e(\alpha), PW^e(\alpha), DW^e(\alpha))$ is equal to $(V/2, V/4, V/4)$ if $\alpha = 0$, and equal to $(0, V, 0)$ if $\alpha = 0$. We can also check that $PW^e(\alpha)$ is increasing in α, whereas $CW^a(\alpha)$ and $DW^e(\alpha)$ are decreasing. If the regulator were to choose $\alpha = 1$ to minimize (eliminate) the deadweight loss, it would unintentionally minimize the expected welfare of consumers, as well. On the other hand, if the regulator were to choose $\alpha = 0$ to maximize the expected welfare of consumers, it would unintentionally maximize the expected deadweight loss. Thus, a benevolent regulator acting on behalf of the society is confronted with a dilemma as to how to choose α in the most plausible way from the viewpoint of consumers and social efficiency. Borrowing from Saglam (2021), we leave the solution to this dilemma to a regulatory bargaining process, between consumers and the monopolist, which integrates the bargaining model of Nash (1950) with a simplified version of BM's (1982) regulatory model, which we have described above. To define this bargaining process, we need some preliminaries.

2.1 Cooperative Bargaining

Consider a society of players $N = \{1, 2\}$, where 1 denotes consumers and 2 denotes the monopolist-monopolist. Following Nash (1950), we define a two-player bargaining problem for this society by a pair (S, d), where $S \subset \mathbb{R}^2$ denotes the bargaining set consisting of von Neumann-Morgenstern utility allocations, and $d \in S$ denotes the disagreement point specifying the utility each player must enjoy if they fail to agree on any other point in S. The set S is assumed to be compact and convex, and it contains a point s with $s > d$. Also, S is d-comprehensive; i.e., for all $s, s' \in \mathbb{R}^2$, $s \in S$ and $s \geq s' \geq d$ only if $s' \in S$. Let Σ^2 denote the set of all two-person bargaining problems that satisfy the assumptions above.

A bargaining rule $F : \Sigma^2 \to \mathbb{R}^2$ is a mapping such that $F(S, d) \in S$ for any $(S, d) \in \Sigma^2$. Notice that $F_1(S, d)$ and $F_2(S, d)$ are the bargaining utilities of player 1 and player 2, respectively.

Below, we define some well-known bargaining rules. The Nash (1950) rule proposes for any problem $(S, d) \in \Sigma^2$ the solution

$$N(S, d) = \mathrm{argmax}_{x \in S}\,(x_1 - d_1)(x_2 - d_2), \tag{18}$$

at which the product of players' net utility gains from agreement attains its maximum.

The Kalai-Smorodinsky rule, proposed by Raiffa (1953) for two-person games and axiomatized by Kalai and Smorodinsky (1975), selects for any problem $(S, d) \in \Sigma^2$ the allocation

$$KS(S, d) = \max\left\{ x \in S \;\middle|\; \frac{x_1 - d_1}{x_2 - d_2} = \frac{a_1(S, d) - d_1}{a_2(S, d) - d_2} \right\}, \tag{19}$$

where for each $i = 1, 2$, $a_i(S, d) = \max\{s_i \,|\, s \in S \text{ and } s_{-i} = d_{-i}\}$ denotes the ideal utility player i can expect from (S, d). Accordingly, the point $a(S, d) = (a_1(S, d), a_2(S, d))$ is called the ideal point for (S, d). The Kalai-Smorodinsky rule selects the maximum point of S on the line segment connecting the points d and $a(S, d)$.

A bargaining rule is called dictatorial for player i, or Dictatorial-i, and denoted by D^i if for each $(S, d) \in \Sigma^2$

$$D^i(S, d) = \max\{x \in S \mid\ x_i \geq d_i \text{ and } x_j = d_j \text{ for } j \neq i\}. \tag{20}$$

The rule D^i chooses for player i the best point in the bargaining set, while providing to the other player its disagreement utility.

A family of solutions, known as proportional solutions (Kalai, 1977), will be sufficient for the analysis in this paper for reasons which will be explained later. Given any $\gamma \geq 0$, a bargaining rule is called γ-proportional, or simply P^γ, if it selects for any $(S, d) \in \Sigma^2$ the allocation

$$P^\gamma(S, d) = d + \Omega(S, d)(\gamma, 1) \text{ and } \Omega(S, d) = \max\{t \mid\ d + t(\gamma, 1) \in S\}. \tag{21}$$

We should notice that the rule P^γ selects the maximum point of S on the line passing through the point d and the point $(\gamma, 1)$. In the definition of Kalai (1977), γ is positive. We have included $\gamma = 0$ for convenience. (Notice that when $\gamma = 0$, the proportional rule we have defined above coincides with a special rule that gives to player 2 full dictatorial power.) When $\gamma = 1$, we obtain a well-known member of the γ-proportional rules, known as the Egalitarian rule, which was first recommended by Rawls (1972). For any bargaining problem, this rule chooses an allocation at which the worst-off player's net utility gain from agreement is maximized. Also, note that for $\gamma = 0$ and $\gamma = \infty$, the rule P^γ coincides with the dictatorial rules D^2 and D^1, respectively.

For any $S \subset \mathbb{R}^2$, we denote by $WPO(S) = \{x \in S \mid y > x \text{ implies } y \notin S\}$ the set of weakly Pareto optimal allocations in S and likewise we denote by $PO(S) = \{x \in S \mid y \geq x \text{ implies } y \notin S\}$ the set of Pareto optimal allocations in S. Below, we present some axioms for an arbitrary solution F on Σ^2.

Weak Pareto Optimality (WPO) If $(S, d) \in \Sigma^2$, then $F(S, d) \in WPO(S)$.

Pareto Optimality (PO) If $(S, d) \in \Sigma^2$, then $F(S, d) \in PO(S)$.

Nash and Kalai-Smorodinsky rules satisfy Pareto Optimality (hence Weak Pareto Optimality), whereas any γ-proportional satisfies Weak Pareto Optimality, but not Pareto Optimality.

2.2 Pre-Donation

We modify Sertel's (1992) definition of pre-donation for our model. A *pre-donation* from player i to player $j \neq i$ is a function $\boldsymbol{\lambda}^{\boldsymbol{k},\boldsymbol{i}} : \mathbb{R}^2 \to \mathbb{R}^2$, parameterized by some number $k \in [0, 1)$, which transforms each $s \in \mathbb{R}^2$ into $\boldsymbol{\lambda}^{\boldsymbol{k},\boldsymbol{i}}(s)$ such that $\boldsymbol{\lambda}_i^{\boldsymbol{k},\boldsymbol{i}}(s) = (1-k)s_i$ and $\boldsymbol{\lambda}_j^{\boldsymbol{k},\boldsymbol{i}}(s) = s_j + ks_i$ if $j \neq i$. Given any bargaining set S and any pre-donation $\boldsymbol{\lambda}^{\boldsymbol{k},\boldsymbol{i}}$, we write

$$\boldsymbol{\lambda}^{\boldsymbol{k},\boldsymbol{i}}(S) = \{\lambda^{k,i}(s) \mid s \in S\} \tag{22}$$

and for the comprehensive closure of $\boldsymbol{\lambda}^{\boldsymbol{k},\boldsymbol{i}}(S)$ we write

$$\underline{\boldsymbol{\lambda}}^{\boldsymbol{k},\boldsymbol{i}}(S) = \{s' \in \mathbb{R}^2_+ \mid s'_i \leq s_i \text{ and } s'_j \leq s_j \text{ if } j \neq i, \text{ for some } s \in \boldsymbol{\lambda}^{\boldsymbol{k},\boldsymbol{i}}(S)\}. \tag{23}$$

Notice that $\underline{\boldsymbol{\lambda}}^{\boldsymbol{k},\boldsymbol{i}}(S)$ is a convex and comprehensive bargaining set as in the model of Nash (1950). Moreover if $d \in S$, then $\boldsymbol{\lambda}^{\boldsymbol{k},\boldsymbol{i}}(d) \in \underline{\boldsymbol{\lambda}}^{\boldsymbol{k},\boldsymbol{i}}(S)$. So, we will assume that the pre-donation $\boldsymbol{\lambda}^{\boldsymbol{k},\boldsymbol{i}}(S)$ transforms the bargaining problem (S, d) into the problem $(\underline{\boldsymbol{\lambda}}^{\boldsymbol{k},\boldsymbol{i}}(S), \boldsymbol{\lambda}^{\boldsymbol{k},\boldsymbol{i}}(d))$.

Given any problem $(S, d) \in \Sigma^2$, any bargaining rule F on Σ^2, any $k \in [0, 1)$, and any $i \in \{1, 2\}$, we say that the pre-donation $\boldsymbol{\lambda}^{\boldsymbol{k},\boldsymbol{i}}$ is

(i) beneficial for player $m \in \{1, 2\}$ if $F_m(\underline{\boldsymbol{\lambda}}^{\boldsymbol{k},\boldsymbol{i}}(S), \boldsymbol{\lambda}^{\boldsymbol{k},\boldsymbol{i}}(d)) > F_m(S, d)$,

(ii) harmful for player $m \in \{1, 2\}$ if $F_m(\underline{\boldsymbol{\lambda}}^{\boldsymbol{k},\boldsymbol{i}}(S), \boldsymbol{\lambda}^{\boldsymbol{k},\boldsymbol{i}}(d)) < F_m(S, d)$,

(iii) ineffective for player $m \in \{1, 2\}$ if $F_m(\underline{\boldsymbol{\lambda}}^{\boldsymbol{k},\boldsymbol{i}}(S), \boldsymbol{\lambda}^{\boldsymbol{k},\boldsymbol{i}}(d)) = F_m(S, d)$.

2.3 Regulatory Bargaining under Pre-donation

Now we can turn to consider the specific bargaining problem in the regulated monopolistic industry. We assume that if the monopolist and consumers fail to agree in the bargaining process, then the monopolist is not allowed to operate and consequently both parties end up with zero utilities. Accordingly, we set the disagreement point to $d^R = (0, 0)$, where the superscript R emphasizes that the bargaining payoffs are related to the 'regulatory' mechanism of BM. Notice that as the parameter α is varied on the interval $[0, 1]$, equations (14) and (15) together define a locus of points in $\mathbb{R}^2_+$. Defining $\hat{u}_1(\alpha) \equiv CW^e(\alpha)$ and $\hat{u}_2(\alpha) \equiv PW^e(\alpha)$, we can write this locus as

$$\hat{u}_1(\alpha) = 2\sqrt{V\hat{u}_2(\alpha)} - 2\hat{u}_2(\alpha). \tag{24}$$

The convex and comprehensive hull of the above locus of points defines the bargaining set, S^R, facing the players in the absence of pre-donation:

$$S^R = \left\{ u(\alpha) \;\middle|\; \begin{array}{l} u_1(\alpha) = \dfrac{2(1-\alpha)V}{(2-\alpha)^2}, \\ \\ 0 \le u_2(\alpha) \le \dfrac{V}{(2-\alpha)^2}, \quad \alpha \in [0,1] \end{array} \right\} \tag{25}$$

Notice that $PO(S^R)$ is the locus of points that satisfy (24). The pair (S^R, d^R) is the (regulatory) bargaining problem in the absence of pre-donation. With pre-donation, the problem (S^R, d^R) is transformed into a new problem which we will describe next.

First recall that we denote consumers and the monopolist by the indices 1 and 2, respectively. Thus, $\boldsymbol{\lambda^{k,1}}$ ($\boldsymbol{\lambda^{k,2}}$) denotes the pre-donation from consumers to the monopolist (from the monopolist to consumers), realized at the rate $k \in [0, 1)$. We should observe that given any $k \in [0, 1)$, the pre-donation $\boldsymbol{\lambda^{k,1}}$ transforms the bargaining problem (S^R, d^R) into the problem $(\underline{\boldsymbol{\lambda}}^{\boldsymbol{k,1}}(S), \boldsymbol{\lambda^{k,1}}(d))$ such that $\boldsymbol{\lambda^{k,1}}(d^R) = (0, 0) = d^R$ and

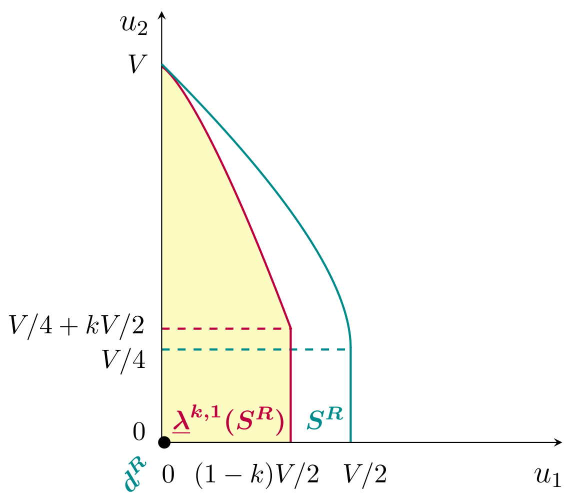
$$\underline{\boldsymbol{\lambda}}^{\boldsymbol{k,1}}(S) = \left\{ u(\alpha) \;\middle|\; \begin{array}{l} u_1(\alpha) = \dfrac{[2(1-k)(1-\alpha)]\,V}{(2-\alpha)^2}, \\ \\ 0 \le u_2(\alpha) \le \dfrac{[1 + 2k(1-\alpha)]\,V}{(2-\alpha)^2}, \quad \alpha \in [0,1] \end{array} \right\}. \tag{26}$$

Likewise, given any $k \in [0, 1)$, the pre-donation $\boldsymbol{\lambda^{k,2}}$ transforms the bargaining problem (S^R, d^R) into the problem $(\underline{\boldsymbol{\lambda}}^{\boldsymbol{k,2}}(S), \boldsymbol{\lambda^{k,2}}(d))$ such that $\boldsymbol{\lambda^{k,2}}(d^R) = (0, 0) = d^R$ and

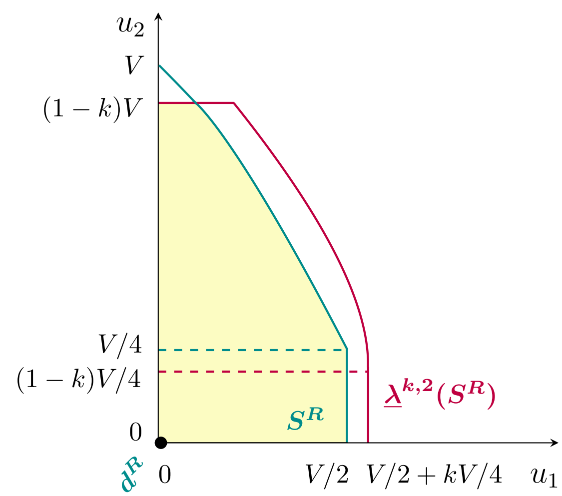
$$\underline{\boldsymbol{\lambda}}^{\boldsymbol{k,2}}(S) = \left\{ u(\alpha) \;\middle|\; \begin{array}{l} 0 \le u_1(\alpha) \le \dfrac{[2(1-\alpha)+k]\,V}{(2-\alpha)^2}, \\ \\ 0 \le u_2(\alpha) \le \dfrac{(1-k)\,V}{(2-\alpha)^2}, \quad \alpha \in [0,1] \end{array} \right\}. \tag{27}$$

In Figure 1, we illustrate the effect of one-sided pre-donation on the bargaining problem. In panel (i), we observe that the original bargaining set S^R shrinks to the modified set $\underline{\boldsymbol{\lambda}}^{\boldsymbol{k,1}}\boldsymbol{(S^R)}$ due to the pre-donation $\boldsymbol{\lambda^{k,1}}$ by consumers. The yellow shaded area illustrates the set of points in $\underline{\boldsymbol{\lambda}}^{\boldsymbol{k,1}}\boldsymbol{(S^R)}$ that are already achievable (feasible) in S^R.

Figure 1. Bargaining Problems Under One-Sided Pre-donation



(i) Player 1 (Consumers) Pre-donates



(ii) Player 2 (Monopolist) Pre-donates

Indeed, this colored area coincides with $\underline{\boldsymbol{\lambda}}^{\boldsymbol{k,1}}(\boldsymbol{S^R})$, suggesting that a pre-donation by consumers cannot bring any Pareto improvement in the players' payoffs. In fact, it would (almost) always reduce their payoffs under any bargaining solution that satisfies the axiom of WPO. Panel (ii) shows that a pre-donation by the monopolist has a different effect than that of consumers. With the pre-donation $\boldsymbol{\lambda}^{\boldsymbol{k,2}}$ made by the monopolist, the original bargaining set S^R shrinks inwards at the top but expands outwards elsewhere, forming the modified set $\underline{\boldsymbol{\lambda}}^{\boldsymbol{k,2}}(\boldsymbol{S^R})$. Now, we observe that the yellow shaded area, which illustrates the set of points in $\underline{\boldsymbol{\lambda}}^{\boldsymbol{k,2}}(\boldsymbol{S^R})$ that are already achievable (feasible) in S^R, is only a proper subset of $\underline{\boldsymbol{\lambda}}^{\boldsymbol{k,2}}(\boldsymbol{S^R})$. That is, the players can obtain a new set of payoffs in the modified bargaining set, i.e. the points between the red and yellow colored frontiers when $u_2 \in [0, (1-k)V]$, which were not available in the original bargaining set, suggesting that a pre-donation by the monopolist may lead to Pareto improvement in the players' payoffs depending upon the bargaining solution. On the other hand, a pre-donation by the monopolist also leads to a distressing effect that a set of payoffs in the original bargaining set, i.e. the points between the red and yellow colored frontiers when $u_2 \in ((1-k)V, V]$, are no longer feasible in the modified bargaining set. This contraction in the bargaining opportunities of players may make them worse off depending upon the bargaining solution, as we will elaborately show in Section 3.

2.4 The Modified BM Mechanism

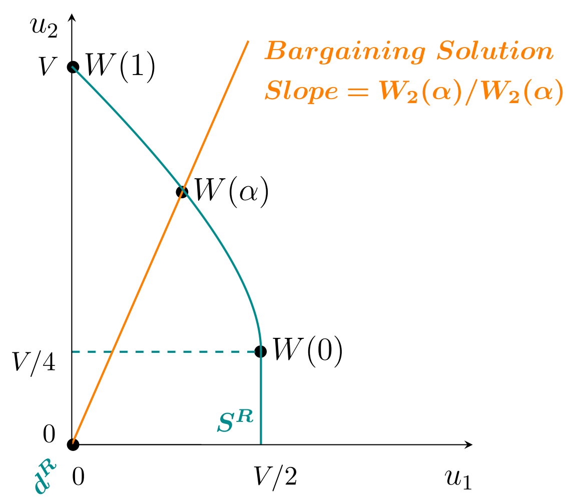
Recall that in the BM mechanism, consumers do not act as decision-makers. The BM mechanism is designed, and publicly declared, by a regulator who is maximizing the expected value of a pre-determined (exogenously given) social welfare function, and the monopolist is expected to act (to reveal its cost and to produce) in accordance with the predictions of this mechanism. We will modify the regulatory mechanism of BM by allowing consumers as well as the monopolist to actively contribute to the design of the mechanism. Briefly, we will let consumers and the monopolist to collectively choose the expected social welfare function that will be maximized by the regulator over the direct-revelation mechanisms proposed by BM. Consumers and the monopolist will solve this choice problem using a cooperative bargaining game under pre-donation (utility transfers from the pre-donating side to the receiver). We will consider this bargaining game separately under two informational situations, namely the *ex-ante* stage and the interim *stage*. The *interim* stage reflects the assumed informational state in the BM model where the monopolist privately knows its marginal cost parameter (which is unknown to consumers and the regulator until the end of the implementation of the regulatory mechanism). What we introduce in this study is an *ex-ante* stage where even the monopolist does not know about its marginal cost parameter, yet. Associated with the two informational stages, our model, and hence our modified regulatory mechanism, will have two variants, in one of which pre-donations occur in the *ex-ante* stage and in the other pre-donations occur in the *interim* stage. However, we will retain one important feature of the BM model assuming that information revelation will always occur in the *interim* stage. We will also assume that the regulator will inform both the monopolist and consumers about the details of the modified regulatory mechanism at the beginning of the stage they are allowed to make pre-donation. Given these assumptions, if consumers should decide whether and how much to pre-donate in the *ex-ante* or *interim* stage, they should always consider the maximization of their *ex-ante* payoffs. In contrast, the monopolist should take into account its *ex-ante* payoff only if it makes pre-donation decision in the *ex-ante* stage. When it is allowed to pre-donate in the *interim* stage, the monopolist should always consider the maximization of its *interim* (equivalently *ex-post*) payoff.

After these observations, we are ready to describe the modified BM mechanism under pre-donation. Below, we will first present it informally to understand, in a non-technical way, how this mechanism will work. We should start by saying that the modified BM mechanism will be equal to the BM mechanism if neither consumers nor the monopolist make any pre-donation. Moreover, the possible outcomes of the BM mechanism will generate a bargaining set that will be used to find the regulatory outcome in case any pre-donation occurs. Recall that the expected welfares (utilities) that consumers and the monopolist obtain from the BM mechanism are $CW^e(\alpha)$ and

$PW^e(\alpha)$, each of which is a function of $\alpha \in [0,1]$ that denotes the relative weight of the monopolist's expected welfare in the social welfare function. For simplicity, we call the vector of these expected utilities by $W(\alpha)$ so that $W_1(\alpha) = CW^e(\alpha)$ and $W_2(\alpha) = PW^e(\alpha)$. As α increases, the utility allocation $W(\alpha)$ changes monotonically: $W_1(\alpha)$ decreases and $W_2(\alpha)$ increases. The set of all possible allocations, $W(\alpha)$, obtained when α is varied in $[0,1]$, forms a strictly concave and downward-sloping curve in $\mathbb{R}^2_+$, as shown in Figure 2. This curve denotes all utility points that can be obtained from the BM mechanism by changing the relative weight parameter α in the social welfare function. For example, the points $W(1) = (0, V)$ and $W(0) = (V/2, V/4)$ correspond to $\alpha = 1$ and $\alpha = 0$, respectively. On the other hand, all utility points in $\mathbb{R}^2_+$ that are weakly dominated by (smaller or equal to) the utility points on the green curve segment between $W(1)$ and $W(0)$ constitute a bargaining set, S^R, for consumers and the monopolist, representing the set of all utility points that are achievable in the BM mechanism by changing the parameter α and/or by freely disposing of some achievable utilities. The strictly concave curve segment between $W(1)$ and $W(0)$ is called the strict Pareto frontier of this bargaining set. The vertical green line below $W(0)$ contains the set of weakly but not strictly Pareto optimal utility points. Given any α chosen by the regulator, the BM mechanism automatically picks the point $W(\alpha)$ on the Pareto frontier of S^R, without using any other information in S^R. Alternatively, if consumers and the monopolist are allowed to choose α or equivalently any $W(\alpha)$ on the Pareto frontier of S^R as in Saglam (2021), they can do this, without appealing to the BM mechanism, by just solving the bargaining problem implied by the bargaining set S^R and the disagreement point $d^R = (0,0)$ under any bargaining rule that the bargaining parties will agree upon. The bargaining set S^R will have another use in our study. We will use this set to construct a modified bargaining set under pre-donation that will enable us to calculate the regulatory outcome again using the bargaining approach introduced by Saglam (2021).

Here, we should notice that even though pre-donation will modify the bargaining set S^R, the welfare parameter α that generates S^R must be kept intact during the bargaining process following pre-donation. In other words, once the BM mechanism or the bargaining players select a point on the strict Pareto frontier of S^R, say the expected utility point $W(\alpha)$ in Figure 2, the α value that induces this expected utility point must also be used to find the expected utility point in the relevant bargaining set when pre-donation occurs. So, given any point $W(\alpha) = (u_1, u_2)$ in the strict Pareto frontier of S^R, we must find a way to calculate the value of the parameter α using u_1 and u_2 and preserve this information to use it later in the bargaining process following pre-donation. We should recall from Section 2 that $W_1(\alpha) = 2(1-\alpha)V/(2-\alpha)^2$ and $W_2(\alpha) = V/(2-\alpha)^2$ with $V = a^2/6$. Then, the ratio of expected utilities in $W(\alpha)$ can be calculated as $W_1(\alpha)/W_2(\alpha) = 2(1-\alpha)$. So, if we pick any utility point u^* on the strict Pareto frontier of S^R (which we know is wholly generated by the BM mechanism), the utility ratio u_1^*/u_2^* must be equal to $2(1-\alpha)$. Thus, a proportional bargaining rule (a ray) passing through the origin $(0,0)$ (representing the disagreement point d^R) crosses the strict Pareto frontier of the bargaining set obtained by the BM mechanism exactly at the regulatory outcome $W(\alpha)$ associated with α if the slope parameter of the bargaining solution is $1/[2(1-\alpha)]$ in the (u_1,u_2) plane. Conversely, we can say that if the BM mechanism chooses an expected utility point u^* in $\mathbb{R}^2_+$, the relevant α value that induces this point can be calculated as $1-(u_1^*/(2u_2^*))$ and to preserve this α value in the regulatory problem allowing pre-donation we need to use in the modified bargaining problem a proportional bargaining rule having the slope u_2^*/u_1^*. Now, we are ready to calculate the regulatory solution under pre-donation.

Figure 2. Bargaining Problem Generated by the BM Mechanism



Recall from Section 2.2 that a *pre-donation* from player 1 (consumers) to player 2 (the monopolist) associated with the amount of pre-donation k is a function $\lambda^{k,1}$ which transforms each utility point $u=(u_1,u_2)$ in the bargaining set S^R to a point u' such that $u_1'=(1-k)u_1$ and $u_2'=u_2+ku_1$, that is player 1 commits to give the fraction k of any utility it can expect to obtain in S^R to player 2 before the bargaining begins. Under this pre-donation, the bargaining set S^R changes to $\lambda^{k,1}(S^R)$ and its comprehensive closure changes to $\underline{\boldsymbol{\lambda}}^{\boldsymbol{k,1}}(S^R)$. We know from Figure 1-(i) that the bargaining set shrinks inwards when consumers pre-donate. Similarly, we know that a *pre-donation* from player 2 (the monopolist) to player 1 (consumers) associated with the amount of pre-donation k is a function $\lambda^{k,2}$ which transforms each utility point $u=(u_1,u_2)$ in the bargaining set S^R to a point u' such that $u_1'=u_1+ku_2$ and $u_2'=(1-k)u_2$. Here, player 2 commits to give the fraction k of any utility it can expect to obtain in S^R to player 1 before the bargaining begins. Under this pre-donation, the bargaining set S^R changes to $\lambda^{k,2}(S^R)$ and its comprehensive closure changes to $\underline{\boldsymbol{\lambda}}^{\boldsymbol{k,2}}(S^R)$. We know from Figure 1-(ii) that when player 2 pre-donates, the bargaining set shrinks inwards at the top but expands outwards elsewhere.

Now, we are ready to find the solution on the modified bargaining set $\underline{\boldsymbol{\lambda}}^{\boldsymbol{k,i}}(S^R)$ facing us when player $i \in \{1, 2\}$ pre-donates a fraction k of its expected utilities in S^R. If the BM mechanism selects a utility point u^* on the Pareto frontier of S^R, it is clear that the modified BM mechanism should select under pre-donation of player i a point in $\underline{\boldsymbol{\lambda}}^{\boldsymbol{k,i}}(S^R)$ that can be found by intersecting the proportional bargaining rule with the slope u_2^*/u_1^* (to respect the underlying α value) with the Pareto frontier of $\underline{\boldsymbol{\lambda}}^{\boldsymbol{k,i}}(S^R)$. If this intersection is a point $\hat{u}$, then we must have $\hat{u}_2/\hat{u}_1 = u_2^*/u_1^*$. Moreover, $\hat{u}$ must be on the 'weak' Pareto frontier of $\underline{\boldsymbol{\lambda}}^{\boldsymbol{k,i}}$ by the nature of a proportional bargaining rule. Notice also that there must exist a point, say s, on S^R such that the solution $\hat{u}$ is obtained by pre-donation from s, i.e., $(1-k)s_i = \hat{u}_i$, since $\hat{u}$ is on the modified bargaining set $\underline{\boldsymbol{\lambda}}^{\boldsymbol{k,i}}$. Then, for each k there must also exist some welfare weight $\tilde{\alpha} \in [0, 1]$ such that the expected welfare vector $W(\tilde{\alpha})$ generated by the BM mechanism at the welfare weight $\tilde{\alpha}$ is the same as the utility point s in the original bargaining set S^R that is known to induce, through pre-donation, the solution $\hat{u}$ in the modified bargaining set.

To briefly summarize, the bargaining players will first choose a value for the welfare weight parameter α. The induced bargaining rule will be a proportional rule, say P^γ, with the slope equal to $\gamma = 1/[2(1-\alpha)]$. Alternatively, the players may choose any proportional rule P^γ and the regulator can calculate the implied α parameter as $\alpha = 1 - (1/2\gamma)$. The regulator and players will then calculate the bargaining problem (S^R, d^R) induced by the BM mechanism. If there is no pre-donation by any player, the regulatory solution can be calculated by applying the bargaining rule P^γ onto the bargaining problem (S^R, d^R). This solution would be equal to $W(\alpha)$ that is generated by the BM mechanism for the welfare weight α. On the other hand, if there is one-sided pre-donation by player $i \in \{1, 2\}$, then this player announces a value for the pre-donation fraction k chosen from the set $[0, 1]$. The regulator then calculates and announces the modified bargaining problem $(\underline{\boldsymbol{\lambda}}^{\boldsymbol{k,i}}, d^R)$ implied by the pre-donation of player i. Applying the bargaining rule P^γ (which was also used in the case of no pre-donation) onto the modified bargaining problem, the regulator and the players calculate the regulatory solution as a point $\hat{u}$ in the Pareto frontier of $\underline{\boldsymbol{\lambda}}^{\boldsymbol{k,i}}$ such that $\hat{u}_2/\hat{u}_1 = \gamma$. This solution point can also obtained from the BM mechanism since the pre-donation of player i implies $(1-k)W_i(\tilde{\alpha}) = \hat{u}_i$ for some $\tilde{\alpha} \in [0, 1]$. Now, we are ready to formally describe the modified BM mechanism.

The Modified BM Mechanism

Step 1: The regulator picks, and announces, from the interval $[0, 1]$ a value, α, to be used for the initial value of the social welfare weight of the monopolist.

Step 2: Given the announced α value, all parties (the regulator, the monopolist, and consumers) calculate the induced expected utility allocation $W(\alpha) = (CW^e(\alpha), PW^e(\alpha))$ implied by the BM mechanism. They also calculate the problem (S^R, d^R) and select a proportional rule P^γ with $\gamma \geq 0$ such that $P^\gamma(S^R, d^R) = W(\alpha)$.

Step 3: The regulator announces the index of the player, say i, which is allowed to make a unilateral pre-donation.

Step 4: The regulator announces a function $\tilde{\alpha} : [0,1]^2 \to [0,1]$ such that if player i were to announce pre-donation parameter, as any $k' \in [0,1)$, the regulator would run the BM mechanism with $\tilde{\alpha}(\alpha, k')$, instead of α, to ensure that $(1-k)W_i(\tilde{\alpha}(\alpha, k')) = P_i^\gamma(\underline{\boldsymbol{\lambda}}^{\boldsymbol{k',i}}(S^R), d^R)$. (Due to the geometries of the bargaining sets S^R and $\underline{\boldsymbol{\lambda}}^{\boldsymbol{k',i}}(S^R)$ and the fact that $W(\alpha) \in PO(S^R)$, we know that $\tilde{\alpha}(\alpha, k')$ exists for all $k' \in [0,1)$.)

Step 5: Given the announced function $\tilde{\alpha}(\alpha, .)$, player i picks, and announces, from the interval $[0,1)$ a value, k, to be used for its pre-donation rate in all relevant calculations.

Step 6: Given the announced k value, all parties calculate the pre-donation function $\boldsymbol{\lambda}^{\boldsymbol{k,i}}$, the social welfare weight $\tilde{\alpha}(\alpha, k)$, the bargaining set $\underline{\boldsymbol{\lambda}}^{\boldsymbol{k,i}}(S^R)$, and the disagreement point $\boldsymbol{\lambda}^{\boldsymbol{k,i}}(d^R) = d^R$. They also calculate the induced bargaining solution $P^\gamma(\underline{\boldsymbol{\lambda}}^{\boldsymbol{k,i}}(S^R), d^R)$. This is the expected utility allocation of the modified BM mechanism and denoted by $\tilde{W}^i(\alpha, k)$.

Recall that for any α chosen by the regulator, the BM mechanism consists of the list of functions $\langle(p^*(.,\alpha), q^*(.,\alpha), r^*(.,\alpha), s^*(.,\alpha)\rangle$. We will denote this mechanism by $\Gamma(\alpha)$. We can then denote the modified BM mechanism we have described above by $\tilde{\Gamma}^i(\alpha, k)$ which is equal to $\Gamma(\tilde{\alpha}(\alpha, k)) \cup \{\boldsymbol{\lambda}^{\boldsymbol{k,i}}, P^\gamma, \tilde{\alpha}\}$. Notice that the BM mechanism, $\Gamma(\alpha)$ generates the welfare allocation $W(\alpha)$ whereas the modified BM mechanism by $\tilde{\Gamma}^i(\alpha, k)$ generates $\tilde{W}^i(\alpha, k)$.

Now we turn to consider the problem of the monopolist in the above bargaining game. Notice that the modified BM mechanism $\tilde{\Gamma}^i(\alpha, k)$ operates through the BM mechanism $\Gamma(\tilde{\alpha}(\alpha, k))$ to extract the private information of the monopolist. Thus, it satisfies ex-post incentive compatibility and individual rationality conditions. Consequently, the monopolist will obtain in the ex-post stage, after the revelation of its private information is realized, the actual profit $\pi(\theta, \tilde{\alpha}(\alpha, k))$. In the *interim* stage, the monopolist can precisely calculate this profit since it completely knows the actual value of θ. In fact, it can calculate the actual gross utility $\pi(\theta, \tilde{\alpha}(\alpha, k))$ it would get under any pre-donation rate $k \in [0, 1)$. When pre-donation occurs, the actual net utility of the monopolist would be

$$\pi^a(\theta, \alpha, k) = \pi(\theta, \tilde{\alpha}(\alpha, k)) - kW_2(\tilde{\alpha}(\alpha, k)). \tag{28}$$

When the monopolist is allowed to make pre-donation in the *interim* stage, it must choose k, in the interval $[0, 1)$, to maximize this actual net utility.

In the *ex-ante* stage, the objective of the monopolist is inevitably different. Since the monopolist does not (yet) know in this stage what the actual value of θ is, it cannot calculate its actual net utility resulting from any pre-donation. We assume that in the *ex-ante* stage the monopolist has the same (incomplete) information about θ as do the regulator and consumers. Thus, it shares their beliefs $f(.)$ about the distribution of θ. Because the monopolist can calculate $\pi^a(\theta, \alpha, k)$ for all possible values of $\theta \in (0, a]$ and $k \in [0, 1)$, it can calculate its expected value under the beliefs $f(.)$. Notice that the expected value of $\pi(\theta, \tilde{\alpha}(\alpha, k))$ is just equal to $W_2(\tilde{\alpha})$. Accordingly, the expected net utility of the monopolist from the bargaining game becomes

$$E[\pi^a(\theta, \alpha, k)|f] = (1 - k)W_2(\tilde{\alpha}(\alpha, k)), \tag{29}$$

for any $k \in [0, 1)$. So, if the monopolist is allowed to make pre-donation only in the *ex-ante* stage, it should maximize the above expected net utility over possible values of k in $[0, 1)$.

On the other hand, consumers who can learn about θ only after the cost-revelation occurs in the *interim* stage, always consider the maximization of their expected utility whenever they are allowed to pre-donate in the *ex-ante* or *interim* stage. This expected utility simply becomes $(1 - k)W_1(\tilde{\alpha}(\alpha, k))$ if consumers choose the pre-donation rate as $k \in [0, 1)$.

3 Results

In this section, our goal is to explore whether there exist any $\alpha \in [0, 1]$ and $i \in \{1, 2\}$ such that the modified BM mechanism $\tilde{\Gamma}^i(\alpha, k)$ can Pareto dominate, in the *ex-ante* or *interim* stage, the BM mechanism $\Gamma(\alpha)$. To achieve this goal, we will first restrict our attention to the bargaining problems with and without pre-donation and explore the effect of pre-donation by consumers or the monopolist on the solutions implied by some bargaining rules that are relevant for our purpose.

Notice that the regulatory outcome that is determined by the BM mechanism is always *ex-ante* Pareto optimal. Since the bargaining solution in the absence of pre-donation must be equivalent to the expected utility allocation generated by the BM mechanism, we will restrict our attention to bargaining rules that respect Weak Pareto Optimality and has the potential to select a Pareto Optimal solution for the problem

(S^R, d^R). On this account, we can restrict ourselves to the class of proportional rules with no loss of generality. To see why that is so, consider any problem $(S, d) \in \Sigma^2$ and any bargaining rule F on Σ^2 that satisfies Weak Pareto Optimality. Define $\gamma \equiv F_1(S, d)/F_2(S, d)$. By the definition of the rule P^γ, we have $P^\gamma(S, d) \in WPO(S)$. Also, $P_1^\gamma(S, d)/P_2^\gamma(S, d) = F_1(S, d)/F_2(S, d)$. Moreover, $F(S, d) \in WPO(S)$ since F satisfies Weak Pareto Optimality. Therefore, we must have $P^\gamma(S, d) = F(S, d)$. So, in order to study the implications of bargaining rules that satisfy Weak Pareto Optimality in any fixed bargaining problem, it is sufficient to consider only the set of proportional bargaining rules.

Before moving to our results, we will borrow, as a preliminary, two helpful results from Saglam (2021).

Proposition 1. (Saglam, 2021) *Given the bargaining problem (S^R, d^R), the bargaining rule P^γ yields the utilities*

$$u_1 = \begin{cases} \dfrac{4\gamma}{(2+\gamma)^2} V & \text{if } \gamma \in (0, 2] \\ \dfrac{1}{2} V & \text{if } \gamma > 2 \end{cases} \quad \text{and} \quad u_2 = \begin{cases} \dfrac{4}{(2+\gamma)^2} V & \text{if } \gamma \in (0, 2] \\ \dfrac{1}{2\gamma} V & \text{if } \gamma > 2. \end{cases} \tag{30}$$

Proof. See the proof of Proposition 10 in Saglam (2021). ■

The above result leads to the following simple corollary.

Corollary 1. (Saglam, 2021) *Given the bargaining problem (S^R, d^R), the bargaining rule P^γ and the BM mechanism lead to the same utility allocation if and only if $\gamma = 2(1 - \alpha)$.*

The proof of the above corollary, which was stated as Corollary 7 in Saglam (2021), rests on the observation that the utility ratio u_1/u_2 is equal to γ under the bargaining rule P^γ while it is equal to $2(1 - \alpha)$ under the BM mechanism, as can be observed from equations (14) and (15).

Now we turn to consider the problem of pre-donation. We will consider this problem in the *ex-ante* and *interim* stages separately.

3.1 Pre-donation in the *Ex-Ante* Stage

Here, we assume that pre-donation by the monopolist or consumers occurs in the *ex-ante* stage, where (even) the monopolist does not know the actual value of its cost parameter. The monopolist shares, like consumers, the regulator's prior beliefs about its cost parameter. At the beginning of the *ex-ante* stage, the monopolist and consumers are both informed by the regulator about the details of the modified BM regulatory mechanism that allows pre-donation. Under this setting, if consumers or the monopolist should decide whether and how much to pre-donate in the *ex-ante* stage, they are able to do so by maximizing their *ex-ante* (expected) payoffs guaranteed by the modified BM regulatory mechanism. As we have represented the potential payoffs of the original and modified BM regulatory mechanisms as the Pareto optimal points in the bargaining sets corresponding to these mechanisms, we will find the optimal pre-donation of consumers and the monopolist in the *ex-ante* stage by calculating the effect of any amount of pre-donation they would make on the bargaining set corresponding to the original BM regulatory mechanism.

The following lemma shows that a positive amount of pre-donation from consumers to the monopolist, $\boldsymbol{\lambda^{k,1}}$ with $k \in (0,1]$, contracts the bargaining set S^R so that $WPO(\underline{\boldsymbol{\lambda}}^{\boldsymbol{k,1}}(S^R))$ is always below $WPO(S^R)$ except for the point $(V,0)$ where the two frontiers intersect.

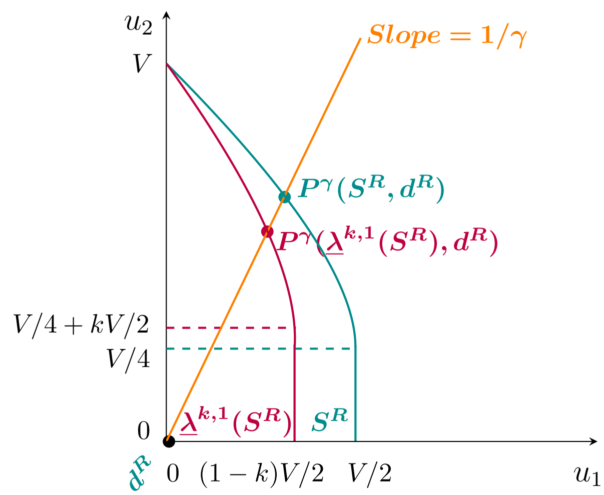
Lemma 1. *For any $k \in (0,1]$ and $u \in WPO(S^R)$ it is true that $\boldsymbol{\lambda^{k,1}}(u) = u$ if $u_1 = 0$ and $\boldsymbol{\lambda^{k,1}}(u) \in S^R \setminus WPO(S^R)$ if $u_1 > 0$.*

Proof. Pick any $k \in (0,1)$ and $u \in WPO(S^R)$. If $u_1 = 0$ then $\boldsymbol{\lambda^{k,1}}(u) = u$, by (26). So, let $u_1 > 0$. Again by (26), we know that $\boldsymbol{\lambda^{k,1}}(u)$ is on the (half-open) line segment $[b, u[$ where $b \equiv (0, u_1 + u_2)$. Consider the line segment $[d, u[$ where $d \equiv (0, V)$. In order to prove $\boldsymbol{\lambda^{k,1}}(u) \in S^R \setminus WPO(S^R)$, it is sufficient to show that b is below d implying $u_1 + u_2 < V$. By (25), there exists a unique $\alpha \in [0,1]$ such that $u_1 = 2(1-\alpha)V/(2-\alpha)^2$. Since $u_1 > 0$, we know that $\alpha \neq 1$; thus $u_2 < V$. Moreover, $u_2 = V/(2-\alpha)^2$ if $u \in PO(S^R)$, and $u_2 \in [0, V/(2-\alpha)^2)$ if $u \in WPO(S^R) \setminus PO(S^R)$ (which occurs when $\alpha = 0$). Therefore, for any $u \in WPO(S^R)$ with $u_1 > 0$, we have $u_1 + u_2 \leq (3-2\alpha)V/(2-\alpha)^2$, and we know that the right-hand-side of this inequality is less than V for all $\alpha \in [0,1)$. ■

Lemma 2. *Given any bargaining rule P^γ with $\gamma > 0$, the pre-donation from consumers to the monopolist via $\boldsymbol{\lambda^{k,1}}$ with any $k \in (0,1]$ is harmful in terms of expected utilities for both consumers and the monopolist.*

Proof. Pick any $\gamma > 0$ and consider the bargaining rule P^γ. Let $u = P^\gamma(S^R, d^R)$ and $u' = P^\gamma(\underline{\boldsymbol{\lambda}}^{\boldsymbol{k,1}}(S^R), d^R)$. By (21), u and u' are on the line connecting d^R and the point $(\gamma, 1)$, and also $u \in WPO(S^R)$ and $u' \in WPO(\underline{\boldsymbol{\lambda}}^{\boldsymbol{k,1}}(S^R))$. By Lemma 1, the set $WPO(\underline{\boldsymbol{\lambda}}^{\boldsymbol{k,1}}(S^R))$ is always below $WPO(S^R)$ except for the point $(V,0)$ where the two sets intersect. Moreover, $\{u, u'\} \cap \{(V,0)\} = \emptyset$ since $\gamma > 0$. Therefore, we must have $u'_1 < u_1$ and $u'_2 < u_2$. ■

Figure 3. Bargaining under the Rule P^{γ} and the Pre-donation $\boldsymbol{\lambda}^{k,1}$



In Figure 3, we illustrate the welfare effect predicted by Lemma 2. This result implies that if consumers make pre-donation the modified BM mechanism becomes always inferior, in terms of expected utilities, to the BM mechanism for both consumers and the monopolist.

Proposition 2. *For any $\alpha \in [0,1]$ and $k \in (0,1]$, the modified BM mechanism $\tilde{\Gamma}^1(\alpha, k)$ under the pre-donation $\boldsymbol{\lambda}^{k,1}$ is always* ex-ante *Pareto inferior to the BM mechanism $\Gamma(\alpha)$.*

Proof. Pick any $\alpha \in [0,1]$ and $k \in (0,1]$. Let $\gamma = 2(1-\alpha)$. By Corollary 1, the BM mechanism yields the expected utility allocation $W(\alpha) = P^{\gamma}(S^R, d^R)$. We also know that the modified BM mechanism $\tilde{\Gamma}^1(\alpha, k)$ yields the expected utility allocation $W^1(\alpha, k) = P^{\gamma}(\underline{\boldsymbol{\lambda}}^{k,1}(S^R), d^R)$. Moreover, Lemma 2 implies that $P_i^{\gamma}(\underline{\boldsymbol{\lambda}}^{k,1}(S^R), d^R) < P_i^{\gamma}(S^R, d^R)$ for each $i = 1, 2$. So, $\tilde{\Gamma}^1(\alpha, k)$ is *ex-ante* Pareto inferior to $\Gamma(\alpha)$. ■

Since the modified BM mechanism $\tilde{\Gamma}^1(\alpha, k)$ and the BM mechanism $\Gamma(\alpha)$ coincide only if $k = 0$, Proposition 2 implies that consumers would choose not to pre-donate under the modified BM mechanism. It also implies that any improvement by the modified mechanism $\tilde{\Gamma}^i(\alpha, k)$ should not be expected unless $i = 2$, i.e., the pre-donating party in the bargaining process is the monopolist. The following lemma shows that the pre-donation from the monopolist to consumers, $\boldsymbol{\lambda}^{k,2}$ for any $k \in (0,1]$, twists the bargaining set S^R around a point u in $WPO(S^R)$ with $u_2 = V(1-k)$.

Lemma 3. *For any $k \in (0,1]$ it is true that* *(i) if $u \in S^R$ is such that $u_2 > (1-k)V$, then $u \notin \underline{\boldsymbol{\lambda}}^{\boldsymbol{k,2}}(S^R)$,*

(ii) if $u \in S^R$ is such that $u_2 \leq (1-k)V$, then there exists $u' \in \underline{\boldsymbol{\lambda}}^{\boldsymbol{k,2}}(S^R)$ such that $u'_1 > u_1$ and $u'_2 = u_2$.

Proof. Consider any $k \in (0, 1]$.

(i). Pick any $u \in \underline{\boldsymbol{\lambda}}^{\boldsymbol{k,2}}(S^R)$. By (26), there exists $u' \in S^R$ such that $u = \boldsymbol{\lambda}^{\boldsymbol{k,2}}(u')$. Notice that $u'_2 \leq V$, implying $(1-k)u'_2 \leq (1-k)V$. Since $u_2 = (1-k)u'_2$, we have $u_2 \leq (1-k)V$, completing the proof of part (i).

(ii). Now pick any $u \in S^R$ such that $u_2 \leq (1-k)V$. First assume that $u \notin WPO(S^R)$. Pick any $u' \in S^R$ such that $u'_1 > u_1$ and $u'_2 = u_2$. Let $u''_2 = u'_1/(1-k)$ and $u''_1 = u'_1 - ku''_2$. Clearly, $u'' \in S^R$ and $u' = \boldsymbol{\lambda}^{\boldsymbol{k,2}}(u'')$. Thus, $u' \in \underline{\boldsymbol{\lambda}}^{\boldsymbol{k,2}}(S^R)$. Now, assume that $u \in WPO(S^R) \setminus PO(S^R)$. By (26), $u_1 = V/2$ and $u_2 \in [0, V/4)$. Let $\hat{u} \in S^R$ be such that $\hat{u}_1 = u_1$ and $\hat{u}_2 = u_2/(1-k)$. Also, let $u' \in \mathbb{R}^2_+$ be such that $u'_1 = \hat{u}_1 + k\hat{u}_2$ and $u'_2 = (1-k)\hat{u}_2$. Notice that $u' = \boldsymbol{\lambda}^{\boldsymbol{k,2}}(\hat{u})$, hence $u' \in \underline{\boldsymbol{\lambda}}^{\boldsymbol{k,2}}(S^R)$. Also, $u'_1 > u_1$ and $u'_2 = u_2$. Finally, assume that $u \in PO(S^R)$. Recall that $u_2 \leq (1-k)V$ by assumption. To prove that there exists $u' \in \underline{\boldsymbol{\lambda}}^{\boldsymbol{k,2}}(S^R)$ such that $u'_1 > u_1$ and $u'_2 = u_2$, it is sufficient to show that for any $x \in PO(S^R)$ the line segment $(x, \boldsymbol{\lambda}^{\boldsymbol{k,2}}(x)]$ is outside S^R. This can be true if the slope of $[x, \boldsymbol{\lambda}^{\boldsymbol{k,2}}(x)]$ (in absolute value), which is 1, is smaller than the slope of $PO(S^R)$ (in absolute value) at any $y \in Y$ where Y is a subset of $PO(S^R)$ satisfying $\max\{y_2 \,|\, y \in Y\} = x_2$ and $\min\{y_2 \,|\, y \in Y\} = \boldsymbol{\lambda}^{\boldsymbol{k,2}}(x)$. For any $y \in PO(S^R)$, we know by (24) and (25) that $y_1 = 2\sqrt{Vy_2} - 2y_2$. Thus, we have $|dy_2/dy_1| = |\sqrt{V/y_2} - 2|^{-1}$. We also know that $y_2 \in [V/4, V]$. Therefore, $|dy_2/dy_1| \in (1, \infty)$ for any $y \in Y$, implying that the line segment $(x, \boldsymbol{\lambda}^{\boldsymbol{k,2}}(x)]$ is outside S^R, which completes the proof. ■

Lemma 4. *Given any bargaining rule P^γ with $\gamma > 0$, the pre-donation from the monopolist to consumers via $\boldsymbol{\lambda}^{\boldsymbol{k,2}}$ is*

(i) ex-ante beneficial for the monopolist and consumers if $k < \bar{k}(\gamma)$,

(ii) ex-ante harmful for the monopolist and consumers if $k > \bar{k}(\gamma)$,

(iii) ex-ante ineffective for the monopolist and consumers if $k = \bar{k}(\gamma)$,

where

$$\bar{k}(\gamma) = \begin{cases} 1 - \dfrac{4}{(2+\gamma)^2} & \text{if } \gamma \in (0, 2] \\[2ex] 1 - \dfrac{1}{2\gamma} & \text{if } \gamma > 2. \end{cases} \tag{31}$$

Proof. Pick any $\gamma > 0$ and consider the bargaining rule P^γ. Let $u(\gamma) \equiv P^\gamma(S^R)$ and $u'(\gamma) \equiv P^\gamma(\underline{\boldsymbol{\lambda}}^{\boldsymbol{k,2}}(S^R))$. If $\gamma \in (0, 2]$, then equations (21), (24), and (25) would imply that $u_1(\gamma) = 2(1-\alpha^*)/(2-\alpha^*)^2$ and $u_2(\gamma) = 1/(2-\alpha^*)^2$ for some α^* such that $u_1(\gamma)/u_2(\gamma) = \gamma = 2(1-\alpha^*)$. It follows that $\alpha^* = 1-\gamma/2$, implying $u_2(\gamma) = 4V/(2+\gamma)^2$. On the other hand, if $\gamma > 2$, then $u_2(\gamma) = V/(2\gamma)$. Now, define $\bar{k}(\gamma) \equiv 1 - u_2(\gamma)/V$ for each $\gamma > 0$. Notice that $u(\gamma)$ and $u'(\gamma)$ are on the same line passing through the points $d^R = (0, 0)$ and $(\gamma, 1)$ and they are the maximal points of $WPO(S^R)$ and $WPO(\underline{\boldsymbol{\lambda}}^{\boldsymbol{k,2}}(S^R))$ on this line. Also, since this line is positively sloped, any pre-donation by the monopolist makes both of the bargaining parties better off if it makes any of them better off. Thus, we observe from (27) and Lemma 3 that $\boldsymbol{\lambda}^{\boldsymbol{k,2}}$ is (i) *ex-ante* beneficial for all parties, i.e., $u_i'(\gamma) > u_i(\gamma)$ for each $i = 1, 2$, if $u_2(\gamma) < (1-k)V$ or $k < \bar{k}(\gamma)$, (ii) *ex-ante* harmful for all parties, i.e., $u_i'(\gamma) < u_i(\gamma)$ for each $i = 1, 2$, if $u_2(\gamma) > (1-k)V$ or $k > \bar{k}(\gamma)$, and (iii) *ex-ante* ineffective for all parties, i.e., $u_i'(\gamma) = u_i(\gamma)$ for each $i = 1, 2$, if $u_2(\gamma) = (1-k)V$ or $k = \bar{k}(\gamma)$. ■

The welfare effect in Lemma 4 is illustrated in Figure 4. Recall that when $\alpha = 1$, the BM mechanism produces the utility allocation $W(1) = (0, V)$, under which player 2 (the monopolist) has no incentive to pre-donate. On the other hand, when $\alpha < 1$, there is always an expected deadweight loss generated by the BM mechanism, as calculated in equation (17). Below, we will explore whether this loss can be reduced by the modification of the BM mechanism under the monopolist's pre-donation, even when it is not optimal.

Proposition 3. *For any $\alpha \in [0, 1]$ and $k \in (0, 1]$, the modified BM mechanism $\tilde{\Gamma}^2(\alpha, k)$ under the pre-donation $\boldsymbol{\lambda}^{\boldsymbol{k,2}}$ is*

(i) ex-ante Pareto superior to the BM mechanism $\Gamma(\alpha)$ if $k < \bar{k}(\alpha)$,

(ii) ex-ante Pareto inferior to the BM mechanism $\Gamma(\alpha)$ if $k > \bar{k}(\alpha)$,

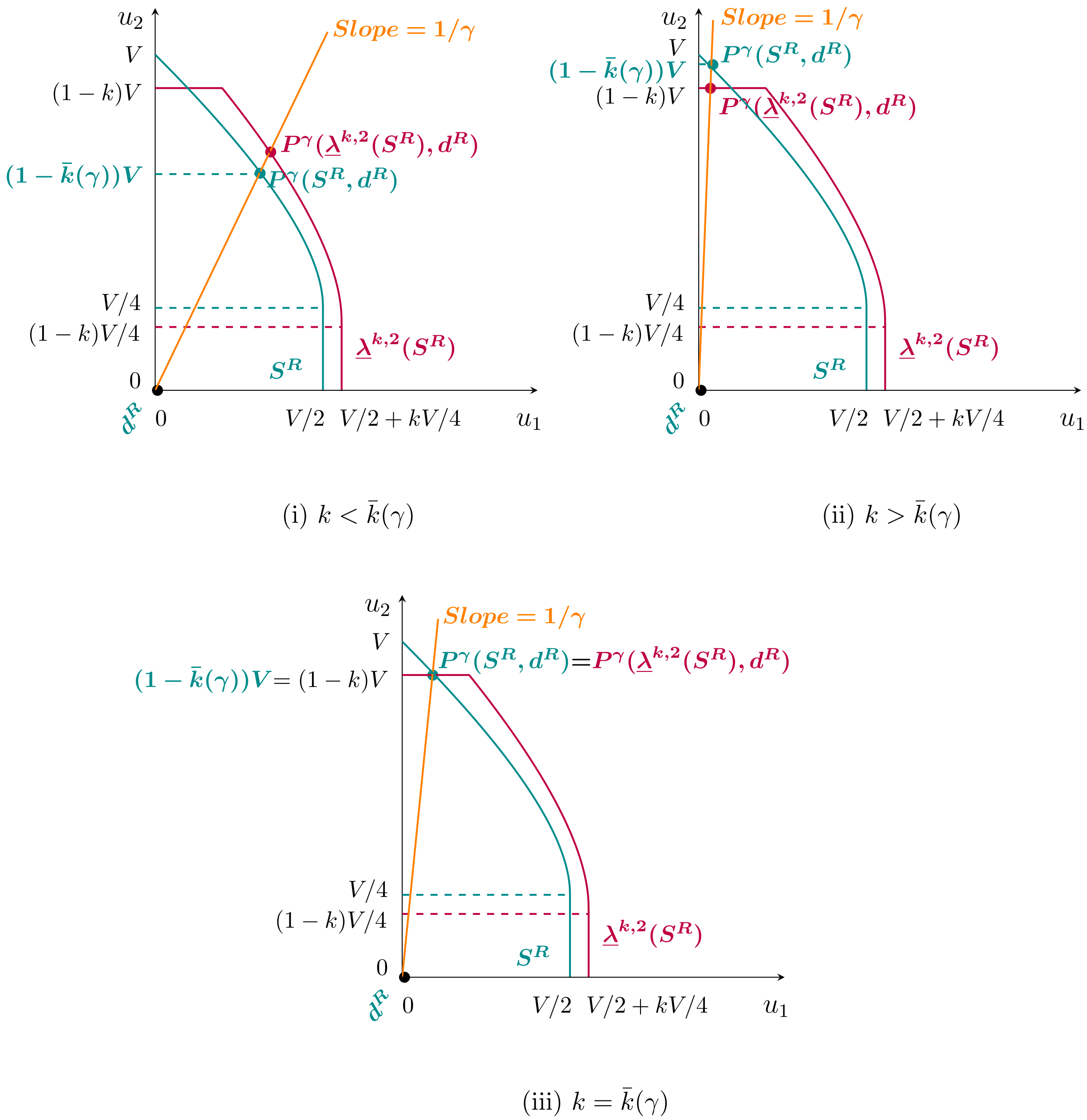
(iii) ex-ante Pareto equivalent to the BM mechanism $\Gamma(\alpha)$ if $k = \bar{k}(\alpha)$,

where $\bar{k}(\alpha) = 1 - 1/(2-\alpha)^2$.

Proof. Pick any $\alpha \in [0, 1]$ and $k \in (0, 1]$. Let $\gamma = 2(1-\alpha)$. Notice that $\gamma \in [0, 2]$. By Corollary 1, $W(\alpha) = P^\gamma(S^R, d^R)$. We also know that the modified BM mechanism $\tilde{\Gamma}^1(\alpha, k)$ yields the expected utility allocation $W^2(\alpha, k) = P^\gamma(\underline{\boldsymbol{\lambda}}^{\boldsymbol{k,2}}(S^R), d^R)$. Notice that the threshold in equation (31) reduces to $\bar{k}(\gamma) = 1 - 4/(2+\gamma)^2$ since $\gamma \in [0, 2]$. Notice also that the equality $\gamma = 2(1-\alpha)$ implies that $k \geq \bar{k}(\gamma)$ if and only if $k \geq \bar{k}(\alpha) = 1 - 1/(2-\alpha)^2$. Thus, Lemma 4 implies that (i) $P_i^\gamma(\underline{\boldsymbol{\lambda}}^{\boldsymbol{k,2}}(S^R), d^R) > P_i^\gamma(S^R), d^R)$ for each $i = 1, 2$ and $\tilde{\Gamma}^2(\alpha, k)$ is *ex-ante* Pareto superior to $\Gamma(\alpha)$ if $k < \bar{k}(\alpha)$, (ii) $P_i^\gamma(\underline{\boldsymbol{\lambda}}^{\boldsymbol{k,2}}(S^R), d^R) < P_i^\gamma(S^R), d^R)$ for each $i = 1, 2$ and $\tilde{\Gamma}^2(\alpha, k)$ is *ex-ante* Pareto inferior to $\Gamma(\alpha)$ if $k > \bar{k}(\alpha)$, (iii) $P_i^\gamma(\underline{\boldsymbol{\lambda}}^{\boldsymbol{k,2}}(S^R), d^R) = P_i^\gamma(S^R), d^R)$ for each $i = 1, 2$ and $\tilde{\Gamma}^2(\alpha, k)$

is *ex-ante* Pareto equivalent to $\Gamma(\alpha)$ if $k > \bar{k}(\alpha)$. ■

Figure 4. Bargaining under the Rule P^{γ} with $\gamma \leq 2$ and the Pre-donation $\lambda^{k,2}$



(i) $k < \bar{k}(\gamma)$

(ii) $k > \bar{k}(\gamma)$

(iii) $k = \bar{k}(\gamma)$

Notice that the modified BM mechanism $\tilde{\Gamma}^2(\alpha, k)$ becomes *ex-ante* inferior to the BM mechanism for the monopolist (and consumers) when the pre-donation fraction k exceeds the threshold $\bar{k}(\alpha)$ for any given $\alpha \in [0, 1]$, Thus, we should never expect an equilibrium where the monopolist pre-donates more than the amount implied by this threshold. Given this observation, our next question is to find the optimal pre-donation by the monopolist in the *ex-ante* stage where it has not learned the actual value of θ, yet. Here, we assume that the monopolist will always prefer no pre-donation to an ineffective pre-donation, which arises if $k = \bar{k}(\alpha)$, because of the actual implementation costs of pre-donation we have assumed to be zero for simplicity. This leaves us with part (i) of Proposition 3 suggesting that the monopolist should restrict itself to pre-donation functions whose parameter, k, lies in the set $[0, 1 - 1/(2-\alpha)^2)$. In this set, the monopolist should choose the value of k to maximize its bargaining utility $P_2^\gamma(\underline{\boldsymbol{\lambda}}^{\boldsymbol{k},\mathbf{2}}(S^R), d^R)$, which is equal to $(1-k)W_2(\tilde{\alpha}(\alpha, k))$ by equation (29).

Lemma 5. *Given any bargaining rule P^γ with $\gamma > 0$, the ex-ante optimal pre-donation from the monopolist to consumers is a function $\boldsymbol{\lambda}^{\boldsymbol{k^*},\mathbf{2}}$ where $k^* = \gamma/(1+\gamma)$. This yields to consumers and the monopolist the bargaining utilities $P_1^\gamma(\underline{\boldsymbol{\lambda}}^{\boldsymbol{k^*},\mathbf{2}}(S^R), d^R) = \gamma V/(1+\gamma)$ and $P_2^\gamma(\underline{\boldsymbol{\lambda}}^{\boldsymbol{k^*},\mathbf{2}}(S^R), d^R) = V/(1+\gamma)$, respectively.*

Proof. Pick any $\gamma > 0$ and consider the rule P^γ. Let $u(\gamma, k) \equiv P^\gamma(\underline{\boldsymbol{\lambda}}^{\boldsymbol{k},\mathbf{2}}(S^R), d^R)$ for any $k \in [0, 1)$. The problem of the monopolist is to find the value of k that maximizes $u_2(\gamma, k)$. Notice that $u(\gamma, k)$ is the point of intersection between the set $WPO(\underline{\boldsymbol{\lambda}}^{\boldsymbol{k},\mathbf{2}}(S^R))$ and the line passing through the points d^R and $(\gamma, 1)$. Notice also that the line segment $[(0, V), (V, 0)]$ is the upper envelope of the sets $WPO(\underline{\boldsymbol{\lambda}}^{\boldsymbol{k},\mathbf{2}}(S^R))$ obtained when k is varied over $[0, 1]$. Therefore, $k = k^*$ maximizes $u_2(\gamma, k)$ only if $u(\gamma, k^*)$ is on the line segment $[(0, V), (V, 0)]$ or equivalently $u_1(\gamma, k^*) + u_2(\gamma, k^*) = V$. Then, using the fact that $u_1(\gamma, k^*)/u_2(\gamma, k^*) = \gamma$ by the definition of P^γ, we obtain $u_2(\gamma, k^*) = V/(1+\gamma)$ and $u_1(\gamma, k^*) = \gamma V/(1+\gamma)$, implying $k^* = \gamma/(1+\gamma)$. One can easily check that $k^* < \bar{k}(\gamma)$ for all $\gamma > 0$. Thus, the pre-donation implied by k^* is optimal for the monopolist. ■

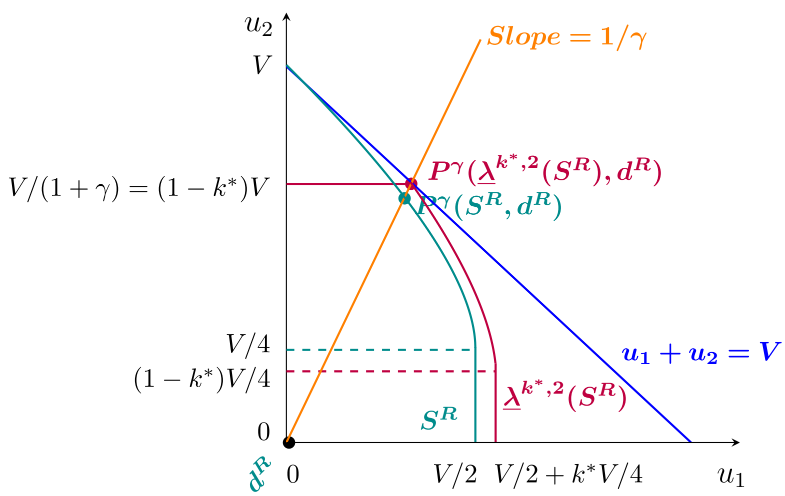
Figure 5 portrays how to find the optimal pre-donation k^* by the monopolist. Below, we calculate the value of k^* under some well-known bargaining rules to give an insight about how widely it can vary in its range when γ changes.

Remark 1. *Given the bargaining rule P^γ, the ex-ante optimal pre-donation from the monopolist to consumers via $\boldsymbol{\lambda}^{\boldsymbol{k^*},\mathbf{2}}$ implies (i) $k^* = 2/3$ if P^γ is outcome-equivalent to the Dictatorial-1 rule, (ii) $k^* = 1/2$ if P^γ is outcome-equivalent to the Egalitarian rule, (iii) $k^* = 2/5$ if P^γ is outcome-equivalent to the Nash rule, (iv) $k^* = 1/3$ if P^γ is outcome-equivalent to the Kalai-Smorodinsky rule, and (v) $k^* = 0$ if P^γ is outcome-equivalent to the Dictatorial-2 rule.*

Proof. We know that the rule P^γ is outcome-equivalent to the Egalitarian rule only if

$\gamma = 1$. Also, we know from Corollary 6 of Saglam (2021) that P^γ is outcome-equivalent to the Nash rule only if $\gamma = 2/3$ and to the Kalai-Smorodinsky rule only if $\gamma = 1/2$. Moreover, for $\gamma = 2$ and $\gamma = 0$, P^γ becomes outcome-equivalent to the Dictatorial-1 and Dictatorial-2 rules, respectively. Inserting each of these five values of γ into the equation $k^* = \gamma/(1+\gamma)$ yields the desired result. ■

Figure 5. Bargaining under the Rule P^γ and the Optimal Pre-donation $\boldsymbol{\lambda}^{k^*,2}$



Recall that for any $\alpha \in [0,1]$, the expected utility allocation produced by the BM mechanism is always Pareto optimal, i.e., $W(\alpha) \in PO(S^R)$. Moreover, Corollary 1 reveals that $W(\alpha) = P^\gamma(S^R, d^R)$ if and only if $\gamma = 2(1-\alpha)$. This implies that any proportional bargaining rule P^γ selects its solution from $PO(S^R)$ if and only if $\gamma \in [0,2]$. It follows that for any proportional rule that selects its solution from $PO(S^R)$ the optimal pre-donation by the monopolist must fall in the interval $[0/(1+0), 2/(1+2)] = [0, 2/3]$. Remark 1 above shows that the lower and upper bounds of this interval are induced by the Dictatorial-2 and Dictatorial-1 rules respectively, while the Egalitarian, Nash, and Kalai-Smorodisnky rules are compatible with k^* values in the interior of the same interval.

Proposition 4. *For any $\alpha \in [0,1]$, the modified BM mechanism $\tilde{\Gamma}^1(\alpha, k^*)$ under the ex-ante optimal pre-donation $\boldsymbol{\lambda}^{k^*,2}$ yields the expected utility allocation $W^2(\alpha, k^*)$ where $W_1^2(\alpha, k^*) = 2(1-\alpha)V/(3-2\alpha)$ and $W_2^2(\alpha, k^*) = V/(3-2\alpha)$.*

Proof. Pick any $\alpha \in [0,1]$. Let $\gamma = 2(1-\alpha)$. We know that the modified BM mechanism $\tilde{\Gamma}^1(\alpha, k^*)$ yields the expected utility allocation $W^2(\alpha, k^*) = P^\gamma(\underline{\boldsymbol{\lambda}}^{k^*,2}(S^R), d^R)$.

Also, by Lemma 5, $P_1^\gamma(\underline{\boldsymbol{\lambda}}^{\boldsymbol{k^*,2}}(S^R), d^R) = \gamma V/(1+\gamma)$ and $P_2^\gamma(\underline{\boldsymbol{\lambda}}^{\boldsymbol{k^*,2}}(S^R), d^R) = V/(1+\gamma)$. Replacing γ in the last two equations with $2(1-\alpha)$, we obtain the desired result. ■

The optimal pre-donation of the monopolist induced by k^* always equates the ratio between the utilities of consumers and the monopolist to the slope γ under any rule P^γ as can be seen from Lemma 5. Since the BM mechanism $\Gamma(\alpha)$ induced by any $\alpha \in [0,1]$ is outcome-equivalent to a bargaining rule P^γ only if $\gamma = 2(1-\alpha)$, we observe that the modified BM mechanism $\tilde{\Gamma}^2(\alpha, k^*)$ induced by α and k^* equates the ratio between the utilities of consumers and the monopolist to $2(1-\alpha)$ as does the BM mechanism. This implies that the same ratio must exist between the utility gains of the two parties generated by the modified BM mechanism with respect to the status quo. The following result shows that these utility gains are always decreasing in α.

Corollary 2. *For any $\alpha \in [0,1]$, the expected utility gain of the modified BM mechanism $\tilde{\Gamma}^2(\alpha, k^*)$ over the BM mechanism $\Gamma(\alpha)$ is equal to $\Delta\tilde{\Gamma}^2(\alpha)$ such that*

$$\Delta\tilde{\Gamma}_1^2(\alpha) = \frac{8(1-\alpha)^3}{(3-2\alpha)(4-2\alpha)^2}V$$

and

$$\Delta\tilde{\Gamma}_2^2(\alpha) = \frac{4(1-\alpha)^2}{(3-2\alpha)(4-2\alpha)^2}V.$$

Moreover, for each $i \in \{1,2\}$ the gain $\Delta\tilde{\Gamma}_i^2(\alpha)$ is always decreasing in α.

Proof. The equations for the utility gains follow from equations (15), (14), and Proposition 4. Differentiating them with respect to α we can straightforwardly reach the desired result. ■

The result that the utility gains implied by the modified BM mechanism under the optimal pre-donation of the monopolist are positive as long as $\alpha < 1$ is not surprising since by Proposition 3-(i) we already know that given any $k \in [0,1)$ and a pre-donation $\boldsymbol{\lambda}^{\boldsymbol{k,2}}$, the induced modified BM mechanism is Pareto superior to the BM mechanism if k is below the threshold $\bar{k}(\alpha)$ while by Lemma 5 and Proposition 4 we know that the optimal pre-donation by the monopolist satisfies this threshold condition for all $\alpha \in (0,1]$. The result that the aforementioned utility gains are decreasing in α is not surprising either, since the expected deadweight loss in the BM mechanism is decreasing in α, as we can recall from (17). Thus, the social benefit of the modified mechanism, though it is always positive for any α less than 1, becomes lower and lower, and eventually totally diminished, as α approaches 1 from below.

Table 1. The Utilities Generated by Various Bargaining Rules under the *Ex-Ante* Optimal Pre-donation $\boldsymbol{\lambda}^{k^*,2}$

Bargaining Rule	α	$\gamma = 2(1-\alpha)$	$k^* = \frac{2(1-\alpha)}{3-2\alpha}$	$P^\gamma(S^R, d^R) = \left(\frac{2(1-\alpha)V}{(2-\alpha)^2}, \frac{V}{(2-\alpha)^2}\right)$	$P^\gamma\left(\underline{\boldsymbol{\lambda}}^{k^*,2}(S^R), \boldsymbol{\lambda}^{k^*,2}(d^R)\right) = \left(\frac{2(1-\alpha)V}{3-2\alpha}, \frac{V}{3-2\alpha}\right)$
Dictatorial-1	0	2	20/30	$\left(\frac{1800V}{3600}, \frac{900V}{3600}\right)$	$\left(\frac{2400V}{3600}, \frac{1200V}{3600}\right)$
Egalitarian	6/12	1	15/30	$\left(\frac{1600V}{3600}, \frac{1600V}{3600}\right)$	$\left(\frac{1800V}{3600}, \frac{1800V}{3600}\right)$
Nash	8/12	8/12	12/30	$\left(\frac{1350V}{3600}, \frac{2025V}{3600}\right)$	$\left(\frac{1440V}{3600}, \frac{2160V}{3600}\right)$
KS	9/12	6/12	10/30	$\left(\frac{1152V}{3600}, \frac{2304V}{3600}\right)$	$\left(\frac{1200V}{3600}, \frac{2400V}{3600}\right)$
Dictatorial-2	1	0	0	$(0, V)$	$(0, V)$

In Table 1, we report the calculated utilities without pre-donation and with an optimal pre-donation under five distinct bargaining rules, including Dictatorial-1, Egalitarian, Nash, Kalai-Smorodinsky, and Dictatorial-2 rules. We should observe that under all five rules, the monopolist's optimal pre-donation, whenever positive, increases the utility of consumers as well. The percentage increase in the bargaining utilities of consumers and the monopolist due to the optimal pre-donation by the monopolist can be calculated as $(16.67, 8.33), (5.56, 5.56), (2.50, 3.75), (1.33, 2.67)$, and $(0, 0)$ for the Dictatorial-1, Egalitarian, Nash, Kalai-Smorodinsky, and Dictatorial-2 rules, respectively. It is interesting to see from Table 1 that under the optimal pre-donation $\boldsymbol{\lambda}^{k^*,2}$, the solution under the Kalai-Smorodinsky rule can be obtained from the solution under the Dictatorial-1 distribution by permuting the expected utilities of the monopolist and consumers. That is to say, when consumers are given the dictatorial power in the regulatory bargaining, they could obtain under the optimal pre-donation $\boldsymbol{\lambda}^{k^*,2}$ only what the monopolist would get under the Kalai-Smorodinsky rule, instead of the diametrically opposed Dictatorial-2 rule under which the monopolist is entitled to the whole surplus V. This result is simply caused by the asymmetry (skewness) in the bargaining problem S^R, which remains to manifest itself in the bargaining problem $\underline{\boldsymbol{\lambda}}^{k^*,2}(S^R)$.

3.2 The Possibility of Rejecting/Reversing Pre-Donation in the Ex-Ante Stage

So far, we have implicitly assumed that when the bargaining party i makes any pre-donation within the rules of the modified BM mechanism $\tilde{\Gamma}^i(\alpha, k)$, its opponent j does not reject or reverse it. Now, we shall see the implications of relaxing this assumption. We have seen the under the mechanism $\tilde{\Gamma}^1(\alpha, k)$, the pre-donating party, consumers, have never incentive to choose the rate of pre-donation k above zero, therefore the implicit assumption that the monopolist never rejects pre-donation has practically no bite. On the other hand, the monopolist has clearly an incentive to make a reverse pre-donation under $\tilde{\Gamma}^1(\alpha, k)$. Whereas consumers would choose their pre-donation rate as $k_1 = 0$, the monopolist would optimally respond in turn by choosing its unasked, and formally unallowed, pre-donation rate as $k_2 = k^* = \gamma/(1+\gamma) = 2(1-\alpha)/(3-2\alpha)$, as implied by Lemma 5. Thus, by informally or illegally, yet optimally, deviating from the modified mechanism $\tilde{\Gamma}^1(\alpha, k)$, the monopolist has always incentive to implement the outcome of the mechanism $\tilde{\Gamma}^2(\alpha, k^*)$. Proposition 3 and the proof of Lemma 5 together imply that consumers always become better-off when the monopolist pre-donates at a rate $k_2 = k^*$ formally under the mechanism $\tilde{\Gamma}^2(\alpha, k_2)$, or informally under the mechanism $\tilde{\Gamma}^1(\alpha, k_1)$ after consumers optimally choose $k_1 = 0$. Thus, a benevolent regulator can be argued to serve the interests of the society by allowing the monopolist to make reverse pre-donation under the mechanism $\tilde{\Gamma}^1(\alpha, .)$. However, still a question remains as to whether consumers could not also improve their welfare by rejecting or reversing some part of the monopolist's formal pre-donation under the mechanism $\tilde{\Gamma}^2(\alpha, .)$, or equivalently some part of the monopolist's informal pre-donation under the mechanism $\tilde{\Gamma}^1(\alpha, .)$.

To answer the above question, notice that for any $k_1 \in [0, 1)$ a reverse pre-donation $\boldsymbol{\lambda}^{\boldsymbol{k_1,1}}$ made by consumers under the mechanism $\tilde{\Gamma}^2(\alpha, k_2)$ when the monopolist make its pre-donation formally according to $\boldsymbol{\lambda}^{\boldsymbol{k_2,2}}$ for any $k_2 \in [0, 1)$ would change the bargaining problem from $\underline{\boldsymbol{\lambda}}^{\boldsymbol{k_2,2}}(S^R)$ to $\underline{\boldsymbol{\lambda}}^{\boldsymbol{k_1,1}}(\boldsymbol{\lambda}^{\boldsymbol{k_2,2}}(S^R))$, whereas it would have no effect on the disagreement point since $\boldsymbol{\lambda}^{\boldsymbol{k_1,1}}(\boldsymbol{\lambda}^{\boldsymbol{k_2,2}}(d^R)) = d^R$.

Lemma 6. *For any $\alpha \in [0, 1]$, $k_2 \in [0, 1)$, and the associated mechanism $\tilde{\Gamma}^2(\alpha, k_2)$, consumers have incentive to make a reverse pre-donation $\boldsymbol{\lambda}^{\boldsymbol{k_1,1}}$ where*

$$k_1 = \begin{cases} 0 & \text{if } k_2 \le k^* \\ 1 - \dfrac{k^*}{k_2} & \text{if } k_2 > k^*. \end{cases} \tag{32}$$

Proof. Pick any $\alpha \in [0, 1]$, $k_2 \in [0, 1)$, and consider the associated mechanism $\tilde{\Gamma}^2(\alpha, k_2)$. First observe that when k_2 is equal to k^*, the optimal rate chosen by the monopolist in the *ex-ante* stage, consumers' expected welfare also reaches its maximal level that any value of k_2 lead to, since the equilibrium utility allocation $(k^*V, (1-k^*)V)$ implied by the mechanism $\tilde{\Gamma}^2(\alpha, k^*)$ is on the Pareto frontier of the transformed bargaining set $\underline{\boldsymbol{\lambda}}^{\boldsymbol{k^*,2}}(S^R)$. Also note that if $u \in \underline{\boldsymbol{\lambda}}^{\boldsymbol{k_2,2}}(S^R)$ and $u' \in \underline{\boldsymbol{\lambda}}^{\boldsymbol{k_1,1}}(\boldsymbol{\lambda}^{\boldsymbol{k_2,2}}(S^R))$ such that $u_2 = u'_2 \neq V$, then $u'_1 < u_1$. That is to say, reverse pre-donation of consumers always contracts the transformed bargaining set $\underline{\boldsymbol{\lambda}}^{\boldsymbol{k_2,2}}(S^R)$ for any $k_2 \in (0,1)$. Therefore, it is not optimal for consumers to contract this set even further if $k_2 \leq k^*$, implying that their optimal response must be $k_1 = 0$.

On the other hand, if $k_2 > k^*$, then the mechanism $\tilde{\Gamma}^2(\alpha, k_2)$ chooses on the bargaining set $\boldsymbol{\lambda}^{\boldsymbol{k_2,2}}(S^R)$ the allocation $\hat{u}$ such that $\hat{u} \in WPO(\boldsymbol{\lambda}^{\boldsymbol{k_2,2}}(S^R)) \setminus PO(\boldsymbol{\lambda}^{\boldsymbol{k_2,2}}(S^R))$ with $\hat{u}_2 = (1-k_2)V$ and $\hat{u}_1 = \gamma(1-k_2)V$. Since $\gamma(1-k_2)V < \gamma(1-k^*)V = k^*V$, we have $\hat{u}_1 < k^*V$; i.e., consumers are worse off under $\tilde{\Gamma}^2(\alpha, k_2)$ than they would be under $\tilde{\Gamma}^2(\alpha, k^*)$. So, consumers have an incentive to contract $\underline{\boldsymbol{\lambda}}^{\boldsymbol{k_2,2}}(S^R)$. Notice that consumers always get γ times what the monopolist obtains under the rule P^γ. Therefore, for consumers the optimal choice of $k_1 \in (0,1)$ must ensure that γ times what the monopolist obtains under the reverse pre-donation $\underline{\boldsymbol{\lambda}}^{\boldsymbol{k_1,1}}$ is equal to k^*V, the highest utility that consumers can obtain under P^γ. So, we must have $\gamma[k_1(k_2V) + (1-k_2)V] = k^*V$. Inserting above $\gamma = k^*/(1-k^*)$ and rearranging the equation yields $k_1 = 1-(k^*/k_2)$. ■

Proposition 5. *Consumers have no incentive to reverse the optimal pre-donation of the monopolist k^* under the modified BM mechanism $\tilde{\Gamma}^2$.*

Proof. Directly follows from Lemma 6. ■

3.3 Pre-donation in the *Interim* Stage

Up until now, we have dealt with the possibility of pre-donation in the *ex-ante* stage. We shall henceforth consider the *interim* stage. Notice that consumers' information about the monopolist's private cost parameter θ is the same in the *ex-ante* and *interim* stages. Therefore, consumers make expected utility calculations in the *interim* stage, as well. Since Lemma 2 implies that the pre-donation from consumers to the monopolist is always harmful for consumers in the *ex-ante* stage, it must remain to be so in the *interim* stage, as well. As for the monopolist, however, this is not true. We know from Proposition 4 that the sum of the total utility under the modified BM mechanism $\Gamma^2(\alpha, k^*)$ is always equal to V. Therefore, the modified welfare weight is always equal to $\tilde{\alpha}(\alpha, k^*) = 1$. That is, the regulator always gives under the modified BM mechanism $\Gamma^2(\alpha, k^*)$ the whole expected surplus V to the monopolist, and out of this the monopolist pre-donates k^*V to consumers. In result, the actual net profit of the monopolist that optimally pre-donates in the *ex-ante* stage is equal to $\tilde{\pi}(\theta, \alpha) =$

$\pi(\theta, \tilde{\alpha}(\alpha, k^*)) - k^* V$ or more explicitly

$$\tilde{\pi}(\theta, \alpha) = \int_\theta^a q^*(x, \tilde{\alpha}(\alpha, k^*))dx - \frac{2(1-\alpha)}{(3-2\alpha)}V = \frac{(a-\theta)^2}{2} - \frac{(1-\alpha)}{(3-2\alpha)}\frac{a^2}{3}. \tag{33}$$

(In Section 2, we saw that the upper bound of the integral in the above equation is $\theta^*(\tilde{\alpha}(\alpha, k^*)) = a/(2-\tilde{\alpha}(\alpha, k^*))$ and the optimal output function is $q^*(x, \tilde{\alpha}(\alpha, k^*)) = a - (2-\tilde{\alpha}(\alpha, k^*))x$ for any $x \in (0, a]$ which reduce to $\theta^*(\tilde{\alpha}(\alpha, k^*)) = a$ and $q^*(x, (\tilde{\alpha}(\alpha, k^*))) = a - x$ since $\tilde{\alpha}(\alpha, k^*) = 1$.)

One can easily show that the expected value of $\tilde{\pi}(\theta, \alpha)$ is just equal to $\tilde{W}_2^2(1, k^*)$, i.e., what the monopolist expects to earn from the modified BM mechanism $\tilde{\Gamma}^2(1, k^*)$ in the *ex-ante* stage. However, in the *interim* stage the expected value of $\tilde{\pi}(\theta, \alpha)$ is nothing but itself for the monopolist, as it has learned the true value of θ. Thus, in the *interim* stage the monopolist must only be interested in maximizing its actual profit, and this profit does not have to be equal to the actual profit $\tilde{\pi}(\theta, \alpha)$ it would obtain in the *ex-ante* stage. Since the optimal pre-donation rate of the monopolist in the *ex-ante* stage is independent of θ (as shown by Lemma 5), the actual profit it would induce runs the risk of becoming negative if θ is sufficiently close to a, or more formally if $\theta > \underline{\theta}(\alpha) = a[1 - \sqrt{2(1-\alpha)/(3(3-2\alpha))}]$, as can be observed from (33). When $\alpha = 0$, this condition reduces to $\theta > \underline{\theta}(0) = a[1 - \sqrt{2}/3] \sim 0.53\, a$, which becomes never binding since we also know that the monopolist is allowed to operate only if $\theta \leq \theta^*(\alpha) = a/(2-\alpha)$ and this second condition reduces to $\theta \leq 0.50\, a$ when $\alpha = 0$. When $\alpha = 1$, pre-donation is not observed ($k^* = 0$). In this limiting case, the threshold $\underline{\theta}(\alpha)$ reduces to a, implying that the actual profit of the monopolist is always non-negative. On the other hand, if $\alpha \in (0.157, 1)$, then $\underline{\theta}(\alpha) < \theta^*(\alpha) < 1$ implying that $\tilde{\pi}(\theta, \alpha) < 0$ for all $\theta \in [\underline{\theta}(\alpha), \theta^*(\alpha)]$. That is, for most values of α, there exists a non-zero measure of θ values where the monopolist will find that the optimal pre-donation it would make in the *ex-ante* stage can no longer be optimal in the *interim* stage.

Remark 2. For any $\alpha \in [0, 1)$ and $\theta \in [0, a)$, the monopolist finds that the outcome of the modified BM mechanism $\tilde{\Gamma}^2(\alpha, k^*)$ with the *ex-ante* optimal pre-donation $\boldsymbol{\lambda}^{\boldsymbol{k^*,2}}$ is

(i) *interim* superior to the BM mechanism $\Gamma(\alpha)$ if $\theta < \bar{\theta}(\alpha)$,

(ii) *interim* inferior to $\Gamma(\alpha)$ if $\theta > \bar{\theta}(\alpha)$, and

(iii) *interim* equivalent to $\Gamma(\alpha)$ if $\theta = \bar{\theta}(\alpha)$,

where $\bar{\theta}(\alpha) = a[1 - \sqrt{2(1-\alpha)/(3(3-2\alpha))}]$.

The observations in the above remark directly imply the following.

Proposition 6. *For any $\alpha \in [0, 1)$, the modified BM mechanism $\tilde{\Gamma}^2(\alpha, k^*)$ under the* ex-ante *optimal pre-donation $\boldsymbol{\lambda}^{\boldsymbol{k^*,2}}$ is interim Pareto non-comparable to the BM mechanism $\Gamma(\alpha)$.*

Our next question is to find the *interim* optimal pre-donation for the monopolist under the modified BM mechanism. Notice that Remark 2-(i) only shows that the monopolist *interim* prefers the *ex-ante* optimal pre-donation $\boldsymbol{\lambda}^{\boldsymbol{k^*,2}}$ to no pre-donation $\boldsymbol{\lambda}^{\boldsymbol{0,2}}$; it does not imply that $\boldsymbol{\lambda}^{\boldsymbol{k^*,2}}$ is *interim* optimal. As we have already discussed in Section 2.4, the objective of the monopolist when it chooses the pre-donation rate in the interim stage is to maximize the actual net profit $\pi^a(\theta, \alpha, k)$ given by equation (28). For any choice of pre-donation rate $k \in [0, 1)$, the modified BM mechanism will be $\tilde{\Gamma}^2(\alpha, k)$. The regulator, to whom θ is yet unknown, will determine the function $\tilde{\alpha}(.,.)$ to satisfy

$$(1-k)W_2(\tilde{\alpha}(\alpha, k)) = P_2^{\gamma}((\underline{\boldsymbol{\lambda}}^{\boldsymbol{k,2}}(S^R), d^R)) \tag{34}$$

at each $k \in [0, 1)$ using the conversion $\gamma = 2(1-\alpha)$. From the viewpoint of the regulator and consumers, the expected value of the gross profit from the modified BM mechanism, $\pi(\theta, \tilde{\alpha}(\alpha, k))$, is still equal to $W_2(\tilde{\alpha}(\alpha, k))$ and the expected value of the actual net profits, $\pi^a(\theta, \alpha, k)$, is therefore still $P_2^{\gamma}((\underline{\boldsymbol{\lambda}}^{\boldsymbol{k,2}}(S^R), d^R))$ as in the *ex-ante* stage.

The monopolist, on the other hand, can fully observe $\pi^a(\theta, \alpha, k)$ for any choice of k. However, the monopolist has now an additional constraint in the *interim* stage. Recall that in the *ex-ante* stage, the interests of the monopolist and consumers were aligned as shown by Lemma 4. A pre-donation by the monopolist is *ex-ante* beneficial (harmful) for itself if and only if it is also so for consumers. In the *interim* stage, this alignment does not necessarily exist. The monopolist may improve its actual profits at the expense of a deterioration in the expected utility of consumers. So, the regulator should be expected to use a modified BM mechanism allowing the monopolist to pre-donate in the *interim* stage only if the outcome of this mechanism yields a higher expected utility to consumers than obtained under the BM mechanism. This however can be true only if the pre-donation rate k is less than the threshold level $k < \bar{k}(\alpha)$, as implied by Proposition 3-(i). In that case, $P^{\gamma}(\underline{\boldsymbol{\lambda}}^{\boldsymbol{k,2}}(S^R), d^R)$ would lie on $PO(\underline{\boldsymbol{\lambda}}^{\boldsymbol{k,2}}(S^R), d^R))$. Together with equation (34), this would imply

$$P_1^{\gamma}(\underline{\boldsymbol{\lambda}}^{\boldsymbol{k,2}}(S^R), d^R) = W_1(\tilde{\alpha}(\alpha, k)) + kW_2(\tilde{\alpha}(\alpha, k)). \tag{35}$$

Since $P_1^{\gamma}(\underline{\boldsymbol{\lambda}}^{\boldsymbol{k,2}}(S^R), d^R) = \gamma P_2^{\gamma}(\underline{\boldsymbol{\lambda}}^{\boldsymbol{k,2}}(S^R), d^R)$, equations (34) and (35) together imply

$$W_1(\tilde{\alpha}) + kW_2(\tilde{\alpha}) = \gamma(1-k)W_2(\tilde{\alpha}), \tag{36}$$

where $\tilde{\alpha} \equiv \tilde{\alpha}(\alpha, k)$. The above equation is satisfied only if

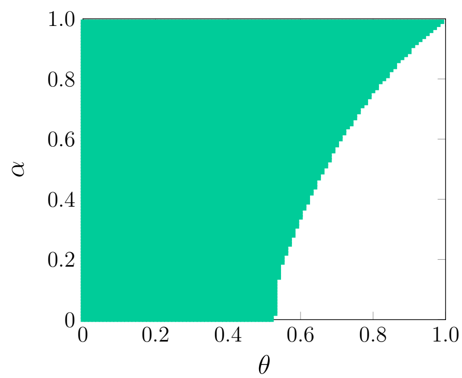
$$\tilde{\alpha} = \alpha + k \left(\frac{3 - 2\alpha}{2} \right). \tag{37}$$

So, using (12) along with (28), we can write the problem of the monopolist as

$$\max_{k \in [0,1]} \ \pi^a(\theta, \alpha, k) = \left(\frac{2 - \tilde{\alpha}}{2} \right) \theta^2 - a\theta + \frac{a^2}{2(2 - \tilde{\alpha})} - \frac{k}{(2 - \tilde{\alpha})^2} \frac{a^2}{6}, \tag{38}$$

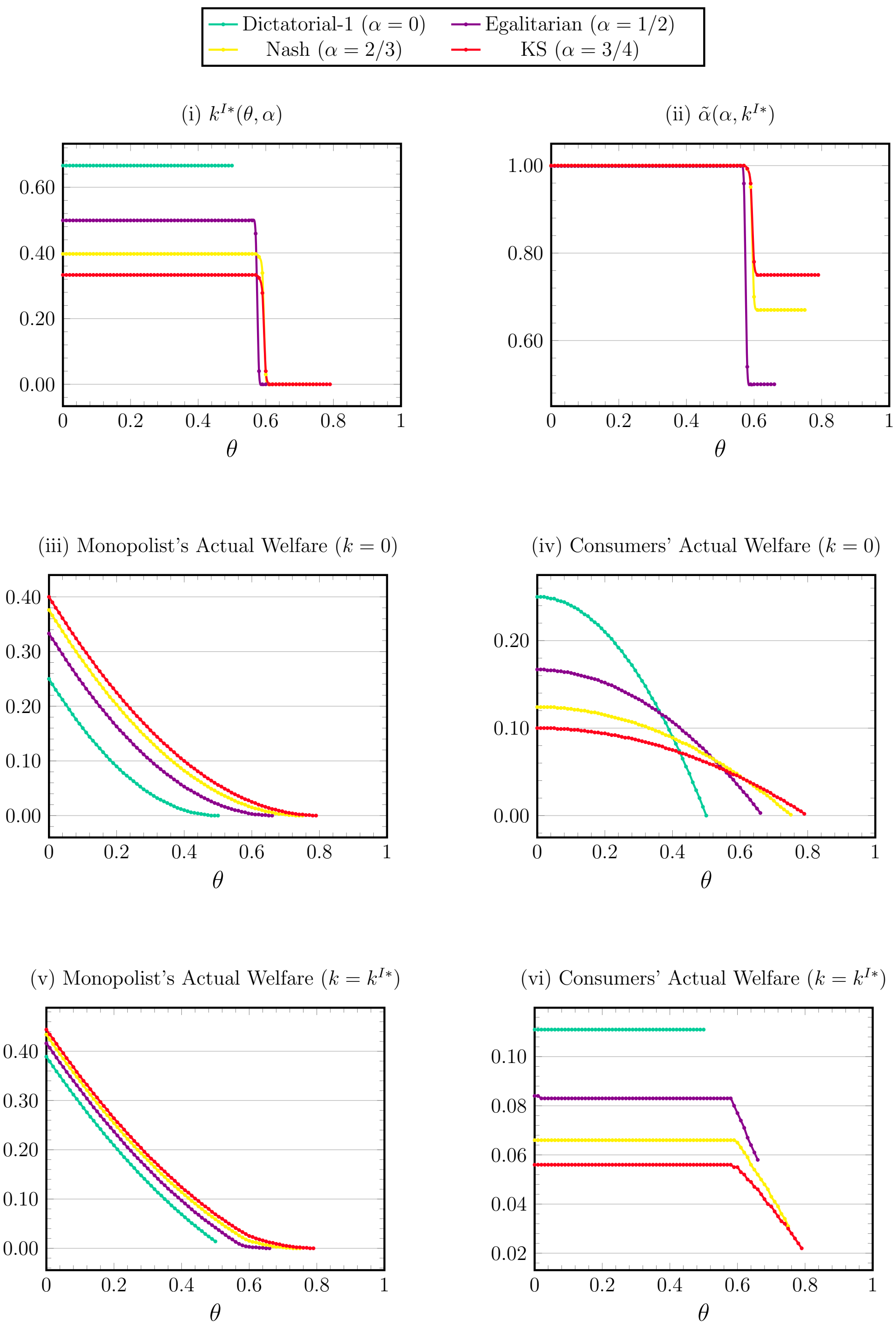
subject to the constraint $k < \bar{k}(\alpha) = 1 - 1/(2-\alpha)^2$ and equation (37) over the parameter values where $\tilde{\alpha} \in [0, 1]$ is satisfied. We solve the above optimization problem with the help of a computer and by setting the parameter a to 1. In Figure 6, we plot the set of (θ, α) pairs that satisfy the aforementioned constraints faced by the monopolist in the *interim* stage. We observe that if α is zero, any θ inside the set $[0, 0.520]$ is consistent with the modified BM mechanism, and this set becomes wider and wider as α increases and eventually coincides with the unit interval when α becomes 1.

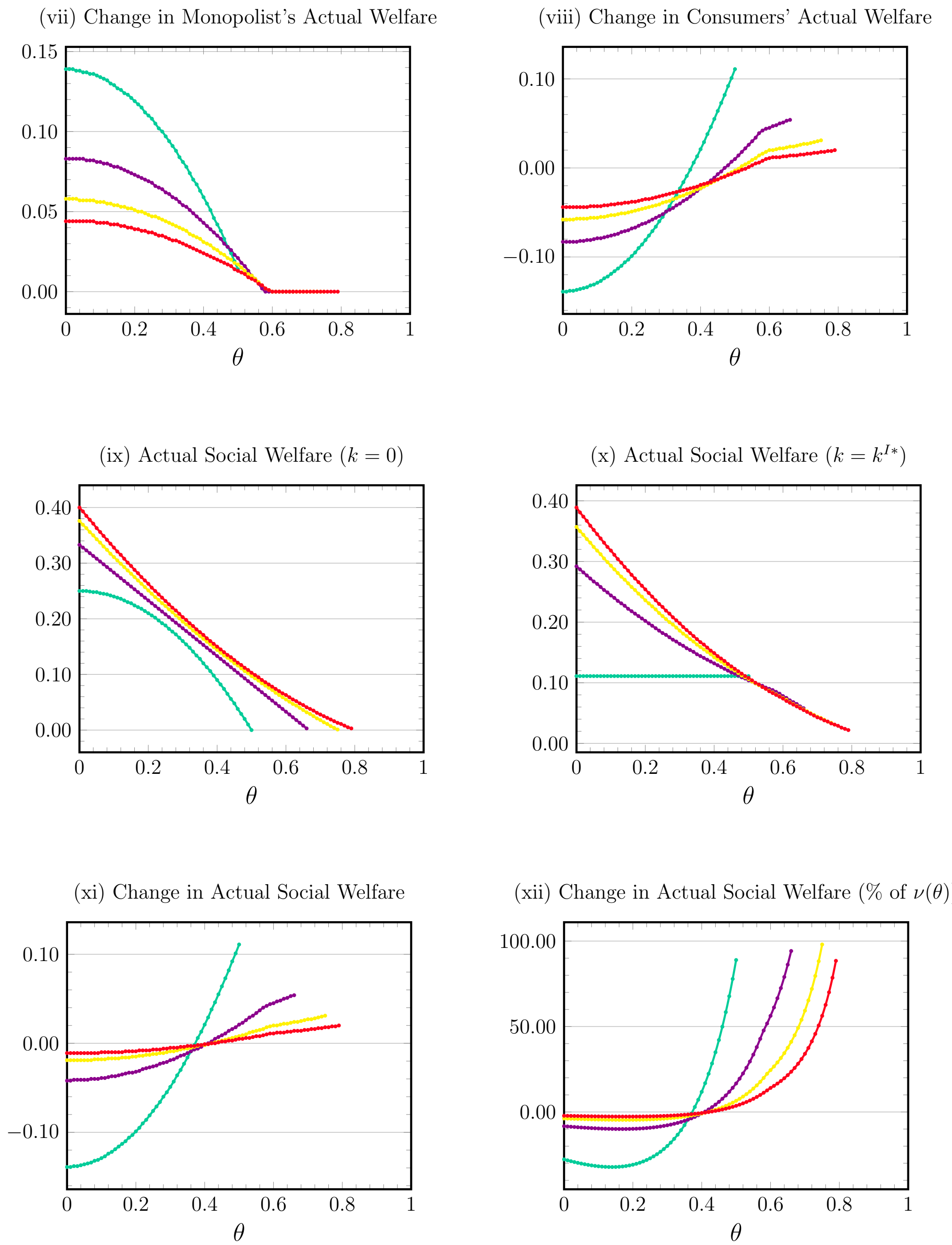
Figure 6. The Set of (θ, α) Pairs Supporting Equilibrium in the Interim Stage



Next, we look for the solution to the constrained maximization problem in (38) over the set of (θ, α) pairs satisfying the constraints of the monopolist. As the first-order condition is not analytically conclusive, we will make simulations for various values of α. Here, we will denote the monopolist's optimal choice of pre-donation rate in the *interim* stage by k^{I*} to distinguish it from its optimal choice k^* made in the *ex-ante* stage. Recall that when $\alpha = 1$, the monopolist has no incentive to make pre-donation since $\tilde{\alpha}(\alpha, k)$ cannot exceed 1. So excluding $\alpha = 1$ (and the Dictatorial-2 rule associated thereof), we consider in our simulations four values of α in the set $\{0, 1/2, 2/3, 3/4\}$ corresponding to the rules in the set {Dictatorial-1, Egalitarian, Nash, Kalai-Smorodinsky}. Our results are portrayed in Figure 7.

Figure 7. The Welfare Effects of Pre-donation in the Interim Stage





As shown by panel (i) of Figure 7, under all four rules the monopolist's optimal pre-donation rate k^{I*} chosen in the *interim* stage is equal to the optimal value $k^* = 2(1-\alpha)/(3-2\alpha)$ it would choose in the *ex-ante* stage if θ is sufficiently small (not exceeding the middle values of θ in the unit interval) and zero otherwise. In result, the modified welfare weight $\tilde{\alpha}(\alpha, k^{I*})$ becomes equal to 1 when $k^{I*} = k^*$ and equal to α when $k^{I*} = 0$, as shown in panel (ii). Thus, there exists a small range of θ

values which are low enough, less than $\theta^*(\tilde{\alpha}(\alpha, k^{I*}))$, to warrant the operation of the monopolist but also high enough to imply that the monopolist chooses not to donate ($k^{I*} = 0$). Panels (iii) and (iv) illustrate the actual monopolist and consumer welfares when the pre-donation rate is zero and the modified mechanism coincides with the BM mechanism. We observe that both welfares always decrease with θ as theoretically predicted. However, the welfare effect of a change in α, and equivalently a change in the bargaining rule, is different for the two parties. The monopolist could rank the four bargaining rules (associated with four α values) from the best to the worst as KS, Nash, Egalitarian, and Dictatorial-1, whereas consumers would rank them in the reverse direction. Notice that the effects of θ and α on the actual welfares appear in the *interim* stage too, as we can inspect in panels (v) and (vi) of Figure 7. We should, however, observe that in the *interim* stage the modified BM mechanism reduces the effect of α on the actual welfare of the monopolist and eliminates the effect of θ on the actual welfare of consumers when θ is not high. Comparing panels (v) and (vi) with the previous two panels, we calculate in the next two panels the actual welfare gains. Panel (vii) shows that the monopolist always benefits from pre-donating when θ is not too high. On the other hand, as shown in the next panel, consumers suffer from the monopolist's pre-donation if θ is low (nearly less than 0.4 or so) and benefit from it otherwise, as long as the monopolist is allowed to operate. In the last four panels, we consider the social welfare analysis. Panels (ix) and (x) together show that the modification of the BM mechanism by allowing the monopolist to pre-donate in the *interim* stage increases the variance of the social welfare with respect to α at low values of θ and reduces this variance otherwise. Finally, the last two panels show (in utility levels and percentage terms) that the actual social welfare always decreases (though not extremely) at low values of θ and substantially increases at high values of θ. Indeed, the last panel illustrates that when the percentage increase can be almost as high as 100% under all four bargaining rules when θ is sufficiently high. This result suggests that the lower the efficiency of the monopolist, the higher the ex-post social benefit we obtain from the modified BM mechanism. The welfare results summarized above imply the following existence result.

Proposition 7. *There exist $\alpha \in [0, 1]$ and $\theta \in [0, a)$ such that by allowing the monopolist to optimally pre-donate in the* interim *stage under the modified BM mechanism the regulator can increase the actual (*ex-post*) welfare of both consumers and the monopolist.*

The simulations in Figure 7 suggest that the set of θ values for which Proposition 7 holds is a continuum. These simulations also suggest that the set of α values leading to the predicted welfare gains must contain the set $\{0, 1/2, 2/3, 3/4\}$ corresponding to four bargaining rules considered in Figure 7. Moreover, the continuity of the BM mechanism and its modified version imply that the set of α for which Proposition 7 holds must be a continuum, too. In fact, our additional simulations not reported in this study suggest that as long as $\alpha < 1$, one can always predict to find some measurable range of θ values under which the studied modification for the BM mechanism leads to *ex-post* welfare improvements for both the monopolist and consumers.

Finally, we should notice that the optimal pre-donation rate k^{I*} of the monopolist in the interim stage is not independent of θ. Thus, it reveals information about the monopolist's private costs. The monopolist can avoid the consequences of this unintentional revelation, before the planned cost revelation, if it can make a contractual agreement with the regulator to prevent her from exploiting any information revealed by pre-donation. To see whether such an agreement would be plausible for the regulator, we should recall that in calculating the optimal *interim* pre-donation rate k^{I*} of the monopolist, we restricted ourselves to a domain where each pre-donation level was *ex-ante* admissible by the regulator and consumers, consequently ensuring $k^{I*} < \bar{k}(\alpha)$. As long as the contractual agreement between the monopolist and the regulator enforces the pre-donation rate in the *interim* stage to lie below the threshold level $\bar{k}(\alpha)$, the regulator may have an incentive to sign this contract as it increases the expected consumer welfare whenever the monopolist chooses to pre-donate. However, we should emphasize here that such a contract may or may not be suboptimal from the viewpoint of consumers, as we do not know whether the modified BM mechanism we propose in this study would coincide with the optimal regulatory mechanism that should be used when the regulator decides to optimally exploit the cost information revealed by the pre-donation of the monopolist in the *interim* stage. While we leave a formal answer to this question to future research, below we will attempt to briefly explore how the outcome of the modified BM mechanism would be affected if the regulator were to exploit the cost information revealed by the pre-donation of the monopolist.

3.4 The Regulator's Attempt to Exploit Information Revealed by Pre-donation in the *Interim* Stage

Suppose the regulator conducts the simulations in Figure 7 and finds out from panel (i) that the monopolist will not pre-donate if its cost parameter θ is above a certain threshold. Notice that this threshold is (slightly) different for different bargaining solutions or equivalently for different α values. So, let us fix a α value and denote the corresponding threshold value $\tilde{\theta}(\alpha)$. Notice that for all θ values below $\tilde{\theta}(\alpha)$, the monopolist makes the same pre-donation; i.e., $k^{I*}(\theta, \alpha)$ is constant if $\theta \leq \tilde{\theta}(\alpha)$. Thus, the regulator cannot learn the true value of θ when she observes $k^{I*}(\theta, \alpha)$. Nevertheless, she learns that the true value of θ can not be above $\tilde{\theta}(\alpha)$. Suppose the regulator responds to this new bit of information acquired in the *interim* stage by updating her uniformly distributed prior belief $f(.)$ about θ to the belief $\tilde{f}(.)$, using the Bayes rule so that $\tilde{f}(\theta) = 1/\tilde{\theta}(\alpha)$ if $\theta \in [0, \tilde{\theta}(\alpha)]$ and $\tilde{f}(\theta) = 0$ otherwise. For the regulatory mechanism to be completely certain and admissible, the regulator should announce her updating rule (that she will make the aforementioned update on her beliefs after she observes a positive amount of pre-donation by the monopolist) at the same time as she announces the modified BM mechanism. Even if the regulator avoids for any reason making this announcement before pre-donation occurs, the monopolist may competently calculate the updated belief $\tilde{f}(\theta)$ and take it into account whenever it anticipates that the regulator will use this updated belief conditionally. To simplify the analysis, we assume that the monopolist is informed (or fully aware) about the updating rule and the updated belief $\tilde{f}(\theta)$ of the regulator before it makes any pre-donation in the *interim* stage. Below, we will investigate how this assumption affects the monopolist's pre-donation decision and the regulatory outcome.

Recall that under the prior beliefs of the regulator, the optimal regulatory price is equal to $p^*(\theta) = \theta + (1-\alpha)F(\theta)/f(\theta) = (2-\alpha)\theta$, since $F(\theta)/f(\theta) = \theta$ for all $\theta \in [0, a)$. We should also notice that the inverse hazard rate does not change after the belief update, i.e., $\tilde{F}(\theta)/\tilde{f}(\theta) = F(\theta)/f(\theta) = \theta$. This implies that the optimal regulatory price $p^*(\theta)$ and consequently the optimal regulatory output $q^*(\theta)$ will be the same irrespective of the pre-donation decision of the monopolist in the *interim* stage. However, the information rent of the monopolist may be affected by the belief update since the support on which the beliefs are defined narrows down from $[0, a)$ to $[0, \tilde{\theta}(\alpha))$. If the monopolist pre-donates, its information rent becomes $\int_\theta^{\theta^m} q^*(x, \alpha)dx$ where $\theta^m = \min\{\theta^*(\alpha), \tilde{\theta}(\alpha)\}$. If $\theta^m = \theta^*(\alpha)$, the monopolist's information rent under the belief $\tilde{f}(.)$ will be the same as its rent under the belief $f(.)$. Thus, the actual net utility of the monopolist in equation (28) will be unchanged, implying that the monopolist, whenever pre-donates, will pre-donate the amount $k^{I,*}$, irrespective of the regulator's updating her beliefs. On the other hand, if $\theta^m = \tilde{\theta}(\alpha)$, then the monopolist's information rent will reduce to

$$\pi(\theta, \alpha) = \int_\theta^{\tilde{\theta}(\alpha)} q^*(x, \alpha)dx = a(\tilde{\theta}(\alpha) - \theta) - \frac{(2-\alpha)}{2}\left((\tilde{\theta}(\alpha))^2 - (\theta)^2\right) \tag{39}$$

and its expected value under the updated beliefs $\tilde{f}(\theta)$ will be equal to

$$W_2(\alpha) = \int_0^{\tilde{\theta}(\alpha)} \pi(\theta, \alpha)\tilde{f}(\theta)d\theta = \frac{(2\alpha - 1)}{6}\left(\tilde{\theta}(\alpha)\right)^2. \tag{40}$$

Thus, the monopolist's actual profit under the modified BM mechanism augmented with the updated beliefs $\tilde{f}(.)$ of the regulator will be equal to

$$\begin{aligned}
\pi^a(\theta, \tilde{\alpha}, k) &= \int_\theta^{\tilde{\theta}(\tilde{\alpha})} q^*(x, \tilde{\alpha})dx - kW_2(\tilde{\alpha}) \\
&= a(\tilde{\theta}(\tilde{\alpha}) - \theta) - \frac{(2-\tilde{\alpha})}{2}\left((\tilde{\theta}(\tilde{\alpha}))^2 - (\theta)^2\right) - \frac{(2\tilde{\alpha} - 1)}{6}\left(\tilde{\theta}(\tilde{\alpha})\right)^2 \quad (41)
\end{aligned}$$

where $\tilde{\alpha}$ is given by (37) and k must satisfy the constraint $k < \bar{k}(\alpha)$, as we have discussed earlier. The monopolist chooses $k \in [0, 1]$ to maximize $\pi^a(\theta, \tilde{\alpha}, k)$. We will solve this problem using a computer by setting $a = 1$ and $\alpha = 3/4$ (inducing the Kalai-Smorodinsky solution). For $\alpha = 3/4$, we observe from panel (i) of Figure 7 that $\tilde{\theta}(\alpha) = \min\{\theta : k^{I,*}(\theta, \alpha) = 0\} = 0.59$. Inserting this value in the above maximization problem of the monopolist along with the other fixed parameters, we calculate the optimal pre-donation of the monopolist and the implied welfares under the modified regulatory mechanism augmented with the updated belief $\tilde{f}(.)$. We illustrate the results of these calculations in the following figure.

Figure 8. The Effects of Belief Updating Conditional on Pre-donation

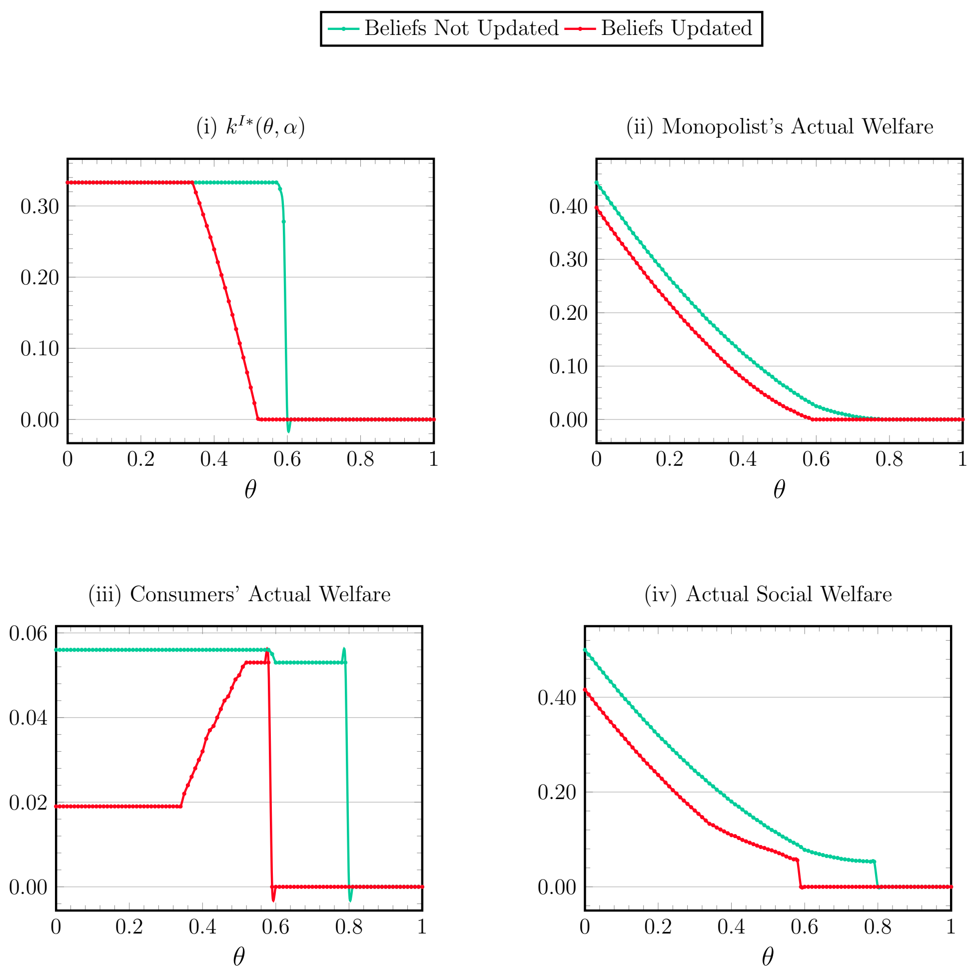


Figure 8 shows that the optimal pre-donation of the monopolist in the *interim* stage is lower when the regulator updates her prior belief (conditional on pre-donation) than when she does not. Consequently, due to belief updating of the regulator, the actual welfares of both the monopolist and consumers are reduced at all cost levels where pre-donation occurs, leading to a reduction in the actual social welfare. We should also observe from panel (i) that the optimal pre-donation of the monopolist falls to zero at lower values of θ under belief updating than it does under no belief updating.

At this point, one may argue that we could push the analysis presented above for an indefinite number of steps further. For example, for the next step we may argue that the regulator, being fully aware that by updating her beliefs she will reduce the threshold value of $\tilde{\theta}(\alpha)$ above which the monopolist will find pre-donation non-optimal, can take this observation also into account before she updates her beliefs. Given our computational results, one may be tempted to guess that the monopolist's optimal response would then be to reduce $\tilde{\theta}(\alpha)$ even further. Consequently, in the limit of such a *k-level* (hierarchical) reasoning it is very likely that the threshold $\tilde{\theta}(\alpha)$ would go down to zero, implying that the monopolist would never pre-donate and the regulator would never learn anything about its cost interval. Finally, we should emphasize that the above analysis assumes that the regulator sticks to the modified BM regulatory mechanism when she indirectly learns about the cost of the monopolist through its pre-donation. Under such an assumption, the negative welfare results obtained by our analysis suggest that the regulator may be reluctant to update her prior beliefs. However, as we have stated earlier, the characterization of the optimal regulatory mechanism under the possibility of pre-donation in the *interim* stage and whether the regulator who implements such an optimal mechanism should respond to any information revealed by pre-donation are open questions waiting to be answered.

4 Conclusions

In this paper, we have proposed a Pareto-improving modification for the BM mechanism. The modification allows consumers and the monopolist to make –in the *ex-ante* or *interim* stage of the regulatory process– contingent utility transfers (pre-donations) between themselves to ensure a bilaterally beneficial improvement upon the expected social welfare function selected by the regulator in the BM mechanism.

We have proved that under the modified regulatory mechanism any amount of pre-donation by the monopolist in the *ex-ante* stage always leads to an *ex-ante* Pareto improvement, while a certain amount of it eliminates expected deadweight loss. Moreover, the optimal pre-donation of the monopolist in the *interim* stage may lead under some cost parameters to an *ex-post* Pareto improvement. Consumers, on the other hand, have never any incentive to make a unilateral pre-donation, nor to reverse the optimal pre-donation of the monopolist.

An interesting question to be asked at this point is as follows:[5] Would the monopolist prefer to pre-donate in the *ex-ante* stage or the *interim* stage if it had the chance to choose between the two, for example, by postponing its pre-donation in the *ex-ante* stage until it received information in the *interim* stage, or alternatively, committing not to view its private cost information in the *interim* stage until after it makes its pre-donation as if it was making this decision in the *ex-ante* stage? To answer this question, we should recall that in the *interim* stage the monopolist finds it optimal to pre-donate if and only if its cost parameter is sufficiently small. Moreover, whenever it decides to pre-donate, the amount of pre-donation is the same as the amount that it would choose in the *ex-ante* stage. So, conditional on the values of the cost parameter that induce the monopolist to pre-donate in the *interim* stage, the monopolist should rationally expect in the *ex-ante* stage that it will not have any loss or benefit from postponing its pre-donation decision to the *interim* stage. But, unconditionally the monopolist may benefit from such a postponement since in the likely event that its private cost parameter is realized in the *interim* stage at a sufficiently high value, the monopolist will have the opportunity to join the bargaining process without making any pre-donation in accordance with its optimal decision rule. Hence, depending on our results, we can also say that it may never be optimal for the monopolist to commit not to view its private cost information in the *interim* stage until after it makes its pre-donation.

Finally, we should notice that an important assumption we have made in modifying the BM mechanism is that the regulator can make a perfectly binding commitment preventing herself from using any information that may be revealed by the monopolist's pre-donation to update her prior beliefs about the monopolist's private cost information or to change the revelation mechanism borrowed from BM. This assumption has no bite when the monopolist makes pre-donation in the *ex-ante* stage. This is because the monopolist's pre-donation turns out to be independent of its private cost information, revealing no undesired information to the regulator. On the other hand, if the monopolist is allowed to pre-donate in the *interim* stage, its decision as to whether it should pre-donate becomes a function of its private cost information. Hence, the monopolist unintentionally reveals some cost information to the regulator before the actual revelation takes place, endangering some part of the informational rents it expects to earn from the regulatory process. However, the regulator has an incentive to be blind to any information revealed by pre-donation in the *interim* stage, as she can verifiably ensure that the optimal pre-donation of the monopolist should be increasing the expected utility of consumers. On the other hand, if the regulator chooses to exploit the information revealed by pre-donation, then the modified mechanism we propose would no longer be incentive-compatible. The monopolist would

[5] The author is grateful to an anonymous reviewer for this question.

have incentives to revise its pre-donation decision strategically to limit the information revealed thereof and also to manipulate its cost report at certain values of its private cost parameter to make it comply with the announced pre-donation. We leave the characterization of the optimal regulatory mechanism in that case and the induced equilibrium pre-donation by the monopolist for future research.

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