Stable Profit Sharing in Patent Licensing: General Bargaining Outcomes[*]

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September, 2005

Abstract

We investigate coalition structures formed by an external licensor of a patented innovation and firms operating in oligopolistic markets, and study licensing agreements reached as the bargaining outcomes under those coalition structures. Every core with coalition structure is empty. If the number of licensees that maximizes their total surplus is greater than the number of non-licensees, each symmetric bargaining set with coalition structure is a singleton and the optimal number of licensees that maximizes the licensor's revenue is uniquely determined. The bargaining set coincides with the core, if the core is nonempty.

JEL Classification Numbers : D45; D43; C71

Keywords : licensing; oligopoly; stable coalition structure; core bargaining set

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[*]The authors wish to thank Yukihiko Funaki, Atsushi Kajii, Mamoru Kaneko, Mikio Nakayama and participants in the 10th Decentralization Conference and the 3rd International Conference on Mathematical Analysis in Economic Theory for helpful comments and suggestions. They are partially supported by the Ministry of Education, Culture, Sports, Science and Technology (MEXT), Grant-in-Aid for 21 Century COE Program.

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1 Introduction

Patent licensing policies in oligopolistic markets have been studied only as non-cooperative mechanisms; upfront fee or royalty in Kamien and Tauman (1984, 1986), and auction in Katz and Shapiro (1985, 1986). After these seminal papers, the main concern of researchers was focused on the optimal licensing mechanism that maximizes the licensor's revenue from a patented innovation. For instance, Kamien, Oren and Tauman (1992) showed that in the Cournot competition for a homogeneous good it is never optimal for an external licensor to license a cost-reducing innovation by means of royalty only. Muto (1993) found that in the Bertrand duopoly with differentiated commodities there is some cases where it is optimal for an external licensor to license by means of the royalty only.

On the other hand, licensing agreements are basically contract terms signed by licensors and licensees as negotiation results (Macho-Stadler et al. (1996)). A role of patent system might be to facilitate the resolution of disputes in the negotiations by pre-determining simplified procedures. This paper hence seeks into the original viewpoint and studies patent licensing as bargaining outcomes.

As the first attempt of this agenda, Tauman and Watanabe (2005) showed that in the Cournot market, when the number of firms is large, the Shapley value of the patent holder in a cooperative approach approximates his payoff in the non-cooperative auction game traditionally studied.[1]

However, they focused on a large industry to obtain the above asymptotic equivalence. For any finite industries, systematic results should be derived. We hence study (1) coalition structures formed by an external licensor of a patented innovation and firms operating in oligopolistic markets and (2) licensing agreements reached as the bargaining outcomes under those coalition structures. For this purpose we use cooperative solutions with coalition structure (Thrall and Lucas (1963), Aumann and Maschler (1964), and Aumann and Drèze (1974)).

[1] It is remarkable that the two approaches asymptotically coincide, since the patent holder does not have full bargaining power in the cooperative approach and the Shapley value measures the fair contribution of the patent holder to the total industry profit.

A key is how to define the worth of a coalition of players. Watanabe and Tauman (2003) proposed a definition that reflects a sophisticated nature of events under a subtle mixture of conflict and cooperation: licensees can form a cartel S to enhance their oligopolistic power, whereas non-licensees may react also by forming some cartels.[2] Then, the licensees in S might not merge into a single entity, but gather as smaller subcartels in S forming the headquarter-subsidiaries relationship.

In this paper, we assume that any forms of cooperation among firms are prohibited (by law). Under some conditions, however, such a group formation appears also in the Watanabe and Tauman's postulate: firms in any groups will decide not to coordinate their strategies, even if they are allowed to cooperate in the market. Hence, the coalition in this paper means that a group of a licensor and potential licensee firms in negotiation, and so the coalition structure provides an implication on how many potential licensees the licensor should negotiate with.

Our main results are as follows. (1) Every core with coalition structure is empty. (2) If the number of licensees that maximizes their total surplus is greater than the number of non-licensees, each symmetric bargaining set with coalition structure is a singleton and the optimal number of licensees that maximizes the licensor's revenue is uniquely determined. (3) The bargaining set coincides with the core, if the core is nonempty.

In the literature, linear demand and cost functions, Cournot or Bertrand oligopoly, cost-reducing or quality-improving innovation, perfect or imperfect patent protection are assumed. We analyze a much less specified model. To that generalized game, kernel or nucleolus might be applied as solutions, but they give "ethical" standards to the bargaining outcomes. To investigate "stable" profit sharing, we selected core and bargaining set as solutions.

The outline of this paper is as follows. For better understanding our generalization, section 2 gives a linear example in the literature. Section 3 formalizes our licensing game and defines the solutions. Section 4 and 5 provide the results and their proofs. The optimal number of licensees is discussed in section 5. Remarks on related papers are stated in section 6.

[2] Tauman and Watanabe (2005) simplified this definition.

2 A Linear Example

There are n firms, $2 \le n < \infty$, each producing an identical commodity. The production cost per unit of output is $c(> 0)$. Let q_i be the output level produced by firm i. The market of the commodity is cleared at the price $p = \max(a - \sum_{i \in N} q_i, 0)$, where $a \in (c, \infty)$ is a constant.

An agent has the patent of an innovation which reduces the unit cost of production from c to $c - \epsilon(> 0)$. The profit of firm i is $pq_i - (c - \epsilon)q_i$ if i has an access to the patented innovation (licensee), while it is $pq_i - cq_i$ if i has no access to that innovation (non-licensee). The agent takes no action in the market but shares the profits of licensees (external licensor).

Suppose that s firms are licensed at the rates of upfront fee F and royalty $r(\le \epsilon)$ per unit of output. The upfront fee is paid to the licensor when the license is purchased. Firms compete *à la* Cournot (i.e., in quantities) in the market, knowing which firms are licensed or not. Finally, the royalty is paid. Let $W(s, \delta)$ and $L(s, \delta)$ denote the equilibrium gross profit (including upfront fee F) of each licensee and that of each non-licensee respectively, where $\delta = \epsilon - r$. We write $L(0, \delta)$ as $L(0)$. Let $\hat{s} := (a - c)/\delta$. Then

$$W(s, \delta) = \begin{cases} ((a - c + (n - s + 1)\delta)/(n + 1))^2 & \text{if } s < \hat{s} \\ ((a - c + \delta)/(\hat{s} + 1))^2 & \text{if } s \ge \hat{s} \end{cases}$$

$$L(s, \delta) = \begin{cases} ((a - c - s\delta)/(n + 1))^2 & \text{if } s < \hat{s} \\ 0 & \text{if } s \ge \hat{s}, \end{cases}$$

which are summarized in the following order:

$$\begin{aligned} W(1, \delta) > \cdots > W(s, \delta) > \cdots > W(n, \delta) > L(0) > \cdots > L(s, \delta) \\ > \cdots > L(\hat{s} - 1, \delta) \ge L(\hat{s}, \delta) = \cdots = L(n - 1, \delta) = 0. \end{aligned}$$

3 A Licensing Game with Coalition Structure

We analyze the following much less specified model that includes a linear case described in section 2. Let $N = \{1, \ldots, n\}$ be the set of identical firms (dealing with a homogeneous good or differentiated goods). An external licensor, player 0, has a patent of an innovation (cost-reducing or quality-improving one). Any non-empty subset of $\{0\} \cup N$ (the set of players) is called a coalition.

Suppose that the licensor chooses a subset $S \subset N$ of firms and form a coalition $\{0\} \cup S$. No firm outside $\{0\} \cup S$ is licensed. The game has two stages. (1) Given a set S of licensee firms, the players in $\{0\} \cup S$ negotiate over how much each firm should pay to the licensor. The payment is made before the next competition stage. (2) Firms compete in the market, knowing which firms are licensed or not. (The market can be of Cournot or Bertrand competition.) No cooperation among firms is allowed in production and in the market, but sidepayments between players are allowed.

Given a subset $S \subseteq N$ of licensee firms, the permissible coalition structure is $P^S = [\{0\} \cup S, \{\{j\}\}_{j \in N \setminus S}]$, since no firm outside $\{0\} \cup S$ is licensed and firms are not allowed to cooperate in the market.

Let $s = |S|$ for any $S \subseteq N$. Given that s firms hold the license, $W(s)$ and $L(s)$ denote the equilibrium gross profits (including payments) of a licensee and a non-licensee, respectively. Since sidepayments are allowed (and there is no uncertainty introduced in this game), there is no substantial difference of payments between before and after market competition, and so royalty payments are included in our analysis. We require

$$W(s) > L(0) \ {}^{\forall}s = 1, \ldots, n, \quad L(0) > L(s) \ {}^{\forall}s = 1, \ldots, n-1, \tag{1}$$

which covers many patterns of piracy, resale and spillover of the patented innovation to non-licensees, since the order of equilibrium gross profits are not concerned except (1).[3] Let s^* be the number of licensees that maximizes their total surplus, i.e., $s^*(W(s^*) - L(0)) \geq s(W(s) - L(0))$ for any $s = 1, \ldots, n$. For any $S \subseteq N$, let $\min_{t=|T|,\, T \subseteq N \setminus S} L(t) = L(t(s))$.

The v-function $v : 2^{\{0\} \cup N} \to \mathbb{R}$ is defined by

$$v(\{0\}) = v(\emptyset) = 0 \quad v(\{0\} \cup S) = sW(s)$$
$$v(S) = sL(t(s)), \ 0 \leq t(s) \leq n - s,$$

where $v(T)$ represents the worth of a coalition $T \subseteq \{0\} \cup N$. The licensor, player 0, can gain nothing without selling the innovation. $v(\{0\} \cup S) = sW(s)$ is the total equilibrium gross profits of licensees in S. $v(S) = sL(t(s))$ is the total equilibrium gross profits that the firms in S can guarantee for themselves even in the worst anticipation that other $t(s) \leq n - s$ firms are licensed when firms in S jointly break off the negotiations.

[3]See Muto (1987) for resale-proofness.

The Solution Concepts for Stable Profit Sharing

The set of imputations under coalition structure P^S is defined as

$$X^S = \{x = (x_0, x_1, \cdots, x_n) \in \mathbb{R}^{n+1} | x_0 + \textstyle\sum_{i \in S} x_i = sW(s),$$
$$x_0 \geq 0,\ x_i \geq L(t(1))\ {}^\forall i \in S, \quad x_j = L(s)\ {}^\forall j \in N \setminus S\}.$$

The core with coalition structure P^S is defined as

$$C^S = \{x \in X^S | \sum_{i \in T} x_i \geq v(T)\ {}^\forall T \subseteq \{0\} \cup N,\ T \cap (\{0\} \cup S) \neq \emptyset\}.$$

We simply call C^N the core.

Let $i, j \in \{0\} \cup S$ and $x \in X^S$. We say that i has an objection (y, T) against j in x if $i \in T$, $j \notin T$, $T \subseteq \{0\} \cup N$, $y_k > x_k\ {}^\forall k \in T$, and $\sum_{k \in T} y_k \leq v(T)$, and that j has a counter objection (z, R) to i's objection (y, T) if $j \in R$, $i \notin R$, $R \subseteq \{0\} \cup N$, $\quad z_k \geq x_k\ {}^\forall k \in R$, $z_k \geq y_k\ {}^\forall k \in R \cap T$, and $\sum_{k \in R} z_k \leq v(R)$. We say that i has a *valid* objection (y, T) against j in x if (y, T) is not countered. The bargaining set with coalition structure P^S is defined as

$$M^S = \{x \in X^S | \text{no player in } \{0\} \cup S \text{ has a valid objection in } x\}.$$

By the definition, $C^S \subset M^S$ under any coalition structure P^S, if $C^S \neq \emptyset$. We simply call M^N the bargaining set.

remark: Negotiation is made within a coalition $\{0\} \cup S$, but the definitions of our solution concepts do never mean that the other players outside $\{0\} \cup S$ are not involved in the negotiation. Any firm i can object against the licensor saying "Lower my payment, otherwise I will preach a boycott of your license to all the firms", whereas the licensor can counter-object against i saying "If you say such a thing, I will license to all the other firms except you at a lower rate".

Let $i, j \in N$. We say that i and j are *substitutes* in v if

$$v(S \cup \{i\}) = v(S \cup \{j\})\ {}^\forall S \subset N \setminus \{i, j\}.$$

Since all the firms in S are substitutes in v, the following symmetric sets facilitate our analysis: $\tilde{X}^S = \{x \in X^S | x_i = x_j\ {}^\forall i, j \in S\}$,

$$\tilde{C}^S = C^S \cap \tilde{X}^S, \quad \tilde{M}^S = M^S \cap \tilde{X}^S.$$

4 The Cores with Coalition Structures

In this section, we give a necessary and sufficient condition for C^S being non-empty.

Lemma 1 *If $C^S \neq \emptyset$, then there exists an $x \in \tilde{C}^S$.*

Proof : Let $y \in C^S \neq \emptyset$. Define $x \in \tilde{X}^S$ by $x_j = y_j$ if $j \notin S$ and $x_i = \bar{x} = (1/s)\sum_{i \in S} y_i$ if $i \in S$. For any $z \in X^S$, we write $\sum_{i \in S' \subseteq \{0\} \cup N} z_i = z(S')$. Fix a coalition $T \subseteq \{0\} \cup N$ such that $T \cap S \neq \emptyset$. Let $l = |T \cap S|$. Then $\min_{U \subset S,\ |U|=l} y(U) \leq (l/s) y(S) = x(T \cap S)$. Hence,

$$\begin{aligned} x(T) = \ & x(T \setminus S) + x(T \cap S) \ \geq \ y(T \setminus S) + \min_{U \subset S,\ |U|=l} y(U) \\ \geq \ & \min_{U \subset S,\ |U|=l} y((T \setminus S) \cup U) \ \geq \ \min_{U \subset S,\ |U|=l} v((T \setminus S) \cup U) = v(T), \end{aligned}$$

since $y(S') \geq v(S')$ by $y \in C^S$ and $v((T \setminus S) \cup U) = v(T)$ $^{\forall} U \subset S$ with $|U| = l$ by the fact that all firms in S are substitutes in v. *Q.E.D.*

Proposition 1 *$C^S = \emptyset$ if $S \neq N$.*

Proof : We first show that $\tilde{C}^S = \emptyset$ if $S \neq N$. Suppose $\tilde{C}^S \neq \emptyset$. Take $x \in \tilde{C}^S$ with $x_i = \bar{x}$ $^{\forall} i \in S$. If $\bar{x} \leq L(0)$, $\sum_{i \in N} x_i = s\bar{x} + (n-s)L(s) < nL(0) = v(N)$ since $L(0) > L(s) = x_j$ $^{\forall} j \in N \setminus S$. Hence, $\bar{x} > L(0)$. Next, take a coalition $\{0\} \cup T$ such that $|T| = |S|$, $T \subseteq N \setminus S$ if $|S| \leq n/2$ and $T \supseteq N \setminus S$ if $|S| > n/2$. Let $t = |T|$. Then, $x_0 + \sum_{i \in T} x_i < sW(s) = tW(t)$, since $x_0 + s\bar{x} = sW(s)$ and $\bar{x} > L(0) > L(s)$. This contradicts $x \in \tilde{C}^S$. Finally, $\tilde{C}^S = \emptyset$ implies $C^S = \emptyset$ by Lemma 1. *Q.E.D.*

Proposition 2 *$C^N \neq \emptyset$ if and only if $s^* = n$.*

Proof : (only if) Suppose $s^* < n$. If $C^N \neq \emptyset$, there is an $x \in \tilde{C}^N \neq \emptyset$ by Lemma 1. Let $x_i = \bar{x}$ $^{\forall} i \in N$ and $x_0 = nW(n) - n\bar{x}$. Then,

$$\bar{x} \geq L(0) \quad \text{and} \quad x_0 + s\bar{x} \geq sW(s),\ s = 0, 1, \cdots, n-1. \tag{2}$$

Letting $s = s^*$ in the latter condition of (2) gives $nW(n) - n\bar{x} + s^*\bar{x} \geq s^*W(s^*)$ or $(n - s^*)\bar{x} \leq nW(n) - s^*W(s^*)$. By the former condition of (2), $(n - s^*)L(0) \leq nW(n) - s^*W(s^*)$ or $s^*(W(s^*) - L(0)) \leq n(W(n) - L(0))$, contradicting that $n > s^* = \arg\max_{s=1,\ldots,n} s(W(s) - L(0))$.

(if) Take x such that

$$x_i = \begin{cases} n(W(n) - L(0)) & \text{if } i = 0 \\ L(0) & \text{if } i \in N. \end{cases}$$

Since $s^* = n$, it is easily shown that $x \in \tilde{C}^N$. *Q.E.D.*

remark: Watanabe and Tauman (2003) showed that the core is empty as the number of firms tends to infinity, although their v-function is slightly different from ours. It is confirmed $s^* < n$ in the linear example described in section 2, and so $C^N = \emptyset$ with any finite set of firms. Even in a linear environment, however, Watanabe and Muto (2005) showed the possibility that $C^N \neq \emptyset$ for the Bertrand duopoly with differentiated goods.

5 The Bargaining Sets with Coalition Structures

By Proposition 1, $C^S = \emptyset$ if $S \neq N$. We next consider M^S, since it is well known that $M^S \neq \emptyset$ for any S. Let us confine attention to $\tilde{M}^S$ for simplicity. Then, it suffices to examine objections and counter objections of the licensor (player 0) and a licensee $i \in S$. Propositions 3 and 4 characterize M^S for any $S \subseteq N$. The former is for $S \subset N$ and the latter is for $S = N$. Proposition 5 is our main result (2) stated in section 1. Proposition 6 ensures $M^{S^*} \neq \emptyset$, where S^* is the set $S \subseteq N$ with $|S| = s^*$.

Proposition 3 *Let $x \in \tilde{M}^S$. Then, we have the following.*
(a) *If $1 \leq s \leq n/2$, and $s(W(s) - W(t)) \leq (n-s)(W(n-s) - L(0))$ and let $W(t^*)$ be the minimum of $W(t)$ that satisfies this inequality, then*

$$s(W(s) - W(t^*)) \leq x_0 \leq s^*(W(s^*) - L(0)).$$

(b) *If $n/2 \leq s < n$ or $s(W(s) - L(0)) \leq (n-s)(W(n-s) - L(0))$, then*

$$s(W(s) - L(0)) \leq x_0 \leq s^*(W(s^*) - L(0)).$$

Proof : Let $x \in \tilde{M}^S$ and $x_i = \bar{x}\ \forall i \in S$. The proposition is proved by the following lemmas. Lemmas 2, 3, 4 suggests the upper (lower) bound of $\bar{x}$ (x_0). Lemma 5 suggests the upper bound of x_0.

Lemma 2 *Suppose* $n/2 \leq s < n$. *If* $x \in \tilde{M}^S$, *then* $\bar{x} \leq L(0)$.

Proof : Suppose $\bar{x} > L(0)$. Take an objection $(y, \{0\} \cup T)$ of the licensor against firm $i \in S$ in x such that $|T| = |S|$, $T \supseteq N \setminus S$ and

$$y_k = \begin{cases} x_0 + \epsilon & \text{if } k = 0 \\ \bar{x} + \epsilon & \text{if } k \in T \cap S \\ L(0) + \epsilon & \text{if } k \in T \cap (N \setminus S), \end{cases}$$

where $\epsilon = (n - s)(\bar{x} - L(0))/(s + 1) > 0$ and $s = |S|$. Note that

$$\begin{aligned} y_0 + \textstyle\sum_{k \in T} y_k &= x_0 + (2s - n)\bar{x} + (n - s)L(0) + (s + 1)\epsilon \\ &= x_0 + (2s - n)\bar{x} + (n - s)L(0) + (n - s)(\bar{x} - L(0)) \\ &= x_0 + s\bar{x} = sW(s). \end{aligned}$$

Since $y_k > L(0)$ $^\forall k \in T$ and $x_k = \bar{x} > L(0)$ $^\forall k \in N \setminus T$, any firm $i \in S$ has no counter objection to the objection, contradicting that $x \in \tilde{M}^S$. *Q.E.D.*

Lemma 3 *Suppose* $1 \leq s \leq n/2$. *If* $x \in \tilde{M}^S$ *and if* $s(W(s) - L(0)) \leq (n - s)(W(n - s) - L(0))$, *then* $\bar{x} \leq L(0)$.

Proof : Let $x \in \tilde{M}^S$. Suppose $\bar{x} > L(0)$. Then $x_0 < s(W(s) - L(0))$, since $x_0 + s\bar{x} = sW(s)$. Take an objection $(y, \{0\} \cup (N \setminus S))$ of the licensor against firm $i \in S$ in x such that

$$y_k = \begin{cases} (n - s)(W(n - s) - L(0)) & \text{if } k = 0 \\ L(0) & \text{if } k \in N \setminus S. \end{cases}$$

Since $y_k = L(0)$ $^\forall k \in N \setminus S$ and $x_k = \bar{x} > L(0)$ $^\forall k \in S$, no counter objection can be made to the objection, contradicting that $x \in \tilde{M}^S$. *Q.E.D.*

Lemma 4 *Suppose* $1 \leq s \leq n/2$. *If* $x \in \tilde{M}^S$ *and if* $0 < s(W(s) - W(t)) \leq (n - s)(W(n - s) - L(0))$, *then* $\bar{x} \leq W(t)$.

Proof : Suppose $\bar{x} > W(t)$. Then $x_0 < s(W(s) - W(t))$. Since $W(t) > L(0)$, the same argument as in the proof of Lemma 2 applies. *Q.E.D.*

Let $W(t^*)$ be the minimum of $W(t)$ that satisfies $0 < s(W(s) - W(t)) \leq (n - s)(W(n - s) - L(0))$. Whenever the licensor proposes x with $\bar{x} = W(t^*)$, licensees accept it, since $\bar{x} \leq W(t^*)$.

Lemma 5 *For any $S \subseteq N$, $x_0 \leq s^*(W(s^*) - L(0))$ if $x \in \tilde{M}^S$.*

Proof : Let $x \in \tilde{M}^S$. Suppose $x_0 > s^*(W(s^*) - L(0))$. By the definition of s^*, $\bar{x} = (sW(s) - x_0)/s < (sW(s) - s^*(W(s^*) - L(0)))/s \leq L(0)$. Take an objection (y, N) of $i \in S$ against the licensor in x with $y_k = L(0)\ {}^\forall k \in N$. If the licensor had a counter objection $(z, \{0\} \cup T)$ to the objection with $z_0 \geq x_0 > s^*(W(s^*) - L(0))$ and $z_k \geq y_k = L(0)\ {}^\forall k \in T$, it should be $z_0 + \sum_{k \in T} z_k > s^*(W(s^*) - L(0)) + tL(0) \geq tW(t)$ by the definition of s^*, where $t = |T|$. Hence, no counter objection can be made, contradicting that $x \in \tilde{M}^S$. *Q.E.D.*

Lemmas 2 to 5 complete the proof. *Q.E.D.*

Proposition 4 *Let $x \in \tilde{M}^N$. Then, we have the following.*
(a) *If $n > s^*$, then $n(W(n) - L(0)) \leq x_0 \leq s^*(W(s^*) - L(0))$.*

(b) *If $n = s^*$, then $\tilde{M}^N = \tilde{C}^N$, where $x \in \tilde{C}^N$ is characterized by the following inequalities: $x_0 + n\bar{x} = nW(n)$, $0 \leq x_0 \leq n(W(n) - L(0))$ and $L(0) \leq \bar{x} \leq \min_{s:s \neq n}(nW(n) - sW(s))/(n - s)$.*

Proof : Consider (a). Let $\bar{x} = L(0) + z > 0$ with $z > 0$. If $x_0 = n(W(n) - \bar{x})$, the licensor can make an objection $(y, \{0\} \cup S^*)$ such that $y_i > x_i$ for any $i \in \{0\} \cup S^*$, since $n(W(n) - \bar{x}) = n(W(n) - L(0)) - nz < s^*(W(s^*) - L(0)) - s^*z = s^*(W(s^*) - \bar{x})$. Since $\bar{x} > L(0)$, no counter objection can be made by any $i \in N \setminus S^*$. Thus $\bar{x} \leq L(0)$. Lemma 5 completes the proof.

Next, we show (b) with the next lemma.

Lemma 6 *If $x \in \tilde{M}^{S^*}$, then $\bar{x} \geq L(0)$.*

Proof : Let $x \in \tilde{M}^{S^*}$. Suppose $\bar{x} < L(0)$. Then, a licensee $i \in S^*$ has an objection (y, N) against the licensor in x, such that $y_k = L(0)\ {}^\forall k \in N$. If the licensor had a counter objection $(z, \{0\} \cup R)$ to the objection (y, N),

$$rW(r) \geq z_0 + \textstyle\sum_{k \in R} z_k, \text{ where } r = |R|$$
$$z_0 \geq x_0, \text{ and } z_k \geq y_k = L(0)\ {}^\forall k \in R.$$

Since $\bar{x} < L(0)$, $x_0 = s^*W(s^*) - s^*\bar{x} > s^*W(s^*) - s^*L(0)$. Hence,

$$rW(r) \geq z_0 + \sum_{k \in R} z_k > s^*W(s^*) - s^*L(0) + rL(0),$$

which violates the definition of s^*. Thus, i's objection (y, N) cannot be countered by the licensor, contradicting that $x \in \tilde{M}^{S^*}$. *Q.E.D.*

Proposition 2 ensures $\tilde{C}^N \neq \emptyset$ if $s^* = n$. Clearly $\tilde{C}^N \subseteq \tilde{M}^N$ by the definitions. We here show $\tilde{C}^N \supseteq \tilde{M}^N$. Let $x \in \tilde{M}^N$. Suppose that there is an $x \in \tilde{M}^N$ and that $x \notin \tilde{C}^N$. Since $x \in \tilde{M}^N$, $\bar{x} \geq L(0)$ by Lemma 6. Since $x \notin \tilde{C}^N$, there exists $\{0\} \cup T$ with $x_0 + \sum_{i \in T} x_i < tW(t)$, where $t < n$. Let $(y, \{0\} \cup T)$ be an objection of the licensor against some $i \in N \setminus T$ in x such that $y_k = x_k + \epsilon \ \forall k \in \{0\} \cup T$ and $(t+1)\epsilon = tW(t) - (x_0 + \sum_{i \in T} x_i) > 0$. Since $\bar{x} \geq L(0)$, i has no counter objection, contradicting that $x \in \tilde{M}^S$. Thus, $\tilde{M}^N = \tilde{C}^N$. From the system (2) of inequalities in the proof of Proposition 2, $L(n-s) \leq \bar{x} \leq (nW(n) - sW(s))/(n-s)$ for any s. By Lemma 6, $\bar{x} \geq L(0)(> L(n-s))$. Thus, $\tilde{C}^N$ is characterized as in (b). *Q.E.D.*

Let S^* be a set $S \subseteq N$ with $|S| = s^*$. Proposition 3 (b) directly implies the next proposition.

Proposition 5 *If $n/2 \leq s^* < n$, then $\tilde{M}^{S^*} = \{x^*\}$, where*

$$x_i^* = \begin{cases} s^*(W(s^*) - L(0)) & \text{if } i = 0 \\ L(0) & \text{if } i \in S^* \\ L(s^*) & \text{if } i \in N \setminus S^*. \end{cases}$$

In the linear example described in section 2, there exists a threshold $\hat{\epsilon}$ of the cost reduction such that $n/2 \leq s^*$ if and only if $\epsilon \leq \hat{\epsilon}$.

Proposition 6 *For any $s^* = 1, \ldots, n$, $x^* \in \tilde{M}^{S^*}$.*

Proof : Consider any objection $(y, \{0\} \cup T)$ of the licensor against $i \in S^*$ in x^* $(T \neq N)$. If $\sum_{k \in T} y_k \geq tL(0)$, it should be $tW(t) \geq y_0 + \sum_{y \in T} y_k > x_0^* + tL(0) = s^*(W(s^*) - L(0)) + tL(0)$, which violates the definition of s^*. Hence, i can make a counter objection (z, N) to the objection such that

$$z_i = \begin{cases} L(0) & \text{if } k \in S^* \setminus T \\ y_k + \epsilon & \text{if } k \in T \\ L(0) & \text{if } k \in (N \setminus S^*) \setminus T, \end{cases}$$

where $\epsilon = (tL(0) - \sum_{k \in T} y_k)/t > 0$. In fact, $\sum_{k \in N} z_k = nL(0)$, $z_k \geq x_k \ \forall k \in N$ and $z_k > y_k \ \forall k \in T$.

Next consider any objection (u, R) of $i \in S^*$ against the licensor in x^* $(0 \notin R)$. Note that there is no objection if $s^* = n$. Let

$$u'_k = \begin{cases} u_k & \text{if } k \in R \\ x^*_k & \text{if } k \in N \setminus R. \end{cases}$$

By the definition of objection, there is at least one firm k with $u_k < L(0)$. Arrange the payoffs of all the firms in non-decreasing order. Take the first $s^*(< n)$ firms and let Q be the set of them. Since $\sum_{k \in Q} u'_k < s^* L(0)$, the licensor can make a counter objection to the objection. *Q.E.D.*

The Optimal Number of Licensees: an Implication

Proposition 4 suggests that if $n/2 \leq s^* < n$, the revenue of the licensor is uniquely determined as $s^*(W(s^*) - L(0))$, since $\tilde{M}^{S^*}$ is a singleton. Hence, the licensor should invite $s^*(< n)$ firms to the negotiation and license his patented innovation to them.

In the other cases, however, the optimal number of licensees cannot be determined completly. By Proposition 5, the licensor can obtain his revenue $s^*(W(s^*) - L(0))$ as negotiation results, but Propositions 3 indicates that the licensor cannot gain more than that amount. When $s^* = n$, for instance, there are some cases where it is better for him not to invite all the n firms to the negotiation, if the (collective) bargaining power of firms is quite large; it is better for the licensor to invite $n - 1$ firms to the negotiations if $(n-1)(W(n-1) - L(0)) > n(W(n) - \bar{x})$ with $\bar{x} > L(0)$.

6 Final Remarks

Other v-Functions and Solutions

In the linear example described in section 2, our v-function assigns the same values to coalitions as v-function defined in Watanabe and Tauman (2003) does, if $s \leq (n+1)/2$ and $s \leq \hat{s}$. When the size of a coalition is not large, firms belonging to the coalition will decide not to act cooperatively, even if firms are allowed to coordinate their strategies in the market. They form a coalition only for negotiating how to split their total profit.

Driessen, Muto and Nakayama (1992) applied another v-function to information trading: information is shared in the most efficient way among a seller and potential buyers of the information. According to their definition, however, the information is not provided to all the potential buyers, although they can share their total profits. Our v-function is defined in a more natural way, since any firms in coalition $\{0\} \cup S$ can be equally licensed.

Some results similar to ours can be obtained with the other solution concepts such as the strong equilibrium and the coalition-proof Nash equilibrium. We will show them precisely in another paper. For the reference, see Muto (1990) and Nakayama and Quintas (1991).

The Shapley Value

Let $\mathrm{Sh}_0(v)$ denote the Shapley value of the licensor and let $x \in \tilde{M}^S$ for some S. By Lemma 5, $x_0 \leq s^*(W(s^*) - L(0))$. Watanabe and Tauman (2003) also showed that in the linear example $\mathrm{Sh}_0(v) \notin M^N$ if the number of firms is quite large. In our model, $\mathrm{Sh}_0(v) \notin M^N$ if and only if

$$\begin{aligned}(1/n+1) \textstyle\sum_{s=1}^{n-\hat{s}} sL(0)) + (1/n+1) \sum_{s=n-\hat{s}+1}^{n-1} s(L(0) - L(n-s)) \\ > s^*(W(s^*) - L(0)) - (1/n+1) \textstyle\sum_{s=1}^{n} s(W(s) - L(0)).\end{aligned}$$

It is well known that the Shapley value is not necessarily in the core, but its relationship with the bargaining set has not been studied comprehensively. With more specified models, we could have proceeded further on that topic.

Limitation of Sidepayments

We could have analyzed an alternative model where sidepayments are not allowed except fee payments to the licensor: in $\{0\} \cup S$, each $i \in S$ pays p_i to the licensor (for all $S \subseteq N$) and there is no money transfer among firms in S. Assume the uniform pricing scheme: $p_i = p \ {}^\forall i \in S$. Below is the addendum to extend our model. The permissible coalition structure is

$$P^S = (\{0\} \cup S, \{\{i\}\}_{i \in N \setminus S}), \ {}^\forall S \subseteq N,$$

and so the characteristic function is given by

$$V(\{0\} \cup S) = \{(x_i)_{i \in \{0\} \cup S} | x_0 \leq sp,\ x_i \leq W(s) - p,\ 0 \leq p \leq W(s)\}$$
$$V(\{0\}) = 0, \quad V(S) = \{(x_i)_{i \in S} | x_i \leq L(n-s)\}.$$

The imputations under a coalition structure P^S is defined by

$$X^S = \{x = (x_0, x_1, \cdots, x_n) \in \mathbb{R}^{n+1} | x_0 = sp,\ x_i = W(s) - p,\ ^\forall i \in S$$
$$0 \leq p \leq W(s) - L(n-1),\ x_i \geq L(n-s),\ ^\forall i \in N \setminus S\}.$$

The core C^S is defined by

$$C^S = \{x \in X^S | \text{ for any } T \subseteq \{0\} \cup N \text{ with } T \cap (\{0\} \cup S) \neq \emptyset,$$
$$\text{there exists no } y \in V(T) \text{ such that } y_k > x_k,\ ^\forall k \in T\}.$$

Let $i, j \in \{0\} \cup S$ and $x \in X^S$. i has an objection (y, T) against j in x if $i \in T$, $j \notin T$, $T \subseteq \{0\} \cup N$, $y_k > x_k\ ^\forall k \in T$, and $y \in V(T)$. j has a counter objection (z, R) to i's objection (y, T) if $j \in R$, $i \notin R$, $R \subseteq \{0\} \cup N$, $z_k \geq x_k\ ^\forall k \in R$, $z_k \geq y_k\ ^\forall k \in R \cap T$, and $z \in V(R)$. i has a *valid* objection (y, T) against j in x if (y, T) is not countered. The bargaining set M^S is defined by

$$M^S = \{x \in X^S | \text{no player in } \{0\} \cup S \text{ has a valid objection in } x\}.$$

Since almost the same results are regained even in this setup, sidepayments do not play important roles in our propositions. With sidepayments, however, licensing by means of royalty only is not substantially different from licensing by means of upfronty fee only, as noted in section 3. Hence, it is significant to analyze the above patent licensing game without sidepayments so as to reconsider "fee versus royalty" that has been studied in the traditional literature. We will show the complete results in another paper.

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