

# 10th International Conference on Game Theory

## Book of Abstracts

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# Collusion Among Bidders can Benefit the Seller

Murali Agastya\* and Arupratan Daripa†

## Abstract

The literature on auctions holds the view that collusive bidding in auctions harm the seller. Our purpose in this paper is to explore if cooperation among buyers can actually benefit the seller in auction-like environments in which the sale of the object confers a benefit to all potential bidders. There is a large class of auctions that admit such externalities. We seek the best selling procedure under the restriction that a bidder's expected payment is zero whenever a bidder's type implies that he will never win. Our main result is that the selling procedure that maximizes the seller's revenue must involve joint bids provided the externalities are all positive and not too insignificant. More importantly, the seller can outdo the maximum revenue that can be generated by independent bidding, simply by running a standard English auction with reserve price, while allowing for joint bids.

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# On Preferences Over Subsets and the Lattice Structure of Stable Matchings

Ahmet Alkan\*

## Abstract

It is first shown that *quotafilling* preferences over subsets are equivalently described by a *consistent* choice function or a *locally* consistent semilattice. A second result shows that certain subsets of such semilattices are lattices. The main theorem of the paper, which uses the above result, says that stable matchings form a *coherent* distributive polar lattice when preferences are quotafilling and locally consistent.

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# Subjective Representation of Complexity

Nabil I. Al-Najjar, Ramon Casadesus-Masanell,  
and Emre Ozdenoren\*

## Abstract

We study how individuals cope with the complexity of their environment by developing subjective models, or representations, to guide their predictions and decisions. Formally, an individual who believes his environment is deterministic, but too complex to permit tractable deterministic representation, builds a probabilistic model embodying perceived regularities of that environment. In this model, the individual's inability to think through all possible instances of the problem is represented by an uncertainty about random states. The resulting behavior is fully rational in the traditional sense, yet consistent with an agent who believes his environment is too complex to warrant precise planing, forgoes finely detailed contingent rules in favor of vaguer plans, and expresses a preference for flexibility. We consider applications to time-inconsistent preferences, delegation, and two-player simultaneous games.

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# Perturbed Zero-Sum Games With Applications to Stochastic and Repeated Games

Eitan Altman<sup>\*</sup>, Eugene A. Feinberg<sup>†</sup>, Jerzy Filar<sup>‡</sup>, and Vladimir  
A. Gaitsgory<sup>§</sup>

## Abstract

This paper deals with perturbed matrix games. The main result is that for a large class of perturbations the sets of solutions of perturbed games converge to subsets of solutions of appropriate lexicographic games. We consider applications of these results to dynamic games with finite numbers of states. In particular, we apply our main result to the following three models: (i) infinite horizon stochastic games with perturbed transition probabilities and rewards when only one player controls transition probabilities, (ii) finite horizon stochastic games with perturbed transition probabilities and rewards when both players control transition probabilities, and (iii) repeated games with weighted discounted criteria.

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# Ordinal versus Cardinal Complementarity: The Case of Cournot Oligopoly

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## Abstract

While ordinal complementarity is more general than cardinal complementarity, the corresponding global sufficient conditions placed on the primitives of a constrained optimization problem are typically not comparable. We explore this unforeseen issue in detail for the special case of a Cournot firm. In particular, we derive necessary and sufficient conditions for downward-sloping best-responses by imposing the ordinal test only for output levels that are actually reached. Furthermore, we show that checking supermodularity of suitably transformed profits can work when the global tests for ordinal and cardinal complementarity both fail.

JEL codes: C72, D43, L13.

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About the Non-Comparability of the  
Egalitarianism of the Prenucleolus and the  
Shapley Value.  
The Weighted Coalition-Lorenz Solutions.

J. Arin\* and V. Feltkamp†

**Abstract**

In this work we investigate the difference in the egalitarian ideas underlying the definition of the prenucleolus and the Shapley value, which are maybe the two most important single-valued solutions for TU games. We find that these solution concepts apply different egalitarian criteria over coalitions, but also differ in the relative weights they assign to coalitions: while for the prenucleolus all coalitions a priori have the same relevance, the Shapley value weights the coalitions by a coefficient depending on their cardinality. The difference is made clear by their pertaining to different,  $\omega$ -CoLoSes.

Keywords: Prenucleolus, Shapley value, TU games, Lorenz domination.

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# Common Knowledge of Proper Rationality

Geir B. Asheim\*

## Abstract

Most contributions on the relation between common knowledge (belief / certainty) of rationality and backward induction in perfect information games perform the analysis in the extensive form of the game. An exception to this rule is Schuhmacher (1997) who - based on Myerson's (1978) concept of a *proper equilibrium*, but without making equilibrium assumptions - defines the concept of *proper rationalizability* and shows that proper rationalizable play leads to backward induction. Schuhmacher defines the set of proper rationalizable strategy vectors to be the limit of the set of  $\epsilon$ -proper rationalizable strategy vectors, where the concept of  $\epsilon$ -proper rationalizability is based on players doing mistakes, but where more costly mistakes are made with a much smaller probability than less costly ones. For a given  $\epsilon$ , he offers a common knowledge foundation for  $\epsilon$ -proper rationalizability. However, for the limiting concept, i.e. proper rationalizability, there has not been a common knowledge basis available. It is the purpose of the present paper to establish how *common knowledge of proper rationality* characterizes proper rationalizability.

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# Optimal Collusion with Private Information

Susan Athey and Kyle Bagwell

## Abstract

We consider an infinitely-repeated Bertrand game, in which prices are perfectly observed and each firm receives a privately-observed, i.i.d. cost shock in each period. We explore a hierarchy of collusive arrangements, each characterized by alternative assumptions about the institutional environment, in particular the ability of firms to communicate, keep track of player identities, and make explicit side-payments (which are potentially inefficient, as when they increase the risk of detection). In order for a cartel to achieve productive efficiency, firms that receive high cost draw in the current period must be induced to give up market share, and firms may need to sacrifice pricing today and/or future profits to provide this incentive. Different assumptions about the institutional environment lead to different resolutions of these tradeoffs. In general, sophisticated cartels will choose to implement some productive efficiency even when side-payments are prohibited, but they do not resort to low prices or future price wars to dissuade high-cost firms from producing: instead, firms with bad cost draws may be favored in a continuation game (e.g., by receiving a higher expected market share in future periods). High-cost firms are more willing accept lower market share in the present in exchange for the future. We further show that for firms of moderate to high patience, if side-payments incur any positive transaction costs, optimal collusive equilibria will be non-stationary and will entail the use of future-market share favors. If types are discrete, there exists a discount factor strictly less than one where first-best can be attained purely through history-dependent reallocation of market share between equally efficient firms.

# A Strategic Analysis of Network Reliability

Venkatesh Bala\* and Sanjeev Goyal†

## Abstract

We develop a non-cooperative model of social communication in networks where communication is costly and not fully reliable. We show that Nash networks, provided they are not empty, ensure that every agent communicates with every other agent with positive probability. If the society is large, and link formation costs are moderate, Nash networks for the most part must be ‘super-connected,’ i.e., one or more agents will maintain multiple paths for information flow to enhance reliability. This contrasts with the properties of a deterministic model of information decay, where Nash networks typically involve unique paths between agents. We also study the welfare properties of communication networks and show that if costs are very low or very high, or if links are highly reliable then there is virtually no conflict between efficiency and stability. However, for intermediate ranges of costs and link reliability, Nash networks may be underconnected relative to the social optimum. As with Nash networks, if the society is large, and link formation costs are moderate, efficient networks will typically have multiple paths between two or more agents to reduce the probability of communication failure.

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# Sequencing Strategically: Wage Negotiations Under Oligopoly

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## Abstract

In a unionized oligopoly, such as the American automobile industry, should the union (such as the United Auto Workers) negotiate new contracts by bargaining with the firms simultaneously, or should it “strategically sequence” its bargaining partners? This paper analyzes two models of noncooperative bargaining and product market oligopoly. In the first, bargaining is over wages and employment, in the second, it is over wages alone; employment and output are determined by the firms in a post negotiation product market game. One effect of sequencing bargaining partners is present in both scenarios : It allows preexisting contracts at the firms not being currently bargained with to act as “status quo points” that influence the bargaining outcome of the negotiations currently on. The better are the preexisting contracts from the union’s point of view, the more attractive is the option of sequencing. In the second model, there is another channel, operating via the post negotiation product market game that tends to make sequencing preferable. By negotiating a relatively high wage with the first firm, the union can raise the profitability of the second firm in the product market game; consequently, it can get a higher wage there as well, as its share in the incremental revenue that accrues.

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# Supporting Others and the Evolution of Power

Salvador Barberà\* and Andrés Perea†

## Abstract

The aim of this paper is to provide a simple model where phenomena as the delay in supporting friends for fear that they will help our opponents in the future, or the converse, can be generated. More than that, we want to have a model that is able to clarify when such phenomena will occur, when they will not, and to discuss the relevant variables that might lead to different types of behavior.

We propose a stylized dynamic model in which forward looking agents consider the possibility of supporting others, knowing that their support will induce changes in the future distribution of power among agents, and that they may not control the impact of their initial actions once these have contributed to strengthen other agents. We assume that the decision to support another agent, or not to, is binary; hence, we do not allow for intensities in the support of others. Moreover, the agents are supposed to plan their support behavior during a finite number of stages.

The presence of every agent  $A$  induces (positive or negative) externalities for the other agents and it is assumed that the magnitude of the externality depends proportionally on  $A$ 's power. If, for instance, agents  $A$  and  $B$  are 'friends',  $B$  derives a positive utility from  $A$ 's presence and this utility grows if  $A$  becomes more powerful. On the other hand, if the effect of  $A$  on  $B$  were negative, the effect becomes worse if  $A$ 's power grows. A similar approach can be found, for instance, in Cowan et al. This paper studies a group of consumers, divided into types, in which the attractivity of consuming the good for a certain type depends linearly on the proportions of other types that consume the same good.

The impact of support on the power distribution among agents is modeled as follows: the power of an agent increases proportionally to the total power of agents that support him. This reflects the idea that powerful agents have more impact than others since their contribution, when supporting, is more significant. The above described relationship between power and support allows us to capture two key phenomena. First of all, supporting another agent produces the direct effect of increasing his power and thereby affecting your own utility (in a positive or negative sense). A second effect is that the receiving agent may use his increase of power to support other agents in the future. As such, by supporting an agent, one implicitly transmits power to other agents, too.

All along the paper, the term 'power' is to be interpreted loosely, since our model is open to several alternative interpretations. In one interpretation, the power of each type of agents is just a measure of its numerical weight within a society which admits new members of different

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types in proportion to the support that each type gets from the existing members. In another interpretation, it might reflect the power of countries that support other countries by entering into different types of cooperative agreements. In yet another, power can be the market power of firms that engage in joint activities which strengthen them in front of others.

The paper proceeds as follows. Section 2 introduces our model. Section 3 shows that the dynamic game, in which the agents plan their support behavior during a finite number of stages, has an essentially unique subgame perfect equilibrium. Moreover, the equilibrium has an appealingly simple structure: the agents' support behavior at each stage is independent of the history up to that stage and is characterized by so-called value functions which can be computed by a recursive formula working backwards from the final stage. Roughly speaking, the value functions reflect at each stage the future increase of utility that results from supporting a given agent. Put formally, the value  $V_t(\theta; x)$  measures the future impact for agent  $\theta$  of supporting  $x$  at stage  $t$ . The equilibrium states that agent  $\theta$  should always support exactly those agents  $x$  having positive value.

In Section 4 we study the equilibrium behavior for large groups of agents as the group converges to some continuum. Approximating large societies by a continuum has the advantage that the latter is easier to work with. For a given continuous society, we focus on so-called limit equilibria which are strategy profiles arising as the limit of subgame perfect equilibria in approximating discrete societies. It is shown that there is an essentially unique limit equilibrium, characterized by value functions which can be viewed as the continuous analogues of the value functions for discrete societies.

In Section 5 we consider societies where all agents have concave preferences over other agents which peak at their own type, and the shapes of the preferences of all agents are identical, up to translation. Concave preferences can be interpreted as arising from situations where agents are ordered according to their type, each agent has strong preferences for similar types and strong objections against different types. The latter is reflected by the fact that concave utility functions decrease faster if one goes further away from the peak. Under these assumptions we prove that agents will never give support to their enemies, but that they may withdraw support from some of their friends. Conditions under which support spreads larger over time are also discussed.

Section 6 produces numerical examples in which the equilibrium is computed explicitly, using the recursive formula for the value functions. We focus on quadratic, exponential and normal preferences. In the former two cases, the limit equilibrium is such the agents' support grows over time, whereas normal utilities lead to situations where the group supported by an agent decreases as time evolves.

In Section 7, we provide a sufficient condition which guarantees that support behavior remains constant over time. We prove that, if the utility profile (the utilities that agents attach to other agents) is an eigenfunction of a certain operator, agents have no incentive to change their support over time.

# Perception Dynamics

Alfonso Barriuso and José Ramón Uriarte\*

## Abstract

In evolutionary games the perturbations have been endogenized by making them responsive to changes in the expected payoffs. We present a model where, at every period, there is a proportion of agents in each player population who use a choice procedure to establish a preference in their particular expected payoff-strategy frequency space. We show that each agent's perception of the game and mistakes are induced by the choice procedure itself and present measures for both perception and perturbations that are sensitive to expected payoffs, strategy frequencies, the degree of risk aversion and the sharpness of a threshold function. A perturbed replicator dynamics is then built and the notion of initial conditions as "initial cultural endowments" is introduced. It is shown that this notion might explain the between countries differences of the Nash equilibria observed in the laboratory experiments with the Ultimatum Game.

Key words.: Choice procedure, correlated similarities, perturbed replicator dynamics, Nash F-equilibrium.

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# An invariant measure of incomplete group preference

Manel Baucells and Lloyd S. Shapley

## Abstract

Most of the modeling of group preferences relies on either the Pareto rule, which produces incomplete group preferences, or on the use of some weighted sum of individual utilities, which produces complete group preferences. We begin by developing a framework in which these two models are the end points of a whole spectrum of intermediate possibilities for group preferences, with varying degrees of incompleteness. The next step is to measure the “degree of incompleteness” of any such group preference. Surprisingly, we discover the existence of a unique additive measure satisfying cardinal invariance.

Our setting invites a stronger form of consistency across coalitions called the extended Pareto rule: if two disjoint coalitions  $S$  and  $T$  prefer  $a$  to  $b$ , then so does the union of  $S$  and  $T$ . By the extended Pareto rule, the measure of incompleteness of a group is bounded by the measure of incompleteness of smaller coalitions. In particular we confirm our previous result that the group preference is complete preference whenever certain pairs of individuals reach complete pair-preferences.

# Antagonistic Games

Jean-Pierre Beaud\*

## Abstract

The purpose of this article is to extend some properties of zero-sum games to different classes of two-person non zero-sum games. The definitions of these classes are based on different notions of antagonism.

We generalize the notion of almost surely competitive games introduced by Aumann while keeping the uniqueness of Nash equilibrium payoff. We show that the set of Nash equilibria of a weakly competitive game coincides with the set of its saddle points. We give also some properties satisfied by correlated equilibrium payoff of a weakly unilaterally competitive game; and by Nash equilibria of a symmetric weakly unilaterally competitive game.

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# Common Knowledge with Monotone Statistics

James Bergin

## **Abstract**

When individual statistics are aggregated through a strictly monotone function to an aggregate statistic, common knowledge of the value of the aggregate statistic does not imply, in general, constancy of the individual statistics. This paper discusses two circumstances where it does occur. The first case arises when the partitions are independently drawn: in this case common knowledge of the aggregator function implies (with probability one) constancy of the individual statistics. The second case is where private statistics are related: affiliation of individual statistics and a lattice condition imply constancy of the individual statistics when the aggregate statistic is constant.

# Sequential Equilibria in the Repeated Prisoners' Dilemma With Private Monitoring

V. Bhaskar\*

## Abstract

We analyze the infinitely repeated prisoners' dilemma with imperfect private monitoring and discounting. The main point of this paper is that one can explicitly construct sequential equilibria for such games. This allows us to invoke the one-step deviation principle of dynamic programming, and prove that one can approximate the symmetric efficient payoff in *any* prisoners' dilemma game provided that the monitoring is sufficiently accurate. Furthermore, for a class of prisoners' dilemma games, one can approximate every individually rational feasible payoff. These results require that monitoring be sufficiently accurate, but only require a uniform lower bound on the discount rate.

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# Gradualism in Trade Agreements with Asymmetric Countries

Eric W. Bond and Jee-Hyeong Park

## Abstract

This paper uses recursive methods to illustrate how “gradualism,” in which tariffs start out at a positive level and are reduced over time to free trade, can be part of an efficient trade agreement between a large and small country. We analyze a partial equilibrium trade model in which small country producers choose their output levels in each period before tariff rates are chosen. We characterize the incentive constrained Pareto frontier, which represent the highest payoffs that can be obtained by repeated interaction between the countries. We show that at points on the frontier where only one country’s incentive constraint binds the optimal agreement will be non-stationary. In the optimal agreement, the payoff to the incentive constrained country rises over time and the large country tariff is gradually reduced over time to the free trade level. These agreements thus involve an optimal trading off of static inefficiencies (i.e. trade distortions) and dynamic inefficiencies (lack of consumption smoothing) over the life of the agreement.

# Competitive Fair Division

Steven J. Brams\* and D. Marc Kilgour†

## Abstract

In the fair-division problem addressed here, the indivisible goods to be divided among two or more players have divisible “bads” associated with them—namely, the prices the players must pay for them. A procedure, called the Gap Procedure, is proposed whereby players bid for the goods, but the bidding competition is balanced by fairness requirements that entitle the players to certain numbers of goods.

Under the Gap Procedure, the prices the players pay for the goods reflect the bids of not only the winners but also those of players that bid less. This market-oriented approach to fair division (1) ensures non-negative prices that never exceed a player’s winning bid, (2) is Pareto-optimal, precluding mutually beneficial trades, though not envy, (3) is monotonic in bids, so higher bids never hurt in obtaining a good, (4) encourages sincere bids, and (5) produces prices that are partially independent of the amounts bid (as in a Vickrey auction).

The analysis is developed in the context of the “housemates problem,” in which the rent for a house (the bad) -must be apportioned among several housemates, each of whom is entitled to receive one room (the good). Each housemate is assumed to have the same endowment, so the auction of rooms is a “relativized” one – only relative, not absolute, bids for each room matter. Other applications of the Gap Procedure, in which player endowments and entitlements may be different, or the procedure may be carried out in rounds, are discussed.

JEL Classification: D44, D61, D63.

Keywords: Fair division, envy-freeness; allocative efficiency-, bidding; Vickrey auction.

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# An Impossibility Theorem on Beliefs in Games

Adam Brandenburger\* and H. Jerome Keisler†

## Abstract

The epistemic program in game theory operates with a description of a game that includes, in addition to the customary strategies and payoffs, the players' beliefs or knowledge about these objects, the players' beliefs or knowledge about each other's beliefs or knowledge about these objects, and so on.

A central question in the epistemic literature has been 't the existence of a space containing, in some sense, all possible beliefs of the players (including beliefs about beliefs, beliefs about beliefs about beliefs, etc.). The question is a natural foundational one, but has also turned out to be a more 'practical' one in the context of current investigations into the epistemic status of the backward- induction algorithm (Battigalli and Siniscalchi 1997, Stalnaker 1998). Positive answers to the question have been given, under a variety of assumptions, by Armbruster and Boge (1979), Boge and Eisele (1979), Mertens and Zamir (1985), Brandenburger and Dekel (1993), Heifetz (1993), Battigalli and Siniscalchi (1998), and others.

These notes re-examine this existence question when the players have 'abstract,' set-theoretic beliefs, rather than probabilistic beliefs as in the above treatments.

The question is formulated in terms of a Harsanyi types model (Harsanyi 1967-68). Fix a two-person strategic-form game, and let  $S_1$  and  $S_2$  be the strategy sets of the two players. A types model appends to this description sets  $T_1$  and  $T_2$ , to be interpreted as the sets of *types* of the two players. An element of  $T_1$ , i.e. a -particular type of player 1, is associated with a belief over  $S_2 \times T_2$ , i.e. a belief about player 2's strategy and type.. Likewise, each element of  $T_2$  is associated with a belief over  $S_1 \times T_1$ , i.e.a belief about player 1's strategy and type.

Of course, to finish the description, it is necessary to say how the players' beliefs are formalized. In these notes, beliefs will be formalized as (nonempty) subsets. Thus, a belief of a particular type of player 1 will be a (nonempty) subset of  $S_2 \times T_2$ , with the interpretation that player 1 then believes possible those strategy-type pairs of player 2 that lie in the given subset. Beliefs of player 2 will be formalized in similar fashion.

A set is said to be *definable* in a model  $M$  if the set can be defined by a formula of set theory with parameters for  $M$  and for elements of  $M$ .

Call a types model *definably complete* if: (i) for each possible definable belief about player 2's strategy and type, there is a type of player 1 with that belief; and (ii) for each possible definable belief about player 1's strategy and type, there is a type of player 2 with that belief.

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The result of these notes is an impossibility theorem, establishing that, if beliefs are formalized as subsets, then a definably complete types model does not exist. (To be precise, this is true provided at least one of the strategy sets  $S_1$  and  $S_2$  contains more than one element.) The idea of the proof can be seen in a simple verbal argument. Indeed, suppose that a definably complete model did exist. Then, since all definable beliefs would be present in such a model, it would contain, in particular, the following configuration of definable beliefs

*Player 1 believes that player 2 believes that player 1 believes that player 2's beliefs (about player 1) are incorrect.*

Now ask: Does player 1 believe that player 2's beliefs are correct? If so, then, since player 1 believes that player 2 believes the opposite, player 1 believes that player 2's beliefs are incorrect. We have reached a contradiction. So, suppose instead that player 1 believes that player 2's beliefs are incorrect. But now, since player 1 believes that player 2 believes this, it follows that player 1 believes that player 2's beliefs are correct. We have again reached a contradiction. Thus, the above configuration of beliefs is impossible. The upshot is that a definably complete types model does not exist.

# Complementarities and Collusion in Open Ascending Auctions with Multiple Objects

Sandro Brusco and Giuseppe Lopomo

## **Abstract**

We show that collusive equilibria exist in English auctions with multiple objects, even with high complementarities in the bidders' utility functions, if the number of bidders is sufficiently small relative to the number of objects.

# On Bargaining Partner Selection When Communication is Restricted

Antoni Calvó Armengol\*

## Abstract

This paper analyzes the optimal selection of a bargaining partner when communication among players is graph-restricted. We define a two-stage selection game. In the first stage, players choose a partner. In the second stage, one player chosen randomly has an idea to be implemented jointly with an associate as a new firm and initiates a round of negotiation with her selected partner. We find that when the population is homogeneous in time preferences, ex post payoffs do not depend on players' relative locations: players who collaborate jointly always share their benefits according to a standard half-half splitting rule. When the population is heterogeneous in time preferences, the equilibrium agreed-upon shares depend both on players' relative positions and on their time preferences. We prove that selecting the more impatient neighbor is an equilibrium strategy for a rich class of communication networks including the complete graph and star-shaped communication structures among many others. We further extend the model allowing for ex post oligopoly competition among several firms created on the basis of the same idea, and check for the robustness of our previous results. Keywords: noncooperative bargaining, bargaining partner, time preferences, communication graph.

Journal of Economic Literature Classification Numbers: C72, C78, D20

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# On the Non-Existence of Reputation Effects in Two-Person Infinitely-Repeated Games

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## Abstract

This paper studies reputation effects in two-person, infinitely-repeated games in which there is some probability that one of the players is a “commitment” type that plays a fixed repeated-game strategy. It is well known that in perfect-information infinitely-repeated games, where it is common knowledge that the players are rational, the folk theorem implies that the minimum payoff for a player can be arbitrarily close to her minmax payoff. Recent research however has shown that for any fixed discount factor of the uninformed player, if the informed player’s discount factor is sufficiently close to one, then, regardless of how small the prior probability that the informed player is a commitment type, her minimum equilibrium payoff is strictly higher and bounded away from her minmax payoff. (Fudenberg and Levine 1989, Schmidt 1993, Cripps, Schmidt and Thomas 1993, Celentani, Fudenberg, Levine and Pesendorfer 1993).

While these results are robust in many aspects, the requirement that the uninformed player’s discount factor be fixed as the informed player’s discount factor goes to one is obviously a restrictive one. As the informed player becomes more patient, she also becomes relatively, and in the limit infinitely, more patient than the uninformed player. In this paper, I separate the effect of increasing relative patience from that of increasing absolute patience: Instead of having the uninformed player’s discount factor fixed as the informed player’s discount factor goes to one, I let the discount factors of both players go up in a way that keeps their relative patience constant.

As the discount factors of the players increase in the way I described above, on the one hand, the rational informed player will have a stronger incentive to imitate the commitment type, and, on the other hand, the uninformed player will have a stronger incentive to “screen out” the rational player. These two effects cancel out each other. In the limit, the strength of reputation effects depends only on the relative patience of the players and the prior probability that the informed player is a commitment type. My main result shows that in any two-person, infinitely-repeated game, except for two special classes of stage games, if the two players are equally and sufficiently patient, the commitment strategy is history independent, and the prior probability that the informed player is a commitment type is sufficiently small, then any strictly individually-rational payoff profile can be supported by a perfect Bayesian equilibrium. Under slightly stronger conditions, the same result holds even when the commitment strategy is history-dependent, the informed player is (finitely ) more patient than the uninformed player, and both players possess private information.

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There is an important exception to the above result. A stage game is called a strongly-dominant-action game if there exists a strictly dominant action  $a_1$ , for the informed player such that if she chooses  $a_1$ , receive her highest payoff in the convex hull of the set of strictly- individually-rational stage-game payoffs. I show that in an infinitely- repeated strongly-dominant-action game, if the only commitment type of the informed player is one who always chooses  $a_1$ , then she will receive her commitment payoff in any perfect Bayesian equilibrium. In the perfect-information version of the game, there usually exists a large set of equilibrium outcomes. In this case, reputation effects select a Unique outcome (the one most favorable to the informed player) from that set. Moreover, unlike all previous results on reputation effects in infinitely-repeated games, which critically depend on the requirement of infinite patience in the limit, this result holds for any strictly positive prior probability of a commitment type and any discount factors less than one.

# Decentralized Business Strategies in a Multi-Unit Firm

Myong-Hun Chang\* and Joseph E. Harrington, Jr.†

## Abstract

Retail chains typically have stores in diverse markets – rural and urban, affluent and poor, Rust Belt and Sun Belt. This diversity of markets suggests that one business strategy – in terms of such components as merchandising, marketing, and pricing – is unlikely to be best for all stores. It may then make sense to give store managers the authority to develop their store’s business strategy to suit their local market. However, there may be a cost to this diversity in business strategies. The discovery of a new practice by one store will often be of value to other stores that deploy comparable business strategies. Thus, if stores in a chain tailor their business strategies to their market, they may find ideas less transferable between them. Furthermore, this learning spillover is not one that a store manager need necessarily internalize. Given these counteracting forces – heterogeneity in markets argues to decentralization and diversity and learning spillovers argue to centralization and uniformity – the objective of this research is to sort out these forces to identify when store managers will act in the best interests of the chain and when business strategies may need to be decided upon and mandated from above.

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# The Separability Principle in Economies With Single-Peaked Preferences

Youngsub Chun\*

## Abstract

We investigate the implications of the “separability principle” for the class of problems allocating an infinitely divisible commodity among a group of agents with *single-peaked* preferences. The separability principle requires that for two problems with the same population, but possibly different social endowments, in which the preferences of agents may change, if there is a subgroup of agents whose preferences are not changed and the total amounts awarded to them are the same, then the amount awarded to each agent in the subgroup should be the same. First, we investigate the logical relations between separability and other axioms. As it turns out, separability is implied by consistency. Then, we present characterizations of the uniform rule on the basis of separability, and also on the basis of other axioms.

Journal of Economic Literature Classification Numbers: D63, D70

Keywords: economies with single-peaked preferences, axiomatic characterization, separability principle, uniform rule.

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# Affirmative Action as an Implementation Problem

Kim-Sau Chung\*

## Abstract

Existing affirmative action policies fail because they generate new undesirable equilibrium outcomes. This paper redesigns affirmative action policies from the implementation perspective. It shows that some simple combinations of unemployment insurance and employment subsidy can selectively eliminate undesirable equilibrium outcomes. This class of policies is also the cheapest among all effective policies.

keywords: affirmative action, statistical discrimination, implementation, policy design, mechanism design, unemployment insurance, employment subsidy

JEL D78, J78

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# Extensive-Form Game Dynamics When Learning From Play

Ross Cressman and Karl Schlag

## **Abstract**

Payoff monotone dynamics have repeatedly been proposed as a plausible class of learning dynamics for games. Recent work of ours has shown that under mild assumptions such dynamics no longer exist when one considers behavioral rules that learn by imitating observed play instead of imitating the entire strategy of others. In our previous work we propose alternative rules that learn by imitating the play of others in a specific way. Here we investigate the ability of such rules to learn to play Nash or subgame perfect equilibria in simple perfect information games. When the game consists of only two stages then we find the backward induction solution to predict long run play better than it does under payoff monotone dynamics. In the Centipede Game of length three we find that dynamics behave as if a player has split identities, one for each decision node in the game.

# Concessions and Choice of Partners in a Bargaining Game

Vicent Calabuig, Antoni Cuñat, and Gonzalo Olcina\*

## Abstract

Schelling (1960) defined a negotiation as a struggle to establish commitments to favorable bargaining positions. Although no formal theory has explored this appealing bargaining view yet, it is true that commitments have received a considerable attention in recent years. An important class of bargaining games that incorporate commitment devices are the “concession games,” where each current offer is a commitment for not demanding a larger payoff in the future. Fershtman and Seidman (1993) and Compte and Jehiel (1997) are the most prominent examples of such games. The remarkable feature of “concession games” is that they explain inefficient delays in a complete information framework.

Particularly, in Fershtman and Seidman (1993)’s bilateral negotiation, the existence of delays in equilibrium is explained by the interaction of two features in the bargaining process. A. deadline and a specific commitment device which they call “endogenous commitment.” This latter feature means that a player will reject any offer less generous than those rejected in previous periods. As we will see throughout this paper, it is, crucial for the FS’s, inefficiency result the implicit assumption that both players. a positive payoff in the deadline.

Our main purpose in this paper is twofold: on the one hand to find whether the delays obtained by FS are robust beyond a bilateral bargaining framework; on the other hand, to capture the competition between buyers in a choice of partners scenario. In particular, we consider a bargaining situation in which a seller, who owns one unit of an indivisible good, faces two buyers. One of them has a higher valuation of the good than the other one. Since the seller only can sell the good to one of the buyers, there is one of them that gets a zero- payoff in the deadline

This class of bargaining situations in thin markets has drawn the literature’s attention recently. One of the reasons for this growing interest is that the results in the existing literature were not rather compelling (see Binmore (1985), Wilson (1994)). In these models, the general result is that the presence of a second buyer does not have any effect on the equilibrium price<sup>1</sup>. The seller’s payoff equals the equilibrium payoff in bilateral bargaining between the seller and B1. The intuition of this result is that only if the trading procedure allows the seller to effectively commit to trade with B2 when he does not reach an agreement with B1, then the presence of B2 enhances the seller’s bargaining power.

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<sup>1</sup>There are some exceptions. For instance, when the seller can switch to the other buyer after making an offer there is a continuum of equilibria and the seller can obtain a price in excess of what he would obtain “locked in” with B1 (see Osborne and Rubinstein (1990), chapter 9, p. 184).

As we will see, in our model the possibility of commitment allows the competition between the buyers. But we depart from the existing literature by considering in the choice of partners framework an explicit commitment (endogenous commitment) instead of an implicit commitments through the bargaining procedure. The explicit commitment has the advantage that allows to isolate the real effects of the commitments in these bargaining situations.

The endogenous commitment assumption can be justified by the existence of representatives. A representative will never accept an offer that is worse to any previously rejected offer because doing so he would have problems explaining this strategy to his principal. Since the seller is locked in with two buyers, the endogenous commitment has a feature not present in FS. The seller, after hearing an offer  $x'$  from one buyer, can use it to obtain a better payoff with the other one given that the threat of rejecting any  $x < x'$  will be credible. FS assumed that in any period a fair lottery determined the identity of the proposer. To simplify, we will depart slightly from FS's assumptions by supposing that only the buyers make offers in any period previous to the deadline. It can be easily checked that our results still hold qualitatively with FS's original assumptions.

Our main result states that the introduction of a: deadline and endogenous commitment in a one seller-two buyers negotiation yields significant differences with respect to FS's model and also with respect- to the choice of partners literature. If Buyer 2 (B2)'s valuation of the good is relatively close to Buyer 1 (B1)'s, on the one hand, the inefficient delay disappears and, on the other hand, the seller obtains a higher share of the surplus than in FS's framework. That is, our model captures the competition between the buyers. Since the available surplus is larger when bargaining with B1, in the deadline the seller sells the good to B1. Then, B2 has nothing to lose by making positive offers in any previous period to the deadline, given that he cannot obtain a positive payoff in the last period. Therefore, if B2's valuation is close enough to B1's, it is worth for B2 to make a serious offer and get the object in the first period. Moreover, if B2's valuation is not too far but also not too close he will not get the object but there exists an equilibrium in which the object is sold in the first period to B1. In this case, an optimal strategy for the seller is to hear first B2's offer and after that to switch to B1, since B2's valuation of the good is not too small. B1 anticipates the seller's strategy and makes in equilibrium an offer to be accepted by the seller. Given that the seller's incurred payoff to accept- is larger than what he would obtain locked in with B1, not only does the inefficient delay disappear but the presence of a second buyer improves the seller's equilibrium payoff.

Moreover, we obtain that even for B2's small valuations of the good, B2's presence makes still a difference. In this case, there is no agreement until the last period. But there will be a multiplicity of equilibria where each possible equilibrium depends on B2'S maximal offer in any period previous to the last one. The intuition of this result is that, since B2's valuation is small, B1 will prefer to delay the agreement until the last period even if B2 offers his valuation. The seller's optimal strategy will be to hear B2's offers and reject them because he can use these offers in the last period to obtain a better payoff. The seller's equilibrium payoff

will depend on B2's maximal offer throughout the game.

# Decision Making under Uncertainty and the Evolution of a Concern for Status

Philip A. Curry\*

## Abstract

This paper finds support for the evolution of interdependent preferences under natural selection from a (perhaps) surprising source: decision making under uncertainty. Individuals choose from sets of risky alternatives. The lotteries may involve either idiosyncratic risk or aggregate uncertainty. Robson (1996) gives the evaluation criterion for lotteries that maximizes reproductive value and shows that it does not satisfy the expected utility theorem. Cooper and Kaplan (1982) have demonstrated that when lotteries are aggregate, the optimal decision rule involves randomization. This paper reexamines Robson's evaluation criterion, using it to solve for the optimal amount of randomization. The solution is characterized and an interpretation is offered that links maximizing reproductive value to maximizing expected relative offspring. It is shown that when agents' preferences correspond to their own offspring relative to the average within the population, a game is constructed out of the choice of lotteries and that the unique Nash equilibrium maximizes reproductive value.

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# Committee Decisions and Contracting II: The Incentives of Money, Minimum Requirements, and Voting Proxies

James Dearden\* and Dorothea Klotz†

## Abstract

Committees make personnel decisions. In corporations, boards of directors decide whether to retain chief executives. In academia, tenured faculty members decide whether to retain a non-tenured faculty member. Committees also make procurement decisions. After having a supplier produce a prototype, a production committee decides whether to accept the prototype and have the supplier start producing copies of the prototype.

Inevitably, committees are composed of individuals (i.e., principals) who value relevant performance dimensions differently. To arrive at a committee decision, the individual committee members often vote and, if some pre-specified number of the members vote favorably, the committee as a whole decides to continue the relationship. Otherwise, the relationship ends. These above anecdotes are similar in that after a probationary period, the committee of principals evaluates the agent's performance and decides whether to continue their relationship.

In this paper, we model these relationships as a multiple principal/single agent problem in which the agent produces a two-dimensional commodity. Within our framework, prior to the probationary period, the individual committee members convey to the agent their expectations and requirements (i.e., the committee members offer the agent contracts) and the principals determine how much to pay the agent during the probationary period. Then the agent performs (i.e., produces a commodity), the principals evaluate the agent's performance, and vote whether to retain the agent (i.e., purchase the produced commodity).

Issues embedded in this seemingly simple relationship ones that we address are the types of contracts that the principals offer the agent, whether stable coalitions of principals offer the agent identical contracts, how the agent allocates her effort in response to the contracts, the economic gains of the principals and agent, and the effects of the verifiability of the agent's performance.

The main thrust of our paper is to analyze the environment in which one dimension of the agent's performance is observable but non-verifiable, while the other dimension is both observable and verifiable.<sup>1</sup>

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<sup>1</sup>If a dimension is observable, the parties in the relationship (here, the principals and the agent) can observe the agent's performance in that dimension. If the dimension is verifiable, parties outside the relationship also can observe the agent's performance in that dimension. On the other hand, if the dimension is non-verifiable, parties outside the relationship cannot observe the agent's performance in that dimension.

Within our environment, each principal has three strategic tools to affect the agent's production. The first two tools concern whether a principal will accept an agent's production. A principal can commit to approve the agent's production only if (i.) the agent produces at least some minimum level of the verifiable dimension that the principal specifies and (ii.) he chooses to designate a different principal and that principal approves the agent's output.<sup>2</sup> This second option, which we call a *negative proxy*, works as follows. If principal  $k$  hands his negative proxy to principal  $k'$ , then  $k$  agrees to accept the agent's produced allocation only if  $k'$  does so; or equivalently if  $k'$  rejects the agent's production, then  $k$  rejects the production also. Hence, if principal  $k$  hands his negative proxy to principal  $k'$ , then  $k'$ 's approval is a necessary but not sufficient condition for  $k$ 's approval of the agent's production. The third tool is the amount of money that the principals pay the agent.

We model of this contracting and production problem as a three-stage process. In the first stage, each principal simultaneously states (i.) the minimum of the verifiable dimension he is willing to accept, (ii.) a negative proxy, if he chooses to use one, and (iii.) the amount the group should pay the agent. A predetermined function specifies how the principals' individual statements of what the group should offer the agent translates into an actual contracted monetary offer. The agent then decides whether to accept the contract. If the agent accepts, then in the second stage she produces a prototype and is compensated. In the third stage, the principals vote whether to accept the agent's production.

We use the university tenure scenario as a running example in this paper because of its familiarity among most readers. To facilitate discussion and to provide some realism in terms of the tenure example, we assume that the two dimensions in which the agent (i.e., the non-tenured faculty member) is evaluated are classroom performance and research. In particular, we take classroom performance as the non-verifiable dimension and research, as measured by the number of paper published, as the verifiable dimension. If a majority of the committee members approves the agent's effort allocation and resulting production, then within our formulation, the committee will recommend in favor of the agent.<sup>3</sup> If the committee recommends the agent, we will assume the agent is granted tenure. Finally, we assume that the principals have employed a capable agent who possesses the ability and desire to earn tenure. This means that, in this paper, we will not address the question of what types of agents do or do not get tenure. In fact, in equilibrium, within our formulation, the agent is always granted tenure. Again, the focus here is on the committee members' actions and the agent's resulting allocation of effort.

Through commitments to minimum criteria, the use of negative prox-

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<sup>2</sup>Whether or not such commitments are credible depends upon the consequences a principal expects to incur if he violates the commitment. Here, we assume that the principals' commitments are literally contractible or that the principals are interested in maintaining reputations and, if a principal violates a commitment, his reputation is adversely effected.

<sup>3</sup>Requiring a majority of favorable votes is one benchmark the committee can use. Alternatives include requiring a two-thirds of the principals to vote to approve the agent, or unanimity of principals voting to approve the agent. The results obtained in this paper remain qualitatively unchanged regardless of the benchmark adopted by the committee.

ies, and monetary payments the principals endogenously coalesce into voting blocks. In particular, groups of principals set identical minimum criteria on research, agree to pay the agent a certain amount of money, and/or hand their negative proxies to a common principal. Although principals take action individually, the fact that the strategies within a group are identical and the fact that the effectiveness of each principal's strategy depends upon the strategies of other group members indicate behavior exhibited by coalitions. Moreover, in deciding the agent's fate as a part of a group, individual committee members may not be willing to approve outcomes that they would have been willing to approve had they been individually deciding the agent's fate. Such behavior typifies what many refer to as the politics of group decisions. This paper shows that when committees make decisions such political maneuvering naturally results in equilibrium.

With our concern for principal voting coalitions and how these coalitions affect the agent's effort allocation, an equilibrium concept that allows for coalitional moves is appropriate. The strong equilibrium concept, which we use, allows for such moves. In particular, given that the principals, in stage 3, vote according to their preferences and stage-1 commitments and that the agent chooses her optimal production in stage 2, we require that the principals' stage-1 offers constitute a strong equilibrium. A strong equilibrium outcome is any allocation (i.e., level of classroom performance and research) that (1.) given the other principals' strategies and (2.) given the agent's effort allocation in response to the principals' strategies, no group of principals can change their strategies and thereby shift the agent's allocation choice in such a way that at least one principal in the group is made better-off and no principals in the group are made worse-off. The strong equilibrium therefore captures the requirements of coalitional stability.

We consider an array of possible principal and agent preference profiles. In particular, for our preference domain, we provide necessary and sufficient conditions for the existence of strong equilibria. That is, within our preference domain, we characterize the set of preference profiles for which strong equilibria exist.

Suppose the principals face an agent who is able to satisfy a majority of principals by producing only research but must expend great effort to produce in the classroom. The majority of principals who most value research coalesce to set a minimum research requirement at the greatest amount of research the agent is willing to produce. Moreover, this majority of principals pay the agent so that she is just indifferent to accepting their contract. The agent responds by producing this amount of research. In this case, the principals do not use negative proxies. As such, the agent puts forth no effort in the classroom. One interesting point about this equilibrium is that the principals with the smallest preference for research may prefer the outside option to the agent's equilibrium production. In fact, the principal-agent preference profiles that give us this research-only outcome are the only profiles for which some principals may strictly prefer the outside option to the equilibrium allocation.

Suppose the principals encounter an agent who is unable to satisfy even a single principal by producing only research but is so talented in

the classroom that she is willing and able to satisfy the most research-oriented principal by a high level of classroom performance. Each and every principal (i.e., the grand coalition of principals) then hands his negative proxy to a common principal—the principal who is most demanding of the agent’s classroom performance. If this common principal’s demands on the agent’s classroom performance are increasing in the amount that the principals pay the agent, then the principals may agree to pay the agent more even if she earns positive economic surplus. Moreover, the principals do not use minimum research requirements because the opportunity cost of doing so for this agent is too high; to meet this increased research demand, the agent would significantly decrease her classroom performance. As a result, the agent may earn economic surplus due to her exceptional ability in the classroom, the principals’ desire to increase her classroom performance, and the principals’ unwillingness to place minimum criteria on research. We demonstrate that the preference profile that gives us this is the type of equilibrium is the only preference structure for which the agent earns positive economic surplus.

If the principals encounter an agent who is moderately talented both at research and in the classroom, if a strong equilibrium exists, then the principals impose a minimum research requirement and use negative proxies. A majority of principals—the principals with the greatest preference for research—set a common minimum research requirement, not at the highest level the agent is willing to produce, but rather at a level that is low enough so she is willing to put the proper effort into classroom performance. Given this minimum research requirement, the grand coalition of principals then hand their negative proxies to the principal, for that level of research, who demands the greatest classroom performance. In addition, the principals agree on a monetary payment where the agent is just indifferent to accepting their contract. The agent produces this minimum research and receives majority approval by satisfying the principal who holds all of the negative proxies. We demonstrate that these preference scenarios in which the agent is moderately talented at both research and teaching are the only ones for which a strong equilibrium may not exist.

# When are Plurality Rule Voting Games Dominance-Solvable?\*

Amrita Dhillon<sup>†</sup> and Ben Lockwood

## Abstract

This paper studies a solution to the problem of multiple Nash equilibria under plurality rule, when voters can vote strategically. We study under what conditions this game is dominance-solvable(DS) by iterated elimination of weakly dominated strategies. For the case of three alternatives and individuals, we find necessary and sufficient conditions for the game to be DS. In the case of three alternatives and many voters, we find “almost” necessary and sufficient conditions, in terms of only one statistic of the game, the largest proportion of voters who agree on which alternative is worst. This last sufficiency condition generalizes to the case of an arbitrary number of alternatives. If the game is DS, the outcome is usually but not always the Condorcet Winner, whenever it exists.

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# Dynamic Price Competition With Persistent Consumer Tastes

Toker Doganoglu\*

## Abstract

We analyze the dynamic price competition in a horizontally differentiated duopoly when consumers value previous market shares. The conditions for the existence of a unique Markov Perfect Equilibrium in linear strategies where the market is covered at every period are established. The equilibrium outcomes can be of two types: monotonic or price war. In the monotonic outcome, the steady state is reached monotonically; while in the price war outcome, firms' prices and market shares oscillate around the steady state values. The model can produce outcomes where the steady state is reached very slowly which provides an alternative explanation for slow emergence of competition when entrants face an established incumbent.

Keywords: Dynamic price competition, long memory, markov perfect equilibrium, stability, product differentiation.

JEL Classification: C73, D21, D43, L13, L21.

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# On the Semivalues and the Power Core of Cooperative TU Games

Irinel Dragan\*

## Abstract

The problem of characterizing the cooperative TU games for which the Shapley value lies in the core was solved in several papers by Iñarra and Usategui (1993), Marin-Solano and Rafels (1996) and Izawa and Takahashi (1998). All these papers were discussing exclusively the Shapley value, which is an efficient value. However, any nonefficient value does not belong to the core, hence the problem has no meaning in this case. Recent papers by Dragan (1996), Calvo and Santos (1997), and Sanchez (1997), have introduced what we call the Power Game of a given game (or the auxiliary game). For any value  $\Psi$ , the Power Game of a game  $(N, v)$  relative to  $\Psi$  is the game  $(N, \pi^\Psi)$  defined by  $\pi^\Psi(N, v) = \sum_{i \in S} \Psi_i(S, v)$ , for all coalitions  $S \subseteq N$ . We call the Power Core of  $(N, v)$  the core of the game  $(N, \pi^\Psi)$ . As the players outcomes are  $\Psi_i(N, v), \forall i \in N$ , the Power Core for various values, including the efficient ones, plays the same role relative to the outcomes as the usual core for efficient values. Therefore, it may be interesting to consider the general problem of characterizing the values which lie in the Power Core relative to those values. In this paper, we discuss the case of the Semivalues introduced by Dubey, Neiman and Weber (1981). In an earlier paper we have shown a formula which gives the Power Game of a game relative to a Semivalue in terms of the given game. So, in this case we can write explicitly the inequalities determining the Power Core. This is helpful in obtaining a characterization of the Semivalues which lie in the Power Core; obviously, the Iñarra/Usategui conditions are derived easily from the characterization, for the efficient values, because the Shapley value is the unique efficient Semivalue. We can also obtain characterizations for other Semivalues like the Banzhaf value and the Marginal contribution value. It is also shown that in the space of all TU games with a given set of players there are games for which the Semivalue is in the Power Core. Therefore, it is interesting to try to get computational formulas for the Semivalues. We prove average per capita formulas for the computation of Semivalues, similar to the formulas we proved earlier for the Shapley value (Dragan, 1992). Recall that these formulas were allowing a computation in parallel of the components of the Shapley value and the same situation occurs for the Semivalues. Further, we considered the potential approach used for the Shapley value by Marin-Solano and Rafels. As the Semivalues have a potential, as shown already by Calvo and Santos, a similar approach can be used. A new proof for the characterization due to Marin-Solano and Rafels leads to the introduction of the concept of weak average convex games; it is proved that for the weak average convex games the Semivalue is in the Power Core, but there may

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be some subgames for which the Semivalue is not in the Power Core. The class of weak average convex games is strictly included in the class of average convex games. We show that every game for which the Potential Game is weak average convex has the Semivalue in the Power Core. In an earlier paper we have shown that the existence of a potential for a value is equivalent to the Balanced Contributions, to the Recursion formula, to what we call the Shapley Blueprint property. We called the values characterized by all these properties Myerson values, because Myerson has introduced first one of the four properties. The Semivalues are Myerson values, but there are also Myerson values which are not Semivalues. The proof of the characterization of games having Semivalues in the Power Core holds also for all Myerson values, hence the characterization has been extended to more general values than the Semivalues.

# Finite Horizon Common Interest Games

Prajit K. Dutta

## **Abstract**

In a game of common interest there is one action vector that all players prefer to every other action vector. In many common interest games there are multiple Pareto-ranked Nash equilibria; rational equilibrium play is unable to rule out Pareto-dominated equilibria. In this paper, I show that two elements - asynchronicity and a finite horizon - are sufficient and minimally necessary to uniquely select the Pareto dominant action vector (in subgame perfect equilibrium play).

# The Shapley Value in a Game of Information Trading

Elina Eguiazarova\*

## Abstract

The present paper deals with the problem of a certain trade of information on technological innovation. The first study of the question goes back to Arrow (1962). The information trading situation may be described as follows: there exists a fixed number of identical producers of which all use the same technology to produce the same product. The profit level of each producer is supposed to be identical. Besides, there is another participant here - the owner of the information on the technological innovation who has no means of production. Consequently, we can consider the owner of the information as the seller of the technological innovation and the producers as the potential buyers of the new technology. Being a seller, the owner of the information on the new technology tries to sell it to some or all of the producers. Here we follow the model described in Driessen et al. (1992), where certain assumptions on the information trading situation had been specified.

As in Driessen et al. (1992), we denote by  $E(t)$  the monetary profit level of any producer who has bought the new technology (supposing that  $t$  represents the number of producers who have purchased the new technology). The profit level of any nonbuyer is denoted by  $E^*(t)$ . Let the total number of producers be  $n$ . The set  $\{E(t), E^*(t) | t \in \{0, 1, \dots, n\}\}$  is supposed to satisfy the following assumptions:

1) The profit of any actual buyer is at least as much as of a nonbuyer:

$$E(t) \geq E^*(t), \forall t \in \{1, 2, \dots, n-1\}.$$

2) If the number of actual buyers grows it brings down the profit level of any type of a producer:

$$E(t) \geq E(t+1), \forall t \in \{1, 2, \dots, n-1\},$$

$$E^*(t) \geq E^*(t+1), \forall t \in \{0, 1, \dots, n-1\}.$$

3) If there is only one buyer, his profit level is greater than the positive profit level obtained without the technological innovation:

$$E(1) > E^*(0) > 0.$$

We assume that  $E(0) = E^*(n) = 0$ . Consider the model as a  $(n+1)$ -person game in cooperative form. Its player set is  $N^0$ ,  $N^0 = N \cup \{0\}$ , where  $N = \{1, 2, \dots, n\}$  - the set of the potential buyers and 0 is the seller. Any nonempty subset  $S$  of the player set  $N^0$  is a coalition,  $S \subset N^0$ ;

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when a coalition includes the seller, it's denoted by  $S^0, S^0 = S \cup \{0\}$ . The number of potential buyers in the coalition  $S^0$  and  $S$  is denoted by  $s$ .

While considering a game in a characteristic form, one should point out the value of maximal joint profit that each coalition  $S$  can guarantee its members. This value is defined by the characteristic function (c. f.)  $v : 2^{N^0} \rightarrow R$ , which is given here by the following formulas:

$$v(S^0) = \max(tE(t) + (s-t)E^*(t) | t \in \{0, 1, \dots, s\}), \quad \forall S^0 \subset N^0$$

$$v(S) = sE^*(n-s), \quad \forall S \subset N$$

The main problem of the cooperative game theory is how to divide between the players the largest possible monetary profit of grand coalition  $N^0$ . There exists a lot of optimality principles, each giving us some sets of optimal solutions. We choose here the one that prescribes a unique optimal way to distribute the total payoff  $v(N^0)$ .

In this game we consider as an optimality principle the Shapley value  $Sh(v) = (Sh_0(v), Sh_1(v), \dots, Sh_n(v))$ .

Consider the dynamic process of the game. The information trading between the seller and  $n$  producers can be considered a multistage game  $\Gamma(n)$ . Suppose that  $n-j$  producers have decided on whether each of them is buying the information. Then the problem of information trading to  $j$  producers left is described by a subgame  $\Gamma(j)$ . Here the distribution of the joint profit  $v(J \cup \{0\})$  (where  $J$  denotes the set of  $j$  producers) is supposed to be carried out according to the Shapley value

$$Sh^j(v) = (Sh_0^j(v), Sh_1^j(v), \dots, Sh_n^j(v)),$$

$$j = 1, 2, \dots, n-1.$$

But the question arises whether it is true, or, to put it more precisely, if all of the players will really follow  $Sh^j(v)$  at each stage  $j$ ? The situation when at some stage a player would rather deviate from the distribution way defined "optimal" at the beginning of the game is quite feasible. Should that happen, the original Shapley value could never be realized. An optimality principle having such an "inconvenient" property is called "time inconsistent" one.

The conception of time consistency (Petrossjan 1995) means that no one player is inclined to deviation from the chosen optimality principle.

**Definition 1** *The Shapley value  $Sh(v) = \{Sh_0(v), Sh_1(v), \dots, Sh_n(v)\}$  is time consistent in the cooperative game  $\Gamma(n)$  of information trading if there exist vector-functions  $\beta(j) = \{\beta_i(j)\}, \beta_i(j) \geq 0, i = 0, 1, \dots, n; j = 1, 2, \dots, n$ , so that the following equality holds*

$$Sh(v) = \sum_{k=1}^{n-j} \beta(k) + Sh^j(v), j = 1, 2, \dots, n.$$

We prove that under certain assumptions the Shapley value in the game fails to be time consistent. The next theorem is valid.

**Theorem 1** *Let  $(N^0; v)$  be the information trading game. If for every  $k$*

$$\begin{aligned} v_k^0 &= \max (tE(t) + (k - t)E^*(t) | t \in \{0, 1, \dots, k\}) = \\ &= E(1) + (k - 1)E^*(1), \end{aligned}$$

*then*

$$\beta_i(n - j) < 0, \quad i = n - j + 1, \dots, n; \quad j = 1, 2, \dots, n - 1,$$

*i. e. the time consistency condition is violated.*

It turns out that this problem can be solved. We propose a way to solve it which consists in a certain regularization of the c. f. We modify, or improve it so that the Shapley value corresponding to the regularized c. f. be consistent.

# Fine value allocations in large exchange economies with differential information

Ezra Einy\*, Diego Moreno†, and Benyamin Shitovitz‡

## Abstract

We show that the set of fine value allocations of a pure exchange economy with a continuum of traders and differential information coincides with the set of competitive allocations of an associated symmetric information economy in which each trader has the “joint information” of all the traders in the original economy.

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# Majority Rule in a Stochastic Model of Bargaining

Hulya K. K. Eraslan\* and Antonio Merlo†

## Abstract

In this paper we consider multilateral stochastic bargaining models with general agreement rules. For  $n$ -player games where in each period a player is randomly selected to allocate a stochastic level of surplus and  $q \leq n$  players have to agree on a proposal to induce its acceptance, we characterize the set of stationary subgame perfect equilibrium payoffs. With this characterization we establish the existence and the monotonicity of the equilibrium payoffs in the proposer selection probabilities. We show that for agreement rules other than the unanimity rule, the equilibrium payoffs need not be unique. Furthermore, even when the equilibrium is unique, it need not be efficient.

JEL Classification numbers: C73, C78, D70

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# The Proportional Value of a Cooperative Game

Barry Feldman\*

## Abstract

The proportional value of a cooperative game is based on the ratio potential, which is analogous to the potential of Hart and Mas-Collel (1989), but is based on ratios instead of differences.

Let  $\nu$  be a TU cooperative game with coalitions  $S \subset N$ , and let  $R$  be the ratio potential. Fix  $R(O) = x > 0$ , where  $O$  is the empty set. The ratio potential is determined by the recursive relationship

$$\nu(S) = \sum_{i \in S} \frac{R(S)}{R(S \setminus \bar{i})}$$

and the proportional value of a player  $i$  is:

$$\varphi(i) = \frac{R(N)}{R(N \setminus \bar{i})}$$

The NTU ratio potential and value are defined similarly to Hart and Mas-Collel (1989).

This paper develops some of the properties of the proportional value and the ratio potential. Some key results: The TU and NTU proportional values are consistent. The ratio potential can be defined for noncooperative games in the manner of Shapley and Monderer (1995) and many of their results obtain. The Myerson (1977) approach to the definition of value through link formation in cooperative games can be applied to axiomatize the proportional value. The TU and NTU proportional values are the expected equilibrium payoffs in variations of the game described in Hart and Mas-Collel (1996) where a player's opportunity to make a proposal is proportional to her expected payoff.

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# Choosing One's Identity

Dragan Filipovich

## **Abstract**

This paper has two parts: In the first part, it presents a way of formalizing identity choice issues. In the main part, it looks at the incentives firms have to differentiate themselves by means of choosing meaningless, worthless 'tags' in the absence of exogenous rules preventing them from imitating each other. It shows that even when one can find a ranking of firms' products (from worse to better sell), 'endogenous' differentiation of this sort might result. A crucial factor in obtaining this result is that not all buyers should be able to observe all tags in the same way, i.e., that observability conditions should vary across buyers.

# An Economic Theory of Leadership Turnover

Maria E. Gallego\* and Carolyn Pitchik†

## Abstract

A majority of the worlds fallen political leaders are not defeated in popular elections (Blondel, 1980, p. 198, Table 1). Of the 1,028 leaders who reached high office between 1945 and 1976, 169 remained in office in 1976; of the 859 who has fallen from power, only 5% were defeated at election. By contrast, 20% of leadership turnovers were due to pressures such as a coup. Despite the paucity of examples of removal from office at election, the attention devoted to modeling coups dtat and other non-democratic processes has been small compared with that devoted to “democratic” electoral processes. The objective of this paper is to understand the relationship between changes in economic conditions and leadership turnover in non-democratic processes. We develop a dynamic model that accounts for the main stylized facts concerning leadership turnover. These are:

1. The probability of leadership turnover in any given year is inversely related to the length of time in office (Bienen and van de Walle, 1991).
2. A high coup propensity in any given period is associated with low per capita lagged real GDP growth (Londregan and Poole, 1990, 1992).
3. A high coup propensity in any given period is associated with low per capita lagged real investment (Gallego, 1998).
4. Low risk risk of leadership turnover is associated with a smaller size of the group whose support is necessary for continued leadership (Blondel, 1980, p. 220, 230).

We study an infinite-horizon model of an economy in which a leader is maintained in power by the support of a finite subgroup of economic agents, called kingmakers. In each period, the leader first receives a fixed payment, then observes the realization of an exogenous independently and identically distributed shock, and finally exerts costly effort. The payoff to each kingmaker depends positively on both the level of the shock and the leaders effort. After observing the shock and the leaders effort, the kingmakers decide individually whether or not to withdraw support from the leader. If the leader is overthrown, a new leader is chosen at random from among the kingmakers, and a member of the population at large is chosen to replace the promoted kingmaker. This model is consistent with the stylized facts 2, 3, and 4. An extension of the model in which the dictator’s type varies with the cost of exerting effort is consistent with the four stylized facts above.

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The model draws on previous principal agent models of both political competition and dictatorship that are based on Green and Porter (1984). The models in this literature assume, as we do, that the leader of a group provides costly effort to maintain a public good that enhances the well-being of a subgroup. The subgroup can discipline the leader by withholding support. We depart from this literature in three ways:

1. Subsequent leaders are chosen at random from the subgroup whose support is necessary for the leaders survival so that leadership turnover confers a benefit on a random member of the subgroup.
2. The subgroup is finite so that each member of the subgroup has a positive chance of being chosen the new leader and therefore may derive a benefit from leadership turnover.
3. Perfect monitoring exists. We assume that the principal can monitor the agent perfectly. We do so in order to provide insights into the effect our first two assumptions above have on the principal-agent model used in the political economy literature. The predictions of our model are consistent with the stylized facts in the literature, and remain so in the presence of imperfect monitoring.

When the supply of future leaders is provided by a finite subgroup as in our model, leadership turnover serves two purposes. In common with the principal-agent models in the political economy literature, one purpose of leadership turnover is to discipline the actions of the leader. Another purpose of leadership turnover is to allow a member of the subgroup a chance to change status and thereby give up the flow of benefits in order to receive the expected capital gain of a change in status. Even in the presence of perfect monitoring, leadership turnover allows a member of the subgroup a chance to change status in equilibrium.

The intuition for the fact that turnover occurs even in the presence of perfect monitoring is as follows. The payoff to the members of the subgroup in any period depends positively on the level of the exogenous shock and on the effort exerted by the dictator. This implies that a given payoff level in a period is maintained only if the effort exerted by the agent increases as the exogenous shock decreases. If the members of the subgroup decide to withdraw support from the leader then each member has a chance to change status and become the future leader and thereby receive the expected payoff of a future leader. The expected payoff to a future leader is independent of the current level of the exogenous shock. When deciding between staging a coup or not, the kingmakers compare the flow of benefits to maintaining the status quo with the expected capital gain of a change in status. The flow of benefits to a kingmaker of maintaining the status quo in a period is larger than the expected capital gain of a change in status if the leaders effort is high enough given the level of the exogenous shock in the period. As the level of the shock decreases, the leaders effort must increase in order to maintain support from the subgroup. Thus, the subgroups reservation standard of effort increases in the expected capital gain of a change in status and decreases with the level of the exogenous shock. By contrast, the leaders payoff in a period decreases with effort and is independent of the level of the exogenous shock in the period. The leader weighs the cost of providing the

effort today with the benefit of remaining as leader tomorrow. The benefit of remaining as leader tomorrow depends on the kingmakers strategy and on the future levels of the exogenous shock. Thus, the benefit of remaining as leader tomorrow is fixed given the strategy of the kingmakers. Therefore, the leaders reservation value of effort is fixed given the strategy of kingmakers. In equilibrium, the leader meets the subgroups variable reservation standard of effort provided it is below the leaders reservation level. Otherwise, the leader exerts zero effort. When the level of the exogenous shock is low enough, the subgroups reservation standard of effort is larger than the leaders equilibrium reservation value. In this case, the leader exerts zero effort and the subgroup withdraws support.

# How to Play With a Biased Coin?

Olivier Gossner\* and Nicolas Vieille†

## Abstract

We characterize the maxmin of repeated zero-sum games in which player one plays in pure strategies conditional on the private observation of a fixed sequence of random variables. Meanwhile we introduce a definition of a strategic distance between probability measures, and relate it to the standard Kullback distance.

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# Explaining Incomplete Contracts by Ambiguity Aversion

Jacob Gyntelberg, Lars Jørgensen, and Birgitte Sloth\*

## Abstract

Introducing ambiguity-averse agents, we offer an explanation of contracts that leave some contractable contingencies unspecified even though the contracting parties have obvious possibilities of gains from trade. Such contracts are usually termed “incomplete”. We demonstrate that if agents are ambiguity-averse and can take unverifiable actions to influence the likelihood of final outcomes, it may be good to create ambiguity about the outcome in undesirable states. Thus, an incomplete contract may help to provide incentives by exploiting the agents’ ambiguity aversion. Ambiguity works as a sort of “money-burning” mechanism that deters opportunistic behaviour

Keywords: Incomplete Contracts, Ambiguity-Aversion,

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# Best Response Dynamics and Neural Networks

Hans Haller\* and Alexander Outkin†

## Abstract

We analyze local as well as global interaction in population games, using the formalism of neural networks. In contrast to the existing literature, a state encodes not only the frequency of play, but also the spatial pattern of play. For our stochastic best response dynamics, the stationary distribution is of Boltzmann type. With a Boltzmann distribution, the long-run equilibria are the minima of a cost function (defined in the paper) which allows us to encompass, generalize, or qualify previous findings.

In the young tradition of Kandori, Mailath, and Rob (1993), abbreviated KMR in the sequel, Ellison (1993) and Rhode and Stegeman (1996), we consider best response dynamics where at each time, every player plays a (static) best response against the empirical distribution of the last strategies played by his neighbors. This constitutes rational behavior impaired by myopia. Myopia in the temporal sense means that the player is not forward looking, does not take into account that other players might be changing their strategies. (This trait is shared, for example, by naive Bayesian learners [Eichberger et al. (1993)].) Myopia in the spatial sense, if applicable, means that the player is influenced only by his local environment.

In the sequel, an "interaction structure" is modeled as an undirected finite graph whose vertices are the members of the player population. Two players are neighbors, if they form an edge of the graph. We assume that the graph and a fortiori the interaction structure is regular, i.e. all players have the same number of neighbors. We finally assume that direct interaction is only possible between neighbors. We are going to analyze local as well as global interaction in population games, using the formalism of neural networks. More specifically, we forward a modeling approach to best response dynamics that

- (a). allows for rather general interaction structures;
- (b). exhibits spatial patterns of play;
- (c). exhibits differentiated noise;
- (d). links stochastic and deterministic dynamics;
- (e). involves asynchronous updating;
- (f). encompasses majority imitation.

Berninghaus and Schwalbe (1996) were the first to demonstrate that the theory of neural networks can be successfully applied to analyze deterministic best response dynamics with global or local interaction. Here

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we go beyond Berninghaus and Schwalbe and introduce noise into the system. This has been done before, notably by KMR and Ellison for what they call “best reply” and other dynamics.

We use a “Boltzmann machine”, a particular kind of stochastic neural network, to model noise. This produces two major innovations. First, a state of the system describes a spatial pattern of play, not merely a summary statistics as in KMR and Ellison. Second, the probability of a “flip”, i.e. of non-best-response play is a continuous and decreasing function of the payoff loss caused by the flip whereas in KMR and Ellison the probability is independent of state and player.

Running the “Boltzmann machine”, we obtain an explicit formula for the stochastic steady state (invariant distribution) in terms of the parameters of the model for any regular interaction structure. We are able to relate the long-run equilibria à la KMR to the deterministic steady states. We can, in principle, compute all long-run equilibria for any symmetric  $2 \times 2$  game and any regular interaction structure, by solving a discrete optimization problem. We not only determine how often a strategy occurs, but also detect the spatial pattern of play in a long-run equilibrium. By and large, the properties of long-run equilibria reported by KMR and Ellison are confirmed which implies a certain robustness of the model. However, Rhode and Stegeman have amended and corrected some of the KMR results, ending up with a richer taxonomy of games.

The distinction between global and local interaction is only interesting, if it makes a difference. Apart from purely descriptive reasons, the comparison of interaction structures has been motivated by the pioneering work of Novak and May (1993) whose simulations of deterministic best performance imitation have generated significant differences across interaction structures. The theoretical analysis of deterministic best response dynamics by Berninghaus and Schwalbe (1996) shows that the size and shape of neighborhoods can affect the nature, number, stability, and attractiveness of limit cycles and steady states. In the case of differentiation games (games with no symmetric pure strategy equilibrium), our analysis of stochastic best response dynamics with asymptotic updating demonstrates a significant difference of long-run equilibria across interaction structures, an important new discovery.

# Dynamic Choices of Hyperbolic Consumers

Christopher Harris and David Laibson

## Abstract

Laboratory and field studies of time preference find that discount rates are much greater in the short-run than in the long-run. Hyperbolic discount functions capture this property. This paper solves the decision problem of a hyperbolic consumer who faces stochastic income and a borrowing constraint. The paper uses the bounded variation calculus to derive the Hyperbolic Euler Relation, a natural generalization of the standard Exponential Euler Relation. The Hyperbolic Euler Relation implies that consumers act as if they have endogenous rates of time preference that rise and fall with the future marginal propensity to consume (e.g., discount rates that endogenously range from 5% to 41% for the calibration discussed in the paper). The Hyperbolic Euler Relation implies that hyperbolic consumers will engage in high interest borrowing even when the consumption path is downward sloping, will experience predictable sharp drops in consumption, and will fail to exhibit precautionary savings effects.

# An Evolutionary Analysis of the Returns to Learning in a Changing Environment

Dana Heller\*

## Abstract

The evolutionary fitness of the ability to learn is investigated in a model with replicator dynamics. Members of the population are randomly paired, and they play one of three types of symmetric  $2 \times 2$  games: the prisoners' dilemma, coordination, or chicken. The population includes types who always play the same stage-game strategy and a type who learns the best response to his individual opponent in the specific game being played. The ability to learn is assumed to carry a biological cost.

Preliminary analysis concerns unchanging environments; if the game is coordination or the prisoners' dilemma, the learners cannot survive in the long run. Notwithstanding, in a stable environment of coordination, the presence of the learners early on can affect the eventual ecological balance; starting from any initial composition of the population, for small enough learning costs, the simple type that plays the payoff-efficient action will eventually come to dominate the population following an intermediate phase where the learners are dominant. If the game is chicken and if learning costs are not too high, the fraction of learners in the population tends to a stable, strictly positive fraction.

We then go on to investigate an environment that shifts stochastically among the different types of games. We find that even extraordinarily rare changes might lead to very different evolutionary outcomes than does a fixed environment. First we show that even if the learners are never the most fit type for any one specific environment they may have the highest average performance over time, and hence they may come to dominate the population. What drives this phenomenon is the observation that as time progresses, it takes a simple type longer and longer periods in which it has an advantage to recover, establishing the learners' superiority. We then demonstrate how "early events" in the environment may leave an imprint on the asymptotic composition of the population. More specifically, how a long epoch in a prisoners' dilemma game might lead to the dominance of the corresponding simple type; this dominance can not be overturned by (low-cost) learners when the environment later changes to a coordination game where this simple type is payoff inefficient. In particular, an "all or none" result is established — either that simple type comes to dominate, or it is doomed to become asymptotically extinct. Finally, a sufficient condition on the set of states of the environment is derived to ensure that the learners do not become asymptotically extinct; if their share ever becomes too small they make "surges" back to the population. This result builds on the observation that any strategic disadvantage of the learners, i.e., gross of learning costs, is vanishing as the share of the learners becomes arbitrarily small.

JEL Classification Number: C72, C73.

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# A Differentiable Homotopy to Compute Nash Equilibria of $n$ -Person Games

P.J.J. Herings and R.J.A.P. Peeters\*

## Abstract

Harsanyi and Selten (1988) have proposed a theory of equilibrium selection that selects a unique Nash equilibrium for any noncooperative  $n$ -person game. The heart of their theory is given by the tracing procedure, a mathematical construct that adjusts arbitrary prior beliefs into equilibrium beliefs. Although the term ‘procedure’ suggests a numerical approach, the tracing procedure itself is a non-constructive method.

In this paper we prove that for a generic game the feasible path of the linear tracing procedure is a compact one-dimensional piecewise differentiable manifold with boundary. Using a well-chosen transformation of variables, this enables us to formulate an everywhere differentiable homotopy function, whose zeros describe that path. Because of differentiability we can use the predictor-corrector method to follow that path from beginning to end.

Our algorithm is an algorithm to compute Nash equilibria. Compared to other algorithms to compute Nash equilibria, it has several advantages. One advantage is that it computes a Nash equilibrium that has a sound game-theoretic underpinning. It computes the Nash equilibrium selected by the Harsanyi-Selten theory. A further advantage is that our method, unlike the commonly used simplicial algorithms, exploits the differentiability in the problem, which leads to great improvements in computing times. The formulation of the algorithm as a differentiable homotopy makes it possible to apply standard path-following techniques that are available in professionally programmed software. This makes that our algorithm is easily implemented on a computer.

JEL classification: C68, C72

Keywords: Computation of equilibria; Non-cooperative game theory; Tracing procedure.

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# Cryptographic Automata and Efficient Outcomes

Penélope Hernández and Amparo Urbano\*

## Abstract

This paper highlights the importance of restricting the computational complexity of automata in two-player finitely repeated games. In particular, we show how to reach efficient outcomes in these games in shorter time and by automata with bigger complexity than the last results in the literature. To this end we use tools of modern cryptography, and we combine together two kinds of bounded rationality: computational and strategic complexity.

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# The Privilege of the Uninformed in Uniform Price Auctions

Angel Hernando\*

## Abstract

This paper studies a multiunit uniform price auction with a common value object where there are informed and uninformed bidders. In general, there are two symmetric equilibria: one where the uninformed always win and another one where always lose. The intuition for the former is that the uninformed can successfully freeride the information of the informed bidders that win. I also study through simulations games with well informed bidders and poorly informed. The limit equilibrium of these games is either the equilibrium where the uninformed always wins or the one where the uninformed always loses depending on the rate of number of bidders to units.

JEL Classification Numbers: D44.

Keywords: uninformed bidder, multiunit auction, common value, uniform price auction.

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# Market Organization and Coordination: An Experimental Investigation

Dorothea K. Herreiner\*

## Abstract

Weisbuch/Kirman/Herreiner (1999) analyze self-organization of a market, where a homogeneous good is bought repeatedly, in the framework of reinforcement learning. They derive strong results about individual behaviour: individuals are either loyalists or searchers. Loyalists buy from the same shop almost all the time, searchers select a shop randomly every time. Whether an individual is loyal or not depends on individual characteristics, in particular on the discrimination rate and a memory constant. Individuals who discriminate strongly between different shops and individuals who remember far back how they were treated in those shops are loyal whilst individuals who lack those characteristics are searchers. These results are interesting because they allow for heterogeneity of individual behaviour in a market which offers the same conditions for everybody. Moreover, the prediction is, that there are only two extreme kinds of behaviour, but no intermediate behaviour of sometimes being loyal and sometimes searching.

The model presented in Weisbuch/Kirman/Herreiner is basically a repeated and somewhat enhanced version of a coordination game between  $n$  players. These  $n$  players have to find  $n$  homogeneous units of a good distributed in  $m$  different sites. In every round players can pick only one site. If there are more players than units, then units are assigned randomly to players. If there are at most as many players as there are units, then all players receive a unit. In the following round units are distributed according to the distribution of players over the sites in the preceding round. Making appropriate assumptions about the beliefs of players, it can be shown that the two extreme kinds of behaviour characterized by Weisbuch et al. emerge as equilibria in this game.

The coordination game version of the model was implemented in an experiment in two different treatments which differed only with respect to the number of sites available (3 or 6). Each treatment was repeated 6 times with different players.

The two different kinds of behaviour, loyalty and searching, emerge also in the experiment. However, there is also some intermediate kind of behaviour. It can be shown that the number of players being loyal, both at the beginning and at the end of the experiment, is significantly different for the two treatments, thereby confirming predictions of the Weisbuch et al paper. Moreover, the degree of searching depends on the degree of coordination failure in the market, again in line with the Weisbuch et al model.

The experiment was supplemented by further tests and a questionnaire, from which it emerges (preliminary results), that loyalty depends

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in fact on memory characteristics again in line with the predictions of the Weisbuch et al. model.

The learning algorithm used by Weisbuch et al is similar to that analyzed by Roth/Erev (1997) and Camerer/Ho (1997). Roth/Erev's model uses a linear choice function (instead of the exponential choice function of Weisbuch et al), enhanced by different additional features. They test their model in the framework of different experiments and by comparing the behaviour with behaviour simulated according to their model. Their reinforcement learning algorithm can explain convergent (and also divergent) behaviour in their experiments. Camerer/Ho suggest an even richer model which includes belief-based models and reinforcement learning as special cases. Like Roth/Erev, they apply their model to experiments and find that it is even better suited to explain the experimental data. Similar to Weisbuch et al, they use a logit choice function, which they confirm to yield a better fit in Camerer/Ho (1998). The results of these papers are compared with the findings of the experiment presented here.

# On the Epistemic Conditions for Nash Equilibrium\*

John Hillas

## Abstract

Aumann and Brandenburger (1995) have given a set of sufficient epistemic conditions for a beliefs of the players in a game to constitute a Nash equilibrium. They emphasise the point that in their conditions common knowledge plays a much smaller role than had been assumed in much earlier work. Indeed, in two player games common knowledge plays no role whatsoever.

This paper does not dispute the correctness of Aumann and Brandenburger's results, either in a mathematical sense or in the sense that they do provide one set of conditions for Nash equilibrium in which common knowledge plays a small role. Nevertheless, there is something a bit mysterious about these results. After all we now understand quite well that common knowledge of the game being played (including the rationality of the players) implies either rationalizability (if no assumption of a common prior is made) or correlated equilibrium (if such an assumption is made). And something approaching the full strength of the *common* knowledge assumption is needed for these results. How is it that something less is required for the stronger condition of Nash equilibrium?

In this paper I show that one can obtain a different set of conditions for Nash equilibrium by adding in a natural way to the conditions that give us correlated equilibrium. One adds to the assumption of common knowledge of the game the assumption of a common understanding of the way in which the game is played. By this I mean that a player has no private information about the actions of another player or that the player's assessment of the way that other players act is the same as that of a hypothetical outside observer. In this case the assessment of the outside observer constitutes a Nash equilibrium.

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\**Date:* March 1999. This note grew out of a brief comment in joint work with Elon Kohlberg on the chapter on the "Foundations of Strategic Equilibrium" for the *Handbook of Game Theory with Economic Applications* edited by Robert J. Aumann and Sergiu Hart.

# Consistency Implies Equal Treatment

Toru Hokari\*

## Abstract

Sobolev (1975) shows that, on the class of all TU games, the prenucleolus is the only *single-valued* solution satisfying *efficiency*, *zero-independence*, *anonymity*, and *max consistency*. Peleg (1986) shows that, on the class of all TU games, the prekernel is the only solution satisfying *efficiency*, *zero-independence*, *equal treatment of equals*, *max consistency*, and *converse max consistency*. In this paper, we study implications of dropping *anonymity* and *equal treatment of equals* from the above systems of axioms. For the two-person case, the prenucleolus and the prekernel coincide with the "standard solution". We define a family of "weighted standard solutions" for the two-person case in a very natural way. We show that, on the class of all TU games, if a solution is *max consistent* and coincides with an weighted standard solution for the two-person case, then it satisfies *equal treatment of equals*. As corollaries, we obtain two interesting refinements of the above characterizations of the prenucleolus and the prekernel. On the class of convex games, there are other *max consistent* solutions that coincide with an weighted standard solution for the two-person case, and we describe all such solutions: namely, we provide a characterization of all *single-valued* solutions on this domain satisfying *efficiency*, *individual rationality*, *zero-independence*, *homogeneity*, and *max consistency*.

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# An Epistemic Analysis of the Harsanyi Transformation

Hong Hu and H. W. Stuart, Jr.\*

## Abstract

Harsanyi (1967-68) proposed a method to transform uncertainty over the strategy sets of the players in a game into uncertainty over their payoffs. At first glance, the transformation appears to rely on an assumption that the players are rational, or, indeed, that they are rational and that there is common belief of rationality. Such an assumption would be awkward from the perspective of the epistemic program, which is often interested in the implications of irrationality or a lack of common belief of rationality. This paper shows that within a belief system model, such an assumption is not necessary. For any belief system with strategy-set uncertainty, an associated belief system without strategy-set uncertainty is constructed. Moreover, in this construction, the player's strategy choices, rationality or irrationality, and beliefs are all preserved. The construction can be extended to uncertainty over the player set.

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# The Choice between Patent and Trade Secret with Product Life Considered

Jin-Li Hu, Yung-Ho Chiu, and Reiko Aoki\*

## Abstract

There are many types of intellectual property rights (IPR) available for protecting innovation. Patent and trade secret are the major two IPR types used to protect innovations with industrial application. How to choose a proper IPR type to maximize the expected profit is an important issue for an innovating firm. Most of the existing economic literature merely focuses on the patent system, such as patent length and breadth. The choice of trade secret, product life, time to imitate, early disclosure, and patent challenge, etc., are not taken into account in most of the existing economic literature. However, these factors are crucially in determining the profit from protecting an innovation through patenting it or keeping it in trade secret. Therefore, in this article we explicitly incorporate (1) the product life and patent length, (2) the probability of acquiring a patent, (3) the probability of successfully challenging a patent, and (4) the expected time of a successful reverse engineering into the model. Then we analyze the choice between patent and trade secret to protect a product innovation with these factors considered. There are two firms in the model. One is the innovating firm with a product innovation and the other is a potential imitator. Our major conclusions are as follows: 1. No matter whether the product life is longer or shorter than patent length, as long as if the expected time of a successful reverse engineering is longer than the patent length, then the innovating firm will choose trade secret. Yet, if the expected time of a successful reverse engineering is shorter than the patent length, then the innovating firm will still choose trade secret unless the time for reverse engineering is short enough. 2. Even when the probability of acquiring a patent is 100% and the potential imitator does not challenge the patent, it is still possible for the innovating firm to choose trade secret. 3. When the probability of a successful challenge is large enough, then the innovating firm will choose trade secret. 4. When the novelty requirement of patenting is high, the innovating firm will choose trade secret. 5. When the innovation can be brought out with a lower R&D cost, a patent system inducing the innovating firm to apply for a patent results in a higher expected social surplus than trade secret.

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# The $p$ -Core

C-Y. Huang and T. Sjoström

## Abstract

In order to calculate the worth of a coalition of players, the coalition needs to predict the actions of outsiders. We propose that, for a given solution concept, such predictions should be made by applying the solution concept to the “reduced society” consisting of the non-members. This leads to a new way of defining coalitional values, the  $p$ -theory. We illustrate the  $p$ -theory using the core and the bargaining set, i.e. we define a  $p$ -core and a  $p$ -bargaining set that can be compared with the well-known  $\alpha$ -core and  $\alpha$ -bargaining set. We focus on the  $p$ -core, which is a subset of the  $\alpha$ -core. With differentiated Bertrand competition among  $n$  firms, the  $p$ -core is non-empty if and only if the parameter of substitutability is greater than a threshold level  $\hat{r}(n)$ , where  $\hat{r}(n)$  is increasing in  $n$ . Hence, cooperation is easier to sustain with more competitive market conditions (higher substitutability), and with fewer firms.

# Players with Limited Memory

Steffen Huck and Rajiv Sarin

## **Abstract**

We study players with limited memory and characterize the set of strategies they learn to play. We also consider players whose limited memory is imperfect. Such players experience noise in their memory. Lastly, we introduce a solution concept, "iterated uniform dominance" and show its relevance to the study of such agents.

# Absorbing Sets in Coalition Formation Systems

E. Iñarra\*, J. Kuipers, and N. Olaizola†

## Abstract

The objective of this paper is to study the problem of endogenous coalition formation, and in particular to analyze what coalition structures are likely to occur. We define a solution concept, the absorbing set, and we prove the uniqueness of this set for the class of symmetric coalition formation systems.

Keywords: coalition formation, stable sets.  
JEL numbers: C71, D63.

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†The authors thank F. Grafe for his participation in the first stages of the paper.

# The Evolution of Social and Economic Networks

Matthew O. Jackson\* and Alison Watts†

## Abstract

We examine the dynamic formation and stochastic evolution of networks connecting individuals. The payoff to an individual from an economic or social activity depends on the network of connections among individuals. Over time individuals form and sever links connecting themselves to other individuals based on the improvement that the resulting network offers them relative to the current network. We call such sequences of networks, ‘improving paths,’ and show that such sequences can include cycles and study conditions on underlying allocation rules that characterize cycles.

Building on the concept of improving paths, we consider a stochastic evolutionary process where in addition to intended changes in the network there is a small probability of unintended changes or errors. Predictions can be made regarding the relative likelihood that the stochastic process will lead to any given network at some time, where the evolutionary process selects from among the statically stable networks and cycles. We apply these results to a series of models including the Gale-Shapley marriage market. In some cases, the evolutionary process selects only inefficient networks even though there are efficient networks that are statically stable, and moreover even if interventions that reallocate value are admissible.

Journal of Economic Literature Classification Numbers: A 14, D20, J00.

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# Information Acquisition in the Stackelberg Model

Maarten C. W. Janssen and Emiel Maasland\*

## Abstract

This paper studies a simple Stackelberg model with random linear demand in which firms have to decide whether or not to do market research on an unknown demand parameter. Depending on the parameter values different outcomes are possible. If uncertainty is small, both Leader and Follower refrain from market research. If uncertainty is large, only the Leader does market research. For the intermediate case, only the Follower does market research. We also show that if the Leader has more information about demand than the Follower there exists a unique D1 equilibrium and that this equilibrium is perfectly revealing.

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# The Structure and Complexity of Strategy-implementing and Choice-learning Automata

Mark Johnson

## Abstract

It now is common to use automata to represent individual behavior in implementing strategies in repeated play games. Less common is to use this same structure in analyzing choice behavior. Further, in virtually all economic models, the automata are analyzed in the directed graph representations. The results presented here demonstrate the construction of a common algebraic model for analyzing strategy-implementing and choice-implementing automata. This algebraic model allows differences in properties and powers of strategy-implementing and choice-implementing automata to be identified and their complexities compared. Notably, most strategy-implementing automata are higher order complexity than is required for implementing choice-rules constrained by economic consistency axioms (e.g., WARP). Specifically, while all automata have representations as semigroups (a closed, associative binary system), it is demonstrated that the most commonly used choice consistency axioms limit the semigroup's structure to one of the simplest classes of semigroup-semilattices. For "small" automata it is shown that the only strategies implementable by automata with the same (limited) "power" as choice-implementing automata are those isomorphic to the "grim trigger" strategy. All other strategies require more powerful classes of semigroups with many requiring full group structure (groups are semigroups with inverses and identity elements). Against this background, a "choice-learning" automaton is introduced and its algebraic structure detailed. This choice-learning automaton is demonstrated to have roughly the same "power" as the broad class of strategy-implementing automata i.e., the choice-learning automaton has group structure.

# Pillage and Property

James Jordan

## Abstract

The existence of property rights is a precondition for most of the phenomena encompassed in the subject of economics. The social benefit of property rights is obvious, but the incentive to create them is not. Any individual or group with enough power to create property rights has an incentive to make exceptions in its favor. Moreover, since wealth enhances power, property acquired in violation of the rights of others increases the power of the violator. Thus the concept of property rights appears to contain an inherent instability.

This paper studies a class of transferable utility games representing anarchy. There is a single commodity, wealth, which must be allocated among a finite number of individuals. The power of any coalition is given by a function that depends at least partly on the wealth of its members. Any coalition can pillage, costlessly and with certainty, any other coalition having strictly less power. Since power is endogenous, the game does not have a characteristic function, but pillage provides the dominance relation. In the case in which each coalition's power is equal to its total wealth, the unique stable set (von Neumann-Morgenstern solution) consists of all allocations in which each player has a binary fraction of the total wealth. In all cases, a stable set must contain only finitely many allocations. However, if economically extrinsic social types are added to the model, a much richer stable set is possible. This result provides some support for the social stability of property even in an environment where the strong can despoil the weak with costless certainty.

# Check Your Partners' Behavior by Randomization

Michihiro Kandori\*

## Abstract

Randomization is an effective way of extracting information from a limited number of observations, as random auditing shows. We employ this idea to support efficient outcomes in repeated games with imperfect monitoring, when information is severely limited. In particular, we show that efficiency can be improved when the players randomize and condition their future actions both on the signal and their actions. Firstly, we show that in a version of Radner, Myerson and Maskin's example of inefficient partnership, efficiency can be achieved by such (i.e., privately mixed strategy) equilibria. Secondly, we show that the folk theorem under imperfect public monitoring can be extended to the case with a small signal space by means of mixed strategy equilibria with communication. In particular, we show that for generic symmetric games with at least four players, we can drop the Fudenberg-Levine-Maskin condition on the number of actions and signals altogether and prove the folk theorem under the same condition as in the perfect monitoring case.

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# The Strategic Use of Seller Information in Private-Value Auctions

Todd R. Kaplan\* and Shmuel Zamir†

## Abstract

The standard model of a private-value auction assumes the common knowledge of the joint distribution of valuations and the private knowledge of only a Buyer's own value. As Landsberger et al. [1998] show any additional information regarding the valuations may change the behavior of the Buyers and consequently the increase the revenue of the Seller.

We study the possibility that a Seller, who has private information about the realized valuations, may strategically transmit messages to Buyers in order to increase his revenue. In an environment where the Seller knows the rankings and lacks any commitment power, we find that the Seller is unable to exploit his information. However, in an environment where the Seller knows the realized valuations and can to either say nothing or credibly announce either the rankings or the values, then it is possible to increase his revenue to even beyond that of the Landsberger et al. model. If the Seller, in addition, can commit to the full signalling strategy, then the expected revenue will be even higher.

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# Innovative Activity and Sunk Cost

Todd R. Kaplan, Israel Luski, and David Wettstein\*

## Abstract

We analyze innovative activity in two different deterministic environments where all firms participating in a patent race incur costs. We start by considering the case of identical firms and show that while no pure-strategy equilibrium exists, there is a unique symmetric mixed-strategy equilibrium that yields zero expected profits for all firms. Furthermore, we show that the expected innovation time is an increasing function of the number of firms and a decreasing function of the size of the reward.

Next, we consider the asymmetric case where one firm has a higher reward than a second firm. We show that there is a unique equilibrium where the higher reward firm always enters the race (invests a positive amount), and is more likely to win, however, there is still a positive probability of the lower reward firm winning. Finally, we contrast our analysis with the all-pay-auction literature.

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# Power Indices Without the Transfer Axiom

Anna B.Khmel'nitskay<sup>‡</sup>

## Abstract

Simple games are often used to study the distribution of power in voting systems. A payoff for a player may be interpreted as the player's perceived political power and is called a power index. Since Shapley's axiom of additivity cannot be applied for the class of simple games, axiomatizations of power indices (see Dubey (1975), Dubey and Shapley (1979), Weber (1988)) have used the transfer axiom suggested by Dubey (1975). But the transfer axiom has no natural interpretation and is pure technical condition. It turns out that for a wide class of power indices — random-order indices (Weber, 1988), and in particular, for the Shapley-Shubik power index (1954) — the transfer axiom may be avoided.

We exploit the property of marginalism introduced by Young (1985) in his characterization of the Shapley value (1953), which means the dependence of a player's payoff only upon his/her marginal utilities. Marginalism, efficiency, and the null-player property are reasonable for simple games. For voting systems with at least three voters we derive an explicit formula for a marginalist and efficient power index that possesses the null-player property. It follows that for societies with at least three members the set of all marginalist, efficient, and monotonic power indices possessing the null-player property coincides with the set of random-order power indices. The last statement extends to simple games the result of Khmel'nitskaya (1999) concerning an axiomatization without the linearity assumption for random-order values for the entire class of TU games. Further every marginalist, efficient, and symmetric power index is just the Shapley-Shubik power index. This provides an axiomatization for the latter similar to that of Young for the Shapley value. It is worth noting that in symmetric case there is no restriction that the number of players be at least three.

An  $n$ -person game in characteristic function form or a transferable utility game (TU game) is a pair  $\langle N, v \rangle$  where  $N = \{1, \dots, n\}$  and  $v$  is a mapping  $v: 2^N \rightarrow R^1$  such that  $v(\emptyset) = 0$ .  $N$  is the set of players and  $2^N$  denotes the family of all coalitions  $S \subset N$ . In this context a class of games with a fixed set  $N$  is naturally identified with the Euclidean space  $R^{2^n-1}$  of vectors  $v$ ,  $v = \{v_S\}_{\substack{S \subset N \\ S \neq \emptyset}}$ . A game  $v$  is said to be simple if it takes only the values 0 and 1 and obeys monotonicity:  $v_S \geq v_T$  whenever  $S \supset T$ . In particular, the zero game, i.e., the game  $v$  such that for all  $S \subset N$   $v_S = 0$ , is a simple game. We denote the set of all simple games by  $\mathcal{P}$ . For any set of games  $\mathcal{G} \subset R^{2^n-1}$ , a value on  $\mathcal{G}$  is a mapping  $\xi: \mathcal{G} \rightarrow R^n$  that associates with each game  $v \in \mathcal{G}$  a vector  $\xi(v) \in R^n$  where the real number  $\xi_i(v)$  represents the payoff to player  $i$  in the game  $v$ ; if  $\mathcal{G} = \mathcal{P}$

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then  $\xi_i(v)$  is called a *power index* of the player  $i$  in  $v$  and an  $n$ -vector  $\xi(v)$  presents a *collection of power indices*.

A collection of power indices  $\xi$  is *efficient* if for all  $v \in \mathcal{P}$

$$\sum_{i \in N} \xi_i(v) = v_N.$$

A collection of power indices  $\xi$  is *marginalist* if for all  $v \in \mathcal{P}$  and every  $i \in N$ ,  $\xi_i(v)$  depends only on the  $i$ th marginal utility vector  $\{v_{S \cup \{i\}} - v_S\}_{S \subset N \setminus \{i\}}$ , i.e.,

$$\xi_i(v) = \phi_i(\{v_{S \cup \{i\}} - v_S\}_{S \subset N \setminus \{i\}}),$$

where  $\phi_i: R^{2^{n-1}} \rightarrow R^1$ .

Let  $\Pi$  be the set of  $n!$  permutations of  $N$  and for all  $v \in \mathcal{P}$  and for any  $\pi \in \Pi$ ,  $\pi N = \{i_1, i_2, \dots, i_n\}$ , let  $x^\pi(v) \in R^n$  denote the  $n$ -dimensional marginal contribution vector

$$x_{i_k}^\pi(v) = v_{\{i_1, \dots, i_{k-1}, i_k\}} - v_{\{i_1, \dots, i_{k-1}\}}.$$

All vectors  $x^\pi(v)$ ,  $\pi \in \Pi$ , are efficient, and therefore for every  $v \in \mathcal{P}$  the set  $\{x_i^\pi(v)\}$ ,  $i \in N$ ,  $\pi \in \Pi$ , forms  $(n \times n!)$ -matrix with elements in each column summing to either 1 for all non-zero simple games or to 0 for the zero game, for the zero game the elements of this matrix are all equal 0. One can easily see that for any non-zero simple game, all elements of the matrix are equal either to 1 or to 0, and in each column there is exactly one 1. Obviously, for any  $i \in N$  every marginalist power index may be represented in the form

$$\xi_i(v) = \phi_i(\{x_i^\pi(v)\}_{\pi \in \Pi}),$$

for some  $\phi_i: R^{n!} \rightarrow R^1$ .

A collection of power indices  $\xi$  is *monotonic* if for any  $i \in N$ ,  $\xi_i(v) \geq 0$ .

A player  $i$  is a *null-player in game  $v$*  if  $v_{S \cup \{i\}} = v_S$  for every  $S \subset N \setminus \{i\}$ . A collection of power indices  $\xi$  possesses *the null-player property* if for all  $v \in \mathcal{P}$ , for every null-player  $i$  in  $v$ , the power index  $\xi_i(v) = 0$ .

A collection of power indices is *symmetric* if for all  $v \in \mathcal{P}$ , for any permutation  $\pi: N \rightarrow N$ , and for all  $i \in N$

$$\xi_i(v^\pi) = \xi_{\pi(i)}(v),$$

where  $v_S^\pi = v_{\pi(S)}$  for all  $S \subset N$ ,  $S \neq \emptyset$ .

A collection of power indices  $\xi$  is a collection of *random-order* power indices if for all  $v \in \mathcal{P}$  and all  $i \in N$ , the power index  $\xi_i(v)$  has the form

$$\xi_i(v) = \sum_{\pi \in \Pi} r_\pi x_i^\pi(v)$$

for some probability distribution  $\{r_\pi\}_{\pi \in \Pi}$  over the set  $\Pi$ , i.e.  $\{r_\pi\} \in R_+^{n!}$ , where  $\sum_{\pi \in \Pi} r_\pi = 1$ .

A *Shapley-Shubik power index* is a Shapley value restricted to the set of simple games  $\mathcal{P}$ . It may be equivalently defined as a random-order power index corresponding to the uniform distribution  $\{r_\pi\}_{\pi \in \Pi}$ , to wit, if by  $\text{Sh}_i(v)$  we denote the Shapley-Shubik power index of a player  $i \in N$  in a game  $v \in \mathcal{P}$ , then

$$\text{Sh}_i(v) = \frac{1}{n!} \sum_{\pi \in \Pi} x_i^\pi(v).$$

**Theorem 1** *Let  $n \geq 3$  and let the mapping  $\xi: \mathcal{P} \rightarrow R^n$  be a collection of marginalist and efficient power indices possessing the null-player property. Then for every  $v \in \mathcal{P}$ ,  $\xi(v)$  has the form*

$$\xi(v) = \sum_{\pi \in \Pi} r_\pi x^\pi(v) \quad (2)$$

for some  $\{r_\pi\}_{\pi \in \Pi} \in R^{n!}$  such that

$$\sum_{\pi \in \Pi} r_\pi = 1; \quad (3)$$

and moreover, for monotonic  $\xi$ ,  $\{r_\pi\}_{\pi \in \Pi} \in R_+^{n!}$ .

Two direct corollaries to Theorem 1 are the following axiomatizations for random-order power indices and for the Shapley-Shubik power index, without the transfer axiom.

**Theorem 2** *Let  $n \geq 3$ . The only set of marginalist, efficient, and monotonic power indices possessing the null-player property is the set of random-order ones.*

Without the symmetry assumption, the condition  $n \geq 3$  is an essential one. In fact, if  $n = 2$  the power indices

$$\xi_1(v) = \begin{cases} \min(v_{\{1\}}, 1 - v_{\{2\}}), & v \neq \mathbf{0}, \\ 0, & v = \mathbf{0}. \end{cases}$$

$$\xi_2(v) = \begin{cases} \max(1 - v_{\{1\}}, v_{\{2\}}), & v \neq \mathbf{0}, \\ 0, & v = \mathbf{0}. \end{cases}$$

provide an example of marginalist, monotonic and efficient collection of power indices also possessing the null-player property, but are not random-order ones.

However, supposing symmetry, the restriction  $n \neq 2$  may be avoided and we can prove the next theorem which is an exact extension of Young's result to simple games.

**Theorem 3** *The Shapley-Shubik power index is the only one that satisfies marginalism, efficiency, and symmetry.*

# Misrepresentation of Utilities in Bargaining: Pure Exchange and Public Good Economies

Özgür Kibris\*

## Abstract

Cooperative bargaining theory is motivated by the assumption that the agents' true utility functions are known to the social planner. If this is not the case and if the agents do not declare their utility functions truthfully, the bargaining outcome suggested by the social planner need not have any of the desirable properties that motivated its choice in the first place. For two underlying economic models (pure exchange and public good economies) we question the “desirability” of the corresponding bargaining outcomes. Following Sobel (1981) we construct a noncooperative “distortion” game in which agents strategically declare utility functions. For an arbitrary number of agents and commodities we analyze the Nash equilibria of this game and relate them to the constrained Walrasian/Lindahl allocations with respect to agents' true utilities.

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# An Algorithm for Envy-Free Allocations in an Economy with Indivisible Objects and Money

Flip Klijn\*

## Abstract

This paper studies envy-free allocations for economies with indivisible objects, quasi-linear utility functions, and an amount of money. We give a polynomially bounded algorithm for finding envy-free allocations. Connectedness of envy-graphs, which are used in the algorithm, characterizes the extreme points of the polytopes of sidepayments corresponding with envy-free allocations.

Classification number (J.E.L.): D63.

Keywords: envy-free allocations, indivisible good economy, algorithm

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# Repeated Games With Lack of Information on one Side: The Dual Differential Approach\*

Rida LARAKI<sup>†‡</sup>

## Abstract

In this paper, we introduce the dual differential game of a repeated game with lack of information on one side as the natural continuous time version of the dual game introduced by De Meyer (1996). A traditional way to study the value of differential games is through discrete time approximations. One uses here the opposite approach: we identify the limit value of a repeated game in discrete time as the value of a differential game. Namely, we use the recursive structure for the finitely repeated version of the dual game to construct a differential game for which the upper values of the uniform discretization satisfy precisely the same property.

The value of the dual differential game exists and is the unique viscosity solution of a first order derivative equation (which appears implicitly in De Meyer and Rosenberg (1997)), with a limit condition. We identify the solution as the Fenchel conjugate of the value of the average game.

This analysis permits to extend naturally the result of Aumann and Maschler to generalized payoffs associated to repeated game with incomplete information.

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# Assortative Matching and Endogenous Stratification

Susan Lee\*

## Abstract

This research project extends existing results on equilibrium selection in evolutionary games to the case of nonuniform matching of agents. The class of common interest games with a risk dominant Nash equilibrium and a payoff dominant Nash equilibrium lends itself well to a study of emergence of heterogeneous behavior in a large population. A stochastic matching function is employed which assumes matches between individuals with similar levels of success in previous games (measured by wealth) are more likely to be paired. This is achieved by borrowing a functional form from statistical mechanics which allows the degree of exogenous social stratification to be parametrized.

While the standard evolutionary model of equilibrium selection examines feedback from the history of population play into current play, the incorporation of nonuniform matching allows feedback effects to arise from a richer set of events. Correlations in agents' experiences, economic characteristics of agents' previous opponents, and the previous actions of opponents will influence current behavior. The resulting dynamics exhibit the emergence of subpopulations whose number and memberships are constantly in flux, and which are defined by similarity in current behavior and success in previous games. The state of the system is the joint distribution of wealth and action taken in the previous period. Analytical results using the theory of Markov chains on the characteristics of sample paths and limiting behavior are undertaken alongside simulations.

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# Stability and Rationality

Xiao Luo\*

## Abstract

Within the context of strategic interaction, this paper provides some decision- theoretic foundations for the criterion of stability, which is first proposed by von Neumann and Morgenstern. Specifically, this paper establishes the following main results: in strategic games, achieving common knowledge of rationality (*CKR*) implies an internally stable set that is contained in an externally stable set. In the case of two-person games, achieving *CKR* implies a stable set. Moreover, when choice sets are mutually known, rationality alone implies a stable set. In extensive games with perfect information, achieving *CKR* implies a unique stable set. On the other hand, in both strategic and extensive games, any of the commonly known stable sets implies *CKR*. In addition, any stable set can be achieved in terms of *CKR*.

JEL Classification Number. C70, C72, D81.

KEYWORDS. General systems, Stable sets, Common knowledge of rationality, Noncooperative games

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# Games as Abstractions of Reality

G. M. MacDonald, Leslie M. Marx, and M. D. Ryall

## **Abstract**

This paper explores the idea that a player might usefully contemplate a game that is not a literal description of the actual situation. Loosely, (a) we develop a framework in which each player behaves as if he is playing a game that may differ markedly from the situation he actually faces, but satisfies some conditions that preserve consistency with the real game; and (b) we show that this consistency is sufficient for best replies in these games to correspond to Nash equilibria in the real situation.

# Joint Outside Options

Paola Manzini\* and Marco Mariotti†

## Abstract

Several contractual situations are such that the parties may ‘step out’ of negotiations and take up outside opportunities only if there is mutual consent to do so. Examples include employer-employee negotiations, divorce and inheritance procedures, and arbitration. To analyse such cases we develop the general concept of a ‘joint outside option’ and study its effect in the standard bargaining game. Examples from the economics of divorce and theory of the firm are considered in some depth.

J.E.L. classification codes. C78

Keywords. Bargaining

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# Maximum Games, Dominance-Solvability and Coordination

Marco Mariotti\*

## Abstract

Inspired by the model of Kalai and Satterthwaite (1994), I define a class of abstract games, called maximum games, which are dominance-solvable. In the leading subclass of games of common interest, they are solvable on the unique Pareto-dominant outcome. Maximum games include, as economic applications, some types of Bertrand games and public good games.

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# Clever agents in Young's evolutionary model of conventions

Alexander Matros\*

## Abstract

In the models of P. Young (1993a,b), boundedly rational individuals are recurrently matched to play a game, and they play myopic best replies to the recent history of play. M. Saez-Marti and J. Weibull (1998) investigate the opportunity to play the myopic best reply to the myopic best reply in the context of Young's (1993b) bargaining model. They introduce "cleverness" of players. I consider such clever individuals first in coordination games and then in general games. In this contest, *stochastically stable equilibria* and *risk dominant equilibria* in 2x2 games could be different. Moreover, "cleverness" in coordination games could be disadvantage in some sense. At the same time, Young's results are robust if the share of clever individuals less than one.

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# Coalitional Negotiation

Ana Mauleon\* and Vincent Vannetelbosch†

## Abstract

We develop a two-stage negotiation model to study the impact of costly inspections on both the coalition formation outcome and the per-member payoffs. In the first stage, the players are forming coalitions and inside each coalition formed the members share the coalition benefits. We adopt the *largest consistent set* (LCS) to predict which coalition structures are possibly stable. We also introduce a refinement, the *largest cautious consistent set* (LCCS). In the second stage, the inspection game takes place inside each coalition. For games with positive spillovers, many coalition structures may belong to the LCS under costless inspection. The grand coalition, which is the efficient coalition structure, always belongs to the LCS and is the unique one to belong to the LCCS. Under costly inspection, the grand coalition does not always belong to the LCS. Nevertheless, there exist inspection cost parameters such that the LCS singles out the grand coalition.

Keywords: coalition formation, inspections, positive spillovers, largest consistent set.

JEL Classification: C70, C71, C72, C78.

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# A General Technique for Computing Evolutionarily Stable Strategies Based on Errors in Decision Making\*

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Tamas Székely<sup>§</sup>, and Alasdair I. Houston<sup>§</sup>

## Abstract

Models of contests between animals often involve a series of state-dependent decisions by the contestants. Computation of evolutionarily stable strategies for such state-dependent dynamic games are usually based on damped iterations of the best response map. Typically this map is discontinuous so that iterations may not converge and even if they do converge it may not be clear if the limiting strategy is a Nash Equilibrium. We present a general computational technique based on errors in decision making that removes these difficulties.

The computational technique can be show to work for a simple example (the Hawk-Dove Game) where an analytic solution is known. The method has also been successfully applied to a dynamic game-theoretic model of parental care decisions.

It can also be argued that including these types of error leads to more realistic models of animal behaviour.

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# Noncooperative Bargaining in Apex Games and the Kernel

Maria Montero\*

## Abstract

This paper studies non-cooperative bargaining with random proposers in apex games. Two different protocols are considered: the egalitarian protocol, which selects each player to be the proposer with the same probability, and the proportional protocol, which selects each player with a probability proportional to his number of votes. Expected equilibrium payoffs coincide with the kernel for the grand coalition regardless of the protocol. Expected payoffs conditional on a coalition may depend on the protocol: Given a coalition of the apex player with a minor player, an egalitarian protocol yields a nearly equal split whereas a proportional protocol leads to a proportional split.

Keywords: noncooperative bargaining, apex games, kernel, random proposers.

JEL classification numbers: C72, C78.

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# Farsighted Stability in the n-Person Prisoner's Dilemma

Shigeo Muto\* and Akihiro Suzuki

## Abstract

The purpose of this paper is to study how cooperation emerges in an n-person prisoner's dilemma. The dilemma, sometimes called a social dilemma, has the following typical structure. Each individual has two choices, "cooperation" and "defection", and gains a higher payoff by taking defection no matter what choices other individuals may take. Thus taking defection is a rational behavior of each individual. If all individuals take defection, however, all of them are worse off than in the case where all take cooperation. Therefore the prisoner's dilemma shows individuals' rational behavior can induce a socially undesirable outcome.

In this paper, we examine whether an agreement to cooperate is obeyed when no institution for enforcement exists. Individuals can negotiate and reach an agreement if they wish. The agreement is, however, not binding; no institution or agency exists that enforces it. An individual (or a group of individuals) can later deviate from the agreement and take defection. Does cooperation emerge even under such occasions?

To analyze this problem, we employ farsighted stability proposed by Chwe (1994), i.e., the farsighted stable set and the largest consistent set. The stable set was originally defined by von Neumann and Morgenstern (1953). Though it was defined as a solution in characteristic function form cooperative games, von Neumann and Morgenstern defined the notion also in a wider class of games. Later Greenberg (1990) cleared a way to apply the notion to noncooperative strategic form games.

The stable set is a set of outcomes satisfying two stability notions: the internal stability and the external stability. The internal stability implies that between any two outcomes in the set there is no group of individuals such that all of its members prefer one to the other and can realize the preferred outcome. The external stability implies that for any outcome outside the set there is a group of individuals such that all of its members have a commonly preferred outcome in the set and can realize it. Both of von Neumann/Morgenstern and Greenberg assume only a single move by a group of individuals in defining the two stabilities.

Chwe criticized that the stable set is too myopic. When a group of individuals moves, another group of individuals moves, then a third group of individuals in turn moves, and so on: a sequence of moves may follow. Thus a group of players moving first must take into account such a sequence of moves that may follow and evaluate their profits in the end. A similar idea was proposed by Harsanyi (1974) though in a cooperative game theory context.

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Chwe incorporated a sequence of moves into the definition of stable set. We call it a farsighted stable set. He defined another solution that he called a consistent set. The notion is very similar to the farsighted stable set. A difference is found just in how deviating individuals expect moves that may follow. In the farsighted stable set, deviating individuals are optimistic and expect that a sequence of moves that are most favorable to them will follow. On the contrary, the consistent set assumes that deviating individuals have pessimistic expectation. The consistent set that contains all other consistent sets is called the largest consistent set.

It is shown that the following hold.

1. Any individually rational and Pareto efficient outcome is itself a farsighted stable set and there is no other farsighted stable set.
2. The largest consistent set consists of all individually rational outcomes.

Thus even if an agreement is not binding and individuals can freely deviate, individuals can necessarily reach a Pareto efficient and individually rational outcome if they have an optimistic view for moves that may follow their deviation. The largest consistent set does not give us a very clear outcome.

# Bayesian Learning in Repeated Games

John H. Nachbar\*

## Abstract

For a large set of two-player discounted infinitely repeated games, there does not exist any Bayesian learning theory in which player beliefs are *weakly cautious*, *symmetric*, and *consistent*. Loosely, weak caution means that if a repeated game strategy is in the support of a player's belief then so are computationally trivial variants of that strategy, symmetry means that the supports of player beliefs contain repeated game strategies of comparable strategic complexity, and consistency means that the support of each player's belief contains one of his opponent's  $\varepsilon$  best responses. This impossibility theorem generalizes the results of an earlier paper, Nachbar, *Econometrica* 1997. The impossibility theorem does not preclude construction of Bayesian learning theories per se, nor does it rule out convergence to equilibrium play in Bayesian theories. But the theorem does cast doubt on the possibility of constructing general theories of, for want of a better term, *rational* Bayesian learning.

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# An Experimental Study of Belief Learning Using Real Beliefs

Yaw Nyarko and Andrew Schotter\*

## Abstract

This paper investigates belief learning. Unlike other investigators who have been forced to use observable proxies to approximate unobserved beliefs, we have, using a belief elicitation procedure (proper scoring rule), elicited subject beliefs directly. As a result we were able to perform a more direct test of the proposition that people behave in a manner consistent with belief learning. What we find is interesting. First to the extent that subjects tend to “belief learn” the beliefs they use are the stated beliefs we elicit from them and not the “empirical beliefs” posited by fictitious play or Cournot models. Second, we present evidence that the stated beliefs of our subjects differ dramatically, both quantitatively and qualitatively, from the type of empirical or historical beliefs usually used as proxies for them. Third, our belief elicitation procedures allow us to examine how far we can be led astray when we are forced to infer the value of parameters using observable proxies for variables previously thought to be unobservable. By transforming a heretofore unobservable into an observable we can see directly how parameter estimates change when this new information is introduced. Again, we demonstrate that such differences can be dramatic.

Key Words: Belief Learning, Game Theory, Experimental Economics  
JEL Classification: D83, C91, C73

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\*New York University. The complete paper can be downloaded at <http://www.nyarko.com/papers.htm>

# Tacit Coordination in a Multi-Sender Signaling Game

Ram Orzach\*

## Abstract

The paper will show that in a multi-sender signaling game, a separating equilibrium may exist even when the payoff function of each sender is independent of his type. It is assumed that all of the senders have the same information partition that consists of two elements: a “strong” type and a “weak” type. Strong type senders wish to reveal their type, whereas weak type senders wish to conceal their type. The “device” used to achieve a separating equilibrium is the tacit coordination between the strong type senders. The paper will give sufficient conditions for the existence of such equilibria.

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# A Model of Professional Advice

Marco Ottaviani\* and Peter Sørensen†

## Abstract

The paper models professional experts – like security analysts or economic forecasters – who offer advice or make decisions with the objective of appearing well informed. While the outcome of strategic information transmission by a partisan expert has been extensively studied since Crawford and Sobel 1982), much less is known of cases where the adviser is simply a professional expert. Professional advice is modeled as a sender-receiver game à la Scharfstein and Stein (1990) with a natural specification with continuous state, signal, ability type spaces. The (labor) market evaluates how informed the expert is, based on the message sent (or action taken) and the realization of the state of the world.

While Laster, Bennett and Geoum (1999) posit that experts are motivated by the accuracy of their forecast, we give a full microeconomic foundation to the objective of professional advisers. Contrary to what one might naively believe, forecasters who wish to be perceived accurate will not truthfully reveal their private information. In particular, the desire to be on the mark gives the expert an incentive to bias the forecast towards the prior belief. Truth telling cannot be an equilibrium. Experts who are motivated by their reputation as good forecasters turns out to provide recommendations (or make decisions) which incorporate only part of the information they possess. Exaggerated and non-informative statements are made and appropriately discounted by the audience. The main finding in our model is that even if the message space is as rich as the signal space, in the most informative equilibrium only two messages are sent at most. At best, the sender can transmit only the direction but not the intensity of the information possessed.

Furthermore, an expert who does not know her ability type cannot reveal any information in equilibrium when the prior on the state of the world is sufficiently skewed relative to the prior reputation of the expert. Developing an insight of Trueman (1994), we also show that there is always an informative equilibrium when the expert knows her own ability type, contrary to what happens with unknown own ability. Finally, we investigate whether relative reputational concerns can improve the problem.

JEL Classification: D82 (Asymmetric and Private Information), D83 (Search, Learning, and Information), G20 (Financial Institutions and Services), J30 (Wages, Compensation, and Labor Costs), D72 (Economic Models of Political Processes).

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# A Generalization of Nash Equilibrium Theorem on Two-Person Non-Zero Sum Stochastic Games With SER-SIT Properties

T.Parthasarathy\* and D. Sampangi Raman†

## Abstract

In this paper we consider a two person non-zero sum stochastic games with SER-SIT properties in which the first player picks a probability distribution  $x$  on  $(1, \dots, m)$  and a row representative matrix  $A$  from a finite set of  $m \times n$  real matrices and the second player picks a probability distribution  $y$  on  $(1, \dots, n)$  and a column representative matrix  $B$  from a finite set of  $m \times n$  real matrices. This leads to the respective costs  $C_1(s) + x^t A y$  and  $C_2(s) + x^t B y$  to the two players. This they do after observing the state of the system. The state moves to a new state according to a given law of motion  $q$  which depends only on  $x, y$ . The game is played over the infinite future. We consider the finite state space stochastic games with SER-SIT properties and prove the existence of Nash equilibrium in the discounted case. Method of proof is based on a result of Gowda and Sznajder on the generalization of bimatrix games.

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# An Ex-Post Efficient Auction

Motty Perry and Philip Reny

## **Abstract**

An analogue of Vickrey's (1961) multi-unit auction is provided when bidders have interdependent values. The analogue is strategically equivalent to a collection of *two-bidder single-unit* second-price auctions and it possesses an ex-post efficient equilibrium. As an application of this result, it is shown that the FCC auction possesses an efficient equilibrium in the case of homogeneous goods. Conditions are provided under which the new auction (and also the FCC auction) revenue-dominates all ex-post equilibria of ex-post efficient individually rational mechanisms.

# Measures of Powerlessness in Simple Games

Thomas Quint\*

## Abstract

Consider a simple game with  $n$  players. Let  $\psi_i$  be the Shapley-Shubik power index for player  $i$ . Then  $1 - \psi_i$  measures his powerlessness. We break down this powerlessness into two components – a “quixote index”  $Q_i$  (which measures how much of a “quixote”  $i$  is), and a “follower index”  $\mathcal{F}_i$  (which measures how much of a “follower” he is). Formulae, properties, and axiomatizations for  $Q$  and  $\mathcal{F}$  are given. Examples are also supplied.

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# Social Comparison in a Three-Person Ultimatum Game: A Strategy Design Experiment

Arno Riedl\* and Jana Vyrastekova†

## Abstract

By today we have an extensive experimental evidence on the fact that players do not act as simple money maximizers. Various explanations are given in the literature for observed behavior deviating from game-theoretical predictions. Such explanations include intrinsic tastes for fairness, equity, or reciprocal behavior. In our experimental study we address the theories of social evaluation that argue that players utility is shaped by social comparison of the monetary outcomes within a group. To the extent that many of the economic decisions we make in our lives lead to observable outcomes, where we are subject to such comparisons, it is important to understand in which way they influence the actual economic decision making.

In order to study the social comparison, we implement a three-person ultimatum game (3PUG). The huge body of literature on two-person ultimatum games refutes the money maximization hypothesis, and suggests that players are willing to forego material payoffs in order to punish unsatisfactory offers. The question remains what offers lead to rejection, in other words what is the price players are willing to pay for the punishment option and if the price does depend on the distribution of income among the players. In a two-person ultimatum game, the implemented sharing rule affects the material payoff outcome and eventual social comparison among the involved players in the same direction: we cannot distinguish situations in which the rejection is due to a low material payoff and in which it is guided by social comparison. Rejection leads in any case to both, punishment of low offers and socially equal standing among the involved players.

The three person ultimatum game we study in our paper allows us to separate these two effects. Similarly to the previous experiments by Okada and Riedl (1999), there is one player, proposer, asked to choose a split of the endowment among himself and the two other players, responders. At the same time, responders state individually and simultaneously whether they want to accept any of the splits the proposer could choose. In this way we collect strategies for both the proposer as well as the responders. Whether and in which way the split chosen by the proposer is implemented depends on the acceptance of this particular split by the responders, as well as on the treatment of the game.

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In order to study the social comparison effects, we designed three treatments of the basic game that differ only in the payoff consequences for the responders in the case of a rejection. As in the two-person ultimatum game, any rejection leads to the same punishment of the proposer and bears the same material cost to the rejecting responder. Rejection simply means nullifying proposers payoff and own payoff. The treatments, however, differ in the social cost of rejection in the sense that they imply different inequality within the responders group (consisting of the two responders). In the T-all treatment, if any responder rejects the offer made by the proposer, all three players earn zero. Consequently, an unsatisfied responder punishes by his rejection the proposer, but nullifies also the payoff to the other responder. In this way, all players earn the same payoff of zero after any rejection. In the T-prown treatment, if a responder rejects an offer he discards only his own payoff and the payoff of the proposer. So, an unsatisfied responder can punish the proposer, but this has no effect on the payoff to the other responder. And finally, in the T-trans treatment, a responder rejecting an offer discards the proposers payoff and transfers the payof offered to him by the proposer to the other responder. In particular, if only one of the responders rejects an offer, the proposer earns zero, the rejecting responder earns zero, whereas the earnings of the not-rejecting responder increase by the amount offered by proposer to the rejecting responder. If both responders reject the offer, they swap the payoffs, and the proposer earns zero. Thus, the rejection of an offer by only one responder punishes the proposer, and improves the material payoff of the not-rejecting responder.

From the game-theoretical viewpoint, assuming money maximizing players, these three treatments of the three-person ultimatum game are strategically equivalent (save for situations in which one or both responders are offered a payoff of zero). If the players care only for monetary payoffs, we should observe the same behavior in all three treatments. This holds even if a money maximizing player is not sure about the utility of his opponent.

In our analysis we focus on the simultaneous move game responders play against each other for any give possible split. To formulate our hypothesis, we assume in accord with reality that responders cannot be sure about the utility of their partners in the game. They can only make a subjective guess on the probability distribution over the strategy set. The formulation of our hypothesis is based only on the assumption that both strategies are given positive weight. Any responder, disregarding the form of his utility function, is then playing a best response against such a completely mixed strategy of his opponent.

An alternative assumption to the money maximizing behavior that can be found in the literature is that the players have a fixed aspiration level, a minimal amount they are willing to accept, or, in other words, the maximal amount they are willing to forego if unsatisfied with the monetary outcome of the game. Using the assumptions that a player with a utility including fixed aspiration level plays his best response against some completely mixed strategy, we can make unambiguous prediction about the aggregate acceptance behavior of a player endowed with such utility function: we predict that he will play the same strategy in the

treatments T-all and T-prown, while his overall acceptance rate will be higher in the T-trans treatment.

Finally, we calculate best response for a player with utility function including social comparison terms – utility respectively disutility from receiving a payoff different from the other responder (as well as proposer). We formalize the social comparison type of player by an inequality aversion utility function formulated by Fehr and Schmidt, which can also adapt the status-seeking behavior. We can again make an unambiguous statement about the aggregate acceptance behavior that leads to the prediction that the social comparison type should on aggregate accept more offers in the T-prown treatment than in T-all treatment.

In this way, we can state the hypothesis on the utility function of players purely in the players aggregate acceptance rate: money maximizers are consistent with constant aggregate acceptance rate, fixed aspiration level types are consistent with acceptance rates that are different between treatments T-prown and T-trans, but not between T-all and T-prown, and acceptance rates of players holding utility function including social comparison terms are consistent with acceptance rates that are different between T-all and T-prown.

To test, if players behave in according with the social comparison hypothesis, or if their behavior is better explained by fixed aspiration level, we run two experiment sessions with 66 participants, undergraduate students in Vienna, Austria, and tested the above hypothesis.

# Yosef Rinott\* and Marco Scarsini†

## Abstract

Any finite normal game admits at least one Nash equilibrium in mixed strategies, but the existence of a Nash equilibrium is not guaranteed if only pure strategies are allowed. It is therefore interesting to ask how many pure Nash equilibria we can expect to have in a finite game chosen at random. Solutions to the above problem have been proposed in some special cases, e.g. when the payoffs are assumed to be all independent, or when the components of each payoff profile are all equal and the profiles are independent. In this paper we retain the hypothesis of independence among the profiles, but we will allow either positive or negative dependence among the players' payoffs given a profile. We provide asymptotic results for the distribution of the number of Nash equilibria when either the number of players or the number of strategies increases. We will show that, when there is negative dependence (e.g., in zero-sum games, possibly with some "noise") the number of Nash equilibria converges in probability to zero; when the payoffs are independent, the expected number of pure Nash equilibria is one and the distribution is asymptotically Poisson; when there is positive dependence the number of pure equilibria diverges and the standardized distribution is asymptotically normal.

AMS 1991 Subject Classification: Primary 60F05, 90D10.

Keywords: negative and positive dependence, Stein's method, central limit theorem, zero-sum games, common-payoff games, multinormal distribution, maxima of exchangeable random variables, orthant probabilities.

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# Citizen Candidacy, Party Formation and Duverger's Law

Anouk Riviere\*

## Abstract

The main empirical feature of elections is that the citizens who become candidates are generally not independent: Politically active citizens are organized in political parties, whose leaders usually become candidates. Therefore, modeling elections should explain the decisions made by citizens to remain independent citizens, to create or to join a political party, to become candidates and eventually to vote for one or another candidate. A formal model including all these decisions has never been proposed, probably because of the complexity of the problem.

The classical approach consists in taking parties as given: Competition between a set of parties has been examined, without distinction between parties or candidates. A major problem is then how many candidates there should be. Duverger's Law, the informal idea that majoritarian systems have a tendency to create two-party systems, incited to consider two-candidate competition, the main result being the median voter theorem by Black (1958). Many researchers examined the robustness of this result (Calvert, Wittman, Romer, Alesina, Plott, McKelvey). . . Another trend of the literature considers three or more candidates.

A more recent approach focuses on the endogenous determination of the set of candidates. The decision to enter is examined for a set of potential candidates (Osborne, Feddersen, Sened and Wright). In citizen candidate models (Osborne and Slivinsky, Besley and Coate), each citizen may become a candidate, but at a certain cost. This kind of modelling derives equilibria in very general set-ups but lacks predictability: Duverger's Law is only one equilibrium among others. Moreover, political parties are left out of the picture

The present paper builds a bridge between these two approaches and proposes a dynamic non cooperative game theoretic model including both endogenous parties and candidates. The primitives of the model are the citizens, their preferences and distribution, the policy alternatives and a single representative first-past-the-post election. I assume that the citizens and candidates are purely ideological and that they implement their preferred policy if elected. For simplicity, I restrict the ideologies to three points on a line. Moreover, before the election, there is uncertainty on the position of the median voter.

I first develop a citizen candidate model with the following results: if the cost of candidacy is very small, each ideology is represented by a candidate in coalition-proof political equilibrium. The number of candidates decreases with an increase of the candidacy cost. Equilibria with two candidates appear to be rare.

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In the second part of the paper, I argue that the cost of candidacy is in general too high to be paid by an isolated individual and I therefore consider the possibility that citizens share the cost of candidacy by the means of political parties. In subgame perfect coalition-proof Nash equilibrium, with a coalition-proofness refinement, there are generally two candidates, one centrist and one extremist. The centrists will strategically form a party whose number of members is large enough to finance its president against one extremist, but not against two. Since extremists prefer not to be represented rather than not to have a centrist candidate (because they are risk-averse), one of the extremist groups will be obliged to stay out of the political scene.

The paper proposes a new explanation for Duverger's Law (usually seen as the result of strategic voting and/or strategic entry): it is the result of the party formation process, more precisely of the strategic choice by the centrist citizens of the number of members in their party. The paper shows that party formation can be modelled and is crucial when trying to understand politics: such a simple thing as the size of a political party is an important strategic variable. This fact should encourage formal theorists to explore political parties as heterogeneous entities.

# The Evolution of Intelligence and the Red Queen

Arthur Robson\*

## Abstract

Suppose strategic complexity is subject to selection in an arms race fashion. Consider a two-person repeated game of incomplete information in which there is one long-run individual who is informed about the state of the world playing against a sequence of short-run opponents who are not. Strategies are noisy and have bounded recall. A suitable equilibrium can be shown to exist. Relative to any such equilibrium, sufficiently greater recall enjoys an advantage which is not decreasing in the original level of recall, thus capturing the Red Queen effect. The selection pressure to reduce a small amount of noise must be less than that to increase recall.

*Journal of Economic Literature*

Classification Numbers: A12, C73, D82.

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# Zero Sum Absorbing Games With Incomplete Information on one Side: Asymptotic Analysis

Dinah Rosenberg\*

## Abstract

We prove the existence of the limit of the values of finitely repeated (resp. discounted) absorbing games with incomplete information on one side, as the number of repetitions goes to infinity (resp. the discount factor goes to zero). The proof is based on the study of the mapping defined by the recursive formula relating the value of the  $n + 1$  stage game to the value of the  $n$  stage game.

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# An operator approach to zero-sum repeated games

Dinah Rosenberg and Sylvain Sorin \*

## Abstract

We consider a two person zero-sum stochastic game. The recursive formula for the values  $v_\lambda$  (resp.  $v_n$ ) of the discounted (resp. finitely repeated) version can be written in terms of a basic operator  $\Phi(\alpha, f)$  where  $\alpha$  is the weight on the present payoff and  $f$  the future payoff. We give sufficient conditions in terms of  $\Phi(\alpha, f)$  and its derivative at 0 for  $\lim v_n$  and  $\lim v_\lambda$  to exist and to be equal.

We apply these results to obtain such convergence properties for absorbing games with compact action spaces and incomplete information games.

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# The Evolution of Preferences and Rapid Social Change

William H. Sandholm\*

## Abstract

We present a dynamic analysis of the evolution of preferences in a strategic environment. In our model, a single population of players repeatedly plays a two-strategy game, the payoffs of which represent evolutionary fitnesses. Each player's utility function combines the common fitness function with his idiosyncratic biases. As the preference distribution is perpetually reshaped by evolutionary forces, behavior adjusts in tandem, as players alter their strategy choices to maintain equilibrium play.

We establish the existence and uniqueness of solution trajectories of this coevolutionary process. Furthermore, we show that the underlying strategic interaction has important qualitative effects on the manner in which society's behavior changes over time. In equilibration games, which possess a unique equilibrium in which both strategies are used, aggregate behavior adjusts continuously in response to changes in the distribution of preferences. In contrast, strategy adjustment in coordination games may be discontinuous: Discrete changes in the overall strategy distribution occur in an instant of evolutionary time. Thus, when society must learn to distribute itself between strategies, the adjustment process is smooth; when society must agree upon a convention, consensus may emerge in leaps and bounds.

We study the evolution of biases, by which we mean idiosyncratic preferences for certain modes of behavior. In the current context, a bias is a predisposition in favor of one of the two strategies. Biases are drawn from a continuous set of possible values.

Holding its biases fixed, the population's behavior is in equilibrium if no player can unilaterally improve his payoffs. Every equilibrium can be characterized by a bias threshold,  $\theta$ , according to which players whose bias towards strategy  $A$  exceeds  $\theta$  choose strategy  $A$ , while the others choose strategy  $B$ . In the equilibrium, a player whose bias is exactly  $\theta$  is indifferent between the two strategies.

While players' decisions are influenced by their bias, their reproductive prospects are not. Thus, differences in fitness may exist even when players' behavior is in equilibrium. (For example, if strategy  $A$  is dominant in the underlying game, strategy  $B$  may nevertheless be played in equilibrium by players who are sufficiently biased towards it.) Suppose that strategy  $A$  yields a higher expected fitness than strategy  $B$ . Then players who choose  $A$  are more fit than the others. Therefore, if behavior is described by the bias threshold  $\theta$ , then biases above  $\theta$  (i.e., those which prompt play of strategy  $A$ ) will become more prevalent at the expense of those below.

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We use a continuous time model to study the joint dynamics of preference evolution and behavior adjustment. To do so, we must specify the relative timing of the two processes. We base our chronology the following observation: While individual agents are able to switch strategies quickly, society's preferences must change slowly. Consequently, we assume that as evolution alters society's biases, players instantly adjust their behavior to maintain equilibrium play. By specifying the relative rates of adjustment of preferences and behavior, we create a model whose dynamic solution is unique.

Apart from our existence and uniqueness results, our main conclusions concern the continuity of the behavior adjustment process. We first study *equilibration games*, which are games in which each strategy's relative payoffs decline in the number of players choosing that strategy, and in which neither strategy is dominant. (Examples of equilibration games include the Hawk-Dove game, commuter selection between routes linking a pair of locations, and consumer choice between goods exhibiting negative consumption externalities.) All such games have a unique equilibrium in which both strategies are used. We show that in all equilibration games, aggregate behavior changes continuously along every coevolutionary solution trajectory. As the distribution of preferences evolves, equilibrium behavior is sustained through a smooth adjustment process.

This result contrasts starkly with our conclusions concerning *coordination games*: games in which each strategy's relative payoffs increase as more players use the strategy, and in which neither strategy is dominant. We show that in coordination games, under many initial preference distributions, coevolution forces aggregate behavior to adjust discontinuously: As the distribution of preferences evolves, a moment is reached at which equilibrium play can only be maintained if a significant fraction of the players simultaneously switch strategies. Thus, when a population benefits from acting in concert, we should expect sudden, seemingly unprovoked shifts in the way it behaves.

We can provide intuition for these results by decomposing the influence of preference evolution on aggregate behavior into two distinct effects. As an example, consider a situation in which strategy  $A$  earns higher fitness than strategy  $B$ . In this case, evolution causes biases above the threshold (i.e., those which prompt strategy  $A$ ) to become more prevalent in the population. The primary effect of this change in preferences is to increase the proportion of players who choose strategy  $A$ .

As preferences change, players adjust their behavior to maintain equilibrium play. If the underlying contest is a equilibration game, then increasing the representation of strategy  $A$  makes this strategy less attractive relative to strategy  $B$ . Thus, the secondary effect of the good performance of strategy  $A$  is that players who had marginally preferred to play  $A$  will begin to play  $B$ . This secondary effect inhibits the growth of strategy  $A$ .

On the other hand, if the players face a coordination game, then increasing the representation of strategy  $A$  makes this strategy more attractive. Hence, the secondary effect of the high fitness of strategy  $A$  is to cause nearly indifferent players to switch from strategy  $B$  to strategy  $A$ . Thus, the secondary effect *reinforces* the growth of strategy  $A$ . The mag-

nitude of reinforcement depends upon the number of indifferent players. Our analysis establishes that if the density of the indifferent players becomes large enough, only a discrete change in aggregate behavior suffices to maintain equilibrium play.

# Calibration With Many Checking Rules

A. Sandroni, R. Smorodinsky, and R. Vohra

## **Abstract**

In this paper, we propose a forecasting scheme which calibrates countably many checking rules and the associated forecast-based checking rules.

# Collusion in a Secret Price Cutting Model

Tadashi Sekiguchi\*

## Abstract

This paper considers an infinitely repeated version of Stigler's (1964) secret price cutting model, where in the stage game oligopolistic firms compete with price but cannot observe the other firms' choice of prices. Instead, each firm can observe the sales of its own product. Since demands for the product of the firm depends both on the price profile and on the state of the market, which we assume is unobservable, the sales only provided imperfect information about the other firms' pricing behavior. One remarkable aspect of this model is that each firm cannot observe the other firms' sales. Thus the sales is an imperfect and private signal about the opponents' actions.

Theoretically, this model is considered as an example of repeated games with private monitoring, a class of repeated games that has not been fully investigated yet. While recent study shows that efficient outcomes can be approximated as an equilibrium in some (discounted) repeated games with private monitoring, those results are limited to a simple framework of repeated prisoners' dilemma. Therefore we do not know whether an efficient outcome or a collusive outcome can be similarly approximated or attained in a general model of secret price cutting, in which we have typically three or more firms with three or more prices to choose (or, possibly, continuum of prices).

The purpose of this paper is to provide a simplest version of secret price cutting model where a nearly collusive outcome is attained as an equilibrium when the firms are sufficiently patient. In this version, we assume duopoly and also assume that each firm has only two prices to choose; High price and Low price. Furthermore, we assume that the sales provide correct information about the prices actually chosen with probability close to 1, and the sales between the firms are correlated to some extent. With those assumptions, we are able to find out an equilibrium where High price profile is played for a sufficiently long period with a probability sufficiently close to 1, which guarantees near efficiency or collusion.

Since the expected stage games payoffs we consider in the model are those of a standard prisoners' dilemma (by interpreting High price as cooperation and Low price as defection), one may wonder if the model is subsumed to one of the existing efficiency results for repeated prisoners' dilemma with private monitoring. However, there is an important difference between the previous model of repeated prisoners' dilemma and our secret price cutting model. In the former model, it is assumed that there are only two possible signals; the one that signals cooperative behavior of the opponent and the one that indicates defection. Put it differently, the signal that is most likely to be observed given an action profile is

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determined entirely by the opponent's action. In contrast, our model has a feature that the signal, here sales, is determined by the state of the market, the opponent's price, and the firm's own price. Thus, when the signals are highly accurate, the sales that is most likely to be seen are also dependent on the firm's own price, too. Therefore we typically must have four or more signals to observe, even if each firm has only two prices to choose. And the previous analysis has not shown that extension to the case with three or more signals is immediate.

The assumptions that the sales provide correct information almost always and that those sales are correlated to some extent need a further explanation, because a more general form of demand fluctuations will not satisfy those assumptions. However, those assumptions are plausible when a demand fluctuation is interpreted as a consequence of an error on the side of consumers. For example, a consumer may not be perfectly rational and may purchase a product that should have not been purchased with a small probability. Then one can conclude that the sales are quite an accurate signal of the price of the other firm. In addition, those errors among consumers can correlate depending on the way each consumer makes a mistake. For example, if wrong purchase is a result of wrong information about quality of the products that is widespread in a particular region, then a firm observing higher sales may as well conclude that the other firm also sells well thanks to the wrong information. Still one should emphasize that the necessary amount of correlation is shown to be small. Indeed, it can be arbitrarily small if we can strengthen the requirement on informativeness of the signals.

Our result has an important implication on whether communication is necessary to sustain collusion in the secret price cutting model. There is a recent literature which emphasizes that communication is a powerful device to facilitate cooperation among players in private monitoring situations. An intuition is that each player's announcement about the past signal serves as a public signal and therefore players can coordinate their future play depending on the announcements. Some results are directly applicable to some secret price cutting model, suggesting the potential role of antitrust law that prohibits communication among competing firms. Therefore one implication of our result is that at least in this example implicit collusion can be sustained even in the presence of antitrust law.

The collusive equilibrium is shown to have the following form. Each firm starts from charging High price and continues to do so if the sales most likely to be observed under the High price profile are actually observed. If the firm observes sales most likely to be observed when the other firm deviates, then the firm switches to Low price. A firm charging Low price in the previous period chooses the same price if the signal in the previous period is the one that is likely to happen under one of the two profiles where the firm plays Low price. Note that this description is incomplete in the sense that behavior at all histories is not specified. Nevertheless, one can show that there exists an equilibrium consistent with the above partial description of play. This is because for any strategy of the other firm consistent with the above "mode of behavior", the firm has a best reply consistent with the mode of behavior. Then existence of an equilibrium

follows by a similar fixed point theorem argument employed in existence of a Nash equilibrium in finite games.

Of course the example provided in the paper is special, so extension to various directions should be explored. While adding more firms and/or more prices to choose is one important direction of future extension, equally important is to extend it to the case with more imperfect monitoring structure. If we are interested in existence of a partially collusive equilibrium, rather than a fully collusive equilibrium, such an extension is possible. We discuss about further possibility of those extensions.

# Allocation of Prizes in All-Pay Contests

B. Moldovanu and Aner Sela\*

## Abstract

In a first-price all-pay auction with one prize every contestant submits a bid and he pays the price he bid, while the prize is awarded exclusively to the contestant with the highest bid. In a first-price all-pay auction of multiple prizes, every contestant submits a bid, he pays his bid and he can only win one prize, where the prizes are not necessarily identical. The prizes are awarded according to the rank of the contestants' bids, that is, the highest prize is awarded to the contestant with the highest bid, the second highest prize is awarded to the contestant with the second highest bid and so on until all the prizes are allocated. In economics, this auction has several applications such as rent seeking and lobbying, technological competition, political contests and job promotion.

In this paper we study all-pay contests of multiple prizes under incomplete information. We assume that the administrator's aim is to maximize the total contestants' bids. For example, consider a firm which is looking to promote  $m$  workers to new jobs. These promotions form the prizes in an all-pay contest, where the one who exerts the highest effort wins the best job, the one who exerts the second highest effort wins the second best job and so on. The aim of the manager is to maximize the expected effort exerted by the workers. Our goal in this paper is to investigate the optimal number of prizes in all-pay contests, namely, how many prizes the administrator should award in order to maximize his revenue. In particular, we study whether it is better to award only one prize or alternatively several prizes in such contests. We indicate the conditions under which it is better to award only one prize and the conditions under which it is better to award more than one prize, given that the number of contestants is large enough. We also study the best way of allocating several prizes. We compare two different ways of allocation: a contest in which all prizes are awarded simultaneously as detailed above, and alternatively, a contest in which the prizes are available sequentially such that the winner in each stage is eliminated from the subsequent contests. We show that if the number of participants is large enough then the sequential all-pay contest will be weakly dominated by the simultaneous all-pay contest.

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# On the Impossibility of Implementation under Incomplete Information\*

Roberto Serrano and Rajiv Vohra<sup>†</sup>

## Abstract

We demonstrate that the condition of Bayesian monotonicity, necessary for implementation in Bayesian equilibrium, is extremely restrictive. Over certain natural classes of economies, only constant social choice functions satisfy it. All our negative results also extend to virtual Bayesian implementation. This is in sharp contrast with the complete information case, where the virtual approach yields extremely permissive implementation results.

JEL Classification: C72, D78, D82.

Key Words: Bayesian implementation, virtual Bayesian implementation, Bayesian monotonicity, incomplete information.

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# Network Formation Models With Costs for Establishing Links

Marco Slikker and Anne van den Nouweland

## Abstract

In this paper we study endogenous formation of communication networks in situations where the economic possibilities of groups of players can be described by a cooperative game. We concentrate on the influence that the existence of costs for establishing communication links has on the communication networks that are formed. The starting points of this paper are two game-theoretic models of the formation of communication links that were studied in the literature fairly recently, the extensive-form model by Aumann and Myerson (1988) and the strategic-form model of Dutta, van den Nouweland and Tys (1998). In both of these papers forming communication links is costless and, once a communication network has been formed, an external allocation rule is used to determine the payoffs to the players in different communication networks. The external allocation rule used by Aumann and Myerson (1988) is the Myerson value (cf Myerson (1977)) and Dutta, van den Nouweland and Tys (1998) considered a class of external allocation rules that contains the Myerson value. We follow their analyses as closely as possible and use an extension of the Myerson value to determine the payoffs to the players in communication situations when forming links is not costless.

The goal of this paper is to study the influence that costs of forming communication links have on the structures that are formed. In order to be able to isolate the influence of the costs, we assume that costs are equal for all possible communication links. Starting from costs equal to zero, we increase the costs and see how these increasing costs induce different equilibrium communication structures.

In the extensive-form game of link formation we consider communication structures that are formed in subgame perfect Nash equilibria. We find that for this game, the pattern of structures formed as costs increase depends on whether the underlying coalitional game is superadditive and/or convex. We find that in case the underlying game is not superadditive or in case it is convex, increasing costs for forming communication links result in the formation of fewer links in equilibrium. However, if the underlying game is superadditive but not convex, then increasing costs initially lead to the formation of fewer links, then to the formation of more links, and finally lead to the formation of fewer links again. This is, in our view, the most surprising result of the paper. It shows that subsidizing the formation of links does not necessarily lead to more links being formed. Hence, authorities wishing to promote more cooperation cannot always rely on subsidies to accomplish this goal. In fact, such subsidies might have an adverse effect.

For the strategic-form game of link formation we briefly discuss the inappropriateness of Nash equilibria and strong Nash equilibria and then

consider coalition-proof Nash equilibria. We find that for this game the pattern of structures formed as costs increase also depends on whether the underlying coalitional game is superadditive and/or convex. In contrast to the results for the extensive-form game of link formation, we find that in all cases increasing costs for forming communication links result in the formation of fewer links in equilibrium. We prove the existence of coalition-proof Nash equilibria to show that the analysis using the coalition-proof Nash equilibrium concept is not vacuous.

# On the Bargaining Set, Kernel and Core of Superadditive Games

Tamás Solymosi\*

## Abstract

We prove that for superadditive games a necessary and sufficient condition for the bargaining set to coincide with the core is that the monotonic cover of the excess game induced by a payoff be balanced for each imputation in the bargaining set. We present some new results obtained by verifying this condition for specific classes of games. For N-zero- monotonic games we show that the same condition required at each kernel element is also necessary and sufficient for the kernel to be contained in the core. We also give examples showing that to maintain these characterizations, the respective assumptions on the games cannot be lifted.

Key words: cooperative games, bargaining set, core, kernel

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# Efficient Sealed-Bid Auctions for Non-Identical Objects With Single-Unit Demands\*

Marilda Sotomayor<sup>†</sup>

## Abstract

This paper considers a class of sealed- bid auctions to sell, simultaneously, non-identical objects, where individuals can only buy one object and the outcome is a competitive equilibrium price. We restrict attention to the complete information case. We show that, whatever equilibrium price rule is used by the auctioneer, the bidders can force the minimum competitive equilibrium price by using Nash equilibrium strategies. Thus, if the bidders play in equilibrium, the revenue for the seller is the same under any auction of this class.

Keywords: optimal allocation, competitive price, optimal stable payoff.

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# A Necessary and Sufficient Condition for Approachability

Xavier Spinat\*

## Abstract

Approachability in games with vector payoffs (Blackwell, 1956) has found many applications in game theory, for example in the study of repeated games with lack of information on one side (Aumann-Maschler, 1995) or in problems of calibration and learning processes (Foster, 1997, Hart-Mas Colell, 1997).

In a repeated game with vector payoffs, a set  $F$  is approachable if a player can guarantee that the average payoff will, after some stages, remain forever close to  $F$  with a large probability. Approachability itself is difficult to prove since its definition involves the existence of a strategy that gives a good asymptotic distribution of the payoff. Blackwell solved this difficulty by introducing a very useful sufficient condition for approachability. We will use two features of this condition.

First, Blackwell's condition, using only geometric considerations, is simple to verify and it is a necessary condition in the case of convex sets. We prove that, in an approachable set, some points can be "secondary" for the approachability. We can thus reduce any approachable set to a subset in which there is no such "useless" (in the sense of approachability) points. By considering minimal approachable sets, we show that a set  $F$  is approachable if and only if a subset of  $F$  satisfies Blackwell's condition.

But Blackwell's condition has another advantage: It gives an explicit way to construct approachability strategies with certain properties. Those strategies are robust to discretization or random perturbation. We use the main result of this paper to prove that the same type of approachability strategy can be found in the general case.

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# Games With Infinitesimal Forgetfulness

Francesco Squintani\*

## Abstract

Unlike formal games, most social interactions are not accompanied by a list of written, fixed rules describing all the actions that can be taken. When repeatedly involved in the same interaction, individuals may hold different awareness of available actions at different times. Such awareness typically depends on their past experiences. This paper allows any player to forget with infinitesimal probability some possible opponents' actions, in particular if they are not observed in previous periods. While large-probability forgetfulness is not a convincing assumption in economic settings, infinitesimal forgetfulness cannot be dismissed. This paper shows that perfect-memory solution concepts may be destabilized by infinitesimal forgetfulness.

Our model is as follows. Players from different large populations are repeatedly matched to play a game. In the first period each player is fully aware of all possible actions, but later, with infinitesimal probability, she may forget some of them. At each period, players coordinate on a Perfect Bayes Equilibrium restricted by their possibly partial awareness. In fact, as we want to present destabilization results due to forgetfulness only, we need to maintain assumptions such as full rationality or equilibrium. Equilibrium play given awareness, together with the evolution of awareness resulting from forgetfulness, defines a dynamic system on the players' perception of the game. The results concern the long-run aggregate distributions of play, calculated by compounding the strategies played by the different awareness types, with the distribution of types.

We first allow players to forget only opponents' actions that were not played in the previous period, to capture the intuition that one usually less likely forgets her own possible choices, or actions recently observed. Full-awareness solutions may be destabilized even under these conservative assumptions. In extensive-form games, we prove that Perfect Equilibria may drift to a *non-Nash* (and not even unitary-beliefs self-confirming) equilibrium. Backward induction solution is destabilized in a class of generic perfect-information games, we characterize as games of *nested threats*. Intuitively, that characterization requires a player to make a credible threat which is itself non-credibly threatened. For any game where the Backward Induction solution is not destabilized, there exists a richer game where the same Backward Induction path is destabilized. Finally, we show that, even without initial full awareness, all stationary points of the system are heterogeneous-beliefs self-confirming Equilibria.

In one-shot simultaneous-move games of normal form<sup>1</sup> forgetfulness of (opponents') actions *unobserved at equilibrium*, may destabilize entire

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<sup>1</sup>We show the normal form representation not to be generally appropriate to deal with forgetfulness. Whenever we refer to normal-form games, we mean one-shot simultaneous moves games of normal form.

Nash Equilibrium components. In the spirit of the refinement literature, we characterize the set of Nash Equilibria not destabilized by forgetfulness. The assumption that observed actions may not be forgotten renders all pure-strategy equilibria stable. However, Weak Dominance, Trembling Hand Perfection, or Properness are shown not to be stronger than our refinement. In fact, the latter is invariant to deletion of strongly dominated strategies (see Kohlberg and Mertens 1982). Finally, all stationary points of the system are Nash Equilibria. Thus whenever a Nash Equilibrium is destabilized by forgetfulness, the system will drift into a different Nash Equilibrium.

Next, we allow also opponents' observed actions to be forgotten. The full-awareness Backward Induction solution is now destabilized also in games of perfect information that display at least a non-credible threat. In normal-form games, "bad" pure strategies equilibria are destabilized, and, despite conservative assumptions such as infinitesimal forgetfulness and equilibrium play, our stability concept refines more (i.e. it is *stronger*) than Trembling Hand Perfection. Finally, we allow forgetfulness of own actions, but, in order to capture the intuition that one more likely forgets an action if she does not recently use it, we maintain the hypothesis that off-path actions may not be forgotten. Some non-self-confirming-equilibrium outcomes may now be stable even in normal-form games, and the full-awareness Backward Induction path is destabilized in all games of perfect-complete information with credible threats.

In order to convey the basic intuition for the simplest results, we present an example where forgetfulness is restricted to opponents' actions off path.

**Example 1** *At each period of play, a player from population 1 is matched with a player from population 2 to play the game depicted in Figure 1. Assume that each player 2 may forget action  $C$  with infinitesimal probability whenever it is off path. The only full-awareness Backward Induction solution is  $((M, C), S)$ ; its path is  $M$ , so  $C$  can be forgotten. Each player 2 unaware of  $C$  will take  $W$ , as she believes that if she takes  $S$ , she will receive  $-1$ . The ratio  $k_t$  of players 2 unaware of  $C$  will increase over time as long as  $k_t \leq \frac{1}{2}$ . In fact, each player 1 expects to face, with probability  $k_t$ , a player 2 who takes  $w$ , and with probability  $1 - k_t$ , one that plays  $S$ . For  $k_t < \frac{1}{2}$ , she prefers to play  $M$ . By doing so, she renders  $C$  off-path, allowing  $k_t$  to increase. However, when  $k_t > \frac{1}{2}$ , each player 1 prefers to play  $(T, C)$ . None of the players 2 unaware of  $C$  may be reminded it, as they play  $W$  which keeps  $C$  off path. None of those aware of  $C$  can now forget it, as the path they observe is  $T - SC$ . The unique full-awareness Perfect Bayes Equilibrium is destabilized, even though, at each period, players coordinate on the only PBE conditional on their awareness. Moreover, the only aggregate stable state of the system is  $((T, C), (W/2, S12))$  which is not a full awareness PBE, nor a full awareness Nash Equilibrium, and it is not even an unitary beliefs self-confirming equilibrium (it is an heterogeneous beliefs self-confirming equilibrium).*

In conclusion, this paper introduces a new question in the literature about learning in games. Traditionally, the analysis is concerned with how players learn to play a given game. Instead, by addressing the evo-

lution of the awareness of actions, we explore how players learn, forget, and recall what they and their opponents can do in a social interaction, (i.e. how they learn the rules of the game). When learning how to deal with a complex economic interaction, the most difficult task is to find out all the relevant possibilities. Once the modeling step is accomplished, to coordinate on a model's equilibrium is a relatively easier step. In order to highlight forgetfulness, this paper focuses on the long-run behaviour reached from initial full awareness. One may analyze our model considering any initial state of the system, and derive a complete picture of how players learn the rules of the game.

# Rental Harmony: Sperner's Lemma in Fair Division

Francis Edward Su\*

## Abstract

We present a constructive method for solving the rent-partitioning problem: given  $N$  housemates and an  $N$  bedroom house, is it always possible to price the rooms so that each person prefers a different room? More generally, we address the allocation of indivisible goods mediated by payments of a divisible medium, such as money.

We show that an envy-free solution exists if the following three conditions hold:

1. (Good House) In any partition of the rent, each person finds some room acceptable.
2. (Miserly Tenants) Each person never chooses the most expensive room if there is a free room available. (This does not require the person to choose the free room.)
3. (Closed Preference Sets) A person who prefers a room for a convergent sequence of prices prefers that room at the limiting price.

In fact, we present a constructive method for approximating an envy-free solution as closely as desired. Moreover, Condition 2 can be dropped if by allowing solutions with negative rents.

Our solution is based on a combinatorial theorem known as Sperner's lemma. It says that in any triangulated  $N$ -dimensional simplex with vertices labelled in a special way by  $\{1, \dots, N\}$ , there exists an elementary simplex carrying all labels. The special labelling can be obtained by numbering every facet, and then requiring vertices on facet  $j$  not be labelled  $j$ .

We show how Sperner's lemma can be used to solve (in an envy-free manner) a large class of fair division problems such as cake-cutting, chore-division, and the rent-partitioning problem. Moreover, a constructive proof of Sperner yields constructive fair division algorithms for obtaining approximate envy-free solutions, up to a pre-specified tolerance. These results are contained in the forthcoming paper Su (1999), to appear in the *American Mathematical Monthly*.

Similar combinatorial results can be used to prove additional fair division theorems. For instance, in joint work with Forest Simmons (Simmons and Su, preprint), we develop a "consensus-halving" algorithm. This is a method which can be used to split an object into two pieces in such a way that all  $N$  players believe the division is a 50-50 split.

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In fact, with Forest Simmons, we have obtained a further generalization of this result which yields a very strong kind of fair division: one that can be used to divide a cake into  $M$  pieces so that each of  $N$  players believe the pieces to be of EQUAL size, to within a pre-specified tolerance. Our approach works whether the cake is desirable, undesirable, a mixture of both, or even if players disagree on what portions of the object are desirable or undesirable. While the algorithm yields an approximate solution, an exact solution can be approached arbitrarily closely by running the algorithm longer. This improvement of equal division over envy-free divisions comes at a small cost— both the number of cuts required, as well as the number of steps in the algorithm are larger, but not significantly so.

All of our fair division algorithms are iterative and work by polling various players for their preferences at proposed cuts. There is no bidding involved, and players do not have to reveal their relative valuations to each other. Being discrete in nature, our algorithms be easily implemented on a computer since the number of steps is bounded. These approximate envy-free solutions are advantageous over exact methods because exact envy-free methods tend to be far more complicated, and do not generalize easily for large  $N$ .

# Multi-Agent Bilateral Bargaining

Sang-Chul Suh and Quan Wen\*

## Abstract

Consider a multi-agent bargaining model where the bargaining is conducted by a sequence of bilateral bargaining sessions. In each bilateral bargaining session, two players may reach a partial agreement that specifies a transfer between the two players as in Rubinstein's (1982) bilateral bargaining framework. After a partial agreement, one player will effectively exit the game and the other player will carry this partial agreement to the next bilateral bargaining session. Depending on which of the two players moves to the next bilateral bargaining session after the partial agreement, we consider two special cases and show that each of them has a unique subgame perfect equilibrium. The unique perfect equilibrium outcomes in both cases converge to the Nash (1950) bargaining solution to the corresponding bargaining problem as the players' common discount factor goes to one.

Keywords: Multilateral bargaining, subgame perfect equilibrium, Nash bargaining solution

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# Evolutionary Selection with Different Modes of Learning in Supermodular Games

Alexander F. Tieman\* and Oddvar M. Kaarboe†

## Abstract

We apply the dynamic stochastic framework proposed by recent evolutionary literature to the class of strict supermodular games when two simple behavior rules coexist in the population, imitation and myopic optimization. Furthermore we pose that myopic optimizers who are able to see how well their payoff does relative to what they can get in the stage game, experiment more in low payoff states. By using this framework a clear-cut equilibrium selection result is obtained: the payoff dominant equilibrium is selected as the unique long run equilibrium. Furthermore, the expected waiting time until the payoff dominant equilibrium is reached is relatively short, even in the limit as the population size grows large.

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# Calibration and Bayesian Learning

Nurlan Turdaliev\*

## Abstract

In a repeated game of incomplete information, myopic players form beliefs on next-period play and choose strategies to maximize next-period payoffs. Beliefs are treated as forecast of future plays. Forecast accuracy is assessed using calibration tests, which measure asymptotic accuracy of beliefs against some realizations. Beliefs are calibrated if they pass all calibration tests. For a positive Lebesgue measure of payoff vectors, beliefs are not calibrated. But, if payoff vector and calibration test are drawn from a suitable product measure, beliefs pass the calibration test almost surely. *Journal of Economic Literature*

Classification Numbers: C10, C70, C72.

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# Utility Equivalence in Sealed Bid Auctions and the Dual Theory of Choice Under Risk

Oscar Volij\*

## Abstract

This paper analyzes symmetric, single item auctions in the private values framework, with buyers whose preferences satisfy the axioms of Yaari's (1987) dual theory of choice under risk. It is shown that when their valuations are independently and identically distributed, buyers are indifferent among all the auctions contained in a big family of mechanisms that includes the standard auctions. It is also shown that in the linear equilibria of the sealed bid double auction, as the degree of players' risk aversion grows arbitrarily large, the ex post inefficiency of the mechanism tends to vanish.

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# Extremism, Campaigning and Ambiguity

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## Abstract

In political science, an important issue is information transmission between political representatives and the electorate. This issue has several aspects. One is the question whether parties send truthful information to voters, or not. This issue has been studied by Banks (1990) and others. Banks finds that, if the realized platform of a party is far away from the median of the voter distribution, voters are able to infer the true platform of that party. If the platform is close to the median of the voter distribution, this is not the case. Martinelli (1997) has studied whether voters can learn from parties that have private information during the electoral process. Schultz (1996) studies a situation where parties possess more information about the true state of the world compared with voters. He finds that polarization leads to non-revealing sequential equilibria. However, in none of these papers is it costly to send messages to voters. An important aspect in campaigning is that it is costly to send information to voters. This motivates the study of a model with costly information transmission.

The main aim of this paper is to analyze how parties use resources for campaigning to inform voters. We describe how the resources spent on campaigning depend on how close parties are to each other and how this in turn affects voters.

We study a general model and are able to show existence of equilibrium. In the model there is an exogenously given number of parties. Each party has a predetermined ideology that is drawn from some distribution. This ideology could be determined by the history of the group, for example. The parties care about the number of votes as well as the consumption of some private good. Each party has access to resources that can be used for campaigning. Initially, voters do not know the platform of the parties. To affect the voters the parties use campaigning to inform the voters about the policy of the party. The parties are assumed to use only truthful messages. If a voter is informed by some party, it is assumed that he knows the platform of the party with certainty. This, combined with risk aversion, makes informed voters on average more positive to the party. Given the platforms and the strategies of the parties, voters update their beliefs and then vote sincerely for the parties.

Then we study a symmetric model with two parties and find that, the farther away parties are from each other (on average), the less information is supplied (on average) in equilibrium. Note that informing a voter eliminates the risk of voting for that party. Then, consider voters that are informed by one party only, say party 1, and assume that the platform of party 1 is close to the median voter. For the indifferent voter, the expected platform of party 2 is closer to the voters' peak than the actual

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platform of party 1. Since voters are risk-averse, the voter would otherwise strictly prefer to vote for party 1. If both parties move closer to each other, then the actual and expected platform moves closer to the indifferent voters peak. Since voter preferences are concave, the closer to the peak a platform is, the flatter preferences are. This implies that the increase in payoff of voting for the party that informed is bigger than the increase in payoff of voting for the other party. Thus, the previously indifferent voter now strictly prefers party 1. The effect makes parties gain more votes by informing when parties are moderate. Since spending increases, voters are (on average) more informed when parties are moderates. Thus, extremism leaves more voters uninformed. Also, the uninformed voters are going to be more uncertain if the parties are further away from each other. The reason is that the variability of spending goes down.

One of the influences of this paper is Harrington and Hess (1996). In Harrington and Hess campaigning is explicitly modeled. Parties are assumed to have a fixed ideology. Parties can use resources either to move their platform closer to the opponent (positive campaigning) or to move their opponents platform further away from the party's own platform (negative campaigning). However, there is no explicit model of why expenditures can affect voter's perceptions of the parties. Thus, the model of influencing voters is modeled as a black box.

The paper by Chappell (1994) has a more sophisticated model of voter behavior. In the model campaigning is assumed to be truthful. There are two parties that can choose either to spend an endowment on campaigning or not. Thus, only two possible levels of campaigning are allowed. Existence of equilibrium cannot be proven even in this simple setup. In contrast, in the model presented here, equilibria generally exist.

# Dual Bargaining, Cost Sharing, and the Talmud Bankruptcy Problem

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## Abstract

In dual bargaining agents try to minimize their losses. If no agreement is reached, agents incur the highest loss at d-point, which bounds the feasible set from above. We first extend bargaining solutions and axioms to dual bargaining, we then solve cost sharing and other linear bargaining problems. Because the set of dual bargaining is larger than the set of utility bargaining, our dual approach brings a new dimension to cost sharing problems. For example, it provides a new interpretation for the Talmud bankruptcy problem: The ancient solutions are either equal (in large and small asset cases) or close (in middle asset cast) to Kalai-Smorodinsky solution of the corresponding dual bargaining problem.

Keywords: Bargaining, core selection, dual bargaining, cost sharing, bankruptcy problem.

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# Vector Decompositions of Additively Separable Preferences

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## Abstract

In a *simple coalition formation game*, the players in a finite set  $N$  have preferences over the coalitions of  $N$  they might join, and form coalitions that constitute a partition  $\varphi$  of  $N$ . Preferences are *additively separable* (AS) if each player can independently assign numerical weights (possibly negative) to the others in such a way that she prefers joining  $A$  to joining  $B$  iff  $A$  has greater total weight, and are *symmetrically additively separable* (SAS) if each player  $p$  can additionally assign the same weight to each  $q$  as  $q$  does to  $p$ .

Bogomolnaia and Jackson (1998) showed from SAS that at least one partition  $\varphi$  must be *Nash stable*: No individual  $p$  in  $N$  will defect (because no  $p$  will strictly prefer joining some other piece of  $\varphi$  to staying where she is). Banerjee, Konishi, and Snmetz (1998) provide a 6-player example satisfying SAS such that for each partition  $\varphi$  there exists some coalition  $D \subset N$  that would defect (because each  $d$  in  $D$  strictly prefers  $D$  to the piece of  $\varphi$  that  $d$  is in); no  $\varphi$  is *core stable*.

We provide a decomposition of the vector space of all weight assignments corresponding to SAS; when one of the components is zero, simultaneously Nash and core stable partitions must exist. Arguments extracted from the analysis provide relaxed conditions, using weaker separability conditions, that also guarantee stability.