Signaling Quality through Price in a Durable Good Market

Evrim Dener*and Aditi Sengupta†

Abstract

We consider a durable good monopoly where the seller has private information about its product quality and buyers infer quality through dynamic prices. We analyze the effect of inability to precommit to future price on the distortion required to signal quality through prices, as well as the effect of signaling through prices on time inconsistency. We show that unlike the complete information case, inability to precommit to future price can decrease both profit and consumer surplus so that commitment devices may be welfare improving. Inability to commit to future price creates an incentive for a low-quality seller to imitate a high-quality type for one period and then reveal its true type in the next period; this tends to increase signaling distortion. On the other hand, greater variation in consumers' valuations of the high quality good implies that the high quality product is affected more by the time inconsistency problem so that inability to commit reduces the incentive of the low quality seller to imitate the high quality seller that, in turn, reduces signaling distortion. The first effect dominates when the unit costs of producing the two quality levels are far apart and the reverse holds when they are close.

JEL Classifications: L15, L12, D42, D82

Keywords: durable goods, signaling, dynamic inconsistency, asymmetric information.

*Sabre International Solutions. Email: evrimdener@gmail.com.
†Department of Economics, Auburn University. Email: azs0074@auburn.edu

1 Introduction

In many markets, buyers are unable to observe product quality prior to purchase. Product quality could differ across firms due to factors which are beyond the immediate control of the seller such as randomness in outcome of product development, technology shocks, random imperfections in input etc. In such situations firms often have more information about their own current product than consumers. One important issue here is the ability of a seller to signal product quality through prices. In a static monopoly model with unobservable quality, Bagwell and Riordan (1991) show that a seller can signal high-quality by charging a sufficiently high price as low-quality firms tend to have lower cost of production and therefore more interested in selling high quantity at lower price rather than low quantity at very high price.

The classic examples of markets with asymmetric information where sellers know product quality but buyers do not are durable good markets. The existing signaling literature has ignored aspects related to durability in analyzing signaling of quality through prices. Dynamic durable good markets with adverse selection have been studied by Hendel and Lizzeri (1999), Janssen and Roy (2002) and others. These papers focus on dynamic sorting of sellers, interaction between new and used good markets etc. but there is no explicit treatment of deliberate signaling of quality through prices in these models. This paper is an attempt to address this gap in the literature. We study the signaling problem of a durable good monopoly that has private information about the quality of its product. The model is a direct extension of Bagwell and Riordan (1991) to the durable good case.

One of the key issues in the literature on pricing in durable good markets under complete information is the dynamic inconsistency problem (Coase (1972) conjecture). When a durable good monopolist is unable to credibly commit to a strategy of not lowering prices in the future, consumers refuse to pay a high price and instead prefer to wait for a lower future price. In equilibrium, the prices and market power are lower than in the situation where monopolist can credibly precommit. An important question in this connection is how the inability to precommit to future prices affects signaling of product quality. A related question is whether the need to signal quality through prices under incomplete information affects the dynamic inconsistency problem itself.

The durable good monopolist with unobservable quality faces two problems in its pricing decision. Time inconsistency problem due to inability to commit to future prices tends to reduce the current price.[1] On the other hand, signaling through prices requires that the low-quality seller be dissuaded from imitating high-quality prices and therefore the high-quality seller needs to distort its price upwards (sell less). Thus, time inconsistency problem and signaling under incomplete

[1] See Bulow (1982) for an analysis of time inconsistency problem in pricing under complete information in a two period setting.

information affect high-quality prices in opposite directions. It is possible to imagine that signaling could mitigate the time inconsistency problem or the existence of time inconsistency problem could reduce signaling distortion.

We consider a two period durable good monopoly model.[2] The seller introduces a product with an exogenously given uncertain quality. Quality is either high or low. High-quality product is produced with a higher marginal cost compared to the low-quality product. Consumers have unit demand and differ in their valuations of the high-quality product. The valuations of consumers are identical for the low-quality product as in Bagwell and Riordan (1991). This implies that there is no time inconsistency problem for the low-quality seller under full information.

We study the signaling problem under two different regimes: commitment and no-commitment to the future prices. The seller with ability to commit announces the binding prices for both periods at the beginning of period 1. Consumers update their beliefs about product quality on the basis of both prices. In the regime with no ability to commit, the seller faces a time inconsistency problem. Consumers have two chances to update their beliefs: first, after observing the price in period 1 and then, after observing the price in period 2. In each period, after observing the price(s) and updating their beliefs consumers decide whether to purchase in the current period or not. No new consumers enter in period 2.

As in the complete information case, we show that, under precommitment, the high-quality seller commits to a high future price in period 1 and that there is no trading in period 2. Therefore, the two period signaling model is essentially identical to the static analysis in Bagwell and Riordan (1991). The seller signals high-quality with high prices. Greater the difference in marginal costs of the high and low quality products smaller is the distortion caused by signaling. If costs are sufficiently distant, signaling occurs at full information prices.

Our analysis of the no-commitment outcome however yields somewhat different and more interesting result. When the seller cannot commit to future price, there is trading in both periods. However, the outcome differs qualitatively from complete information durable good market as well as incomplete information static market. The inability to commit to future price has two opposing effects on the incentive of the low-quality seller to imitate the high-quality seller. On the one hand, it reduces the incentive of the low-quality seller to imitate the high-quality seller because the time inconsistency problem reduces the profit of the high-quality seller (as there is no time inconsistency problem for the low-quality seller under our assumptions). On the other hand, it also creates an incentive for the low-quality seller to imitate high-quality seller in period 1 and then reveal its type

[2] The monopoly model of Bagwell and Riordian (1991) has been extended to more competitive market structures by Daughety and Reinganum (2007, 2008) and Janssen and Roy (2009). In order to understand the effect of the dynamic inconsistency problem which arises in the existence of market power, we stick with the monopoly.

in period 2 by choosing a low price in period 2. This introduces an additional incentive constraint and therefore may increase signaling distortion relative to the commitment outcome.

We show that when the marginal costs of two types are sufficiently apart, the second effect described above dominates. Signaling distortion and cost of signaling is greater with no-commitment compared to that with commitment. In contrast, when marginal costs are sufficiently close, the opposite result holds, namely, the cost of signaling is greater with commitment.

We also examine the effects of signaling on the time inconsistency problem in durable good markets. In any situation when the cost of signaling is higher with no-commitment, it is also true that unobservability in quality alleviates time inconsistency problem and vice versa. In fact, time inconsistency problem and cost of signaling are the two sides of the same coin.

In the literature on durable good monopoly with complete information, it has been shown that the devices[3] that allow the seller to precommit increases its profit at the expense of the consumer welfare. However, under incomplete information, commitment could increase *both* profit and consumer welfare. The argument is as follows. Commitment eliminates the incentive of the low-quality seller to imitate the high-quality seller for one period and then reveal its quality in the next period and this, in turn, reduces signaling distortion in pricing which increases profit. This reduction in price distortion is also beneficial to consumers as they face lower level of high-quality price relative to what they would under no-commitment. Thus, under incomplete information the ability to commit may increase social welfare as well as market power.

The remainder of the paper is organized as follows. The next section presents the two period durable good monopoly model. Section 3 and 4 give the commitment and no-commitment equilibria respectively. Section 5 analyses the implications of commitment.

2 Model

Consider a durable good monopoly where the seller has private information about the quality of the product. The quality denoted by s is either high or low: $s \in \{H, L\}$. Seller knows the quality but buyers cannot observe the quality prior the purchase. There are two periods indexed by $t = 1, 2$ and the quality is not changed over time. Buyers who buy in period 1 leave[4] the market and do not communicate the quality to other potential buyers.[5]

The production technology is common knowledge. The marginal cost of supplying the product of quality s is c_s. We assume that $c_L = 0$ and $c_H = c > 0$ without loss of generality.

[3] Devices that allow firms to make price guarantees such as buybacks, best-price provisions etc.

[4] We assume that first period buyers leave the market. This allows us to keep the analysis clean and focus on the signaling of quality through prices by avoiding the issues that arise from the possibility of retrading.

[5] Daughtey and Reinganum (2005) argue that the secret settlement of disputes ensures that other potential buyers can not come to know the quality.

The demand structure will be a direct extension of the one in Bagwell and Riordan (1991) to a durable good market.[6] There is a continuum of consumers whose total mass is normalized to 1. Each consumer uses either 0 or 1 unit of the good every period. All consumers have identical per period valuation of θ for using the low-quality product. Consumers have heterogeneous per period valuations v for the use of the high-quality product which is uniformly distributed between θ and $1+\theta$.

We assume that $c < \theta < 1$. The assumption $c < \theta$ implies that seller has incentive to sell even to the lowest valuation consumer. This ensures a significant time inconsistency problem for the high-quality product. The assumption $\theta < 1$ ensures that there is a non-trivial signaling distortion.

Let $p = (p_1, p_2)$ denote equilibrium prices. A priori, all consumers believe that quality is high with probability $r_0 \in (0, 1)$. We study the two different versions of the problem. One is a situation where seller can credibly commit to p_2 at the beginning of period 1. The seller with ability to commit sets (p_1, p_2) at the beginning of period 1. Consumers update their beliefs about product quality on the basis of both prices. As there is no transmission of information between consumers that buy in period 1 and those that do not buy in period 1, beliefs of the consumers do not change between periods 1 and 2.

In the version of the model where the seller has no ability to commit, it sets price p_1 at the beginning of period 1. Consumers update their beliefs on observing the price in period 1 and decide whether or not to buy. Those who buy the product leave the market. At the beginning of period 2, the seller sets price p_2. After observing this price, the residual consumers once again update their beliefs and then make their purchasing decisions.

Consumers maximize the expected net surplus. The objective of the seller is to maximize expected profit. We use the solution concept of Perfect Bayesian equilibrium. We solve for a fully separating equilibrium that satisfies the intuitive criterion (Cho and Kreps (1987)).

Lemma 1 *In any equilibrium $p_1 \geq 2\theta$ and $p_2 \geq \theta$. Further, in any equilibrium at which quality is revealed before purchase (separating and full information equilibria) the low-quality seller charges 2θ in period 1 and sells to all the consumers and obtains a profit of 2θ. Low-quality seller does not trade in period 2.*

In the event that the low-quality type is revealed, it cannot do better than to charge at its full information price and obtain its full information profit. Once the quality is revealed Coasian problem of having an incentive to reduce price to sell to low-valuation consumers does not exist for the low-quality seller because everyone has identical valuation for the low-quality product. Only the

[6] The only difference is that all consumers are uninformed in our model.

high-quality seller is punished by the dynamic inconsistency problem. Low-quality seller obtains the full information commitment monopoly profit.

3 Commitment Equilibria

First, consider the case of full commitment. The seller with ability to commit sets prices for periods 1 and 2 at the beginning of period 1. Notice that in any commitment equilibrium if $p_2 \geq \frac{p_1}{2}$, no consumer buys in period 2. As the seller commits to a sufficiently higher p_2 compared to p_1 consumers prefer to buy in period 1. We will see that the commitment equilibria will be the one where there is no trading in period 2.

3.1 Full Information Commitment Equilibrium

As a benchmark, first, we characterize the full information commitment equilibrium. Under full information consumers know whether the quality is high or low. Denote the full information commitment equilibrium prices and profit of the seller with quality s by $p^{s,c} = (p_1^{s,c}, p_2^{s,c})$ and $\pi^{s,c}$ respectively.

Proposition 1 *Full information commitment equilibrium is as follows. In period 1, the seller with quality s commits to a price $p_2^{s,c} \geq \frac{p_1^{s,c}}{2}$, where $s \in \{H, L\}$, so that there is no trading in period 2. If $s = L$, the seller charges $p_1^{L,c} = 2\theta$ and obtains a profit of $\pi^{L,c} = 2\theta$. If $s = H$, the seller charges $p_1^{H,c} = 1 + \theta + \frac{c}{2}$ and obtains a profit of $\pi^{H,c} = \frac{(2(1+\theta)-c)^2}{8}$.*

3.2 Separating Commitment Equilibrium

We now consider the incomplete information case where consumers are uncertain about the product's quality. In the beginning of period 1, the seller sets the prices. After observing (p_1, p_2) consumers update their beliefs and believe that quality is high with probability $r_1(p_1, p_2)$. Buyers who do not buy in period 1 maintain their beliefs in period 2.

If the seller commits to a high p_2 such that $p_2 \geq \frac{p_1}{2}$, no consumer waits in period 1 in order to buy in period 2. Therefore, the commitment demand in periods 1[7] and 2 are characterized as follows.

$$D_1^c(p \,|\, p_2 \geq \frac{p_1}{2}, r_1) = \begin{cases} 0 & \text{, if } 2(\theta + r_1) < p_1 \\ 1 + \frac{2\theta - p_1}{2r_1} & \text{, if } 2\theta < p_1 \leq 2(\theta + r_1) \\ 1 & \text{, if } p_1 \leq 2\theta \end{cases} \tag{1}$$

$$D_2^c(p \,|\, p_2 \geq \frac{p_1}{2}) = 0$$

[7] $p_1 = r_1[2(1 + \theta - q_1)] + (1 - r_1)[2\theta]$

On the other hand, if $p_2 < \frac{p_1}{2}$, the seller trades in period 2. In this case the demand in period 1[8] is characterized by fraction

$$D_1^c(p \mid p_2 < \frac{p_1}{2}, r_1) = \begin{cases} 0 & \text{, if } 2\theta + r_1(1 + p_2 - \theta) < p_1 \\ 1 + p_2 - \theta + \frac{2\theta - p_1}{r_1} & \text{, if } 2\theta < p_1 \leq 2\theta + r_1(1 + p_2 - \theta) \\ 1 & \text{, if } p_1 \leq 2\theta \end{cases} \tag{2}$$

consumers buying. The residual demand[9] can be characterized by fraction

$$D_2^c(p \mid p_2 < \frac{p_1}{2}, r_1, q_1) = \begin{cases} 1 - q_1 + \frac{\theta - p_2}{r_1} & \text{, if } \theta < p_2 \\ 1 - q_1 & \text{, if } p_2 \leq \theta \end{cases} \tag{3}$$

consumers buying in period 2.

Given the demand in periods 1 and 2, the commitment profit of a seller with quality s and prices $p = (p_1, p_2)$ facing consumers with belief r_1 denoted by $\pi^c(s, r_1, p)$ can be given as follows.

$$\pi^c(s, r_1, p) = (p_1 - c_s)D_1^c(p \mid r_1) + (p_2 - c_s)D_2^c(p \mid r_1, q_1)$$

We will characterize a separating equilibrium in which prices serve as signals of quality. Let $p^c(s) = (p_1^c(s), p_2^c(s))$ denote the commitment separating equilibrium prices charged by an s-quality seller. In a separating equilibrium, the low-quality seller has no incentive to imitate the high-quality type, i.e., $p^c(H)$ is such that

$$\pi^c(L, 0, p^{L,c}) \geq \pi^c(L, 1, p^c(H)) \tag{4}$$

The low-quality seller should make higher profit by charging its full-information prices and revealing itself as a low-quality seller than by charging the high-quality seller's equilibrium price and being perceived as a high-quality seller. It can be shown that the high-quality full information prices may or may not satisfy (4). The shaded region[10] in Figure 1; the set

$$S = \left\{ p \mid \frac{p_1 - \sqrt{(2 - p_1)(p_1 - 2\theta)}}{2} < p_2 \right\} \cup \tag{5}$$
$$\left\{ p \mid 1 + \theta + p_2 < p_1 \text{ and } \frac{1 + \theta - \sqrt{1 - 6\theta + \theta^2}}{2} < p_2 < \frac{1 + \theta + \sqrt{1 - 6\theta + \theta^2}}{2} \right\}$$

shows the prices at which (4) does not hold, i.e., the $p = (p_1, p_2)$ that the low-quality seller has

[8] $p_1 = r_1[1 + \theta + p_2 - q_1] + (1 - r_1)[2\theta]$

[9] $p_2 = r_1[1 + \theta - q_1 - q_2] + (1 - r_1)[\theta]$

[10] The figure is drawn for $\theta > 0.1716$ at which low-quality seller has no incentive to imitate the $p_1 > 1 + \theta + p_2$ where there is no trading in period 1, i.e., $\left\{ p \mid 1 + \theta + p_2 < p_1 \text{ and } \frac{1+\theta-\sqrt{1+6\theta+\theta^2}}{2} < p_2 < \frac{1+\theta+\sqrt{1+6\theta+\theta^2}}{2} \right\} = \varnothing$

incentive to imitate. In this region, the gain from being perceived as a high-quality type outweighs the loss from charging a high-quality price and limiting output.

The full information prices (p^H, p^L) are also separating if $p^H \notin S$. This occurs if $c \geq 2(1-\theta)$. In the following we characterize the separating equilibrium under the commitment regime.

Proposition 2 *When the seller is able to commit, there exists a separating equilibrium. In period 1, the seller with quality s commits to a price $p_2^c(s) \geq \frac{p_1^c(s)}{2}$, where $s \in \{H, L\}$, so that there is no trading in period 2. Low-quality seller charges its full information price and obtains its full information profit. High-quality seller charges $p_1^c(H) = \max\left\{p_1^{H,c}, 2\right\}$, where $p_1^{H,c}$ is the full information price, and obtains its full information profit if $c \geq 2(1-\theta)$ and obtains $\pi^c(H) = (2-c)\theta$ if $c < 2(1-\theta)$. The beliefs supporting this equilibrium are $r_1 = 0$ when $p_1^c < 2$ and $r_1 = 1$ when $p_1^c \geq 2$ and they satisfy the intuitive criterion.*

As in the complete information case, under precommitment, the seller commits to a high future price in period 1 so that there is no trading in period 2. Therefore, the two period signaling model reduces into the static analysis. As in the static Bagwell and Riordan (1991) model, in a situation where full information prices fail to be separating, the high-quality seller has to separate with a price higher than its full information price so as to avoid the low-quality seller's mimicking. In this case, the high-quality seller separates at a cost of signaling.

4 No-Commitment Equilibria

Next, we consider a situation where there is dynamic inconsistency because the seller cannot precommit to p_2 in period 1. In the beginning of period 1, seller sets the p_1 and then consumers make their purchases. Since the good is durable, period 1 buyers leave the market. In period 2, the market consists of the residual demand with consumers who value the good less, thus, the seller has an incentive to reduce price in period 2 to be able to sell to lower valuation consumers. In period 1, anticipating this incentive to reduce price, consumers tend to wait for the low future prices. This reduces the profit, that is, the seller faces the dynamic inconsistency problem described by Coase (1972).

Lemma 1 shows that the low-quality seller charges 2θ, sells to all consumers in period 1 and obtains a profit of 2θ. Below, we confine attention to the high-quality seller's behavior.

4.1 Full Information No-Commitment Equilibrium

First, we characterize a full information no-commitment equilibrium where consumers know the quality prior to purchase. Suppose $s = H$ and fraction q_1 consumers buy for the price p_1 in period 1. Denote the per period valuation of the consumer who is indifferent between purchasing in the

first period and waiting to purchase in the second period (marginal consumer) by v_1. Consumers with per period valuations exceeding v_1 are better off by purchasing in period 1. That is the marginal consumer in period 1 has per period valuation of

$$v_1 = 1 + \theta - q_1 \tag{6}$$

The consumers with per period valuation less than v_1 constitute the residual demand in period 2. Therefore, if $p_2 \geq v_1$, there is no trading in period 2. If $v_1 > p_2 \geq \theta$, fraction $v_1 - p_2$ consumers buy in period 2 and if $p_2 < \theta$ all the remaining consumers buy in period 2. The period 2 full information demand for the high-quality product denoted by D_2^H is as follows.

$$D_2^H = \begin{cases} 0 & \text{, if } 1 + \theta - q_1 < p_2 \\ 1 + \theta - q_1 - p_2 & \text{, if } \theta < p_2 \leq 1 + \theta - q_1 \\ 1 - q_1 & \text{, if } p_2 \leq \theta \end{cases} \tag{7}$$

Given $q_1(p_1)$, we can set up the period 2 profit maximization problem of the high-quality seller as follows.

$$\max_{p_2} \ (p_2 - c) D_2^H$$

Profit maximization shows that

$$p_2^H(q_1) = \max \left\{ \frac{1 + \theta + c - q_1}{2}, \theta \right\} \tag{8}$$

In period 1, observing p_1, consumers have an expectation of the price in period 2 denoted by $E(p_2 \mid p_1)$. The marginal consumer's indifference between purchasing in period 1 and waiting to purchase in period 2 implies the equality of the surplus from purchasing in the first period and expected surplus from purchasing in period 2.

$$2v_1 - p_1 = v_1 - E(p_2 \mid p_1) \tag{9}$$

When $s = H$ and consumers have full information about quality, period 2 price is expected to be equal to the profit maximizing price given q_1.

$$E(p_2 \mid p_1) = p_2^H(q_1) \tag{10}$$

In period 1, rational consumers consider the effect of their current purchases on the future residual demand and consequently on the future price in forming their expectations about the period 2 price.

Using (10), we can rewrite the equation (9) to express v_1 in terms of p_1 and $p_2^H(q_1)$ as follows.

$$v_1 = p_1 - p_2^H(q_1) \tag{11}$$

Given (6) and (11), we can derive the period 1 full information demand for the high-quality product[11] denoted by D_1^H as follows.

$$D_1^H = \begin{cases} 0 & \text{, if } \frac{3(1+\theta)+c}{2} < p_1 \\ 1+\theta+\frac{c-2p_1}{3} & \text{, if } 3\theta - c < p_1 \leq \frac{3(1+\theta)+c}{2} \\ 1+2\theta-p_1 & \text{, if } 2\theta < p_1 \leq 3\theta - c \\ 1 & \text{, if } p_1 \leq 2\theta \end{cases} \tag{12}$$

Given D_1^H, we can write the full-information profit in period 2 denoted by π_2^H in terms of p_1 as follows.

$$\pi_2^H = \begin{cases} \frac{(1+\theta-c)^2}{4} & \text{, if } \frac{3(1+\theta)+c}{2} < p_1 \\ \frac{(p_1-2c)^2}{9} & \text{, if } 3\theta - c < p_1 \leq \frac{3(1+\theta)+c}{2} \\ (\theta-c)(p_1-2\theta) & \text{, if } 2\theta < p_1 \leq 3\theta - c \\ 0 & \text{, if } p_1 \leq 2\theta \end{cases} \tag{13}$$

The period 1 profit maximization problem of the high-quality seller is as follows.

$$\max_{p_1} \ (p_1 - c)D_1^H + \pi_2^H$$

Profit maximization shows that

$$p_1^H = \begin{cases} \frac{1+3\theta}{2} & \text{, if } \theta \geq \frac{6+5c-\sqrt{5}(2+c)}{4} \\ \frac{9(1+\theta)+5c}{10} & \text{, if } \theta < \frac{6+5c-\sqrt{5}(2+c)}{4} \end{cases}$$

Given p_1^H, from (8) and (12) it is easy to see that if $\theta \geq \frac{6+5c-\sqrt{5}(2+c)}{4}$, $p_2^H = \theta$, otherwise $p_2^H = \frac{3(1+\theta)+5c}{10}$.

Proposition 3 *Full information no-commitment equilibrium is as follows. The low-quality seller charges exactly the same prices as in the commitment equilibrium. The high-quality seller charges*

$$p^H = \begin{cases} (\frac{1+3\theta}{2}, \theta) & \text{, if } \theta \geq \frac{6+5c-\sqrt{5}(2+c)}{4} \\ (\frac{9(1+\theta)+5c}{10}, \frac{3(1+\theta)+5c}{10}) & \text{, if } \theta < \frac{6+5c-\sqrt{5}(2+c)}{4} \end{cases}$$

If $\theta \geq \frac{6+5c-\sqrt{5}(2+c)}{4}$, $\pi^H = \frac{1+6\theta+\theta^2-4c}{4}$, *otherwise* $\pi^H = \frac{9(1+\theta)^2-5c(2(1+\theta)-c)}{20}$.

[11] Unlike the static analysis, with no-commitment demand is a function of c as p_2^H increases by an increase in c.

When θ is large, even the high-quality seller serves all consumers. In period 1, the high-quality seller sells to higher valuation consumers and then in period 2 he reduces the price to θ and sells to all the residual demand. On the other hand, when θ is small high-quality seller does not serve all consumers.

4.2 Separating No-Commitment Equilibrium

We now consider the incomplete information case where consumers are uncertain about quality and try to infer quality from the prices charged by the seller. The seller announces prices p_1 and p_2 at the beginning of the first and second periods respectively. Consumers have two chances to update their beliefs about quality. After observing p_1, consumers believe that quality is high with probability $r_1(p_1)$ and fraction q_1 consumers purchase in period 1 and after observing p_2 and q_1, consumers remaining in the market believe that quality is high with probability $r_2(p_1, p_2, q_1)$.

In order to describe the behavior of the high-quality seller we partition the parameter space into four regions. Let

$$f_1 = \begin{cases} \frac{1+c}{2} & \text{, if } c \leq \frac{1}{3} \\ \frac{11+5c-2\sqrt{5(2+c-c^2)}}{9} & \text{, if } c > \frac{1}{3} \end{cases}$$

$$f_2 = \begin{cases} \frac{1+c}{2} & \text{, if } c \leq \frac{1}{3} \\ 9 - c - 2\sqrt{3(6-c)} & \text{, if } c > \frac{1}{3} \end{cases}$$

$$f_3 = \frac{1 - 4c + \sqrt{1+8c}}{4}$$

It is easy to see that

$$f_3 \leq f_2 \leq f_1$$

Figure 2 illustrates the partitioning.

The no-commitment separating equilibrium is a collection of the seller's strategies, consumers' beliefs and consumers' actions in periods 1 and 2: $\{p(s) = (p_1(s), p_2(s \mid p_1, q_1))$ for $s \in \{H, L\}$, $r = (r_1(p_1), r_2(p_1, p_2, q_1))$, $a(v)$ for $v \in [\theta, 1+\theta]\}$ where $a(v)$ can be buying in period 1, buying in period 2 or not buying at all.

We first describe the equilibrium for the parameter values $f_3 \leq \theta < f_2$. This range of parameter values expands as c increases and it covers the entire parameter space when c is sufficiently high. The strategy profile of the high-quality seller and consumers' actions in the separating no-commitment equilibrium can be characterized as follows. Let $\overline{p_1} = \frac{3+5\theta+c+3\sqrt{c^2+6c+2\theta c+9-18\theta+\theta^2}}{4}$. In period 1, high-quality seller charges

$$p_1(H) = \overline{p_1} \tag{14}$$

and the consumers with valuations $v \in [\frac{2\overline{p_1}-c}{3}, 1+\theta]$ buy, i.e., fraction

$$q_1(H) = 1 + \theta + \frac{c - 2\overline{p_1}}{3} \tag{15}$$

consumers buy. We will now describe the equilibrium for every possible continuation game in period 2. In period 2, if $p_1 < \overline{p_1}$, high-quality seller charges

$$p_2(H \mid p_1 < \overline{p_1}, q_1) = \theta \tag{16}$$

and all the remaining consumers buy[12] and if $p_1 \geq \overline{p_1}$, high-quality seller charges

$$p_2(H \mid p_1 \geq \overline{p_1}, q_1) = \max\left\{p_2^H(q_1), 1 - q_1\right\} \tag{17}$$

where $p_2^H(q_1)$ is the full information profit maximizing price given in (8). Among the remaining consumers those who have valuation $v \in [p_2, \frac{2\overline{p_1}-c}{3})$ buy in period 2[13] and those with valuation $v \in [\theta, p_2)$ do not buy at all.

The beliefs supporting this equilibrium are as follows. If $p_1 < \overline{p_1}$, in both periods consumers believe that the product is of low-quality; $r_1 = r_2 = 0$. If $p_1 \geq \overline{p_1}$, in period 1 consumers believe that product is of high-quality; $r_1 = 1$. In this case, if $p_2 \geq 1 - q_1$, in period 2 the residual consumers maintain their beliefs; $r_2 = 1$, else if $p_2 < 1 - q_1$, the residual consumers change their beliefs and believe that the product is of low-quality; $r_2 = 0$.

Now, we show that given this belief system, neither the seller nor the buyers have an incentive to deviate from the described strategy profile and the actions respectively. First, suppose that seller charges $p_1 \geq \overline{p_1}$ and further suppose that fraction q_1 consumers buy in period 1. The next lemma describes continuation play of the high-quality seller in period 2.

Lemma 2 *Given that $p_1 \geq \overline{p_1}$ and fraction $q_1 \in [0, 1]$ consumers buy in period 1. In period 2, the high-quality seller does not deviate from the equilibrium strategy given in (17) and the residual consumers do not deviate from the actions described above. The continuation profit of the high-*

[12] A fraction of $q_2(p_2 \mid p_1 < \overline{p_1}, q_1) = 1 - q_1$ consumers buy in period 2.

[13] A fraction of $q_2(p_2 \mid p_1 < \overline{p_1}, q_1) = \min\left\{1 - q_1, \frac{1+\theta-c-q_1}{2}, \theta\right\}$ consumers buy in period 2.

quality seller in period 2 is

$$\pi_2(H \mid p_1 \geq \overline{p_1}, q_1) = \begin{cases} (1-q_1-c)\theta \text{ , } if \;\; q_1 < 1-\theta-c \\ \pi_2^H(q_1) \qquad \text{ , } if \;\; q_1 \geq 1-\theta-c \end{cases} \tag{18}$$

where $\pi_2^H(q_1)$ is the continuation profit in the full information no-commitment solution.

Notice that $p_2^H(q_1) \geq 1-q_1$ when $q_1 > 1-\theta-c$. In this case, it is easy to see intuitively that the high-quality seller would prefer the equilibrium price which is the profit maximizing price $p_2^H(q_1)$. On the other hand, $1-q_1 > p_2^H(q_1)$ when $q_1 < 1-\theta-c$. In this case, the high-quality seller makes less than $\pi_2^H(q_1)$ by adopting the signaling price $1-q_1$. Therefore, the high-quality seller may consider deviating from $1-q_1$ to a lower price. For such low prices, we assign beliefs that put zero probability on the high-quality type[14]. Thus, the best profit that the high-quality seller can make by deviating (charging θ) is $(\theta-c)(1-q_1)$. Since $\pi_2(H \mid p_1 \geq \overline{p_1}, q_1) \geq (\theta-c)(1-q_1)$, the high-quality seller who charges $p_1 \geq \overline{p_1}$ and sells fraction q_1 units in period 1 has no incentive to deviate from the strategy given in (17) in period 2[15].

In period 1, upon observing the price $p_1 \geq \overline{p_1}$, consumers rationally expect period 2 price to be equal to the high-quality profit maximizing price.

$$E(p_2 \mid p_1 \geq \overline{p_1}) = p_2(H \mid p_1 \geq \overline{p_1}, q_1)$$

Given this expectation in period 1, if the period 1 price is sufficiently high compared to the expected period 2 price, demand in period 1 would be zero, that is, if $1+\theta+E(p_2 \mid p_1 \geq \overline{p_1}) < p_1$, even the highest valuation consumer (with per period valuation $1+\theta$) postpones his purchase. On the other hand if $2\theta < p_1 \leq 1+\theta+E(p_2 \mid p_1 \geq \overline{p_1})$, fraction $1+\theta+E(p_2 \mid p_1 \geq \overline{p_1}) - p_1$ consumers buy in period 1 and exit the market. If $p_1 \leq 2\theta$, all the consumers buy in period 1. Therefore, when $p_1 \geq \overline{p_1}$ the demand in period 1 denoted by $D_1(p_1 \mid p_1 \geq \overline{p_1})$ can be given as follows

$$D_1(p_1 \mid p_1 \geq \overline{p_1}) = \begin{cases} 0 \qquad\qquad \text{, if } \alpha < p_1 \\ 1+\frac{\theta-p_1}{2} \text{ , if } \delta < p_1 \leq \alpha \\ D_1^H \qquad\quad \text{, if } p_1 \leq \delta \end{cases} \tag{19}$$

where $\alpha = \max\left\{2+\theta, \frac{3(1+\theta)+c}{2}\right\}$, $\delta = \min\left\{3\theta+2c, \frac{3(1+\theta)+c}{2}\right\}$ and D_1^H is given by (12).[16]

[14] Later, we show that these beliefs are sustained by the intuitive criterion.

[15] When $c > 1-\theta$ this second case where $1-q_1 > p_2^H(q_1)$ disappears as $1-\theta-c < 0$. In period 2, as in the static Bagwell and Riordan (1991) model, when $c > 1-\theta$ there is no need to distort prices to signal quality.

[16] One can show that $\alpha = \delta$ when $c \geq 1-\theta$ that is, in period 2 high-quality seller can signal with its profit maximizing price given q_1, so the demand in period 1 reduces to the full information demand.

Given the demand in period 1, the continuation profit of the high-quality seller in (18) can be rewritten in terms of p_1 as

$$\pi_2(H \mid p_1 \geq \overline{p_1}) = \begin{cases} (1-c)\theta & \text{, if } \alpha < p_1 \\ (\frac{p_1-\theta}{2} - c)\theta & \text{, if } \delta < p_1 \leq \alpha \\ \pi_2^H & \text{, if } p_1 \leq \delta \end{cases} \tag{20}$$

where π_2^H is the full information period 2 profit given by (13).

We can set up the period 1 profit maximization problem of the high-quality seller as follows.

$$\max_{p_1} \ (p_1 - c)D_1(p_1 \mid p_1 \geq \overline{p_1}) + \pi_2(H \mid p_1 \geq \overline{p_1}) \tag{21}$$
$$\text{subject to } \overline{p_1} \leq p_1$$

When $f_3 \leq \theta < f_2$, profit maximization shows that

$$p_1(H \mid p_1 \geq \overline{p_1}) = \overline{p_1}$$

From $D_1(p_1 \mid p_1 \geq \overline{p_1})$ given by (19) we see that at this price, it is rational for fraction $q_1(H)$ (given in (15)) consumers buy in period 1. From $p_2(H \mid p_1 \geq \overline{p_1}, q_1)$ given by (17) we can see that high-quality seller signals by its profit maximizing price

$$p_2(H \mid p_1 = \overline{p_1}) = \frac{\overline{p_1} + c}{3} \tag{22}$$

in period 2. These prices yield a profit of
$\pi(H \mid p_1 = \overline{p_1}) = \frac{25\theta^2 - 34\theta c + 114\theta - 54c + 13c^2 + 9 + (3+5c-7\theta)\sqrt{c^2+6c+2\theta c+9-18\theta+\theta^2}}{72}$.

Now, we are going to check whether the high-quality seller has an incentive to deviate from $\overline{p_1}$. Suppose the high-quality seller deviates to a price $p_1 < \overline{p_1}$. Given the belief system, consumers believe that the product is of low-quality in both periods, thus, the consumers are not willing to pay more than θ in period 2. Given that $p_2 \geq \theta$ by Lemma 1, the high-quality seller has no incentive to deviate from the strategy given in (16). Thus, the high-quality seller could make at most $2\theta - c$ by deviating (charging 2θ in period 1). Since $\pi(H \mid p_1 = \overline{p_1}) > 2\theta - c$ there is no incentive to deviate to $p_1 < \overline{p_1}$.

Suppose the high-quality seller deviates to a price $p_1 > \overline{p_1}$. At such prices high-quality seller would obtain a profit of $\pi(H \mid p_1 \geq \overline{p_1})$. We have seen that when $f_3 \leq \theta < f_2$, $\pi(H \mid p_1 \geq \overline{p_1})$ is maximized at $\overline{p_1}$, therefore in period 1 the high-quality seller has no incentive to deviate from the price given in (14).

Similarly, for other regions in the parameter space, we solve for the equilibrium in the Appendix. The following proposition gives the no-commitment separating equilibrium for the entire parameter space.

Proposition 4 *When the seller is not able to commit, there exists a separating equilibrium satisfying the intuitive criterion. The beliefs supporting this equilibrium are* $r_1 = r_2 = 0$ *when* $p_1 < \overline{p_1}$. *When* $p_1 \geq \overline{p_1}$, $r_1 = 1$. *In this case, if* $p_2 < 1 - q_1$, $r_2 = 0$, *otherwise* $r_2 = 1$. *In this equilibrium* $p(L) = (2\theta, \theta)$, $q(L) = (1, 0)$ *and* $\pi(L) = 2\theta$.
(i) If $f_1 \leq \theta$, $p(H) = (\overline{p_1}, \theta)$, $q(H) = (\theta, 1-\theta)$ *and* $\pi(H) = 2\theta - c$ *where* $\overline{p_1} = 1 + \theta$.
(ii) If $f_2 \leq \theta < f_1$, $p(H) = (\frac{9(1+\theta)+5c}{10}, \frac{3(1+\theta)+5c}{10})$, $q(H) = (\frac{2(1+\theta)}{5}, \frac{3(1+\theta)-5c}{10})$ *and* $\pi(H) = \frac{9(1+\theta)^2-5c(2(1+\theta)-c)}{20}$ *where* $\overline{p_1} = 1 + \theta$.
(iii) If $f_3 \leq \theta < f_2$, $p(H) = (\overline{p_1}, \frac{\overline{p_1}+c}{3})$, $q(H) = (1+\theta+\frac{c-2\overline{p_1}}{3}, \frac{\overline{p_1}}{3})$ *and* $\pi(H) = \frac{25\theta^2-34\theta c+114\theta-54c+13c^2+9+(3+5c-7\theta)\sqrt{c^2}}{72}$ *where* $\overline{p_1} = \frac{3+5\theta+c+3\sqrt{c^2+6c+2\theta c+9-18\theta+\theta^2}}{4}$.
(iv) If $\theta < f_3$, $p(H) = (\overline{p_1}, \frac{\overline{p_1}-\theta}{2})$, $q(H) = (1 + \frac{\theta-\overline{p_1}}{2}, \theta)$ *and* $\pi(H) = \frac{2\theta(2-c)-c(1-\sqrt{1-2\theta})}{2}$, *where* $\overline{p_1} = 1 + \theta + \sqrt{1-2\theta}$.

To separate, high-quality seller charges prices at which the low-quality seller has no incentive to imitate. As marginal cost, consequently, the full information prices are greater for the high-quality seller, charging a high price is less costly for the high-quality seller than it is for the low-quality seller. Depending on the parameter values, there is a high enough price $\overline{p_1}$ that makes the low-quality seller indifferent between imitation and revealing its quality by charging its full information price in period 1. In period 2, given q_1 the low-quality seller is indifferent between imitation and revealing at $p_2(H) = 1 - q_1$.

Similar to the full information equilibrium, when θ is large (case (i)) even the high-quality seller charges θ and sells to all the residual consumers in period 2. In period 1, high-quality seller signals quality by distorting its price upwards and charging $p_1(H) = \overline{p_1} > p_1^H$.

In case (ii) where θ is slightly lower, we have two interesting situations. When $f_2 \leq \theta < \frac{6+5c-\sqrt{5}(2+c)}{4}$, the low-quality seller has no incentive to imitate the full information prices of the high-quality seller so that the high-quality seller does not need to distort its prices to be recognized and charges its full information prices in both periods. Interestingly, when $\frac{6+5c-\sqrt{5}(2+c)}{4} < \theta < f_1$ the high-quality seller distorts its prices more than required to signal its quality; $p_1(H) > \overline{p_1} > p_1^H$. This is due to the kinked demand curve in period 1. The kinks in the demand curve lead to a profit function which is not single-peaked. As the low-quality seller has an incentive to imitate the global profit maximizing price of the high-quality type, the high-quality seller chooses a local maximum.

In case (iii) which is described above in detail θ is medium. In period 2, the low-quality seller has no incentive to imitate the profit maximizing price of the high-quality seller so that high-quality

seller signals by its profit maximizing price which is an increasing function of c. This leads to a high period 1 demand for large values of c. In period 1, as in case (i), high-quality seller distorts its price upwards.

In case (iv), θ is low. The profit of the low-quality seller without imitation is so low that its incentive to imitate is much higher; therefore, the high-quality seller distorts its prices upwards in both periods.

5 Implications of Commitment

We have solved the signaling problem of the seller under two different regimes: commitment and no-commitment. In this section we consider how the interaction of signaling and durability affects the cost of signaling and welfare.

5.1 Cost of Signaling

In this part, we compare the signaling distortions in commitment and no-commitment solutions. We focus on the comparison for the high-quality seller, because the low-quality seller's behavior is same under commitment and no-commitment equilibria. To facilitate the comparison, we define the cost of signaling as the profit differential between the full information and the separating equilibria. The cost of signaling with commitment and no-commitment denoted by $\Delta\pi^c$ and $\Delta\pi$ can be defined as

$$\Delta\pi^c = \pi^{H,c} - \pi^c(H)$$
$$\Delta\pi = \pi^H - \pi(H)$$

where $\pi^{H,c}$, $\pi^c(H)$, π^H and $\pi(H)$ are given in the Propositions 1-4.

Ability to commit helps to signal quality and reduces the cost of signaling if

$$\Delta\pi^c < \Delta\pi \tag{23}$$

Commitment leads to a greater cost of signaling if the inequality is reversed. Now, we consider the conditions under which this inequality holds.

From the Propositions 1 and 2 describing the commitment equilibria we can see that as the difference in the marginal costs of high and low quality increases, cost of signaling reduces. When the difference in marginal costs is sufficiently large, signaling occurs at the full information price, with no distortion.

With no-commitment, contrary to the static commitment model, cost of signaling may increase with an increase in the difference in marginal costs of high and low quality. The intuition is as

follows.

When the high-quality seller is unable to credibly commit to a strategy of not lowering prices in the future, consumers refuse to pay a high price and wait for a lower future price. In equilibrium, the profit is lower than in the commitment profit. As there is no variation in consumers' valuations of the low-quality good, the low-quality product is not affected by this time inconsistency problem. To that extent, inability to commit reduces the incentive of the low-quality seller to imitate the high-quality seller that, in turn, reduces the signaling distortion.

On the other hand, inability to commit to future price creates an incentive for the low-quality seller to imitate the high-quality type for one period and then reveal its true type by charging its full information price (θ) in the next period. Given that the residual demand in period 2 is smaller compared to the demand in period 1, the low-quality seller has much lower incentive to imitate the price charged by the high-quality seller in period 2 and it is more likely that the high-quality seller signals quality by simply charging its full information profit maximizing price (given the residual demand). This option to imitate in one period and then reveal in the next tends to increase the signaling distortion in the dynamic two period problem.

The overall effect of the inability to commit on the signaling distortion hinges on the balance of these two effects that is determined by the marginal cost difference between the high and low quality products (c). First, a high c implying a high profit maximizing price (as in case (iii)) reduces the incentive of the high-quality seller to serve lower valuation consumers in period 2. Mitigating the time inconsistency problem, this increases the signaling distortion. Next, an increase in c, implying a higher future price leads to a greater demand in period 1 for the high-quality product. Thus, with a higher c, the low-quality seller who imitates in the first period and reveals in the next faces a higher demand in period 1 with no future costs. This also adds to the signaling distortion. Consequently, when c is sufficiently high cost of signaling is greater with no-commitment compared to the one with commitment.

Whenever cost of signaling is greater with no-commitment (the inequality (23) holds), it is also true that the time inconsistency problem with full information is alleviated under incomplete information, i.e.,

$$\pi^{H,c} - \pi^{H} \leq \pi^{c}(H) - \pi(H)$$

When c is sufficiently low, the opposite results hold. While it is difficult to explicitly characterize the border between these two regions, i.e. the solution to $\Delta\pi^{c} = \Delta\pi$, we can graphically illustrate it by plotting. Figure 3 illustrates the regions where the inequality (23) holds and the reverse holds.

5.2 Welfare

In the literature on durable good monopoly with complete information, it has been shown that precommitment to future prices increases the profit at the expense of the consumer surplus and social welfare. Now, we examine the welfare consequences of the ability to commit under incomplete information. In particular, we investigate the relative welfare performances of the commitment and no-commitment separating equilibria. Social welfare is defined as the sum of the consumer surpluses in both periods and the profit. Since the commitment model reduces into the static model, social welfare with ability to commit denoted by W^c can be defined as follows.

$$\begin{aligned} W^c &= \int_0^{q_1} [2(1+\theta-q_1)-c]dq_1 \\ &= q_1[2(1+\theta)-q_1-c] \end{aligned}$$

Given $p^c(H)$ in Proposition 2, we can find the quantity sold in the commitment separating equilibrium as $q_1^c(H) = \min\left\{\theta, \frac{2(1+\theta)-c}{4}\right\}$ by (1) and rewrite W^c as follows.

$$W^c = \begin{cases} \theta(2+\theta-c) & \text{, if } c < 2(1-\theta) \\ \frac{3}{16}(2(1+\theta)-c)^2 & \text{, if } c \geq 2(1-\theta) \end{cases} \tag{24}$$

With no-commitment, social welfare denoted by W can be defined as follows.

$$\begin{aligned} W &= \int_0^{q_1} [2(1+\theta-q_1)-c]dq_1 + \int_0^{q_2} [1+\theta-q_1-q_2-c]dq_2 \\ &= q_1[2(1+\theta)-q_1-c] + q_2[1+\theta-q_1-\frac{q_2}{2}-c] \end{aligned}$$

Given the quantities traded in the no-commitment separating equilibrium (Proposition 4) we can rewrite W as follows.

$$W = \begin{cases} \frac{1+4\theta+\theta^2-2c}{2} & \text{, if } f_1 < \theta \\ \frac{31+31\theta^2+15c^2+62\theta-34c-34\theta c}{40} & \text{, if } f_2 < \theta \leq f_1 \\ \frac{99+79\theta^2+43c^2+258\theta-126c-94\theta c+(11c-25\theta-15)\sqrt{c^2+6c+2\theta c+9-18\theta+\theta^2}}{144} & \text{, if } f_3 < \theta \leq f_2 \\ \frac{1+4\theta-c+\theta^2-2\theta c-(1+\theta-c)\sqrt{1-2\theta}}{2} & \text{, if } \theta \leq f_3 \end{cases} \tag{25}$$

Precommitment to the future price increases the social welfare if

$$W^c > W \tag{26}$$

As discussed above, the incentive to imitate in one period and then reveal in the next dominates when the marginal costs of high and low quality products are sufficiently apart. Commitment helps to signal quality with lower prices by eliminating this incentive. The high-quality seller with ability to commit is able to signal by using a price that is not too high compared to its full information monopoly price. With incomplete information about quality, contrary to Coase Conjecture, prices with commitment may even be smaller than the prices with no-commitment.

Proposition 5 *Consider the separating equilibria with commitment and no-commitment. If* $c > \max\left\{\frac{3\theta^2-4\theta+2}{2-\theta}, \frac{4\theta-2}{1+\theta}\right\}$, $p_1(H) > p_1^c(H)$. *In this case, precommitment to the future price increases welfare, i.e.,* $W^c > W$.

While reducing the time inconsistency problem, commitment under incomplete information may reduce signaling distortion and market power. The reduction in price distortion is also beneficial to the consumers as they face lower high-quality price relative to what they would under no-commitment. When marginal costs of high and low quality products are sufficiently distant, ability to commit increases the social welfare, i.e., inequality (26) holds. Figure 4 illustrates the regions where the inequality (26) holds and the reverse holds.

Proposition 6 *Consider the separating equilibria with commitment and no-commitment. If* $c > \gamma(\theta)$, $W^c > W$ *where* $\gamma(\theta)$ *is the solution to* $W^c = W$.

This welfare analysis shows that under incomplete information about quality, the loss of monopoly power due to time inconsistency does not always improve social welfare. In contrast to the literature on Coase conjecture, our analysis implies that commitment devices may be welfare improving. Policies banning the use of commitment devices should be evaluated by considering the technology of production (c).

Appendix

All supporting proofs which are not provided in the text are given below.

Proof of Lemma 1. $\pi_2(s,r,p) = (p_2 - c_s)(1 - q_1)$ when $p_2 \in [0,\theta]$ that is $\pi_2(s,r,p)$ is strictly increasing in p_2 when $p_2 < \theta$. Therefore $p_2 \geq \theta$. Given that $p_2 \geq \theta$, $\pi(s,r,p) = p_1 - c_s$ when $p_1 \in [0, 2\theta]$ that is $\pi(s,r,p)$ is strictly increasing in p_1 when $p_1 < 2\theta$. Therefore $p_1 \geq 2\theta$. When consumers infer the true quality, they would not be willing to pay more than 2θ for a low-quality in period 1, as per period valuation for the low-quality product equals to θ for all consumers. Given that $p_2 \geq \theta$ and $p_1 \geq 2\theta$, the low-quality seller charges 2θ. At this price all consumers purchase in period 1 so the profit is equal to 2θ. ■

Proof of Proposition 1. Lemma describes the equilibrium for $s = L$. Suppose $s = H$, the seller charges p_1, commits to a price p_2 and fraction q_1 consumers buy in period 1. If the seller commits to a high p_2 such that $p_2 \geq \frac{p_1}{2}$, no consumer waits in period 1 in order to buy in period 2. Therefore, the full information commitment demand for the high-quality product in periods 1 and 2 are characterized as follows.

$$D_1^{H,c}(p \,|\, p_2 \geq \frac{p_1}{2}) = \begin{cases} 0 & \text{, if } 2(1+\theta) < p_1 \\ 1 + \theta - \frac{p_1}{2} & \text{, if } 2\theta < p_1 \leq 2(1+\theta) \\ 1 & \text{, if } p_1 \leq 2\theta \end{cases}$$

$$D_2^{H,c}(p \,|\, p_2 \geq \frac{p_1}{2}) = 0$$

On the other hand, if $p_2 < \frac{p_1}{2}$, there is trading in period 2. In this case, the demand in period 1 is characterized by fraction

$$D_1^{H,c}(p \mid p_2 < \frac{p_1}{2}) = \begin{cases} 0 & \text{, if } 1 + \theta + p_2 < p_1 \\ 1 + \theta + p_2 - p_1 & \text{, if } 2\theta < p_1 \leq 1 + \theta + p_2 \\ 1 & \text{, if } p_1 \leq 2\theta \end{cases}$$

consumers buying. The residual demand can be characterized by fraction

$$D_2^{H,c}(p \mid p_2 < \frac{p_1}{2}, q_1) = \begin{cases} 1 + \theta - q_1 - p_2 & \text{, if } \theta < p_2 \\ 1 - q_1 & \text{, if } p_2 \leq \theta \end{cases}$$

consumers buying in period 2. Given the demand in periods 1 and 2, we can set up the profit maximization problem of the high-quality seller with ability to commit as follows.

$$\max_{p_1, p_2} \; (p_1 - c) D_1^{H,c}(p) + (p_2 - c) D_2^{H,c}(p \mid q_1)$$

Suppose $p_2 \geq \frac{p_1}{2}$. Given $D_1^{H,c}(p \mid p_2 \geq \frac{p_1}{2})$ and $D_2^{H,c}(p \mid p_2 \geq \frac{p_1}{2})$, the profit maximization shows that $p_1^H = 1+\theta+\frac{c}{2}$. Given p_1^H, it is easy to see that $q_1^H = \frac{2(1+\theta)-c}{4}$ and $\pi^c(H,1,p^H) = \frac{(2(1+\theta)-c)^2}{8}$. Suppose $p_2 < \frac{p_1}{2}$. Given $D_1^{H,c}(p \mid p_2 < \frac{p_1}{2})$ and $D_2^{H,c}(p \mid p_2 < \frac{p_1}{2}, q_1)$, if $2\theta < p_1 \leq 1+\theta+p_2$, profit maximization shows that $p_2^H = \frac{p_1}{2}+\frac{c}{4}$. As $p_2^H > \frac{p_1}{2}$, there is a contradiction, therefore $p_1 > 1+\theta+p_2$. If $p_1 > 1+\theta+p_2$, profit maximization shows that $p_2^H = \frac{1+\theta+c}{2}$. Given p_2^H, it is easy to see that $q_2^H = \frac{1+\theta-c}{2}$ and $\pi^c(H,1,p^H) = \frac{(1+\theta-c)^2}{4}$. As $\frac{(1+\theta-c)^2}{4} < \frac{(2(1+\theta)-c)^2}{8}$, profit is maximized when $p_2 \geq \frac{p_1}{2}$ and $p_1^H = 1+\theta+\frac{c}{2}$. ■

Proof of Proposition 2.

Step 1: Necessary conditions for a separating equilibrium: In any separating equilibrium with commitment $p^c(H) \notin S$, where S is given by (5).

The $p = (p_1, p_2)$ is imitated by the low-quality seller if (4) does not hold. By Lemma $\pi^c(L,0,p^{L,c}) = 2\theta$, $p_1 > 2\theta$ and $p_2 > \theta$. Suppose $p_2 \geq \frac{p_1}{2}$. Given (1), $\pi^c(L,1,p) = 0$ for $p_1 > 2(1+\theta)$ and $\pi^c(L,1,p) = p_1(1+\theta-\frac{p_1}{2})$ for $2(1+\theta) \geq p_1 > 2\theta$. That is (4) does not hold if $p_1 \in (2\theta, 2)$. Suppose $p_2 < \frac{p_1}{2}$. Given (3) and (2), $\pi^c(L,1,p) = p_2(1+\theta-p_2)$ for $p_1 > 1+\theta+p_2$ and $\pi^c(L,1,p) = p_1(1+\theta+p_2-p_1)+p_2(p_1-2p_2)$ for $1+\theta+p_2 > p_1 > 2\theta$. For $2\theta < p_1 < 1+\theta+p_2$, (4) does not hold if $p_2 \in (\underline{\underline{p_2}}, \overline{\overline{p_2}})$ where $\underline{\underline{p_2}}, \overline{\overline{p_2}} = \frac{p_1 \pm \sqrt{(2-p_1)(p_1-2\theta)}}{2}$. These equations have no solution for values $p_1 \notin [2\theta, 2]$, i.e., when $p_1 \notin [2\theta, 2]$, (4) holds. We consider only $\underline{\underline{p_2}}$, since $\overline{\overline{p_2}} > \frac{p_1}{2}$. The low-quality seller would not mimic any $p \in \left\{p \mid 1+\theta+p_2 > p_1 \text{ and } \frac{p_1-\sqrt{(2-p_1)(p_1-2\theta)}}{2} < p_2\right\}$. For $p_1 > 1+\theta+p_2$, (4) does not hold if $p_2 \in (\frac{1+\theta-\sqrt{1+6\theta+\theta^2}}{2}, \frac{1+\theta+\sqrt{1+6\theta+\theta^2}}{2})$. The low-quality seller would not mimic any $p \in \left\{p \mid 1+\theta+p_2 < p_1 \text{ and } \frac{1+\theta-\sqrt{1+6\theta+\theta^2}}{2} < p_2 < \frac{1+\theta+\sqrt{1+6\theta+\theta^2}}{2}\right\}$. The range of prices the low-quality seller would not mimic i.e.
$\left\{p \mid 1+\theta+p_2 > p_1 \text{ and } \frac{p_1-\sqrt{(2-p_1)(p_1-2\theta)}}{2} < p_2\right\} \cup$
$\left\{p \mid 1+\theta+p_2 < p_1 \text{ and } \frac{1+\theta-\sqrt{1+6\theta+\theta^2}}{2} < p_2 < \frac{1+\theta+\sqrt{1+6\theta+\theta^2}}{2}\right\}$ can be written simply as $S = \left\{p \mid \frac{p_1-\sqrt{(2-p_1)(p_1-2\theta}}{2}\right.$
$\left\{p \mid 1+\theta+p_2 < p_1 \text{ and } \frac{1+\theta-\sqrt{1+6\theta+\theta^2}}{2} < p_2 < \frac{1+\theta+\sqrt{1+6\theta+\theta^2}}{2}\right\}$.

Step 2: Existence

When $c \geq 2(1-\theta)$, i.e., $p_1^{H,c} \geq 2$, it is easy to see that $p_1^{H,c} \notin S$ and the high-quality seller would always prefer the equilibrium price which is the optimum full information price $p^{H,c}$. In this case the profit of the high-quality seller is equal to its full information profit. When $c < 2(1-\theta)$, the high-quality seller makes less than its full information profit by adopting the signaling price $p_1(H) = 2$. Therefore, the seller may consider deviating from 2 to a lower price in period 1. For such prices, we assign beliefs that put zero probability on the high-quality seller. (Step 3 shows that these beliefs are sustained by the intuitive criterion.) With such a belief, the best profit that the high-quality seller can make by deviating is $\pi^c(H,0,p^{L,c}) = 2\theta - c$. When $c < 2(1-\theta)$ and

$p(H) \notin S$, $p(H) = \{p \mid p_1 = 2 \text{ and } p_2 \geq 1\}$ are the prices which sacrifice least relative to the full information profit. In this case the profit is $\pi^c(H, 1, p(H)) = (2-c)\theta$. Since $(2-c)\theta > 2\theta - c$, the high-quality seller will not consider deviating from the prices given in Proposition 2.

Step 3: Applying the Intuitive Criterion

A commitment equilibrium with prices $p^c(H)$ and $p^c(L)$ satisfies the intuitive criterion if there is no price p' such that (i) $\pi^c(H, 1, p') > \pi^c(H, 1, p^c(H))$ and (ii) $\pi^c(L, 1, p') < \pi^c(L, 0, p^c(L))$. We will show that the equilibrium with prices $p^c(H) = \left\{p \mid p_1 = \max\left\{p_1^{H,c}, 2\right\} \text{ and } p_2 \geq \frac{p_1}{2}\right\}$ and $p^c(L) = \left\{p \mid p_1 = 2\theta \text{ and } p_2 \geq \frac{p_1}{2}\right\}$ satisfies the intuitive criterion. Lemma shows that $p^c(L) = \left\{p \mid p_1 = 2\theta \text{ and } p_2 \geq \frac{p_1}{2}\right\}$. Suppose $p^c(H) \neq \left\{p \mid p_1 = \max\left\{p_1^H, 2\right\} \text{ and } p_2 \geq \frac{p_1}{2}\right\}$. If $c \geq 2(1-\theta)$, the intuitive criterion fails by setting $p' = p^{H,c}$. On the other hand suppose $p^c(H) = p^{H,c}$. The intuitive criterion is not violated for any off-equilibrium price p'. If $c < 2(1-\theta)$, there is no separating equilibrium with $p^c(H) \in S$ by Step 1. If $p^c(H) \notin S$ the intuitive criterion fails by setting $p' = \{p \mid p_1' = 2 \text{ and } p_2' \geq 1\}$ since p' sacrifices least relative to the full information profit. Moreover if $p^c(H) = \{p \mid p_1 = 2 \text{ and } p_2 \geq 1\}$, the intuitive criterion cannot fail for any p'. ■

Proof of Lemma 2. Suppose that seller charges $p_1 \geq \overline{p_1}$. Consumers believe that the product is of high-quality. Further suppose that fraction q_1 consumers buy in period 1. If $p_2 \geq 1 - q_1$, the residual consumers continue to believe that the product is of high-quality, and if $p_2 < 1 - q_1$, they change their beliefs and believe that the product is of low-quality, thus, the demand in period 2 is as follows

$$D_2(p_2 \mid p_1 \geq \overline{p_1}, q_1) = \begin{cases} 0 & \text{, if } 1 + \theta - q_1 < p_2 \\ 1 + \theta - q_1 - p_2 & \text{, if } \max\{\theta, 1 - q_1\} < p_2 \leq 1 + \theta - q_1 \\ 0 & \text{, if } \theta < p_2 \leq 1 - q_1 \\ 1 - q_1 & \text{, if } p_2 \leq \theta \end{cases}$$

The high-quality seller's second period problem is

$$\max_{p_2} \ (p_2 - c) D_2(p_2 \mid p_1 \geq \overline{p_1}, q_1)$$

When $p_1 \geq \overline{p_1}$, the profit in period 2 is maximized by (17). Therefore, when $p_1 \geq \overline{p_1}$, the high-quality seller has no incentive to deviate from (17) in period 2. Replacing (17) to the profit function we get (18). ■

Proof of Proposition 4. We derive the equilibrium for case (iii) in the text. Here, first we show that the equilibrium in case (iii) satisfies the intuitive criterion. A no-commitment equilibrium with prices $p(H)$ and $p(L)$ satisfies the intuitive criterion if there is no price p' such that (i) $\pi(H, (1,1), p') > \pi(H, (1,1), p(H))$ and (ii) $\pi(L, (1,1), p') < \pi(L, (0,0), p(L))$. Suppose

$f_3 \leq \theta < f_2$ and $p(H) \neq (\overline{p_1}, \frac{\overline{p_1}+c}{3})$ where $\overline{p_1} = \frac{3+5\theta+c+3\sqrt{c^2+6c+2\theta c+9-18\theta+\theta^2}}{4}$. The intuitive criterion fails by setting $p' = (\overline{p_1}, \frac{\overline{p_1}+c}{3})$. On the other hand suppose $p(H) = (\overline{p_1}, \frac{\overline{p_1}+c}{3})$. The intuitive criterion is not violated for any off-equilibrium price p'.

Now, we find the equilibrium for the other parameter values. First, notice that given $\overline{p_1}$, p_1 and q_1, the period 2 solution for the parameter values $f_3 \leq \theta < f_2$ holds for all parameter values. As Lemma 2 holds for all the parameter values, $D_1(p_1 \mid p_1 \geq \overline{p_1})$ can be specified by (19) and period 1 profit maximization problem of the high-quality seller for $p_1 \geq \overline{p_1}$ is as in (21). Now, we show that the high-quality seller has no incentive to deviate from the equilibrium specified in Proposition 4. Given $\overline{p_1}$ and the beliefs, high-quality seller has no incentive to deviate to prices $p_1 < \overline{p_1}$. The proof is identical to the one in case (iii).

First, we solve for the case (i). Suppose $\theta > f_1$. The beliefs supporting the equilibrium are same as the one in case (iii) except that $\overline{p_1} = 1 + \theta$. When $\theta > f_1$, (21) is maximized by $p_1 = \overline{p_1}$. Therefore, there is no incentive to deviate to prices $p_1 > \overline{p_1}$. From (19), we can see that at $p_1 = \overline{p_1}$ it is rational, for fraction $q_1 = \theta$ consumers buy in period 1. From (17), we can see that high-quality seller signals by its profit maximizing price $p_2(H \mid p_1 = \overline{p_1}) = \theta$ in period 2. These prices yield a profit of $\pi(H) = 2\theta - c$. Suppose $p(H) \neq (\overline{p_1}, \theta)$. The intuitive criterion fails by setting $p' = (\overline{p_1}, \theta)$. On the other hand suppose $p(H) = (\overline{p_1}, \theta)$. The intuitive criterion is not violated for any off-equilibrium price p'.

Next, we solve for the case (ii). Suppose $f_2 < \theta \leq f_1$. The beliefs supporting the equilibrium are same as the one in case (i). When $f_2 < \theta \leq f_1$, (21) is maximized by $p_1 = \frac{9(1+\theta)+5c}{10} > \overline{p_1}$. Therefore, there is no incentive to deviate to prices $p_1 > \frac{9(1+\theta)+5c}{10}$ and $\frac{9(1+\theta)+5c}{10} > p_1 > \overline{p_1}$. From (19), we can see that at $p_1 = \frac{9(1+\theta)+5c}{10}$ it is rational, for fraction $q_1 = \frac{2(1+\theta)}{5}$ consumers buy in period 1. From (17), we can see that high-quality seller signals by its profit maximizing price $p_2(H \mid p_1 = \overline{p_1}) = \frac{3(1+\theta)+5c}{10}$ in period 2. These prices yield a profit of $\pi(H) = \frac{9(1+\theta)^2-5c(2(1+\theta)-c)}{20}$. Suppose $p(H) \neq (\frac{9(1+\theta)+5c}{10}, \frac{3(1+\theta)+5c}{10})$. The intuitive criterion fails by setting $p' = (\frac{9(1+\theta)+5c}{10}, \frac{3(1+\theta)+5c}{10})$. On the other hand suppose $p(H) = (\frac{9(1+\theta)+5c}{10}, \frac{3(1+\theta)+5c}{10})$. The intuitive criterion is not violated for any off-equilibrium price p'.

Finally, we solve for the case (iv). Suppose $\theta < f_3$. The beliefs supporting the equilibrium are same as the one in case (iii) except that $\overline{p_1} = 1 + \theta + \sqrt{1 - 2\theta}$. Given the beliefs, high-quality seller has no incentive to deviate to prices $p_1 > \overline{p_1}$. The proof is identical to the one in case (iii). When $\theta < f_3$, (21) is maximized by $p_1 = \overline{p_1}$. Therefore, there is no incentive to deviate to prices $p_1 > \overline{p_1}$. From (19), we can see that at $p_1 = \overline{p_1}$ it is rational, for fraction $q_1 = 1 + \frac{\theta-\overline{p_1}}{2}$ consumers buy in period 1. From (17), we can see that high-quality signals by a price greater than its profit maximizing price, i.e., $p_2(H \mid p_1 = \overline{p_1}) = \frac{\overline{p_1}-\theta}{2}$ in period 2. These prices yield a profit

of $\pi(H) = \frac{2\theta(2-c)-c(1-\sqrt{1-2\theta})}{2}$. Suppose $p(H) \neq (\overline{p_1}, \frac{\overline{p_1}-\theta}{2})$. The intuitive criterion fails by setting $p' = (\overline{p_1}, \frac{\overline{p_1}-\theta}{2})$. On the other hand suppose $p(H) = (\overline{p_1}, \frac{\overline{p_1}-\theta}{2})$. The intuitive criterion is not violated for any off-equilibrium price p'. ■

Proof of Proposition 5. Proposition 2 gives $p_1^c(H) = \max\left\{p_1^H, 2\right\}$. Case (iii) of Proposition 4 gives $p_1(H) = \frac{3+5\theta+c+3\sqrt{c^2+6c+2\theta c+9-18\theta+\theta^2}}{4}$ for $f_3 \leq \theta < f_2$. If $c > \max\left\{\frac{3\theta^2-4\theta+2}{2-\theta}, \frac{4\theta-2}{1+\theta}\right\}$, $\frac{3+5\theta+c+3\sqrt{c^2+6c+2\theta c+9-18\theta+\theta^2}}{4}$ $> \max\left\{p_1^H, 2\right\}$. For $f_3 \leq \theta < f_2$, welfare with no-commitment is given by (25) as $W = \frac{99+79\theta^2+43c^2+258\theta-126c-94\theta c+(}{}$ Welfare with commitment W^c is given by (24). If $c > \max\left\{\frac{3\theta^2-4\theta+2}{2-\theta}, \frac{4\theta-2}{1+\theta}\right\}$, $W^c > W$. ■

Proof of Proposition 6. W^c and W are given by (24) and (25). For welfare comparison we partition the parameter space into four as in the Proposition 4. When $\theta > f_2$, $W^c < W$. Consider the case $\theta < f_3$. If $c > \frac{(1+\theta)(1-\theta-\sqrt{1-2\theta})}{1-\sqrt{1-2\theta}}$, $W^c > W$, otherwise $W^c < W$. Finally, consider the case $f_3 \leq \theta < f_2$. For this case the solution to $W^c = W$ cannot be characterized explicitly, however we can show that $\frac{d(W^c-W)}{dc} > 0$ and when $c = \theta$, $W^c > W$. Therefore, there is a $c = \gamma(\theta)$ at which $W^c = W$. ■

References

[1] Bagwell, Kyle and Riordan, Michael H., "High and Declining Prices Signal Product Quality," *American Economic Review*, 1991, 81:1, pp. 224–239.

[2] Balachander, Subramanian and Srinivasan, Kannan, "Modifying Customer Expectations of Price Decreases for a Durable Product," *Management Science*, 1998, 44:6, pp. 776-786.

[3] Bulow, Jeremy. "Durable Goods Monopolists." *Journal of Political Economy.* 1982, April, 90:2, pp. 314–32.

[4] Cho, I.-K. and D. Kreps. "Signaling Games and Stable Equilibria." *Quarterly Journal of Economics,* 1987, 102, pp. 179-222.

[5] Coase, Ronald. "Durability and Monopoly." *Journal of Law and Economics*, 1972, April, 15:1, pp. 143–149.

[6] Daughety, A. and J. Reinganum. "Secrecy and Safety." *American Economic Review. 2005,* 95, pp. 1074-91.

[7] Daughety, A. and J. Reinganum. "Competition and Confidentiality: Signaling Quality in a Duopoly Model." *Games and Economic Behavior,* 2007, 58, pp. 94-120.

[8] Daughety, A. and J. Reinganum. "Imperfect Competition and Quality Signaling." *The RAND Journal of Economics*, 2008, 39-1, pp. 163-183.

[9] Janssen and Roy, "Dynamic Trading in a Durable Good Market with Asymmetric information." *International Economic Review,* 2002, 43, pp. 257 - 282.

[10] Janssen and Roy, "Signaling Quality Through Prices in an Oligopoly." *SMU Working Papers,* 2009.

[11] Hendel, I. and Lizzeri, A. "Adverse Selection in Durable Goods Markets." *American Economic Review*, 1999, 89:5, pp. 1097–1115.