Non-Common Priors, Incentives, and Promotions: The Role of Learning

Matthias Fahn*and Nicolas Klein†

This version: April 6, 2022
Extended Abstract

1 Motivation and Overview of Results

Humans systematically overestimate their abilities. We think that we are better drivers than the average, more intelligent than others, and better at predicting political outcomes. Moreover, there is increasing evidence that such "overconfidence" is prevalent in the workplace as well – among managers (Huffman etl.,), but also among non-executives (Hoffman and Burks,). We are only beginning to understand the extent and perseverance of workers' overconfidence, though, and the effect it has on how firms optimally organize long-term employment relationships. Some (theoretical) studies argue that it may be cheaper for firms to incentivize overconfident workers who overestimate the chances of achieving a successful outcome. But the relevance of such "exploitation contracts" relies on their ongoing use over an extended period of time. If workers learn and update their assessments (as evidence suggests; see Yaouanq and Schwardmann,), the exploitation of workers may quickly become infeasible.

In this paper, we show that a firm's exploitation of a worker's overconfidence may *intensify* over time, even though workers incorporate informative signals and update beliefs using Bayes' rule. This result implies that employing a worker might only be profitable if he is believed to be sufficiently unproductive. Based on this, we also derive an implication for a firm's optimal promotion policy. It can be optimal to base a promotion decision on success in the current job, even if the task requirements in current and new job are *entirely unrelated.* Thereby, we provide a microfoundation for the so-called Peter Principle, that past successes are a bigger driver of promotion decisions than what appears to be optimal (see Benson etl., for recent evidence), and show that the resulting pattern can actually be optimal for firms.

*JKU Linz and CESifo, matthias.fahn@jku.at.
†Université de Montreal and CIREQ, kleinnic@yahoo.com.

2 Model Description

Our results are derived in a setting where a risk-neutral principal can hire a risk-neutral agent to work on a task, in a repeated continuous-time setting. Working on the task is costly for the agent, thus he needs to be compensated accordingly. The agent's value to the principal depends on both the agent's effort and additionally his talent (or match quality), which might either be high or low. If talent is high, the agent's effort generates some extra benefit to the principal with probability $a \in (0, 1)$ at each instant of time. If talent is low, the extra benefit is never generated. The agent's talent is initially uncertain, with each player updating his own belief about it via Bayes' rule: Once the extra benefit materializes for the first time, beliefs jump to 1. Otherwise, beliefs decrease over time. The agent is *overconfident* about his talent, i.e., his starting belief of being talented exceeds the principal's.

The agent's effort as well as the realization of the extra benefit of high talent are verifiable, thus can be part of a formal, court-enforceable, employment contract. Therefore, it is possible to incentivize the agent by deterministically compensating him for his effort cost. Because of the agent's overconfidence, though, this is not optimal. Instead, as long as there is uncertainty (i.e., until a first success has been realized), the principal finds it optimal only to pay the agent conditionally on success. The reason is that the principal-expected payment given this arrangement is below the effort cost, whereas the agent's overconfidence makes him believe that his costs are covered. Only once all uncertainty has been resolved after a success, and the agent's overconfidence has disappeared, a constant payment becomes optimal.

The expected payment from principal to agent, and therefore the extent of the agent's exploitation, depend on the ratio between the principal's and the agent's belief. This ratio decreases over time if there is no success, and with it the agent's expected compensation. Therefore, even though the agent learns and becomes increasingly pessimistic, he is exploited *more*. The reason is that, although his overconfidence diminishes in absolute terms (and converges to zero), it increases in relative terms.

This implies that the principal's profits from incentivizing the agent to exert effort increase as long as there is no success, because the required compensation decreases in expectation. However, the total profits from hiring the agent also contain the extra benefit in case he is talented, and this component decreases (in expectation) as long as no success is observed. It is possible, though, that the effect of more exploitation dominates the impact of the lower extra benefit in the principal's objective. This is the case if the latter is not too high, or if the initial extent of the agent's overconfidence is sufficiently large. In this case, the principal's expected profits *increase with failures* (if no success has so far been realized). Moreover, if the principal's outside option is so attractive that hiring an agent known to be talented is not profitable, it can then be optimal to hire him with *sufficiently pessimistic* beliefs.

After deriving these results, we extend the benchmark model and assume the principal's firm encompasses two stages at which the agent can work, and that he starts in the first. Both stages contain the same setting as before, and the probability that the agent is talented in the second stage is independent of the agent's talent in the first stage. Nevertheless, it might be optimal to make "promotion" to the second stage conditional on a success in the first (and even though someone else is potentially better suited). The reason is that a first-stage success reveals the agent to be talented there. Although this may increase the principal's profits when keeping the agent at his original position, it wipes out exploitation opportunities. Promoting him to the second stage again introduces uncertainty regarding his talent, and thus again creates room to exploit his overconfidence. Moreover, a worker who is currently not successful but who is expected to be talented in the second stage may instead not be promoted because his continued lack of success increases the firm's profits by exploiting him.

This mechanism might render a promotion policy optimal in which agents are promoted to their first level of incompetence. Therefore, we provide a micro-foundation for the Peter Principle and show that following it might actually be optimal for firms.

3 Model

In our baseline model (where we do not consider promotion opportunities), a principal and an agent interact in continuous time over an infinite horizon. At each instant $t \in \mathbb{R}_+$, the principal can either hire the agent or produce himself. If he produces himself in $[t, t+dt)$, he receives a profit flow of $\bar{\pi} dt \geq 0$. If the agent is hired at instant t, the agent chooses his effort level at instant t, $e_t \in \{0, 1\}$; we impose that e_t be left-continuous as a function of t. Choosing an effort of $e_\tau = 1$ ($e_\tau = 0$) for a.a.$\tau \in [t, t+dt)$ entails a cost of $cdt > 0$ (0) to the agent. The choice of effort is observable and contractible. The agent's time-invariant talent $\theta \in \{0, 1\}$ determines, together with the agent's effort choice, the principal's profit flow over those time intervals in which the agent is hired.

Indeed, if the agent is hired at a flow wage of $W \in \mathbb{R}_+$, and exerts effort e, over a time interval $[t, t+dt)$, the principal's profit flow over that period is given by $(e-W)dt + \eta$ with probability $\theta a dt + o(dt)$, and $e - W$ with the counter-probability, for some $a \in (0, 1)$ and $\eta > 0$. Thus, a talented agent (i.e., one with $\theta = 1$) yields the principal an extra profit of η at an instantaneous rate of $adt + o(dt)$. The principal initially believes that the agent is talented with probability $p_0^P \in (0, 1)$; the agent initially believes that he is talented with probability $p_0^A \in [p_0, 1)$. We thus assume that $p_0^A \geq p_0^P$, i.e., the agent is *over-confident* (whether this relates to his general ability or the match quality is not important for us). Both players update their respective beliefs according to Bayes' rule: as soon as an extra profit has been observed, both players' beliefs jump to 1, and stay there. If no extra profit has arrived by period t, the party i's belief can be written as $p_t^i = \frac{p_0^i e^{-a\int_0^t e_\tau \, d\tau}}{p_0^i e^{-a\int_0^t e_\tau \, d\tau} + 1 - p_0}$. In what follows, we shall write beliefs in the form of the odds ratio $x_t^i = \frac{p_t^i}{1-p_t^i} = x_0^i e^{-a\int_0^t e_\tau \, d\tau}$. Thus, $\frac{x_t^P}{x_t^A} = \Psi \in (0,1]$ is constant over time; Ψ is an inverse measure of the agent's over-confidence, with $\Psi = 1$ corresponding to the case of common priors. In the following, we shall simply write x_t (Ψx_t) for the agent's (principal's) belief at instant t.

Contracts Only spot contracts are possible. These specify the agent's instantaneous wage payment as a function of the agent's current effort and the principal's current profit. Clearly, it is not optimal to pay the agent anything in case he does not exert any effort. Moreover, with $\Psi \leq 1$, it is (weakly) optimal only to pay the agent conditionally on his exerting effort *and* producing an extra profit for the principal.[1] Since it is suboptimal to leave the agent a rent, the lump-sum wage payment in this case will amount to $W_t = \frac{1+x_t}{a x_t} c$. Both parties discount future payoffs at the rate of $r > 0$.

Lemma: *W_t is decreasing in x_t (hence increasing in time t if there is no success). The principal-expected cost of hiring the agent*

$$\frac{p_t^P}{p_t^A} c = a\Psi \frac{x_t}{1+\Psi x_t} W_t = \frac{1+x_t}{1+\Psi x_t} \Psi c$$

is increasing in x_t (hence decreasing in time t if there is no success). While $\frac{x_t^P}{x_t^A} = const$, $\frac{p_t^P}{p_t^A}$ decreases over time if there is no success.

This result implies that an agent for whom a success is not observed does not benefit from learning, to the contrary: As time proceeds and negative signals accumulate, his expected compensation goes down as well.

Now, the principal's strategy boils down to, in each period, choosing whether to hire the agent,[2] as a function of the previous history. Clearly, it is without loss to restrict him to choosing a Markov strategy, i.e., an effort process $(e_t)_{t \in \mathbb{R}_+}$ such that $e_t = e(x_t)$ for all $t \in \mathbb{R}_+$, where $e : \mathbb{R}_+ \cup \{\infty\} \to \{0,1\}$ is a time-invariant function of beliefs. In summary, the principal chooses a Markov strategy so as to maximize

$$\Pi(x) = \mathbb{E}\left[\int_0^\infty r e^{-rt} \left(1 - \frac{\Psi x_0}{1+\Psi x_0}\left(1 - e^{-a\int_0^t e(x_\tau)\, d\tau}\right)\right) e(x_t) \right.$$
$$\left. \times \left(1 - \bar{\pi} - \frac{1+x_t}{\frac{1}{\Psi} + x_t} c + \frac{\Psi x_t}{1+\Psi x_t} a \left(\eta + \max\left\{0, \frac{1-\bar{\pi}+a\eta-c}{r}\right\}\right)\right) \middle| x_0 = x \right]. \quad (1)$$

Noting that $1 - \frac{\Psi x_0}{1+\Psi x_0}\left(1 - e^{-a\int_0^t e(x_\tau)\, d\tau}\right) = \frac{1+\Psi x_t}{1+\Psi x_0}$, and neglecting the constant factor $\frac{\Psi}{1+\Psi x_0}$, allows us to re-write the objective as

$$\mathbb{E}\left[\int_0^\infty re^{-rt}e(x_t)\left\{\left(\frac{1}{\Psi}+x_t\right)(1-\bar{\pi})-(1+x_t)c+x_t a\left(\eta+\max\left\{0,\frac{1-\bar{\pi}+a\eta-c}{r}\right\}\right)\right\}|x_0=x\right].$$

Bellman Equation

We now set up the Bellman equation for the problem. It is given by

$$\begin{aligned} rV^*(x) = \max\Bigg\{0, r\left[1-\bar{\pi}=\frac{1+x}{1+\Psi x}\Psi c+\frac{\Psi x}{1+\Psi x}a\eta\right] \\ +\frac{\Psi x a}{1+\Psi x}\left[\max\{0,1-\bar{\pi}+a\eta-c\}-V^*(x)\right]-axV^{*'}(x)\Bigg\}. \end{aligned}$$

The value function $V^*(x)=\max\{0,V(x)\}$, where V satisfies the ODE

$$\begin{aligned} &ax(1+\Psi x)V'(x)+(r+\Psi x(r+a))V(x) \\ &\qquad = r\left[(1+\Psi x)(1-\bar{\pi})-(1+x)\Psi c+\Psi x a\eta\right]+\Psi x a\max\left\{0,1-\bar{\pi}+a\eta-c\right\}, \end{aligned}$$

which is solved by

$$V(x)=1-\bar{\pi}+\frac{\Psi x}{1+\Psi x}a\eta-c\Psi\frac{1+x}{1+\Psi x}-\mathbb{1}_{\{1-\bar{\pi}+a\eta-c<0\}}\frac{a}{a+r}\frac{\Psi x}{1+\Psi x}(1-\bar{\pi}+a\eta-c)+C\frac{1}{x^{\frac{r}{a}}(1+\Psi x)}.$$

We furthermore note that[3]

$$V(0)=1-\bar{\pi}-\Psi c;$$
$$V(\infty)=(1-\bar{\pi}+a\eta-c)\left(1-\mathbb{1}_{\{1-\bar{\pi}+a\eta-c<0\}}\frac{a}{a+r}\right);$$

while the payoff from playing risky forever is given by

$$\hat{V}(x)=1-\bar{\pi}+\frac{\Psi x}{1+\Psi x}a\eta-c\Psi\frac{1+x}{1+\Psi x}-\mathbb{1}_{\{1-\bar{\pi}+a\eta-c<0\}}\frac{a}{a+r}\frac{\Psi x}{1+\Psi x}(1-\bar{\pi}+a\eta-c).$$

First Results

Proposition:

1. *If* $\min\{1-\bar{\pi}-c\Psi,1-\bar{\pi}+a\eta-c\}\geq 0$, $e(x)=1$ *for all* $x\in\mathbb{R}_+\cup\{\infty\}$ *is optimal. The value function in this case is strictly increasing and strictly concave, and is given by* $V^*(x)=1-\bar{\pi}+\frac{\Psi x}{1+\Psi x}a\eta-\frac{1+x}{1+\Psi x}c\Psi$.

2. *If* $\max\{1-\bar{\pi}-c\Psi,1-\bar{\pi}+a\eta-c\}\leq 0$, $e(x)=0$ *for all* $x\in\mathbb{R}_+\cup\{\infty\}$ *is optimal. The value function is* $V^*=0$ *in this case.*

3. *If* $1-\bar{\pi}-c\Psi < 0 < 1-\bar{\pi}+a\eta-c$, $e = \mathbb{1}_{(x^*,\infty]}$, *with* $x^* = -\frac{r}{r+a}\frac{1-\bar{\pi}-\Psi c}{\Psi(1-\bar{\pi}+a\eta-c)}$, *is optimal. The value function is given by*

$$V^*(x) = \mathbb{1}_{[0,x^*]}(x)\left[\frac{x^{-\frac{r}{a}}C}{1+\Psi x} + \frac{1-\bar{\pi}-c\Psi+\frac{(1-c+a\eta-\bar{\pi})\Psi x(a+r)}{a+r}}{1+\Psi x}\right],$$

   for $x > x^*$, *where* $C = -\frac{a(1-\bar{\pi}-c\Psi)}{a+r}x^{*\frac{r}{a}}$ *is a constant of integration determined by value matching at* $x = x^*$. *On* (x^*,∞), V^* *is strictly increasing, and strictly convex (concave) on* $(x^*,\tilde{x})$ $((\tilde{x},\infty))$, *for some inflection point* $\tilde{x} \in (x^*,\infty)$.

4. *If* $1-\bar{\pi}+a\eta-c < 0 < 1-\bar{\pi}-c\Psi$, $e = \mathbb{1}_{[0,\check{x}]}$, *with* $\check{x} = -\frac{1-\bar{\pi}-c\Psi}{\Psi(1-\bar{\pi}+a\eta-c)} = \left(1+\frac{a}{r}\right)x^*$. *The value function in this case is given by* $V^*(x) = \mathbb{1}_{[0,\check{x}]}(x)\left[1-\bar{\pi}+\frac{\Psi x}{1+\Psi x}a\eta - \frac{1+x}{1+\Psi x}c\Psi\right]$; *it is flat on* $[\check{x},\infty)$, *and strictly decreasing and strictly convex on* $(0,\check{x})$.

The value function is C^1 *in all four cases.*

Proof: This follows from the Bellman equation via a standard verification argument. ∎

Remark In case (3.), $\frac{\partial x^*}{\partial \Psi} > 0$. In case (4.), $\frac{\partial \check{x}}{\partial \Psi} < 0$.

Thus, more similar beliefs (higher Ψ) lead to less experimentation.

References

Benson, A., Li, D., and Shue, K. (2019). Promotions and the peter principle$*. The Quarterly Journal of Economics, 134(4) : 2085--2134$.

Hoffman, M. and Burks, S.. (2020). Worker overconfidence: Field evidence and implications for employee turnover and firm profits. *Quantitative Economics*, 11(1):315–348.

Huffman, D., Raymond, C., and Shvets, J. (2021). Persistent overconfidence and biased memory: Evidence from managers.

Yaouanq, Y.. and Schwardmann, P. (2022). Learning about one's self. *Journal of the European Economic Association.*