Political Bargaining under Incomplete Information about Public Reaction

Deepal Basak*

April 2021

Abstract

In a democracy, when two leaders bargain over a policy, they need to compromise. The public reaction determines the tolerances of the constituents, which in turn determines the cost of compromise for the leaders. A leader may bluff when she learns that the public reaction will move unfavorably, but the opponent may not know this. Under continuous-time canonical Rubinstein bargaining, I show that there is a unique equilibrium that resembles a war of attrition with multiple non-committed types. When the probability that the leaders do not know the public response ε converges to zero, the probability of bluff converges to zero in any given bargaining environment. However, given any ε (however small), we can find bargaining environments where one of the leaders will bluff with probability one and keep doing so for a positive duration.

JEL Classification Numbers: *C78, D82*

Key Words: *Bargaining, Higher-order uncertainty, Public Reaction, Reputation*

*I am grateful to David Pearce, Ennio Stacchetti, Joyee Deb, Jack Fanning, Archishman Chakraborty, Aaron Kolb, Urmee Khan, Rick Harbaugh, Marilyn Pease, and the seminar participants at Kelley School of Business, Econometric Society meetings, SAET for helpful comments and suggestions. I thank Samyak Jain for his excellent research assistance. Basak: Indiana University, Kelley School of Business, Email: dbasak@iu.edu

Introduction

Legislating in a democratic system often requires policy compromises. This paper studies political bargaining between two leaders, while there is incomplete information about the public reaction to this policy issue. The public reaction determines the tolerances of the constituents and, accordingly, the cost of compromises for the leaders. We build a simple tractable model and determine what type of equilibrium emerges in such a political bargaining game. We then look into the case where the probability that the leaders do not know the public reaction is small. We study whether, in equilibrium, it is likely that the leaders will immediately agree on the same policy position as they do when the public reaction is commonly known.

Consider two political leaders - left (L) and right (R) - who represent constituencies known to stand on two opposite extremes on this issue. A compromise is necessary to reach an agreement. The constituents may not tolerate compromises, especially a large compromise. A constituent k of leader i has a tolerance threshold x_i^k, varying across constituents. Nature selects a state $\omega \in \{\mathscr{L}, \mathscr{R}\}$ which shifts the distribution of x_i^k. We call this state - public reaction. In state $\mathscr{L}$ the public reaction moves to the left and makes some of the more tolerant left constituents less so. In state $\mathscr{R}$ the public reaction moves to the right and makes some of the more tolerant right constituents less so.[1]

We consider the canonical Rubinstein (1982) bargaining game under continuous-time and assume that when the public reaction moves favorably, the constituents are sufficiently intolerant. If it is commonly known that on this policy issue, the public reaction favors L (say), then L can credibly commit to not making a large compromise because it is indeed costly for her. In equilibrium, we get a *deal me out* result (See Binmore, Shaked and Sutton (1989)), where leader R concedes right away. We relax the complete information assumption. Suppose that leader R privately learns (for instance, bought data from some data broker) that the public reaction on this issue will move unfavorably. If leader R believes that leader L may not know this, she may bluff rather than concede right away.

We assume the following information structure. Nature draws the state or public reaction from a commonly known prior, and each leader privately learns the public reaction with a known probability. A bargaining environment, denoted by θ, captures the following:

[1]For instance, consider the clean air act 1990. The scientific community conclusively proved that acid rain is caused by industrial pollution. The left constituents who used to care about the cost of business (moderates) would not have tolerated if the left leader compromised and agreed to a low environment standard for businesses.

(1) impatience of the leaders, (2) tolerance distributions of the constituents in each state, and (3) the prior belief over the states. While the information environment is captured by $\varepsilon = (\varepsilon_L, \varepsilon_R)$, where ε_i is the probability that leader i does not know the public reaction. There is almost complete information when ε_i is close to zero for each leader $i = L, R$.

I show that a unique equilibrium emerges that resembles a war of attrition. In equilibrium, a leader who knows that the public reaction will move favorably (the I type) is committed to the same policy position they agree to when the public reaction is commonly known. While a leader who knows the public reaction will move unfavorably (the I' type), or does not know the public reaction (the $\mathcal{U}$ type), may bluff and masquerade as the type who knows that public response will favor her and wait for the opponent to concede.

In equilibrium, how long a leader bluff depends on a notion of bargaining strength (which I will explain shortly). I show that given any political bargaining environment θ, the bargaining strengths become almost balanced when $\varepsilon \to (0, 0)$. Accordingly, the leaders do not bluff when they learn that the public reaction will move unfavorably. Thus, almost immediately, they agree to the same policy position as they do when the public reaction is commonly known. However, given ε (however small), we can find θ where the bargaining strengths are far from balanced. Accordingly, one of the leaders will bluff with a probability 1 and may keep bluffing for a positive duration. Thus, in one of the states, it is impossible to reach an immediate agreement on the same policy position as they do under complete information.

The reputational bargaining literature pioneered by Abreu and Gul (2000) (hereafter AG) gives us a unique war of attrition equilibrium under continuous time. In AG, there is one commitment type and one non-committed rational type. Abreu, Pearce and Stacchetti (2015) (henceforth APS) extend this equilibrium when one of the agents has two non-committed types who differ in their discount rates. In equilibrium, a one-sided skimming property holds — the more patient type concedes later. There are two non-committed types (I' and $\mathcal{U}$) on both sides in our setup. Although the non-committed types do not differ in their discount rates, they differ in their beliefs. Compared to the $\mathcal{U}$ type, the I' type is more reluctant to bluff and less optimistic about the opponent conceding. Consequently, in equilibrium, first, the I' type concedes, followed by the $\mathcal{U}$ type. Thus, the skimming property holds on both sides.

As is standard in reputational bargaining, either a non-committed leader does not bluff, and an agreement is reached immediately, or in the absence of immediate agreement, both sides keep bluffing with such probabilities that keep the types who are randomizing indifferent. As time progresses and there is no concession, they will become more convinced that the opponent is committed and not bluffing. Once an uninformed leader becomes fully convinced that the opponent is committed, she stops bluffing and concedes. If she no longer bluffs, the uninformed opponent will also know that she is committed. This means that the updated beliefs that the opponent is the committed type must become 1 at the same time, and there is no bluffing beyond this time.

Suppose that in equilibrium, the I' types never bluffs — that is, when a leader learns that the public reaction will move unfavorably, she concedes immediately. This means after seeing no immediate concession; the uninformed leaders understand that the opponent is either the committed I type or uninformed. This resulting game is similar to AG, with one non-committed type on both sides. Recall that for this to be an equilibrium, the updated beliefs that the opponent is committed must become 1 at the same time. If (θ, ε) are such that this property holds, we call this the *balanced strength* situation. If the strengths are balanced, leaders do not bluff after learning that public reaction will move unfavorably. Notice that as long as the I' types concede immediately with positive probability, the $\mathcal{U}$ type opponent will definitely bluff (strictly prefers bluffing over conceding right away).

If they do not take the same time to build their reputation (assuming that the I' types do not bluff), then the strengths are not balanced. Then, it must be the case that the leader who takes less time to build her reputation will bluff even after learning that the public reaction will move unfavorably. We call this leader the *strong bargainer*, and the other leader the *weak bargainer*. Notice that both the leaders cannot bluff when they know that public reaction will move unfavorably. If they do, then it follows from the skimming property that there is a time interval in which only the I' types from both sides randomize between conceding and waiting. However, if a leader knows that public reaction will move unfavorably (the I' type), her opponent cannot be the I' type. Therefore, the I' type cannot be indifferent and thus, will not randomize if only the I' type opponent randomizes.

Suppose that the strong bargainer always bluffs (probability 1) and the weak bargainer never bluffs (probability 0) when they learn that public reaction will move unfavorably. Then, after time zero, it is clear that the weak bargainer is either committed or uninformed, while the strong bargainer can be committed, uninformed, or the I' type. This resulting game is similar to APS, and as in APS, it has two phases. In the first phase, the uninformed weak bargainer and the I' type of the strong bargainer will randomize between keep bluffing and conceding. The uninformed strong bargainer will never concede and always bluff in this first phase. In the second phase, the uninformed types from both sides will mix. Recall that in equilibrium, the updated beliefs of the uninformed type that the opponent is stubborn must become 1 simultaneously.

- If (θ, ε) are such that the strong bargainer takes longer, then we say that the bargaining strengths are *moderately unbalanced*. This means the strong bargainer is strong enough to bluff even after learning that the public reaction will move unfavorably, but not so strong that she always bluffs.

- However, if (θ, ε) are such that the weak bargainer still takes longer, then we say that the bargaining strengths are *extremely unbalanced*. This means that the strong bargainer will always bluff, and even the uninformed weak bargainer may not always bluff.

I show that for any θ when ε is sufficiently small, the bargaining strengths are never extremely unbalanced. In fact, an almost balance of strengths is reached. Consequently, the probability that even the strong bargainer will bluff after learning that public reaction will move unfavorably converges to zero. Recall that the I' type of the weak bargainer does not bluff. Thus, for any θ, when ε is sufficiently small, in any state, the probability that the leader agree on the same policy position as they do when they commonly know the public reaction is almost 1.[2]

However, this does not mean that for any given small ε, the above probability will be close to 1 regardless of the bargaining environment θ. Consider any ε. Without loss of generality, suppose that $\varepsilon_L \geq \varepsilon_R$. Consider a bargaining environment where the left constituents are extremely intolerant when the public reaction favors the left. This means R will get almost zero if she concedes. This makes R very reluctant to concede, which weakens L's bargaining strength. I show that for sufficiently low tolerance, the strengths become extremely unbalanced. Therefore, in equilibrium, the strong bargainer R always bluffs and keeps bluffing for a positive duration. Suppose the public reaction favors L. Although there is only a small probability that L will ever concede, because R keeps bluffing, it will take time to reach an agreement. In fact, the probability that they will immediately reach an agreement on the same policy position as they do when they commonly know that the state favors L is 0.

[2]Notice that under almost complete information, in either state, one of the leaders is likely to be committed. This limiting case is different from the limiting case studied under reputational bargaining, where the probability that either agent is committed converges to 0.

Related Literature

Public reaction shapes policy compromises in political bargaining. When the public reaction is commonly known, Muthoo (1992), Levenotoğlu and Tarar (2005) show that a higher cost of compromise helps a leader get a larger share of the pie in equilibrium. More recently, Basak and Deb (2020) study such a political bargaining game where the public reaction is unknown and gets publicly revealed at a later date. Since there is no private information, there is no scope for bluffing. The authors show that in equilibrium, the leaders wait and gamble over the public opinion even though at least one of them has to bear the cost of compromise later. This paper considers private information about public reaction, which creates a much richer equilibrium dynamics involving bluffs. We investigate when a leader bluffs, for how long, and will they keep doing so when there is almost complete information.

Alesina and Drazen (1991) build a classic political war of attrition game between two groups who fight over tax burden and cause a delay in adopting a stabilization policy to reduce the large budget deficit. The uncertainty is about the private cost of continuing the war of attrition for the groups. In our setup, the uncertainty is about a common state that moves in one direction, introducing a negative correlation in the cost of compromise. Also, notice that the authors directly impose a war of attrition structure, while we consider a canonical bargaining game and the war of attrition arises as a unique equilibrium.

The early bargaining literature (See Ausubel, Cramton and Deneckere (2002) for a survey) shows that in a canonical bargaining game, under two-sided asymmetric information, multiple equilibria can be constructed using belief-based threats. AG resolved this multiplicity by assuming commitment types immune to belief-based threats. This makes the war of attrition equilibrium unique. In our setup, a leader who knows that the state favors her behaves like a commitment type. However, this commitment arises endogenously. In Section 2, we construct the war of attrition equilibrium assuming that the I type is committed, while In Section 3.1, we show why commitment arises endogenously. We also discuss the features of our setup that drives this result and when this may be violated.

The literature that most closely resembles the equilibrium dynamics is reputational bargaining. I have already mentioned the relation to AG and APS in constructing the war of attrition equilibrium. For some recent developments in this literature, see Kambe (1999), Compte and Jehiel (2002), Abreu and Pearce (2007), Wolitzky (2012), Atakan and Ekmekci (2014), Özyurt (2015), Fanning (2016, 2021), Sanktjohanser (2018), Ekmekci and

Zhang (2021). Fanning and Wolitzky (2020) provide an excellent survey of this literature. There are some crucial differences in the nature of the equilibrium here. For instance, when the two leaders have balanced strengths, the non-committed leaders never give up immediately under reputational bargaining. In our setup, in contrast, both leaders immediately give up whenever they learn that the public reaction will move unfavorably. Moreover, there is higher-order uncertainty. Feinberg and Skrzypacz (2005), Tsoy (2018) and Madarász (2021) also study how higher-order uncertainty influences the equilibrium dynamics in bargaining. In Section 3.2, I discuss some of the important differences with the reputational bargaining literature. In Section 3.3, I describe a special case when one of the leaders almost surely learns the public reaction, but the other leader does not. Thus, there is higher-order uncertainty but only on one side.

This paper also studies the robustness of the complete information bargaining result. Weinstein and Yildiz (2013) study this question but takes a different approach. They establish a general result that for any bargaining outcome, one can introduce a small amount of incomplete information in such a way that the resulting type profile has a unique rationalizable action profile that leads to this bargaining outcome. However, the authors mention *"the types constructed in our article are complicated, and it is not easy to interpret how they are related to economic parameters"* (p 380). In contrast, this paper considers a perturbation with a simple interpretation — small probability that the leaders do not know which way the public reaction will move. Under this simple perturbation, this paper shows why or why not the leader almost immediately agree at the same policy position as they do when the state is commonly known.

1 Model

There are two political leaders $N = \{L, R\}$ and a status quo policy that gives 0 to both leaders. An opportunity has arrived to bring a change, but the two leaders need to agree. The two leaders represent two constituencies with a complete opposite preferences on this issue. We assume that the policy space is $p \in [0, 1]$. While the L constituents want $p = 1$, the R constituents want $p = 0$. Therefore, a compromise is necessary to reach an agreement. A compromise is costly because some constituents may not *tolerate* such compromise.

Tolerance and Public Reaction: There is a mass 1 of constituents for both leaders. We refer to the leaders as $i \in \{L, R\}$ and a constituent as $k \in [0, 1]$. A constituent

$k \in [0,1]$ of leader i is endowed with a tolerance threshold x_i^k. If the distance between the policy position p that the leader i agrees to and the ideal policy position is higher than x_i^k, the constituent k stops supporting leader i.

Nature selects a state $\omega \in \{\mathscr{L}, \mathscr{R}\}$. We say that the public reaction favors L when $\omega = \mathscr{L}$, and the public reaction favors R when $\omega = \mathscr{R}$. The state affects the distribution of tolerance in the following sense. Some constituents are tolerant, and some constituents are intolerant. When the public reaction moves favorably, even the tolerant constituents become intolerant.

We capture this in a simple parametric fashion. We assume that conditional on the state ω, for any leader i, x_i^k are identically and independently distributed for all $k \in [0,1]$. In state $\mathscr{L}$, while $x_R^k \sim \mathscr{U}[0,1]$, the distribution of tolerance thresholds x_L^k is

$$P(x_L^k \leq x|\mathscr{L}) = x \cdot \mathbb{1}(x < 1 - x_L) + \mathbb{1}(x \geq 1 - x_L).$$

Analogously, in state $\mathscr{R}$, while $x_L^k \sim \mathscr{U}[0,1]$, the distribution of tolerance thresholds x_R^k is

$$P(x_R^k \leq x|\mathscr{R}) = x \cdot \mathbb{1}(x < 1 - x_R) + \mathbb{1}(x \geq 1 - x_R).$$

It is important to note how the distribution of tolerance thresholds changes when the public reaction moves favorably. Consider, for instance, the L constituents. When the public reaction moves unfavorably ($\omega = \mathscr{R}$), the tolerance thresholds are uniformly distributed in $[0,1]$. Think of the L constituents who tolerate a compromise larger than $1 - x_L$ as the tolerant constituents and the other constituents as intolerant. When the public reaction moves favorably ($\omega = \mathscr{L}$), the only difference is that the tolerant L constituents are no longer tolerant, and their tolerance thresholds drop to $1 - x_L$.[3]

To go back to the example, consider the issue of imposing an environmental standard on business. While R constituents care more about the cost of business and want a lower standard (ideally $p = 0$), the L constituents care about the pollution and want a higher standard (ideally $p = 1$). Some of the L constituents are more tolerant of compromises because they are sympathetic to the difficulty of raising the cost of business. However, in 1990, when the scientific community conclusively proved that acid rain is caused by industrial pollution, such L constituents turned less tolerant. We assume that when the state is $\mathscr{L}$, the L constituents who used to tolerate a compromise larger than $1 - x^L$ no

[3] This feature plays an important role in the equilibrium uniqueness argument. In Section 3.1, we discuss this feature and show another alternative setup that may violate the uniqueness result.

longer do so.

We refer to x_L and x_R as the tolerance parameters and assume they are commonly known. A higher x_L (or x_R) means that when the public reaction moves favorably, the L (or R) constituents are more intolerant.

Assumption 1 *When the public reaction moves favorably, the constituents becomes sufficiently intolerant: $x_L \geq x^*$, $x_R \geq 1 - x^*$, where $x^* = \frac{r_R}{r_L + r_R}$ is the Rubinstein equilibrium policy position.*

This assumption ensures that these tolerance levels will influence the equilibrium policy position if the public reaction is commonly known. We will discuss the role of this assumption in Section 2.1.

Leaders' Payoff: The leaders prefer an agreement sooner than later, captured by stationary discounting r_i, $i \in N$. If the leaders agree, then both get 1, but the larger compromise a leader makes to reach this agreement, the more supporter she loses. Consider leader L. Recall that the ideal policy for the L constituents is 1. Therefore, if she agrees to policy position p, it means a compromise of size $1 - p$. Constituents with tolerance thresholds strictly lower than $1 - p$ will not support her. Thus, if she agrees to a policy position p after negotiating for time t and the state is ω then her payoff is $e^{-r_L t} u_L(p, t, \omega)$, where

$$u_L(p, t, \omega) = 1 - \underbrace{P(x_L^k < 1 - p|\omega)}_{\text{lost support}}.$$

Accordingly,

$$u_L(p, t, \mathscr{R}) = p \text{ and } u_L(p, t, \mathscr{L}) = p \cdot \mathbb{1}(p \geq x_L).$$

Notice that the only difference between the payoffs in the two states is that if L agrees to a policy position $p < x_L$, then in state $\mathcal{R}$ (unfavorable public reaction), she will still have the support of p most tolerant constituents, however, in state $\mathcal{L}$ (favorable public reaction), she will lose even these tolerant constituents.

Analogously, when R agrees to policy position p, it means a compromise of size p. If leader R agrees to a policy position p after negotiating for time t and the state is ω then her payoff is $e^{-r_R t} u_R(p, t, \omega)$, where

$$u_R(p, t, \omega) = 1 - \underbrace{P(x_R^k < p|\omega)}_{\text{lost support}}.$$

Accordingly,

$$u_R(p, t, \mathscr{L}) = 1 - p \text{ and } u_R(p, t, \mathscr{R}) = (1 - p) \cdot \mathbb{1}(1 - p \geq x_R).$$

Notice that the only difference between the payoffs in the two states is that if R agrees to a policy position $p > 1 - x_R$, then in state $\mathcal{L}$ (unfavorable public reaction), she will still have the support of $(1-p)$ most tolerant constituents, however, in state $\mathcal{R}$ (favorable public reaction), she will lose even these tolerant constituents.

We assume that a leader prefers the status quo over a policy compromise that will cost her all the support. We can interpret this as this policy compromise on this issue (which costs her all the support) being used against her in the next primary.

The bargaining game: The bargaining proceeds as an alternating offer bargaining game 'a la Rubinstein (1982). One of the leaders is picked randomly with probability $\frac{1}{2}$ to make the first policy proposal $p \in [0, 1]$. If the other leader accepts this proposal, an agreement is reached, and the game ends; otherwise, it continues to the next round. The other leader proposes the next round, and the same process continues until an agreement is reached. Thus, potentially the game can go on forever. We assume the time interval between two rounds $\Delta \to 0$. That is, the discount factor $e^{-r_i \Delta} \to 1$.

Information Structure: We say that there is complete information when the public reaction and the resulting compromise cost for both leaders are commonly known. We are interested in a world where there is a positive probability ε_i that leader i does not know the underlying public reaction and, accordingly, whether a compromise will be more costly for her or her opponent. Formally, we assume that nature draws a state $\omega \in \Omega = \{\mathscr{L}, \mathscr{R}\}$ from a commonly known prior: $\pi_L = P(\omega = \mathscr{L})$, $\pi_R = P(\omega = \mathscr{R})$, and $\pi_L + \pi_R = 1$. Each leader $i \in N$ receives a private signal $\omega_i \in \Omega_i = \Omega \cup \{\mathcal{U}\}$, where $\mathcal{U}$ represents an uninformative signal realization. The signal structure of leader $i \in N = \{L, R\}$ is denoted by the conditional probability distributions $q_i : \Omega \to \mathbb{P}(\Omega_i)$, where

$q_i(\omega_i \mid \omega)$	$\omega_i = \mathscr{L}$	$\omega_i = \mathcal{U}$	$\omega_i = \mathscr{R}$
$\omega = \mathscr{L}$	$1 - \varepsilon_i$	ε_i	0
$\omega = \mathscr{R}$	0	ε_i	$1 - \varepsilon_i.$

Thus, the information structure can be represented by a vector $\varepsilon = (\varepsilon_L, \varepsilon_R)$, where ε_i captures the probability that leader i does not know the public reaction. When ε_i is close to 0 for all $i \in N$, we say that there is almost complete information.

Beliefs: Given the information structure, at the initial history $\emptyset$, a leader i who receives a signal $\mathscr{L}$ or $\mathscr{R}$ believes that her opponent $j \neq i$ receives the same signal with probability $1 - \varepsilon_j$ and signal $\mathcal{U}$ with probability ε_j. On the other hand, a leader i who receives the signal $\mathcal{U}$ does not know how the public reaction will move. She believes that it will move unfavorably, and the opponent knows this with probability $\pi_j(1-\varepsilon_j)$; it will move favorably, and the opponent knows this with probability $\pi_i(1-\varepsilon_j)$; or, the opponent is also uninformed with probability ε_j.

For convenience, I relabel the types as I (learns that the public reaction will move favorably), I' (learns that the public reaction will move unfavorably), and $\mathcal{U}$ (uninformed). Since the state is either $\mathscr{L}$ or $\mathscr{R}$, the I type does not believe that her opponent can be the I type, and the I' type does not believe that her opponent can be the I' type. At the initial node $\emptyset$, let $\alpha_j^{\omega_i}(\emptyset)(\omega_j)$ be the belief of the ω_i type of leader i that her opponent j is the ω_j type, where $i, j \in N$ and $i \neq j$. Then,

$\alpha_j^{\omega_i}(\emptyset)(\omega_j)$	$\omega_j = I$	$\omega_j = I'$	$\omega_j = \mathcal{U}$
$\omega_i = I$	0	$1 - \varepsilon_j$	ε_j
$\omega_i = I'$	$1 - \varepsilon_j$	0	ε_j
$\omega_i = \mathcal{U}$	$\pi_j(1-\varepsilon_j)$	$\pi_i(1-\varepsilon_j)$	$\varepsilon_j.$

Recall that the I and I' types know the public reaction, and only the $\mathcal{U}$ type is uncertain whether the public reaction will move favorably or unfavorably. I use $\pi_j(.)$ for $\omega_i = \mathcal{U}$ type's belief that the public reaction will move unfavorably.

Reluctance to compromise: It is important to understand how the public reaction affects the cost of compromise and how different types vary in their reluctance to compromise. Consider leader L (say). A compromise to policy position $p \geq x_L$ is considered to be a sufficiently small compromise. Regardless of the public reaction, such a small compromise only costs her the support of some intolerant constituents, and thus, she gets the same payoff in either state. Accordingly, the reluctance to compromise is the same for any type. On the other hand, a compromise to policy position $p < x_L$ is considered to be a sufficiently large compromise. Such a large compromise definitely cost her the support of the intolerant constituents and some tolerant constituents. When the public reaction moves favorably, not even the most tolerant constituent will support such compromise, which could lead to a challenge in the next primary.

Thus, the I type who knows that the public reaction will move favorably will rather stick with the status quo than agree to a compromise $p < x_L$. However, the leader may make such a compromise when she knows that the public reaction will move unfavorably (I' type) or does not know the public reaction ($\mathcal{U}$ type). The $\mathcal{U}$ type is more reluctant to make such a compromise than the I' type. To see this, note that if L agrees to $p < x_L$ after bargaining for time t, then the I' type gets $e^{-r_L t}p$ while the $\mathcal{U}$ type gets $(1 - \pi_R(t)) \cdot 0 + \pi_R(t) \cdot e^{-r_i t}p$, where $\pi_R(t)$ is the probability she assigns to the public reaction moving unfavorably. This means that if the $\mathcal{U}$ type keeps bargaining, she discounts a smaller share than the I' type. This makes the $\mathcal{U}$ type more reluctant to compromise than the I' type.

Solution Concept: Under complete information, we use sub-game perfect Nash equilibrium, and under incomplete information, we use perfect Bayesian equilibrium as our solution concept. Throughout this paper, we only consider bargaining under the continuous-time limit ($\Delta \to 0$).[4]

2 Main Result

2.1 Complete Information Benchmark

Consider the benchmark where it is commonly known that the public reaction favors L (say). Therefore, even the tolerant L constituents are no longer tolerant. If the leaders agree to a policy p after bargaining for time t, then R gets $e^{-r_R t}(1 - p)$ and L gets $e^{-r_L t}p \cdot \mathbb{1}(p \geq x_L)$. Notice that if leader L agrees to a policy position $p < x_L$, then such a compromise on this issue will cost her all the support. Thus, leader L will rather stick to the status quo than accept such a policy position.

This game is the same as in Binmore, Shaked and Sutton (1989) where x_L works as an outside option or the minimum share of the pie L is willing to accept. Given assumption 1, it follows from Binmore, Shaked and Sutton (1989), that under continuous-time limit ($\Delta \to 0$), they will immediately agree on $p = x_L$. We call it immediate concession from

[4]AG shows that the equilibrium outcome under discrete-time converges in distribution to the continuous-time war of attrition. The same argument applies here (See Section 3.2).

R. Analogously, if it is commonly known that the public reaction favors R, then they immediately agree on $p = 1 - x_R$. We call it immediate concession from L.

Note that when the public reaction moves favorably, leader L (say) will not agree to $p < x_L$. However, in equilibrium, L cannot get anything better either. The best L can do is to say *deal me out* unless $p \geq x_L$. It follows from assumption 1 that this x_L is at least as high as the Rubinstein equilibrium share x^*. If the L constituents were not sufficiently intolerant, that is, $x_L < x^*$, then x_L will have no effect on equilibrium, and the leader will immediately agree on the standard Rubinstein solution x^*. Note that the deal me out result in this political bargaining follows from the crucial feature that when the public reaction moves favorably, the only difference it makes to the distribution of tolerance is that the tolerant constituents are no longer tolerant.

Note that when the public reaction moves favorably, leader L (say) will not agree to $p < x_L$. However, in equilibrium, L cannot get anything better either. The best L can do is to say *deal me out* unless $p \geq x_L$. It follows from assumption 1 that this x_L is at least as high as the Rubinstein equilibrium share x^*. If the L constituents were not sufficiently intolerant, that is, $x_L < x^*$, then x_L will have no effect on equilibrium, and the leader will immediately agree on the standard Rubinstein solution x^*. Note that the deal me out result in this political bargaining follows from the crucial feature that when the public reaction moves favorably, the only difference it makes to the distribution of tolerance is that the tolerant constituents are no longer tolerant.

2.2 War of Attrition

This section constructs the war of attrition equilibrium in the spirit of reputational bargaining. We assume that a leader who knows that the public reaction will move favorably (the I type) is "committed" to demanding the policy position they agree to when the public reaction is commonly known. She accepts a better policy with probability 1, and a worse policy with probability 0.[5]

Assuming such a commitment type, AG first showed that a unique equilibrium emerges under continuous-time, resembling a war of attrition. In AG, there is only one non-commitment type on both sides. APS builds on this result and introduces two non-commitment types on one side. We will build on both these results and construct the war of attrition equilibrium

[5] We will explain later why the I type endogenously chooses to behave like this commitment type in Section 3.1.

when there are two non-commitment types on both sides. Formally, I show that under the continuous-time limit, given the commitment of the I types, a unique equilibrium emerges that resembles a war of attrition with multiple non-commitment types (I' and $\mathcal{U}$) on both sides who bluff and masquerade as the type who knows that the public reaction will move favorably.

An important feature of continuous-time reputational bargaining is that once a leader reveals that she is not committed, while the opponent has not done so, under continuous-time, Coase conjecture holds — the leader concedes immediately.[6] Given this Coasian result, bargaining becomes a war of attrition, where the non-commitment types only decide how long to keep bluffing. They concede and agree to the opponent's proposal when they stop bluffing. Let $F_i^{\omega_i}(t)$ be the probability that leader i stops bluffing by time t. If $F_i^{\omega_i}(0) = 0$, we say that ω_i always bluffs. If $F_i^{\omega_i}(0) = 1$, we say that ω_i never bluffs. Define

$$T_i[\omega_i] := \inf\{t | F_i^{\omega_i}(t) = \lim_{t\to\infty} F_i^{\omega_i}(t)\} \tag{1}$$

as the time by which $\omega_i \in \{I', \mathcal{U}\}$ finish bluffing, and

$$T_i := \max\{T_i[I'], T_i[\mathcal{U}]\}. \tag{2}$$

Therefore, both non-stubborn types of leader i finish bluffing by time T_i, and after time T_i, it is revealed that leader i will never concede.

Consider a non-stubborn type. Suppose she can concede right now or bluff for the next instance and then concede. Since delay is costly, she bluffs only if she believes the opponent may concede in the meantime. In equilibrium, they randomize with such probabilities that the type of opponent who randomizes remains indifferent between keep bluffing and conceding.

Let $G_j^{\omega_i}(t)$ be the belief of ω_i that her opponent $j \neq i$ will stop bluffing by time t. Recall that both $\omega_i = I'$ and $\omega_i = \mathcal{U}$ believe that the opponent is uninformed with probability ε_j. However, while $\omega_i = I'$ believes that the opponent cannot be the I' type, $\omega_i = \mathcal{U}$ assigns probability $\pi_i(1 - \varepsilon_j)$ that the opponent is the I' type. Therefore,

$$\begin{aligned} G_j^{I'}(t) &= \varepsilon_j F_j^{\mathcal{U}}(t) & (3)\\ G_j^{\mathcal{U}}(t) &= \varepsilon_j F_j^{\mathcal{U}}(t) + \pi_i(1-\varepsilon_j) F_j^{I'}(t). & (4) \end{aligned}$$

[6] I will discuss this result in Section 3.2.

Lemma 1 *In the continuous time war of attrition with multiple non-stubborn types on both sides, the following properties must hold true:*

(P1) $\omega_i = I'$ *concedes before* $\omega_i = \mathcal{U}$ *concedes.*

(P2) The two leaders will finish bluffing at the same time — that is,

$$T_L = T_R =: T.$$

(P3) If a leader believes that her opponent may not always bluff, then she will always bluff. That is, for all $i \in N, \omega_i \in \{I', \mathcal{U}\}$ *and* $j \neq i$,

$$F_i^{\omega_i}(0) \cdot G_j^{\omega_i}(0) = 0.$$

(P4) It is impossible that both leaders bluff after they learn that the public reaction will move unfavorably — that is,

$$T_L[I'] \cdot T_R[I'] = 0.$$

We can see from (3) and (4) that $G_j^{\mathcal{U}}(t) \geq G_j^{I'}(t)$. This means conditional on no agreement until time t; the uninformed type is at least as optimistic as the I' type regarding the opponent conceding in the next instance.[7] Moreover, unlike the I' type, the uninformed type does not know the state. Thus, when the uninformed type keeps bargaining, she discounts a smaller share of the surplus than the I' type. This makes the uninformed type less reluctant to keep bargaining compared to the I' type (See the reluctance to compromise discussion in Section 1). Together these two features imply that at any t if the uninformed type of leader i may concede, then the I' type of leader i must have already conceded.

In APS, two non-stubborn types on one side differ in terms of their discount rates. The authors show a one-sided skimming property where the more patient type concedes later in equilibrium. Although the different types have the same discount rate in our setup, they differ in terms of their beliefs. We refer to this first property (P1) as the two-sided skimming property.

The second (P2) and the third (P3) properties are standard in reputational bargaining. Below I provide the main intuition. For the formal argument, see AG. Consider (P2). Recall

[7]The uninformed type is strictly more optimistic than the I' type if either $F_j^{I'}(t) > 0$ or the I' opponent may concede in the next instance.

that beyond time T_i, it is evident that i is stubborn and will never concede. Suppose for contradiction $T_i < T_j$. This means j keeps bluffing even after it is clear that i is stubborn and will never concede. However, a leader will only bluff if there is a positive probability that the opponent may concede in the meantime, which is a contradiction. Next, consider (P3). Suppose that a leader i of type $\omega_i \in \{I', \mathcal{U}\}$ believes that her opponent does not always bluff. That is, leader j may concede right away with positive probability ($G_j^{\omega_i}(0) > 0$). This means she strictly prefers bluffing over conceding right away. Therefore, $F_i^{\omega_i}(0) = 0$.

Finally, the fourth property (P4) says that at least one leader who learns that the public reaction will move unfavorably will never bluff. Assume for contradiction that $T_i[I'] > 0$ for both $i \in N$. It follows from the skimming property (property (P1)) that until time $\min_{i \in N} T_i[I']$, only the I' types on both sides concede. However, since the I' type does not believe that her opponent could be the I' type, she cannot be indifferent between conceding and keep bluffing if only the I' type opponent concedes. This contradicts the fact that in equilibrium, the types who are randomizing must be indifferent.[8]

Notice that given the opponent's strategy, it is optimal for I type of any leader to insist on the same policy position that they agree to when the public reaction is commonly known. Recall that she would rather stick to the status quo than compromise more. Thus, the I type will never concede, while the $\omega_i = I', \mathcal{U}$ may concede or bluff. In equilibrium, they will bluff with such probabilities that make the types of leaders who randomize indifferent between keep bluffing and conceding. Below, I argue that there is a unique solution to $\{F_i^{\omega_i}(t)\}_{i \in N, \omega_i \in \{\mathcal{U}, I'\}}$ that satisfies the indifference conditions, and the above four properties in Lemma 1.

According to property (P4), at least one of the leaders never bluffs when she learns that the public reaction will move unfavorably (the I' types). We say that the two leaders have balanced strengths when this is true for both leaders ($T_L[I'] = T_R[I'] = 0$), and the strengths are unbalanced when one of them never bluffs, but the other bluffs after learning the public reaction will move unfavorably. We call the leader who bluffs the strong bargainer and the leader who does not bluff the weak bargainer — that is, leader i is the strong bargainer and leader $j \neq i$ is the weak bargainer when $T_i[I'] > 0$ and $T_j[I'] = 0$.

[8] A similar property holds in Fanning (2021) in a very different context. The author considers a reputational bargaining model where the leaders may privately tell a mediator that they are not committed to their demands and are willing to compromise. The mediator discloses this information when both sides reveal their willingness to compromise. The author shows that at least one leader has to give up immediately after telling the mediator that she is willing to compromise, but the mediator stays silent.

Below, we first find the condition that makes bargaining strengths balanced. Notice that when the bargaining strengths are balanced, after no concession at time 0, there is only one non-stubborn type on each side (the $\mathcal{U}$ type). This game is similar to AG, and as in AG, we can uniquely identify the condition that makes this an equilibrium.

Second, we consider unbalanced bargaining strengths. We start with the extreme case where one leader (strong bargainer) always bluffs and one (weak bargainer) never bluffs after learning that the public reaction will move unfavorably. This means after no concession at time 0; there is only one non-stubborn type of the weak bargainer ($\mathcal{U}$), while two non-stubborn types of the strong bargainer (I' or $\mathcal{U}$). This game is similar to APS, and as in APS, we can uniquely identify the condition that makes this an equilibrium.

Third, based on the above two conditions, we then classify the unbalanced environment as moderately or extremely unbalanced. When the bargaining strengths are extremely unbalanced, the strong bargainer always bluffs, and when it is only moderately unbalanced, the strong bargainer only sometimes bluffs.

Balanced Strengths

If both leaders never bluff after learning that the public reaction will move unfavorably, then

$$F_i^{I'}(0) = F_j^{I'}(0) = 1$$

or $T_i[I'] = T_j[I'] = 0$. Then, it follows from (P3) that the uninformed types will always bluff ($F_i^{\mathcal{U}}(0) = F_j^{\mathcal{U}}(0) = 1$).

After no immediate concession, the uninformed types understand that they are either facing a stubborn I type or an uninformed type. This game is same as in AG with one non-stubborn type on each side. There are two differences: (1) both leaders cannot be the stubborn type, (2) the uninformed type $\omega_i = \mathcal{U}$ also updates her belief over time regarding which way the public reaction will move:

$$\pi_j(t) \quad = \quad \frac{\pi_j\left[(1-\varepsilon_j) + \varepsilon_j(1 - F_j^{\mathcal{U}}(t))\right]}{1 - G_j^{\mathcal{U}}(t)}. \tag{5}$$

The denominator is the probability that the opponent does not concede by time t, and the numerator is the initial probability that the public reaction will move unfavorably times the probability that the opponent does not concede by time t given that the public reaction moves unfavorably. Suppose that j has not conceded until time t. If $\omega_i = \mathcal{U}$ concedes she gets $\pi_j(t)(1-x_j)$, and if she keeps bluffing and concede after time Δ she gets $e^{-r_i\Delta}\pi_j(t+\Delta)(1-x_j)$. However, if the opponent concedes in the mean time she will get x_i. Accordingly, $\omega_i = \mathcal{U}$ is indifferent if

$$\frac{G_j^{\mathcal{U}}(t+\Delta)-G_j^{\mathcal{U}}(t)}{1-G_j^{\mathcal{U}}(t)}\cdot x_i+\left(1-\frac{G_j^{\mathcal{U}}(t+\Delta)-G_j^{\mathcal{U}}(t)}{1-G_j^{\mathcal{U}}(t)}\right)e^{-r_i\Delta}\pi_j(t+\Delta)(1-x_j)=\pi_j(t)(1-x_j).$$

Rearranging and taking $\Delta \to 0$, we get[9]

$$\frac{\frac{dG_j^{\mathcal{U}}(t)}{dt}}{1-G_j^{\mathcal{U}}(t)}=\frac{r_i\pi_j(t)(1-x_j)-\pi_j'(t)(1-x_j)}{x_i-\pi_j(t)(1-x_j)}. \tag{6}$$

Differentiating $\pi_j(t)$ in (5) and substituting $\pi_j(t)$ and $\pi_j'(t)$ in the above, we get (See the Appendix)

$$\frac{\frac{dG_j^{\mathcal{U}}(t)}{dt}}{1+\pi_i(1-\varepsilon_j)-G_j^{\mathcal{U}}(t)}=\frac{r_i(1-x_j)\pi_j}{x_i-(1-x_j)\pi_j}=:\frac{1}{\eta_j}. \tag{η_j}$$

Solving this differential equation, we get

$$G_j^{\mathcal{U}}(t)=1+\pi_i(1-\varepsilon_j)-\exp\left(-\frac{1}{\eta_j}t\right).$$

When $\omega_j = \mathcal{U}$ finishes bluffing at time T_j, we have $G_j^{\mathcal{U}}(T_j)=1-\pi_j(1-\varepsilon_j)$. This gives us

$$T_j=\ln\left((1-\varepsilon_j)^{-\eta_j}\right).$$

Define, for any $j \in N$,

$$B_j^1(\theta,\varepsilon):=\frac{(1-\varepsilon_i)^{-\eta_i}}{(1-\varepsilon_j)^{-\eta_j}}. \tag{B_j^1}$$

Recall that property (P2) requires $T_L = T_R$. Therefore, such balanced of strength occurs when

$$B_j^1(\theta,\varepsilon)=1.$$

Notice that $B_L^1(\theta,\varepsilon) = 1/B_R^1(\theta,\varepsilon)$. Thus, when the bargaining strength is balanced, $B_j^1(\theta,\varepsilon)=1$ for all $j \in N$.

[9]The argument for the differentiability of $G_j^{\omega_i}(t)$ is standard in the reputational bargaining literature. See AG for details.

Unbalanced Strengths

Next, we consider the case where leader j never bluffs and leader i always bluffs when they learn that the public reaction will move unfavorably. That is,

$$F_j^{I'}(0) = 1 \text{ and } F_i^{I'}(0) = 0.$$

It follows from property (P3) that the uninformed leader i will always bluff ($F_i^{\mathcal{U}}(0) = 0$). However, the uninformed leader j may not always bluff. Let us assume that $\omega_j = \mathcal{U}$ always bluffs — that is, $F_j^{\mathcal{U}}(0) = 0$.

This game is similar to APS. If there is no concession at time 0, then there is only one non-stubborn type of leader j (type $\mathcal{U}$) but two non-stubborn types (type I' and $\mathcal{U}$) of leader i. It follows from property (P1) that there are two phases:

- Phase 1: for a time interval $[0, T_i[I']]$, $\omega_i = I'$ and $\omega_j = \mathcal{U}$ will randomize, and
- Phase 2: thereafter in the time interval $[T_i[I'], T]$, $\omega_i = \mathcal{U}$ and $\omega_j = \mathcal{U}$ will randomize.

Phase 1: Conditional on no concession from j until time t, $\omega_i = I'$ is indifferent between conceding and keep bluffing for the next Δ time if

$$\frac{G_j^{I'}(t+\Delta) - G_j^{I'}(t)}{1 - G_j^{I'}(t)} \cdot x_i + \left(1 - \frac{G_j^{I'}(t+\Delta) - G_j^{I'}(t)}{1 - G_j^{I'}(t)}\right) e^{-r_i \Delta}(1 - x_j) = (1 - x_j).$$

Notice that unlike the uninformed type, type I' knows that the public reaction will move unfavorably. Therefore, $\pi_j(t)$ does not affect this indifference condition. Rearranging and taking $\Delta \to 0$, we get (See the Appendix)

$$\frac{\frac{dG_j^{I'}(t)}{dt}}{1 - G_j^{I'}(t)} = r_i \frac{(1 - x_j)}{x_i - (1 - x_j)} =: \frac{1}{\lambda_j}. \qquad (\lambda_j)$$

Solving this differential equation, we get

$$G_j^{I'}(t) = 1 - \exp\left(-\frac{1}{\lambda_j} t\right).$$

On the other hand, $\omega_j = \mathcal{U}$ is indifference when (6) holds true (interchanging the subscripts i and j). However, unlike under the balanced strength case, $\omega_i = \mathcal{U}$ has not yet started conceding ($F_i^{\mathcal{U}}(t) = 0$). Therefore,

$$\pi_i(t) = \frac{\pi_i}{1 - G_i^{\mathcal{U}}(t)}.$$

Differentiating this, and substituting $\pi_i(t)$ and $\pi_i'(t)$ in the indifference condition (6), we get (see the appendix)

$$\frac{dG_i^{\mathcal{U}}(t)}{dt} = \frac{r_j(1-x_i)\pi_i}{x_j} = \frac{1}{\zeta_i}. \qquad (\zeta_i)$$

Solving this differential equation, we get

$$G_i^{\mathcal{U}}(t) = \frac{1}{\zeta_i}t.$$

When $\omega_i = I'$ finish bluffing at time $T_i[I']$, we have $G_i^{\mathcal{U}}(T_i[I']) = \pi_j(1-\varepsilon_i)$. This gives us

$$T_i[I'] = \zeta_i \pi_j (1 - \varepsilon_i). \qquad (7)$$

Phase 2: As is the case under balanced strength (see equation (η_j)), the $\mathcal{U}$ types from both side randomize starting from $T_i[I']$. Therefore for any leader $j \in N$

$$G_j^{\mathcal{U}}(t) = 1 + \pi_i(1-\varepsilon_j) - \left(1 + \pi_i(1-\varepsilon_j) - G_j^{\mathcal{U}}(T_i[I'])\right) \exp\left(-\frac{1}{\eta_j}(t - T_i[I'])\right). \qquad (8)$$

To specify these equilibrium beliefs for all $j \in N$, we need to know $G_j^{\mathcal{U}}(T_i[I'])$. Recall that $\omega_j = \mathcal{U}$ believes that by time $T_i[I']$, $\omega_i = I'$ has finished bluffing but the $\omega_i = \mathcal{U}$ has not even started conceding yet. Therefore,

$$G_i^{\mathcal{U}}(T_i[I']) = \pi_j(1 - \varepsilon_i).$$

However, $\omega_i = \mathcal{U}$ does not randomize in the first phase. This is because she believes that $\omega_j = I'$ never bluffs, and $\omega_j = \mathcal{U}$ is conceding too fast (or not bluffing enough). $\omega_i = I'$ randomizes in the time interval $[0, T_i[I']]$ and believes that leader j stops bluffing by time $T_i[I']$ with probability

$$G_j^{I'}(T_i[I']) = 1 - \exp\left(-\frac{1}{\lambda_j}T_i[I']\right).$$

Unlike the $\omega_i = I'$ type the uninformed type also assign probability $\pi_i(1-\varepsilon_j)$ that leader j will never bluff and concede immediately. Therefore,

$$G_j^{\mathcal{U}}(T_i[I']) = 1 + \pi_i(1-\varepsilon_j) - \exp\left(-\frac{1}{\lambda_j}T_i[I']\right).$$

Substituting $G_i^{\mathcal{U}}(T_i[I'])$ and $G_j^{\mathcal{U}}(T_i[I'])$ in (8), we get that in the time interval $[T_i[I'], T]$ the belief of the uninformed types must be as follows.

$$G_i^{\mathcal{U}}(t) = 1 + \pi_j(1-\varepsilon_i) - \exp\left(-\frac{1}{\eta_i}(t - T_i[I'])\right)$$

$$G_j^{\mathcal{U}}(t) = 1 + \pi_i(1-\varepsilon_j) - \exp\left(-\frac{1}{\lambda_j}T_i[I'] - \frac{1}{\eta_j}(t - T_i[I'])\right).$$

For any $j \in N$, when $\omega_j = \mathcal{U}$ finishes bluffing at time T_j, we have $G_j^{\mathcal{U}}(T_j) = 1 - \pi_j(1-\varepsilon_j)$. Solving this and substituting $T_i[I']$ using (7), we get

$$T_i = \ln((1-\varepsilon_i)^{-\eta_i}) + \zeta_i\pi_j(1-\varepsilon_i).$$

$$T_j = \ln((1-\varepsilon_j)^{-\eta_j}) + \zeta_i\pi_j(1-\varepsilon_i)\left(1 - \frac{\eta_j}{\lambda_j}\right).$$

Define

$$B_j^2(\theta,\varepsilon) := \frac{(1-\varepsilon_i)^{-\eta_i} \cdot \chi_j(\theta,\varepsilon)}{(1-\varepsilon_j)^{-\eta_j}}, \tag{B_j^2}$$

where

$$\chi_j(\theta,\varepsilon) := \exp\left(\frac{\eta_j}{\lambda_j}\zeta_i\pi_j(1-\varepsilon_i)\right).$$

Recall that property (P2) requires $T_i = T_j$. Therefore, the above specification constitutes an equilibrium when

$$B_j^2(\theta,\varepsilon) = 1.$$

Since $B_R^1(\theta,\varepsilon) = 1/B_L^1(\theta,\varepsilon)$, for any (θ,ε), $B_j^1 \leq 1$ for some $j \in N$. Moreover, since for any (θ,ε), $\chi_j \geq 1$, $B_j^2 \geq B_j^1$ for any $j \in N$. For any (θ,ε), B_j^1 and B_j^2 capture the relative bargaining strength of leader j in the following two events respectively: (1) Neither leader bluffs after learning that the public reaction will move unfavorably ($F_i^{I'}(0) = F_j^{I'}(0) = 1$). (2) Leader j never bluffs and leader i always bluffs after learning that the public reaction will move unfavorably ($F_i^{I'}(0) = 0$ and $F_j^{I'}(0) = 1$). When $B_j^2 \geq 1 \geq$

B_j^1, we say that leader j is moderately weak or the bargaining strengths are moderately unbalanced. When $1 \geq B_j^2$, we say that leader j is extremely weak or the bargaining strengths are extremely unbalanced.

Moderately Unbalanced Strengths

Suppose (θ, ε) is such that

$$B_j^2 \geq 1 \geq B_j^1$$

for some $j \in N$. If the second equality holds, then the strengths are exactly balanced, and neither leader bluffs after learning that the public reaction will move unfavorably ($F_i^{I'}(0) = F_j^{I'}(0) = 1$). Figure 1 plots the uninformed types' beliefs over time that the opponent is stubborn. When $F_i^{I'}(0) = F_j^{I'}(0) = 1$, the dotted curves capture these beliefs. Property (P2) requires that they reach 1 at the same time. This holds true when $B_j^1 = 1$. However, for $B_j^1 < 1$, j will take longer.

If the first equality holds, then leader j never bluffs and leader i always bluffs after learning that the public reaction will move unfavorably ($F_i^{I'}(0) = 0$ and $F_j^{I'}(0) = 1$). The dashed curves in Figure 1 capture these beliefs. They reach 1 at the same time when $B_j^2 = 1$. However, for $B_j^2 > 1$, i will take longer.

Thus, when the inequalities are strict, leader j is weak enough that leader i bluffs after learning that the public reaction will move unfavorably, but not so weak that leader i always bluffs. That is, it must be that $F_i^{I'}(0) \in (0,1)$. As $F_i^{I'}(0)$ increases $T_i[I']$ falls, and accordingly, T_i falls and T_j increases. I show that to reach an equilibrium path of belief, we must have

$$F_i^{I'}(0) = 1 + \left(\frac{\ln B_j^1}{\ln B_j^2 - \ln B_j^1} \right).$$

Notice that as long as $F_i^{I'}(0) > 0$, the uninformed types will always bluff (property (P3)).

Extremely Unbalanced Strengths

Suppose (θ, ε) is such that

$$1 \geq B_j^2.$$

Since $B_j^2 \geq B_j^1$, leader j is the weak bargainer. We can see in Figure 2 that when $1 > B_j^1$ the dotted lines do not reach 1 at the same time — that is, j takes longer to build her reputation. When $1 = B_j^2$, leader j never bluffs and leader i always bluffs after learning

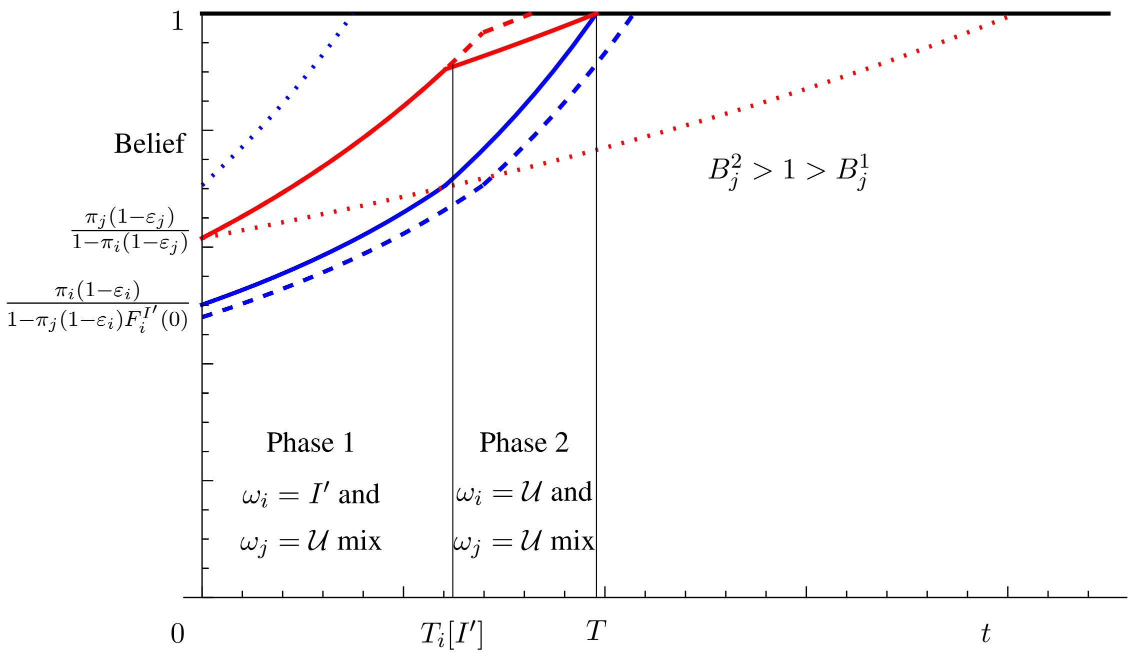

Figure 1: The vertical axis shows the belief of the uninformed type that the opponent is the I type. Red captures the belief about j and blue captures the belief about i. The dotted curves represent the belief if $F_i^{I'}(0) = F_j^{I'}(0) = 1$. We can see that j will take longer to build her reputation. This means j is a weak bargainer. The dashed curves represent the belief if $F_i^{I'}(0) = 0, F_j^{I'}(0) = 1$. We can see that i will take longer to build her reputation. This means j is only a moderately weak bargainer. The solid curves show the equilibrium belief. Specification: $r = (1, 1), \pi = (0.6, 0.4), x = (0.65, 0.915), \varepsilon = (0.2, 0.2)$.

that the public reaction will move unfavorably ($F_i^{I'}(0) = 0$ and $F_j^{I'}(0) = 1$). The dashed curves in Figure 2 capture the resulting beliefs. When $B_j^2 = 1$, they reach 1 at the same time. However, for $1 > B_j^2$, j still takes longer. This means even the uninformed type of leader j cannot always bluff ($F_j^{\mathcal{U}}(0) > 0$). As $F_j^{\mathcal{U}}(0)$ increases T_j falls but $T_i[I']$ or T_i remain unaffected. I show that to reach the equilibrium path of belief, we must have

$$F_j^{\mathcal{U}}(0) = \frac{1}{\varepsilon_j}\left(1 - (B_j^2)^{1/\eta_j}\right).$$

Notice that when $B_j^2 = 1$, we have $F_j^{\mathcal{U}}(0) = 0$ and $F_i^{I'}(0) = 0$, and when $B_j^1 = 1$, $F_i^{I'}(0) = 1$.

Finally, from $G_j^{\omega_i}(t)$, using (3), (4), we can uniquely identify $F_i^{\omega_i}(t)$ for all $i \in N$ and $\omega_i \in \{I', \mathcal{U}\}$ as shown in the following theorem. Recall that our model primitives are the bargaining environment θ and the information environment ε. To describe the unique war of

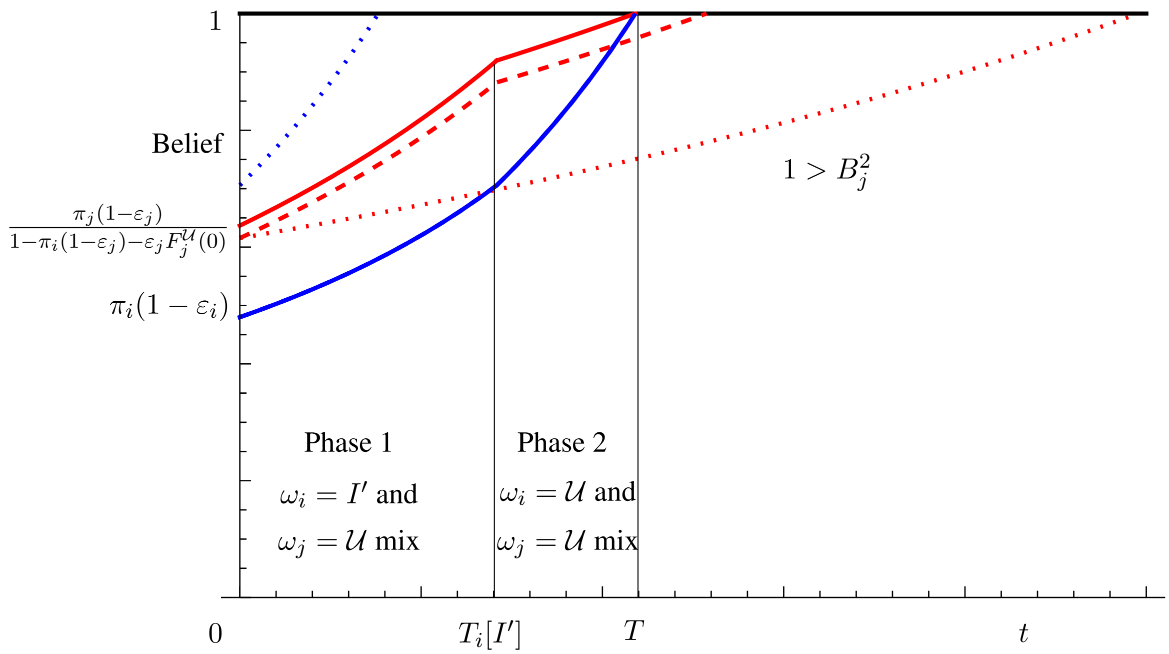

Figure 2: The vertical axis shows the belief of the uninformed type that the opponent is the I type. Red captures the belief about j and blue captures the belief about i. The dotted curves represent the belief if $F_i^{I'}(0) = F_j^{I'}(0) = 1$. We can see that j will take longer to build her reputation. This means j is a weak bargainer. The dashed curves represent the belief if $F_i^{I'}(0) = 0, F_j^{I'}(0) = 1$. We can see that j still takes longer to build her reputation (the dashed curve coincides with the solid blue curve). This means j is a very weak bargainer. The solid curves show the equilibrium belief. Specification: $r = (1,1), \pi = (0.6, 0.4), x = (0.65, 0.93), \varepsilon = (0.2, 0.2)$.

attrition equilibrium, we use the following expressions: (1) (η_j), which captures how much a leader $j \in N$ bluffs to keep $\omega_i = \mathcal{U}$ indifferent when two uninformed types randomize, (2) (λ_j), which captures how much a leader $j \in N$ bluffs to keep $\omega_i = I'$ indifferent when $\omega_i = I'$ and $\omega_j = \mathcal{U}$ randomizes, (3) (ζ_i), which captures how much a leader $i \in N$ bluffs to keep $\omega_j = \mathcal{U}$ indifferent when $\omega_i = I'$ and $\omega_j = \mathcal{U}$ randomizes, (4) (B_j^1), which captures the bargaining strength of j when neither leader bluffs after learning that the public reaction will move unfavorably, and (5) (B_j^2), which captures the bargaining strength of j when leader j never bluffs but leader i always bluffs after learning that the public reaction will move unfavorably. Notice that η_j, λ_j and ζ_i depend only on the bargaining environment θ, while B_j^1 and B_j^2 depend on both the bargaining environment θ and the information environment ε.

Theorem 1 *Under continuous time bargaining, assuming the I type is committed to the policy position that they agree to when public reaction is commonly known, a unique equilibrium emerges that resembles a war of attrition with multiple non-commitment types on both sides. In equilibrium, these non-commitment types $\omega_i \in \{I', \mathcal{U}\}$ may bluff and masquerade as the I type, and stops bluffing by time t with probability $F_i^{\omega_i}(t)$ which is specified as follows.*

1. *If (θ, ε) is such that $B_j^2(\theta, \varepsilon) \geq 1 \geq B_j^1(\theta, \varepsilon)$ for some $j \in N$, then*

$$
\begin{aligned}
&F_i^{I'}(t) = F_i^{I'}(0) + t/(\pi_j(1-\varepsilon_i)\zeta_i) \\
&F_i^{\mathcal{U}}(t) = \frac{1}{\varepsilon_i}\left(1 - \exp\left(-\frac{(t - T_i[I'])}{\eta_i}\right)\right)\mathbb{1}(t \geq T_i[I']) \\
&F_j^{I'}(t) = \mathbb{1}(t \geq 0) \\
&F_j^{\mathcal{U}}(t) = \begin{cases} \frac{1}{\varepsilon_j}\left(1 - \exp\left(-\frac{t}{\lambda_j}\right)\right) \text{ if } t \leq T_i[I'] \\ \frac{1}{\varepsilon_j}\left(1 - \exp\left(-\frac{T_i[I']}{\lambda_j} - \frac{(t - T_i[I'])}{\eta_j}\right)\right) \text{ if } t \geq T_i[I'] \end{cases}
\end{aligned}
$$

where $F_i^{I'}(0) = 1 + \left(\frac{\ln B_j^1}{\ln B_j^2 - \ln B_j^1}\right)$, $T_i[I'] = \zeta_i \pi_j (1-\varepsilon_i)(1 - F_i^{I'}(0))$, and $T = \ln((1-\varepsilon_i)^{-\eta_i}) + \zeta_i \pi_j (1-\varepsilon_i)(1 - F_i^{I'}(0))$.

2. *If (θ, ε) is such that $1 \geq B_j^2(\theta, \varepsilon)$ for some $j \in N$, then*

$$
\begin{aligned}
&F_i^{I'}(t) = t/(\pi_j(1-\varepsilon_i)\zeta_i) \\
&F_i^{\mathcal{U}}(t) = \frac{1}{\varepsilon_i}\left(1 - \exp\left(-\frac{(t - T_i[I'])}{\eta_i}\right)\right)\mathbb{1}(t \geq T_i[I']) \\
&F_j^{I'}(t) = \mathbb{1}(t \geq 0) \\
&F_j^{\mathcal{U}}(t) = \begin{cases} \frac{1}{\varepsilon_j}\left(1 - (1 - \varepsilon_j F_j^{\mathcal{U}}(0))\exp\left(-\frac{t}{\lambda_j}\right)\right) \text{ if } t \leq T_i[I']] \\ \frac{1}{\varepsilon_j}\left(1 - (1 - \varepsilon_j F_j^{\mathcal{U}}(0))\exp\left(-\frac{T_i[I']}{\lambda_j} - \frac{(t - T_i[I'])}{\eta_j}\right)\right) \text{ if } t \geq T_i[I'] \end{cases}
\end{aligned}
$$

where $F_j^{\mathcal{U}}(0) = \frac{1}{\varepsilon_j}\left(1 - (B_j^2)^{1/\eta_j}\right)$, $T_i[I'] = \zeta_i \pi_j (1-\varepsilon_i)$, and $T = \ln((1-\varepsilon_i)^{-\eta_i}) + \zeta_i \pi_j (1-\varepsilon_i)$.

In case 1, we say that the bargaining strengths are moderately unbalanced, and in case 2, we say that the bargaining strengths are extremely unbalanced. The result follows from the discussion preceding the theorem. For the formal derivations, see the appendix. Below, we highlight some of the important features of this equilibrium.

First, consider the equilibrium strategy of the strong bargainer (leader i). She bluffs even after learning that the public reaction will move unfavorably — that is, $F_i^{I'}(0) \leq 1$. When the bargaining strengths are extremely unbalanced, she always bluffs — that is $F_i^{I'}(0) = 0$. She may continue bluffing until time $T_i[I']$, where $T_i[I']$ is such that $F_i^{I'}(T_i[I']) = 1$. The uninformed strong bargainer always bluffs — that is, $F_i^{\mathcal{U}}(0) = 0$. In fact, $F_i^{\mathcal{U}}(t) = 0$ for $t \leq T_i[I']$. That is, in the first phase, while $\omega_i = I'$ randomizes between conceding and keep bluffing, the uniformed strong bargainer strictly prefers bluffing. She starts randomizing between conceding and keep bluffing after time $T_i[I']$ and may keep doing so until time T, where T is such that $F_i^{\mathcal{U}}(T) = 1$.

Second, consider the weak bargainer (leader j). She never bluffs when she learns that the public opinion will move unfavorably — that is, $F_j^{I'}(t) = \mathbb{1}(t \geq 0)$. The uninformed weak bargainer bluffs between $[0, T]$, where T is such that $F_j^{\mathcal{U}}(T) = 1$. When the bargaining strengths are extremely unbalanced, she may concede right away rather than bluffing ($F_j^{\mathcal{U}}(0) > 0$). In the first phase (time interval $[0, T_i[I']]$) she bluffs with such probability that makes the I' type of the opponent indifferent, while in the second phase (time interval $[T_i[I'], T]$), she bluffs with such probability that makes the $\mathcal{U}$ type of her opponent indifferent.

Notice that the first phase lasts for time $T_i[I']$, where $T_i[I'] = \zeta_i \pi_j (1 - \varepsilon_i)(1 - F_i^{I'}(0))$. The strong bargainer does not bluff beyond this time when she learns that the public reaction will move unfavorably. When $B_j^1 = 1$, we have balanced strengths, and $F_i^{I'}(0) = 1$, which makes $T_i[I'] = 0$. That is, there is no first phase. While if $1 \geq B_j^2$, the strengths are extremely unbalanced, and $F_i^{I'}(0) = 0$. Thus, the first phase lasts for $T_i[I'] = \zeta_i \pi_j (1 - \varepsilon_i) > 0$.

It is also important to note how the two cases are determined. If $B_j^1 = 1$, then

$$F_i^{I'}(t) = F_j^{I'}(t) = \mathbb{1}(t \geq 0).$$

That is, a leader never bluffs when she learns that the public reaction will move unfavorably. In this case, we say that the bargaining strengths are exactly balanced. If $B_j^2 = 1$, then

$$F_i^{I'}(0) = F_j^{\mathcal{U}}(0) = 0.$$

In this case, leader i always bluffs and leader j never bluffs when they learn that the public reaction will move unfavorably. This is the demarcation of leader j being moderately weak and extremely weak.

Recall from property (P3) in Lemma 1 that

$$F_i^{I'}(0) \cdot F_j^{\mathcal{U}}(0) = 0.$$

That is, if one of them ($\omega_i = I'$ and $\omega_j = \mathcal{U}$) does not always bluff, the other will always bluff. If $B_j^2 > 1 > B_j^1$, then leader j is weak enough that leader i bluffs, but not so weak that leader i always bluffs after learning that public reaction will move unfavorably. Accordingly, in equilibrium, the uninformed weak bargainer always bluffs. Thus, we have

$$F_i^{I'}(0) \in (0,1) \text{ and } F_j^{\mathcal{U}}(0) = 0.$$

On the other hand, if $1 > B_j^2$, then leader j is so weak that leader i always bluffs after learning that public reaction will move unfavorably. In addition, the uninformed weak bargainer may concede immediately rather than bluffing. Thus, we have

$$F_i^{I'}(0) = 0 \text{ and } F_j^{\mathcal{U}}(0) > 0.$$

2.3 Almost Complete Information

We have seen that when it is commonly known that the public reaction will move unfavorably, a leader concedes immediately (See Section 2.1). However, when there is a positive probability that the opponent may not know this, the leader may bluff and keep bluffing for a positive duration. For any (θ, ε), under continuous-time and commitment of the I type, Theorem 1 characterizes the resulting unique equilibrium, which resembles a war of attrition. Next, we consider the case of almost complete information — the probability that the leaders do not know the public reaction $\varepsilon = (\varepsilon_L, \varepsilon_R)$ is close to $(0,0)$. We investigate whether the distribution of the equilibrium outcome (p,t) is close to that when the public reaction is commonly known.

Notice that the I types always insist on the same policy p they agree on when the public reaction is commonly known. Since it is very likely that a leader knows the public reaction, the probability that they will reach an agreement at a different policy position is negligible. However, this does not mean they will reach such an agreement immediately. It depends on how long a leader bluffs even after learning that the public reaction will move unfavorably.

Recall (from Theorem 1) that a weak bargainer never bluffs after learning that the public reaction will move unfavorably, while a strong bargainer may bluff. So, we need to understand when ε is small, whether the strong leader keeps bluffing, and how long she keeps bluffing. Suppose, under almost complete information, she ($\omega_i = I'$, say) does not bluff for long and concedes immediately with probability $F_i^{I'}(0)$ close to 1. Then, we should see almost immediate agreement on the same policy position as we see under complete information. However, if she bluffs and keeps bluffing for a positive duration (even though there is only a small chance of concession from the opponent), we can see a delay in reaching an agreement.

As we can see from Theorem 1 whether the leader bluffs even after learning that the public reaction will move unfavorably depends on the bargaining strengths. For any given bargaining environment θ, consider a sequence of $\{\varepsilon_n\}$ that converges to zero as $n \to \infty$ and consider the corresponding equilibrium strategies $F_{in}^{\omega_i}(t)$.

Corollary 1 *Under continuous time bargaining, given any bargaining environment θ, as $\{\varepsilon_n\} \to 0$, in equilibrium, the leaders almost never bluffs when they learn that the public reaction will move unfavorably — that is, for all $i \in N$*

$$\lim_{n\to\infty} F_{in}^{I'}(t) = \mathbb{1}(t \geq 0).$$

This means that the probability that the leaders will immediately agree on policy position x_L in state $\mathscr{L}$ and $1 - x_R$ in state $\mathscr{R}$ converges to 1.

The argument involves three simple steps. First, I show that for any θ, when n is sufficiently large, the bargaining strengths cannot be extremely unbalanced. That is, for sufficiently large n, for any $j \in N$, $B_j^2(\theta, \varepsilon_n) > 1$.

Second, under such large n, I show that the bargaining strengths are almost balanced — that is, $B_j^1(\theta, \varepsilon_n)$ is close to 1 for any j. This means that even the strong bargainer, say leader i, almost never bluffs when she learns that the public reaction will move unfavorably — that is, $\lim_{n\to\infty} F_{in}^{I'}(0) = 1$. Recall that the weak bargainer never bluffs when she learns that the public reaction will move unfavorably. Therefore, $\lim_{n\to\infty} F_{in}^{I'}(t) = \mathbb{1}(t \geq 0)$ for all $i \in N$. Third, notice that when ε is small, both leaders are very likely to know the public reaction (that is, either type I or I'). If the leaders almost never bluff after learning that the public reaction will move unfavorably, then the probability that they will immediately agree on the same policy as they do when the public reaction is commonly known is close to 1.

It is important to note that we consider the bargaining environment θ as fixed in the above robustness argument. It is not necessary that for a given small ε, the bargaining strengths will be nearly balanced regardless of the bargaining environment θ. In fact, as the following corollary shows, the bargaining strengths can be extremely unbalanced. Recall that when the bargaining strengths are extremely unbalanced, the strong bargainer always bluffs and keeps bluffing for a positive duration. This means that even for small ε, there are bargaining environments in which when the public reaction favors the weak bargainer, it is impossible that the leaders will agree on the same policy position as they do when they commonly know the public reaction.

Corollary 2 *Under continuous-time bargaining, given any ε (however small), there exists a bargaining environment $\theta = (r, x, \pi)$ such that the bargaining strengths are extremely unbalanced. Accordingly, in equilibrium, the strong bargainer, (say) leader i, always bluffs when she learns that the public reaction will move unfavorably ($F_i^{I'}(0) = 0$), and may keep bluffing for a positive duration, where*

$$F_i^{I'}(t) = \frac{r_j \pi_i (1 - x_i)}{x_j \pi_j (1 - \varepsilon_i)} \cdot t.$$

This means when the public reaction moves in favor of the weak bargainer, the probability that they immediately reach an agreement on the same policy position as they do when they commonly know the public reaction is 0*.*

To understand this result, consider without loss of generality some ε such that $\varepsilon_L \geq \varepsilon_R > 0$. Imagine a bargaining environment θ in which the L constituents are extremely intolerant when $\omega = \mathscr{L}$ — that is, x_L is close to 1. This means if leader R concedes, she gets almost nothing. This makes leader R very reluctant to concede. Therefore, in equilibrium, to keep leader i indifferent between conceding and keep bluffing, the leader L must concede at a very slow rate. Otherwise, R will always bluff rather than concede. In fact, as $x_L \to 1$, $1/\lambda_L, 1/\zeta_L, 1/\eta_L$ converge to 0. Also, note that higher x_L means leader L gets a high payoff if the opponent concedes. This makes leader L more willing to keep bluffing. This means the equilibrium concession rate of leader R is also lower — that is, a higher x_L lowers $1/\lambda_R, 1/\zeta_R$ and $1/\eta_R$. However, this effect is not so severe. I show that for any ε, such that $\varepsilon_L \geq \varepsilon_R > 0$, we can find x_L sufficiently close to 1 such that

$$1 > \frac{(1 - \varepsilon_R)^{-\eta_R}}{(1 - \varepsilon_L)^{-\eta_L}} \cdot \exp\left(\frac{\eta_L}{\lambda_L} \cdot \zeta_R \pi_L (1 - \varepsilon_R)\right).$$

Thus, leader L is a very weak bargainer. Therefore, in equilibrium, leader R always bluffs ($F_R^{I'}(0) = 0$), and may keep bluffing for a positive dusration. It follows from Theorem 1 that

$$F_R^{I'}(t) = \frac{1}{\pi_L(1-\varepsilon_R)\zeta_R} \cdot t = \frac{r_L \pi_R (1-x_R)}{x_L \pi_L (1-\varepsilon_R)} \cdot t$$

which reaches 1 at $T_R[I']$, where $T_R[I'] = \zeta_R \pi_L (1-\varepsilon_R) = \frac{x_L \pi_L (1-\varepsilon_R)}{r_L \pi_R (1-x_R)} > 0$. This implies that when $\omega = \mathscr{L}$, the probability that the R leader will immediately agree to x_L is 0.

3 Discussion

3.1 Endogenous Commitment

In section 2, we assume that the I types are committed to the same policy that they agree to when the public reaction is commonly known. However, since the I type is not a commitment type (as in AG), the readers may wonder that the I type may play a very different strategy. It may constitute an equilibrium if the leaders are harshly punished through beliefs for deviating from such a proposed equilibrium strategy — identified as the weakest type and given a low continuation value. As the early literature on bargaining with two-sided asymmetric information (See, for instance, Fudenberg and Tirole (1983), Chatterjee and Samuelson (1987, 1988), Cramton (1984, 1992), Watson (1998)) pointed out such belief-based threats can lead to many equilibria. However, in our political bargaining setup, under the assumption that the leaders never make a demand which is rejected by all types of her opponent at all history of the game, we can show that the I types endogenously choose to be committed to such demands. This makes them immune to such belief-based threats.

To understand this endogenous commitment, consider leader L (say). When the leader learns that the public reaction will move favorably (I type), she will never agree to a policy position $p < x_L$. However, unlike the stubborn type in AG, she is free to reject a policy position $p \geq x_L$. Recall that when the public reaction is commonly known, it follows from Binmore, Shaked and Sutton (1989) that under $\Delta \to 0$, they immediately agree on the policy position $p = x_L$. Notice that she cannot get anything more than the worst offer she considers acceptable. I show that under incomplete information, when $\Delta \to 0$, leader L will accept any policy position $p \geq x_L$ with probability 1 and reject any policy position $p < 1 - x_R$ with probability 1. Analogously, leader R will accept any policy position $p \leq 1 - x_R$ with probability 1 and reject any policy position $p > x_L$ with probability 1.

This argument is similar in spirit to Watson (1998). The author considers two-sided asymmetric information bargaining in which the leaders are uncertain about their opponent's discount rate. He shows that a leader will accept an offer above the maximum share she can get under complete information with probability 1, and accept an offer lower than the minimum share she can get under complete information with probability 0.[10]

Recall that the $\omega_L = I$ accepts $p < x_L$ with probability 0. The above argument shows she accepts $p \geq x_L$ with probability 1. Moreover, if she ever demands a policy position $p > x_L$, it will never be accepted by any type of her opponent at any history of the game. We assume that in equilibrium, the leaders never make such demands. Thus, $\omega_L = I$ becomes endogenously committed to x_L.

Two specific features of our setup make the I types committed to the policy that they agree to when the public reaction is commonly known. The first feature is how the public reaction affects the distribution of tolerance thresholds. Consider the L leader. Recall that

$$P(x_L^k \leq x | \mathscr{L}) = x \cdot \mathbb{1}(x < 1 - x_L) + \mathbb{1}(x \geq 1 - x_L).$$

That is, when the public reaction moves favorably, the tolerant L constituents ($x_L^k > 1 - x_L$) are no longer tolerant, and their tolerance thresholds drop to $1 - x_L$, while the intolerant constituents remain as they were. To understand the role of this specific feature, consider an alternative setup where under state $\mathscr{L}$, the tolerance thresholds of each L constituent are drawn from $\mathscr{U}[0, x_L]$.

Suppose that it is commonly known that the state is $\mathscr{L}$. Then, the payoff functions of L is $e^{-r_L t}(p - x_L)/(1 - x_L)$ and that of R is $e^{-r_R t}(1 - p)$. Notice that similar to our current set up (where the payoff of L is $e^{-r_L t} p \cdot \mathbb{1}(p \geq x_L)$), L will not agree to $p < x_L$. However, unlike in our current setup, x_L also affects the intertemporal preference over $p > x_L$. A higher x_L means when L makes a small compromise ($p > x_L$), she loses more intolerant constituents. Put differently, when L keeps bargaining, she discounts a lower share $(p - x_L)/(1 - x_L)$. This makes her more willing to keep bargaining. Accordingly, under complete information, in equilibrium, she gets

$$x_L + \frac{r_R}{r_L + r_R}(1 - x_L).$$

Notice that she gets more than x_L. This result is commonly referred to as *split the differ-*

[10] Basak (2021) also makes a similar argument. For completeness, I provide the proof in the online appendix (See Lemma B.1).

ence. Each side takes the surplus they can get outside (x_L for leader L and 0 for leader R) plus her Rubinstein share of the net surplus $(1 - x_L)$. Binmore, Rubinstein and Wolinsky (1986) first pointed out the conceptual difference between "deal me out" and "split the difference" solutions while making the connection between the Rubinstein bargaining and Nash bargaining solution.

If under complete information equilibrium L gets a better policy than x_L, then under incomplete information, she may reject an offer $p \geq x_L$ when she knows that the public reaction will move favorably. This makes the I type susceptible to belief-based threats. Under optimistic conjecture, we can construct a similar war of attrition equilibrium where the I types insist on the same policy position they agree to when the public reaction is commonly known. Chatterjee and Samuelson (1988) constructed a similar war of attrition equilibrium but showed that other equilibria could be constructed as well.

The second feature is that each leader may know that the public reaction will move favorably. It follows from the first feature that the best offer leader L can get is x_L. This second feature implies that the I type never accepts a worse offer. Together, these two features give us stubbornness. In contrast, suppose that $\omega_L = I$ assigns a positive probability that the public reaction may move unfavorably. She is then willing to accept an offer lower than $p < x_L$, making her susceptible to belief-based threats.

3.2 Reputational Bargaining

The war of attrition style equilibrium is a common feature of the reputational bargaining literature. This literature is agnostic about what drives the commitment. Instead, it allows for many commitment types and finds out which commitment types the rational agent will mimic. In contrast, this paper considers a political bargaining problem with uncertainty about a common state — the public reaction. Under binary states, commitment arises endogenously (See Section 3.1), which leads to a war of attrition similar in spirit to the reputational bargaining.

AG also establishes that as the time interval between offers $\Delta \to 0$, the outcome under discrete-time bargaining equilibrium converges in distribution to the war of attrition equilibrium under the continuous-time limit (Proposition 4 in AG). Since under $\Delta \to 0$, in equilibrium, the I types always behave like the commitment types, it follows from the same argument that this limiting result holds in our setup.[11]

[11]AG establishes this limiting result under more general bargaining protocol, where the players do not

AG pointed out that this distributional convergence to the continuous-time war of attrition equilibrium outcome crucially depends on a Coasian result in Myerson (1991) (See Theorem 8.4). Formally, this result states that under $\Delta \to 0$, when a leader reveals that she is not the committed I type and the opponent has not done so, she immediately concedes. This may not be true under discrete-time. She may gradually reduce her demand rather than concede immediately. However, when $\Delta \to 0$, the author shows that (1) the bargaining must end in finite time, and (2) the non-stubborn leader always prefers to concede now over keep bargaining if she knows that the bargaining will end in the next ϵ time, for some small positive ϵ. Thus, the effect of asymmetric information overwhelms the effect of impatience. This result is commonly referred to as the reputational Coasian result. There are two no-stubborn types in our setup, and the uninformed type also updates her belief about the state. However, these differences do not qualitatively affect the reputational Coasian result.[12]

It is important to highlight some interesting differences between the nature of the war of attrition in this political bargaining and typical reputational bargaining.

First, under reputational bargaining, there is no immediate agreement when the leaders have balanced strengths. In contrast, when the leaders have balanced strengths in our setup, they give up immediately whenever they learn that the public reaction will move unfavorably.

Second, if both sides are stubborn in reputational bargaining, they will never reach an agreement. However, in our setup, the public reaction favors one side or the other. Therefore, only one of the leaders can be the stubborn type. Thus, regardless of the state, an agreement will be reached eventually.

Third, under reputational bargaining, perhaps somewhat surprisingly, the stubborn leaders get a lower expected payoff than their rational counterparts (See Sanktjohanser (2018)). This comes from the fact that, unlike the rational leader, the stubborn leader remains stubborn even in the end (at time T) when the opponent does not give up. However, the stubborn leader is stubborn in our setup because she privately learns that the state favors her. Unlike the uninformed leader, she knows that the opponent cannot be actually stubborn. Therefore, in equilibrium, the stubborn leader's expected payoff is higher than that of the uninformed leader.[13]

necessarily move alternately.

[12]For completeness, I provide the formal proof in the online appendix (See Lemma B.2).

[13]For the formal comparison of equilibrium payoff, see the online appendix.

3.3 One-sided uncertainty

Consider a sequence $\{\varepsilon_{in}\} \to 0$ and $\{\varepsilon_{jn}\} \to \varepsilon_j > 0$. That is, leader i is an informed type (I or I') with a probability of almost 1, but leader j could be uninformed.[14]

Notice that $\lim_{n\to\infty} B^1_{jn} = (1-\varepsilon_j)^{\eta_j} < 1$ — that is, in the limit, leader j is the weak bargainer. Therefore, leader j never bluffs when she learns that the public reaction will move unfavorably

$$\lim_{n\to\infty} F^{I'}_{jn}(t) = \mathbb{1}(t \geq 0).$$

Regardless of the bargaining strengths, $\lim_{n\to\infty} T_{in} = \lim_{n\to\infty} T_{in}[I']$ — that is, in the limit, the game only lasts the first phase ($[0, \lim_{n\to\infty} T_{in}[I']]$). In this first phase, $\omega_i = I'$, and $\omega_j = \mathcal{U}$ randomize between bluffing and conceding.

If $\varepsilon_j \in (1-\exp(-\zeta_i\pi_j/\lambda_j), 1)$, then

$$\lim_{n\to\infty} B^2_{jn} = (1-\varepsilon_j)^{\eta_j} \exp\left(\frac{\eta_j}{\lambda_j}\zeta_i\pi_j\right) < 1$$

Therefore, it follows from theorem 1 that in the limit, leader j is extremely weak, and

$$\lim_{n\to\infty} F^{I'}_{in}(t) = \frac{t}{\zeta_i\pi_j}$$

$$\lim_{n\to\infty} F^{\mathcal{U}}_{jn}(t) = \frac{1}{\varepsilon_j}\left(1-(1-\varepsilon_j)\exp\left(-\frac{(t-\zeta_i\pi_j)}{\lambda_j}\right)\right).$$

Notice that leader i always bluffs while even the uninformed leader j does not always bluff. In the limit, the bargaining may last until $\lim_{n\to\infty} T_{in}[I'] = \zeta_i\pi_j$.

If $\varepsilon_j \in (0, 1-\exp(-\zeta_i\pi_j/\lambda_j))$, then

$$\lim_{n\to\infty} B^2_{jn} = (1-\varepsilon_j)^{\eta_j} \exp\left(\frac{\eta_j}{\lambda_j}\zeta_i\pi_j\right) > 1.$$

Therefore, it follows from theorem 1 that in the limit, leader j is moderately weak, and

$$\lim_{n\to\infty} F^{I'}_{in}(t) = 1 + \frac{\lambda_j}{\zeta_i\pi_j}\ln(1-\varepsilon_j) + \frac{t}{\zeta_i\pi_j}$$

$$\lim_{n\to\infty} F^{\mathcal{U}}_{jn}(t) = \frac{1}{\varepsilon_j}\left(1-\exp\left(-\frac{t}{\lambda_j}\right)\right).$$

[14] Think of leader i having access to excellent data, but not leader j.

Notice that the uninformed leader j always bluffs, while leader i does not always bluff when she learns that the public reaction will move unfavorably. In the limit, the bargaining may last until $\lim_{n\to\infty} T_{in}[I'] = -\lambda_j \ln(1-\varepsilon_j)$.

4 Conclusion

This paper considers a canonical frequent offer Rubinstein bargaining game between two political leaders. The leaders are uncertain about which way the public reaction will move. The public reaction determines the tolerance of the constituents and accordingly determines the cost of compromise for the leaders. The leaders may not know the public reaction with a positive probability ε. This results in a war of attrition: a leader who knows that the public reaction will move favorably always insists on the same policy position that they agree to when the public reaction is commonly known; while a leader who does not know the public reaction or knows that it will move unfavorably bluffs and masquerade as the type who knows that the public reaction will move favorably.

We then look into whether under almost complete information (small ε), in equilibrium, the leaders keep bluffing, or almost immediately agree to the same policy position that they agree to when the public reaction is commonly known. I show that this is determined by a notion of balance of bargaining strengths. For any given bargaining environment θ, when $\varepsilon \to 0$, the leaders have almost balanced strengths, leading to no bluff and almost immediate agreement at the same policy position that they agree to when they commonly know the public reaction. However, for a given ε (however small), we can find a bargaining environment θ such that the bargaining strengths are far from balanced. Accordingly, one of the leaders (we call her the strong bargainer) will always bluff and keep doing so for a positive duration when she learns that the public reaction will move unfavorably. Thus, when the public reaction favors the weak bargainer, it is impossible that the two leaders will immediately agree at the same policy position that they agree to when the public reaction is commonly known.

Appendix

Proof of Theorem 1

It follows from Lemma 1 ((P1) and (P4)) that in equilibrium, (1) in phase 1 (if it exists), the I' type of the strong bargainer and the $\mathcal{U}$ type of the weak bargainer randomize; and (2) in phase 2, the $\mathcal{U}$ type from both sides randomize. The types who randomize must be indifferent between conceding and keep bluffing. First, we solve for these indifference conditions, and then we show that there is a unique solution to $\{F_i^{\omega_i}(t)\}$ for $i \in N$ and $\omega_i \in \{I', \mathcal{U}\}$ that satisfy these indifference conditions and Lemma 1.

Step 1: Indifference Conditions

Indifference of $\omega_i = I'$:

Conditional on no agreement until time t, the $\omega_i = I'$ is indifferent between conceding now and conceding after Δ time if

$$\frac{G_j^{I'}(t+\Delta) - G_j^{I'}(t)}{1 - G_j^{I'}(t)} \cdot x_i + \left(1 - \frac{G_j^{I'}(t+\Delta) - G_j^{I'}(t)}{1 - G_j^{I'}(t)}\right) e^{-r_i \Delta}(1 - x_j) = (1 - x_j).$$

When $\Delta \to 0$, the indifference condition boils down to

$$\frac{\frac{dG_j^{I'}(t)}{dt}}{1 - G_j^{I'}(t)} = r_i \frac{(1 - x_j)}{x_i - (1 - x_j)} = \frac{1}{\lambda_j}. \tag{A.1}$$

Notice that the hazard rate is constant as in AG. Solving differential equation (A.1) with some starting point t_0, we get

$$G_j^{I'}(t) = 1 - \left(1 - G_j^{I'}(t_0)\right) \exp\left(-\frac{1}{\lambda_j}(t - t_0)\right). \tag{A.2}$$

Indifference of $\omega_i = \mathcal{U}$:

While $\omega_i = I'$ knows that the state favors the opponent, $\omega_i = \mathcal{U}$ is uncertain about the state. After seeing no concession from leader j, she updates her belief that the state favors the opponent with probability

$$\pi_j(t) = \frac{\pi_j\left[(1-\varepsilon_j)+\varepsilon_j(1-F_j^{\mathcal{U}}(t))\right]}{1-G_j^{\mathcal{U}}(t)} = \pi_j + \frac{\pi_j\pi_i(1-\varepsilon_j)F_j^{I'}(t)}{1-G_j^{\mathcal{U}}(t)}. \tag{A.3}$$

Unlike the I' type, the $\mathcal{U}$ type gets $\pi_j(t)(1-x_j)$ if she concedes now and $\pi_j(t+\Delta)(1-x_j)$ if she concedes after time Δ. Accordingly, the $\mathcal{U}$ type is indifferent if

$$\frac{G_j^{\mathcal{U}}(t+\Delta)-G_j^{\mathcal{U}}(t)}{1-G_j^{\mathcal{U}}(t)}\cdot x_i+\left(1-\frac{G_j^{\mathcal{U}}(t+\Delta)-G_j^{\mathcal{U}}(t)}{1-G_j^{\mathcal{U}}(t)}\right)e^{-r_i\Delta}\pi_j(t+\Delta)(1-x_j) = \pi_j(t)(1-x_j).$$

Rearranging and taking $\Delta\to 0$, we get

$$\lim_{\Delta\to 0}\frac{\frac{G_j^{\mathcal{U}}(t+\Delta)-G_j^{\mathcal{U}}(t)}{\Delta}\left(x_i-e^{-r_i\Delta}\pi_j(t+\Delta)(1-x_j)\right)}{1-G_j^{\mathcal{U}}(t)}$$

$$= \lim_{\Delta\to 0}(1-x_j)\left[\frac{\pi_j(t)-e^{-r_i\Delta}\pi_j(t)}{\Delta} - e^{-r_i\Delta}\frac{\pi_j(t+\Delta)-\pi_j(t)}{\Delta}\right],$$

$$\frac{\frac{dG_j^{\mathcal{U}}(t)}{dt}}{1-G_j^{\mathcal{U}}(t)} = \frac{r_i\pi_j(t)(1-x_j)-\pi_j'(t)(1-x_j)}{x_i-\pi_j(t)(1-x_j)} = \frac{1}{\lambda_j(t)}. \tag{A.4}$$

Since $\pi_j'(t)\geq 0$ and $\pi_j(t)\leq 1$, $\frac{1}{\lambda_j} > \frac{1}{\lambda_j(t)}$. Moreover, since $\mathcal{U}$ is always more optimistic than I' that the opponent may concede in the next instance (See equations (3) and (4)), the LHS in equation (A.1) is lower than the LHS in (A.4). Thus, when the I' type is indifferent between conceding and bluffing, the $\mathcal{U}$ type strictly prefers bluffing (consistent with (P1) in lemma 1).

Recall that if $\omega_i = \mathcal{U}$ randomizes in the first phase, then it is the I' opponent who randomizes, while if $\omega_i = \mathcal{U}$ randomizes in the second phase, then it is the $\mathcal{U}$ opponent who randomizes. Thus, depending on the phase, $\omega_i = \mathcal{U}$ updates her belief $\pi_j(t)$ differently. Next, we look into these two cases separately.

I' opponent randomizes: Consider the case where the opponent of type $\omega_j = I'$ randomizes. Then, $\omega_j = \mathcal{U}$ has not started conceding yet; that is, $F_j^{\mathcal{U}}(t) = 0$. Accordingly, from equation (A.3), we get $\pi_j(t) = \frac{\pi_j}{1-G_j^{\mathcal{U}}(t)}$. Differentiating w.r.t t, we get $\pi_j'(t) = \pi_j\frac{dG_j^{\mathcal{U}}(t)}{dt}/\left(1-G_j^{\mathcal{U}}(t)\right)^2$. Substituting $\pi_j(t)$ and $\pi_j'(t)$ in the equation (A.4) and then simplifying, we get

$$\frac{\frac{dG_j^{\mathcal{U}}(t)}{dt}}{1-G_j^{\mathcal{U}}(t)} = \frac{r_i \frac{\pi_j(1-x_j)}{1-G_j^{\mathcal{U}}(t)} - \frac{\pi_j \frac{dG_j^{\mathcal{U}}(t)}{dt}(1-x_j)}{\left(1-G_j^{\mathcal{U}}(t)\right)^2}}{x_i - \frac{\pi_j(1-x_j)}{1-G_j^{\mathcal{U}}(t)}}$$

$$= \frac{r_i\pi_j(1-x_j)}{x_i(1-G_j^{\mathcal{U}}(t)) - \pi_j(1-x_j)} - \frac{\frac{dG_j^{\mathcal{U}}(t)}{dt}}{1-G_j^{\mathcal{U}}(t)}\left[\frac{\pi_j(1-x_j)}{x_i(1-G_j^{\mathcal{U}}(t)) - \pi_j(1-x_j)}\right]$$

$$\Rightarrow \frac{\frac{dG_j^{\mathcal{U}}(t)}{dt}}{1-G_j^{\mathcal{U}}(t)}\left[1 + \frac{\pi_j(1-x_j)}{x_i(1-G_j^{\mathcal{U}}(t)) - \pi_j(1-x_j)}\right] = \frac{r_i\pi_j(1-x_j)}{x_i(1-G_j^{\mathcal{U}}(t)) - \pi_j(1-x_j)}$$

$$\Rightarrow \frac{dG_j^{\mathcal{U}}(t)}{dt} = \frac{r_i(1-x_j)\pi_j}{x_i} = \frac{1}{\zeta_j}.$$

Solving this differential equation with some starting point t_0, we get

$$G_j^{\mathcal{U}}(t) = G_j^{\mathcal{U}}(t_0) + \frac{1}{\zeta_j}(t-t_0). \tag{A.5}$$

$\mathcal{U}$ opponent randomizes: Consider the case where $\omega_j = \mathcal{U}$ randomizes. Then, $\omega_j = I'$ has already conceded; that is, $F_j^{I'}(t) = 1$. Accordingly, from equation (A.3), we have $\pi_j(t) = \pi_j + \frac{\pi_j\pi_i(1-\varepsilon_j)}{1-G_j^{\mathcal{U}}(t)}$. Differentiating w.r.t t, we get $\pi_j'(t) = \pi_j\pi_i(1-\varepsilon_j)\frac{dG_j^{\mathcal{U}}(t)}{dt} / \left(1-G_j^{\mathcal{U}}(t)\right)^2$. Substituting $\pi_j(t)$ and $\pi_j'(t)$ in the equation (A.4) and then simplifying, we get

$$\frac{\frac{dG_j^{\mathcal{U}}(t)}{dt}}{1-G_j^{\mathcal{U}}(t)} = \frac{r_i(1-x_j)\frac{\pi_j\left[1+\pi_i(1-\varepsilon_j)-G_j^{\mathcal{U}}(t)\right]}{1-G_j^{\mathcal{U}}(t)} - \frac{\pi_j\pi_i(1-\varepsilon_j)\frac{dG_j^{\mathcal{U}}(t)}{dt}(1-x_j)}{\left(1-G_j^{\mathcal{U}}(t)\right)^2}}{x_i - (1-x_j)\frac{\pi_j\left[1+\pi_i(1-\varepsilon_j)-G_j^{\mathcal{U}}(t)\right]}{1-G_j^{\mathcal{U}}(t)}}$$

$$= \frac{r_i(1-x_j)\pi_j\left[1+\pi_i(1-\varepsilon_j)-G_j^{\mathcal{U}}(t)\right]}{x_i(1-G_j^{\mathcal{U}}(t)) - (1-x_j)\pi_j\left[1+\pi_i(1-\varepsilon_j)-G_j^{\mathcal{U}}(t)\right]}$$
$$- \frac{\frac{dG_j^{\mathcal{U}}(t)}{dt}}{1-G_j^{\mathcal{U}}(t)}\left[\frac{\pi_j\pi_i(1-\varepsilon_j)(1-x_j)}{x_i(1-G_j^{\mathcal{U}}(t)) - (1-x_j)\pi_j\left[1+\pi_i(1-\varepsilon_j)-G_j^{\mathcal{U}}(t)\right]}\right]$$

$$\Rightarrow \frac{\frac{dG_j^{\mathcal{U}}(t)}{dt}}{1-G_j^{\mathcal{U}}(t)}\left[1 + \frac{\pi_j\pi_i(1-\varepsilon_j)(1-x_j)}{x_i - (1-x_j)\pi_j(1+\pi_i(1-\varepsilon_j)) - (x_i-(1-x_j)\pi_j)G_j^{\mathcal{U}}(t)}\right]$$
$$= \frac{r_i(1-x_j)\pi_j\left[1+\pi_i(1-\varepsilon_j)-G_j^{\mathcal{U}}(t)\right]}{x_i - (1-x_j)\pi_j(1+\pi_i(1-\varepsilon_j)) - (x_i-(1-x_j)\pi_j)G_j^{\mathcal{U}}(t)}$$

$$\Rightarrow \frac{\frac{dG_j^{\mathcal{U}}(t)}{dt}}{1-G_j^{\mathcal{U}}(t)} =$$

$$\frac{r_i(1-x_j)\pi_j\left[1+\pi_i(1-\varepsilon_j)-G_j^{\mathcal{U}}(t)\right]}{x_i-(1-x_j)\pi_j(1+\pi_i(1-\varepsilon_j))+\pi_j\pi_i(1-\varepsilon_j)(1-x_j)-(x_i-(1-x_j)\pi_j)G_j^{\mathcal{U}}(t)}$$

$$\Rightarrow \frac{\frac{dG_j^{\mathcal{U}}(t)}{dt}}{1+\pi_i(1-\varepsilon_j)-G_j^{\mathcal{U}}(t)} = \frac{r_i(1-x_j)\pi_j}{x_i-(1-x_j)\pi_j} = \frac{1}{\eta_j}.$$

Solving this differential equation with starting some point t_0, we get

$$\int_{t_0}^{t} \frac{dG_j^{\mathcal{U}}}{1+\pi_i(1-\varepsilon_j)-G_j^{\mathcal{U}}} = \frac{1}{\eta_j}\int_{t_0}^{t} dt$$

$$\text{or, } -\ln\left(1+\pi_i(1-\varepsilon_j)-G_j^{\mathcal{U}}(t)\right)+\ln\left(1+\pi_i(1-\varepsilon_j)-G_j^{\mathcal{U}}(t_0)\right) = \frac{1}{\eta_j}(t-t_0)$$

$$\text{or, } \left(1+\pi_i(1-\varepsilon_j)-G_j^{\mathcal{U}}(t)\right) = \left(1+\pi_i(1-\varepsilon_j)-G_j^{\mathcal{U}}(t_0)\right)\exp\left(-\frac{1}{\eta_j}(t-t_0)\right)$$

This gives us

$$G_j^{\mathcal{U}}(t) = 1+\pi_i(1-\varepsilon_j)-\left(1+\pi_i(1-\varepsilon_j)-G_j^{\mathcal{U}}(t_0)\right)\exp\left(-\frac{1}{\eta_j}(t-t_0)\right). \tag{A.6}$$

Step 2: Balanced Strengths

Recall that $T_i[I'] \cdot T_j[I'] = 0$ (Lemma 1 (P4)). Let us fist consider the case where $T_i[I'] = T_j[I'] = 0$, that is, a leader never bluffs when she learns that the public reaction will move unfavorably. This means $F_i^{I'}(t) = \mathbb{1}(t \geq 0)$ for all $i \in N$. We will look for (θ, ε) such that this will hold true.

Note that since $F_i^{I'}(0) \cdot G_j^{I'}(0) = 0$ (Lemma 1 (P3)). Since $F_i^{I'}(0) > 0$ for all $i \in N$, we must have $G_j^{I'}(0) = \varepsilon_j F_j^{\mathcal{U}}(0) > 0$ for all $j \in N$. Therefore, $F_j^{\mathcal{U}}(0) = 0$ for all $j \in N$. Accordingly, $G_j^{\mathcal{U}}(0) = \pi_i(1-\varepsilon_j)$ and $G_i^{\mathcal{U}}(0) = \pi_j(1-\varepsilon_i)$. Only the uninformed types randomizes after $t = 0$. Starting from the initial time $t_0 = 0$, the uninformed types on both sides will update their beliefs that the opponent will bluff until time t according to (A.6). This gives us

$$G_j^{\mathcal{U}}(t) = 1+\pi_i(1-\varepsilon_j)-\exp\left(-\frac{1}{\eta_j}t\right).$$

This means that leader j may continue bluffing until time T_j where

$$G_j^{\mathcal{U}}(T_j) = 1 + \pi_i(1-\varepsilon_j) - \exp\left(-\frac{1}{\eta_j}T_j\right) = 1 - \pi_j(1-\varepsilon_j).$$

Solving this, we get

$$\exp\left(T_j\right) = (1-\varepsilon_j)^{-\eta_j}.$$

Since in equilibrium, $T_i = T_j$ (Lemma 1 (P2)), (θ, ε) must be such that

$$B_j^1(\theta, \varepsilon) = \frac{(1-\varepsilon_i)^{-\eta_i}}{(1-\varepsilon_j)^{-\eta_j}} = 1. \tag{A.7}$$

Notice that for any (θ, ε), $B_L^1 = 1/B_R^1$. When (θ, ε) is such that $B_L^1 = B_R^1 = 1$, we say that the bargaining strengths are balanced.

Step 3: Unbalanced Strengths

Suppose (θ, ε) is such that $B_j^1(\theta, \varepsilon) < 1$. Then, if both the I' types do not bluff, we have $T_i < T_j$ and Lemma 1 (P2) does not hold. We say that the bargaining strengths are unbalanced — leader j is the weak bargainer and leader i is the strong bargainer. To makes $T_i = T_j$, it must be that $F_i^{I'}(0) < 1$. This is because Lemma 1 (P4) means $F_i^{I'}(0) = 1$ for at least one $i \in N$, and $F_j^{I'}(0) < 1$ will widen the gap between T_i and T_j. Since $F_i^{I'}(0) < 1$, in the first phase $[0, T_i[I']]$, $\omega_i = I'$ and $\omega_j = \mathcal{U}$ randomize, and in the second phase $[T_i[I'], T]$, $\omega_i = \mathcal{U}$ and $\omega_j = \mathcal{U}$ randomize. Since $F_j^{I'}(0) = 1$, we must have $F_i^{\mathcal{U}}(0) = 0$ (Lemma 1 (P3)). Notice that $F_j^{\mathcal{U}}(0)$ can be positive. However, it follows from Lemma 1 (P3) that $F_i^{I'}(0) \cdot F_j^{\mathcal{U}}(0) = 0$.

Belief about leader i:

$\omega_j = \mathcal{U}$ believes that leader i will concede immediately with probability $G_i^{\mathcal{U}}(0) = \pi_j(1-\varepsilon_i)F_i^{I'}(0)$, and starting from $t_0 = 0$, she updates her belief that the opponent will bluff until time t according to (A.5) (interchange i and j). This gives us

$$G_i^{\mathcal{U}}(t) = \pi_j(1-\varepsilon_i)F_i^{I'}(0) + \frac{1}{\zeta_i}t. \tag{A.8}$$

By time $T_i[I']$, the I' type opponent finish bluffing. Therefore,

$$G_i^{\mathcal{U}}(T_i[I']) = \pi_j(1-\varepsilon_i)F_i^{I'}(0) + \frac{1}{\zeta_i}T_i[I'] = \pi_j(1-\varepsilon_i).$$

Solving this, we get

$$T_i[I'] = \zeta_i\pi_j(1-\varepsilon_i)(1-F_i^{I'}(0)) \tag{A.9}$$

Starting at time $t_0 = T_i[I']$, and given $G_i^{\mathcal{U}}(T_i[I']) = \pi_j(1-\varepsilon_i)$, $\omega_j = \mathcal{U}$ update her belief that the opponent will bluff until time t according to (A.6) (interchange i and j). This gives us

$$G_i^{\mathcal{U}}(t) = 1 + \pi_j(1-\varepsilon_i) - \exp\left(-\frac{1}{\eta_i}(t - T_i[I'])\right) \tag{A.10}$$

This means that leader i will keep bluffing until time T_i, where

$$G_i^{\mathcal{U}}(T_i) = 1 + \pi_j(1-\varepsilon_i) - \exp\left(-\frac{1}{\eta_i}(T_i - T_i[I'])\right) = 1 - \pi_i(1-\varepsilon_i).$$

Substituting (A.9), we get

$$\exp\left(-\frac{1}{\eta_i}(T_i - \zeta_i\pi_j(1-\varepsilon_i)(1-F_i^{I'}(0)))\right) = (1-\varepsilon_i).$$

This gives us

$$T_i = \ln((1-\varepsilon_i)^{-\eta_i}) + \zeta_i\pi_j(1-\varepsilon_i)(1-F_i^{I'}(0)). \tag{A.11}$$

Note that if $F_i^{I'}(0) = 1$, then $\exp(T_i) = (1-\varepsilon_i)^{-\eta_i}$ as in the balanced strength situation. As $F_i^{I'}(0)$ falls, it take leader i more time to finish building her reputation.

Belief about leader j:

$\omega_i = I'$ believes that leader j will concede immediately with probability $G_j^{I'}(0) = \varepsilon_j F_j^{\mathcal{U}}(0)$, and starting from $t_0 = 0$, she updates her belief that the opponent will bluff until time t according to (A.2). This gives us

$$G_j^{I'}(t) = 1 - (1-\varepsilon_j F_j^{\mathcal{U}}(0))\exp\left(-\frac{1}{\lambda_j}t\right). \tag{A.12}$$

Since, $F_j^{I'}(0) = 1$, we have

$$G_j^{\mathcal{U}}(t) = \pi_i(1-\varepsilon_j) + 1 - (1-\varepsilon_j F_j^{\mathcal{U}}(0)) \exp\left(-\frac{1}{\lambda_j} t\right). \tag{A.13}$$

Therefore, $\omega_i = \mathcal{U}$ believes that leader j will bluff until time $t = T_i[I']$ with probability

$$G_j^{\mathcal{U}}(T_i[I']) = \pi_i(1-\varepsilon_j) + 1 - (1-\varepsilon_j F_j^{\mathcal{U}}(0)) \exp\left(-\frac{1}{\lambda_j} T_i[I']\right).$$

Starting at $t_0 = T_i[I']$, and given $G_j^{\mathcal{U}}(T_i[I'])$, $\omega_i = \mathcal{U}$ believes that leader j will bluff until time t according to (A.6). This gives us

$$G_j^{\mathcal{U}}(t) = 1+\pi_i(1-\varepsilon_j)-(1-\varepsilon_j F_j^{\mathcal{U}}(0)) \exp\left(-\frac{1}{\lambda_j} T_i[I']\right) \exp\left(-\frac{1}{\eta_j}(t - T_i[I'])\right). \tag{A.14}$$

This means that leader j will continue bluffing until time T_j, where

$$\begin{aligned} G_j^{\mathcal{U}}(T_j) &= 1 + \pi_i(1-\varepsilon_j) - (1-\varepsilon_j F_j^{\mathcal{U}}(0)) \exp\left(-\frac{1}{\lambda_j} T_i[I']\right) \exp\left(-\frac{1}{\eta_j}(T_j - T_i[I'])\right) \\ &= 1 - \pi_j(1-\varepsilon_j) \end{aligned}$$

This simplifies to

$$\begin{aligned} \exp\left(-\frac{1}{\eta_j}(T_j - T_i[I'])\right) &= \frac{(1-\varepsilon_j)\exp\left(T_i[I']\right)^{\frac{1}{\lambda_j}}}{(1-\varepsilon_j F_j^{\mathcal{U}}(0))} \\ \implies \exp\left(T_j\right) &= (1-\varepsilon_j)^{-\eta_j} \cdot (1-\varepsilon_j F_j^{\mathcal{U}}(0))^{\eta_j} \cdot \exp\left(T_i[I']\right)^{1-\frac{\eta_j}{\lambda_j}} \end{aligned}$$

Substituting (A.9), we get

$$\begin{aligned} T_j &= \ln((1-\varepsilon_j)^{-\eta_j}) + \ln((1-\varepsilon_j F_j^{\mathcal{U}}(0))^{\eta_j}) \\ &+ \left(\zeta_i \pi_j (1-\varepsilon_i)(1-F_i^{I'}(0))\right)\left(1 - \frac{\eta_j}{\lambda_j}\right). \end{aligned} \tag{A.15}$$

Note that as $F_j^{\mathcal{U}}(0)$ increases T_j falls. Also, since $\lambda_j < \eta_j$, as $F_i^{I'}(0)$ increases T_j increases.

Since in equilibrium $T_i = T_j$ (Lemma 1 (P2)), it follows from equations (A.11) and

(A.15) that $F_i^{I'}(0)$ and $F_j^{\mathcal{U}}(0)$ must be such that

$$(1-\varepsilon_i)^{-\eta_i}\cdot\exp\left(\frac{\eta_j}{\lambda_j}\zeta_i\pi_j(1-\varepsilon_i)(1-F_i^{I'}(0))\right)=(1-\varepsilon_j)^{-\eta_j}(1-\varepsilon_j F_j^{\mathcal{U}}(0))^{\eta_j} \quad \text{(A.16)}$$

It follows from Lemma 1 (P3) that $F_i^{I'}(0)\cdot F_j^{\mathcal{U}}(0)=0$. If in equilibrium, $F_i^{I'}(0)=F_j^{\mathcal{U}}(0)=0$, then it must be that

$$(1-\varepsilon_i)^{-\eta_i}\cdot\exp\left(\frac{\eta_j}{\lambda_j}\zeta_i\pi_j(1-\varepsilon_i)\right)=(1-\varepsilon_j)^{-\eta_j}$$

$$\text{or, } B_j^2(\theta,\varepsilon)=\frac{(1-\varepsilon_i)^{-\eta_i}}{(1-\varepsilon_j)^{-\eta_j}}\cdot\exp\left(\frac{\eta_j}{\lambda_j}\zeta_i\pi_j(1-\varepsilon_i)\right)=1. \quad \text{(A.17)}$$

Moderately Unbalanced Strengths: Leader j is weak enough ($1>B_j^1$) that leader i may bluff even when she learns that the public reaction will move unfavorably. However, leader j is not so weak ($B_j^2>1$) that leader i always bluffs when she learns that the public reaction will move unfavorably. That is, $F_i^{I'}(0)\in(0,1)$. This implies $F_j^{\mathcal{U}}(0)=0$. Therefore, solving (A.16), we get

$$F_i^{I'}(0)=1+\left(\frac{\ln B_j^1}{\ln B_j^2-\ln B_j^1}\right).$$

Consider the strong bargainer i. In the first phase $[0,T_i[I']]$, $\omega_i=\mathcal{U}$ always bluffs ($F_i^{\mathcal{U}}(t)=0$), and $\omega_i=I'$ bluffs exactly so much that $G_i^{\mathcal{U}}(t)$ is as in equation (A.8) ($\omega_j=\mathcal{U}$ is indifferent). Recall that $G_i^{\mathcal{U}}(t)=\varepsilon_i F_i^{\mathcal{U}}(t)+\pi_j(1-\varepsilon_i)F_i^{I'}(t)$ (See equation (4)). Substituting ($F_i^{\mathcal{U}}(t)=0$), we get

$$F_i^{I'}(t)=\frac{G_j^{\mathcal{U}}(t)}{\pi_j(1-\varepsilon_i)}=F_i^{I'}(0)+t/(\pi_j(1-\varepsilon_i)\zeta_i).$$

Since $\omega_i=I'$ finish bluffing by time $T_i[I']$, $F_i^{I'}(T_i[I'])=1$. In the second phase $[T_i[I'],T]$, $\omega_i=\mathcal{U}$ bluffs exactly so much that $G_i^{\mathcal{U}}(t)$ is as in equation (A.10) ($\omega_j=\mathcal{U}$ is indifferent). Substituting $F_i^{I'}(t)=1$ in $G_i^{\mathcal{U}}(t)=\varepsilon_i F_i^{\mathcal{U}}(t)+\pi_j(1-\varepsilon_i)F_i^{I'}(t)$, we get

$$F_i^{\mathcal{U}}(t)=\frac{1}{\varepsilon_i}\left(G_i^{\mathcal{U}}(t)-\pi_j(1-\varepsilon_i)\right)=\frac{1}{\varepsilon_i}\left(1-\exp\left(-\frac{(t-T_i[I'])}{\eta_i}\right)\right).$$

Next, consider the weak bargainer j. $\omega_i = I'$ never bluffs — that is,

$$F_j^{I'}(t) = \mathbb{1}(t \geq 0).$$

In the first phase $[0, T_i[I']]$, $\omega_j = \mathcal{U}$ bluffs exactly so much that $G_j^{I'}(t)$ is as in equation (A.12) ($\omega_i = I'$ is indifferent). Recall that $G_j^{I'}(t) = \varepsilon_j F_j^{\mathcal{U}}(t)$ (See equation (3)). Therefore, for $t \in [0, T_i[I']]$,

$$F_j^{\mathcal{U}}(t) = \frac{G_j^{I'}(t)}{\varepsilon_j} = \frac{1}{\varepsilon_j}\left(1 - \exp\left(-\frac{t}{\lambda_j}\right)\right)$$

In the second phase $[T_i[I'], T]$, $\omega_j = \mathcal{U}$ bluffs exactly so much that $G_j^{\mathcal{U}}(t)$ is as in equation (A.14) ($\omega_i = \mathcal{U}$ is indifferent). Substituting $F_j^{I'}(t) = 1$ in $G_j^{\mathcal{U}}(t) = \varepsilon_j F_j^{\mathcal{U}}(t) + \pi_i(1 - \varepsilon_j)F_j^{I'}(t)$ (See equation (4)), we get that for $t \in [T_i[I'], T]$,

$$F_j^{\mathcal{U}}(t) = \frac{G_j^{\mathcal{U}}(t) - \pi_i(1-\varepsilon_j)}{\varepsilon_j} = \frac{1}{\varepsilon_j}\left(1 - \exp\left(-\frac{T_i[I']}{\lambda_j} - \frac{(t - T_i[I'])}{\eta_j}\right)\right).$$

Extremely Unbalanced Strengths: Leader j is so weak ($1 > B_j^2$) that leader i always bluffs when she learns that the public reaction will move unfavorably. That is, $F_i^{I'}(0) = 0$. Since $1 > B_j^2$, if $F_i^{I'}(0) = F_j^{\mathcal{U}}(0) = 0$, we have $T_j > T_i$. which violates Lemma 1 (P2). To make $T_j = T_i$, $F_j^{\mathcal{U}}(0)$ must be such that equation (A.16) holds. Solving this, we get

$$F_j^{\mathcal{U}}(0) = \frac{1}{\varepsilon_j}\left(1 - (B_j^2)^{1/\eta_j}\right).$$

Consider the strong bargainer i. In the first phase $[0, T_i[I']]$, $\omega_i = \mathcal{U}$ always bluffs ($F_i^{\mathcal{U}}(t) = 0$), and $\omega_i = I'$ bluffs exactly so much that $G_i^{\mathcal{U}}(t)$ is as in equation (A.8), which gives us

$$F_i^{I'}(t) = \frac{G_j^{\mathcal{U}}(t)}{\pi_j(1-\varepsilon_i)} = t/(\pi_j(1-\varepsilon_i)\zeta_i).$$

In the second phase $[T_i[I'], T]$, $\omega_i = \mathcal{U}$ bluffs exactly so much that $G_i^{\mathcal{U}}(t)$ is as in equation (A.10), which gives us

$$F_i^{\mathcal{U}}(t) = \frac{1}{\varepsilon_i}\left(G_i^{\mathcal{U}}(t) - \pi_j(1-\varepsilon_i)\right) = \frac{1}{\varepsilon_i}\left(1 - \exp\left(-\frac{(t - T_i[I'])}{\eta_i}\right)\right).$$

Next, consider the weak bargainer j. $\omega_i = I'$ never bluffs — that is,

$$F_j^{I'}(t) = \mathbb{1}(t \geq 0).$$

In the first phase $[0, T_i[I']]$, $\omega_j = \mathcal{U}$ bluffs exactly so much that $G_j^{I'}(t)$ is as in equation (A.12), which gives us

$$F_j^{\mathcal{U}}(t) = \frac{G_j^{I'}(t)}{\varepsilon_j} = \frac{1}{\varepsilon_j}\left(1 - (1 - \varepsilon_j F_j^{\mathcal{U}}(0)) \exp\left(-\frac{t}{\lambda_j}\right)\right)$$

for $t \in [0, T_i[I']]$. In the second phase $[T_i[I'], T]$, $\omega_j = \mathcal{U}$ bluffs exactly so much that $G_j^{\mathcal{U}}(t)$ is as in equation (A.14), which give us

$$F_j^{\mathcal{U}}(t) = \frac{1}{\varepsilon_j}\left(1 - (1 - \varepsilon_j F_j^{\mathcal{U}}(0)) \exp\left(-\frac{T_i[I']}{\lambda_j} - \frac{(t - T_i[I'])}{\eta_j}\right)\right)$$

for $t \in [T_i[I'], T]$. $\square$

Proof of Corollary 1

Step 1: Given θ, for sufficiently large n, the bargaining strengths cannot be extremely unbalanced.

Consider $\varepsilon_i \in (0, \tilde{\varepsilon})$ for all $i \in N$. Suppose that leader L is the strong bargainer. Then, $B_L^1(\theta, \varepsilon) = (1 - \varepsilon_R)^{-\eta_R}/(1 - \varepsilon_L)^{-\eta_L} \leq (1 - \tilde{\varepsilon})^{-\eta_R}$. The inequality follows from setting $\varepsilon_L = 0$ and $\varepsilon_R = \tilde{\varepsilon}$. Similarly, if leader R is the strong bargainer, then $B_R^1(\theta, \varepsilon) = (1 - \varepsilon_L)^{-\eta_L}/(1 - \varepsilon_R)^{-\eta_R} \leq (1 - \tilde{\varepsilon})^{-\eta_L}$. Therefore,

$$\max_{i \in \{L,R\}} B_i^1(\theta, \varepsilon) \leq (1 - \tilde{\varepsilon})^{-\max\{\eta_L, \eta_R\}}.$$

Suppose that L is very weak — that is, $1 \geq B_L^2(\theta, \varepsilon)$. Then,

$$1 \geq \frac{(1 - \varepsilon_R)^{-\eta_R} \cdot \chi_L}{(1 - \varepsilon_L)^{-\eta_L}},$$

$$\text{or,} \quad \left(\frac{(1 - \varepsilon_R)^{-\eta_R}}{(1 - \varepsilon_L)^{-\eta_L}}\right)^{\frac{1}{(1-\varepsilon_L)}} \geq \exp\left(\frac{\eta_R}{\lambda_R} \cdot \zeta_L \pi_R\right).$$

This is possible only if

$$(1-\tilde{\varepsilon})^{-\frac{1}{1-\tilde{\varepsilon}}} \geq \exp\left(\frac{1}{\max\{\eta_L,\eta_R\}}\cdot\frac{\eta_R}{\lambda_R}\cdot\zeta_L\pi_R\right).$$

Similarly, R is very weak — that is, $1 \geq B_R^2(\theta,\varepsilon)$ only if

$$(1-\tilde{\varepsilon})^{-\frac{1}{1-\tilde{\varepsilon}}} \geq \exp\left(\frac{1}{\max\{\eta_L,\eta_R\}}\cdot\frac{\eta_L}{\lambda_L}\cdot\zeta_R\pi_L\right).$$

Given the bargaining environment θ, let us define

$$k^1 := \frac{1}{\max\{\eta_L,\eta_R\}}\cdot\min\left\{\frac{\eta_R\zeta_L\pi_R}{\lambda_R},\frac{\eta_L\zeta_R\pi_L}{\lambda_L}\right\}$$

$$\text{and } \phi(\tilde{\varepsilon}) := -\frac{\ln(1-\tilde{\varepsilon})}{1-\tilde{\varepsilon}}.$$

Therefore, given θ, the bargaining strengths are extremely unbalanced only if

$$\phi(\tilde{\varepsilon}) \geq k^1.$$

Note that for a given θ, k^1 is a positive constant, while $\phi(\tilde{\varepsilon})$ is increasing in $\tilde{\varepsilon}$, and $\phi(\tilde{\varepsilon}) \to 0$ as $\tilde{\varepsilon} \to 0$. Define ε^1 such that $\phi(\varepsilon^1) = k^1$. Then, for $\tilde{\varepsilon} < \varepsilon^1$, $\phi(\tilde{\varepsilon}) < k^1$. There exists n^1 such that when $n > n^1$, $\varepsilon_{in} < \tilde{\varepsilon} < \varepsilon^1$ for all $i \in \{L,R\}$, which implies the bargaining strengths cannot be extremely unbalanced.

Step 2: Assume that n is sufficiently large such that the bargaining strengths are not extremely unbalanced. Given (θ,ε_n), let X_{in} be the random variable that captures when, in equilibrium, leader i stops bluffing after she learns that the public reaction will move unfavorably. We have $Pr(X_{in} \leq t) = F_{in}^{I'}(t)$. We show that the sequence of random variable X_n converges to the constant 0 — that is,

$$\lim_{n\to\infty} F_{in}^{I'}(t) = \mathbb{1}(t \geq 0).$$

Consider $\varepsilon_i < \tilde{\varepsilon}$ for all $i \in \{L,R\}$. If leader i is the weak bargainer, then $F_i^{I'}(t) = \mathbb{1}(t \geq 0)$. Therefore, the above claim is trivially true. Suppose that leader i is the strong bargainer. Then we have

$$|1 - F_{in}^{I'}(t)| \leq - \left(\frac{\ln B_j^1}{\ln B_j^2 - \ln B_j^1} \right)$$

$$= \frac{1}{\pi_j(1-\varepsilon_i)} \left(\frac{\lambda_j}{\zeta_i \eta_j} \right) [\eta_i \ln(1-\varepsilon_i) - \eta_j \ln(1-\varepsilon_j)]$$

$$\leq \frac{1}{\pi_j} \left(\frac{\lambda_j}{\zeta_i} \right) \left[-\frac{\ln(1-\tilde{\varepsilon})}{1-\tilde{\varepsilon}} \right].$$

The last inequality follows since $\frac{1}{1-\varepsilon_i} \leq \frac{1}{1-\tilde{\varepsilon}}$ and $\eta_i \ln(1-\varepsilon_i) - \eta_j \ln(1-\varepsilon_j) \leq -\eta_j \ln(1-\tilde{\varepsilon})$. Define

$$k^2 := \max\{\frac{\lambda_R}{\pi_R \zeta_L}, \frac{\lambda_L}{\pi_L \zeta_R}\}.$$

Recall that $\phi(\tilde{\varepsilon})$ in increasing, and as $\tilde{\varepsilon} \to 0$, $k^2\phi(\tilde{\varepsilon}) \to 0$. For any $\delta > 0$, define ε^2 such that $k^2\phi(\varepsilon^2) := \delta$. This means for $\tilde{\varepsilon} < \varepsilon^2$,

$$|1 - F_{in}^{I'}(t)| \leq k^2\phi(\tilde{\varepsilon}) < \delta.$$

Let n^2 be such that when $n > n^2$, $\varepsilon_{in} < \tilde{\varepsilon} < \varepsilon^2$ for all $i \in \{L, R\}$. Therefore, for any $\delta > 0$, when $n > \max\{n^1, n^2\}$, $\varepsilon_{in} < \tilde{\varepsilon} < \min\{\varepsilon^1, \varepsilon^2\}$ for all $i \in \{L, R\}$ and accordingly,

$$|1 - F_{in}^{I'}(t)| < \delta.$$

Step 3: Given θ, consider $n > \max\{n^1, n^2\}$ such that the probability that the I' type bluffs is at most δ. We show that as $n \to \infty$, the probability that the leader will not immediately agree on the same policy position as they do when public reaction is commonly known converges to 0.

Suppose that the state is $\mathscr{L}$, then the probability that the leaders will not immediately agree on policy x_L is at most $\varepsilon_{Rn} + (1-\varepsilon_{Rn})(1-F_{Rn}^{I'}(0)) \leq \varepsilon_{Rn} + (1-\varepsilon_{Rn})\delta$. As $n \to \infty$, this upper bound converges to 0. Similarly if the state is $\mathscr{R}$, the probability that the leaders will not immediately agree on policy $1 - x_R$ is at most $\varepsilon_{Ln} + (1-\varepsilon_{Ln})(1-F_{Ln}^{I'}(0)) \leq \varepsilon_{Ln} + (1-\varepsilon_{Ln})\delta$. As $n \to \infty$, this upper bound converges to 0. $\square$

Proof of Corollary 2

Suppose, without loss of generality, $\varepsilon_j \geq \varepsilon_i$. Consider a bargaining environment θ where x_j is close to 1. Then, $\frac{1}{\lambda_j}, \frac{1}{\zeta_j}, \frac{1}{\eta_j} \to 0$ and $\frac{1}{\lambda_i}, \frac{1}{\zeta_i}, \frac{1}{\eta_i}$ converges to a number bounded away from 0. Given continuity w.r.t x_j, we can find x_j close to 1 such that $\eta_j - \eta_i > 0$. Since $\varepsilon_j \geq \varepsilon_i$, and $\eta_j > \eta_i$, $B_j^1(\theta, \varepsilon) = \frac{(1-\varepsilon_i)^{-\eta_i}}{(1-\varepsilon_j)^{-\eta_j}} < 1$. That is, leader j is the weak bargainer. Leader j is a very weak bargainer when $1 > B_j^2(\theta, \varepsilon)$,

$$\text{or, } (1-\varepsilon_j)^{-\eta_j} > (1-\varepsilon_i)^{-\eta_i} \cdot \exp\left(\frac{\eta_j}{\lambda_j}\zeta_i\pi_j(1-\varepsilon_i)\right),$$

$$\text{or, } \frac{\eta_j}{\lambda_j}\zeta_i\pi_j < \frac{1}{1-\varepsilon_i}(\eta_i \ln(1-\varepsilon_i) - \eta_j \ln(1-\varepsilon_j)).$$

Rearranging this and using $\varepsilon_j \geq \varepsilon_i$, we get that if

$$\left(\frac{\eta_j}{\eta_j - \eta_i}\right)\frac{\zeta_i}{\lambda_j}\pi_j < \frac{-\ln(1-\varepsilon_i)}{1-\varepsilon_i} = \phi(\varepsilon_i),$$

then leader j is a very weak bargainer. Note that

$$\lim_{x_j \to 1} \frac{\eta_i}{\eta_j} = 0, \text{ and } \lim_{x_j \to 1} \frac{\zeta_i}{\lambda_j} = 0.$$

Thus, when $x_j \to 1$, the LHS converges to 0. Therefore, for any $\varepsilon_i > 0$, it follows from continuity that when x_j is sufficiently close to 1, the above inequality holds. Since leader j is a very weak bargainer, in equilibrium, leader i always bluffs — that is $F_i^{I'}(0) = 0$. We can see from Theorem 1 that for $t \in [0, T_i[I']]$, where $T_i[I'] = \zeta_i\pi_j(1-\varepsilon_i) > 0$,

$$F_i^{I'}(t) = \frac{r_j\pi_i(1-x_i)}{x_j\pi_j(1-\varepsilon_i)} \cdot t.$$

Suppose that leader L is the very weak bargainer. When $\omega = \mathscr{L}$, the probability that the leaders will immediately agree on x_L is 0. Analogously, when leader R is the very weak bargainer, and $\omega = \mathscr{R}$, the probability that the leader will immediately agree on $1 - x_R$ is 0. □

References

Abreu, Dilip, and David Pearce. 2007. "Bargaining, reputation, and equilibrium selection in repeated games with contracts." *Econometrica*, 75(3): 653–710.

Abreu, Dilip, and Faruk Gul. 2000. "Bargaining and reputation." *Econometrica*, 68(1): 85–117.

Abreu, Dilip, David Pearce, and Ennio Stacchetti. 2015. "One-sided uncertainty and delay in reputational bargaining." *Theoretical Economics*, 10(3): 719–773.

Alesina, Alberto, and Allan Drazen. 1991. "Why Are Stabilizations Delayed?" *American Economic Review*, 81(5): 1170–1188.

Atakan, Alp E., and Mehmet Ekmekci. 2014. "Bargaining and reputation in search markets." *Review of Economic studies*, 81(1): 1–29.

Ausubel, Lawrence M, Peter Cramton, and Raymond J Deneckere. 2002. "Bargaining with incomplete information." *Handbook of game theory with economic applications*, 3: 1897–1945.

Basak, Deepal. 2021. "Social value of Public Information in Bargaining." *Working Paper*.

Basak, Deepal, and Joyee Deb. 2020. "Gambling over Public Opinion." *The American Economic Review*, 110.11: 3492–3521.

Binmore, Ken, Ariel Rubinstein, and Asher Wolinsky. 1986. "The Nash bargaining solution in economic modelling." *The RAND Journal of Economics*, 176–188.

Binmore, Ken, Avner Shaked, and John Sutton. 1989. "An outside option experiment." *The Quarterly Journal of Economics*, 753–770.

Chatterjee, Kalyan, and Larry Samuelson. 1987. "Bargaining with two-sided incomplete information: An infinite horizon model with alternating offers." *The Review of Economic Studies*, 54(2): 175–192.

Chatterjee, Kalyan, and Larry Samuelson. 1988. "Bargaining under two-sided incomplete information: The unrestricted offers case." *Operations Research*, 36(4): 605–618.

Compte, Olivier, and Philippe Jehiel. 2002. "On the role of outside options in bargaining with obstinate parties." *Econometrica*, 70(4): 1477–1517.

Cramton, Peter C. 1984. "Bargaining with incomplete information: An infinite-horizon model with two-sided uncertainty." *The Review of Economic Studies*, 51(4): 579–593.

Cramton, Peter C. 1992. "Strategic delay in bargaining with two-sided uncertainty." *The Review of Economic Studies*, 205–225.

Ekmekci, Mehmet, and Hanzhe Zhang. 2021. "Reputational Bargaining with Ultimatum Opportunities." *arXiv preprint arXiv:2105.01581*.

Fanning, Jack. 2016. "Reputational Bargaining and Deadline." *Econometrica*, 84(3): 1131–1179.

Fanning, Jack. 2021. "Mediation in reputational bargaining."

Fanning, Jack, and Alexander Wolitzky. 2020. "Reputational Bargaining." http://economics.mit.edu/files/21210.

Feinberg, Yossi, and Andrzej Skrzypacz. 2005. "Uncertainty about uncertainty and delay in bargaining." *Econometrica*, 73(1): 69–91.

Fudenberg, Drew, and Jean Tirole. 1983. "Sequential bargaining with incomplete information." *The Review of Economic Studies*, 221–247.

Kambe, Shinsuke. 1999. "Bargaining with imperfect commitment." *Games and Economic Behavior*, 28(2): 217–237.

Levenotoğlu, Bahar, and Ahmer Tarar. 2005. "Prenegotiation public commitment in domestic and international bargaining." *American Political Science Review*, 99(3): 419–433.

Madarász, Kristóf. 2021. "Bargaining under the Illusion of Transparency." *American Economic Review*, 111(11): 3500–3539.

Muthoo, Abhinay. 1992. "Revocable commitment and sequential bargaining." *The Economic Journal*, 102(411): 378–387.

Myerson, Roger B. 1991. “Game theory: analysis of conflict.” *Cambridge: Mass, Harvard University*.

Özyurt, Selçuk. 2015. “Searching for a bargain: power of strategic commitment.” *American Economic Journal: Microeconomics*, 7(1): 320–53.

Rubinstein, Ariel. 1982. “Perfect equilibrium in a bargaining model.” *Econometrica*, 50(1): 97–109.

Sanktjohanser, Anna. 2018. “Optimally Stubborn.” *Working Paper*.

Tsoy, Anton. 2018. “Alternating-offer bargaining with the global games information structure.” *Theoretical Economics*, 13(2): 869–931.

Watson, Joel. 1998. “Alternating-offer bargaining with two-sided incomplete information.” *The Review of Economic Studies*, 65(3): 573–594.

Weinstein, Jonathan, and Muhamet Yildiz. 2013. “Robust predictions in infinite-horizon games—An unrefinable folk theorem.” *Review of Economic Studies*, 80.1: 365–394.

Wolitzky, Alexander. 2012. “Reputational bargaining with minimal knowledge of rationality.” *Econometrica*, 80(5): 2047–2087.

Online Appendix

Lemma B.1 *Under assumption 1, in the continuous time limit, leader L accepts policy $p \geq x_L$ with probability 1, and rejects policy $p < 1 - x_R$ with probability 1; and leader R accepts policy $p \leq 1 - x_R$ with probability 1, and rejects policy $p > x_L$ with probability 1.*

Proof.

Let $R = R_1 \times R_2$ be the set of strategy profile such that any strategy of leader i, $\sigma_i \in R_i$ is undominated with respect to R for some type ω_i.[15] Suppose that after some history h, the opponent makes an offer p. Let (h, p) be this history and $\mathcal{A}$ be the event in which a leader accepts the current offer. Let Z_i be the set of offers any $i \in N$ rejects with positive probability after some history while playing a strategy from the undominated set of strategies, R_i, that is, $Z_i := \{p \in [0,1] | \exists h \in H, \sigma_i \in R_i : \sigma_i((h,p))(\mathcal{A}) < 1\}$. Let us define

$$\overline{p}_L := \sup Z_L \text{ and } \underline{p}_R := \inf Z_R.$$

Let Y_i be the set of offers any $i \in N$ accepts with positive probability after some history while playing some strategy from R_i, that is, $Y_i := \{p \in [0,1] | \exists h \in H, \sigma_i \in R_i : \sigma_i((h,p))(\mathcal{A}) > 0\}$. Let us define

$$\underline{p}_L := \inf Y_L \text{ and } \overline{p}_j := \sup Y_j.$$

This means that leader L will accept any policy $p \geq \overline{p}_L$ with probability 1 and reject any policy $p < \underline{p}_L$ with probability 1. Similarly, leader R will accept any policy $p \leq \underline{p}_R$ with probability 1 and reject any policy $p > \overline{p}_R$ with probability 1. Since, there is a type of leader L who never accepts an offer $p < x_L$, it must be that $\overline{p}_L \geq x_L$. Also, since there is a type of a leader R who never accepts an offer $p > 1 - x_R$, it must be that $\underline{p}_R \leq 1 - x_R$.

If leader L rejects an offer and makes a counter offer $\min\{1 - x_R, \underline{p}_R\}$, it will be accepted. On the other hand, if leader R rejects an offer and makes a counter offer $\underline{p}_L$, it will be definitely rejected. Therefore,

$$\underline{p}_L \geq e^{-r_L \Delta} \min\{1 - x_R, \underline{p}_R\} \text{ and } 1 - \underline{p}_R \leq e^{-r_R \Delta}(1 - \underline{p}_L). \tag{B.1}$$

[15] Strategy σ_i is said to be conditionally dominated for leader i of type ω_i with respect to R if $\exists\ \sigma_i' \in \Sigma_i$ (where Σ_i the set of all behavioral strategies of leader i), such that after any history $h \in H$ (where H is the set of all possible histories), the expected payoff $U_i(\sigma_i', \sigma_j, \omega_i | h) \geq U_i(\sigma_i, \sigma_j, \omega_i | h)$, $\forall\ \sigma_j \in R_j$, and there exists some $h \in H$ such that $U_i(\sigma_i', \sigma_j, \omega_i | h) > U_i(\sigma_i, \sigma_j, \omega_i | h)$. A strategy is called *undominated* if it is not conditionally dominated.

Analogously, if leader R rejects an offer and makes a counter offer $\max\{x_L, \overline{p}_L\}$, it will be accepted. On the other hand, if leader L rejects an offer and makes a counter offer $\overline{p}_R$, it will be definitely rejected. Therefore,

$$1 - \overline{p}_R \geq e^{-r_R\Delta}(1 - \max\{x_L, \overline{p}_L\}) \text{ and } \overline{p}_L \leq e^{-r_L\Delta}\overline{p}_R. \tag{B.2}$$

Assume, for contradiction, that $\underline{p}_R < 1 - x_R$. Then from (B.1)

$$\underline{p}_L \geq e^{-r_L\Delta}\underline{p}_R \geq e^{-r_L\Delta}(1 - e^{-r_R\Delta}(1 - \underline{p}_L))$$

$$\text{or, } \underline{p}_L(1 - e^{-(r_L+r_R)\Delta}) \geq e^{-r_L\Delta}(1 - e^{-r_R\Delta})$$

$$\text{or, } \underline{p}_L \geq \frac{e^{-r_L\Delta}(1 - e^{-r_R\Delta})}{(1 - e^{-(r_L+r_R)\Delta})}.$$

Taking $\Delta \to 0$, we get

$$\underline{p}_L \geq \frac{r_R}{r_L + r_R}$$

Similarly, $\underline{p}_R \geq 1 - e^{-r_R\Delta}(1 - \underline{p}_L) \geq 1 - e^{-r_R\Delta}(1 - e^{-r_L\Delta}\underline{p}_R)$. Taking $\Delta \to 0$, we get

$$\underline{p}_R \geq \frac{r_R}{r_L + r_R}.$$

However, assumption 1 gives us $1 - x_R \leq \frac{r_R}{r_L+r_R}$, which contradicts $\underline{p}_R < 1 - x_R$. Therefore, $\underline{p}_R = 1 - x_R$. This implies $1 - e^{r_R\Delta}x_R \geq \underline{p}_L \geq e^{-r_L\Delta}(1 - x_R)$. As $\Delta \to 0$, we have $\underline{p}_L = 1 - x_R$.

Similarly, if $\overline{p}_L > x_L$, then from (B.2) and taking $\Delta \to 0$, we get

$$\overline{p}_L \leq \frac{r_R}{r_L + r_R},\ \overline{p}_R \leq \frac{r_R}{r_L + r_R}.$$

However, assumption 1 gives us $x_L \geq \frac{r_R}{r_L+r_R}$, which contradicts $\overline{p}_L > x_L$. Therefore, $\overline{p}_L = x_L$. This implies $1 - e^{-r_R\Delta}(1 - x_L) \geq \overline{p}_R \geq e^{r_L\Delta}x_L$. As $\Delta \to 0$, we have $\overline{p}_R = p^L$.

Thus, leader L always accept policy $p \geq x_L$ and always rejects policy $p < 1 - x_R$, and leader R always accept policy $p \leq 1 - x_R$ and always rejects policy $p > x_L$. ■

Lemma B.2 *In the continuous time limit, if leader i ever reveals that she is not committed, while leader j has not done so, then leader i immediately concedes.*

Proof.

Suppose that the leader i has revealed that she is not the I type. However, the I' of $\mathcal{U}$ type of leader i assigns a strictly positive probability that leader j could be the I type. It follows from Myerson (1991) (Theorem 8.4) that the $\omega_i = I'$ will concede immediately. However, unlike $\omega_i = I'$, the $\omega_i = \mathcal{U}$ is uncertain about the state. She may expect to get a higher payoff by keep bluffing. Nevertheless, I show below that the same argument can be extended to this case as well.

Step 1: The game must end in finite time with probability 1.

$\omega_j = I$ always insists on getting x_j share of the budget. The non-I types of leader j can always pretend to be the I type and also insist on the same. Let $\psi_j^{\omega_j}(t)$ be the probability that type ω_j of leader j insists on getting x_j until some time t. Then, $\omega_i = \mathcal{U}$ believes that leader j will insist on getting x_j with probability $\psi_j(t) = \pi_j(1-\varepsilon_j) + \pi_i(1-\varepsilon_j)\psi_j^{I'}(t) + \varepsilon_j \psi_j^{\mathcal{U}}(t)$. After reaching t, she believes that the state is down with probability

$$\pi_j(t) = \frac{\pi_j((1-\varepsilon_j) + \varepsilon_j \psi_j^{\mathcal{U}}(t))}{\psi_j(t)}.$$

Consider $t_2 > t_1$. Suppose leader j has been insisting on x_j until time t_1. If the $\omega_i = \mathcal{U}$ accepts leader j's offer then she gets $e^{-r_i t_1}\pi_j(t_1)(1-x_j)$. If she keeps bargaining until t_2, then the most she can get is

$$\frac{\psi_j(t_2)}{\psi_j(t_1)} e^{-r_i t_2}\pi_j(t_2)(1-x_j) + \left(1 - \frac{\psi_j(t_2)}{\psi_j(t_1)}\right) e^{-r_i t_1} x_i.$$

Therefore, after no agreement until time t_1, the $\omega_i = \mathcal{U}$ will keep bargaining until t_2 only if

$$\frac{\psi_j(t_2)}{\psi_j(t_1)} e^{-r_i(t_2-t_1)}\pi_j(t_2)(1-x_j) + \left(1 - \frac{\psi_j(t_2)}{\psi_j(t_1)}\right) x_i \geq \pi_j(t_1)(1-x_j).$$

This simplifies to

$$\frac{\psi_j(t_2)}{\psi_j(t_1)} \leq \frac{x_i - \pi_j(t_1)(1-x_j)}{x_i - e^{-r_i(t_2-t_1)}\pi_j(t_2)(1-x_j)} = \frac{x_i - \frac{\pi_j((1-\varepsilon_j)+\varepsilon_j\psi_j^{\mathcal{U}}(t_1))}{\psi_j(t_1)}(1-x_j)}{x_i - e^{-r_i(t_2-t_1)}\frac{\pi_j((1-\varepsilon_j)+\varepsilon_j\psi_j^{\mathcal{U}}(t_2))}{\psi_j(t_2)}(1-x_j)}$$

$$\implies \psi_j(t_2)x_i - e^{-r_i(t_2-t_1)}\pi_j((1-\varepsilon_j) + \varepsilon_j\psi_j^{\mathcal{U}}(t_2))(1-x_j)$$
$$\leq \psi_j(t_1)x_i - \pi_j((1-\varepsilon_j) + \varepsilon_j\psi_j^{\mathcal{U}}(t_1))(1-x_j)$$

$$\implies \psi_j(t_1) - \psi_j(t_2) \geq$$

$$\frac{\pi_j(1-x_j)}{x_i} \cdot \left((1-\varepsilon_j)(1-e^{-r_i(t_1-t_2)}) + \varepsilon_j(\psi_j^{\mathcal{U}}(t_1) - e^{-r_i(t_2-t_1)}\psi_j^{\mathcal{U}}(t_2))\right)$$

$$\implies \psi_j(t_1) - \psi_j(t_2) \geq \frac{\pi_j(1-x_j)}{x_i} \cdot \left((1-\varepsilon_j)(1-e^{-r_i(t_2-t_1)})\right) =: \kappa.$$

Consider $t_2 - t_1 = \tau$. Starting at time 0, the $\omega_i = \mathcal{U}$ will keep bargaining for next τ time only if she believes that leader j will insists on getting x_j with probability less than $1 - \kappa$. Repeating the same argument, she will keep bargaining for 2τ time only if she believes that leader j will insists on getting x_j with probability less than $1 - 2\kappa$, and so on. There exists K such that $1 - K\kappa < \pi_j(1 - \varepsilon_j)$. Therefore, $\omega_i = \mathcal{U}$ will keep bargaining until time $K\tau$ only if she believes that leader j will insist on getting x_j with probability less than $\pi_j(1 - \varepsilon_j)$. This contradicts that fact that leader j could be type I and always insists on getting x_j.

Step 2: $\omega_i = \mathcal{U}$ must concede immediately.

Suppose, for contradiction, that this is not true. Let $\bar{t} > 0$ be the supremum of the time such that the $\omega_i = \mathcal{U}$ has not conceded and accepted leader j's offer. Consider the last ϵ time interval, $(\bar{t} - \epsilon, \bar{t})$. Let x be the sup of $\omega_i = \mathcal{U}$'s payoff if leader j agrees to take less than x_j in $(\bar{t} - \epsilon, \bar{t} - (1 - \beta)\epsilon)$, where $\beta \in (0, 1)$. Let y be the sup of $\omega_i = \mathcal{U}$'s payoff if leader j does not do so. Let $\xi = \frac{\psi_j(\bar{t}-(1-\beta)\epsilon)}{\psi_j(\bar{t}-\epsilon)}$ be the probability $\omega_i = \mathcal{U}$ assigns to leader j not accepting anything below x_j in $(\bar{t} - \epsilon, \bar{t} - (1 - \beta)\epsilon)$.

At any t, leader j can behave like the $\omega_j = I$ and insist on getting x_j, and thus, guarantee herself $e^{-r_j(\bar{t}-t)}x_j$. So the maximum share leader j can get is $(1 - e^{-r_j(\bar{t}-t)}x_j)$. Therefore, $\omega_i = \mathcal{U}$'s expected payoff can not be higher than $\pi_j(t)(1 - e^{-r_j(\bar{t}-t)}x_j)$. This gives us

$$x \leq \pi_j(\bar{t} - \epsilon)(1 - e^{-r_j\epsilon}x_j),$$

$$y \leq e^{-r_j\beta\epsilon}\pi_j(\bar{t} - (1 - \beta)\epsilon)(1 - e^{-r_j(1-\beta)\epsilon}x_j).$$

If the $\omega_i = \mathcal{U}$ accepts leader j's offer she gets $\pi_j(\bar{t} - \epsilon)(1 - x_j)$ and if she keeps bargaining until $\bar{t}$, then she gets at most

$$(1 - \xi)\pi_j(\bar{t} - \epsilon)(1 - e^{-r_j\epsilon}x_j) + \xi e^{-r_j\beta\epsilon}\pi_j(\bar{t} - (1 - \beta)\epsilon)(1 - e^{-r_j(1-\beta)\epsilon}x_j).$$

Therefore, after no agreement until time $\bar{t}-\epsilon$, $\omega_i = \mathcal{U}$ keeps bargaining until $\bar{t}$ only if

$$\xi \leq \frac{\pi_j(\bar{t}-\epsilon)(1-e^{-r_j\epsilon}x_j-(1-x_j))}{\pi_j(\bar{t}-\epsilon)(1-e^{-r_j\epsilon}x_j)-e^{-r_j\beta\epsilon}\pi_j(\bar{t}-(1-\beta)\epsilon)(1-e^{-r_j(1-\beta)\epsilon}x_j)}$$

$$\text{or, } \frac{\psi_j(\bar{t}-(1-\beta)\epsilon)}{\psi_j(\bar{t}-\epsilon)} \leq \frac{1-e^{-r_j\epsilon}x_j-(1-x_j)}{1-e^{-r_j\epsilon}x_j-e^{-r_j\beta\epsilon}\frac{\pi_j(\bar{t}-(1-\beta)\epsilon)}{\pi_j(\bar{t}-\epsilon)}(1-e^{-r_j(1-\beta)\epsilon}x_j)}$$

After substituting $\pi_j(.)$, this simplifies to

$$(1-e^{-r_j\epsilon}x_j)\frac{\psi_j(\bar{t}-(1-\beta)\epsilon)}{\psi_j(\bar{t}-\epsilon)} - e^{-r_j\beta\epsilon}(1-e^{-r_j(1-\beta)\epsilon}x_j)\frac{(1-\varepsilon_j)+\varepsilon_j\psi_j^{\mathcal{U}}(\bar{t}-(1-\beta)\epsilon)}{(1-\varepsilon_j)+\varepsilon_j\psi_j^{\mathcal{U}}(\bar{t}-\epsilon)}$$

$$\leq 1-e^{-r_j\epsilon}x_j-(1-x_j).$$

$$\text{or, } A\xi - B\xi^j \leq \frac{1-e^{-r_j\epsilon}x_j-(1-x_j)}{(1-e^{-r_j\epsilon}x_j)-e^{-r_j\beta\epsilon}(1-e^{-r_j(1-\beta)\epsilon}x_j)}, \tag{B.3}$$

where,

$$\xi^j = \frac{(1-\varepsilon_j)+\varepsilon_j\psi_j^{\mathcal{U}}(\bar{t}-(1-\beta)\epsilon)}{(1-\varepsilon_j)+\varepsilon_j\psi_j^{\mathcal{U}}(\bar{t}-\epsilon)}$$

is the conditional probability of insistence by leader j until $\bar{t}-(1-\beta)\epsilon$ from $\bar{t}-\epsilon$ given the state favors j, and

$$A = \frac{(1-e^{-r_j\epsilon}x_j)}{(1-e^{-r_j\epsilon}x_j)-e^{-r_j\beta\epsilon}(1-e^{-r_j(1-\beta)\epsilon}x_j)}$$

$$B = \frac{e^{-r_j\beta\epsilon}(1-e^{-r_j(1-\beta)\epsilon}x_j)}{(1-e^{-r_j\epsilon}x_j)-e^{-r_j\beta\epsilon}(1-e^{-r_j(1-\beta)\epsilon}x_j)}.$$

Note that

$$A - B = 1.$$

It is easy to check that $0 \leq \xi \leq \xi^j \leq 1$.

Claim 1 *For $\beta \in (x_j, 1)$, $\exists \delta_\beta < 1$ such that when $\epsilon \to 0$, $\xi < \delta_\beta$.*

Proof. Assume for contraction that the above claim does not hold true. Then, there exists a subsequence of $\xi(\epsilon_n)$ that converges to 1 while $\epsilon_n \to 0$. If $\xi = 1$, then it must be that $\xi^j = 1$. This implies that the LHS of equation B.3 is $A - B = 1$. Taking $\epsilon \to 0$ and using L'Hospital rule, the right hand side of equation B.3 becomes

$$\frac{r_j x_j}{r_j x_j + r_j \beta(1-x_j) - r_j(1-\beta)x_j} = \frac{x_j}{\beta}.$$

Therefore, when $\beta \in (x_j, 1)$, for any sequence of $\epsilon_n \to 0$, the RHS of equation B.3 is strictly less than 1. This contradicts the inequality in equation B.3. ■

Therefore, $\omega_i = \mathcal{U}$ will play a strategy that can continue the bargaining from $(\bar{t} - \epsilon)$ to $(\bar{t} - (1-\beta)\epsilon)$ only if $\psi_j(\bar{t} - (1-\beta)\epsilon) < \delta_\beta \psi_j(\bar{t} - \epsilon)$. Repeating the same argument, the $\omega_i = \mathcal{U}$ will keep bargaining from $\bar{t} - (1-\beta)\epsilon$ to $\bar{t} - (1-\beta)^2\epsilon$ only if $\psi_j(\bar{t} - (1-\beta)^2\epsilon) < \delta_\beta \psi_j(\bar{t} - (1-\beta)\epsilon) < \delta_\beta^2 \psi_j(\bar{t} - \epsilon)$. Repeating the argument K times we get, $\psi_j(\bar{t} - (1-\beta)^K\epsilon) < \delta_\beta^K \psi_j(\bar{t} - \epsilon)$. Since leader j could be the I type and always insists on getting x_j, $\psi_j(.) \geq \pi_j(1-\varepsilon_j)$. However, for K such that $\delta_\beta^K < \pi_j(1-\varepsilon_j)$, the above inequality cannot hold true. Note that leader i always get the chance to make offers sufficiently close to $\bar{t} - (1-\beta)^m\epsilon$ for all $m = 1, 2 \dots K$. Therefore, $\omega_i = \mathcal{U}$ will never play a strategy that will continue the bargaining until $\bar{t}$. This contradicts the definition of $\bar{t}$.

■

Equilibrium payoff comparison

Let us define $V_i^{\omega_i}$ as the expected payoff of type ω_i of leader i in equilibrium. Suppose that the leader j is a weak bargainer. $\omega_j = \mathcal{U}$ plays the following strategy with positive probability — wait until T and then concede if leader i has not conceded already. Therefore,

$$V_j^{\mathcal{U}} = \left[\pi_j(1-\varepsilon_i)\int_0^{T_i[I']} e^{-r_j t} dF_i^{I'}(t) + \varepsilon_i \int_{T_i[I']}^{T} e^{-r_j t} dF_i^{\mathcal{U}}(t)\right] x_j + \pi_i(1-\varepsilon_i)e^{-r_j T}(1-x_i).$$

In contrast, $\omega_j = I$ always insists, and hence her payoff is

$$V_j^{I} = \left[(1-\varepsilon_i)\int_0^{T_i[I']} e^{-r_j t} dF_i^{I'}(t) + \varepsilon_i \int_{T_i[I']}^{T} e^{-r_j t} dF_i^{\mathcal{U}}(t)\right] x_j.$$

$$= V_j^{\mathcal{U}} + \pi_i(1-\varepsilon_i)\left(\int_0^{T_i[I']} e^{-r_j t} dF_i^{I'}(t) x_j - e^{-r_j T}(1-x_i)\right) > V_j^{\mathcal{U}}.$$

Next, consider the strong bargainer i. Unlike $\omega_j = \mathcal{U}$, the $\omega_i = \mathcal{U}$ keeps waiting after seeing no immediate concession from her opponent. She waits until $T_i[I']$ before starting to concede. She plays the following strategy with positive probability — wait until T and then concede if leader j has not conceded already. Therefore,

$$V_i^{\mathcal{U}} = \left[\pi_i(1-\varepsilon_j) + \varepsilon_j \int_{T_i[I']}^{T} e^{-r_i t} dF_j^{\mathcal{U}}(t)\right] x_i + \pi_j(1-\varepsilon_j)e^{-r_i T}(1-x_j).$$

$\omega_i = I$ always insists, and hence her payoff is

$$V_i^{I} = \left[(1-\varepsilon_j) + \varepsilon_j \int_{T_i[I']}^{T} e^{-r_i t} dF_j^{\mathcal{U}}(t)\right] x_i = V_i^{\mathcal{U}} + \pi_j(1-\varepsilon_j)\left(x_i - e^{-r_j T}(1-x_j)\right) > V_i^{\mathcal{U}}.$$

Thus, the stubborn type gets a higher expected payoff compared to the uninformed type. An uninformed leader j believes that that leader i could be stubborn with probability $\pi_i(1-\varepsilon_i)$. Unlike the stubborn type, the uninformed leader j can raise her offer at the last minute and recovers $e^{-r_j T}(1-x_i)$. In contrast, the stubborn type knows that the opponent cannot be stubborn and assigns this probability $\pi_i(1-\varepsilon_i)$ to leader i being the I' type. Therefore, with probability $\pi_i(1-\varepsilon_i)$ she gets $\int_0^{T_i[I']} e^{-r_j t} dF_i^{I'}(t) x_j$, which is strictly higher than what the uninformed type can recover by giving up at the last minute. $\square$