Vagueness in Multi-Issue Proposals

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Abstract

In many situations such as electoral competition, decision makers choose between agents based on the information provided by those agents. I study how two competing agents reveal information about multiple issues to a decision maker when they are allowed to be vague. I call an agent's *unbiased issue* an issue on which he and the decision maker agree on the optimal action. An agent's *biased issue* is an issue on which he and the decision maker disagree on the optimal action. I find that an agent is disadvantaged by revealing information about his opponent's biased issue because doing so allows his opponent to undercut him. I also show that there is an equilibrium in which each agent is vague about his opponent's biased issue and specific about his opponent's unbiased issue. The model can be applied to electoral competition when candidates are policy experts as well as other delegation settings.

Keywords: Principal-agent, delegation, vagueness, signaling, intuitive criterion.

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1 Introduction

How should two competing agents reveal information about multiple issues to a decision maker? Should they reveal specific information? Or is it better to be vague? In this paper, I show that an agent is disadvantaged by revealing information about certain issues to the decision maker, those on which his opponent is weak. The reason is that doing so enables his opponent to anchor on the revealed information and offer the decision maker a concession. In other words, agents should be vague about their opponents' weaknesses.

Consider an example of two political candidates, Candidate 1 and Candidate 2, competing for the vote of a representative voter. Each player's payoff is determined by the policy outcome on two issues, issue a and issue b. Players' preferences over policies are determined by a state. The voter's ideal policy is equal to the state, but candidates are biased. After observing the state, candidates simultaneously make policy proposals on the two issues to the voter. The voter, who does not observe the state, tries to infer it from the proposals and elects the candidate who she believes will enact better policies. Given that the voter is aware of candidates' biases, is it possible for a candidate to convince her that his proposal is indeed better than his opponent's?

Further suppose that a proposal, though covering both issues, can be either vague or specific on each issue. A proposal that is vague on an issue allows its proposer full freedom with respect to this issue if he is elected. A proposal that is specific on an issue commits its proposer to adopt the corresponding policy on that issue if elected. Given the state, Candidate 1 has the same preference over policies for issue a as the voter, while Candidate 2 does not share this preference. Similarly, Candidate 2 has the same preference over policies for issue b as the voter, while Candidate 1 does not.

I argue that in this situation, a candidate's choice to be vague on the issue on which his opponent's interest is misaligned with that of the voter can help him win the election. To see this, suppose that Candidate 1 commits on issue a in a way that reveals its state to the voter. Since Candidate 2 is already aligned with the voter on issue b, he can now offer a concession on issue a, so that overall the voter prefers him to Candidate 1. On the other hand, if Candidate 1 is vague about issue a, then he is immune to this kind of undercutting. This is because the voter knows that Candidate 2 does not share her views on that issue and thus believes that his policy on it will be unfavorable. Therefore Candidate 2 cannot convince the voter that he is the better candidate.

While electoral competition serves as the motivating example, the model can be applied to other settings in which an uninformed decision maker needs to delegate a task to one of two self-interested, informed agents. Examples of this include a patient choosing between two doctors and a CEO choosing which of two divisions in her company will carry out a project.

The game is as follows. There are three players: a decision maker (DM) and two informed agents (Agent 1 and Agent 2). Nature chooses a payoff-relevant state of the world. Agents observe the state, then simultaneously announce proposals. The DM, who does not observe the state, tries to infer it from the proposals and selects one agent. The selected agent then takes an action to implement his proposal. The outcome of an action depends on the state. Each player cares solely about the outcome.

I model uncertainty by distinguishing the outcome space from the action space à la Austen-Smith and Riker (1987) and Gilligan and Krehbiel (1989). My model differs from theirs in two important respects. First, players care about two issues rather than one. The DM's ideal action is normalized to be the state for each issue. To capture the agents' biases, Agent 1's ideal action coincides with that of the DM on issue a, but differs on issue b. Agent 2's ideal action coincides with that of the DM on issue b, but differs on issue a. I call issue a Agent 1's *unbiased issue* and issue b Agent 1's *biased issue*; the converse is true for Agent 2. Second, agents are allowed to be vague on any issue in their proposals. Being vague on an issue allows an agent freedom of action on that issue; otherwise he is committed to the policy that he proposes. The way I model vagueness is similar to Meirowitz (2005) in that vagueness represents a complete lack of commitment. Commitment to a proposed policy follows Austen-Smith and Riker (1987) and the modified rule in Gilligan and Krehbiel (1989). This distinguishes the present paper from papers studying cheap talk recommendations such as Krishna and Morgan (2001) and Battaglini (2002).

Typically, there are many equilibria in signaling games. However, a two-agent extension of a standard refinement *intuitive criterion* (Cho and Kreps, 1987, hereforth CK) narrows down the set of equilibria to those supported by reasonable off-path beliefs of the DM. In order to get a characterization, for most of the paper I focus on a simple class of intuitive equilibria in which agents' choices of vagueness are symmetric and coincide across all states. I show that in all such equilibria in which vagueness occurs, agents must be vague on their opponents' biased issues. I also provide a constructive proof of the existence of such an equilibrium. In order to characterize asymmetric as well as symmetric equilibria, I analyze the game with a smaller state space. Here I focus on intuitive equilibria, symmetric or asymmetric, in which agents' choices of vagueness coincide across states. The previous results hold.

It is worth mentioning that the restriction on the set of equilibria is not a restriction on the strategies to which players can deviate. In particular, a player who considers incentives to deviate can potentially deviate to any strategies. Therefore, the equilibria that I characterize remain equilibria when the restriction is lifted.

The next section contains an example that illustrates the intuition of my main result. Section 3 discusses related literature. Section 4 introduces the setup. Section 5 discusses the refinement solution concept. Section 6 gives the main results of the paper. Section 7 gives the results for general strategies in a binary state space. Section 8 shows that the undercutting intuition illustrated in Section 2 applies to general environments such as one in which the DM values one issue more than the other, and one with general biases of the agents and $N > 2$ issues. Proofs are rendered in Section 9. Section 10 contains conclusive remarks.

2 An Example

To see how being specific about the opponent's biased issue allows the opponent's undercutting, consider the following example from electoral competition. Suppose that the two issues are the tax rate and the minimum wage. The state for each issue can be any real number. [1]

Now suppose that the state for tax rate is 10%, representing the optimal tax rate from the voter's perspective. The state for the minimum wage is \$5, similarly representing the optimal minimum wage from the voter's perspective. Given the state $(10\%, \$5)$, Candidate 1's ideal tax rate is 10% and his ideal minimum wage is \$4. Candidate 2's ideal tax rate is 9% and his ideal minimum wage is \$5. Therefore, Candidate 1 is unbiased on the tax rate and biased on the minimum wage; conversely for Candidate 2. For simplicity, assume that for each player, a 1% deviation from his/her ideal tax rate results in the same payoff loss from a \$1 deviation from his/her ideal minimum wage. Since the payoff losses for the voter balance out, she is indifferent between the outcomes from both candidates' ideal policies at each state.

Consider a putative equilibrium in which each candidate plays the following strategy: being vague about his biased issue and committing to his ideal policy on his unbiased issue at each state. This means that when the state is (10%, \$5), and $\varnothing$ denotes vagueness, Candidate 1 proposes $(10\%, \varnothing)$ while Candidate 2 proposes $(\varnothing, \$5)$. Since candidates are rational, vagueness means that the candidate will implement his ideal policy if elected. Therefore each candidate will implement his own ideal policy for each issue. The voter is indifferent between two candidates, and is assumed to randomize 50-50. So each candidate gets his own ideal outcome half of the time and his opponent's ideal outcome the other half of the time.

Now suppose that Candidate 1 deviates to $(\varnothing, \$4.01)$. Therefore the voter observes $(\varnothing, \$4.01)$ from Candidate 1 and $(\varnothing, \$5)$ from Candidate 2. She knows that Candidate 1 has

[1] For the ease of exploration, I specify the state space to be $\mathbb{R}^2$. One can also transform the original state space $(0,1) \times (0,\infty)$ to $\mathbb{R}^2$ by taking $\log(\tan \frac{\pi}{2}\text{tax rate})$ and $\log(\text{minimum wage})$, for example.

deviated because in all states Candidate 1 should be vague about the minimum wage and specific about the tax rate. What should she believe about Candidate 2's proposal? I argue that she should continue to believe that Candidate 2 is playing according to the equilibrium and that the optimal minimum wage is \$5. The reason is as follows. Suppose that Candidate 2 is lying about the minimum wage and therefore the state for the minimum wage is not \$5. In this case, if Candidate 2 deviates to $(\varnothing, \$5)$ while Candidate 1 makes a proposal according to the equilibrium, the voter will not detect this deviation and simply believe that the optimal minimum wage is \$5. Therefore she continues to randomize 50-50. Therefore by lying about the optimal minimum wage, Candidate 2 wins with the same probability as before. Moreover, he gets a worse outcome whenever he wins. Therefore such a deviation can never be profitable for him. This justifies the voter's belief that any on-path proposal from a candidate is the equilibrium proposal of that candidate.

Now that the voter believes that Candidate 2 has not deviated and that the optimal minimum wage is \$5, which candidate is better? Candidate 2 will implement his ideal policies on both issues since he is vague on the tax rate and proposes his ideal policy for the minimum wage according to the equilibrium. Candidate 1 will implement his ideal policy for the tax rate issue since he is vague on it. For the minimum wage issue, given that the state is \$5, by committing to \$4.01 Candidate 1 is offering a concession by moving slightly towards the voter's ideal minimum wage. So overall Candidate 1 is better than Candidate 2. Since deviating to $(\varnothing, \$4.01)$ makes Candidate 1 win for sure with an outcome very close to his own ideal outcome, it is a profitable deviation.

The key reason that Candidate 1's deviation is profitable is that Candidate 2 has revealed the optimal minimum wage to the voter. Suppose, on the other hand, that Candidate 2 is vague on the minimum wage. Since this is Candidate 1's biased issue, irrespective the policy that he proposes, the voter is free to believe that it is in fact his own ideal policy. Therefore being vague about one's opponent's biased issue prevents undercutting on the part of the opponent.

3 Relation to Existing Literature

To my knowledge, this is the first paper to address competition for delegation in a multidimensional issue space. Ambrus et al. (2015), the first study of competition for delegation, uses a one-dimensional setup. Battaglini (2002) studies a multidimensional setting, but communication takes the form of cheap talk. Neither studies the role of vagueness in information disclosure.

Early work on quality disclosure (Grossman, 1981; Milgrom, 1981) provides an "unraveling result" that informed parties should always voluntarily disclose their qualities. Unraveling obtains partly because, in their papers, products are vertically differentiated along a single dimension of quality (Dranove and Jin, 2010). In the present paper, agents cannot disclose outcomes, only their actions. The DM remains ignorant of what the outcomes of those actions will be and need to infer them. Therefore this paper illustrates one instance in which the unraveling argument breaks down.

On the other hand, the question of what drives parties' campaign issue choice has been studied extensively. Early evidence and "issue-ownership" theory suggest that parties focus on issues on which they have an advantage and ignore those on which their opponents have an advantage (Riker, 1993; Petrocik, 1996). Historical counterexamples and more recent empirical evidence show, however, that parties do spend considerable effort on issues advantageous to the opposing party (Sigelman and Buell, 2004).

Some papers seek to reconcile this conflict by focusing on how campaigns shift voters' preferences (Amorós and Puy, 2013; Aragonès et al., 2015; Dragu and Fan, 2015). I take a more traditional approach, assuming that the voter's preference is fixed. As it turns out, incomplete information alone can get a sharper result. Krasa and Polborn (2010) study a game in which specialized candidates choose future efforts on each policy area if elected. They specify precise conditions for voters' preferences that determine when parties should focus on the same or different issues. The driving force of my model is the revelation of information rather than the voter's preference.

I join Ash et al. (2015) and Egorov (2015) in attributing issue choice to information asymmetry. Ash et al. (2015) study how an incumbent's desire to signal his policy preference leads him to pursue issues on which the voters disagree. In contrast, I study a competition setting in which candidates simultaneously make proposals. Moreover, the uncertainty in my model is a policy-relevant state of the world. Lastly, I focus on how a candidate's issue advantage, rather than issues' relative divisiveness, affects the issue choice. Since there is only one voter, divisiveness is not a factor. Egorov (2015) studies how the desire to signal competence makes a challenger signal on the same issue following an incumbent's issue choice. He assumes that voters get better information when both signal on the same issue. As a result, a challenger may have an incentive to signal on an issue that he is less competent on, if this gives the voters better information. In my paper, the informativeness of a proposal is endogenously determined in equilibrium. Therefore the underlying intuition in my model is equilibrium reasoning rather than the trade-off for candidates between the favorableness and informativeness of signals.

Other papers model the problem of campaign issue choice through channels other than information asymmetry. The mechanisms they offer include social agreement on an issue as well as its social discontent (Colomer and Llavador, 2012), non-expected utility (Berliant and Konishi, 2005), and shifting of the issue focus in the re-election campaign (Glazer and Lohmann, 1999). In my paper, there is only one voter so there is always social consensus. All players are expected utility maximizers. The game is one-shot instead of sequential. Therefore we offer completely different explanations.

There are papers that study vagueness[2] in a one-dimensional issue space. To formalize the concept of vagueness, Meirowitz (2005) as mentioned before uses the same setup as mine and models vagueness as a complete lack of policy commitment. Others model vagueness as a commitment to a set of policies instead of a single policy (Alesina and Cukierman, 1990; Aragonès and Neeman, 2000; Aragonès and Postlewaite, 2002; Alesina and Holden, 2008;

[2]Vagueness in the political science literature is usually labeled as ambiguity. To avoid confusion with ambiguity in the decision theory literature, I use the term vagueness.

Callander and Wilson, 2008; Kamada and Sugaya, 2014). Several papers examine the role of vagueness in coordination and signaling games (De Jaegher, 2003; Lipman, 2009; Serra-Garcia et al., 2011; Agranov and Schotter, 2012; Blume and Board, 2014). Similarly, I also show that vagueness can be supported in equilibrium. In addition, the multidimensional setup allows me to pinpoint the dimension on which vagueness is likely to occur.

Broadly, this paper belongs to the literature that studies the communication of private information in a competition setting. Communication can take different forms. Many papers, this paper included, study communication through binding policy proposals (Roemer, 1994; Schultz, 1996; Martinelli, 2001; Martinelli and Matsui, 2002; Heidhues and Lagerlöf, 2003; Laslier and Van der Straeten, 2004; Loertscher, 2012; Morelli and Van Weelden, 2013; Jensen, 2013; Kartik et al., 2015; Ambrus et al., 2015). Others study communication through cheap talk (Austen-Smith, 1990; Krishna and Morgan, 2001; Battaglini, 2002, 2004; Schnakenberg, 2014; Kartik and Van Weelden, 2014). Banks (1990) and Callander and Wilkie (2007) analyze models in which misrepresenting one's policy intentions is costly. Gentzkow (2011) and Gul and Pesendorfer (2012) study the provision of information by competing persuaders.

4 The Model

A policy decision needs to be made. A decision maker (DM), who is unable to make the decision, has to delegate it to one of two agents, Agent 1 and Agent 2. The consequences of the decision depend on a random variable called the state. The state is observed by the agents but not by the DM. Agents simultaneously announce proposals. Based on the proposals, the DM chooses an agent. The chosen agent then implements his proposal.

The state, denoted by $\theta = (\theta^a, \theta^b)$, concerns two issues: θ^a is the state on issue a and θ^b on issue b. θ is distributed over $\Theta \equiv \mathbb{R}^2$ according to some continuous distribution function F with density f. f has full support on $\mathbb{R}^2$. The space of proposals is $M = (Y \cup \{\varnothing\})^2$, where $Y \equiv \mathbb{R}$ denotes the action space and $\varnothing$ denotes vagueness. For $i = 1, 2$, $\mathbf{m_i} = (m_i^a, m_i^b)$

denotes Agent i's proposal and $\mathbf{m} = (\mathbf{m_1}, \mathbf{m_2})$ the proposal profile.

If Agent i is chosen, he implements an action $y_i = (y_i^a, y_i^b)$. For each issue $k \in \{a, b\}$, the set of feasible actions depends on m_i^k as follows. If $m_i^k \in Y$, then $y_i^k = m_i^k$; if $m_i^k = \varnothing$, then $y_i^k \in Y$. Therefore when an agent proposes an action, he has full commitment to this action. When he is vague, he has full freedom to implement any action.

The DM and the agents have conflicting interests. When the state is θ and the action is y, the DM's payoff is

$$u_d(\theta, y) = -(\theta^a - y^a)^2 - (\theta^b - y^b)^2.$$

Agent 1 and 2's payoffs are

$$u_1(\theta, y) = -(\theta^a - y^a)^2 - (\theta^b - y^b - 1)^2,$$
$$u_2(\theta, y) = -(\theta^a - y^a - 1)^2 - (\theta^b - y^b)^2.$$

Given the state (θ^a, θ^b), the DM's ideal action equals to the state (θ^a, θ^b). Agent 1's ideal action is equal to $(\theta^a, \theta^b - 1)$. Agent 2's ideal action is equal to $(\theta^a - 1, \theta^b)$. Issue a is called Agent 1's *unbiased issue* and Agent 2's *biased issue*; the converse is true for issue b. The rules of the game, all players' utility functions (and therefore their ideal actions conditional on the state), and the state distribution are common knowledge.

Discussion of the roles of various assumptions are relegated to Section 10. Here I discuss how the model corresponds to real-life situations. First consider the two-candidate electoral competition. I take the view of Austen-Smith and Riker (1987) that policies and outcomes are different objects. A policy combined with a state gives an outcome, over which the candidates and the voter's payoffs are defined.[3] The voter knows the connection between policies and outcomes only with uncertainty. Candidates, on the other hand, are informed. This is because they have more incentives to acquire information, as well as access to some information unavailable to the public. For example, when considering implementing a health

[3]See Kartik and McAfee (2007) for an alternative modeling choice. In their model, some candidates with "characters" care directly about policy platforms per se.

insurance plan, one needs to know its cost to hospitals, insurance companies and tax payers, as well as its social benefits. This information is summarized by the state. Although candidates' ideals about the interests of different groups may be well-known, their knowledge regarding this information may not be.

A second key assumption is candidates' full commitment to policy promises. Evidence on this is somewhat conflicting. On the one hand, well-known broken campaign promises abound. On the other hand, studies show that although there are no regulations punishing candidates' flip-flopping, in reality elected presidents try to carry out their campaign promises (Fishel, 1985; Krukones, 1984). Moreover, Tomz and Van Houweling (2009) presents empirical evidence demonstrating that voters do punish candidates for flip-flopping. Voters are shown to react negatively to the flip-flopping itself, as well as to discount the current promises of elected candidates who flip-flopped in the past.

The model also applies to some situations in which the DM lacks the ability to take action and has to delegate. Examples include a patient choosing between two doctors, and a CEO choosing between two divisions in her company to delegate a project to. In these examples, the optimal decision often depends on many different factors. Although the DM knows, for example, that low costs and high performance are desirable, she may not understand the trade-offs exactly. Consequently she does not know the optimal action to take. The agents providing the services are likely more informed about these trade-offs. But their private interests may be misaligned with that of the DM in certain ways. I assume that the DM knows the agents' preferences. A CEO, for example, may learn through word-of-mouth that one division is more dedicated to cost-saving while the other one is more dedicated to performance. Lastly, the assumption that proposals are binding corresponds to common law (see Treitel, 2003).

5 Strategies and Equilibrium

In the last stage of the game, a chosen agent implements his ideal policy for any issue that he has been vague on. Therefore the state θ and Agent i's proposal $\mathbf{m_i}$ together determine everyone's payoff from the DM choosing Agent i. This means that an agent's strategy is fully captured by his proposal at each state: once a proposal is stated, the action upon delegation is fully determined.

A pure strategy for Agent i is a function $s_i : \Theta \to M$ mapping states into proposals. A strategy for the DM is a function $\beta : M \times M \to [0, 1]$ mapping proposal profiles into the probabilities that Agent 1 is selected. The DM's posterior belief $\mu(\cdot \mid \mathbf{m})$ maps observed proposal profiles into distributions over Θ.

I focus on a class of Perfect Bayesian equilibria (PBE) $\big(s_1, s_2, \beta, \mu(\cdot \mid \mathbf{m})\big)$ in which agents play pure strategies and the DM randomizes $50-50$ whenever she is indifferent between the agents. Any PBE leaves the DM's off-path beliefs, i.e. beliefs conditional on off-path proposal profiles from the agents unspecified. As a result, some PBEs are sustained by implausible off-path beliefs held by the DM. To rule out such equilibria, I propose the following equilibrium refinement.

I extend the intuitive criterion by CK to the two-agent case. The intuitive criterion applies to the case when there is only one agent and restricts the domain of the DM's off-path beliefs. According to the intuitive criterion, the DM assigns positive probabilities only to certain states, those at which the highest payoff a deviator can get is at least as high as his equilibrium payoff. No restrictions are placed on the DM's beliefs conditional on on-path deviations by the agent, since those beliefs are specified by equilibrium. However, two complications arise when there are two agents. First, since the DM observes two proposals, one from each agent, even an on-path deviation by one agent may lead to an off-path proposal profile. This is the case when both agents send on-path proposals, but the proposal profile is not an equilibrium profile for any state. Therefore, off-path profiles due to unilateral deviations include those resulting from off-path as well as possibly on-path deviations. Off-path profiles due to bilateral deviations need not be considered because beliefs conditional on those profiles do not play a role in sustaining the equilibrium.

Second, after a unilateral deviation, the DM should extract the information from the non-deviator's proposal and use it to infer the state. The question is how to identify the non-deviator. There are effectively two cases to consider. When one proposal is on-path and the other is off-path, the DM believes that the on-path agent has not deviated. This is necessarily true if indeed only one agent has deviated. Moreover, for some equilibria such as that in Section 2, this belief is justified since an on-path deviation is never profitable.[4] When both proposals are on-path but the proposal profile is off-path, the DM cannot determine the deviator and therefore considers both possibilities.

After resolving the complications above, I apply the idea of CK. More specifically, when one agent sends an on-path proposal while the other sends an off-path proposal, the DM's belief about the state is supported on the intersection of two sets. The first set contains the states at which the on-path agent's observed proposal is his equilibrium proposal. The second set contains the states at which the off-path agent's equilibrium payoff is weakly lower than the highest payoff he can get by deviating to the observed off-path proposal. When both agents send on-path proposals but the proposal profile is off-path, the DM's belief about the state is supported on the union of two sets. The first set is the intersection of the two sets above, with Agent 1 being the agent playing according to the equilibrium and Agent 2 being the deviator. The second set is the intersection of the same two sets above, with Agent 2 being the agent playing according to the equilibrium and Agent 1 being the deviator. Whenever the support of the DM's belief is empty, I place no restrictions on her beliefs and allows it to be arbitrary.

To state the refinement, I need some additional notation. Given the DM's belief $\mu(\cdot \mid \mathbf{m})$ conditional on observing $\mathbf{m}$, let π_i denote the DM's expected payoff from choosing Agent i:

[4]Two-sender signaling equilibrium refinements similar to mine were proposed by Bagwell and Ramey (1991) and Schultz (1996). See Section 10 for a discussion.

$$\pi_i = \sum_{\theta \in \Theta} \mu(\theta \mid \mathbf{m}) \cdot u_d\big(\theta, y_i(\theta, \mathbf{m_i})\big).$$

Let

$$\widetilde{\mathrm{BR}}\,(\mu, \mathbf{m}) = \begin{cases} \{0\} & \text{if } \pi_1 < \pi_2, \\ \{\frac{1}{2}\} & \text{if } \pi_1 = \pi_2, \\ \{1\} & \text{if } \pi_1 > \pi_2, \end{cases}$$

denote the DM's best response given $\mu(\cdot \mid \mathbf{m})$ and $\mathbf{m}$.

For a non-empty subset T of Θ, let $\mathrm{BR}\,(T, \mathbf{m})$ be the set of the DM's best responses to $\mathbf{m}$ given beliefs $\mu(\cdot \mid \mathbf{m})$ that concentrate on T:

$$\mathrm{BR}\,(T, \mathbf{m}) = \bigcup_{\mu:\mu(T|\mathbf{m})=1} \widetilde{\mathrm{BR}}\,(\mu, \mathbf{m}),$$

For a PBE $\big(s_1, s_2, \beta, \mu(\cdot \mid \mathbf{m})\big)$, let $u_i^*(\theta)$ with $i \in \{1, 2\}$ denote Agent i's equilibrium payoff at state θ. Similarly, $u_d^*(\theta)$ denotes the DM's equilibrium payoff at state θ. Given a proposal $\mathbf{n} = (n^a, n^b)$ and $l \in \{1, 2, d\}$, I use $u_l\big(\theta, y_i(\mathbf{n})\big)$ to denote l's payoff from at state θ when Agent i is chosen to implement his proposal $\mathbf{n}$. For any equilibrium, a proposal profile $\mathbf{m}$ is off-path if there is no θ such that $\big(s_1(\theta), s_2(\theta)\big) = \mathbf{m}$; on-path otherwise. Similarly, $\mathbf{m_i}$ is off-path if there is no θ such that $s_i(\theta) = \mathbf{m_i}$; on-path otherwise. Let Agent j denote Agent i's opponent.

For each $\mathbf{m} = (\mathbf{m_1}, \mathbf{m_2})$ and each $i = 1$, 2, define

$$\Theta^i(\mathbf{m}) = \left\{\theta \mid s_j(\theta) = \mathbf{m_j}, u_i^*(\theta) \le \max_{\beta \in \mathrm{BR}\,(\Theta, \mathbf{m})} \beta u_i\big(\theta, y_i(\mathbf{m_i})\big) + (1-\beta) u_i\big(\theta, y_j(\mathbf{m_j})\big)\right\}. \quad (1)$$

$\Theta^i(\mathbf{m})$ is the set of states at which Agent j's equilibrium proposal is $\mathbf{m_j}$ and Agent i has deviated to $\mathbf{m_i}$. Moreover, the highest possible payoff Agent i can get by deviating to $\mathbf{m_i}$ is weakly higher than his equilibrium payoff. The payoff Agent i gets by deviating is calculated given that the DM best-responds to $\mathbf{m}$ given beliefs supported on Θ, the entire state space.

Definition 1 *(Equilibrium dominance) A belief system* $\mu(\cdot \mid \mathbf{m})$ *satisfies equilibrium dominance if the following conditions are satisfied for each off-path proposal profile* $\mathbf{m} = (\mathbf{m_1}, \mathbf{m_2})$.

(1) *If* $\mathbf{m_i}$ *is off-path while* $\mathbf{m_j}$ *is on-path, then* $\mu \in \Delta\big(\Theta^i(\mathbf{m})\big)$ *whenever* $\Theta^i(\mathbf{m})$ *is nonempty.*

(2) *If both* $\mathbf{m_1}$ *and* $\mathbf{m_2}$ *are on-path, then* $\mu \in \Delta\big(\Theta^1(\mathbf{m}) \cup \Theta^2(\mathbf{m})\big)$ *whenever* $\Theta^1(\mathbf{m}) \cup \Theta^2(\mathbf{m})$ *is nonempty.*

Whenever a unilateral deviation has indeed occurred, the DM suspects only unilateral deviations. Cases (1) in Definition 1 pertains to the situation in which exactly one agent makes an off-path proposal. Case (2) pertains to the situation in which each agent makes an on-path proposal but the proposal profile is off-path. For both cases, the DM believes that the highest possible payoff the deviator can get is weakly higher than his equilibrium payoff. The deviator's payoff is computed assuming that the DM does not choose dominated actions. These two cases are the only possible types of proposal profiles following a unilateral deviation.

A PBE $\big(s_1, s_2, \beta, \mu(\cdot \mid \mathbf{m})\big)$ is called an *intuitive equilibrium* if $\mu(\cdot \mid \mathbf{m})$ satisfies equilibrium dominance. A deviation by Agent i is called *equilibrium-dominated* if the inequality in Equation 1 is violated.

6 Main Results

The first result establishes the existence of an intuitive equilibrium in which both agents are vague on their own unbiased issues, which are their opponents' biased issues. Proofs are collected in Section 9.

Proposition 1 *There exists an intuitive equilibrium in which each agent is vague on his unbiased issue and commits on his biased issue at all states.*

The equilibrium is as follows: at each state, each agent commits to his ideal policy for his biased issue. For each on-path proposal profile, the DM randomizes 50-50 between agents.

For every off-path proposal profile that results from a unilateral deviation, the DM believes that the on-path agent has not deviated. In particular, if the off-path agent deviates to commit on both issues, then the DM believes that he is proposing his own ideal action on his biased issue.

It is easy to see that the DM's beliefs satisfy equilibrium dominance. Moreover, given the DM's beliefs, a deviating agent's highest probability of winning is $\frac{1}{2}$. Since this is the same as his equilibrium winning probability and he proposes his own ideal in equilibrium, no agent has an incentive to deviate.

To further characterize equilibria, I focus on a class of intuitive equilibria that satisfy two conditions. The first condition, *symmetry*, states that an agent's choice of vagueness depends on his bias in the same way as his opponent's.

Assumption 1 (Symmetry) *A PBE $\big(s_1, s_2, \beta, \mu(\cdot \mid \mathbf{m})\big)$ satisfies symmetry if at any state θ, the followings hold:*

(1) *if $s_1(\theta) = (\varnothing, \varnothing)$, then $s_2(\theta) = (\varnothing, \varnothing)$;*

(2) *if $s_1(\theta) = (\varnothing, y)$ for some $y \in Y$, then $s_2(\theta) = (z, \varnothing)$ for some $z \in Y$;*

(3) *if $s_1(\theta) = (y, \varnothing)$ for some $y \in Y$, then $s_2(\theta) = (\varnothing, z)$ for some $z \in Y$.*

In other words, if Agent i is vague on his own biased issue, then Agent j is also vague on his own biased issue. If Agent i is vague on his own unbiased issue, then Agent j is also vague on his own unbiased issue. Similarly when Agent i commits on an issue. This assumption will be lifted in the next section, where I discuss the results in a smaller state space. The second condition, *invariance*, states that each agent's choice of vagueness coincides across all states.

Assumption 2 (Invariance) *A PBE $\big(s_1, s_2, \beta, \mu(\cdot \mid \mathbf{m})\big)$ satisfies invariance if for each $i = 1,\ 2$ and for each $j = a,\ b$, the followings hold:*

(1) *if for some $\overline{\theta}$, $s_i^k = \varnothing$, then $s_i^k(\theta) = \varnothing$ for all θ;*

(2) *if for some* $\overline{\theta}$, $s_i^k \neq \varnothing$, *then* $s_i^k(\theta) \neq \varnothing$ *for all* θ.

In other words, if an agent is vague on an issue at some state, then he is vague on this issue at all states. Similarly, if an agent commits on an issue at some state, then he commits on this issue at all states. This assumption may be interpreted as follows: before learning the state, each agent chooses for each issue whether to be vague on it. Then each agent observes the state and chooses his policy for the issue(s) that he has chosen to be specific about. Under either this specification of the game or the specification that has been used in the paper so far, the next result states that there is no equilibria in which agents are specific on their unbiased issues.

Proposition 2 *In any intuitive equilibrium that satisfies symmetry and invariance, an agent who is vague on some issue must be vague on his unbiased issue.*

The proof rules out the following as agents' intuitive equilibrium strategies: for each θ,

$$s_1(\theta) = (y(\theta), \varnothing),$$
$$s_2(\theta) = (\varnothing, z(\theta)),$$

where y, $z : \Theta \to \mathbb{R}$. There are essentially three steps to rule out the above strategies. First, in equilibrium each agent wins with probability $\frac{1}{2}$. Second, each agent proposes his ideal policy on his unbiased issue. Lastly, at each state θ, Agent 1 has an incentive to deviate to $(\varnothing, z(\theta) - 1 + \varepsilon)$ where ε is a very small positive number. The last step follows the same intuition as that used in Section 2.

Proposition 2 shows that being specific on one's unbiased issue makes one vulnerable in two ways: it allows opponent's undercutting as well as mimicking. First, if an agent reveals the state for his unbiased issue, it allows his opponent to anchor on this information and offer a concession to the DM. For this type of anchoring to work, it is essential that after one agent has deviated, the DM continues to trust the non-deviating agent and uses his proposal to evaluate the deviator's proposal. This is guaranteed by the refinement. Second, by being specific on his unbiased issue and vague on his biased issue, an agent is prone to his opponent's mimicking. For example, whenever Agent 1 proposes $(y, \varnothing)$, Agent 2 can make the exact same proposal and wins for sure. This is because on issue a, both agents commit to the same policy. For issue b, on which both agents are vague and therefore implement their own ideal policies, Agent 2 beats Agent 1 (since it is Agent 2's unbiased issue and Agent 1's biased issue). As a result, the same proposal from both agents gives different outcomes. In order to make Agent 2 unwilling to mimic Agent 1 and win, it must be that the payoff from mimicking is sufficiently low. This intuition guarantees that in equilibrium, whenever agents commit on their unbiased issues, they must commit on their own ideal policies for those issues. This guarantees state-revelation and provides the prerequisite for undercutting.

Proposition 1 and 2 illustrate two points. First, vagueness arises as a natural consequence of competition through information disclosure. This can be seen from the existence of equilibrium with vagueness. Second, although vagueness on either issue is strategically possible, it is more likely to arise on agents' unbiased issues. This can be seen by observing that the equilibrium in which agents are vague on their unbiased issues survives the refinement but the one in which agents are specific on their unbiased issues does not. In other words, when an agent has an advantage on an issue, he should preserve this advantage by being vague on it.

7 General Strategies in a Binary State Space

In order to relax the assumption that agents' choices of vagueness are symmetric, I study the following setup as a special case: θ is uniformly distributed over $\Theta = \{(0,0),(0,1),(1,0),(1,1)\}$. The policy space Y is $[-1,1]$. Therefore the proposal space is $M = ([-1,1] \cup \{\varnothing\})^2$. The next proposition illustrates the effect of biases on where vagueness is likely to occur. All proofs are collected in Section 9.

Proposition 3

(1) *There exists an intuitive equilibrium in which each agent is vague on his unbiased issue and commits on his biased issue at all states.*

(2) *There does not exist an intuitive equilibrium in which, at some state, each agent is vague on his biased issue and commits on his unbiased issue.*

For part (1), I construct a semi-revealing equilibrium in which the DM learns the state at $(1, 1)$, but assigns equal probabilities to the rest of the states when the state is not $(1, 1)$. The construction relies on the following two properties of any equilibrium. First, if both agents propose the same policy at a state, then no deviation is equilibrium-dominated. Even if Agent i has deviated to a less-preferred policy, as long as the DM selects Agent i with zero probability, he gets the same payoff as in equilibrium. Second, let $\mathbf{m}$ be an equilibrium proposal at some state such that: (i) there is only one state at which Agent i proposes $\mathbf{m_i}$, and (ii) $\mathbf{m_j}$ is off-path. Then Agent j's deviation does not alter the DM's belief about the state. In other words, the DM continues to believe that the state is the one at which Agent i proposes $\mathbf{m_i}$.

Part (2) follows the same steps as those in Proposition 2. The only difference is that with the restricted state space, I do not need to resort to equilibrium refinement to rule out such strategies. After showing that each agent, when he commits, must commit to his own ideal policy, I show that there is a profitable deviation for Agent 1 regardless of the off-path beliefs of the DM. In other words, the above strategy profile is not a PBE strategy profile. The next result characterizes intuitive equilibria that satisfies invariance.

Proposition 4 *In any intuitive equilibrium that satisfies invariance, an agent who is vague on some issue must be vague on his unbiased issue.*

I prove this by first supposing that Agent 1 is vague on his biased issue and commits on his unbiased issue. Then, by going through each possible choice of vagueness by Agent 2, I show that the proposal profile cannot be sustained in equilibrium. The case in which Agent 2 is vague on his biased issue and commits on his unbiased issue is symmetric.

The reason for the stronger result to obtain is partly because the domain of the DM's belief is restricted. This makes it possible to go through the different possibilities of the DM's posterior beliefs. Moreover, since there are altogether only four different states, each agent can only make four possibly different proposals. This makes it easy to pin down agents' equilibrium strategies based on the posterior beliefs of the DM.

8 General Environments

In this section, I show that the undercutting intuition applies to general environments. I focus on two generalizations: one in which there are N issues in total and Agent i's bias is captured by a vector $b_i \in \mathbb{R}^N$, another in which the DM values one issue more than the other.

8.1 N issues and general biases

I focus on the case in which for each issue $j \in N$, there is $i \in \{1, 2\}$ such that $|b_i^j| < |b_{-i}^j|$. That is, for each issue, one agent is more advantaged than his opponent. I call such issue j Agent i's advantaged issue and Agent $-i$'s disadvantaged issue. The following result shows that in this environment, agents should not be vague on their disadvantaged issues and commit on their advantaged issues.

Proposition 5 *For $i \in \{1, 2\}$, suppose that $s_i(\theta)$ satisfies the following conditions: $\forall j \in N$ such that $|b_i^j| < |b_{-i}^j|$, $s_i^j(\theta) \in \mathbb{R}$; otherwise $s_i^j(\theta) = \varnothing$. Then (s_1, s_2) is not part of an intuitive equilibrium.*

The intuition of the result is similar as the previously-analyzed environment with two issues and completely (mis)aligned agents. First, if such (s_1, s_2) is part of an intuitive equilibrium, then the outcome must be a tie, otherwise the losing agent can mimic the winning agent's proposal and win. Second, each specific commitment must be its proposer's ideal policy. To see this, suppose that Agent i commits on issue j and that his commitment is not equal to $\theta^j - b_i^j$. Then let Agent i deviate to be vague on all issues. The DM then believes that Agent i will implement his own ideal policy, whereas Agent $-i$ will at best implement his own ideal policy. Therefore Agent i's probability of winning after the deviation is no worse than in equilibrium and his winning proposal is strictly better than in equilibrium. Lastly, given that specific commitment is revealing about the state, the undercutting argument destroys the equilibrium.

8.2 General preferences of the DM

Now we focus back on the case in which there are two issues and agents' ideal outcomes are $(0, 1)$ and $(1, 0)$. Let the DM's payoff function be $u_d(\theta, y) = -(\theta^a - y^a)^2 - \alpha(\theta^b - y^b)^2$, where $0 < \alpha < 1$. Agents' payoff functions are as before:

$$u_1(\theta, y) = -(\theta^a - y^a)^2 - (\theta^b - y^b - 1)^2, u_2(\theta, y) = -(\theta^a - y^a - 1)^2 - (\theta^b - y^b)^2.$$

Since $u_d(\theta, y_1(\varnothing, \varnothing)) = -\alpha$, $u_V(\theta, y_2(\varnothing, \varnothing)) = -1$, $\forall\theta$, Agent 1 has an overall advantage. The next result shows that in this case, agents should not be vague on their disadvantaged issues and commit on their advantaged issues.

Proposition 6 *Suppose that $s_1(\theta) = (y(\theta), \varnothing)$, $s_2(\theta) = (\varnothing, z(\theta))$, $\forall\theta$, where $y, z : \Theta \to \mathbb{R}$. Then s_1, s_2 is not part of an intuitive equilibrium.*

The intuition is simple. Same as before, the outcome must be a tie, which means that Agent 1 cannot get his own ideal policy with probability 1. Now let Agent 1 deviate to be vague on both issues. Then the DM believes that Agent 1 will implement his own ideal policy, while Agent 2 will at best implement his own ideal policy, which is strictly less preferred to Agent 1's. Therefore Agent 1 wins with probability 1 and gets his own ideal policy.

9 Proofs

Throughout the proof, I use $u_1^{dev}(\theta, \mathbf{m_1}^{\mathbf{dev}}, \mathbf{m_2})$ to denote Agent 1's payoff after deviating to $\mathbf{m_1}^{\mathbf{dev}}$ at state θ, at which Agent 2's equilibrium proposal is $\mathbf{m_2}$. Similarly, I use $u_2^{dev}(\theta, \mathbf{m_1}, \mathbf{m_2}^{\mathbf{dev}})$ to denote Agent 2's payoff after deviating to $\mathbf{m_2}^{\mathbf{dev}}$ at state θ, at which Agent 1's equilibrium proposal is $\mathbf{m_1}$.

9.1 A Useful Lemma

The next lemma shows that if an agent wins with positive probability, then he weakly prefers the outcome from his own proposal to that from his opponent's proposal.

Lemma 1 *Let* $\mathbf{m}$ *be any on-path equilibrium proposal profile.*

If $\beta(\mathbf{m}) > 0$*, then* $u_1(\theta, y_1(\mathbf{m_1})) \geq u_1(\theta, y_2(\mathbf{m_2}))$*.*

If $\beta(\mathbf{m}) < 1$*, then* $u_2(\theta, y_2(\mathbf{m_2})) \geq u_2(\theta, y_1(\mathbf{m_1}))$*.*

Proof of Lemma 1. I prove the result for 1 only; 2 is symmetric.

Fix any θ. Let $(s_1(\theta), s_2(\theta))$ be denoted by $\mathbf{m}$. Suppose that $\beta(\mathbf{m}) > 0$ and $u_1(\theta, y_1(\mathbf{m_1})) < u_1(\theta, y_2(\mathbf{m_2}))$. Therefore $u_1^*(\theta) < u_1(\theta, y_2(\mathbf{m_2}))$. Now let Agent 1 deviate to $\mathbf{m_1}^{\mathbf{dev}} = (\varnothing, \varnothing)$. Then $u_1^{dev}(\theta, \mathbf{m_1}^{\mathbf{dev}}, \mathbf{m_2}) \geq u_1(\theta, y_2(\mathbf{m_2})) > u_1^*(\theta)$. ■

9.2 Proof of Proposition 1

Consider the following equilibrium. Agents' strategies are as follows: for each θ,

$$s_1(\theta) = (\varnothing, \theta^b - 1),$$
$$s_2(\theta) = (\theta^a - 1, \varnothing).$$

The DM's belief and strategy is as follows:

1. For any $\mathbf{m} = ((\varnothing, y), (z, \varnothing))$ where y, $z \in \mathbb{R}$, $\mu(z+1, y+1) = 1$ and $\beta(\mathbf{m}) = \frac{1}{2}$.

2. For any $\mathbf{m} = ((\varnothing, \varnothing), (z, \varnothing))$ where $z \in \mathbb{R}$, $\mu\{\tilde{\theta} \mid \tilde{\theta}^a = z + 1\} = 1$ and $\beta(\mathbf{m}) = \frac{1}{2}$.

3. For any $\mathbf{m} = ((y, \varnothing), (z, \varnothing))$ where $y, z \in \mathbb{R}$, $\mu\{\tilde{\theta} \mid \tilde{\theta}^a = z + 1\} = 1$. If $y = z + 1$, then $\beta(\mathbf{m}) = \frac{1}{2}$; otherwise $\beta(\mathbf{m}) = 0$.

4. For any $\mathbf{m} = ((y, w), (z, \varnothing))$ where $y, w, z \in \mathbb{R}$, $\mu\{\tilde{\theta} \mid \tilde{\theta}^a = z + 1, \tilde{\theta}^b = w + 1\} = 1$. If $y = z + 1$, then $\beta(\mathbf{m}) = \frac{1}{2}$; otherwise $\beta(\mathbf{m}) = 0$.

5. For any $\mathbf{m} = ((\varnothing, y), (\varnothing, \varnothing))$ where $y \in \mathbb{R}$, $\mu\{\tilde{\theta} \mid \tilde{\theta}^b = y + 1\} = 1$ and $\beta(\mathbf{m}) = \frac{1}{2}$.

6. For any $\mathbf{m} = ((\varnothing, y), (\varnothing, z))$ where $y, z \in \mathbb{R}$, $\mu\{\tilde{\theta} \mid \tilde{\theta}^b = y + 1\} = 1$. If $y = z - 1$, then $\beta(\mathbf{m}) = \frac{1}{2}$; otherwise $\beta(\mathbf{m}) = 1$.

7. For any $\mathbf{m} = ((\varnothing, y), (w, z))$ where $y, w, z \in \mathbb{R}$, $\mu\{\tilde{\theta} \mid \tilde{\theta}^a = w + 1, \tilde{\theta}^b = y + 1\} = 1$. If $y = z - 1$, then $\beta(\mathbf{m}) = \frac{1}{2}$; otherwise $\beta(\mathbf{m}) = 1$.

8. For any other $\mathbf{m}$, μ is defined arbitrarily and $\beta(\mathbf{m})$ is required to be best-response to $\mathbf{m}$ against μ.

It is easy to see that the DM is best-responding against μ. To see that no agents have incentives to deviate, consider the possible deviations by Agent 1 given that Agent 2 is playing according to the equilibrium (Agent 2 is symmetric):

(a) If Agent 1 deviates to $(\varnothing, y)$ where $y \neq \theta^b - 1$, the DM does not detect the deviation and continues to randomize 50-50. Therefore his probability of winning remains unchanged and he gets a worse outcome than his winning outcome in equilibrium when he wins.

(b) If Agent 1 deviates to $(\varnothing, \varnothing)$, his probability of winning remains unchanged and he gets the same winning outcome as that in equilibrium.

(c) If Agent 1 deviates to $(y, \varnothing)$ where $y \in \mathbb{R}$, he wins with positive probability only when y is the true state θ^a. His winning outcome is the same as that in equilibrium.

(d) If Agent 1 deviates to (y, w) where $y,\ w \in \mathbb{R}$, the highest payoff from deviating is that when he deviates to his own ideal policies for both issues. Since his winning probability is the same as that in equilibrium, he gets the same payoff by deviating.

None of the deviations render a higher-than-equilibrium payoff for Agent 1. Therefore the above strategies constitutes a PBE. To see it is also an intuitive equilibrium, note than whenever one agent is on-path and the other is off-path, the DM believes that the on-path agent has not deviated and the off-path agent has deviated to his own ideal policy, whenever believing so is possible. Therefore the above strategies constitutes an intuitive equilibrium.

9.3 Proof of Proposition 2

I give a proof by contradiction. Suppose that the following are agents' intuitive equilibrium strategies: for each θ,

$$s_1(\theta) = (y(\theta), \varnothing),$$
$$s_2(\theta) = (\varnothing, z(\theta)),$$

where $y,\ z : \Theta \to \mathbb{R}$.

Step 1. $\beta(s_1(\theta), s_2(\theta)) = \frac{1}{2}$ for all θ.

Suppose that $\beta(s_1(\theta), s_2(\theta)) = 0$ for some θ. Then 1 has a profitable deviation $s_2(\theta)$. To see this, first note that

$$u_1^*(\theta) = u_1((\theta^a, \theta^b), y_2(\varnothing, z(\theta))) = u_1((\theta^a, \theta^b), (\theta^a - 1, z(\theta))).$$

Now let 1 deviate to $s_2(\theta)$. Given $\mathbf{m} = (\mathbf{m_1}^{\mathbf{dev}}, s_2(\theta)) = ((\varnothing, z(\theta)), (\varnothing, z(\theta)))$ and any $\mu \in \Delta(\Theta)$, we have that $\pi_1 > \pi_2$. Therefore $\beta(\mathbf{m}) = 1$ and $u_1^{dev}(\theta, \mathbf{m_1}^{\mathbf{dev}}, s_2(\theta)) = u_1(\theta, y_1(\varnothing, z(\theta))) = u_1((\theta^a, \theta^b), (\theta^a, z(\theta))) > u_1^*(\theta)$.

Similarly, if $\beta(s_1(\theta), s_2(\theta)) = 1$ for some θ, then 2 has a profitable deviation $s_1(\theta)$.

Step 2. $y(\theta) = \theta^a$, $z(\theta) = \theta^b$ for all θ.

Suppose that $y(\theta) \neq \theta^a$ for some θ. Then 1 has a profitable deviation $(\varnothing, \varnothing)$. To see this, first note that

$$\begin{aligned} u_1^*(\theta) &= \frac{1}{2} u_1(\theta, y_1(y(\theta), \varnothing)) + \frac{1}{2} u_1(\theta, y_2(\varnothing, z(\theta))) \\ &< \frac{1}{2} u_1((\theta^a, \theta^b), (\theta^a, \theta^b - 1)) + \frac{1}{2} u_1(\theta, y_2(\varnothing, z(\theta))) \end{aligned}$$

Now let 1 deviate to $\mathbf{m_1}^{\mathbf{dev}} = (\varnothing, \varnothing)$. Given $\mathbf{m} = (\mathbf{m_1}^{\mathbf{dev}}, s_2(\theta)) = ((\varnothing, \varnothing), (\varnothing, z(\theta)))$ and any $\mu \in \Delta(\Theta)$, $\pi_1 \geq \pi_2$. Therefore $\beta(\mathbf{m}) \geq \frac{1}{2}$ and

$$\begin{aligned} u_1^{dev}(\theta, \mathbf{m_1}^{\mathbf{dev}}, s_2(\theta)) &\geq \frac{1}{2} u_1(\theta, y_1(\varnothing, \varnothing)) + \frac{1}{2} u_1(\theta, y_2(\varnothing, z(\theta))) \\ &= \frac{1}{2} u_1((\theta^a, \theta^b), (\theta^a, \theta^b - 1)) + \frac{1}{2} u_1(\theta, y_2(\varnothing, z(\theta))) \\ &> u_1^*(\theta). \end{aligned}$$

Similarly if $z(\theta) \neq \theta^b$ for some θ, then 2 has a profitable deviation $(\varnothing, \varnothing)$.

Step 3. $u_1^*(\theta) = -1$ for all θ.

From the steps above,

$$\begin{aligned} u_1^*(\theta) &= \frac{1}{2} u_1(\theta, y_1(y(\theta), \varnothing)) + \frac{1}{2} u_2(\theta, y_2(\varnothing, z(\theta))) \\ &= \frac{1}{2} u_1((\theta^a, \theta^b), (\theta^a, \theta^b - 1)) + \frac{1}{2} u_1((\theta^a, \theta^b), (\theta^a - 1, \theta^b)) \\ &= -1. \end{aligned}$$

Step 4. At any θ, 1 has a profitable deviation $(\varnothing, \theta^b - 1 + \varepsilon)$, where $\varepsilon > 0$ is small.

Let 1 deviate to said proposal. The DM observes $\mathbf{m} = (\mathbf{m_1}^{\mathbf{dev}}, (\varnothing, z)) = ((\varnothing, z - 1 + \varepsilon), (\varnothing, z))$ for some $z \in \mathbb{R}$ and that only 1 has deviated. Therefore her belief is supported on

$$\Theta^1(\mathbf{m}) = \left\{ \tilde{\theta} \mid s_2(\tilde{\theta}) = (\varnothing, z), u_1^*(\tilde{\theta}) \leq \max_{\beta \in BR(\Theta, \mathbf{m})} \beta u_1\big(\tilde{\theta}, y_1(\varnothing, z - 1 + \varepsilon)\big) + (1 - \beta) u_1\big(\theta, y_2(\varnothing, z)\big) \right\}.$$

For all $\mu \in \Delta(\Theta^1(\mathbf{m}))$, $\pi_2 = u_d((\theta^a, \theta^b), (\theta^a - 1, \theta^b)) = -1 > -(1-\varepsilon)^2 = u_d((\theta^a, \theta^b), (\theta^a, \theta^b - 1 + \varepsilon)) = \pi_1$. Therefore $\beta(\mathbf{m}) = 1$ and $u_1^{dev}(\theta, \mathbf{m_1}^{\mathbf{dev}}, s_2(\theta)) = u_1((\theta^a, \theta^b), (\theta^a, \theta^b - 1 + \varepsilon)) = -\varepsilon^2 > -1 = u_1^*(\theta)$. So $(\varnothing, \theta^b - 1 + \varepsilon)$ is a profitable deviation.

9.4 Proof of Proposition 3

(1) Consider a proposal profile

$$s_1(\theta) = (\varnothing, y(\theta)),$$
$$s_2(\theta) = (z(\theta), \varnothing)$$

in which y, $z : \Theta \to [-1, 1]$ are as follows:

$$y(\theta) = z(\theta) = \begin{cases} 1 & \text{if } \theta = (1,1) \\ 0 & \text{otherwise.} \end{cases}$$

Let $\beta : M \times M \to [-1, 1]$ and $\mu : M \times M \to \Delta(\Theta)$ be as follows:

- If $\mathbf{m}$ is on-path, $\beta(\mathbf{m}) = \frac{1}{2}$.

$$\mu((1,1) \mid (\varnothing, 1), (1, \varnothing)) = 1;$$
$$\mu((0,0) \mid (\varnothing, 0), (0, \varnothing)) = \mu((0,1) \mid (\varnothing, 0), (0, \varnothing)) = \mu((1,0) \mid (\varnothing, 0), (0, \varnothing)) = \frac{1}{3}.$$

- If $\mathbf{m}$ is off-path and there is at least one Agent i with $\mathbf{m_i}$ on-path, $\mu(\cdot \mid \mathbf{m})$ is defined as follows:

If $\mathbf{m_1} \neq (\varnothing, 1)$ and $\mathbf{m_2} \neq (1, \varnothing)$, $\mu((0,0) \mid \mathbf{m}) = 1$; otherwise $\mu((1,1) \mid \mathbf{m}) = 1$.

- If $\mathbf{m} = (\mathbf{m_1}, \mathbf{m_2})$ where both $\mathbf{m_1}$ and $\mathbf{m_2}$ are off-path, $\mu(\mathbf{m})$ is unrestricted.

Specifically, $\mu(\cdot \mid \mathbf{m})$ and $\beta(\mathbf{m})$ for off-path $\mathbf{m}$ are defined as follows:

(a) For $\mathbf{m} = (\mathbf{m_1}^{\mathbf{dev}}, (0, \varnothing))$ where $\mathbf{m_1}^{\mathbf{dev}}$ is off-path: $\mu((0,0) \mid \mathbf{m}) = 1$. If $\mathbf{m_1}^{\mathbf{dev}} = (0,0)$, then $\beta(\mathbf{m}) = \frac{1}{2}$; otherwise $\beta(\mathbf{m}) = 0$.

(b) For $\mathbf{m} = (\mathbf{m_1}^{\mathbf{dev}}, (1, \varnothing))$ where $\mathbf{m_1}^{\mathbf{dev}}$ is off-path: $\mu((1,1) \mid \mathbf{m}) = 1$. If $\mathbf{m_1}^{\mathbf{dev}} = (1,1)$, then $\beta(\mathbf{m}) = \frac{1}{2}$; otherwise $\beta(\mathbf{m}) = 0$.

(c) For $\mathbf{m} = ((\varnothing, 0), \mathbf{m_2}^{\mathbf{dev}})$ where $\mathbf{m_2}^{\mathbf{dev}}$ is off-path: $\mu((0,0) \mid \mathbf{m}) = 1$. If $\mathbf{m_2}^{\mathbf{dev}} = (0,0)$, then $\beta(\mathbf{m}) = \frac{1}{2}$; otherwise $\beta(\mathbf{m}) = 1$.

(d) For $\mathbf{m} = ((\varnothing, 1), \mathbf{m_2}^{\mathbf{dev}})$ where $\mathbf{m_2}^{\mathbf{dev}}$ is off-path: $\mu((1,1) \mid \mathbf{m}) = 1$. If $\mathbf{m_2}^{\mathbf{dev}} = (1,1)$, then $\beta(\mathbf{m}) = \frac{1}{2}$; otherwise $\beta(\mathbf{m}) = 1$.

(e) For $\mathbf{m} = (\mathbf{m_1}^{\mathbf{dev}}, \mathbf{m_2}^{\mathbf{dev}})$ where both $\mathbf{m_1}^{\mathbf{dev}}$ and $\mathbf{m_2}^{\mathbf{dev}}$ are off-path, $\mu(\cdot \mid \mathbf{m})$ is unrestricted. $\beta(\mathbf{m}) \in \widetilde{\mathrm{BR}}(\mu, \mathbf{m})$.

(f) For $\mathbf{m} = ((\varnothing, 0), (1, \varnothing))$, $\mu((1,1) \mid \mathbf{m}) = 1$, $\beta(\mathbf{m}) = 0$.

(g) For $\mathbf{m} = ((\varnothing, 1), (0, \varnothing))$, $\mu((1,1) \mid \mathbf{m}) = 1$, $\beta(\mathbf{m}) = 1$.

It is easy to verify that for all $\mathbf{m}$, $\beta(\mathbf{m}) \in \widetilde{\mathrm{BR}}(\mu(\mathbf{m}), \mathbf{m})$. To see that μ satisfies equilibrium dominance, note that for all $\mathbf{m}$ in Case (1)a, $(0,0) \in \Theta^1(\mathbf{m})$. To see this, note that

$$\Theta^1(\mathbf{m}) = \{\theta \mid s_2(\theta) = (0, \varnothing), u_1^*(\theta) \leq \max_{\beta \in \mathrm{BR}(\Theta, \mathbf{m})} \beta u_1(\theta, y_1(\mathbf{m_1})) + (1-\beta) u_1(\theta, y_2(\mathbf{m_2}))\}.$$

Since $u_1^*(0,0) = -1 = u_1((0,0), y_2(\mathbf{m_2}))$, the above inequality always holds for $(0,0)$, with equality reached by $\beta = 0$. Similarly, $(0,0) \in \Theta^2(\mathbf{m})$ for all $\mathbf{m}$ in Case (1)c.

For all $\mathbf{m}$ in Case (1)b, the DM believes that 2 has not deviated and $(1, \varnothing)$ pins down the state $(1,1)$. Moreover, if 1's deviation satisfies equilibrium dominance, then $(1,1) \in \Theta^1(\mathbf{m})$; otherwise, $\Theta^1(\mathbf{m})$ is the empty set and $\mu(\cdot \mid \mathbf{m})$ is unrestricted. Therefore $\mu((1,1) \mid \mathbf{m})$ is not equilibrium-dominated. Similarly for all $\mathbf{m}$ in Case (1)d.

For $\mathbf{m}$ in Case (1)f, the DM believes that 1 has deviated and therefore $\theta = (1,1)$. At $(1,1)$, 1's deviation $(\varnothing, 0)$ is not equilibrium-dominated because $u_1((1,1), y_1(\varnothing, 0)) =$

$u_1((1,1),(1,0)) = 0 > -1 = u_1^*(1,1)$. Therefore $(1,1) \in \Theta^1(\mathbf{m})$. Similarly, $(1,1) \in \Theta^2(\mathbf{m})$ for $\mathbf{m}$ in Case (1)g.

Now I show that 1 has no incentives to deviate; 2 is symmetric.

- At state $(0,0)$, $u_1^*(0,0) = -1$.

(i) If $\mathbf{m_1}^{\mathbf{dev}} = (\varnothing, 1)$, $u_1^{dev}((0,0), \mathbf{m_1}^{\mathbf{dev}}, \mathbf{m_2}) = u_1((0,0), y_1(\varnothing, 1)) = u_1((0,0),(0,1)) = -4 < -1 = u_1^*(0,0)$.

(ii) If $\mathbf{m_1}^{\mathbf{dev}} = (0,0)$, $u_1^{dev}((0,0), \mathbf{m_1}^{\mathbf{dev}}, \mathbf{m_2}) = \frac{1}{2}u_1((0,0), y_1(0,0)) + \frac{1}{2}u_1((0,0), y_2(0,\varnothing)) = -1 = u_1^*(0,0)$.

(iii) For any other off-path $\mathbf{m_1}^{\mathbf{dev}}$, $u_1^{dev}((0,0), \mathbf{m_1}^{\mathbf{dev}}, \mathbf{m_2}) = u_1((0,0), y_2(0,\varnothing)) = u_1((0,0),(0,0)) = -1 = u_1^*(0,0)$.

- At state $(0,1)$, $u_1^*(0,1) = -\frac{1}{2}$.

(i) If $\mathbf{m_1}^{\mathbf{dev}} = (\varnothing, 1)$, $u_1^{dev}((0,1), \mathbf{m_1}^{\mathbf{dev}}, \mathbf{m_2}) = u_1((0,1), y_1(\varnothing, 1)) = u_1((0,1),(0,1)) = -1 < u_1^*(0,1)$.

(ii) If $\mathbf{m_1}^{\mathbf{dev}} = (0,0)$, $u_1^{dev}((0,1), \mathbf{m_1}^{\mathbf{dev}}, \mathbf{m_2}) = \frac{1}{2}u_1((0,1), y_1(0,0)) + \frac{1}{2}u_1((0,1), y_2(0,\varnothing)) = -\frac{1}{2} = u_1^*(0,1)$.

(iii) For any other off-path $\mathbf{m_1}^{\mathbf{dev}}$, $u_1^{dev}((0,1), \mathbf{m_1}^{\mathbf{dev}}, \mathbf{m_2}) = u_1((0,1), y_2(0,\varnothing)) = -1 < u_1^*(0,1)$.

- At state $(1,0)$, $u_1^*(1,0) = -\frac{3}{2}$.

(i) If $\mathbf{m_1}^{\mathbf{dev}} = (\varnothing, 1)$, $u_1^{dev}((1,0), \mathbf{m_1}^{\mathbf{dev}}, \mathbf{m_2}) = -4 < u_1^*(1,0)$.

(ii) If $\mathbf{m_1}^{\mathbf{dev}} = (0,0)$, $u_1^{dev}((1,0), \mathbf{m_1}^{\mathbf{dev}}, \mathbf{m_2}) = \frac{1}{2}u_1((1,0), y_1(0,0)) + \frac{1}{2}u_1((0,0), y_2(0,\varnothing)) = -2 < u_1^*(1,0)$.

(iii) For any other off-path $\mathbf{m_1}^{\mathbf{dev}}$, $u_1^{dev}((1,0), \mathbf{m_1}^{\mathbf{dev}}, \mathbf{m_2}) = u_1((0,0), y_2(0,\varnothing)) = -2 < u_1^*(1,0)$.

- At state $(1,1)$, $u_1^*(1,1) = -1$.

(i) If $\mathbf{m_1}^{\mathbf{dev}} = (\varnothing, 0)$, $u_1^{dev}((1,1), \mathbf{m_1}^{\mathbf{dev}}, \mathbf{m_2}) = u_1((1,1), y_2(1,\varnothing)) = -1 = u_1^*(1,1)$.

(ii) If $\mathbf{m_1}^{\mathbf{dev}} = (1,1)$, $u_1^{dev}((1,1), \mathbf{m_1}^{\mathbf{dev}}, \mathbf{m_2}) = \frac{1}{2}u_1((1,1), y_1(1,1)) + \frac{1}{2}u_1((1,1), y_2(1,\varnothing)) = -1 = u_1^*(1,1)$.

(iii) For any other off-path $\mathbf{m_1}^{\mathbf{dev}}$, $u_1^{dev}((1,1), \mathbf{m_1}^{\mathbf{dev}}, \mathbf{m_2}) = u_1((1,1), y_2(1,\varnothing)) = -1 = u_1^*(1,1)$.

Therefore 1 has no incentives to deviate at any state. Similarly for 2. I have established that above is an equilibrium.

(2) I prove this by showing that any strategy profile in which each agent is vague on his biased issue and commits on his unbiased issue at some state is not a PBE strategy profile. I prove by contradiction: suppose that there exists θ and y, $z \in [-1,1]$ such that

$$s_1(\theta) = (y, \varnothing),$$
$$s_2(\theta) = (\varnothing, z)$$

is a PBE proposal profile when state is θ.

Step 1. $\beta(s_1(\theta), s_2(\theta)) = \frac{1}{2}$.

If $\beta(s_1(\theta), s_2(\theta)) = 0$ for some θ, then 1 has a profitable deviation $s_2(\theta)$. To see this, first note that

$$u_1^*(\theta) = u_1((\theta^a, \theta^b), y_2(\varnothing, z)) = u_1((\theta^a, \theta^b), (\theta^a - 1, z)).$$

Now let 1 deviate to $s_2(\theta)$. Given $\mathbf{m} = (\mathbf{m_1}^{\mathbf{dev}}, s_2(\theta)) = ((\varnothing, z), (\varnothing, z))$ and any $\mu \in \Delta(\Theta)$, $\pi_1 > \pi_2$. Therefore $\beta(\mathbf{m}) = 1$ and $u_1^{dev}(\theta, \mathbf{m_1}^{\mathbf{dev}}, s_2(\theta)) = u_1(\theta, y_1(\varnothing, z)) = u_1((\theta^a, \theta^b), (\theta^a, z)) > u_1^*(\theta)$.

Similarly, if $\beta(s_1(\theta), s_2(\theta)) = 1$ for some θ, then 2 has a profitable deviation $s_1(\theta)$.

Step 2. $y = \theta^a$, $z = \theta^b$.

Suppose $y \neq \theta^a$. Then 1 has a profitable deviation $(\varnothing, \varnothing)$. To see this, first note that

$$\begin{aligned} u_1^*(\theta) &= \frac{1}{2}u_1(\theta, y_1(y, \varnothing)) + \frac{1}{2}u_1(\theta, y_2(\varnothing, z)) \\ &< \frac{1}{2}u_1((\theta^a, \theta^b), (\theta^a, \theta^b - 1)) + \frac{1}{2}u_1(\theta, y_2(\varnothing, z)) \end{aligned}$$

Now let Agent 1 deviate to $\mathbf{m_1}^{\mathbf{dev}} = (\varnothing, \varnothing)$. Given $\mathbf{m} = (\mathbf{m_1}^{\mathbf{dev}}, s_2(\theta)) = ((\varnothing, \varnothing), (\varnothing, z))$ and any $\mu \in \Delta(\Theta)$, $\pi_1 \geq \pi_2$. Therefore $\beta(\mathbf{m}) \geq \frac{1}{2}$ and

$$\begin{aligned} u_1^{dev}(\theta, \mathbf{m_1}^{\mathbf{dev}}, s_2(\theta)) &\geq \frac{1}{2}u_1(\theta, y_1(\varnothing, \varnothing)) + \frac{1}{2}u_1(\theta, y_2(\varnothing, z)) \\ &= \frac{1}{2}u_1((\theta^a, \theta^b), (\theta^a, \theta^b - 1)) + \frac{1}{2}u_1(\theta, y_2(\varnothing, z)) \\ &> u_1^*(\theta). \end{aligned}$$

Similarly if $z \neq \theta^b$, then 2 has a profitable deviation $(\varnothing, \varnothing)$.

Step 3. $u_1^*(\theta) = -1$.

From the steps above,

$$\begin{aligned} u_1^*(\theta) &= \frac{1}{2}u_1(\theta, y_1(y, \varnothing)) + \frac{1}{2}u_2(\theta, y_2(\varnothing, z)) \\ &= \frac{1}{2}u_1((\theta^a, \theta^b), (\theta^a, \theta^b - 1)) + \frac{1}{2}u_1((\theta^a, \theta^b), (\theta^a - 1, \theta^b)) \\ &= -1. \end{aligned}$$

Step 4. If $\theta^b = 0$, then 1 has a profitable deviation $(\varnothing, -0.4)$. If $\theta^b = 1$, then 1 has a profitable deviation $(\varnothing, 0.6)$.

I first consider the case in which $\theta^b = 0$. By Step 2, $z = 0$. Given $\mathbf{m} = (\mathbf{m_1}^{\mathbf{dev}}, s_2(\theta)) = ((\varnothing, -0.4), (\varnothing, 0))$ and any $\mu \in \Delta(\Theta)$, $\pi_1 > \pi_2$. Therefore $\beta(\mathbf{m}) = 1$.

$u_1^{dev}(\theta, \mathbf{m_1}^{\mathbf{dev}}, s_2(\theta)) = u_1(\theta, y_1(\varnothing, -0.4)) = u_1((\theta^a, 0), (\theta^a, -0.4)) = -0.36 > -1 = u_1^*(\theta)$.

Now I consider the case in which $\theta^b = 1$. By Step 2, $z = 1$. Given $\mathbf{m} = (\mathbf{m_1}^{\mathbf{dev}}, s_2(\theta)) = ((\varnothing, 0.6), (\varnothing, 1))$ and any $\mu \in \Delta(\Theta)$, $\pi_1 > \pi_2$. Therefore $\beta(\mathbf{m}) = 1$. $u_1^{dev}(\theta, \mathbf{m_1}^{\mathbf{dev}}, s_2(\theta)) = u_1(\theta, y_1(\varnothing, 0.6)) = u_1((\theta^a, 1), (\theta^a, 0.6)) = -0.36 > -1 = u_1^*(\theta)$.

I have established that 1 has a profitable deviation at θ. Therefore for no $\theta \in \Theta$, y, $z \in [-1, 1]$ is

$$s_1(\theta) = (y, \varnothing),$$

$$s_2(\theta) = (\varnothing, z)$$

a PBE proposal profile.

9.5 Proof of Proposition 4

Let $s_1(\theta) = (y(\theta), \varnothing)$ for all θ, where $y : \Theta \to [-1, 1]$. I show that s_1 is not a strategy in any intuitive equilibrium. The arguments for 2 is symmetric. There are 4 cases:

1. $s_2(\theta) = (\varnothing, z(\theta))$ for all θ, where $z : \Theta \to [-1, 1]$.

2. $s_2(\theta) = (\varnothing, \varnothing)$ for all θ.

3. $s_2(\theta) = (z(\theta), w(\theta))$ for all θ, where z, $w : \Theta \to [-1, 1]$.

4. $s_2(\theta) = (z(\theta), \varnothing)$ for all θ, where $z : \Theta \to [-1, 1]$.

For each of the 4 cases, I show that (s_1, s_2) is not an equilibrium proposal profile.

1. This is shown in Proposition 3.

2. I prove this by contradiction. Suppose the proposal profile is a PBE.

   Step 1. $\forall \theta$, $\beta(s_1(\theta), s_2(\theta)) = \frac{1}{2}$.

If $\beta(s_1(\theta), s_2(\theta)) = 0$ for some θ, then 1 has a profitable deviation $(\varnothing, \varnothing)$. To see this, note that

$$u_1^*(\theta) = u_1(\theta, y_2(\varnothing, \varnothing)) = u_1((\theta^a, \theta^b), (\theta^a - 1, \theta^b)).$$

Now let 1 deviate to $(\varnothing, \varnothing)$. Given $\mathbf{m} = (\mathbf{m_1}^{\mathbf{dev}}, s_2(\theta)) = ((\varnothing, \varnothing), (\varnothing, \varnothing))$, $\beta(\mathbf{m}) = \frac{1}{2}$ and

$$\begin{aligned} u_1^{dev}(\theta, \mathbf{m_1}^{\mathbf{dev}}, s_2(\theta)) &= \frac{1}{2} u_1(\theta, y_1(\varnothing, \varnothing)) + \frac{1}{2} u_1(\theta, y_2(\varnothing, \varnothing)) \\ &= \frac{1}{2} u_1((\theta^a, \theta^b), (\theta^a, \theta^b - 1)) + \frac{1}{2} u_1((\theta^a, \theta^b), (\theta^a - 1, \theta^b)) \\ &> u_1^*(\theta). \end{aligned}$$

If $\beta(s_1(\theta), s_2(\theta)) = 1$ for some θ, then 2 has a profitable deviation $(y(\theta), \varnothing)$. The argument is identical to the one used in Step 1 of the proof for part (2) of the Proposition 3.

Step 2. $y(\theta) = \theta^a, \forall \theta$.

Fix any θ. Let μ denote the DM's belief at θ. Since $\beta(s_1(\theta), s_2(\theta)) = \frac{1}{2}$, $\pi_1 = \pi_2 = u_d((\theta^a, \theta^b), (\theta^a - 1, \theta^b)) = -1$ given μ. Moreover, given any $\tilde{\mu} \in \Delta(\Theta)$, $\pi_1 \leq -1$ with equality reached when $\tilde{\mu}\{\tilde{\theta} \mid \tilde{\theta}^a = y(\theta)\} = 1$. Therefore $\mu\{\tilde{\theta} \mid \tilde{\theta}^a = y(\theta)\} = 1$. Since μ is consistent with s_1 and s_2, $y(\theta) = \theta^a$.

Step 3. 2 has a profitable deviation $(\frac{1}{2}, \varnothing)$ at state $(1, 0)$.

From Step 2, $(s_1(1,0), s_2(1,0)) = ((1, \varnothing), (\varnothing, \varnothing))$. From Step 1 and 2, $u_2^*(1,0) = \frac{1}{2} u_2((1,0), y_1(1, \varnothing)) + \frac{1}{2} u_2((1,0), y_2(\varnothing, \varnothing)) = -1$.

Now let 2 deviate to $(\frac{1}{2}, \varnothing)$. Given $\mathbf{m} = (s_1(1,0), \mathbf{m_2}^{\mathbf{dev}}) = ((1, \varnothing), (\frac{1}{2}, \varnothing))$, for all $\mu \in \Delta(\Theta)$, $\pi_1 < \pi_2$. To see why, note that if $\theta^a = 0$,

$$u_d(\theta, y_2(\frac{1}{2}, \varnothing)) = u_d((0, \theta^b), (\frac{1}{2}, \theta^b)) > u_d((0, \theta^b), (1, \theta^b - 1)) = u_d(\theta, y_1(1, \varnothing)).$$

On the other hand, if $\theta^a = 1$,

$$u_d(\theta, y_2(\frac{1}{2}, \varnothing)) = u_d((1, \theta^b), (\frac{1}{2}, \theta^b)) > u_d((1, \theta^b), (1, \theta^b - 1)) = u_d(\theta, y_1(1, \varnothing)).$$

So, given any $\mu \in \Delta(\Theta)$, $\pi_1 < \pi_2$ and $\beta(\mathbf{m}) = 0$. Therefore $u_2^{dev}((1,0), s_1(1,0), \mathbf{m_2}^{\mathbf{dev}}) = u_2((1,0), y_2(\frac{1}{2}, \varnothing)) = u_2((1,0),(\frac{1}{2},0)) = -\frac{1}{4} > -1 = u_2^*(1,0)$.

3. I prove this by contradiction. Suppose the proposal profile is an equilibrium proposal profile. First I prove the following 4 claims.

Claim 1 *$\forall \theta$, $\beta(s_1(\theta), s_2(\theta)) \in \{0, \frac{1}{2}\}$.*

Proof of Claim 1. If for some θ, $\beta(s_1(\theta), s_2(\theta)) = 1$, then 2 has a profitable deviation $s_1(\theta)$. The argument is identical to the one used in Step 1 of the proof for part (2) of the Proposition 3. ■

Claim 2 *If for some θ, $\beta(s_1(\theta), s_2(\theta)) = \frac{1}{2}$, then $z(\theta) = \theta^a - 1$ and $w(\theta) = \theta^b$.*

Proof of Claim 2. Suppose on the contrary that either $z(\theta) \neq \theta^a - 1$ or $w(\theta) \neq \theta^b$. Then

$$\begin{aligned} u_2^*(\theta) &= \frac{1}{2} u_2(\theta, y_2(z(\theta), w(\theta))) + \frac{1}{2} u_2(\theta, y_1(y(\theta), \varnothing))) \\ &< \frac{1}{2} u_2((\theta^a, \theta^b), (\theta^a - 1, \theta^b)) + \frac{1}{2} u_2(\theta, y_1(y(\theta), \varnothing)). \end{aligned}$$

Now let 2 deviate to $(\varnothing, \varnothing)$. Given $\mathbf{m} = (s_1(\theta), \mathbf{m_2}^{\mathbf{dev}}) = ((y(\theta), \varnothing), (\varnothing, \varnothing))$ and any

$\mu \in \Delta(\Theta)$, $\pi_1 \leq \pi_2$. Therefore $\beta(\mathbf{m}) \leq \frac{1}{2}$ and

$$\begin{aligned} u_2^{dev}(\theta, s_1(\theta), \mathbf{m_2}^{\mathbf{dev}}) &\geq \frac{1}{2}u_2(\theta, y_2(\varnothing, \varnothing)) + \frac{1}{2}u_2(\theta, y_1(y(\theta), \varnothing)) \\ &= \frac{1}{2}u_2((\theta^a, \theta^b), (\theta^a - 1, \theta^b)) + \frac{1}{2}u_2(\theta, y_1(y(\theta), \varnothing)) \\ &> u_2^*(\theta). \end{aligned}$$

■

Claim 3 *If for some θ, $\beta(s_1(\theta), s_2(\theta)) = 0$, then $z(\theta) = \theta^a$.*

Proof of Claim 3. Suppose on the contrary that $z(\theta) \neq \theta^a$. Then

$$\begin{aligned} u_1^*(\theta) &= u_1(\theta, y_2(z(\theta), w(\theta))) \\ &= u_1((\theta^a, \theta^b), (z(\theta), w(\theta))) \\ &< u_1((\theta^a, \theta^b), (\theta^a, w(\theta))). \end{aligned}$$

Now let 1 deviate to $(\varnothing, w(\theta))$. Given $\mathbf{m} = (\mathbf{m_1}^{\mathbf{dev}}, s_2(\theta)) = ((\varnothing, w(\theta)), (z(\theta), w(\theta)))$ and any $\mu \in \Delta(\Theta)$, $\pi_1 \geq \pi_2$. Therefore $\beta(\mathbf{m}) \geq \frac{1}{2}$ and

$$\begin{aligned} u_1^{dev}(\theta, \mathbf{m_1}^{\mathbf{dev}}, s_2(\theta)) &\geq \frac{1}{2}u_1(\theta, y_1(\varnothing, w(\theta))) + \frac{1}{2}u_1(\theta, y_2(z(\theta), w(\theta))) \\ &= \frac{1}{2}u_1((\theta^a, \theta^b), (\theta^a, w(\theta))) + \frac{1}{2}u_1((\theta^a, \theta^b), (z(\theta), w(\theta))) \\ &> u_1((\theta^a, \theta^b), (z(\theta), w(\theta))) \\ &= u_1^*(\theta). \end{aligned}$$

■

Claim 4 *If for some θ, $\beta(s_1(\theta), s_2(\theta)) = 0$, then $u_2^*(\theta) \leq -1$.*

Proof of Claim 4. From Claim 3,

$$\begin{aligned}
u_2^*(\theta) &= u_2(\theta, y_2(s_2(\theta))) \\
&= u_2((\theta^a, \theta^b), (z(\theta), w(\theta))) \\
&= u_2((\theta^a, \theta^b), (\theta^a, w(\theta))) \\
&\leq u_2((\theta^a, \theta^b), (\theta^a, \theta^b)) \\
&= -1.
\end{aligned}$$

■

For the rest of the proof, I fix any θ and show that $(s_1(\theta), s_2(\theta)) = ((y(\theta), \varnothing), (z(\theta), w(\theta)))$ cannot be sustained in equilibrium for any y, z, $w : \Theta \to [-1, 1]$. We divide the discussion into 3 cases.

Case 1 *For all $\tilde{\theta}$ such that $s_1(\tilde{\theta}) = s_1(\theta) = (y(\theta), \varnothing)$, $\tilde{\theta}^a = y(\theta)$.*

If $\beta(s_1(\theta), s_2(\theta)) = \frac{1}{2}$, then $u_2^*(\theta) = \frac{1}{2}u_2(\theta, y_1(s_1(\theta))) + \frac{1}{2}u_2(\theta, y_2(s_2(\theta))) = \frac{1}{2}u_2((\theta^a, \theta^b), (\theta^a, \theta^b - 1)) + \frac{1}{2}u_2((\theta^a, \theta^b), (\theta^a - 1, \theta^b)) = -1$. On the other hand, if $\beta(s_1(\theta), s_2(\theta)) = 0$, then $u_2^*(\theta) \leq -1$ by Claim 4. Therefore in both situations we have $u_2^*(\theta) \leq -1$.

Now let 2 deviate to $(y(\theta) - 1 + \varepsilon, \varnothing)$, where ε is a very small positive number. Given $\mathbf{m} = (s_1(\theta), \mathbf{m_2}^{\mathbf{dev}}) = ((y(\theta), \varnothing), (y(\theta) - 1 + \varepsilon, \varnothing))$, we have that for all $\tilde{\theta} \in \Theta^2(\mathbf{m})$,

$\tilde{\theta}^a = y(\theta)$. Since for any $\mu \in \Delta(\Theta^2(\mathbf{m}))$,

$$\begin{aligned}
\pi_1 &= u_d((\theta^a, \theta^b), y_1(y(\theta), \varnothing)) \\
&= u_d((\theta^a, \theta^b), (\theta^a, \theta^b - 1)) \\
&< u_d((\theta^a, \theta^b), (\theta^a - 1 + \varepsilon, \theta^b)) \\
&= u_d((\theta^a, \theta^b), y_2(y(\theta) - 1 + \varepsilon, \varnothing)) \\
&= \pi_2,
\end{aligned}$$

we have $\beta(\mathbf{m}) = 0$. Therefore $u_2^{dev}(\theta, s_1(\theta), \mathbf{m_2}^{\mathbf{dev}}) = u_2(\theta, y_2(\mathbf{m_2}^{\mathbf{dev}})) = u_2((\theta^a, \theta^b), (\theta^a - 1 + \varepsilon, \theta^b)) = -\varepsilon^2 > -1$.

Therefore any $s_1(\theta)$ that belongs to Case 1 cannot be sustained in equilibrium.

Case 2 *For all $\tilde{\theta}$ such that $s_1(\tilde{\theta}) = s_1(\theta) = (y(\theta), \varnothing)$, $\tilde{\theta}^a \neq y(\theta)$.*

If $\beta(s_1(\theta), s_2(\theta)) = \frac{1}{2}$, then $s_2(\theta) = (\theta^a - 1, \theta^b)$. Therefore given DM's belief μ at θ, $\pi_2 = u_d((\theta^a, \theta^b), (\theta^a - 1, \theta^b)) > \pi_1$, contradicting $\beta(s_1(\theta), s_2(\theta)) = \frac{1}{2}$. Therefore $\beta(s_1(\theta), s_2(\theta)) = 0$ and $u_2^*(\theta) \leq -1$.

Now let 2 deviate to $(\varnothing, \varnothing)$. Given $\mathbf{m} = (s_1(\theta), \mathbf{m_2}^{\mathbf{dev}}) = ((y(\theta), \varnothing), (\varnothing, \varnothing))$ and any $\mu \in \Delta(\Theta^2(\mathbf{m}))$, $\pi_1 < -1 = \pi_2$. Therefore $\beta(\mathbf{m}) = 0$ and $u_2^{dev}(\theta, s_1(\theta), \mathbf{m_2}^{\mathbf{dev}}) = u_2(\theta, y_2(\mathbf{m_2}^{\mathbf{dev}})) = u_2((\theta^a, \theta^b), (\theta^a - 1, \theta^b)) = 0 > u_2^*(\theta)$.

Therefore any $s_1(\theta)$ that belongs to Case 2 cannot be sustained in equilibrium.

Case 3 *There exists $\tilde{\theta}$ such that $s_1(\tilde{\theta}) = s_1(\theta) = (y(\theta), \varnothing)$ and $\tilde{\theta}^a = y(\theta)$, and there exists $\tilde{\theta}$ such that $s_1(\tilde{\theta}) = s_1(\theta) = (y(\theta), \varnothing)$ and $\tilde{\theta}^a \neq y(\theta)$.*

In this case, $y(\theta) \in \{0, 1\}$. We further divide this case into 6 subcases.

Subcase 1 $y(\theta) = 1$. *For all $\tilde{\theta}$ such that $(s_1(\tilde{\theta}), s_2(\tilde{\theta})) = ((1, \varnothing), s_2(\theta))$, $\tilde{\theta}^a = 1$.*

If $\beta(s_1(\theta), s_2(\theta)) = \frac{1}{2}$, then $u_2^*(\theta) = \frac{1}{2}u_2(\theta, y_1(s_1(\theta))) + \frac{1}{2}u_2(\theta, y_2(s_2(\theta))) = -1$. If $\beta(s_1(\theta), s_2(\theta)) = 0$, then $u_2^*(\theta) \leq -1$. Therefore for both situations we have $u_2^*(\theta) \leq -1$.

Now let 2 deviate to $(1-\varepsilon, \varnothing)$, where ε is a very small positive number. Given $\mathbf{m} = (s_1(\theta), \mathbf{m_2}^{\mathbf{dev}}) = ((1, \varnothing), (1-\varepsilon, \varnothing))$ and any $\mu \in \Delta(\Theta)$, $\pi_2 > \pi_1$. To see why, note that when $\theta^a = 1$,

$$u_d(\theta, y_1(1, \varnothing)) = u_d((1, \theta^b), (1, \theta^b - 1)) = -1,$$

while

$$u_d(\theta, y_2(1-\varepsilon, \varnothing)) = u_d((1, \theta^b), (1-\varepsilon, \theta^b)) = -\varepsilon^2 < -1.$$

On the other hand, when $\theta^a = 0$,

$$u_d(\theta, y_1(1, \varnothing))) = u_d((0, \theta^b), (1, \theta^b - 1)) = -2,$$

and

$$u_d(\theta, y_2(1-\varepsilon, \varnothing)) = u_d((0, \theta^b), (1-\varepsilon, \theta^b)) = -(1-\varepsilon)^2 > -2.$$

So $\beta(\mathbf{m}) = 0$ and

$$\begin{aligned} u_2^{dev}(\theta, s_1(\theta), \mathbf{m_2}^{\mathbf{dev}}) &= u_2(\theta, y_2(\mathbf{m_2}^{\mathbf{dev}})) \\ &= u_2((1, \theta^b), (1-\varepsilon, \theta^b)) \\ &= -(1-\varepsilon)^2 \\ &> -1 = u_2^*(\theta). \end{aligned}$$

Therefore any $s_1(\theta)$ that belongs to Subcase 1 cannot be sustained in equilibrium.

Subcase 2 $y(\theta) = 1$. *For all $\tilde{\theta}$ such that $(s_1(\tilde{\theta}), s_2(\tilde{\theta})) = ((1, \varnothing), s_2(\theta))$, $\tilde{\theta}^a = 0$.*

If $s_1(\theta)$ falls into this subcase, then there exists $\theta_1 = (1, \theta_1^b)$ such that $(s_1(\theta_1), s_2(\theta_1)) = ((1, \varnothing), s_2(\theta_1))$ and $s_2(\theta_1) \neq s_2(\theta)$; otherwise it by definition contradicts $(y(\theta), \varnothing)$ belongs to Case 3.

Moreover, the DM's belief μ at θ_1 must assign probability $0 < p < 1$ to $\{\tilde{\theta} \mid \tilde{\theta}^a = 1\}$. The reason is that, if $p = 0$, then μ is inconsistent with equilibrium strategy; if $p = 1$, then $s_1(\theta_1)$ belongs to Subcase 1, which we have established as unsustainable in equilibrium.

Since μ is consistent with the equilibrium strategies, there exists $\theta_2 = (0, 1 - \theta^b)$ such that $(s_1(\theta_2), s_2(\theta_2)) = ((1, \varnothing), s_2(\theta_1))$.

The DM elects 2 with the same probability at states θ_1 and θ_2 since $(s_1(\theta_1), s_2(\theta_1)) = (s_1(\theta_2), s_2(\theta_2))$. If $\beta(s_1(\theta_1), s_2(\theta_1)) = \frac{1}{2}$, then $z(\theta_1) = \theta_1^a - 1 = \theta_2^a - 1$. This is impossible since $\theta_1^a = 1$ and $\theta_2^a = 0$. On the other hand, if $\beta(s_1(\theta_1), s_2(\theta_1)) = 0$, then $z(\theta_1) = \theta_1^a = \theta_2^a$ which is also impossible. Now we have reached a contradiction. Therefore any $s_1(\theta)$ that belongs to Subcase 2 cannot be sustained in equilibrium.

Subcase 3 $y(\theta) = 1$. *There exists* $\tilde{\theta}$ *such that* $(s_1(\tilde{\theta}), s_2(\tilde{\theta})) = ((1, \varnothing), s_2(\theta))$ *and* $\tilde{\theta}^a = 1$. *There exists* $\tilde{\theta}$ *such that* $(s_1(\tilde{\theta}), s_2(\tilde{\theta})) = ((1, \varnothing), s_2(\theta))$ *and* $\tilde{\theta}^a = 0$.

This subcase is ruled out using the argument in the last paragraph of subcase 2.

Subcase 4 $y(\theta) = 0$. *For all* $\tilde{\theta}$ *such that* $(s_1(\tilde{\theta}), s_2(\tilde{\theta})) = ((0, \varnothing), s_2(\theta))$, $\tilde{\theta}^a = 1$.

We first prove the following claim:

Claim 5 *In any equilibrium with* $(s_1(\theta), s_2(\theta)) = ((y(\theta), \varnothing), (z(\theta), w(\theta)))$, *there does not exist* θ *such that* $\theta^a = 1$ *and* $s_1(\theta) = (0, \varnothing)$.

Proof of Claim 5. Suppose on the contrary that $s_1(\theta) = (0, \varnothing)$ for $\theta = (1, \theta^b)$. First note that $u_2^*(\theta) = 0$, otherwise let 2 deviate to $(0, \varnothing)$. Given $\mathbf{m} = ((0, \varnothing), (0, \varnothing))$, since for any $\mu \in \Delta(\Theta)$, $\pi_1 < \pi_2$, we have that $\beta(\mathbf{m}) = 0$. Therefore $u_2^{dev}(\theta, s_1(\theta), \mathbf{m_2}^{\mathbf{dev}}) = u_2(\theta, y_2(\mathbf{m_2}^{\mathbf{dev}})) = u_2(1, 0) = 0$.

Since $u_2(\theta, y_1(s_1(\theta))) < 0$, we must have $\beta(s_1(\theta), s_2(\theta)) = 0$ and $s_2(\theta) = (\theta^a - 1, \theta^b)$. This contradicts Claim 3. ■

Now let's come back to subcase 4. By definition of subcase 4, $\theta^a = 1$. By Claim 5, $s_1(\theta)$ that falls into subcase 4 cannot be sustained in equilibrium.

Subcase 5 $y(\theta) = 0$. *For all* $\tilde{\theta}$ *such that* $(s_1(\tilde{\theta}), s_2(\tilde{\theta})) = ((0, \varnothing), s_2(\theta))$, $\tilde{\theta}^a = 0$.

The argument in first paragraph for case 1 shows that $u_2^*(\theta) \leq -1$.

Now let 2 deviate to $(-\frac{1}{2}, \varnothing)$. Given $\mathbf{m} = (s_1(\theta), \mathbf{m_2}^{\mathbf{dev}}) = ((0, \varnothing), (-\frac{1}{2}, \varnothing))$, by Claim 5, for any $\tilde{\theta} \in \Theta^2(\mathbf{m})$, $\tilde{\theta}^a = 0$. So for any $\mu \in \Delta(\Theta^2(\mathbf{m}))$, $\pi_1 = -1 < -\frac{1}{4} = \pi_2$ and $\beta(\mathbf{m}) = 0$. So $u_2^{dev}(\theta, s_1(\theta), \mathbf{m_2}^{\mathbf{dev}}) = u_2(\theta, y_2(\mathbf{m_2}^{\mathbf{dev}})) = u_2((0, \theta^b), (\frac{1}{2}, \theta^b)) = -\frac{1}{4} > u_2^*(\theta)$. Therefore any $s_1(\theta)$ that belongs to Subcase 5 cannot be sustained in equilibrium.

Subcase 6 $y(\theta) = 0$. *There exists* $\tilde{\theta}$ *such that* $(s_1(\tilde{\theta}), s_2(\tilde{\theta})) = ((0, \varnothing), s_2(\theta))$ *and* $\tilde{\theta}^a = 1$. *There exists* $\tilde{\theta}$ *such that* $(s_1(\tilde{\theta}), s_2(\tilde{\theta})) = ((0, \varnothing), s_2(\theta))$ *and* $\tilde{\theta}^a = 0$.

This subcase is ruled out using the argument in the last paragraph of subcase 2.

Therefore all cases have been shown to be unsustainable in equilibrium.

4. I prove this by contradiction. Suppose the proposal profile is an equilibrium proposal profile. I first prove the following 3 claims.

Claim 1 $\beta(s_1(\theta), s_2(\theta)) \leq \frac{1}{2}$, $\forall \theta$.

Proof of Claim 1. Suppose on the contrary that for some θ, $\beta(s_1(\theta), s_2(\theta)) = 1$. Then $u_2^*(\theta) = u_2(\theta, y_1(s_1(\theta))) = u_2((\theta^a, \theta^b), (y(\theta), \theta^b - 1))$.

Now let 2 deviate to $s_1(\theta)$. Given $\mathbf{m} = (s_1(\theta), \mathbf{m_2}^{\mathbf{dev}}) = ((y(\theta), \varnothing), (y(\theta), \varnothing))$ and any $\mu \in \Delta(\Theta)$, $\pi_2 > \pi_1$. Therefore $\beta(\mathbf{m}) = 0$ and $u_2^{dev}(\theta, s_1(\theta), \mathbf{m_2}^{\mathbf{dev}}) = u_2(\theta, y_2(\mathbf{m_2}^{\mathbf{dev}})) = u_2((\theta^a, \theta^b), (y(\theta), \theta^b)) > u_2^*(\theta)$. ■

Claim 2 *If for some θ, $\beta(s_1(\theta), s_2(\theta)) = \frac{1}{2}$, then $z(\theta) = \theta^a - 1$.*

Proof of Claim 2. Suppose on the contrary that $z(\theta) \neq \theta^a - 1$. Therefore

$$\begin{aligned} u_2^*(\theta) &= \frac{1}{2}u_2(\theta, y_1(s_1(\theta))) + \frac{1}{2}u_2(\theta, y_2(s_2(\theta))) \\ &< \frac{1}{2}u_2(\theta, y_1(s_1(\theta))) + \frac{1}{2}u_2((\theta^a, \theta^b), (\theta^a - 1, \theta^b)) \\ &= \frac{1}{2}u_2(\theta, y_1(s_1(\theta))). \end{aligned}$$

Now let 2 deviate to $(\varnothing, \varnothing)$. Given $\mathbf{m} = (s_1(\theta), \mathbf{m_2}^{\mathbf{dev}}) = ((y(\theta), \varnothing), (\varnothing, \varnothing))$ and any $\mu \in \Delta(\Theta)$, $\pi_1 \leq \pi_2$. Therefore $\beta(\mathbf{m}) \leq \frac{1}{2}$ and

$$\begin{aligned} u_2^{dev}(\theta, s_1(\theta), \mathbf{m_2}^{\mathbf{dev}}) &\geq \frac{1}{2}u_2(\theta, y_1(s_1(\theta))) + \frac{1}{2}u_2(\theta, y_2(\mathbf{m_2}^{\mathbf{dev}})) \\ &= \frac{1}{2}u_2(\theta, y_1(s_1(\theta))) \\ &> u_2^*(\theta). \end{aligned}$$

■

Claim 3 $\beta((1, \varnothing), (\delta, \varnothing)) = 0$, $\forall \delta \in (0, 1)$.

Proof of Claim 3. Let $\mathbf{m} = ((1, \varnothing), (\delta, \varnothing))$. If $\theta^a = 1$, then

$$u_d(\theta, y_1(\mathbf{m_1})) = u_d((1, \theta^b), (1, \theta^b - 1)) = -1,$$

while

$$u_d(\theta - y_2(\mathbf{m_2})) = u_d((1, \theta^b), (\delta, \theta^b)) = -(1 - \delta)^2 > -1.$$

On the other hand, if $\theta^a = 0$, then

$$u_d(\theta - y_1(\mathbf{m_1})) = u_d((0, \theta^b), (1, \theta^b - 1)) = -2,$$

while

$$u_d(\theta - y_2(\mathbf{m_2})) = u_d((0, \theta^b), (\delta, \theta^b)) = -\delta^2 > -2.$$

Therefore for any $\mu \in \Delta(\Theta)$, $\pi_1 < \pi_2$ and $\beta(\mathbf{m}) = 0$. ■

For the rest of the proof, we fix any θ and divide the discussion into three cases to show $(s_1(\theta), s_2(\theta)) = ((y(\theta), \varnothing), (z(\theta), \varnothing))$ cannot be sustained in equilibrium for any y, $z : \Theta \to [-1, 1]$.

Case 1 *For all $\tilde{\theta}$ such that $s_1(\tilde{\theta}) = s_1(\theta) = (y(\theta), \varnothing)$, $\tilde{\theta}^a = y(\theta)$.*

There are two possibilities: $\beta(s_1(\theta), s_2(\theta)) = \frac{1}{2}$ and $\beta(s_1(\theta), s_2(\theta)) = 1$.

- If $\beta(s_1(\theta), s_2(\theta)) = \frac{1}{2}$, then by Claim 2,

$$\begin{aligned} u_2^*(\theta) &= \frac{1}{2}u_2(\theta, y_1(s_1(\theta))) + \frac{1}{2}u_2(\theta, y_2(s_2(\theta))) \\ &= \frac{1}{2}u_2((\theta^a, \theta^b), (\theta^a, \theta^b - 1)) + \frac{1}{2}u_2((\theta^a, \theta^b), (\theta^a - 1, \theta^b)) \\ &= -1. \end{aligned}$$

Now let 2 deviate to an off-path $(y(\theta) - 1 + \varepsilon, \varnothing)$, where ε is a very small positive number. This can be done since 2 makes at most 4 different proposals in equilibrium. Given $\mathbf{m} = (s_1(\theta), \mathbf{m_2}^{\mathbf{dev}}) = ((y(\theta), \varnothing), (y(\theta) - 1 + \varepsilon, \varnothing))$ and any $\tilde{\theta} \in \Theta^2(\mathbf{m})$, $\tilde{\theta}^a = y(\theta)$. Therefore for any $\mu \in \Delta(\Theta^2(\mathbf{m}))$, $\pi_1 = -1 < -(1 - \varepsilon)^2 = u_d((\theta^a, \theta^b), (\theta^a - 1 + \varepsilon, \theta^b)) = \pi_2$. Therefore $\beta(\mathbf{m}) = 0$ and $u_2^{dev}(\theta, s_1(\theta), \mathbf{m_2}^{\mathbf{dev}}) = u_2(\theta, y_2(\mathbf{m_2}^{\mathbf{dev}})) = u_2((\theta^a, \theta^b), (\theta^a - 1 + \varepsilon, \theta^b)) = -\varepsilon^2 > -1 = u_2^*(\theta)$.

- If $\beta(s_1(\theta), s_2(\theta)) = 0$, then

$$u_2^*(\theta) = u_2(\theta, y_2(s_2(\theta))) = u_2(\theta, y_2(z(\theta), \varnothing)) = -(\theta^a - z(\theta) - 1)^2.$$

Moreover, given the DM's belief μ at θ, $\pi_1 < \pi_2$. Since $(y(\theta), \varnothing)$ belongs to Case 1, the DM learns $\theta^a = y(\theta)$ at θ. Therefore $\pi_1 = -1 < \pi_2 = u_d((\theta^a, \theta^b), (z(\theta), \theta^b))$. Hence $|\theta^a - z(\theta)| < 1$.

Now let 2 deviate to an off-path $(z(\theta) - \varepsilon, \varnothing)$, where $z(\theta) - \varepsilon > \theta^a - 1$. Therefore $\theta^a - z(\theta) + \varepsilon \in (-1, 1)$.

Given $\mathbf{m} = (s_1(\theta), \mathbf{m_2}^{\mathbf{dev}}) = ((y(\theta), \varnothing), (z(\theta) - \varepsilon, \varnothing))$ and any $\tilde{\theta} \in \Theta^2(\mathbf{m})$, $\tilde{\theta}^a = y(\theta)$. Therefore for any $\mu \in \Delta(\Theta^2(\mathbf{m}))$, $\pi_1 = -1 < -(\theta^a - z(\theta) + \varepsilon)^2 = u_d((\theta^a, \theta^b), (z(\theta) - \varepsilon, \theta^b)) = \pi_2$. Therefore $\beta(\mathbf{m}) = 0$ and

$$\begin{aligned} u_2^{dev}(\theta, s_1(\theta), \mathbf{m_2}^{\mathbf{dev}}) &= u_2(\theta, y_2(\mathbf{m_2}^{\mathbf{dev}})) \\ &= u_2((\theta^a, \theta^b), (z(\theta) - \varepsilon, \theta^b)) \\ &= -(\theta^a - z(\theta) + \varepsilon - 1)^2 \\ &> -(\theta^a - z(\theta) - 1)^2 \\ &= u_2^*(\theta). \end{aligned}$$

Therefore $s_1(\theta)$ that falls into Case 1 cannot be sustained in equilibrium.

Case 2 *For all $\tilde{\theta}$ such that $s_1(\tilde{\theta}) = s_1(\theta)$, $\tilde{\theta}^a \neq y(\theta)$.*

I show that if $s_1(\theta)$ falls into this case, then $z(\theta) = \theta^a - 1$ and $\beta(s_1(\theta), s_2(\theta)) = 0$. To see this, suppose the contrary. Then $u_2^*(\theta) < 0$. Now let 2 deviate to $(\varnothing, \varnothing)$. Given $\mathbf{m} = (s_1(\theta), \mathbf{m_2}^{\mathbf{dev}}) = ((y(\theta), \varnothing), (\varnothing, \varnothing))$ and any $\mu \in \Delta(\Theta^2(\mathbf{m}))$, $\pi_1 < -1 = \pi_2$. Therefore $\beta(\mathbf{m}) = 0$ and $u_2^{dev}(\theta, s_1(\theta), \mathbf{m_2}^{\mathbf{dev}}) = u_2(\theta, y_2(\mathbf{m_2}^{\mathbf{dev}})) = 0 > u_2^*(\theta)$.

Therefore for any $s_1(\theta)$ that falls into Case 2, 2 is proposing his ideal policy $(\theta^a - 1, \varnothing)$

and wins with probability 1.

Case 3 *There exists* $\tilde{\theta}$ *such that* $s_1(\tilde{\theta}) = s_1(\theta) = (y(\theta), \varnothing)$ *and* $\tilde{\theta}^a = y(\theta)$*, and there exists* $\tilde{\theta}$ *such that* $s_1(\tilde{\theta}) = s_1(\theta) = (y(\theta), \varnothing)$ *and* $\tilde{\theta}^a \neq y(\theta)$*.*

In this case, $y(\theta) \in \{0, 1\}$. I further divide this case into 6 subcases.

Subcase 1 $y(\theta) = 1$*. For all* $\tilde{\theta}$ *such that* $(s_1(\tilde{\theta}), s_2(\tilde{\theta})) = ((1, \varnothing), s_2(\theta))$*,* $\tilde{\theta}^a = 1$*.*

There are two possibilities: $\beta(s_1(\theta), s_2(\theta)) = \frac{1}{2}$ and $\beta(s_1(\theta), s_2(\theta)) = 1$.

- If $\beta(s_1(\theta), s_2(\theta)) = \frac{1}{2}$, then by Claim 2, $z(\theta) = \theta^a - 1$ and $u_2^*(\theta) = \frac{1}{2}u_2(\theta, y_1(s_1(\theta))) + \frac{1}{2}u_2(\theta, y_2(s_2(\theta))) = -1$.

Now let 2 deviate to an off path $(1 - \varepsilon, \varnothing)$, where ε is a very small positive number. Given $\mathbf{m} = (s_1(\theta), \mathbf{m_2}^{\mathbf{dev}}) = ((1, \varnothing), (1 - \varepsilon, \varnothing))$, by Claim 3, $\beta(\mathbf{m}) = 0$. Therefore $u_2^{dev}(\theta, s_1(\theta), \mathbf{m_2}^{\mathbf{dev}}) = u_2(\theta, y_2(\mathbf{m_2}^{\mathbf{dev}})) = u_2((1, \theta^b), (1 - \varepsilon, \theta^b)) = -(1 - \varepsilon)^2 > -1 = u_2^*(\theta)$.

- If $\beta(s_1(\theta), s_2(\theta)) = 1$, then $u_2^*(\theta) = u_2(\theta, y_2(s_2(\theta))) = u_2((1, \theta^b), (z(\theta), \theta^b)) = -(z(\theta))^2$.

Moreover, given the DM's belief μ at θ, $\pi_1 < \pi_2$. Since $s_1(\theta)$ belongs to Subcase 1, the DM learns $\theta^a = y(\theta) = 1$ at θ. Therefore $\pi_1 = -1$ and $\pi_2 = u_d((1, \theta^b), (z(\theta), \theta^b)) = -(1 - z(\theta))^2$, where $|1 - z(\theta)| < 1$. Therefore $z(\theta) \in (0, 1]$.

Now let 2 deviate to an off-path $(z(\theta) - \varepsilon, \varnothing)$, where $z(\theta) - \varepsilon \in (0, 1)$. Given $\mathbf{m} = (s_1(\theta), \mathbf{m_2}^{\mathbf{dev}}) = ((1, \varnothing), (z(\theta) - \varepsilon, \varnothing))$, by Claim 3, $\beta(\mathbf{m}) = 0$. Therefore

$$
\begin{aligned}
u_2^{dev}(\theta, s_1(\theta), \mathbf{m_2}^{\mathbf{dev}}) &= u_2(\theta, y_2(\mathbf{m_2}^{\mathbf{dev}})) \\
&= u_2((1, \theta^b), (z(\theta) - \varepsilon, \theta^b)) \\
&= -(z(\theta) - \varepsilon)^2 \\
&> -(z(\theta))^2 \\
&= u_2^*(\theta).
\end{aligned}
$$

Therefore $s_1(\theta)$ that falls into Subcase 1 cannot be sustained in equilibrium.

Subcase 2 $y(\theta) = 1$. *For all* $\tilde{\theta}$ *such that* $(s_1(\tilde{\theta}), s_2(\tilde{\theta})) = ((1, \varnothing), s_2(\theta))$, $\tilde{\theta}^a = 0$.

If $s_1(\theta)$ falls into this subcase, then there exists $\theta_1 = (1, \theta_1^b)$ such that $(s_1(\theta_1), s_2(\theta_1)) = ((1, \varnothing), s_2(\theta_1))$ and $s_2(\theta_1) \neq s_2(\theta)$; otherwise it contradicts the definition of Case 3 and Subcase 2.

Moreover, the DM's belief μ at θ_1 must assign probability $0 < p < 1$ to $\{\tilde{\theta} \mid \tilde{\theta}^a = 1\}$. The reason is that, if $p = 0$, then μ is inconsistent with equilibrium strategies; if $p = 1$, then $s_1(\theta_1)$ belongs to Subcase 1, which we have established as unsustainable in equilibrium.

Since μ is consistent with equilibrium strategy, there exists $\theta_2 = (0, 1 - \theta^b)$ such that $(s_1(\theta_2), s_2(\theta_2)) = (s_1(\theta_1), s_2(\theta_1)) = ((1, \varnothing), s_2(\theta_1))$.

The DM elects 2 with the same probability at states θ_1 and θ_2 since $(s_1(\theta_1), s_2(\theta_1)) = (s_1(\theta_2), s_2(\theta_2))$. If $\beta(s_1(\theta_1), s_2(\theta_1)) = \frac{1}{2}$, then by Claim 2 we have $z(\theta_1) = \theta_1^a - 1 = \theta_2^a - 1$. But this is impossible since $\theta_1^a = 1$ and $\theta_2^a = 0$. Therefore $\beta(s_1(\theta_1), s_2(\theta_1)) = 0$ and $u_2^*(\theta_1) = u_2((1, \theta_1^b), (z(\theta_1), \theta_1^b)) = -(z(\theta_1))^2$.

I argue that $z(\theta_1) = z(\theta_2) = 0$; otherwise at state θ_1, let 2 deviate to an off-path $(\varepsilon, \varnothing)$ such that $0 < \varepsilon < |z(\theta_1)|$. Then given $\widetilde{\mathbf{m}} = ((1, \varnothing), (\varepsilon, \varnothing))$, $\beta(\widetilde{\mathbf{m}}) = 0$ by Claim 3.

Therefore $u_2^{dev}(\theta_1, (1, \varnothing), (\varepsilon, \varnothing)) = u_2(\theta, y_2(\mathbf{m_2}^{\mathbf{dev}})) = -\varepsilon^2 > -(z(\theta_1))^2 = u_2^*(\theta_1)$.

Therefore we have that $\beta((1, \varnothing), (0, \varnothing)) = 0$ since $(s_1(\theta_1), s_2(\theta_1)) = ((1, \varnothing), (0, \varnothing))$ and $\beta(s_1(\theta_1), s_2(\theta_1)) = 0$.

This means for any $\tilde{\theta}$ such that $\tilde{\theta}^a = 1$ and $s_1(\tilde{\theta}) = (1, \varnothing)$, $s_2(\tilde{\theta}) = (0, \varnothing)$ and $u_2^*(\tilde{\theta}) = 0$; otherwise, 2 has incentive to deviate to $(0, \varnothing)$.

Now I show that 2 has a profitable deviation at θ_2. First note that $u_2^*(\theta_2) = u_2(\theta_2, y_2(s_2(\theta_2))) = u_2((0, \theta_2^b), (0, \theta_2^b)) = -1$. Now let 2 deviate to an off-path $(\varepsilon - 1, \varnothing)$ where ε is a very small positive number. Then given $\mathbf{m} = (s_1(\theta), \mathbf{m_2}^{\mathbf{dev}}) = ((1, \varnothing), (\varepsilon - 1, \varnothing))$,

$$\Theta^2(\mathbf{m}) = \{\tilde{\theta} \mid s_1(\tilde{\theta}) = (1, \varnothing),$$
$$u_2^*(\tilde{\theta}) \leq \max_{\beta \in \mathrm{BR}\,(\Theta, \mathbf{m})} (1 - \beta) u_2(\tilde{\theta}, y_2(\varepsilon - 1, \varnothing)) + \beta u_2(\tilde{\theta}, y_1(1, \varnothing))\}.$$

For $\tilde{\theta}$ such that $\tilde{\theta}^a = 1$ and $s_1(\tilde{\theta}) = (1, \varnothing)$, since

$$u_2^*(\tilde{\theta}) = 0 > u_2(\tilde{\theta}, y_1(1, \varnothing))$$

and

$$u_2^*(\tilde{\theta}) = 0 > u_2(\tilde{\theta}, y_2(\varepsilon - 1, \varnothing)),$$

we have

$$u_2^*(\tilde{\theta}) > \max_{\beta \in \mathrm{BR}\,(\Theta, \mathbf{m})} (1 - \beta) u_2(\tilde{\theta}, y_2(\varepsilon - 1, \varnothing)) + \beta u_2(\tilde{\theta}, y_1(1, \varnothing)).$$

Therefore for any $\tilde{\theta} \in \Theta^2(\mathbf{m})$, $\tilde{\theta}^a = 0$. Hence, for any $\mu \in \Delta(\Theta^2(\mathbf{m}))$,

$$\pi_1 = u_d((0, \tilde{\theta}^b), y_1(1, \varnothing)) = u_d(0, \tilde{\theta}^b), (1, \tilde{\theta}^b - 1)) = -2$$

while

$$\pi_2 = u_d((0,\tilde{\theta}^b), y_2(\varepsilon - 1, \varnothing)) = u_d((0,\tilde{\theta}^b), (\varepsilon - 1, \tilde{\theta}^b)) = -(1-\varepsilon)^2 > -2.$$

So $\beta(\mathbf{m}) = 0$ and

$$\begin{aligned} u_2^{dev}(\theta_2, (1,\varnothing), (\varepsilon - 1, \varnothing)) &= u_2((0,\theta_2^b), y_2(\varepsilon - 1, \varnothing)) \\ &= u_2((0,\theta_2^b), (\varepsilon - 1, \theta_2^b)) \\ &= -\varepsilon^2 \\ &> -1 \\ &= u_2^*(\theta_2). \end{aligned}$$

Therefore $s_1(\theta)$ that falls into subcase 2 cannot be sustained in equilibrium.

Subcase 3 *$y(\theta) = 1$. There exists $\tilde{\theta}$ such that $(s_1(\tilde{\theta}), s_2(\tilde{\theta})) = ((1,\varnothing), s_2(\theta))$ and $\tilde{\theta}^a = 1$. There exists $\tilde{\theta}$ such that $(s_1(\tilde{\theta}), s_2(\tilde{\theta})) = ((1,\varnothing), s_2(\theta))$ and $\tilde{\theta}^a = 0$.*

Let $\theta_1 = (0,\theta_1^b)$ and $\theta_2 = (1,\theta_2^b)$ be such that $(s_1(\theta_1), s_2(\theta_1)) = (s_1(\theta_2), s_2(\theta_2))$. The same argument from Subcase 2 shows that 2 has incentive to deviate at θ_2. Therefore $s_1(\theta)$ that falls into Subcase 3 cannot be sustained in equilibrium.

Subcase 4 *$y(\theta) = 0$. For all $\tilde{\theta}$ such that $(s_1(\tilde{\theta}), s_2(\tilde{\theta})) = ((0,\varnothing), s_2(\theta))$, $\tilde{\theta}^a = 0$.*

I first prove the following claim:

Claim 4 *If for some $\theta = (1,\theta^b)$, $s_1(\theta) = (0,\varnothing)$, then $s_2(\theta) = (0,\varnothing)$ and $\beta(s_1(\theta), s_2(\theta)) = 0$.*

Proof of Claim 4. $u_2^*(\theta) = 0$; otherwise, 2 has a profitable deviation $(0,\varnothing)$. To see this, note that given $\mathbf{m} = ((0,\varnothing),(0,\varnothing))$, for any $\mu \in \Delta(\Theta)$, $\pi_1 < \pi_2$ and therefore $\beta(\mathbf{m}) = 0$. Therefore $u_2^{dev}(\theta, s_1(\theta), \mathbf{m_2}^{\mathbf{dev}}) = u_2(\theta, y_2(\mathbf{m_2}^{\mathbf{dev}})) = 0$.

Since $u_2(\theta, y_1(s_1(\theta))) < 0$, we have that $\beta(s_1(\theta), s_2(\theta)) = 0$ and that $s_2(\theta) = (0, \varnothing)$. ■

Now let's come back to Subcase 4. There are two possibilities: $z(\theta) > -1$ and $z(\theta) = -1$.

- If $z(\theta) > -1$, then 2 has a profitable deviation to an off-path $(z(\theta) - \varepsilon, \varnothing)$ where $z(\theta) - \varepsilon > -1$. To see this, note that given $\mathbf{m} = (s_1(\theta), \mathbf{m_2}^{\mathbf{dev}}) = ((0, \varnothing), (z(\theta) - \varepsilon, \varnothing))$, by Claim 4, any $\tilde{\theta}$ with $\tilde{\theta}^a = 1$ does not belong to $\Theta^2(\mathbf{m})$. Therefore if $\tilde{\theta} \in \Theta^2(\mathbf{m})$, then $\tilde{\theta}^a = 0$. Hence, for any $\mu \in \Delta(\Theta^2(\mathbf{m}))$, $\pi_1 < \pi_2$ and $\beta(\mathbf{m}) = 0$. So

$$\begin{aligned} u_2^{dev}(\theta, s_1(\theta), \mathbf{m_2}^{\mathbf{dev}}) &= u_2(\theta, y_2(\mathbf{m_2}^{\mathbf{dev}})) \\ &= u_2((0, \theta^b), y_2(z(\theta) - \varepsilon, \varnothing)) \\ &= -(z(\theta) + 1 - \varepsilon)^2 \\ &> -(z(\theta) + 1)^2 \\ &= u_2(\theta, y_2(s_2(\theta))) \\ &\geq u_2^*(\theta). \end{aligned}$$

The last inequality follows from Lemma 1.

- $z(\theta) = -1$. Let μ be the DM's belief at θ. Then by the definition of Subcase 4, $\mu\{\tilde{\theta} \mid \tilde{\theta}^a = 0\} = 1$. Given $\mathbf{m} = (s_1(\theta), s_2(\theta)) = ((0, \varnothing), (-1, \varnothing))$ and μ, $\pi_1 = \pi_2$. Therefore $\beta(s_1(\theta), s_2(\theta)) = \frac{1}{2}$ and $u_2^*(\theta) = \frac{1}{2} u_2(\theta, y_1(s_1(\theta))) + \frac{1}{2} u_2(\theta, y_2(s_2(\theta))) = -1$.

Now let 2 deviate to an off-path $(\varepsilon - 1, \varnothing)$. Given $\mathbf{m} = (s_1(\theta), \mathbf{m_2}^{\mathbf{dev}}) = ((0, \varnothing), (\varepsilon - 1, \varnothing))$, same argument for the $z(\theta) > -1$ case above shows that for any $\tilde{\theta} \in \Theta(\mathbf{m})$, $\tilde{\theta}^a = 0$. Therefore $\beta(\mathbf{m}) = 0$ and $u_2^{dev}(\theta, s_1(\theta), \mathbf{m_2}^{\mathbf{dev}}) = u_2(\theta, y_2(\mathbf{m_2}^{\mathbf{dev}})) = u_2((0, \theta^b), (\varepsilon - 1, \theta^b)) = -\varepsilon^2 > u_2^*(\theta)$.

Therefore if $s_1(\theta)$ falls into subcase 4, then it cannot be sustained in equilibrium.

Subcase 5 $y(\theta) = 0$. *For all* $\tilde{\theta}$ *such that* $(s_1(\tilde{\theta}), s_2(\tilde{\theta})) = ((0, \varnothing), s_2(\theta))$, $\tilde{\theta}^a = 1$.

By the definition of Case 3 and Subcase 5, there exists $\theta_1 = (0, \theta_1^b)$ such that $s_1(\theta_1) = (0, \varnothing)$ and $s_2(\theta_1) \neq s_2(\theta)$. Then the DM's belief μ at θ must assign probability $0 < p < 1$ to $\{\tilde{\theta} \mid \tilde{\theta}^a = 0\}$: if $p = 0$, then μ is inconsistent with equilibrium strategies; if $p = 1$, then $s_1(\theta_a)$ falls into subcase 4, which we have established as unsustainable in equilibrium.

Therefore there exists $\theta_2 = (1, \theta_2^b)$ such that $(s_1(\theta_2), s_2(\theta_2)) = (s_1(\theta_1), s_2(\theta_1))$. The same argument in Subcase 2 shows that 2 has a profitable deviation at θ_1.

Therefore if $s_1(\theta)$ falls into subcase 5, then it cannot be sustained in equilibrium.

Subcase 6 $y(\theta) = 0$. *There exists* $\tilde{\theta}$ *such that* $(s_1(\tilde{\theta}), s_2(\tilde{\theta})) = ((0, \varnothing), s_2(\theta))$ *and* $\tilde{\theta}^a = 1$. *There exists* $\tilde{\theta}$ *such that* $(s_1(\tilde{\theta}), s_2(\tilde{\theta})) = ((0, \varnothing), s_2(\theta))$ *and* $\tilde{\theta}^a = 0$.

Let $\theta_1 = (0, \theta_1^b)$ and $\theta_2 = (1, \theta_2^b)$ be such that $(s_1(\theta_1), s_2(\theta_1)) = (s_1(\theta_2), s_2(\theta_2))$. Same argument for Subcase 5 shows that 2 has a profitable deviation at θ_1.

Therefore if $s_1(\theta)$ falls into Subcase 6, then it cannot be sustained in equilibrium.

So the only sustainable case is Case 2. Therefore in any equilibrium in which $(s_1(\theta), s_2(\theta)) = ((y(\theta), \varnothing), (z(\theta), \varnothing))$ for all θ, $z(\theta) = \theta^a - 1$, $y(\theta) \neq \theta^a$, and $\beta(s_1(\theta), s_2(\theta)) = 0$.

Now let 1 deviate to $(\varnothing, \varnothing)$ at any state θ. Given $\mathbf{m} = (\mathbf{m_1}^{\mathbf{dev}}, s_2(\theta)) = ((\varnothing, \varnothing), (\theta^a - 1, \varnothing))$, for any $\tilde{\theta} \in \Theta^1(\mathbf{m})$, $\tilde{\theta}^a = 1 + z(\theta)$. Therefore for any $\mu \in \Delta(\Theta^1(\mathbf{m}))$, $\pi_1 = -1 = \pi_2$ and $\beta(\mathbf{m}) = \frac{1}{2}$. Hence, $u_1^{dev}(\theta, \mathbf{m_1}^{\mathbf{dev}}, s_2(\theta)) = -1 > -2 = u_1^*(\theta)$. $(\varnothing, \varnothing)$ is a profitable deviation for 1.

9.6 Proof of Proposition 5

Suppose $s = (s_1, s_2)$ satisfies: $\forall j$, $|b_i^j| < |b_{-i}^j|$ implies that $s_i^j(\theta) \in \mathbb{R}$ and $s_i^j(\theta) = \varnothing$ for all θ.

Step 1. $\beta(s(\theta)) = \frac{1}{2}$, $\forall \theta$.

Suppose $\beta(s(\theta)) = 1$ for some θ. Then Agent 2 can deviate to $s_1(\theta)$. Since both agents are committing to the same policies on Agent 2's disadvantaged issue and vague on Agent 2's advantaged issues, $\beta(s_1(\theta), s_1(\theta)) = 0$.

Step 2. We proceed through two cases.

Case 1. $\|b_1\| = \|b_2\|$.

First we show that for any $j \in N$ such that $|s_i^j(\theta)| \in \mathbb{R}$, $s_i^j(\theta) = \theta^j - b_i^j$. If the equality does not hold for some $j \in N$ at some θ, then let Agent i deviate to m_i such that $m_i^j = \varnothing$, $\forall j \in N$. Given $\mathbf{m}' = (m_i, s_{-i}(\theta))$, $\mu(\mathbf{m}')$ must satisfy:

$$\mathrm{E}_{\mu(\mathbf{m}')}[u_d(\tilde{\theta}, y_i(m_i))] = u_d(b_i),$$

$$\mathrm{E}_{\mu(\mathbf{m}')}[u_d(\tilde{\theta}, y_{-i}(s_{-i}(\theta)))] \leq u_d(b_{-i}) = u_d(b_{-i}).$$

Therefore the winning probability for Agent i is at least $\frac{1}{2}$. Since Agent i has deviated to a strictly better proposal, m_i is a profitable deviation for him.

Since any specific policy is its proposer's ideal policy given the state, for any θ and any j such that $s_i^j(\theta) \in \mathbb{R}$, let Agent i deviate to $\theta^j - b_i^j + \varepsilon$, with $\varepsilon > 0$ small. The DM realizes this as a compromise and chooses him with probability 1. Agent i then gets arbitrarily close to his own ideal policy instead of a 50-50 lottery between his and Agent $-i$'s ideal policy. Therefore this constitutes a profitable deviation.

Case 2. $\|b_1\| < \|b_2\|$. ($\|b_1\| > \|b_2\|$ is similar.)

Let Agent 1 deviate to m_1 such that $m_1^j = \varnothing$, $\forall j \in N$. Agent 1 wins with probability 1 and gets his own ideal policy for sure. Since in equilibrium each agent wins with probability $\frac{1}{2}$, and Agent 2 is vague for some issue, Agent 1 cannot get his own ideal policy for sure. Therefore m_1 is a profitable deviation for Agent 1.

9.7 Proof of Proposition 6

Similar to Proposition 5, $\beta(s(\theta)) = \frac{1}{2}$, $\forall\theta$. Therefore $u_1^*(\theta) < 0$. Now let Agent 1 deviate to $(\varnothing, \varnothing)$. $\beta((\varnothing, \varnothing), s_2(\theta)) = 1$ since Agent 1 has an advantage. Therefore $u_1^{dev}(\theta, (\varnothing, \varnothing), s_2(\theta)) = 0 > u_1^*(\theta)$.

10 Discussion

Commitment to a single policy. Throughout the paper, the proposals are binding: any policy proposed commits its proposer to it. This is a polar case. Another polar case would be when proposals are cheap talk. In cheap talk communication, messages have no intrinsic meaning except from that determined in equilibrium. Therefore a vague message is no different from a specific message.

Proposals in my model incorporate two extreme types of commitment: full commitment or complete lack of commitment. As mentioned in Section 3, one can think of incorporating a third type of commitment: committing to a set of policies. This would change the meaning and implications of vagueness and I focus on the two extreme cases in my paper.

General biases. We have seen from Section 8 that the undercutting argument can be applied to the case in which agents' biases are more general. The key factor for the undercutting argument is agents' comparative advantages. More formally, as long as Agent 1's bias on issue a is smaller than that of Agent 2 and vice-versa for issue b, it is not an equilibrium for both agents to be vague on their opponents' less-biased issue and commit to their own ideal policies on their opponents' more-biased issue. Therefore, undercutting does not rely on the agents' total biases being equal in size, or each agent having a zero bias on some issue.

Moreover, the strategy profile in Proposition 1 can also be sustained when the biases are more general. In particular, it remains to be an intuitive equilibrium strategy profile for the following two classes of biases:

1. On issue a, Agent 1 is less biased than Agent 2; the converse is true on issue b. In addition, Agent 2's bias on issue a alone is larger than Agent 1's total bias.

2. Agent 1 is unbiased on issue a and biased on issue b. Agent 2 is unbiased on issue b and biased on issue a. Agent 1's total bias is equal to that of Agent 2.

The difference between these two classes of biases is that, in the first case, Agent 1 always wins. In the second case, each agent wins with probability $\frac{1}{2}$.

Refinement. Refinement for two-agent signaling games was first studied by Bagwell and Ramey (1991). They first consider refinement for equilibria in which each agent plays a separating strategy. In this kind of equilibria, they restrict the beliefs to be *unprejudiced* and *open-minded*. Beliefs are unprejudiced if a deviation is rationalized by the minimal number of deviations possible. A consequence of this restriction is that, seeing an off-path and an on-path proposal, the DM believes that the on-path agent has not deviated. Therefore our refinements coincide on this respect. Recent work shows that any pure-strategy equilibria satisfying Kohlberg and Mertens (1986)'s notion of strategic stability must have unprejudiced beliefs (Vida and Honryo, 2015). In addition, beliefs are open-minded if the following is satisfied: if at all states the signal profile can be justified by the same number of deviations, the DM does not believe that any state is certain. This corresponds to the case in which both agents make on-path proposals but the proposal profile is off-path in my paper. My restriction for this case is stronger than theirs. Rather than requiring the belief to be non-degenerate, I require the DM to believe one of the agents has made a deviation that is not equilibrium dominated.

Then they consider equilibria in which each agent plays a pooling strategy. In this kind of equilibria, beliefs are required to be ε-*intuitive*: whenever an off-path profile can be rationalized by the same number of deviations for all states, instead of open-mindedness, they require the beliefs to attach a much greater probability to equilibrium-admissible than equilibrium-dominated deviations. This refinement is a combination of CK's intuitive criterion and open-mindedness.

Instead of imposing restrictions for different types of equilibria, I impose the same refinement procedure, i.e. intuitive criterion to different kinds of equilibria. Schultz (1996) takes an approach similar to mine. The difference is that, when the DM cannot identify the deviator, no restriction is placed. Whereas in this paper, the DM continues to hold unprejudiced beliefs and uses the minimal number of deviations to rationalize the observed profile. Moreover, she continues to apply the intuitive criterion to the deviations.

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