COST SHARING IN PUBLIC GOODS GAME IN NETWORKS

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Abstract

The existing literature studying the effect of network structure on public good provision reports a negative relationship between the number of neighbors an individual has and their likelihood of investing. The evidence points to the lack of incentives that individuals in central network positions have to invest in the local public good. This paper uses a laboratory experiment to test the relative efficacy of two cost sharing rules in raising efficiency across three network structures in a best shot public goods game. Across the three network structures, I vary the asymmetry in the number of neighbors each position has in the network. The two cost sharing rules are designed to incentivize individuals with more neighbors to invest. The first rule is a local cost sharing, where individuals who invest receive transfers from each of their neighbors who do not invest. The second is a global cost sharing rule, where the total cost of investment is equally divided among individuals who benefit from the public good. The efficiency of provision is the lowest in absence of cost sharing rules. The low efficiency is driven by the under-provision of the public good. Introducing the two cost sharing rules increases the provision of the public good. The local cost sharing rule increases efficiency across all three network structures. The effectiveness of the global cost sharing rule in raising efficiency decreases as the asymmetry of the network structure increases.

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1 Introduction

The decisions made by individuals in central positions in social and geographical networks have asymmetric effects on social welfare when actions exhibit local positive externalities Zenou (2016). Costly investment decisions by individuals in central positions benefit a large number of people who are connected to them. These decisions could range from farmers deciding whether to invest in a new crop or farming technique[1] to physicians deciding whether to adopt a new procedure or protocol[2] or R&D decisions (Bramoullé, 2007, Bramoullé et al., 2014). There is no a priori reason to expect central individuals to behave pro-socially. In fact, if actions are substitutable the incentive is to not invest and free-ride on investment decisions of people connected to them[3].

The experimental literature studying the effect of network structure on public good provision finds an inverse relationship between the number of neighbors and the likelihood of making an investment (Charness et al., 2014, Rosenkranz and Weitzel, 2012). There is little empirical evidence on mechanisms that can motivate individuals in central positions to make costly pro-social investment decisions. Dufwenberg and Patel (2017) and Jackson and Wilkie (2005) show that theoretically cost sharing agreements can raise efficiency in public good provision. Dufwenberg and Patel (2017) argues that reciprocity alone is not sufficient but combined with cost sharing can enhance the private provision of public goods.

Therefore, I investigate the efficacy of two cost sharing rules on efficiency in a best shot public good game[4] (Hirshleifer, 1983) across three network structures in a laboratory experiment. In the *local cost sharing rule* individuals who invest get a transfer of a fixed proportion of the cost from each of their neighbors who dont invest. This rule exploits the number of connections of the individuals to align their incentives with social efficiency, since for a given cost sharing proportion the cost of investing is decreasing as the number of neighbors increases. The *global cost sharing rule* is based on the component-wise egalitarian allocation rule (Jackson, 2005); here, the total cost of provision is equally shared among individuals who benefit from the public good. Payoff for everyone in the group is maximized when everyone in the group benefits from the public good and the least number of individuals are investing.

[1]There is a large body of literature highlighting the importance of social learning in technology adoption and diffusion (Chuang and Schechter, 2015, Foster and Rosenzweig, 2010, Conley and Udry, 2010).

[2]Tasselli (2014) provides an excellent overview of the literature studying the effect of social networks on physicians decisions.

[3]An equilibrium where the maximum number of unlinked agents are investing is stochastically stable in a best-shot public good games (Boncinelli and Pin, 2012, Bramoullé and Kranton, 2007).

[4]See Harrison and Hirshleifer (1989) for experiments on a best-shot game.

I vary the number of neighbors that individuals have across the network structures. The Circle network is symmetric, and everyone has two neighbors, whereas the Line and Asymmetric networks are not symmetric. In the Line, individuals in the periphery have one neighbor, and others have two neighbors, and in the Asymmetric network, individuals have either one, two or three neighbors. This asymmetry provides a rich environment to test the relative efficacy of the two cost sharing rules in increasing efficiency across different network structures.

Across all network structures, the efficiency[5] is the lowest in the treatment with no cost sharing rules. The low level of efficiency is driven by under-provision[6] only 72% of the group member has access to a public good compared to 92% when the two cost sharing rules are introduced. Consistent with earlier research (Charness et al., 2014, Rosenkranz and Weitzel, 2012) I find a negative relationship between the number of neighbors an individual has and their likelihood of investing in the non-symmetric networks. This pattern of investment lowers efficiency due to excess investment in the public good. The local cost sharing treatment is successful in increasing efficiency across all network structures by incentivizing individuals with more neighbors to invest. In the global cost sharing rule, as the asymmetry in the network increases, the effectiveness of the rule in raising efficiency decreases. Compared to the baseline the access to public good increases with introducing the global cost sharing rule however, it is mired by over investment.

Comparing across networks, I find the efficiency is highest in the Circle network, which is in line with the previous findings (Fatas et al., 2010, Carpenter et al., 2012, Rosenkranz and Weitzel, 2012, Leibbrandt et al., 2015, Boosey and Isaac, 2016)'.

This chapter contributes to the small literature on public goods game in network[7]. In particular, Rosenkranz and Weitzel (2012) and Charness et al. (2014) find that individuals with more neighbors contribute less than players with fewer neighbors. The closest to my work is Charness et al. (2014) which reports that in a best shot public goods game groups start by coordinating on the efficient equilibria and eventually drift towards the *stable inefficient equilibrium*[8]. There is little evidence pointing at mechanisms that incentivize central individuals to make costly pro-social investments. Caria and Fafchamps (2018)

[5]Efficiency is defined as the ratio of the realized group profit and maximum possible total profit in each round.

[6]Under-provision is defined as the ratio of the number of subjects who benefit from the public good and the total number of group members.

[7](Fatas et al., 2010, Carpenter et al., 2012, Rosenkranz and Weitzel, 2012, Leibbrandt et al., 2015, Charness et al., 2014, Boosey and Isaac, 2016, Caria and Fafchamps, 2018). See Choi et al. (2016) for an excellent overview of laboratory experiments in networks.

[8]In a stochastically stable equilibrium maximum number of agents are investing Boncinelli and Pin (2012).

reports results from an artifactual field experiment with farmers in India, where they exploit guilt aversion as the mechanism to induce pro-social behavior. They find that individuals in the center of a star network are more likely to invest once subjects are made aware of the expectations of the individuals in the periphery. I contribute to this literature by offering two cost sharing rules which facilitate groups to coordinate on the efficient equilibrium by incentivizing central individuals to invest in the public good.

In the next section, I present a theoretical analysis and derive testable hypotheses. Section 3 presents the experiment design and procedures. I present the empirical results in Section 4 and Section 5 concludes.

2 Predictions

Consider n agents and let the set of agents be $N = \{1, \ldots, n\}$. Each agent i simultaneously chooses to either invest (1) or not invest (0) in a local public good. Let $\boldsymbol{a} = (a_1, \ldots, a_n)$ denote the action profile of all agents, where $a_i \in \{0, 1\}$ is agent is action. Agent is action determines her payoff and affects payoffs of the agents to whom she is linked through positive externalities. Agents are assigned on an undirected graph. Any two agents i and j who share a local public good are represented by a link: $g_{ij} = g_{ji} = 1$. For two agents who are not linked $g_{ij} = g_{ji} = 0$. Let the collection of all links be represented by $n \times n$ matrix $\boldsymbol{G}$. Let N_i denote the set of agents who are directly linked to agent i, called agent is *neighbors*: $N_i = \{j \in N/i : g_{ij} = 1\}$. Agents is *neighborhood* is defined as herself and the set of her neighbors; i.e., $\{i\} \cup N_i$.

An agent gets a benefit of *(b)* from the local public good if she or any of her direct neighbors invest. The cost of providing the local public good, c, is positive but smaller than b. Let $\boldsymbol{a_j}$ denote the set of actions of all agents $j \neq i$. An agent is payoff:

$$u_i(a_i, \boldsymbol{a_j}, \boldsymbol{G}) = b \times \mathbb{1}\left\{\sum_j g_{ij} a_j + a_i \geq 1\right\} - c \times a_i \tag{1}$$

It is straightforward to show that each agent i's best reply is: $a_i = 1$ if no one in the neighborhood invest and (ii) $a_i = 0$, if at least one of her neighbors invest.

I consider three networks of five agents – Line, Asymmetric, and Circle (see Figure 1). Table 1 reports the pure strategy Nash equilibrium for each network structure. One of the central features of this game is the multiplicity of equilibria. Boncinelli and Pin (2012) refine the equilibrium set through stochastic stability, and show that if the source of error affects both investing and not investing agents or agents randomize using a logistic response, then the only stochastically stable states are Nash equilibria with the largest number of unlinked agents investing. Based on the empirical evidence in favor of Boncinelli and Pin (2012) in public good experiments in networks (Charness et al., 2014), I expect that when there is no cost sharing groups are more likely to coordinate on the stochastically stable equilibrium.

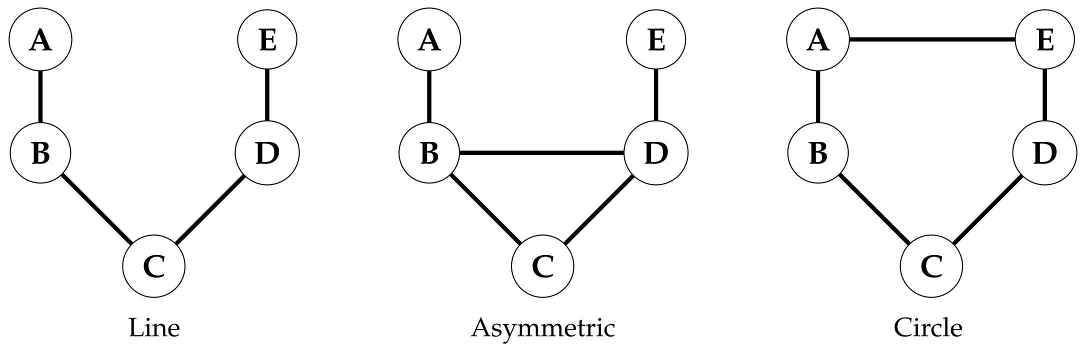


Figure 1: Network structures in the experiment

To analyze the welfare implications of different equilibria, I use a standard utilitarian measure of social welfare, $W(\cdot)$, defined as:

$$W(\boldsymbol{a}, \boldsymbol{G}) = \sum_{i \in N} b \times \mathbb{1} \left\{ \sum_{j} g_{ij} a_j + a_i \geq 1 \right\} - c \times \sum_{i \in N} a_i$$

Note that groups maximize the total social welfare in any equilibrium when all agents in the network have access to the local public good and the minimum number of agents are investing. Across all the network structures, equilibria with only two unlinked agents investing maximizes total welfare for the group (see Table 1).

Table 1: Pure Strategy Nash Equilibrium

Network (equilibrium)	Investment Choice					$W(\cdot)$
Line	**A**	**B**	**C**	**D**	**E**	
(L1)	1	0	1	0	1	$5b - 3c$
(L2)	0	1	0	1	0	$5b - 2c$
(L3)	0	1	0	0	1	$5b - 2c$
(L4)	1	0	0	1	0	$5b - 2c$
Asymmetric						
(A1)	1	0	1	0	1	$5b - 3c$
(A2)	0	1	0	0	1	$5b - 2c$
(A3)	1	0	0	1	0	$5b - 2c$
Circle						
(C1)	1	0	0	1	0	$5b - 2c$
(C2)	1	0	1	0	0	$5b - 2c$
(C3)	0	1	0	0	1	$5b - 2c$
(C4)	0	0	1	0	1	$5b - 2c$
(C5)	0	1	0	1	0	$5b - 2c$

See Appendix **??** *for proof and mixed strategy Nash equilibrium profiles.*

2.1 Network Structures

Since investments are substitutes, adding a link has two opposite effects, it increases access to the local public good, but, it also increases incentives to free-ride[9]. Starting with the Line network adding a link between B & D leads to the Asymmetric network. This increases the number of neighbors of B and D has. However, adding the link does not alter the inefficient equilibrium, but it reduces the set of Nash equilibria, which could help with coordination. It is an empirical question whether an additional link leads to efficiency gains.

Adding a link between the agents on the periphery of the Line network results in the Circle network. In the Circle network, theoretically all pure strategy Nash equilibria are efficient, but the pure strategy Nash set is larger, which might lead to coordination problems. Given the two countervailing effects of adding a link, it is unclear a priori if it leads to efficiency gains.

Hypothesis 1. ***The efficiency is the same in Line and Asymmetric networks and higher in Circle network in the baseline.***

[9] Galeotti et al. (2010) show that in a symmetric Bayes-Nash equilibria in a best shot public goods game with monotone (threshold) strategies, agents invest only if the number of neighbors they have is below a certain threshold. The threshold decreases as the number of neighbors is increasing.

2.2 Local Cost Sharing Rule

The investment decision of agents with more neighbors disproportionately affects the total social welfare, since a large number of agents connected to them benefit from their investment. The local cost sharing rule incentivizes individuals in central positions to invest by introducing transfers from each of their neighbors who do not invest.

Definition 1. *The local cost sharing rule is defined as following:*

$$u_i(1, \boldsymbol{a_j}, \boldsymbol{G}) = b - c + c\eta \left(\sum_{j \in N_i} (1 - a_j) \right)$$
$$u_i(0, \boldsymbol{a_j}, \boldsymbol{G}) = b \times \mathbb{1} \left\{ \sum_{j \in N_i} a_j \geq 1 \right\} - c\eta \sum_{j \in N_i} a_j \tag{2}$$

Where $1 \geq \eta > 0$ *determines the fixed proportion of the cost, an investor receives from each of her neighbors who choose not to invest.*

The local cost sharing incentivizes agents with more neighbors to invest by lowering the cost of investing via transfers. Conversely, the transfers incentivize agents with fewer neighbors to free-ride. In the baseline given the parameter values[10], in the inefficient equilibrium, the payoff for agents in positions B and D in the Line and Asymmetric network is 100 (see Table 2). Whereas, with the local cost sharing rule, the payoff in the inefficient equilibrium for B and D is 50; they get a payoff of 100 as they have access to the public good and transfer 25 to two neighbors who invest. In the Line network the payoff for all agents in equilibria where either B or D are investing (*L3 and L4 see table* 2), is higher or equal to the payoff in the inefficient equilibrium. In the Asymmetric network, the inefficient equilibrium is Pareto dominated by the equilibria in which either B or D is investing (A2 and A3 see Table 2). In the Asymmetric network the local cost sharing rule creates a competition between B and D to invest, since in equilibria where either B or D are investing, the agent who invests gets back the whole cost of provision as a transfer from three of her neighbors who are not investing, while the agent who is not investing gets a lower payoff of 50. This competition leads to the rise of a fourth equilibrium profile in the Asymmetric network in which only B & D are investing in the group, this equilibrium is efficient but does not Pareto dominate the inefficient equilibrium since the payoff of C is lower.

[10] $b = 100$, $c = 75$ and $\eta = \frac{1}{3}$.

Proposition 1. *Given the local cost sharing rule and $\eta \leq \frac{1}{3}$, the inefficient equilibrium in the Line and Asymmetric network is Pareto dominated by an efficient equilibrium.*[11]

The local cost sharing rule incentivizes the groups to coordinate away from the inefficient equilibrium to the set of the Pareto dominant efficient equilibria in the Line and Asymmetric network. Note that the local cost sharing rule does not alter the set of efficient equilibria in the Circle network.

Hypothesis 2. ***Introducing the local cost sharing rule will increase efficiency in the Line and Asymmetric networks.***

In comparison to the Circle network, the set of Pareto dominant equilibria is smaller in the Line and Asymmetric network. The smaller equilibrium set could help to reduce frictions in coordination, which can lead to higher efficiency.

Hypothesis 3. ***Given the local cost sharing rule the efficiency in Line and Asymmetric networks will be higher than in the Circle network.***

2.3 Global Cost Sharing Rule

I base the global cost sharing rule on the component-wise egalitarian allocation rule discussed in (Jackson, 2005, Page 150). In the global cost sharing rule, the total cost of investments in the network is divided equally among agents who benefit from the local public goods.

Definition 2. *The global cost sharing rule is defined as the following:*

$$u_i(a_i, a_j, \boldsymbol{G}) = \left[b - \frac{c \times \sum_k^N a_k}{\sum_k^N \mathbb{1}\left\{\sum_{j \in N_k} g_{kj} a_j + a_k \geq 1\right\}} \right] \times \mathbb{1}\left\{ \sum_{j \in N_i} g_{ij} a_j + a_i \geq 1 \right\} \tag{3}$$

where

1. $\sum_k^N a_k$ *– the total number of agents who chose to invest,*
2. $\sum_k^N \mathbb{1}\left\{\sum_{j \in N_k} g_{kj} a_j + a_k \geq 1\right\}$ *– total number of agents who benefit from the public good.*

The global cost sharing rule aligns an agent's payoff with the maximum group payoff. The rule removes incentives to free-ride as well as over-investments for all group members because the payoff for everyone in the group decreases as the total number of

[11] For proof see Appendix B.

investments increases. I can rank equilibria based on the total group payoffs. The inefficient equilibrium where the maximum number of agents are contributing yields the lowest payoff for all agents in the network, while any efficient equilibria yield a higher payoff (*see, efficiency column Table 2*).

Proposition 2. *Given the global cost sharing rule, the inefficient equilibrium in the Line and Asymmetric network is Pareto dominated by an efficient equilibrium.*[12]

The global cost sharing rule incentivizes groups to coordinate away from the inefficient equilibrium in the Line and Asymmetric network. The global cost sharing rule in the Circle network by subsidizing the cost of investment reduces the incentive to free-ride compared to the baseline, which can help groups to coordinate and raise efficiency.

Hypothesis 4. ***In comparison to the game without cost-sharing the efficiency is higher across all three network structures.***

The three network structures can be ranked based on the number of Pareto dominant Nash equilibria, a smaller set of equilibria could help groups to coordinate.

Hypothesis 5. ***With the introduction of the global cost sharing rule, the efficiency will be highest in the Asymmetric network, followed by Line and Circle.***

Both cost sharing rules incentivize agents to coordinate on the set of efficient equilibria across all network structures. However, the global cost sharing rule induces transfers across unconnected agents even though there are no externalities that flow between them. This feature of the global cost sharing rule might lead to coordination failure compared to the local cost sharing rule.

3 Experiment Design and Procedures

3.1 Experiment Game

Experiment design crosses the three network structures: Line, Asymmetric, and Circle with the baseline and the two cost sharing rules: local cost sharing rule, and the global cost sharing rule. I implement a finitely repeated version of a best-shot game in stages of ten rounds. In each stage, subjects interact in groups of five in one of the three network structures. The subjects know their position in the network. A subjects position and the group remain fixed within a stage. Subjects make simultaneous decisions on whether to

[12] For proof see Appendix B

invest in the public good. The benefit of the public good is available to everyone in the neighborhood. The benefit from investing is $b = 100$ cents, and the cost of investing is $c = 75$ cents. The payoff for each round in the baseline game is, therefore:

$$u_i(a_i, \boldsymbol{a_j}, \boldsymbol{G}) = 100 \times \mathbb{1}\left\{\sum_j g_{ij}a_j + a_i \geq 1\right\} - 75 \times a_i \tag{4}$$

In the two cost sharing treatments, I keep the parameter values of b and c fixed at 100 cents and 75 cents. In the *local cost sharing rule* the cost sharing parameter $\eta = \frac{1}{3}$. Thus, each subject who invests gets a transfer of 25 cents from each of her neighbors who did not invest in the local public good. The payoff for the subjects in the local cost sharing rule is given by:

$$\begin{aligned} u_i(1, \boldsymbol{a_j}, \boldsymbol{G}) &= (100 - 75) + 25 \sum_{j \in N_i} (1 - a_j) \\ u_i(0, \boldsymbol{a_j}, \boldsymbol{G}) &= 100 \times \mathbb{1}\left\{\sum_{j \in N_i} a_j \geq 1\right\} - 25 \sum_{j \in N_i} a_j \end{aligned} \tag{5}$$

In the *global cost sharing rule* the total cost of investment is equally divided[13] among subjects who benefit from the local public good. The payoff for the subjects of the global cost sharing rule is given by:

$$u_i(\boldsymbol{a}, \mathbf{G}) = \left[100 - \frac{75 \times \sum_{k=1}^{5} a_k}{\sum_{k=1}^{5} \mathbb{1}\left\{\sum_{j \in N_k} g_{kj}a_j + a_k \geq 1\right\}}\right] \times \mathbb{1}\left\{\sum_{j \in N_i} g_{ij}a_j + a_i \geq 1\right\} \tag{6}$$

Table 2 presents the round payoffs across all pure strategy Nash equilibria.

[13]the cost of one investment is the same as the baseline, 75 cents.

Table 2: Theoretical profits in equilibrium for each round.

Network (equi.)	Baseline					Profit in cents every round Local Cost Sharing					Global Cost Sharing	Efficiency
Line	A	B	C	D	E	A	B	C	D	E	Each Agent	
(L1)	25	100	25	100	25	50	50	75	50	50	55	0.79
(L2)	100	25	100	25	100	75	75	50	75	75	70	1
(L3)	100	25	100	100	25	75	75	75	75	50	70	1
(L4)	25	100	100	25	100	50	75	75	75	75	70	1
Asymmetric												
(A1)	25	100	25	100	25	50	50	75	50	50	55	0.79
(A2)	100	25	100	100	25	75	100	75	50	50	70	1
(A3)	25	100	100	25	100	50	50	75	100	75	70	1
(A4)*	—	—	—	—	—	75	75	50	75	75	—	1
Circle												
(C1)	25	100	100	25	100	75	75	75	75	50	70	1
(C2)	25	100	25	100	100	75	50	75	75	75	70	1
(C3)	100	25	100	100	25	50	75	75	75	75	70	1
(C4)	100	100	25	100	25	75	75	75	50	75	70	1
(C5)	100	25	100	25	100	75	75	50	75	75	70	1

*In the Asymmetric network A4 is an equilibrium only in the *local cost sharing rule*.

3.2 Experiment Procedures

The experiment was conducted at the ExCEN lab at Georgia State University in February - March 2018. A total of 240 subjects participated in the experiment over twelve sessions. The subjects were recruited via email using the ExCEN automated system. Upon arrival at the lab, the subjects reviewed and signed the consent form and were randomly assigned seats in the lab. Throughout the session, subjects were not allowed to communicate with each other[14]. Each session was conducted in three stages, followed by a demographic survey. At the start of each stage, subjects were instructed to read the experimental instruction (*see appendix B*) at their own pace[15]. Before the start of each stage in order to gauge a better understanding of the game, subjects had the option to explore a game simulator at their own pace; they also played a set of practice rounds. Each session[16] lasted for roughly one hour and fifteen minutes.

Subjects only took part in one of the three network structures: Line, Asymmetric, or Circle. Each session consisted of three stages of ten rounds each. At the start of each stage, *su*bjects were randomly matched to form groups of five, and each subject was randomly assigned a network position. The group and the position remained fixed within each stage. Across stages, treatments were introduced. In the first stage, subjects played the

[14]To ensure privacy, each computer terminal in the lab is enclosed with dividers.

[15]A summary of instructions was read out loud which was available for subjects to see on their computer screens.

[16]computerized using z-Tree (Fischbacher, 2007).

baseline game, followed by the two cost sharing rules. Across sessions, the order in which the two cost sharing rules were introduced was randomized.

As explained above, in the baseline treatment, the benefit from an investment in the neighborhood common fund was 100 cents for all individuals in the neighborhood of the investor, and the cost was 75 cents. In the local cost sharing rule, each subject who invests receives a transfer of 25 cents from each neighbor who did not invest. The cost sharing rule was enforced after all subjects made their investment decisions. In the global cost sharing treatment subjects were informed that the benefit of investing was still 100 cents, but all subjects who received a benefit from the neighborhood common fund would equally share the cost of total investment.

At the end of every round, the subjects were provided a summary of the number of the contributing neighbors, their position on the network, their investment decisions and their earnings; this information was available for all the 30 periods. I chose to repeat the stage game for ten rounds[17] to allow for enough time for groups to understand the game and coordinate on an equilibrium.

At the end of each session, each subject answered a questionnaire asking on demographics and some context-specific risk and social preferences questions. Subjects were paid for all 30 periods in cash privately right after the experiment session. The average payoff in the experiment was \$14.50 per subject, with a minimum of \$ 9.60 and maximum earning of \$ 21.60.

4 Results

4.1 Efficiency across cost-sharing rules

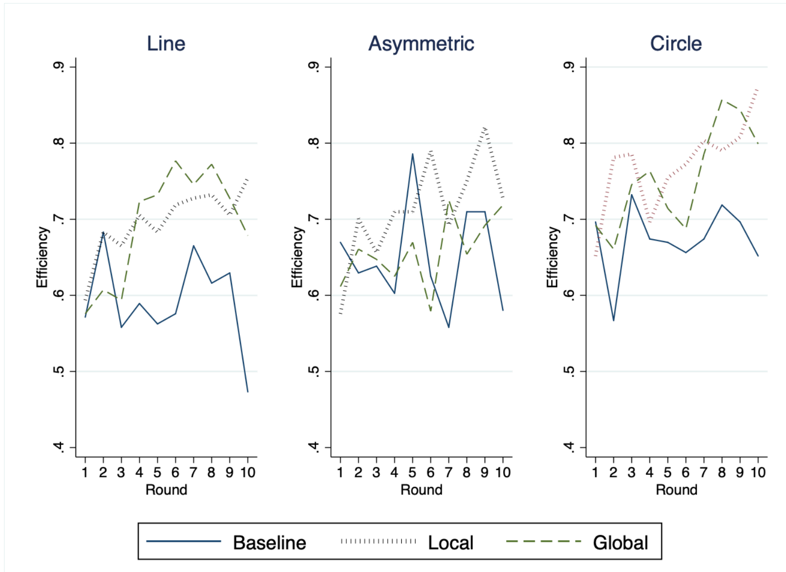
The two cost sharing rules incentivize groups to coordinate away from the inefficient equilibrium in the Line and Asymmetric networks. Hypotheses 2 and 4 state that compared to the baseline, the two cost sharing rules will increase efficiency in both the network structures. The set of pure strategy Nash equilibria are efficient in the Circle network; the two cost sharing rules subsidize the cost of investing compared to the baseline. The lower cost has two countervailing effects in the Circle network; it could lead to an increase in efficiency because of an increase in the provision of the public good or a loss in efficiency due to over-investment compared to the baseline.

To test the effect of two cost sharing rules on efficiency, first I compute the efficiency

[17] Best response dynamics converge to a pure strategy Nash equilibrium within at most $2 \times N$ (Komarovsky et al., 2015, Proposition 2); in our case 10 rounds.

for each round using a traditional measure[18]. Figure 2 shows efficiency across the rounds in the baseline and the two cost sharing rules across the three network structures.

Figure 2: Efficiency Across Treatments



Inspecting figure 2 one can observe that in the first round across all the three treatments groups start with a similar level of efficiency. In the Line and Circle, compared to the baseline efficiency is higher in the two cost sharing rules. However, in the Asymmetric network, the effect of the two cost sharing rules is not clear. To identify the effect of the two cost sharing rules on the efficiency of provision, I run a linear panel regression model with efficiency as the dependent variable and a categorical variable for baseline, and the two cost sharing rules as the variable of interest with a random effect at the group level. To account for learning about the group members due to the repetition of the game, I use round numbers as a control. Table 3[19] reports for each network structure the change in efficiency in the local and the global cost sharing rule compared to the treatment without cost sharing (baseline).

[18] $\text{Efficiency} = \frac{\text{Realized Payoff}_{\text{round}}}{\text{Theoretical Max. Possible Payoff}}$

[19] I find similar results if I use $1/Round$ as an independent variable, see Appendix A.2

Table 3: OLS Estimates - Efficiency Across Treatment

	Line	Asymmetric	Circle
Local	0.104***	0.0625**	0.0982***
	(0.031)	(0.032)	(0.032)
Global	0.101***	0.00748	0.0813**
	(0.031)	(0.032)	(0.032)
Round	0.00825**	0.00799***	0.0118***
	(0.004)	(0.003)	(0.003)
Observations	480	480	480

Robust standard errors in parentheses
$*p < 0.10, **p < 0.05, ***p < 0.01$

In the Line network, the local cost sharing rule increases the efficiency of the play by 17% compared to 59.2% efficiency in the baseline. Compared to the baseline efficiency of 65% in the Asymmetric network, the local cost sharing rule increased efficiency by 9.75%. The local cost sharing rule increases efficiency by 14.5% in the Circle network. I summarize the effect of the local cost sharing rule on efficiency in the following result.

Result 1. ***Compared to the baseline, the local cost sharing rule is successful in raising efficiency across the three network structures.***

Hypothesis 4 states that the global cost sharing rule would increase efficiency in all three network structures. Compared to the baseline, the global cost sharing rule in the Line network increases efficiency by 17% and by 12% in the Circle network. However, there are no gains in efficiency in the Asymmetric network. The following results summarize the findings for the global cost sharing rule.

Result 2. ***Compared to the baseline, the global cost sharing rule is successful in raising efficiency in the Line and Circle network structure but it has no effect on efficiency for Asymmetric network.***

Both the local and global cost sharing rules align individual incentives with higher group efficiency. Unlike the local cost sharing rule, global cost sharing induces transfers among unlinked agents. The cross-subsidization of investment, even when there is no flow of externalities could lead to coordination failure since choices of subjects outside their neighborhood determine an individuals payoffs. This could lead to lower efficiency in the global cost sharing rule when compared to the local cost sharing rule. To test if there is any difference in efficiency between the local and global cost sharing treatments in each of the network structure, I run a Wald test for equality of the two estimated coefficients.

In the Line and Circle, there is no statistically significant difference between efficiency in the local and global cost sharing rules; however, in the Asymmetric network, the local cost sharing rule is successful in raising efficiency compared to the global cost sharing rule ($p = 0.075$).

4.2 Efficiency across networks

Hypotheses 1, 3, and 5 state differences in efficiency across network structures in the three treatments: baseline, local cost sharing, and global cost sharing. To test these hypotheses, I run a linear panel regression with efficiency as the dependent variable and a categorical variable for the three network structures as the explanatory variable of interest with random effects at the group level. To account for learning, I use the round number as a control variable. For each treatment: baseline, local cost sharing, and global cost sharing table 4[20] reports the differences in efficiency in Asymmetric and Circle comparison to the Line network.

Table 4: OLS Estimates - Efficiency Across Networks

	Baseline	Local	Global
Baseline (mean)	0.592	0.697	0.693
	(.021)	(0.016)	(0.017)
Asymmetric	0.0585**	0.0165	-0.0349
	(0.028)	(0.034)	(0.030)
Circle	0.0812***	0.0750**	0.0616**
	(0.030)	(0.032)	(0.030)
Round	-0.000974	0.0148***	0.0142***
	(0.004)	(0.003)	(0.003)
Observations	480	480	480

Robust standard errors in parentheses
$*p < 0.10, **p < 0.05, ***p < 0.01$

In the baseline, hypothesis 1 states that the efficiency in the Circle will be higher compared to the Line and Asymmetric networks. Comparing between the Line and Asymmetric networks, there should not be any differences in efficiency if groups coordinate on the inefficient equilibrium[21]. However, if a smaller pure strategy Nash equilibrium set can aid with coordination, then in the Asymmetric, the efficiency could be higher because

[20] I find similar results if I use $1/Round$ as an independent variable, see Appendix A.2.

[21] Charness et al. (2014) find that stochastic stability drives equilibrium selection in best shot games in a network.

of lower coordination failure. The efficiency in the Asymmetric network is 9.8% higher than the Line network.

The Circle network is 13.7% more efficient compared to the Line network. To compare the difference in efficiency between the Asymmetric to Circle network, I run a Wald test with the null hypothesis that the two regression coefficients are equal. I fail to reject the null hypothesis; there is no statistically significant difference between the efficiency of the Asymmetric and Circle networks.

In the Line and Asymmetric networks, the two cost sharing rules align individual incentives with group efficiency. Recall that all equilibria in Circle network are efficient. However, in both the cost sharing rules compared to the Line and Asymmetric, the set of Pareto dominant Nash equilibria is the larger in the Circle network. If the larger set makes it hard for groups to coordinate on an equilibrium, then one might expect a lower level of efficiency in Circle network, as stated in Hypothesis 3 and 5.

In both the local and global cost sharing rules, the efficiency in Line and Asymmetric networks are not statistically different from each other. Whereas compared to the Line, the Circle network is 10% more efficient in the local cost sharing rule and 8.8% more efficient in the global cost sharing rule. A Wald test of equality of regression coefficient of Asymmetric and Circle network. The Circle is more efficient than the Asymmetric network in local cost sharing ($p = 0.076$) and global cost sharing ($p = 0.0003$). I summarize these observations in the following result.

Result 3. ***The Circle is the most efficient network, with or without the cost sharing of investments.***

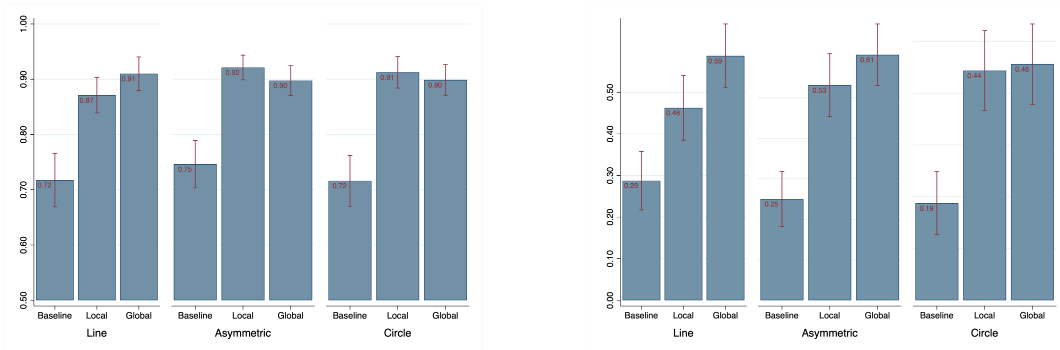
4.3 What are the sources of inefficiency?

There are two sources of inefficiency: under-provision and over-provision of the local public good. The welfare loss because of under-provision is higher than overprovision because the benefits from the public good exceed the cost of provision. Under-provision implies a lack of access to the public good. To capture underprovision of the public good, I construct the following measure; *access to public good*, as the ratio between the total number of subjects in the group who receive a benefit from the public good and total group size. An increase in access to the public good could lead to over-investment in a round. To capture over-investment, I define overprovision in a round as a binary variable which takes the value one if all five group members receive a benefit from the public good but there are over two investments in the group.

Figure 3: Sources of Inefficiency

Access to Public Good

Overprovison of Public Good



Inspection of Figures 3a and 3b suggests that the two cost sharing rules increase access to the public good, but they also increase excess investment. Table 5[22] reports results from a linear panel regression with access to public and excess provision as the dependent variable and a categorical variable for the three treatments: baseline, local cost sharing, and global cost sharing as independent variables of interest with random effects at the group level. I use the round as control variables to account for learning. Across all the network structures, the two cost sharing rules increase access to the public good, which improves efficiency. However, greater access comes with overprovision, which lowers efficiency.

Table 5: Access to Public Good and Excess Provision

	Access to Public Good			Over Provision		
	Line	Asymmetric	Circle	Line	Asymmetric	Circle
Local	0.154***	0.175***	0.196***	0.175***	0.281***	0.256***
	(0.029)	(0.025)	(0.028)	(0.058)	(0.039)	(0.051)
Global	0.192***	0.151***	0.182***	0.300***	0.356***	0.269***
	(0.022)	(0.030)	(0.030)	(0.069)	(0.046)	(0.068)
round	0.000379	0.00167	0.00379	-0.00833	-0.00934	-0.0119
	(0.003)	(0.004)	(0.003)	(0.009)	(0.007)	(0.008)
Observations	480	480	480	480	480	480

Robust standard errors in parentheses
$*p < 0.10, **p < 0.05, ***p < 0.01$

Result 4. ***The two cost sharing rules increase the access to public good however that leads to over investment across the three network structures.***

[22] I find similar results if I use $1/Round$ as an independent variable, see Appendix A.2.

4.3.1 What equilibria are groups more likely to coordinate on?

In this section, I look at equilibrium selection. Figure 4 shows the share of observations consistent with an equilibrium across three network structures. Table 6 reports the percentage of group decisions consistent with equilibrium across three network structures.

Figure 4: Share of Rounds Consistent with Equilibrium

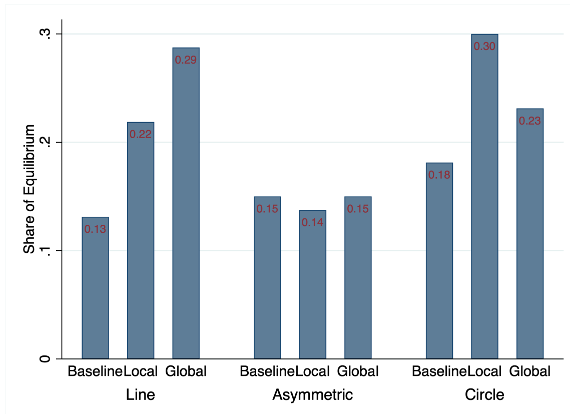


Table 6: Frequency of Equilibrium

Network		**Equilibrium**	**Baseline**	**Local**	**Global**
Line	Equilibrium		13%	22%	29%
	Inefficient	*L1*	29%	23%	35%
	Efficient	*L2*	10%	3%	24%
		L3	10%	9%	22%
		L4	52%	66%	20%
Asymmetric	Equilibrium		15%	14%	15%
	Inefficient	*A1*	46%	0%	79%
	Efficient	*A2*	38%	45%	8%
		A3	17%	5%	13%
		A4	–	50%	–
Circle	Equilibrium		18%	30%	23%
	Efficient	*C1*	21%	35%	14%
		C2	28%	29%	32%
		C3	7%	8%	30%
		C4	24%	13%	22%
		C5	21%	15%	3%

Inspecting figure 4 shows that the two cost sharing rules increase the number of decisions consistent with an equilibrium in the Line and Circle networks. To further investigate the effect the two cost sharing rules have on the likelihood of observing an equilibrium, I run a logistic panel model where the dependent variable in a dummy variable which takes the value one when a group decision is consistent with equilibrium, and zero otherwise. The independent variable of interest is the categorical variable for each of the three treatments: baseline, local cost sharing, and global cost sharing. I also use the round number to account for learning. Table 7[23] reports the marginal effect of the two cost sharing rules compared to the baseline for each of the three network structures.

Table 7: Marginal Effect of Probability of Coordinating on Equilibrium

	Line	Asymmetric	Circle
Baseline	0.132	0.153	0.184
	(0.031)	(0.043)	(0.040)
Local	0.0859*	-0.0212	0.117*
	(0.050)	(0.058)	(0.063)
Global	0.155***	-0.000941	0.0454
	(0.054)	(0.060)	(0.059)
Round	0.0154**	0.0155***	0.0227***
	(0.006)	(0.005)	(0.006)
Observations	480	480	480

Robust standard errors in parentheses
$*p < 0.10, **p < 0.05, ***p < 0.010$

Table 8: Mean Investment by Position

	Baseline					**Local Cost Sharing**					**Global Cost Sharing**				
Network	**A**	**B**	**C**	**D**	**E**	**A**	**B**	**C**	**D**	**E**	**A**	**B**	**C**	**D**	**E**
Line	57%	26%	36%	39%	44%	66%	44%	44%	56%	45%	61%	55%	46%	56%	66%
Asymmetric	51%	31%	38%	24%	50%	41%	78%	58%	56%	48%	71%	45%	59%	44%	73%
Circle	36%	29%	39%	26%	34%	70%	46%	42%	54%	36%	53%	38%	53%	43%	60%

Line

In the baseline, 13% of group decisions are consistent with an equilibrium prediction. Only 29% of the observations are at the inefficient equilibrium. 52% of group decisions consistent with the equilibrium is the efficient equilibrium where subjects in positions A and D are investing. The local cost sharing rule increases the likelihood of coordinating on any equilibrium by 8.6 percentage points (see Table 7). Similar to the baseline in the local cost sharing rule, 66% of equilibrium profiles are ones where subjects in positions A and D are investing. The global cost sharing rule increases the probability of coordinating

[23] I find similar results if I use $1/Round$ as an independent variable, see Appendix A.2

on equilibrium by 15.5 percentage points, groups are equally likely to coordinate on any of the four pure strategy equilibrium profile[24].

Asymmetric

15% of the group decisions are consistent with an equilibrium prediction in the baseline. The two cost sharing rules do not increase the likelihood of coordinating on an equilibrium (see Table 7). However, the two cost sharing rules influence the equilibrium profile groups to coordinate on. In the baseline, groups are coordinating either on inefficient equilibrium (46%) or on the equilibria where subjects in positions B and E are investing (38%).The local cost sharing rule induces groups to coordinate away from the inefficient equilibrium to either the one where B and E are investing or B and D are investing[25]. In the global cost sharing rule in 79% of the observation consistent with equilibrium, groups coordinate on the inefficient equilibrium where A, C, and E are investing.

Circle

In the baseline 18% of the group decisions are consistent with equilibrium. Groups are equally likely to coordinate on all equilibria except C3, where B and E are investing[26]. The local cost sharing rule increases the likelihood of equilibrium play by 11.7 percentage points. Groups are more likely to coordinate on the equilibrium where either A and E (C1) are investing, or A and C (C2) are investing[27]. Compared to the baseline, the global cost sharing rule has no effect on the likelihood of equilibrium play; groups are more likely to coordinate on C1, C2 and C3[28].

4.3.2 Effect of cost sharing rule on free-riding behavior

Charness et al. (2014) and Rosenkranz and Weitzel (2012) find a negative relationship between the number of neighbors a subject has and their likelihood of investing in the local public good. Figure 5 for the Asymmetric and Line networks shows the average investment for subjects with one, two, and three neighbors. Examining figure 5 there is evidence of an inverse relationship between the number of neighbors and likelihood

[24]Subjects across all nodes are investing on average 57% of rounds (see Table 8).

[25]This is driven by investment behavior of subjects in position B who invest 78% of rounds (see Table 8).

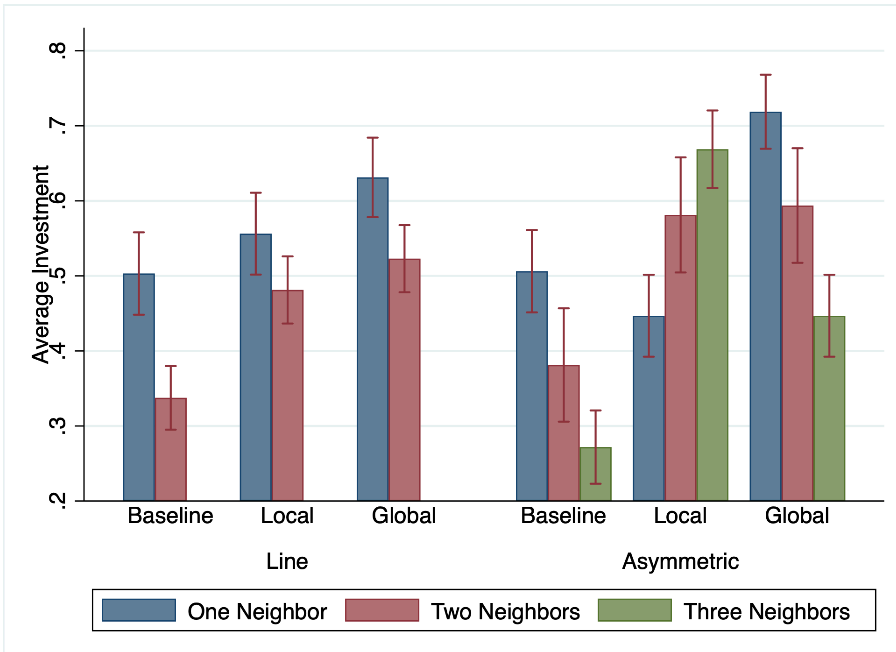
[26]This pattern is consistent with the observation that subjects across all nodes are contributing 33% of the times.

[27]This is driven by the investment behavior of subjects in position A who invest in 70% of the rounds (see Table 8).

[28]Subjects across all positions except on position B are investing 50% of the rounds (Table 8).

of investing in both the Line and Asymmetric networks. The two cost sharing rules in the Line network increase the probability of investment independent of the number of neighbors. In the Asymmetric network, the local cost sharing rule induces subjects with more neighbors to invest more than subjects with fewer neighbors, whereas in the global cost sharing rule one can see the negative relationship between the number of neighbors and their likelihood of investing.

Figure 5: Mean Investment by Neighbors



I further investigate the effect of the two cost sharing rules on the likelihood of investing in the Asymmetric network[29]. I run a logistic panel regression where the binary decision to invest is the dependent variable, and the number of neighbors is the primary variable of interest. In addition to the number of neighbors, I take into account the previous period decision, the total investments by the neighbors in the previous period, round, session fixed effect, and demographic variables – gender and race, and as well as random effects at the subject level. I report the estimated marginal effect by the number of neighbors in Asymmetric network in Table 9[30] for the three treatments: baseline, local cost sharing, and global cost sharing.

[29]The results for Line network structure are available on request.

[30]I find similar results if I use $1/Round$ as an independent variable, see Appendix A.2.

Table 9: Marginal Effect of Neighbors on Investment

	Baseline	Local	Global
One Neighbor (mean)	0.499	0.386	0.678
	(.039)	(.049)	(.041)
Two Neighbors	-0.118*	0.209**	-0.0934
	(0.072)	(0.094)	(0.072)
Three Neighbors	-0.222***	0.276***	-0.207***
	(0.054)	(0.077)	(0.077)
# Neighbors invest lag	0.0157	-0.0283	-0.0478*
	(0.031)	(0.037)	(0.027)
Invest lag	-0.116***	-0.0629	-0.0662
	(0.041)	(0.046)	(0.043)
Round	Yes	Yes	Yes
Race	Yes	Yes	Yes
Gender	Yes	Yes	Yes
Session Fixed Effects	Yes	Yes	Yes
Observations	720	720	720

Robust standard errors in parentheses; $p^{*} < 0.10$, $p^{**} < 0.05$, $p^{***} < 0.010$

In the baseline, subjects with two neighbors and subjects with three neighbors are 23% and 44% less likely to invest compared to subjects with one neighbor. Comparing subjects with two or three neighbors, I find no statistically significant difference between the likelihood of investing (p = 0.1490).

The local transfer rule incentivizes subjects with more neighbors to invest in the local public good. Subjects with three neighbors are 71% more likely to invest compared to subjects with one neighbor.

The global cost sharing rule aligns individual incentive with total social welfare, but unlike the local cost sharing rule it does not directly induces individuals with more neighbors to invest.. The subsidized cost of investment could lead to over provision of the public good in the network. In global cost sharing, the subjects with three neighbors are 30% less likely to invest compared to subjects with one neighbor.

5 Conclusion

In this chapter, I study a repeated best shot public goods game in three network structures. The primary aim of the chapter is to test mechanisms that can raise the total social welfare. I introduce two cost sharing rules and show theoretically that the inefficient equilibrium in the Line, and Asymmetric network is Pareto dominated by an efficient equilibrium. I used a laboratory experiment to test the empirical validity of the theoretical predictions. Compared to the baseline, which is mired by under-provision of the public good, the two cost sharing rules are successful in increasing access to the public good. However, the increase in access leads to over-investment, which leads to a loss in efficiency. The local cost sharing rule increases efficiency across all network structures, whereas global cost sharing is successful in increasing efficiency only in the Line and Circle networks.

These findings have implications for policy interventions that involve costly investment decisions and have local positive externalities. Examples include the adoption of new farming techniques or making expensive time investments to adopt new medical practices. It is imperative not only to identify the central players who can help with diffusion but also provide incentives in the form of cost sharing rules which lower the cost of experimentation and increase take-up of new products and technology. The local cost sharing rule is successful in raising efficiency across the three network structures. To implement the local cost sharing rule, it suffices to know the maximum degree for any network structure to calibrate the cost sharing proportion. There is evidence from field experiments in India that social networks can be effectively used to implement and uphold contracts (Breza and Chandrasekhar, 2019). Implementing the local cost rule in large networks might be procedurally cumbersome. For a larger network structures, the global cost sharing rule might be more effective since it is easy to implement, and it can help with increasing access to local public goods although there is over-investment in the public good.

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A Instructions

WELCOME!

No Talking Allowed
Once the experiment begins, we request that you do not to talk until the end of the experiment. If you have any questions, please raise your hand.

Three Stages
There are 3 stages in this experiment. Each stage consists of 10 rounds. So, there are a total of 30 rounds in this experiment. At the beginning of each stage, you will be:

1. Randomly matched with four other individuals in the room. Group composition remains fixed within each stage but differs across stages.

2. At the end of each round, you will be provided a summary of your earnings in the experiment.

Payment
You will earn in cents for the decisions you make in each round of the experiment. At the end of the experiment, you will be paid in cash your total earnings from all the 30 rounds.

Decision Environment
Members of each group are randomly assigned to one of the five positions, {A, B, C, D or E}, as shown, in Figure 1, below at the beginning of each stage. Assigned positions remains fixed within each stage but differs across stages.

Your assigned position determines which members of the group you are connected to. A connection between two positions is represented by a line. For example, in Figure 1, if your position is at C, then you are connected to two of your group members, the ones assigned to positions B and D. We will call B and D **your neighbors** and {B, C, D} **your neighborhood**.

Decision Task and Payoffs
Stage 1:
There is a neighborhood common fund that you share with your neighbors. At the beginning of each round, everyone is asked to make a decision on whether to INVEST in the neighborhood common fund at a cost of 75 cents. If there is at least one investment in the neighborhood common fund then the individual who invested, as well as each of his neighbors, earns 100 cents.

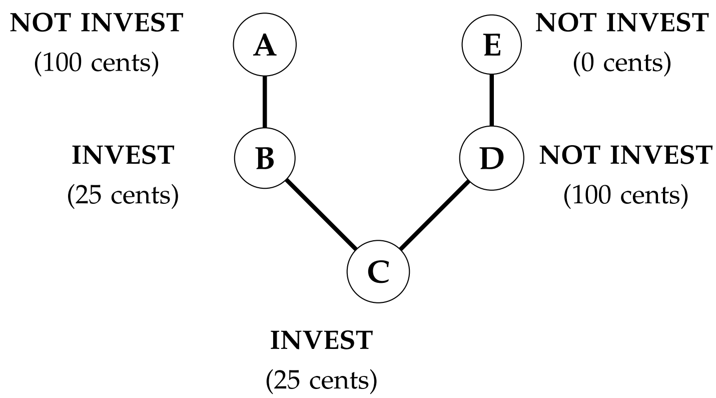
Line

1. You earn 25 cents: 100 cents (from the neighborhood common fund) minus 75 cents (the cost of investing).

2. Your neighbor at D earns 100 cents: 100 cents (from the neighborhood common fund as you and B invested).

3. Your neighbor at B earns 25 cents: 100 cents (from neighborhood common fund) minus 75 cents (the cost of investing)

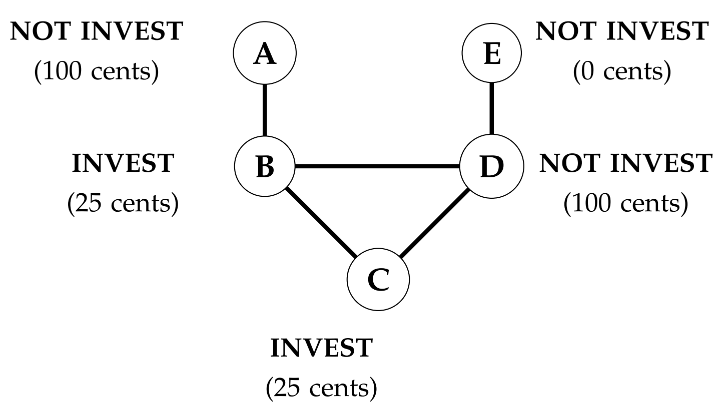
Figure 6: Stage 1 - Instruction - Line



Asymmetric

1. You earn 25 cents: 100 cents (from the neighborhood common fund) minus 75 cents (the cost of investing).
2. Your neighbor at D earns 100 cents: 100 cents (from the neighborhood common fund as you and B invested).
3. Your neighbor at B earns 25 cents: 100 cents (from neighborhood common fund) minus 75 cents (the cost of investing)

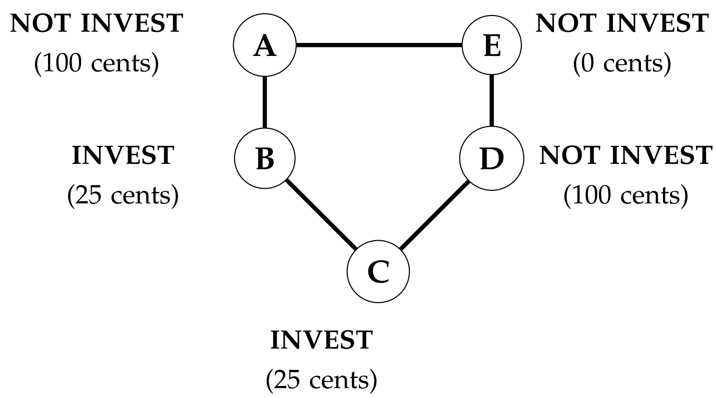
Figure 7: Stage 1 - Instruction - Asymmetric



Circle

1. You earn 25 cents: 100 cents (from the neighborhood common fund) minus 75 cents (the cost of investing).
2. Your neighbor at D earns 100 cents: 100 cents (from the neighborhood common fund as you and B invested).
3. Your neighbor at B earns 25 cents: 100 cents (from neighborhood common fund) minus 75 cents (the cost of investing)

Figure 8: Stage 1 - Instruction - Circle



Stage 2:

Transfers are introduced in stage 2: Individuals who invest receive a transfer of 25 cents from each neighbor who does not invest. Decision tasks are the same as in Stage 1.

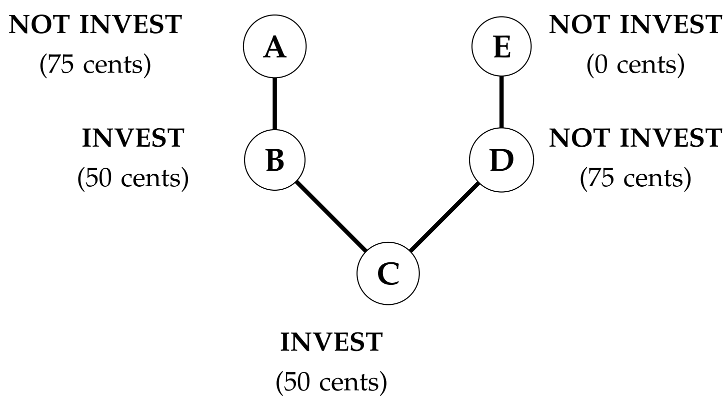
Example: Suppose your position is C. If C (you), and B invest but A, D, and E do not invest, then both you and B receive a transfer of 25 cents from D, in addition B receives a transfer of 25 cents from A. Payoffs (Figure 2) are:

Line

1. You earn 50 cents: 100 cents (from neighborhood common fund), minus 75 cents (the cost of investing) plus 25 cents (25 cents transfer from D your neighbor who did not invest).
2. Your neighbor at D earns 75 cents: 100 cents (from the neighborhood common fund as you invested) minus 25 cents (D's transfer to you).

3. Your neighbor at B earns 50 cents: 100 cents (from neighborhood common fund), minus 75 cents (the cost of investing) plus 25 cents (25 cents transfer from A, B's neighbors who did not invest).

Figure 9: Local Cost Sharing - Instruction - Line



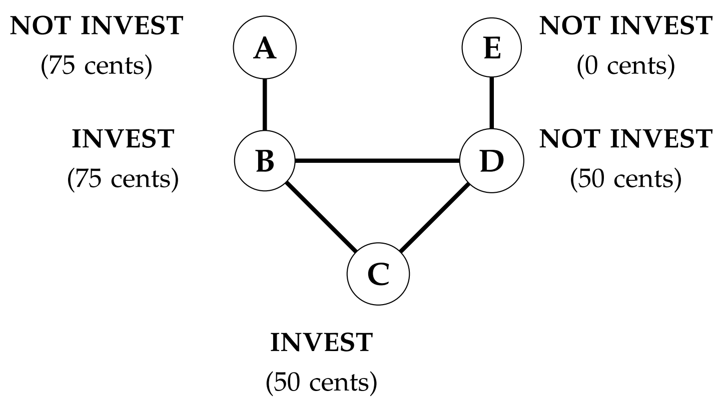
Asymmetric

1. You earn 50 cents: 100 cents (from neighborhood common fund), minus 75 cents (the cost of investing) plus 25 cents (25 cents transfer from D your neighbor who did not invest).

2. Your neighbor at D earns 50 cents: 100 cents (from the neighborhood common fund as you invested) minus 50 cents (D's transfers to you and B).

3. Your neighbor at B earns 75 cents: 100 cents (from neighborhood common fund), minus 75 cents (the cost of investing) plus 50 cents (25 cents transfers from A and D, B's neighbors who did not invest).

Figure 10: Local Cost Sharing - Instruction - Asymmetric



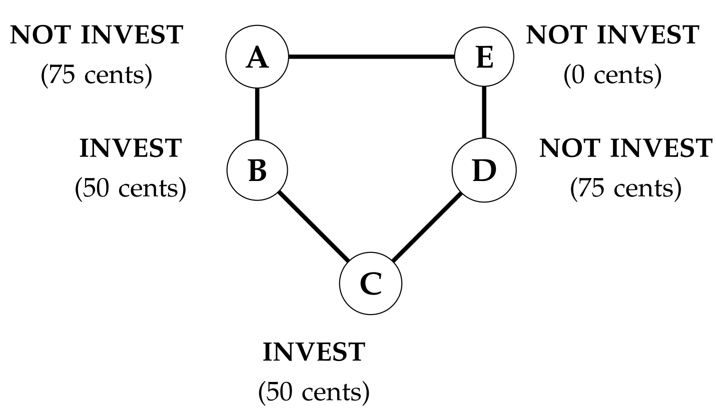
Circle

1. You earn 50 cents: 100 cents (from neighborhood common fund), minus 75 cents (the cost of investing) plus 25 cents (25 cents transfers from D your neighbor who did not invest).

2. Your neighbor at D earns 75 cents: 100 cents (from the neighborhood common fund as you invested) minus 25 cents (D's transfer to you).

3. Your neighbor at B earns 50 cents: 100 cents (from neighborhood common fund), minus 75 cents (the cost of investing) plus 25 cents (25 cents transfers from A, B's neighbors who did not invest).

Figure 11: Local Cost Sharing - Instruction - Circle



Stage 3:

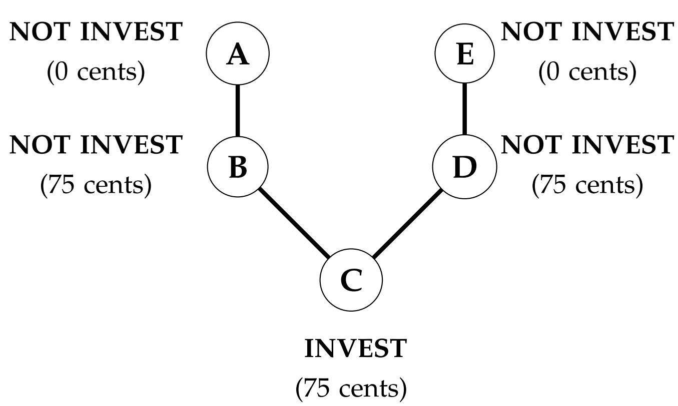
Cost sharing is introduced in stage 3: The total cost of investments for the group is equally shared by individuals who benefit from the common fund. Decision tasks are the same as in Stage 1.

Example: Suppose your position is C. Suppose C (you) invest but A, B, D, and E do not invest. Then the total cost of investment is 75 cents. C (you), B and D benefit from the common fund and each pay 25 cents (= 75/3). Then payoffs (Figure 3) are:

Line

1. You earn 75 cents: 100 cents (from the neighborhood common fund) minus 25 cents (the cost of investing).
2. Your neighbor at D earns 75 cents: 100 cents (from the neighborhood common fund) minus 25 cents (the cost of investing).
3. Your neighbor at B earns 75 cents: 100 cents (from the neighborhood common fund) minus 25 cents (the cost of investing).

Figure 12: Global Cost Sharing - Instruction - Line



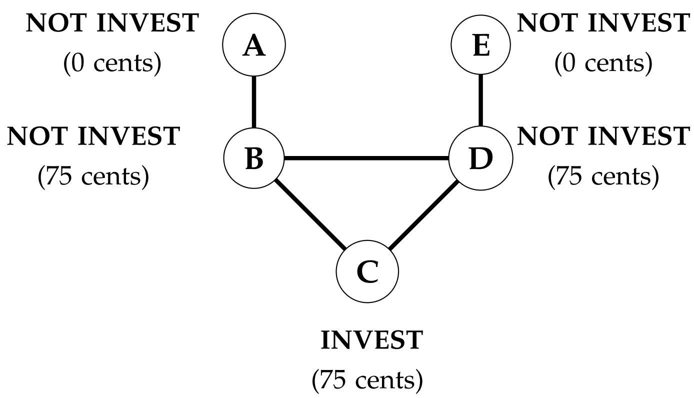
Asymmetric

1. You earn 75 cents: 100 cents (from the neighborhood common fund) minus 25 cents (the cost of investing).

2. Your neighbor at D earns 75 cents: 100 cents (from the neighborhood common fund) minus 25 cents (the cost of investing).

3. Your neighbor at B earns 75 cents: 100 cents (from the neighborhood common fund) minus 25 cents (the cost of investing).

Figure 13: Global Cost Sharing - Instruction - Asymmetric



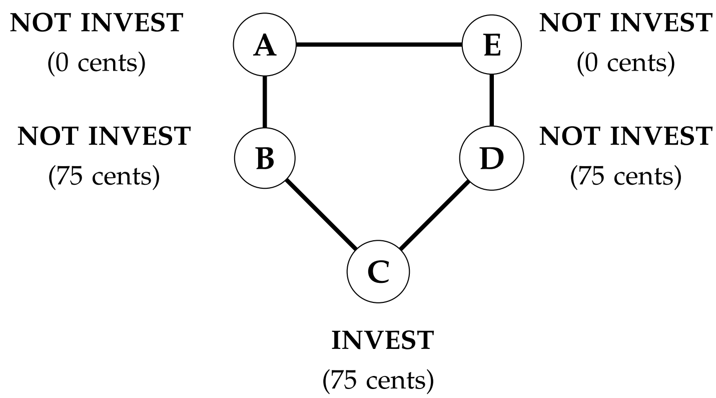
Circle

1. You earn 75 cents: 100 cents (from the neighborhood common fund) minus 25 cents (the cost of investing).

2. Your neighbor at D earns 75 cents: 100 cents (from the neighborhood common fund) minus 25 cents (the cost of investing).

3. Your neighbor at B earns 75 cents: 100 cents (from the neighborhood common fund) minus 25 cents (the cost of investing).

Figure 14: Global Cost Sharing - Instruction - Circle



sectionExtra Tables

Table 10: OLS Estimates - Efficiency Across Treatments (1/Rounds)

	Line	Asymmetric	Circle
Local	0.104***	0.0625*	0.0982***
	(0.024)	(0.033)	(0.032)
Global	0.101***	0.00748	0.0813***
	(0.025)	(0.037)	(0.031)
$1/Round$	-0.115***	-0.0859***	-0.106***
	(0.038)	(0.026)	(0.040)
Observations	480	480	480

Robust standard errors in parentheses

* p¡0.10, ** p¡0.05, *** p¡0.01

Table 11: OLS Estimates - Efficiency Across Network (1/Rounds)

	Baseline	Local	Global
Asymmetric	0.0585**	0.0165	-0.0349
	(0.028)	(0.034)	(0.030)
Circle	0.0812***	0.0750**	0.0616**
	(0.030)	(0.032)	(0.030)
$1/Round$	0.00401	-0.169***	-0.141***
	(0.040)	(0.030)	(0.030)
Observations	480	480	480

Robust standard errors in parentheses

* p¡0.10, ** p¡0.05, *** p¡0.01

Table 12: Access to Public Good and Excess Provision (1/Rounds)

	Access to Public Good			Excess Provision		
	Line	Asymmetric	Circle	Line	Asymmetric	Circle
Local	0.154***	0.175***	0.196***	0.0963**	0.271***	0.163***
	(0.029)	(0.025)	(0.028)	(0.041)	(0.038)	(0.041)
Global	0.192***	0.151***	0.182***	0.150***	0.224***	0.174***
	(0.022)	(0.030)	(0.030)	(0.055)	(0.032)	(0.049)
$1/Round$	0.00614	0.0210	-0.00322	0.193***	0.156***	0.195***
	(0.036)	(0.033)	(0.037)	(0.030)	(0.043)	(0.047)
Observations	480	480	480	480	480	480

Robust standard errors in parentheses

* p¡0.10, ** p¡0.05, *** p¡0.01

Table 13: Marginal Effect of Probability of Coordinating on Equilibrium (1/Rounds)

	Line	Asymmetric	Circle
Baseline (mean)	0.131***	0.150***	0.181***
	(0.028)	(0.028)	(0.036)
Local	0.0875**	-0.0125	0.119**
	(0.042)	(0.039)	(0.046)
Global	0.156***	0	0.0500
	(0.045)	(0.040)	(0.044)
$1/Round$	-0.156*	-0.202**	-0.253***
	(0.080)	(0.086)	(0.089)
Observations	480	480	480

Robust standard errors in parentheses

* p¡0.10, ** p¡0.05, *** p¡0.010

Table 14: Marginal Effect of Neighbors on Investment (1/Rounds)

	Baseline	Local	Global
One neighbor	0.500	0.385	0.679
	(0.039)	(0.050)	(0.041)
Two neighbors	-0.118*	0.210**	-0.0941
	(0.072)	(0.094)	(0.072)
Three neighbors	-0.222***	0.279***	-0.208***
	(0.054)	(0.077)	(0.077)
# neighbors invest lag	0.0157	-0.0303	-0.0465*
	(0.031)	(0.037)	(0.027)
Invest lag	-0.116***	-0.0659	-0.0660
	(0.041)	(0.045)	(0.043)
1/*Round*	Yes	Yes	Yes
Race	Yes	Yes	Yes
Gender	Yes	Yes	Yes
Session	Yes	Yes	Yes
Observations	720	720	720

Robust standard errors in parentheses

* p¡0.10, ** p¡0.05, *** p¡0.01

B Proof – Proposition

Lemma 1. *In the baseline game without cost sharing rules, in the Line network, the pure strategy Nash equilibria are (1,0,1,0,1), (0,1,0,1,0), (1,0,0,1,0), and (0,1,0,0,1). In the Asymmetric network, the pure-strategy Nash equilibria are (1,0,1,0,1), (1,0,0,1,0), and (0,1,0,0,1). In the Circle network, the pure strategy Nash equilibria are (1,0,0,1,0), (1,0,1,0,0), (0,1,0,0,1), (0,0,1,0,1) and (0,1,0,1,0)*

Proof. In a Nash equilibrium, we show that:

1. $a_i = 1$ if and only if $\forall j \in N_i$, $a_j = 0$

2. $a_i = 0$ if and only if $\exists j \in N_i$ s.t. $a_j = 1$

Let's consider the first condition, consider a profile of actions, a_j such that, $\forall j \in N_i, a_j = 0$. Then the best reply for agent i is $a_i = 1$ because $u_i(0, \boldsymbol{a_j}, G) = 0$ and $u_i(1, \boldsymbol{a_j}, G) = b - c$, $u_i(1, \boldsymbol{a_j}, G) > u_i(0, \boldsymbol{a_j}, G)$ since $b > c > 0$. Consider a profile of actions , a_j such that, $\exists j \in N_i$ s.t. $a_j = 1$, then the best reply of agent i is $a_i = 0$ because $u_i(0, \boldsymbol{a_j}, G) = b$ and $u_i(1, \boldsymbol{a_j}, G) = b - c$ and $u_i(0, \boldsymbol{a_j}, G) > u_i(1, \boldsymbol{a_j}, G)$ since $b > b - c$. Assume a Nash equilibrium $a_i = 0$ and $\forall j \in N_i$, $a_j = 0$, then the best reply for agent i is $a_i = 1$ because $u_i(0, \boldsymbol{a_j}, G) = 0$ and $u_i(1, \boldsymbol{a_j}, G) = b - c$, a contradiction. Assume a Nash equilibrium $a_i = 1$ and $\exists j \in N_i$ s.t. $a_j = 1$, then the best reply for agent i is $a_i = 0$ because $u_i(0, \boldsymbol{a_j}, G) = b$ and $u_i(1, \boldsymbol{a_j}, G) = b - c$, a contradiction. Based on the best reply it is straightforward to verify that the pure strategy Nash equilibria listed are the set of all possible Nash equilibria ■

Proposition 3. *Suppose $\eta \leq \frac{1}{\max_G\{N_k\}}$:*

1. *If it is optimal for an agent to invest in the baseline, then it is optimal for the agent to invest in the game with local cost sharing.*

2. *If it is optimal for an agent to not invest in the baseline, then it is optimal for the agent to not invest in the game with local cost sharing.*

3. *Inefficient equilibrium in the Line and Asymmetric network is Pareto dominated by an efficient equilibrium.*

Proof. The following are the proofs for the three claims in Proposition 1.

Part 1: From Lemma 1 we know that in the baseline game without cost sharing, $a_i = 1$ if and only if $\forall j \in N_i$, $a_j = 0$. The best reply is the same in the game with local cost sharing. In the game with local cost sharing rule, consider a profile of action, a_j such that, $\forall j \in N_i$, $a_j = 0$. Then the best reply for player i is $a_i = 1$ because $u_i(0, \boldsymbol{a_j}, G) = 0$ and $u_i(1, \boldsymbol{a_j}, G) = b - c + N_i \eta c$, $u_i(1, \boldsymbol{a_j}, G) > u_i(0, \boldsymbol{a_j}, G)$ since $b - c > 0$ and $N_i \eta c > 0$.

Part 2: From Lemma 1 we know that in the baseline game without cost sharing, $a_i = 0$ if and only if $\exists j \in N_i$ s.t. $a_j = 1$. In the game with local cost sharing rule, consider the following two cases:

Case 1: For $\eta < \frac{1}{\max_G\{N_k\}}$ the best reply in the game with local cost sharing is the same as in the baseline. Assume a Nash equilibrium where, r agents in agent i's neighborhood are investing, then the best reply of agent i is $a_i = 0$ because $u_i(0, \boldsymbol{a_j}, G) = b - r\eta c$ and $u_i(1, \boldsymbol{a_j}, G) = b - c + (N_i - r)\eta c$, $u_i(0, \boldsymbol{a_j}, G) > u_i(1, \boldsymbol{a_j}, G)$ since $\eta < \frac{1}{\max_G\{N_k\}}$.

Case 2: For $\eta = \frac{1}{\max_G\{N_k\}}$, if it is optimal for an agent to not invest in the baseline, it is optimal for the agent to not invest in the game with local cost sharing. In the baseline, agent i best reply is $a_i = 0$ if and only if $\exists j \in N_i$ s.t. $a_j = 1$. Assume a Nash equilibrium, r agents in agent i's neighborhood are investing, then the best reply of agent i is $a_i = 0$ because $u_i(0, \boldsymbol{a_j}, G) = b - r\eta c$ and $u_i(1, \boldsymbol{a_j}, G) = b - c + (N_i - r)\eta c$, $u_i(0, \boldsymbol{a_j}, G) < u_i(1, \boldsymbol{a_j}, G)$ if $N_i < \max_G N_i N_k$ and $u_i(0, \boldsymbol{a_j}, G) = u_i(1, \boldsymbol{a_j}, G)$ for $N_i = N_k$. Therefore, agents with the maximum number of neighbors will be indifferent between investing or not investing when there are r investments in their neighborhood. Therefore if it was optimal for an agent to not invest in the baseline, it still will be weakly optimal for them to invest in the game with local cost sharing rule.

Applying Parts 1 and 2 it can be verified that pure strategy Nash equilibrium that in the Line network are: (1,0,1,0,1), (0,1,0,1,0), (1,0,0,1,0), and (0,1,0,1,0). In the Asymmetric network, the pure-strategy Nash equilibria are (1,0,1,0,1), (1,0,0,1,0), and (0,1,0,0,1). In the Circle network, the pure strategy Nash equilibria are (1,0,0,1,0), (1,0,1,0,0), (0,1,0,0,1), (0,0,1,0,1) and (0,1,0,1,0).

Part 3: For Line $\max_G\{N_k\} = 2$ and Asymmetric $\max_G\{N_k\} = 3$, so for η[31] used in the experiment satisfies $\eta \leq \frac{1}{\max_G\{N_i\}}$. Table 15 reports the pure strategy Nash equilibria in Line and Asymmetric networks and the corresponding payoffs.

Table 15: Equilibrium Profits

		Investment Decision						Theoretical Profits				
		A	B	C	D	E		A	B	C	D	E
Line	(L1)	1	0	1	0	1		$b-(1-\eta)c$	$b-2\eta c$	$b-(1-2\eta)c$	$b-2\eta c$	$b-(1-\eta)c$
	(L2)	0	1	0	1	0		$b-\eta c$	$b-(1-2\eta)c$	$b-\eta c$	$b-(1-2\eta)c$	$b-\eta c$
	(L3)	0	1	0	0	1		$b-\eta c$	$b-(1-2\eta)c$	$b-\eta c$	$b-\eta c$	$b-(1-\eta)c$
	(L4)	1	0	0	1	0		$b-(1-\eta)c$	$b-\eta c$	$b-\eta c$	$b-(1-2\eta)c$	$b-\eta c$
Asymmetric	(A1)		1	0	1	0	1	$b-(1-\eta)c$	$b-2\eta c$	$b-(1-2\eta)c$	$b-2\eta c$	$b-(1-\eta)c$
	(A2)	0	1	0	0	1		$b-\eta c$	$b-(1-3\eta)c$	$b-\eta c$	$b-2\eta c$	$b-(1-\eta)c$
	(A3)	1	0	0	1	0		$b-(1-\eta)c$	$b-2\eta c$	$b-\eta c$	$b-(1-3\eta)c$	$b-\eta c$
	(A4)	0	1	0	1	0		$b-\eta c$	$b-(1-2\eta)c$	$b-\eta c$	$b-(1-2\eta)c$	$b-\eta c$

In the Line network, the payoff of each agent in pure strategy Nash equilibria: L3 and L4 either remains the same or increases in comparison to L1 (inefficient equilibrium). Similarly in the Asymmetric network, the payoff for each agent in pure

[31] $\eta = \frac{1}{3}$

strategy Nash equilibria: A2 and A3 either remains the same or increases in comparison to A1 (inefficient equilibrium,).

■

Proposition 4. *In the game with the global cost sharing rule and a given strategy profile for a_j such that everyone in the network has access to the public good then :*

1. *If it is optimal for agent i to invest in the baseline, then it is optimal for agent i to invest in the game with global cost sharing rule.*
2. *If it is optimal for agent i to not invest in the baseline, then it is optimal for agent i to not invest in the game with global cost sharing in the Line, Asymmetric and Circle networks.*
3. *Inefficient equilibrium is Pareto dominated by the efficient equilibrium.*

Proof. The following are the proofs for the three claims in Proposition 2.

Part 1 If agent i invest in the baseline then no one in her neighborhood is investing (see *Lemma 1*). If so then her decision with global cost shairng remains optimal because of investing with global cost sharing is less than or equal to the cost of investing, c, in the baseline.

Part 2 If agent i does not invest in the baseline then it must be the case that is there is at least one of her neighbors investing (see *Lemma 1*). If agent i invests then the benefit remains the same but the total cost of provision increases since there is an additional investment which generates no new benefits.

Part 3 Consider an inefficient equilibrium profile, α, where there are c_0 investments in the network,and an efficient equilibrium profile, β, where there are c_1 investments in the network. The payoff of agent i is:

$$u_i(\alpha) = b - \frac{c \times c_0}{N}$$
$$u_i(\beta) = b - \frac{c \times c_1}{N}$$

Since $c_0 > c_1$ it implies that

$$u_i(\beta) > u_i(\alpha)$$

■

Mixed Strategy Nash Equilibrium

The decision whether to invest or not depends on the evaluation of the following condition:

$$b - c \geq \left(1 - \prod_{i=1}^{N_i}(1 - p_i)\right) b$$
$$\left(\prod_{i=1}^{N_i}(1 - p_i)\right) \geq \frac{c}{b}$$

Where p_i be the probability of investing for agent i in a network.

Line Network

To solve for the mixed strategy Nash equilibrium we look for values of p_i for $i \in \{A, B, C, D, E\}$ such that the following conditions are satisfied implying that each player is indifferent between investing and not investing.

$$\text{Player A}: \quad (1 - p_B) = \frac{c}{b} \tag{7}$$
$$\text{Player B}: \quad (1 - p_A)(1 - p_C) = \frac{c}{b} \tag{8}$$
$$\text{Player C}: \quad (1 - p_B)(1 - p_D) = \frac{c}{b} \tag{9}$$
$$\text{Player D}: \quad (1 - p_C)(1 - p_E) = \frac{c}{b} \tag{10}$$
$$\text{Player E}: \quad (1 - p_D) = \frac{c}{b} \tag{11}$$

Case 1: We start with $p_B = 1 - \frac{c}{b}$ (see condition in equation 8). Substituting $p_B = 1 - \frac{c}{b}$ in equation 9 we derive that $p_D = 0$, which implies $p_E = 1$. To solve for p_A and p_C the condition in equation 8 simplifies to:

$$(1 - p_A)(1 - p_C) = \frac{c}{b}$$
$$\implies p_A = 1 - \frac{c}{b(1 - p_C)}$$

There are many values of $p_C \in (0, 1 - \frac{c}{b}]$ as long $p_A \in (0, 1 - \frac{c}{b}]$. So a mixed strategy Nash equilibria: $\left(p_A, 1 - \frac{c}{b}, 1 - \frac{c}{b(1-p_A)}, 0, 1\right)$ and by symmetry we have 1, $\left(1, 0, 1 - \frac{c}{b(1-p_E)}, 0.5, p_E\right)$. For $p_A = 1 - \frac{c}{b}$, equation 8 implies that $p_C = 0$. Hence $(1 - \frac{c}{b}, 1 - \frac{c}{b}, 0, 1 - \frac{c}{b}, 1 - \frac{c}{b})$, $(1 - \frac{c}{b}, 1 - \frac{c}{b}, 0, 1, 0)$, and $(1 - \frac{c}{b}, 1 - \frac{c}{b}, 0, 0, 1)$ are Nash equilibria. By symmetry, $(0,1,0,1-\frac{c}{b},v)$ and $(1,0,0,1-\frac{c}{b},1-\frac{c}{b})$ are also Nash equilibria.

Case2: For $p_B = 1-\frac{c}{b}$ and lets assume $p_A = 0$, we know from equation 9, $p_D = 0$ which implies $p_E = 1$. From equation 8 we know $p_C = 1-\frac{c}{b}$. Hence, $(0,1-\frac{c}{b},1-\frac{c}{b},0,1)$ is a Nash equilibrium. By symmetry, $(1,0,1-\frac{c}{b},1-\frac{c}{b}0)$is also an equilibrium.

Asymmetric Network

To solve for the mixed strategy Nash equilibrium we look for values of p_i for $i \in \{A,B,C,D,E\}$ such that the following conditions are satisfied implying that each player is indifferent between investing and not investing.

$$\text{Player A}: \quad (1-p_B) = \frac{c}{b} \tag{12}$$

$$\text{Player B}: \quad (1-p_A)(1-p_C)(1-p_D) = \frac{c}{b} \tag{13}$$

$$\text{Player C}: \quad (1-p_B)(1-p_D) = \frac{c}{b} \tag{14}$$

$$\text{Player D}: \quad (1-p_B)(1-p_C)(1-p_E) = \frac{c}{b} \tag{15}$$

$$\text{Player E}: \quad (1-p_D) = \frac{c}{b} \tag{16}$$

Case 1: We start with $p_B = 1-\frac{c}{b}$ (see condition in equation 12). Substituting $p_B = 1-\frac{c}{b}$ in equation 14 we derive that $p_D = 0$, which implies $p_E = 1$. To solve for p_A and p_C the condition in equation 13 simplifies to:

$$(1-p_A)(1-p_C) = \frac{c}{b}$$
$$\implies p_C = 1-\frac{c}{b(1-p_A)}$$

There are many values of $p_C \in (0,1-\frac{c}{b}]$ as long $p_A \in (0,1-\frac{c}{b}$. The following are the mixed strategy Nash equilibria: $\left(p_A, 1-\frac{c}{b}, 1-\frac{c}{b(1-p_A)}, 0, 1\right)$ and by symmetry we have $\left(1,0,1-\frac{c}{b(1-p_E)},1-\frac{c}{b},p_E\right)$.

Case 2: For $p_B = 1-\frac{c}{b}$ we know from equation 14, $p_D = 0$ which implies $p_E = 1$. Lets assume $p_A = 0$, according to equation 13, $p_C = 1-\frac{c}{b}$. We have a mixed strategy Nash equilibrium $(0,1-\frac{c}{b},1-\frac{c}{b},0.1)$. By symmetry we have $(1,0,1-\frac{c}{b},1-\frac{c}{b},0)$ as a mixed strategy Nash equilibrium. Assume, $p_B = p_D = 1-\frac{c}{b}$, based equation 14 we have $p_C = 0$, $(0,1-\frac{c}{b},0,1-\frac{c}{b},0)$ is a mixed strategy Nash equilibrium.

Circle Network

To solve for the mixed strategy Nash equilibrium we look for values of p_i for $i \in \{A, B, C, D, E\}$ such that the following conditions are satisfied implying that each player is indifferent between investing and not investing.

$$\text{Player A}: \quad (1-p_E)(1-p_B) = \frac{c}{b} \tag{17}$$
$$\text{Player B}: \quad (1-p_A)(1-p_C) = \frac{c}{b} \tag{18}$$
$$\text{Player C}: \quad (1-p_B)(1-p_D) = \frac{c}{b} \tag{19}$$
$$\text{Player D}: \quad (1-p_C)(1-p_E) = \frac{c}{b} \tag{20}$$
$$\text{Player E}: \quad (1-p_A)(1-p_D) = \frac{c}{b} \tag{21}$$

Case 1: Symmetric mixed strategy Nash equilibrium, lets assume $p_i = p \; \forall i \in \{A, B, C, D, E\}$ in that case any of the equation 17 - 21 simplifies to

$$(1-p)^2 = \frac{c}{b}$$
$$p = 1 - \sqrt{\frac{c}{b}}$$

Symmetric mixed strategy Nash equilibrium, $\left(1-\sqrt{\frac{c}{b}}, 1-\sqrt{\frac{c}{b}}, 1-\sqrt{\frac{c}{b}}, 1-\sqrt{\frac{c}{b}}, 1-\sqrt{\frac{c}{b}}\right)$

Case 2: Lets assume $p_C = p_E = 0$, equation 20 would imply $p_D = 1$, solving equation17 we get $p_B = 1 - \frac{c}{b}$ and solving equation 18 we get $p_A = 1 - \frac{c}{b}$, therefore $(1-\frac{c}{b}, 1-\frac{c}{b}, 0, 1, 0)$. Similarly $(0, 1, 0, 1-\frac{c}{b}, 1-\frac{c}{b})$, $(0, 1-\frac{c}{b}, 1-\frac{c}{b}, 0, 1)$, $(1-\frac{c}{b}, 0, 1, 0, 1-\frac{c}{b})$, $(1, 0, 1-\frac{c}{b}, 1-\frac{c}{b}, 0)$ are mixed strategy Nash equilibria.