PROBABILISTIC SPATIAL POWER INDICES

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1. Extended abstract

Cooperative game theory has been applied successfully to measure the power of agents in voting situations, which are represented by simple games. A winning coalition is assigned a worth of one, and a losing one a worth of zero. The Shapley-Shubik index (1954) and the Banzhaf index (1965) can be seen as the best known indices for measuring power. They both take into account whether the presence of an agent changes a losing coalition into a winning one, i.e. whether an agent is pivotal.

A paper by Owen (1971) inspired Shapley (1977) to propose a spatial power index: the Owen-Shapley power index (see also Owen and Shapley, 1989). In this new model, ideological differences between the agents can be taken into account. It is formalized by means of a spatial game, which is a simple game together with a constellation of agent profiles, i.e., a set of vectors in the Euclidean space $\mathbb{R}^m$ that represents the ideological locations of voters. The different dimensions can be seen as ideological considerations or criteria, so each position represents the "ideal point" (of highest preference) in the space. Shapley (1977) writes that the use of the Euclidean space $\mathbb{R}^m$ "seems to leave us ample scope for capturing many kinds of political and ideological parameters without an excess of abstraction and generality".

An issue is formalized by Shapley (1977) through a vector $r \in \mathbb{R}^m$. A player in position x is more in favor of r than a player in position y if the scalar product $r \cdot x$ is less than or equal to $r \cdot y$. Therefore, players can be ordered from the most to the least enthusiastic with respect to an issue, which implies that one of them is pivotal in that ordering. When all issues are equally likely, the probability of a player being pivotal is his or her Owen-Shapley spatial power index.

A natural variation of the Owen-Shapley spatial power index is to consider that not all issues are equally probable. For example, think about a regional parliament of a country, in which local issues are more relevant than the state ones. Barr and Passarelli (2009) consider that there is a probability distribution over issues, defined by a continuous density function. They analyze the distribution of power in the Council of the EU with two dimensions. The stances toward the EU on international issues and domestic issues are the two dimensions that are taken into account.

In this work we also consider the variation of the Owen-Shapley spatial power index with two dimensions when there is a(ny) probability distribution over issues. We call these spatial power indices *probabilistic Owen-Shapley spatial power indices.* We give a formula for calculating the indices for unanimity games. Therefore, the index of any spatial game can be easily calculated by means of linear combinations of indices of unanimity games. We conduct an axiomatic study and prove that the family of probabilistic Owen-Shapley spatial power indices can be obtained by means of the axioms employed by Peters and Zarzuelo (2017) to characterize the Owen-Shapley spatial power index, dropping an invariance axiom and adding continuity. Therefore, an equal power change property, a dummy axiom, anonymity, a positional invariance axiom and continuity generate the family of probabilistic Owen-Shapley spatial power indices.

We also consider the model in which there is a finite number of issues $r \in \mathbb{R}^m$. In this case continuity is not satisfied any more and we show that the entire family of probabilistic Owen-Shapley spatial power indices is characterized by means of only three axioms: an equal power change property, a dummy axiom and anonymity.

We also give some illustrative examples, including an application to the Basque Parliament.

2. References

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