

**Cournot competition among multiproduct firms:  
specialization through licensing**

Luigi Filippini<sup>1</sup>

Abstract

In a duopoly where each firm produces substitute goods, we show that under process innovation, specialization is the equilibrium attained with cross-licensing. Each firm produces only the good for which it has an advantage. Patent pool extension confirms the results.

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<sup>1</sup> Università Cattolica - Largo Gemelli 1 - 20123 Milano - Italy  
e-mail: luigi.filippini@unicatt.it

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## **Introduction**

Antitrust law historically has viewed cross-licensing or pooling agreements with suspicion because these mechanisms are potentially capable of promoting collusion in the product market.

The literature on cross-licensing has in fact stressed that it facilitates collusion. C.Shapiro (1985 p.26) states that: “two rivals (with or without innovations) alternately could design a cross-licensing agreement whereby each would pay the other a royalty per unit of output, ostensibly for the right to use the other’s technology. By imposing a “tax” on each other ....., the firms could again achieve the fully collusive outcome. A cross-licensing contract may be required to achieve the fully collusive outcome if the firms produce different products or are otherwise heterogeneous”.

M.Eswaran (1993) assumes that the firms license their technologies to each other but tacitly agree not to produce from the acquired technology as long as the contracting firm does not defect. In an infinitely repeated game it is shown that collusion can be sustained from tacitly restricted level of production by credibly introducing the threat of increased rivalry in the market for each firm’s product .

P.Ling (1996) is close to M.Eswaran’s contribution as fixed fee licensing makes firms’ costs symmetric and increases the licensee’s scope for retaliation.

C.Fershtman and M.Kamien (1992) deals with cross licensing of complementary technologies, that may be independently developed by different firms. Relevant to this note is the problem the firms face about how to design a cross licensing agreement such that the resultant non-cooperative game, yields equilibrium profits identical to the cooperative outcome.

This Note studies product specialization in a duopoly where each firm produces two imperfect-substitute goods. We show that under process innovation, specialization is the equilibrium attained under optimal cross-licensing arrangements. The optimum licensing contracts are royalty-contracts. Royalties are set so as to implement the joint-profit maximization (monopoly) outcome as the unique Nash equilibrium of the competition game. The monopoly-First-Best optimum is attained: i) each firm produces solely the good for which it has a technological advantage; ii) the quantities of goods which are produced are the monopoly levels; iii) firms' joint profits attain the First Best optimum, but social welfare do not improve with respect to no licensing. We show that the same results are attained with patent pool.

The plan of the paper is as follows. Section 1 describes the basic framework where the two firms diversify their production and considers the introduction of the process innovation that may lead to product specialization. Section 2 discusses the cross licensing and the product specialization which

results from that. Section 3 analyzes the welfare effects. Section 4 extends the analysis to patent pool.

### 1. Two firms diversifying their production

Let's consider a model of an industry composed by two symmetric firms, and two imperfect substitute goods, good 1, good 2. Each firm can produce both goods. We assume linear demand functions:

$$\begin{aligned} p_1 &= a - \theta (q_{21} + q_{22}) - (q_{11} + q_{12}) \\ p_2 &= a - \theta (q_{11} + q_{12}) - (q_{21} + q_{22}), \end{aligned} \quad [1]$$

where  $p_i$  is the price of good  $i$ ,  $i=1,2$ ,  $q_{ij}$  the quantity of good  $i$  produced by firm  $j$ , and  $\theta \in (0, 1]$  represents the degree of product differentiation.

Firm cost functions are linear and symmetric: each firm produces good  $i$ ,  $i=1,2$ , at the constant marginal cost,  $c$ . We assume  $c < a$  in order to avoid a corner solution.

Firm profit functions are:

$$\begin{aligned} \Pi_1 &= p_1 q_{11} + p_2 q_{21} - c q_{11} - c q_{21} \\ \Pi_2 &= p_1 q_{12} + p_2 q_{22} - c q_{12} - c q_{22}. \end{aligned}$$

Let's assume Cournot competition. Firm 1 chooses its outputs:

$$\begin{aligned} &\text{Max}_{q_{11}, q_{21}} \{ q_{11}(p_1 - c) + q_{21}(p_2 - c) \} \\ &\text{s.t. [1]} \\ &q_{11} \geq 0, q_{21} \geq 0, \end{aligned} \quad [2]$$

and Firm 2 chooses its outputs :

$$\begin{aligned} &\text{Max}_{q_{12}, q_{22}} \{ q_{12}(p_1 - c) + q_{22}(p_2 - c) \} \\ &\text{s.t. [1]} \\ &q_{12} \geq 0, q_{22} \geq 0, \end{aligned}$$

Equilibrium outputs, prices and profits of system [2] are given by:

$$\begin{aligned} q_{11} &= q_{12} = q_{21} = q_{22} = (a - c) / [3(1+\theta)] \\ p_1 &= p_2 = (a + 2c)/3. \end{aligned}$$

$$\Pi_1 = \Pi_2 = 2(a - c)^2 / [9(1+\theta)].$$

We then have:

**Proposition 1** In a duopoly composed by two symmetric firms that both produce two imperfect substitute goods and linear demand functions [1], there exists a unique Nash equilibrium where both firms produce positive quantities, for  $c < a$ .

Both firms are active in both markets and there exists limited specialization.

### 1.1 A process innovation

Let's now suppose that Firm 1 discovers and patents a cost reducing technology for good 1 such that  $c = 0$ , and Firm 2 for good 2 such that  $c = 0$ .

The profits functions are (the subscript P denotes process innovation) :

$$\begin{aligned}\Pi_{1P} &= p_1 q_{11} + p_2 q_{21} - c q_{21} \\ \Pi_{2P} &= p_1 q_{12} + p_2 q_{22} - c q_{12}.\end{aligned}$$

In Cournot competition, Firms 1 and 2 again choose their (individual) profit-maximizing outputs:

$$\begin{aligned}\text{Max } \{ & q_{11} p_1 + q_{21}(p_2 - c) \} \\ & q_{11}, q_{21} \\ \text{s.t. [1], } & q_{11} \geq 0, q_{21} \geq 0\end{aligned} \tag{3}$$

$$\begin{aligned}\text{Max } \{ & q_{12}(p_1 - c) + q_{22} p_2 \} \\ & q_{12}, q_{22} \\ \text{s.t. [1], } & q_{12} \geq 0, q_{22} \geq 0\end{aligned}$$

Solving system [3] leads to:

1) if

$$c < [a(1-\theta)/(2+\theta)] , \tag{4}$$

then there is limited specialization (differentiation):

Equilibrium outputs are strictly positive, and are given by:

$$q_{11} = q_{22} = [a + c - a\theta + 2\theta c] / [3(1-\theta^2)], \tag{4.a}$$

$$q_{12} = q_{21} = [a - 2c - a\theta - \theta c] / [3(1-\theta^2)] . \tag{4.b}$$

Prices and profits (the subscript LC denotes limited specialization and Cournot prices) are:

$$p_1 = p_2 = (a + c)/3 ,$$

$$\Pi_{iLC} = \Pi_{2LC} = [(a + c)^2 + (a - 2c)^2 - 2\theta (a + c)(a - 2c)] / [9(1-\theta^2)]. \quad [4.c]$$

2) if

$$c > [a(1-\theta)/(2+\theta)] , \quad [5]$$

then there is full specialization:

Equilibrium outputs, prices and profits (the subscript FM denotes full specialization and monopoly pricing) are:

$$q_{12}^* = q_{21}^* = 0 , \quad [5.a]$$

$$q_{11}^* = q_{22}^* = a / (2 + \theta) \quad [5.b]$$

$$p_1^* = p_2^* = a / (2 + \theta)$$

$$\Pi_{1FM} = \Pi_{2FM} = [a / (2 + \theta)]^2 . \quad [5.c]$$

Case 2 is the case of drastic innovation <sup>2</sup>. That is, there is specialization if and only if inequality [5] holds.

Clearly, if the innovation is drastic (inequality [5] holds), then firms earn monopoly profits: i) each firm produces solely the good for which it has a technological advantage; ii) the quantities of good 1 and 2 which are produced are the monopoly levels as given by [5.b]. When the innovation is non-drastic, i.e. inequality [4] holds, then both firms produce both goods, and firms' profits fall below monopoly levels (by [4.c], [5.c]).<sup>3</sup>

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<sup>2</sup> It is an adaptation of the drastic and non drastic innovation differences discussed by Arrow (1962). A drastic innovation arises in case the monopoly price by means of the new technology does not exceed the competitive price under the old technology (Kamien and Tauman, 1986 p.472).

<sup>3</sup> It suffices to note that both equilibria are symmetric, so that in both cases each firm gains half of the industry profit. The result then follows from the fact that industry profit must be higher when each segment of the market is monopolised by the firm who is more efficient in producing the corresponding good.

In a formal way, for all feasible  $c$  and all  $\theta$ ,  $\Pi_{iFM} > \Pi_{iLC}$ ,  $i = 1,2$ . This follows because: a)  $\Pi_{iLC}$  is decreasing in  $c$ , for  $c < [a(1-\theta)/(5+4\theta)]$ . It is increasing in  $c$  for  $[a(1-\theta)/(5+4\theta)] < c < [a(1-\theta)/(2+\theta)]$ ; b)  $\Pi_{iLC} < \Pi_{iFM}$ , for  $c \in [0, a(1-\theta)/(5+4\theta)]$ . Whence,  $\Pi_{iFM} > \Pi_{iLC}$  for all  $\theta$ ; and  $\Pi_{iFM} = \Pi_{iLC}$ , iff  $c = [a(1-\theta)/(2+\theta)]$ . ■

**Proposition 2:** I.) If the innovation is drastic, if condition [5] holds, then the Nash-Cournot equilibrium entails full specialization: i) each firm produces solely the good for which it has a technological advantage; ii) the quantities of goods 1 and 2 which are produced are the monopoly levels as given by [5.b]; iii) firms' joint profits attain the First Best optimum.

II) If the innovation is non-drastic, if condition [4] holds, then the Nash-Cournot equilibrium entails limited specialization: each firm produces both goods, output levels are given by [4.a]-[4.b], firms' joint profits fall below the First Best optimum.

Clearly, in the case of non-drastic innovation, firms would be better off if they could commit to joint profit maximization.

**Corollary 1.** Let the innovation be non-drastic. Suppose firms can commit to joint profit maximization. Then: i) each firm produces solely the good for which it has a technological advantage; ii) the quantities of goods 1 and 2 which are produced are the monopoly levels as given by [5.b]; iii) firms' joint profits attain the First Best optimum.

This immediately follows from Proposition 2.

However, the only credible commitments are those that are incentive compatible, and  $q_{12}^* = q_{21}^* = 0$ , are not. Indeed, the unique Nash-Cournot equilibrium has

$$q_{11} = q_{12} = q_{21} = q_{22} > 0 \quad (\text{by II. of Proposition 2}).$$

We show below that there exists a cross - licensing scheme that implements the collusive outcome: the unique Nash-Cournot equilibrium entails full specialization, and firm profits attain the First Best optimum level.

## 2. Cross-Licensing

We now consider the possibility of a technology transfer from firm 1 to firm 2 for good 1 and vice versa for firm 2 under licensing by means of a two part tariff (fixed fee,  $F_i$  and royalty,  $r_i$ ).

We assume that the innovation is observable and verifiable, and similarly for output. Contracts of technology transfer from firm 1 to firm 2 (and vice versa) are then enforceable and the payments by the recipient can be conditioned on recipient's output. We shall refer to technology transfer contracts as to licensing contracts, and name the party that makes the technology transfer the licensor and the recipient the licensee. More specifically, a licensing contract states parties' obligations as follows:

the licensor discloses the new technology to the licensee. The licensee pays the licensor a fixed fee and/or a royalty per unit of its output. Contract offers are made by one firm, the other either rejects the offer or accepts it. If the latter rejects it then it will necessarily use the old technology, if it accepts it then royalty-payment obligations are due independently of the technology used and therefore its profit-maximizing choice is necessarily to adopt the new (cost-reducing) technology.

The game played by the two firms is a non-cooperative two-stage game. In the first stage each firm simultaneously offers a licensing contract to its rival who then chooses whether to accept it or reject it. We shall make the conventional assumption that when each firm is indifferent between accepting the rival's licensing offer and rejecting it, it chooses to accept the offer. In the second stage firms engage in quantity Cournot competition as described in Section 1.

The profits functions are (the subscript Lic denotes licensing):

$$\Pi_{1Lic} = p_1q_{11} + p_2q_{21} + r_1q_{12} - r_2q_{21} + F_1 - F_2$$

$$\Pi_{2Lic} = p_1q_{12} + p_2q_{22} - r_1q_{12} + r_2q_{21} + F_2 - F_1.$$

where outputs,  $q_{11}$ ,  $q_{21}$ ,  $q_{12}$ ,  $q_{22}$ , are the outcome of the Cournot competition second stage game, given the royalty rates set at the first stage. Specifically, for any given royalty rates, equilibrium outputs, prices and profits are:

$$q_{11} = [a + r_1 - a\theta + 2\theta r_2] / [3(1-\theta^2)]$$

$$q_{12} = [a - 2r_1 - a\theta - \theta r_2] / [3(1-\theta^2)]$$

$$q_{21} = [a - 2r_2 - a\theta - \theta r_1] / [3(1-\theta^2)]$$

[6]

$$q_{22} = [a + r_2 - a\theta + 2\theta r_1] / [3(1-\theta^2)]$$

$$p_1 = (a + r_1)/3$$

$$p_2 = (a + r_2)/3$$

At the first stage, each firm  $i$  chooses  $(r_i, F_i)$  in order to maximize its profits subject to rival's participation constraint, and output non-negativity constraints. That is:

firm 1

$$\text{Max}_{r_1, F_1} \Pi_{1Lic}(r_1, r_2, \dots)$$

$$\text{s.t. } \Pi_{2Lic}(r_1, r_2, \dots) \geq \Pi_{2LC}, \quad q_{11} \geq 0, \quad q_{21} \geq 0, \quad q_{12} \geq 0, \quad q_{22} \geq 0, \quad \text{and } r_1, F_1 \geq 0 ;$$

firm 2

$$\text{Max } \Pi_{\text{ILic}}(r_1, r_2, \dots)$$

$$r_2, F_2$$

$$\text{s.t. } \Pi_{\text{ILic}}(r_1, r_2, \dots) \geq \Pi_{\text{ILC}}, \quad q_{11} \geq 0, q_{21} \geq 0, q_{12} \geq 0, q_{22} \geq 0, \text{ and } r_2, F_2 \geq 0,$$

where:

$$\Pi_{\text{ILic}} \equiv 1/3 \{ (a + r_1) [a + r_1 - a\theta + 2\theta r_2] / [3(1-\theta^2)] + (a + r_2) [a - 2r_2 - a\theta - \theta r_1] / [3(1-\theta^2)] \} + r_1 [a - 2r_1 - a\theta - \theta r_2] / [3(1-\theta^2)] - r_2 [a - 2r_2 - a\theta - \theta r_1] / [3(1-\theta^2)] + F_1 - F_2;$$

$$\Pi_{\text{2Lic}} \equiv 1/3 \{ (a + r_1) [a - 2r_1 - a\theta - \theta r_2] / [3(1-\theta^2)] + (a + r_2) [a + r_2 - a\theta + 2\theta r_1] / [3(1-\theta^2)] \} + r_2 [a - 2r_2 - a\theta - \theta r_1] / [3(1-\theta^2)] - r_1 [a - 2r_1 - a\theta - \theta r_2] / [3(1-\theta^2)] + F_2 - F_1.$$

In the unique Nash equilibrium, licensing contracts are:

$$r_1 = r_2 = a(1-\theta)/(2+\theta), \quad F_1 = F_2 > 0,$$

these are payoff equivalent to pure royalty contracts:

$$r_1 = r_2 = a(1-\theta)/(2+\theta), \quad F_1 = F_2 = 0. \quad [7]$$

For any given  $c$  that satisfies inequality [4], i.e. for non-drastic innovation, the royalty rate exceeds the cost reduction (by [7]). Using [7], and solving for outputs, prices and profits, leads to:

$$q_{11}^* = q_{22}^* = a/(2+\theta)$$

$$q_{12}^* = q_{21}^* = 0$$

$$p_1^* = p_2^* = a/(2+\theta)$$

$$\Pi_{\text{ILic}} = \Pi_{\text{2Lic}} = a^2/(2+\theta)^2$$

This leads to:

### Proposition 3

The optimum licensing contracts are the royalty- contracts defined by [7]. These implement the monopoly-First-Best optimum: i) each firm produces solely the good for which it has a technological advantage (full specialization); ii) the quantities of goods 1 and 2 which are produced are the monopoly levels, identical to [5.b]; iii) firms' joint profits attain the First Best optimum.

Royalty-licensing contracts act as an incentive-compatible commitment device for attaining joint-profit maximization. The firm that has a technological (cost) advantage in the production of good  $j$ , let say firm  $j$ , licenses its technology to its rival, i.e. firm  $i$ , by means of a royalty contract: a) the royalty is set to a level such that the licensee (rival firm  $i$ ) finds it optimal to abstain from producing good  $j$  (in equilibrium, royalties are not paid); b) royalty-licensing contracts are designed so as to act

as off-equilibrium threats to implement the joint-profit maximization (monopoly) outcome as the unique Nash equilibrium of the Cournot-competition game.

### 3. Welfare effects

We now compare social welfare between cross-licensing and the process innovation status quo.

We have:

#### Proposition 4

Social welfare in the cross-licensing case,  $W_{iLic} = 3a^2/(2+\theta)^2$ , is lower than with no-licensing,  $W_{iLC} = [(2a-c)/3(1+\theta)]^2 + 2[(a+c)^2 + (a-2c)^2 - 2\theta(a+c)(a-2c)] / [9(1-\theta^2)]$ , for all feasible  $c$  and all  $\theta$ .

#### Proof

Notice that the maximum value attainable by is  $c_{max} = [a(1-\theta)/(2+\theta)]$  and that  $W_{iLic} = W_{iLC}(c_{max})$ .

Notice that  $W_{iLic} < W_{iLC}(c=0)$ .

The result then follows from observing that  $W_{iLC}(c)$  decreases in  $c$ , it attains a minimum at  $c = c_1$ ,

$$c_1 = 8a(1-\theta)/(5\theta+13),$$

which is greater than  $[a(1-\theta)/(2+\theta)]$ . ■

### 4. Patent pools

The patent pool game differs from the cross-licensing game above in that at the first stage (i.e. the licensing stage), firms act cooperatively: firms 1 and 2 choose  $(r_i, F_i)$  that maximize joint profits. In the second stage, firms again engage in quantity Cournot competition. The solution is again:

$$r_i = a(1-\theta)/(2+\theta), \quad F_i = 0,$$

That is the royalty rate is identical to that derived for the cross licensing case. The same holds for outputs, prices and profits: each firm produces solely the good for which it has a technological advantage (full specialization), and the quantities of goods 1 and 2 which are produced are the monopoly levels.

### Conclusion

We have studied product specialization in a duopoly where each firm produces two imperfect-substitute goods. We have shown that under process innovation, specialization is the equilibrium attained under optimal cross-licensing arrangements, as well as under patent pool. The optimum

licensing contracts are royalty contracts. These are designed so as to implement the joint-profit maximization (monopoly) outcome as the unique Nash equilibrium of the competition game. The monopoly-First-Best optimum is attained: Each firm produces solely the good for which it has a technological advantage, firms' joint profits attain the First Best optimum, but social welfare does not improve with respect to no licensing.

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