

The Benefit of Collective Reputation

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Abstract

We study a model of collective reputation. Consumers form beliefs about the expected quality of a good that is produced by a firm that belongs to a collective of firms who operate under a shared brand name. Consumers' limited ability to distinguish between firms in the collective and to monitor firms' investment decisions creates incentives to free-ride on other firms' investment efforts. Nevertheless, we show that collective brands induce stronger incentives to invest in quality than individual firms under two types of circumstances: if the main concern is with quality control and the baseline reputation of the collective is low, or if the main concern is with the acquisition of specialized knowledge and the baseline reputation of the collective is high. Our results can be applied to country-of-origin, but also appellation or other collective brands.

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1 Introduction

Appellation names of wines, such as “Bordeaux”, “Riesling”, or “Malbec,” play an important role when we evaluate the quality of a wine. Average wine drinkers are typically not familiar with specific vineyards but do have an idea about the quality of different wine types. Similarly, the country of origin of a product indicates something about its quality. How does such a collective brand name operate and how can it sustain its brand value? Why, for example, does the car manufacturer Volkswagen advertise “The power of German engineering,” and the watch manufacturer TAG Heuer emphasizes “Swiss made,” while a German appliance manufacturer such as Bosch fosters its brand with non-country specific slogans such as “made for life”? Why do so many successful Chinese suppliers emphasize their country of origin?¹

A brand can be thought of as a means to build good reputation. When building reputation, a firm faces a *moral hazard problem*; its investment in quality is unobserved by current consumers, and the reputational return on its investment can only be collected in the future.

The benefits of reputation differ in a collective brand and an individual brand. At first glance, collective brands may seem like a bad idea. If several firms operate under one brand name, each firm has an incentive to free-ride on other firm’s investments. And the impact of a firm’s investment in its own quality is weaker in a collective brand because consumers are uncertain whether quality is generated by the firm itself or one of the other firms in the brand. Thus, the “precision” of the signal that is generated by investments in quality of a firm is smaller with collective reputation. This weakens the incentive to invest in quality in a collective brand.

Nevertheless, under some circumstances, a collective brand can serve as a *commitment device* for investment in high quality. If a brand is very successful (for example, as a result of previous large investments), then a firm might be discouraged from investing more because the returns from additional investment become small, i.e., the firm rests on its laurels.

¹For example, thousands of Chinese manufacturers advertise on the platform www.made-in-china.com.

Similarly, if a brand develops a bad reputation (possibly as a result of no investment), then returns on investment are also low and the firm might altogether give up. As we show, collective reputation can mitigate these “discouragement effects” faced by individual firms after very good or very bad histories by making extreme beliefs about them less likely.

Exactly how extreme beliefs about a brand (good or bad) can be depends both on the structure of signals that consumers obtain about firms’ investments in quality and the baseline reputation of firms in the industry. For example, in manufacturing *quality control* is important and consumers can easily learn that a firm is incompetent (or has failed to invest) when a product has low quality. In contrast, in industries that require a special *technological know-how*, a good quality realization reveals that a firm has that technology and is a “good type.”

We analyze a model of reputation that can incorporate both individual and collective reputation with multiple sellers in the vein of Mailath and Samuelson (2001). The model has the following features. There are two types of firms, competent and incompetent. Only competent firms have the option to invest in quality.² Consumers observe noisy signals of past investment decisions. Given these features, competent types can differentiate themselves from incompetent types by investing over time and generating better signals. If consumers believe that competent types invest, then they infer that a firm with good signals is indeed more likely to be competent. And, given this belief, they are willing to pay more for goods produced by firms with better past signals. This, in turn, provides an incentive for a competent firm to invest in quality.

Accordingly, we define a firm’s reputation as the consumers’ posterior belief that it is competent. The best possible equilibrium is the one where competent firms always invest, after every history, and we call an equilibrium where this is the case the *reputational equilibrium*. In most of the paper, we restrict our analysis to the properties of this equilibrium.

²Incompetent firms are often referred to in the literature as bad “Stackelberg” or “commitment” types. Their incompetence can be due to either prohibitively high cost of investment or lack of access to the technology necessary for investment.

As pointed out by Mailath and Samuelson (2001), such an equilibrium exists only if beliefs are bounded. If beliefs are not bounded, then as the competent type continues to invest and to produce favorable signals, consumers eventually learn almost perfectly that the firm must be competent. This destroys the firm's incentive to invest, which leads to a collapse of the equilibrium. This cannot happen in our model as we assume bounded memory of buyers as in Moav and Neeman (2010).³

Our model of reputation for a collective of firms is a natural extension of this basic model. Firm's types are independently drawn from a given distribution once and for all. In each period, a short-lived consumer is randomly matched with one firm. With individual reputation, each firm establishes its brand, which allows the consumer to observe firm-specific past signals. With collective reputation, the consumer observes signals only at the group level.

The reputational equilibrium exists in each model if the benefit of investment exceeds its cost after every possible history. A firm has both short-run and long-run incentives to invest or refrain from investment. In the *short-run*, a firm may want to exploit its current reputation. In the *long-run*, the firm may want to free-ride on future efforts by itself and or other possible members of the brand. Collective reputation can improve the short-run incentives to invest because the best possible collective reputation is weaker than the best possible individual reputation so the incentive to cash in on existing good reputation is weaker. However, the very fact that the best possible individual reputation is better than the best possible collective reputation also implies that individual reputation induces stronger incentives to invest in the long-run because it allows the firm to establish a stronger reputation individually.

³Mailath and Samuelson (2001) instead allow firms to exit and be replaced by another whose type consumers are uncertain about. alternative is to assume that a firm's type randomly over time, as discussed in Holmström (1999). In this paper, we make the assumption of limited memory because it allows us to compute the necessary and sufficient conditions for the existence of the reputational equilibrium in a closed form, which we use to compare two models of reputation (individual and collective). We also believe the assumption captures the nature of the market's limited memory of, and/or attention paid to, an arbitrarily long history.

In the case of “quality control,” the discouragement effect is stronger for individual brands after a firm has produced low quality because in this case consumers infer that the firm is likely to be incompetent, regardless of what else is observed. We show that in this case, collective reputation can help overcome the firm’s moral hazard problem if the baseline reputation for the firm or brand is low. The mechanism is the following: if baseline reputation is low, then observation of a good signal moves beliefs significantly, while observation of bad signals do not have a big effect on beliefs because even if one firm is incompetent, other firms in the brand may be competent.

In the case of “exclusive know-how,” we show that collective reputation is beneficial if the baseline reputation for the firm or brand is high. The reason is that it is easier for a competent individual firm to re-establish its good reputation after cashing in on it compared to a firm that is part of a collective brand, because even if consumers observe high signals, it is still possible that some members of the brand are incompetent. This implies that individual firms have a stronger incentive to shirk than firms that belong to a collective brand.

It is important to note that in order for collective reputation to function as a commitment device, investment decisions cannot be made too frequently, or equivalently, the discount factor cannot be too large. The reason is that a firm has a stronger incentive to free-ride on future efforts by itself or other members of the brand with collective reputation (long-run incentive). Thus, the short-run advantage of collective reputation can only outweigh the long-run free-riding incentive if firms do not care too much about the future. We first establish these results in a model in which consumers have a two-period memory, and then extend them to a model with any given finite memory.

Finally, we also address the issue of brand formation. If firms can freely choose with whom to brand then it is important to understand whether the commitment value of a collective brand is sufficiently high to encourage competent firms to brand with an incompetent firm. We show that this is indeed the case for some parameter values.

The paper is structured as follows. In Section 2, we set up the model and discuss the

equilibrium concept. In Section 3, we first analyze the model of individual reputation and collective reputation separately, and then make comparisons to present our main results. Then, we end the section with a discussion of the results and applications. Section 4 relaxes the assumption of two-period memory. We conclude with a discussion of endogenous brand formation. All proofs are in Appendix. Next, we discuss the related literature.

Related Literature.

This paper on the one hand relates to the theoretical economics literature on reputation, and on the other hand to the more applied literature on branding and career concerns, the two most common applications of reputation models.

The theoretical paper on reputation that is most relevant to the present work is by Mailath and Samuelson (2001). The authors examine a market with two types of firms, where the competent type can choose between high- and low-level of effort, while the incompetent type can only put in low effort. They find conditions under which a high-effort equilibrium in which competent firms always choose high effort. They also document difficulties with sustaining the high-effort equilibrium. If consumers can observe all the past history of a firm, then they eventually learn almost perfectly about the firm's type. Then, the firm's investment incentive falls close to zero, unraveling the equilibrium. Therefore, the model requires firms to be replaced by new firms, which prevents consumers' posterior beliefs from being driven to extremes. Sustainability of the high-effort equilibrium is important because it provides the highest profit to competent firms. In this paper, we suggest collective brand as a possible device to provide more commitment power. Cripps et al. (2004)

Tirole (1993) is to the best of our knowledge the first to model and analyze the idea of collective reputation. He studies incentives that a new member of an organization faces in deciding his or her effort levels when the reward depends on the past record of the organization's elders. With employees being randomly replaced by new ones, an employee can be given either of two tasks, one of which provides a greater incentive to work hard. If the group's reputation is good, the principal benefits from assigning a task that will induce a

hard work from the employee, which reinforces a good reputation for the group. Due to the complementarity between past and future reputations, multiple stationary equilibria arise, depending on the past reputation of the organization. One of the striking results is that a single period of corruption shock in which every employee “shirks” can be followed by the low-reputation steady state, leading to a persistence of corruption. Levin (2009) extends Tirole’s work by incorporating dynamics through stochastically evolving cost of high effort. Unlike in Tirole (1993) where a group reputation can change drastically, Levin (2009) finds that with dynamics a group’s reputation has lingering effects.

A concurrent strand of the literature in marketing and economics has been concerned with umbrella branding and brand extension. Reputation of an umbrella brand is determined by performance of different products within the brand. Unlike in the environment of our focus and those of collective reputation, consumers can distinguish between products under an umbrella brand. Works by Wernerfelt (1988), Choi (1998), Cabral (2000), Miklós-Thal (2012), and Moorthy (2012) study a decision to extend a brand as a device to pool reputation of multiple products within a firm and find that the decision can signal for a high quality.⁴

Collective reputation has also been studied in marketing in different topics, such as county of origin (COO) and umbrella branding. Because consumers imperfectly observe quality investment of each firm, firms in COO have incentives to free-ride on other firms’ efforts. Because in emerging markets COO is perceived as a stamp for low quality, high quality firms consider an option of breaking away from COO. Zhang (2015) shows that high quality firms’ efforts to dissociate themselves from COO may improve the reputation for COO, as the free-riding incentive of remaining firms decreases, and hence they invest more in quality. Fleckinger (2014) considers collective reputation under a Cournot oligopoly setting where consumers only learn the average quality. He studies the effect of number of firms in the market on the welfare. He shows that the quality is decreasing in the number of firms due to free-riding incentives (Holmstrom (1982)), while the quantity is increasing.

⁴Moorthy (2012) finds that the signaling may only work under a set of stringent conditions and posits that a brand extension may provide information through an alternative mechanism other than signaling.

The closest other work to consider the benefits of collective reputation is that of Fishman et al. (2014), which considers consumers of a collective brand observe the quality produced by all members of the brand prior to their purchase, and focus on how a collective brand can benefit each member by improving the brand's visibility. They show that this benefit of collective reputation can outweigh each member's incentive to free-ride on other members' investment efforts as long as the number of brand members is not too large. Firms in their model invest only once. Thus, they abstract away from issues of commitment and dynamics, which are addressed in our model.

Reputation in teams has also been analyzed in the context of career concerns ala Holmström (1999) and the theory of the firm. The most related paper is by Bar-Isaac (2007) who looks at an overlapping generations model to highlight the benefit of senior entrepreneurs to work with young juniors who lack any reputation. Our paper is also related to Tadelis (1999) and Tadelis (2002) who considers the firm as a bearer of reputation who can sell the name.

2 The Model

Basics. Consider a market with two firms (female) $i \in \{1, 2\}$ that produce vertically differentiated experience goods at zero marginal cost. In every period over an infinite discrete time horizon $t = \dots, -1, 0, 1, \dots$, exactly one short-lived customer (male) with unit demand arrives and is randomly matched with one of the firms.

Each firm is either competent C or incompetent I , but the type $\theta_i \in \{C, I\}$ of firm i is not observed by customers. Each firm's type is independently drawn being C with probability μ and I otherwise. Unlike an incompetent firm, a competent firm can invest to improve the probability of producing a product that performs well by paying a cost $c > 0$. If a competent firm makes an investment in period t , then her product performs well (G) with probability π_H . Otherwise, a good performance occurs with probability π_L , where $\pi_L < \pi_H$.

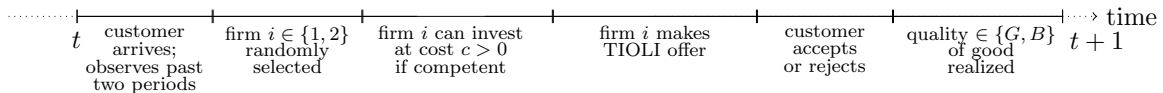


Figure 1: Timing of the game

A firm's investment decision is her private information, but a customer observes the past two outcomes. Using this information, he updates his beliefs about the type of the firm. A firm that is matched to a customer makes a take-it-or-leave-it (TIOLI) offer p . The customer can accept or reject the offer and leaves the market. Figure 1 summarizes the timing of the game.

One can think of the observability of product quality in past periods resulting from word-of-mouth between customers or customer reviews. Similarly, the model is consistent with a long-lived, myopic customer who forgets everything that has happened more than two periods ago.

Payoffs. We normalize the utility from rejection to 0. If the customer accepts, he receives utility 1 from a product that performs well, and 0 otherwise. A firm that sells in period t at a price p_t receives a profit of

$$v_t = p_t - c \cdot \mathbf{1}(\text{invest})$$

in period t where $\mathbf{1}(\text{invest}) = 1$ if she invests and $\mathbf{1}(\text{invest}) = 0$ otherwise. The firm's discount factor is $\delta \in (0, 1)$. Thus, at the beginning of any period t , the seller's expected continuation value is given by

$$V_t = \mathbb{E} \left[\sum_{s=t}^{\infty} \delta^{s-t} v_s \right]$$

where v_s are random variables that depend on whether the firm is chosen or not and on the realization of the quality of the product.

Information Structure. We consider two different information structures. In the case of *individual reputation*, firms establish and manage their reputation independently. If a consumer is matched with firm i , then the set of payoff-relevant histories is given by

$$\mathcal{H}_b^{\text{ind}} = \{G, B, \emptyset\}^2$$

where \emptyset represents the past event in which firm $i \in \{1, 2\}$ has not been chosen. Firm i only gets a chance to sell if she is exogenously chosen. Hence, for each firm i the set of payoff-relevant histories are given by $\mathcal{H}_i^{\text{ind}} = \mathcal{H}_b^{\text{ind}} \times \{1, \emptyset\}$ where 1 stands for a visit and \emptyset for no visit.

Under the case of *collective reputation*, consumers cannot distinguish between the two firms. Thus, the set of payoff-relevant histories is given by

$$\mathcal{H}_b^{\text{col}} = \{G, B\}^2$$

and firms' payoff-relevant histories are given by

$$\mathcal{H}_i^{\text{col}} = \mathcal{H}_b \times \{1, \emptyset\},$$

where the last entry of the history is 1 if firm i is visited in the current period and \emptyset otherwise. We denote a history produced i periods ago by h_i . Thus, a customer's history can be written as $h = h_2 h_1 \in \mathcal{H}_b^x$ ($x \in \{ind, col\}$), with h_1 being the most recent history, and h_2 the history produced two periods ago.

Equilibrium. We are interested in *stationary equilibria* where all strategies only depend on the histories specified above. It is given by a purchasing strategy of buyers $\sigma : \mathcal{H}_b^x \rightarrow \{0, 1\}$ where $\sigma(h) = 1$ if the consumer buys, a pricing strategy $p : \mathcal{H}_i^x \rightarrow \mathbb{R}$, and investment strategy $\mathcal{I} : \mathcal{H}_i^x \rightarrow \mathbb{R}$, and buyers' beliefs over the type of the firm(s) they are facing.

With *individual reputation*, those posterior beliefs given a history $h \in \mathcal{H}_b^{\text{ind}}$ are given by

a probability $\hat{\mu}^{\text{ind}}(h)$ that the firm is C . With *collective reputation*, those beliefs are given by a probability measure $\hat{\eta}$ over the pair of types of two firms, i.e., the set of states is $\{C, I\}^2$. We denote the posterior belief for the state being $s \in \{C, I\}^2$ given a history $h \in \mathcal{H}_b^{\text{col}}$ by $\hat{\eta}_s(h)$.

In equilibrium, given the other players' strategies and beliefs, each player's strategy must maximize his/her payoffs. The belief about the the state of the world is derived from the realized histories and the seller's strategy by Bayes' rule whenever possible.

We focus most of our attention on the “good” equilibrium in which competent firms always invest in quality. Let us call this a **reputational equilibrium**, and **RE** in short. This equilibrium is the socially optimal one if and only if

$$\Delta\pi \equiv \pi_H - \pi_L \geq c. \tag{1}$$

3 Main Analysis

3.1 Individual Reputation

The equilibrium price after a history h must be equal to the expected value of the consumer as firms make TIOLI offers to short-lived consumers, i.e.,

$$p^{\text{ind}}(h) = \hat{\mu}(h)\pi_H + (1 - \hat{\mu}(h))\pi_L. \tag{2}$$

Given an equilibrium, we denote the continuation value after a history $(h, y) \in \mathcal{H}_i^{\text{ind}}$ by $V^{\text{ind}}(h, y)$. Then, after a history $(h, 1)$, the continuation value after an investment is

$$V^{\text{ind}}(h, 1) \equiv p^{\text{ind}}(h) - c + \frac{\delta}{2}(\pi_H(V^{\text{ind}}(h_1G, 1) + V^{\text{ind}}(h_1G, \emptyset)) + (1 - \pi_H)(V^{\text{ind}}(h_1B, 1) + V^{\text{ind}}(h_1B, \emptyset)))$$

while the continuation value after no investment is (assuming the firm follows the equilibrium strategy after the deviation) is

$$\tilde{V}^{\text{ind}}(h, 1) \equiv p^{\text{ind}}(h) + \frac{\delta}{2} (\pi_L (V^{\text{ind}}(h_1 G, 1) + V^{\text{ind}}(h_1 G, \emptyset)) + (1 - \pi_L) (V^{\text{ind}}(h_1 B, 1) + V^{\text{ind}}(h_1 B, \emptyset))).$$

First, note that as in Mailath and Samuelson (2001), a competent firm never investing is always an equilibrium. In the following we show under which condition a **RE** can be sustained. Since the firm extracts all the surplus in any equilibria, this is also the equilibrium preferred by the firm. There can also be other equilibria which we analyze in Section 5 and Appendix.

The **RE** exists if and only if there are no one-shot deviations, i.e., $V^{\text{ind}}(h, 1) \geq \tilde{V}^{\text{ind}}(h, 1)$. Analogously to Lemma 1 in Moav and Neeman (2010), we can rewrite this necessary and sufficient condition for a **RE** to exist in a closed form thanks to the limited memory of customers.

Lemma 1. *The **RE** exists if and only if*

$$c \leq \hat{c}^{\text{ind}} \equiv \hat{c}^{\text{ind}}(\mu, \pi_H, \pi_L) \equiv \delta \cdot \frac{\Delta\pi}{2} \cdot \left(\min_{h_1 \in \{G, B, \emptyset\}} \hat{d}^{\text{ind}}(h_1) \right)$$

where

$$\hat{d}^{\text{ind}}(h_1) \equiv \underbrace{p^{\text{ind}}(h_1 G) - p^{\text{ind}}(h_1 B)}_{\text{short-run benefits}} + \underbrace{\frac{\delta}{2} (\pi_H (p^{\text{ind}}(GG) - p^{\text{ind}}(BG)) + (1 - \pi_H) (p^{\text{ind}}(GB) - p^{\text{ind}}(BB)) + p^{\text{ind}}(G\emptyset) - p^{\text{ind}}(B\emptyset))}_{\text{long-run benefits}}. \quad (3)$$

Lemma 1 shows that the necessary and sufficient condition for the existence of **RE** simplifies to a cutoff-rule: the investment cost c must be less than its benefit. Since this rule should hold for all histories, the cutoff $\hat{c}^{\text{ind}}(\mu, \pi_H, \pi_L)$ is determined by taking the minimum

of benefits over all possible histories. Although there are a total of nine possible histories in $\{G, B, \emptyset\}^2$, only the most recent history $h_1 \in \{G, B, \emptyset\}$ affects a firm's investment decision. This is because today's older history is being forgotten by the time the benefit of investment materializes in the next period. By investing in period t , the firm can improve the chance of producing a good outcome by $\Delta\pi$ and the gains in profit are given by differences in prices after a good versus a bad history. This gain is captured by $\hat{d}^{\text{ind}}(h_1)$, which we explain in detail next.

By investing in period t , the firm can receive a price premium in period $t + 1$ given by $p^{\text{ind}}(h_1G) - p^{\text{ind}}(h_1B)$ with a higher probability. If she saves the investment cost today, she gives up this benefit and *free-rides on the current reputation*. By period $t + 2$, consumers no longer remember h_1 . Instead, they observe outcomes of the investment decision in period t and $t + 1$. Therefore, the long-run incentive for investment hinges on how much the firm wants to *free-ride on efforts by its future self*. In other words, if the future investment can recover the damage to the brand's reputation that may be caused by the current deviation, the firm is less afraid of shirking now. Then, the realized benefit in $t + 2$ depends on the outcome of the investment decision in period $t + 1$, which are G, B and \emptyset with probability $\frac{\pi_H}{2}$, $\frac{1-\pi_H}{2}$, and $\frac{1}{2}$, respectively. By period $t + 3$, the investment decision in period t is no longer part of the customer's memory and does not affect firm's payoff.

Figure 2 depicts \hat{c} as a function of μ . It shows that different histories are binding for different values of priors and that it is harder to sustain investment for extreme priors, i.e., close to 0 or 1. This is because for extreme beliefs the firm cannot lose much by shirking as pointed out by Mailath and Samuelson (2001) and hence, they are discouraged to invest. If beliefs are high, they rest on their past laurels and if beliefs are low, they give up on trying to build reputation in that period. This is discuss in more detail next.

To fully characterize \hat{c}^{ind} , we need to identify which history provides the binding constraint for different parameter regions. As h_1 only enters in the first two terms of $\hat{d}^{\text{ind}}(h_1)$, it is necessary and sufficient to compare $p^{\text{ind}}(h_1G) - p^{\text{ind}}(h_1B)$ for $h_1 \in \{G, \emptyset, B\}$. We show that

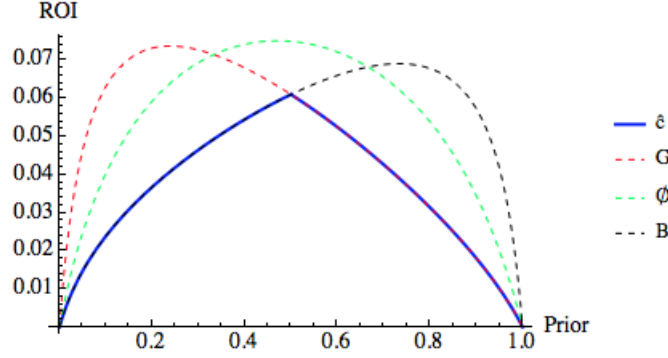


Figure 2: Return on Investment after each history and the minimum

$$\pi_H = 0.8, \pi_L = 0.2, \delta = 0.5$$

$h_1 = \emptyset$ is never binding and the history $h_1 = G$ attains the minimum return from investment if and only if

$$p(GG)^{\text{ind}} - p(GB)^{\text{ind}} \leq p(BG)^{\text{ind}} - p(BB)^{\text{ind}}. \quad (4)$$

Lemma 2. *We have that*

$$\hat{c}^{\text{ind}} = \begin{cases} \delta \cdot \frac{\Delta\pi}{2} \cdot \hat{d}^{\text{ind}}(G) & \text{if } \mu \geq \bar{\mu}^{\text{ind}} \equiv \frac{\pi_L(1-\pi_L)}{\pi_H(1-\pi_H) + \pi_L(1-\pi_L)}, \\ \delta \cdot \frac{\Delta\pi}{2} \cdot \hat{d}^{\text{ind}}(B) & \text{otherwise} \end{cases}$$

As apparent from Equation 2, prices are determined by posterior beliefs. Therefore, for a given history h_1 , $p^{\text{ind}}(h_1G) - p^{\text{ind}}(h_1B)$ quantifies the informativeness of a good signal versus a bad one. By the nature of Bayesian updating, the benefit of a signal reduces as there is less uncertainty in the ex-ante belief. Thus, if consumers' prior beliefs are optimistic, then for $h_1 = G$ the benefit of an additional signal is the smallest. Put differently, for priors μ close to 1 the price difference between GG and GB , as well as GB and BB gets smaller as can be seen in Figure 3 that plots prices after histories $\{GG, GB, BB\}$ as a function of priors. Similarly, the price differences are getting smaller as the prior decreases close to 0.

This results in $h_1 = G$ being binding for high μ and $h_1 = B$ being binding for low μ . $\bar{\mu}^{\text{ind}}$ is the cutoff level of the prior belief where the value of additional signal is the same following

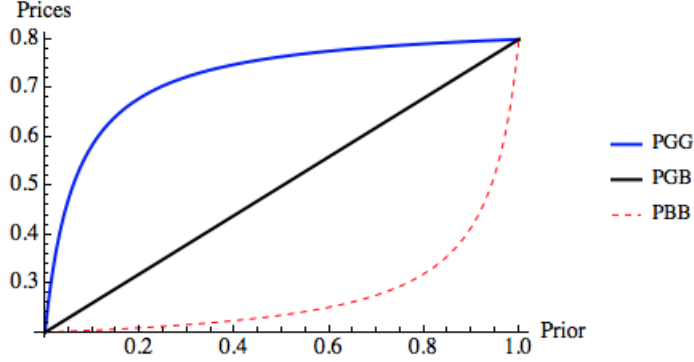


Figure 3: Prices for $\pi_H = 0.8$, $\pi_L = 0.2$

a good history and a bad history.

3.2 Collective Reputation

In the model of collective reputation, two firms form a brand and build reputation together. Since consumers cannot distinguish the two firms, they form beliefs over the joint types of the two firms – $\{C, C\}$, $\{C, I\}$, $\{I, C\}$, and $\{I, I\}$. Note that in this case a firm’s investment incentives depend on the type of the other member of the group because the other firm’s investment decision affects the brand’s reputation, which in turn influences the firm’s future payoffs. In this subsection, we establish under which conditions the **RE** can be sustained and how trade-offs here may be different from that under individual reputation.

We denote the prior belief about a state of the group’s quality $s \in \{C, I\}^2$ by η_s . Since the type of the two firms are independently drawn, the prior beliefs are given by $\eta_{CC} = \mu^2$, $\eta_{IC} = \eta_{CI} = \mu(1 - \mu)$ and $\eta_{II} = (1 - \mu)^2$. Recall that the posterior after having observed a history $h \in \mathcal{H}_b^{\text{col}} = \{G, B\}^2$ is denoted by $\hat{\eta}_s(h)$, which we derive using Bayes’ rule. Thus, given a history h , the price offered by a firm must be

$$p(h) = \Pr(C|h) \cdot \pi_H + (1 - \Pr(C|h)) \cdot \pi_L, \quad (5)$$

where $\Pr(C|h) = \hat{\eta}_{CC}(h) + \frac{1}{2}(\hat{\eta}_{CI}(h) + \hat{\eta}_{IC}(h))$ is the probability that after history h the

chosen firm is competent. We again only need to consider one-shot deviations, i.e., for all histories $h \in \mathcal{H}_b^{\text{col}}$ and all types of the other firm ($\theta \in \{C, I\}$), the continuation payoff following investment should be greater or equal than that following a single deviation. Let us denote the present discounted expected equilibrium profit of a competent firm under branding with a θ -type firm after history $(h, d) \in \mathcal{H}_i^{\text{col}}$ by $V(h, d; \theta)$ and the continuation payoff after no investment (assuming the firm follows the equilibrium strategy after the deviation) by $\tilde{V}(h, d; \theta)$. Then, a **RE** exists if and only if $V(h, d; \theta) \geq \tilde{V}(h, d; \theta)$ for all h, d, θ . The equivalent to Lemma 1 is the following.

Lemma 3. *In the case of collective reputation, the **RE** exists if and only if*

$$c \leq \hat{c}(\mu, \pi_H, \pi_L) \equiv \delta \cdot \frac{\Delta\pi}{2} \cdot \left(\min_{h_1 \in \{G, B\}, \theta \in \{C, I\}} \hat{d}(h_1, \theta) \right) \quad (6)$$

where

$$\begin{aligned} \hat{d}(h_1, \theta) = & \underbrace{p(h_1G) - p(h_1B)}_{\text{short-run benefits}} \\ & + \delta \underbrace{\left(\frac{\pi_H + \pi(\theta)}{2} (p(GG) - p(BG)) + \left(1 - \frac{\pi_H + \pi(\theta)}{2}\right) (p(GB) - p(BB)) \right)}_{\text{long-run benefits}}, \end{aligned} \quad (7)$$

and $\pi(\theta) = \pi_H$ if $\theta = C$ and π_L if $\theta = I$.

Analogously to the case of individual firms, $\hat{d}(h_1, \theta)$ summarizes the future benefit of investment when the group's most recent outcome is h_1 and the type of the other firm is $\theta \in \{C, I\}$. If the short-run benefit $p(h_1G) - p(h_1B)$ is small, the firm's incentive to invest is low and it wants to *free-ride on its current reputation*. The long-run benefit to be realized in period $t + 2$ depends on the outcome generated by the group in period $t + 1$. If the group produces a good outcome, the firm would enjoy a price premium $p(GG) - p(BG)$. This event occurs with probability $\frac{\pi_H + \pi(\theta)}{2}$ because each firm is visited equally likely. The group produces a bad signal in period $t + 1$ with the remaining probability and receives a

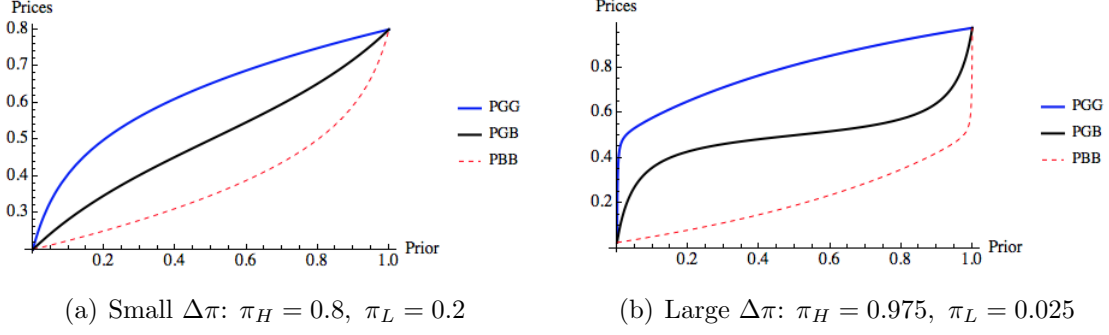


Figure 4: Prices as a function of prior μ

premium of $p(GB) - p(BB)$. Therefore, the long-run benefits capture the *free-riding on future investments by the collective*. An individual brand only produces a good outcome in period $t + 1$ with a probability $\frac{\pi_H}{2}$. However, a collective brand does with a greater probability $\frac{\pi_H + \pi(\theta)}{2}$ because the other firm can produce a good signal, too. In the next section we compare two regimes of reputation in detail and show how collective reputation can provide better incentives for investment despite this free-riding incentives.

Next, we identify the pair $(h_1, \theta) \in \{G, B\} \times \{C, I\}$ that minimizes $\hat{d}(\cdot, \cdot)$. First, for any given θ , $h_1 = G$ is binding if and only if

$$p(GG) - p(GB) \leq p(BG) - p(BB), \quad (8)$$

Moreover, if (8) then $\theta = C$ is binding because it places a higher probability on $h_1 = G$. Thus, $(h_1, \theta) = (G, C)$ attains the minimum for \hat{d} in that case, and otherwise (B, I) does. By plugging in the beliefs for prices, we obtain the following characterization of \hat{d} :

Lemma 4. (i) If $\Delta\pi$ is small, there exists $\mu_1 \in (0, 1)$ such that $\arg \min_{h_1 \in \{G, B\}, \theta \in \{G, B\}} \hat{d}(h_1; \theta) = (G, C)$ if and only if $\mu \geq \mu_1$ and $\arg \min_{h_1 \in \{G, B\}, \theta \in \{G, B\}} \hat{d}(h_1; \theta) = (B, I)$ if and only if $\mu < \mu_1$. (ii) If $\Delta\pi$ is large, there exists μ_2, μ_3, μ_4 such that $0 < \mu_2 \leq \mu_3 \leq \mu_4 < 1$ such that $\arg \min_{h_1 \in \{G, B\}, \theta \in \{G, B\}} \hat{d}(h_1; \theta) = (G, C)$ if and only if $\mu \in [\mu_2, \mu_3] \cup [\mu_4, 1]$, and (B, I) if and only if $\mu \in [0, \mu_2) \cup [\mu_3, \mu_4)$.

Lemma 4 identifies sufficient conditions under which either of two environments— (G, C)

or (B, I) —provides the binding constraint for the cutoff, \hat{c} . Figure 4 contains plots of three prices, $p(GG)$, $p(GB)$, $p(BB)$, for a small and large $\Delta\pi$. If $\Delta\pi$ is small, then signals relatively uninformative, and thus, the prior belief plays a dominant role in shaping consumers' beliefs, which then determine prices. Recall that (8) implies that the group does not find obtaining the best history GG as attractive as avoiding the worst history BB . Thus, the optimistic environment (G, C) attains the cutoff level if and only if consumers' prior is sufficiently high, and otherwise (B, I) does.

Even for a large $\Delta\pi$, at extreme values of μ , the binding environment is the same as with a small $\Delta\pi$. That is, for a very large μ (close to 1), (G, C) attains the minimum return on investment, while for a very small μ (close to 0) (B, I) does. This is because μ is relatively more informative, and hence, plays a dominant role in shaping consumers' posterior beliefs.

The simple monotonic characterization in μ breaks down in the intermediate range of the prior belief as is illustrated in Figure 5 which depicts the return on investment for large and small $\Delta\pi$. We find that in an intermediate-low range, $[\mu_2, \mu_3)$, the optimistic environment (G, C) attains the minimum level of return on investment, while in an intermediate-high range, $[\mu_3, \mu_4)$, the pessimistic environment (B, I) does. In the intermediate-low region, consumers' initial beliefs are mostly placed on the group's quality being either the lowest (II) or mixed (CI) , and almost none on the best quality (CC) . Therefore, a competent firm's investment decision hinges on whether an additional investment will move consumers' beliefs away from II and towards CI . With accurate signals, each signal is indicative of the corresponding firm's type. Therefore, to prove to consumers that the group is of mixed quality, the group needs just one good outcome. Thus, the investment incentive is the lowest for (G, C) . Analogously if μ is in an intermediate-high range, consumers' initial belief is divided between CC and CI , and the group's desire to convince consumers that they are a group of two competent firms drives its investment incentive. Then, the group really needs all histories to be good. In an environment where this is improbable, a competent firm is discouraged from investing. Therefore, (B, I) provides the minimum return on investment.

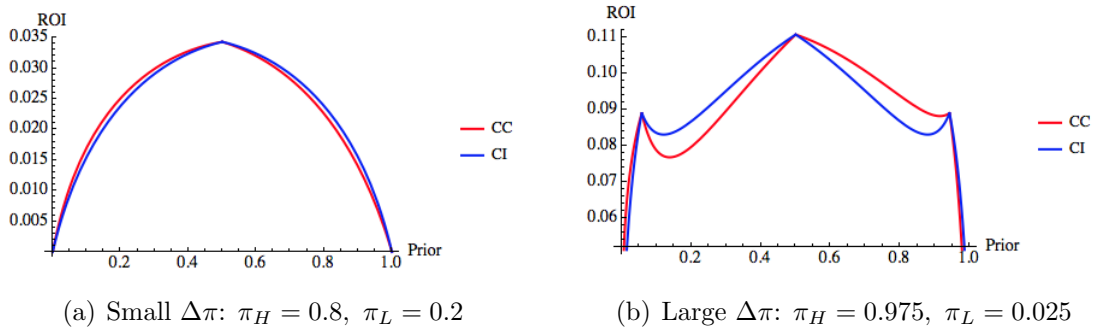


Figure 5: Return on Investment under (G, C) or (B, I) as a function of prior μ

Note that this analysis shows that it is not straight forward to compare collective reputation building with the individual one. Nevertheless, we can derive some economically interpretable results in the next section.

3.3 Comparing Individual and Collective Brands

We have examined two regimes of reputation and identified conditions under which the **RE** exists. In this section, we compare the two and investigate which regime provides better incentives for the following two different limiting signal structures:

1. **Exclusive knowledge** ($\pi_L \approx 0$): If $\pi_L = 0$, then without investment a firm cannot produce a good product. Thus, a good history completely reveals that the firm is competent. If competence represents the possession of a special technology or some advanced expertise, such as watches, automobiles, electronics, this seems to be a reasonable approximation.
2. **Quality control** ($\pi_H \approx 1$): If $\pi_H = 1$, then a firm always produces a good product if it invests, which implies that one bad outcome completely reveals that the firm is incompetent. If competence is about the ability to conduct quality control, such as manufacturing of generic products, e.g., clothes or skewers, this seems to be a reasonable approximation.

Note that in these two extreme cases, the fact that one observation fully reveals competence

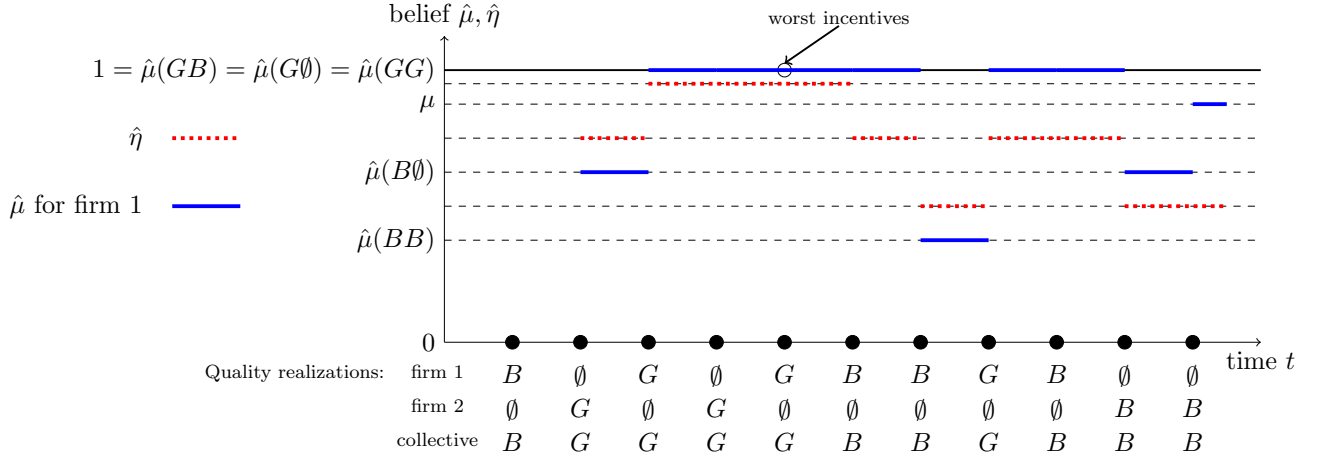


Figure 6: Belief realization for $\pi_L = 0$

or incompetence of a firm is bad for incentives of competent firms to overcome the moral hazard problem. We show under which conditions collective brands can help the firm to commit to invest.

3.3.1 Exclusive knowledge ($\pi_L \approx 0$)

If π_L is close to 0, the short-run benefit of investment in (3) after a history $h_1 = G$ vanishes for individual reputation: $p^{\text{ind}}(GG) - p^{\text{ind}}(GB) \rightarrow_{\pi_L \rightarrow 0} \pi_H - \pi_H = 0$. In contrast, under collective reputation, consumers remain uncertain about the group's quality as they believe that with positive probability only one firm is competent.

Figure 6 illustrates how beliefs evolve over time given the history realization presented at the bottom of the graph. The joint quality realizations of the two firms are outlined in the lowest row, while the realizations by individual firms are set out in the rows above. The solid blue line represents the beliefs of consumers if firm 1 builds reputation by itself. In that case, after the history $\emptyset G$ the belief must remain 1 independently of the realization in that period. This point, where the short-run benefit of investment of an individual firm vanishes, is marked by a circle.

The question is when does this benefit of collective reputation building outweigh the cost

of free-riding on future investments of the other firm and of having less precise signals. Note that less precise signals lead to lower prices for a competent firm after a good signals because the customer cannot distinguish between the two firms.

Let us consider the marked history G with the worst incentives. If the prior μ is relatively optimistic, as in the graph, then the posterior $\hat{\eta}$ at this point is already very high even for a brand. However, a bad realization increases the belief that the collective brand is of type CI which causes $\hat{\eta}$ to drop significantly. Thus, in a collective the incentives of investment are much better. On the contrary, if the prior μ was very low, then even after the history G , buyers would place a high belief on the brand being of type CI , so that the cost of generating a bad quality product in the period after is relatively small.

The following proposition makes this observation formal. We show that the benefit of collective brands outweighs the free-riding cost and cost of weaker signal if the prior about the firm being competent is relatively high.

Proposition 1. *Let $\pi_L = 0$. Then, the following holds:*

- i) A collective brand sustains a **RE** for higher investment costs than an individual brand if consumers' prior belief μ about the firm's type is sufficiently optimistic and δ is not too large. Formally, $\hat{c} > \hat{c}^{ind}$ for sufficiently small π_L if $\delta \leq \frac{1}{3}$ and μ sufficiently close to 1.*
- ii) An individual brand sustains a **RE** for higher investment costs than a collective brand if the prior belief μ is sufficiently low. Formally, $\hat{c} < \hat{c}^{ind}$ for sufficiently small π_L and μ .*

One caveat of this result is that δ cannot be too large in order to make collective reputation a good commitment for investment. The reason for why δ cannot be too large is that it ensures that short-run incentives dominate long-run incentives. For the examples we have in mind, small δ can be a reasonable assumption if investment decisions are only made relatively infrequently.

Figure 7 depicts the return on investment for collective and individual brands if $\pi_H = 0.9$ and $\delta = 0.4$. One can see that collective reputation building dominates individual reputation building for a wide range of priors while for tractability we only show the result for large

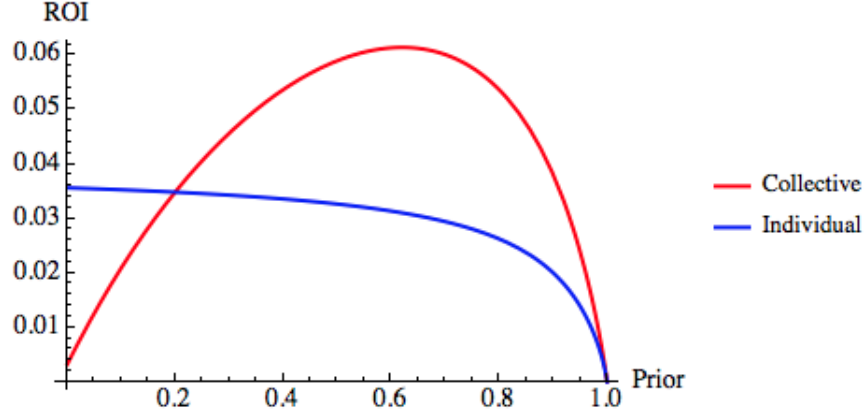


Figure 7: Comparison of Returns on Investment with $\pi_L = 0$
 $\pi_H = 0.9$, $\delta = 0.4$

and small μ .

3.3.2 Quality control ($\pi_H \approx 1$)

If π_H close to 1, the short-run benefit of investment in 3 after a history $h_1 = B$ vanishes as $p^{\text{ind}}(BG) - p^{\text{ind}}(BB) \rightarrow_{\pi_L \rightarrow 0} \pi_L - \pi_L = 0$. On the contrary, for collective reputation, i.e., in 7, a bad outcome does not eliminate uncertainty entirely because the other firm may be good or bad and hence, $p(BG) - p(BB) > 0$.

Using the same history of realizations as in Figure 6, Figure 8 depicts the evolution of beliefs over time if $\pi_H = 0$ and for low prior μ . In that case, incentives for investment are the worst after a history with $h_1 = B$ which is marked by a cycle. At that point, the beliefs in the next period remain 0 independently of the quality realization today.

Again, the question is under which circumstances the benefit of collective reputation can outweigh the cost of free-riding and of having a less precise signal. Let us consider the circled history. If the prior μ is relatively pessimistic, as in the graph, then the posterior $\hat{\eta}$ is relatively low after $h_1 = B$. A good realization in the next period, however, would increase the belief that the collective brand is of type CI significantly. In contrast, if μ was small, the belief that the brand type is CI would be relatively large to start with after a history $h_1 = 1$. The following proposition is analogous to Proposition 1 and makes this observation

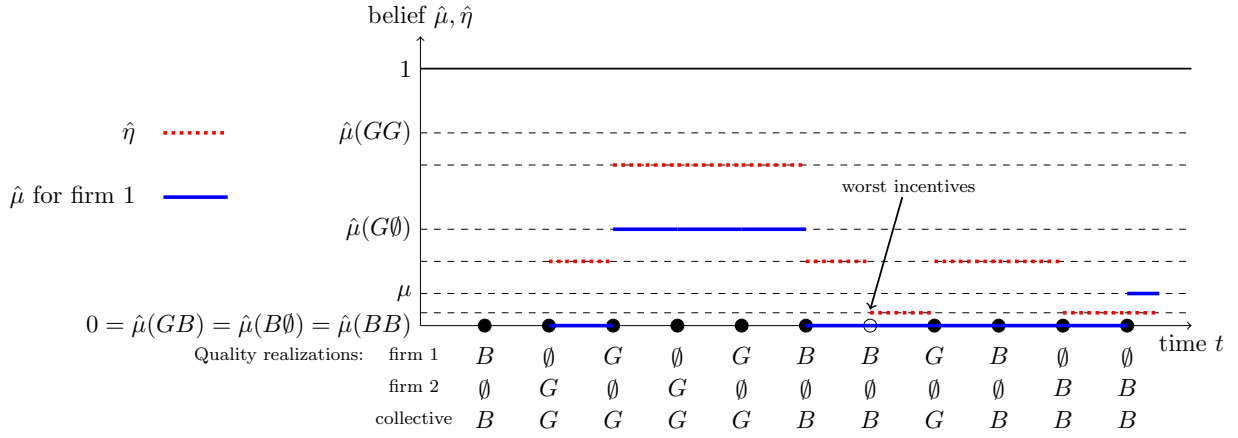


Figure 8: Belief realization for $\pi_H = 1$

formal.

Proposition 2. *Let $\pi_H = 1$. Then, the following holds:*

- i) A collective brand sustains a **RE** for higher investment costs than an individual brand if consumers' prior belief μ about the firm's type is sufficiently pessimistic and δ is not too large. Formally, $\hat{c} > \hat{c}^{ind}$ if $\delta \leq \frac{2\pi_L}{3+\pi_L}$ and μ sufficiently close to 0.*
- ii) An individual brand sustains a **RE** for higher investment costs than a collective brand if the prior belief μ is sufficiently high. Formally, $\hat{c} < \hat{c}^{ind}$ for sufficiently large μ .*

Figure 9 depicts the return on investment for collective and individual brands if $\pi_H = 1$, $\pi_L = 0.6$, and $\delta = 0.2$. One can see that collective reputation building dominates individual reputation building for a wide range of priors while for tractability we only show the result for large and small μ .

Analogously to the “exclusive knowledge” case, we need δ to be not too large for collective reputation building being a good commitment device, as otherwise the short-run benefit is dominated by the long-run free-riding incentives.

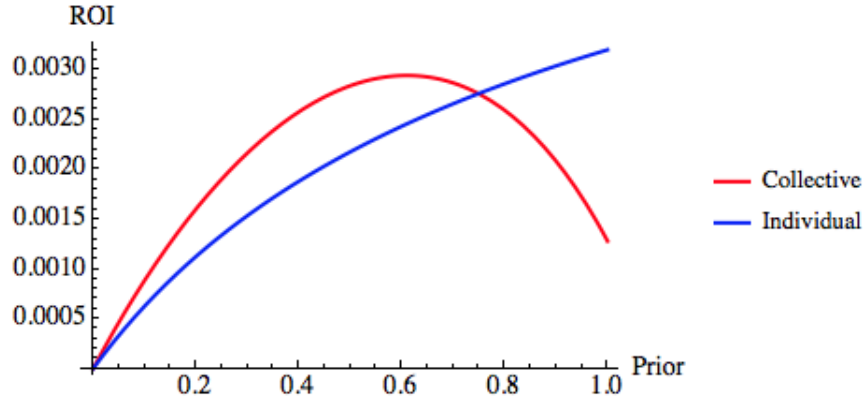


Figure 9: Comparison of Returns on Investment with $\pi_H = 1$
 $\pi_L = 0.6$, $\delta = 0.2$

3.4 Interpretation of Results

So far, we have only stated the formal results and the intuition for why those results hold. Here, we summarize the economic implications and interpretation of the results in the context of country of origin.

First, the parameter μ can be thought of as a “base reputation” of a country. The question we would like to investigate here is: In a country with high μ , which industries have an incentive to make use of country of origin advertising. Which industries should advertise country of origin for countries with low μ ?

By Propositions 1 and 2 for high μ , industries with exclusive knowledge, such as French wine, Swiss watches, German automobiles, Japanese electronics, US software, etc. can benefit from advertising country of origin. In contrast, producers of generic products such as screws, basic clothes, etc., are better off advertising their own brand only. In countries with low base reputation μ , those generic manufacturers can instead benefit from country of origin advertising. Producers of products that require exclusive knowledge are instead better off building their own brand. These interpretations are summarized in Table 3.4

These predictions are consistent with anecdotal evidence. For example the collective brand “Made in China” is advertised by subsuppliers on platforms such as ‘Made-in-China.com’,

	Exclusive knowledge ($\pi_H \approx 1$)	Quality control ($\pi_L \approx 0$)
High base reputation ($\mu \approx 1$)	collective reputation has commitment value	individual reputation always better
Low base reputation ($\mu \approx 0$)	individual reputation always better	collective reputation has commitment value

Table 1: Summary of results

while successful high-tech companies such as Huawei rather try to build their own brand names. Instead German subsuppliers such as ThyssenKrupp rather count on their own brand reputation. Our results could also be applied to the labeling of “Made in Germany” versus “Made in Europe”.

There are two ways to think about these observations. First, one can argue that companies that pick the correct branding strategies will be the ones that survive and thus on average we should observe a selection of companies that apply the correct strategy. Another interpretation is that firms are actually choosing the best strategy taking into account the commitment value of country of origin.

Finally, these insights can play a role for the regulation of labeling of country of origin. While the classic argument is that companies should be required to label their product with certain information for consumer protection, imposing what aspects are emphasized and what implications this has on the brand and customer beliefs and moral hazard problem of firms involved.

4 General Analysis with T -Period Memory

In this section, we extend the model to a memory of an arbitrarily finite periods and verify that our results are robust. Intuitions for the results do not depend on the two-period assumption. In fact, we expect our results to be stronger with a longer memory. We saw in the main analysis that an individual brand is better at reaching a high or low level of reputation at which point it faced a low incentive for investing in quality. An individual brand that has built a very good or bad reputation is discouraged from investing further

because it cannot improve consumers' beliefs sufficiently. Consumers' longer memory would worsen this problem as it will allow firms to be in a more extreme level of reputation after a longer streak of good or bad outcomes.

Now a relevant history is an element of $\mathcal{H}^{\text{ind}} := \{G, \emptyset, B\}^T$ for an individual brand and $\mathcal{H}^{\text{col}} := \{G, B\}^T$ for a collective brand. Suppose the focal investment decision is made at $t = 0$, and denote an outcome produced by an investment decision at any period t by $h_t \in \{G, \emptyset, B\}$. Then, the T -period history is a vector $\mathbf{h}_{\text{old}} = (h_{-T}h_{-T+1}\dots h_{-2}h_{-1})$. As the firm continues to make a sequence of investment decisions after $t = 0$, outcomes h_1, h_2, \dots are realized. We find it useful to denote a vector of consecutive outcomes between $t = i$ and $t = j$ by $\mathbf{h}^{i:j}$. So, for example $\mathbf{h}_{\text{old}} = \mathbf{h}^{-T:-1}$. Otherwise, we use notations that are natural extensions from those of the two-period model.

4.1 Individual reputation

The equilibrium analysis is analogous to that for the two-period model. The reputational equilibrium exists if and only if investing is an optimal decision after all possible histories. Equivalently, the benefit from investment should be greater than its cost for every history. Given a history \mathbf{h}_{old} , the benefit of the focal investment decision is realized through the next T periods until consumers forget h_0 , or its outcome. By investing in quality, the firm can improve the outcome produced, thereby improving consumers' willingness to pay. Thus, the expected benefit is a present-discounted sum of price premium over T periods. We now state a lemma that characterizes the cutoff level for individual reputation.

Lemma 5. *For an individual firm, there exists a constant $\hat{c}^{\text{ind}} > 0$ such that a **RE** exists if and only if $c \leq \hat{c}^{\text{ind}}$ where*

$$\hat{c}^{\text{ind}} = \frac{\delta \Delta \pi}{2} \cdot \min_{\mathbf{h}_{\text{old}} \in \{G, \emptyset, B\}^T} \sum_{k=0}^{T-1} \delta^k \left(\sum_{\mathbf{h}^{1:k} \in \{G, \emptyset, B\}^k} Pr(\mathbf{h}^{1:k}) (p(\mathbf{h}^{-T+k+1:-1} G \mathbf{h}^{1:k}) - p(\mathbf{h}^{-T+k+1:-1} B \mathbf{h}^{1:k})) \right).^5 \quad (9)$$

⁵When $k = 0$, $\mathbf{h}^{1:k}$ is defined to be an emptyset. Therefore, $\mathbf{h}^{-T+k+1:-1} G \mathbf{h}^{1:k}$ is equivalent to $\mathbf{h}^{-T+1:1} G$.

This lemma is essentially a general version of lemma 1. As time proceeds to $t = k$, where $1 \leq k \leq T - 1$, of the elements of the history \mathbf{h}_{old} , old ones are forgotten ($\mathbf{h}^{-T+k+1:-1}$), while new ones enter the memory ($\mathbf{h}^{1:k}$).

The source of the price premium is the differential element G and B in between older and newer outcomes. By investing in the focal period $t = 0$, the firm manages to have one more good outcome in the history observed by consumers. Therefore, the premium realized in $t = k$ conditional on the sequence of new outcomes $\mathbf{h}^{1:k}$ is $p(\mathbf{h}^{-T+k+1:-1}G\mathbf{h}^{1:k}) - p(\mathbf{h}^{-T+k+1:-1}B\mathbf{h}^{1:k})$.

In the short-run, most of the original history remains in consumers' memory ($\mathbf{h}^{-T+k+1:-1}$). Therefore, if a firm has built a very high level of reputation with many good outcomes in the past, the firm does not gain much through premiums realized in the near future by investing in $t = 0$. Similarly, a firm that has a very bad reputation at $t = 0$ has a low short-run incentive to invest.

What may motivate such firms with extreme reputation is the long-run incentive. A firm with good reputation may fear that a decision not to invest today may lead to a bad future reputation, especially if outcomes of its future investment ($\mathbf{h}^{1:k}$) is likely to be bad. On the other hand, if the firm expects them to be good, then the long-run incentive would be too low to discipline the firm's action today.

Recall that we are trying to compute the exact cutoff level \hat{c}^{ind} . This requires of knowing the history that provides the minimum benefit for given parameters. For this purpose, we consider again two special signal structures: exclusive technological know-how ($\pi_L = 0$) and quality control ($\pi_H = 1$). The former provides an environment where building an extremely high level of reputation is easy for a competent firm, as one good outcome completely reveals its type. Therefore, we can attain a small benefit from investment by choosing a history that has a lasting damage to the firm's incentives. This implies that any history $\mathbf{h}^{-T:1}$ with $h_{-1} = G$ does the job. Since the most recent outcome is good, consumers know perfectly the

Also, when $k = T - 1$, $\mathbf{h}^{-T+k+1:-1}$ is equivalent to an emptyset so that $\mathbf{h}^{-T+k+1:-1}G\mathbf{h}^{1:k}$ is $G\mathbf{h}^{1:T-1}$.

firm's type to be good until $t = T - 2$. This eliminates any incentive for the firm to invest at $t = 0$ other than the incentive realized at the very last period $t = T - 1$. In other words, $p(\mathbf{h}^{-T+k+1;-1}G\mathbf{h}^{1:k}) - p(\mathbf{h}^{-T+k+1;-1}B\mathbf{h}^{1:k}) = 0$ for all $0 \leq k \leq T - 2$.

Under the structure of quality control, one bad outcome completely reveals a firm to be an incompetent type. Since the incentives are lower on the lower end of the reputation ladder, we find a history that has a lasting negative influence on the history observed by consumers. Then, $h_{-1} = B$ does the job. The most recent outcome reveals the firm to be incompetent to consumers from $t = 0$ to $t = T - 2$.

The following lemma summarizes analyses on the characterization of the cutoff-levels under two special signal structures.

Lemma 6. *1. Under the environment of exclusive technology, a competent firm expects the lowest benefit from an investment immediately following a good outcome. In this case, all of the firm's investment incentives vanish, except for the one realized in the longest run, $t = T - 1$, and the benefit is equivalent to:*

$$\lim_{\pi_L \rightarrow 0} \hat{c} = \frac{\delta^T \cdot \pi_H^2 (1 - \mu)}{2^T} \cdot \left(\sum_{k=0}^{T-1} \binom{T-1}{k} \cdot \frac{(1 - \pi_H)^k}{\mu(1 - \pi_H)^{k+1} + (1 - \mu)} \right). \quad (10)$$

2. Under the environment of quality control, an investment decision immediately following a bad outcome provides the lowest benefit and it is equivalent to:

$$\lim_{\pi_H \rightarrow 1} \hat{c}^{ind} = \frac{\delta^T (1 - \pi_L)^2 \mu}{2^T} \cdot \left(\sum_{k=0}^{T-1} \binom{T-1}{k} \frac{1}{\mu + (1 - \mu)\pi_L^{k+1}} \right). \quad (11)$$

4.2 Collective reputation

A longer memory may allow a collective brand to achieve either a high or low level of reputation and cause a commitment problem to firms. However, as we saw in the analysis of the two-period model, consumers' limited observability for a collective brand alleviates this problem; As consumers cannot observe history at firm-level, they can never learn perfectly

about the types of two firms in the group. Therefore, a competent firm can always improve the brand reputation by investing in quality.

The next lemma establishes the necessary and sufficient condition for the existence of reputational equilibrium. Let $\Pr(\mathbf{h}^{1:k}; \theta)$ for $\mathbf{h}^{1:k} \in \{G, B\}^k$ and $\theta \in \{C, I\}$ be the probability that the brand of type θ produces a sequence of outcome $\mathbf{h}^{1:k}$ in k periods if a competent firm always invests.

Lemma 7. *For a competent firm within a collective brand, there exists a constant $\hat{c} > 0$ such that a **RE** exists if and only if $c \leq \hat{c}$ where*

$$\hat{c} = \frac{\delta \Delta \pi}{2} \cdot \min_{\mathbf{h}_{old}, \theta} \sum_{k=0}^{T-1} \delta^k \left(\sum_{\mathbf{h}^{1:k} \in \{G, B\}^k} \Pr(\mathbf{f}; \theta) (p(\mathbf{h}^{-T+k+1:-1} G \mathbf{h}^{1:k}) - p(\mathbf{h}^{-T+k+1:-1} B \mathbf{h}^{1:k})) \right), \quad (12)$$

where $\mathbf{h}_{old} \in \{G, B\}^T$ and $\theta \in \{C, I\}$.

This lemma is a general version of lemma 3. And the distinction we drew in the two-period model between the individual and collective reputation clearly applies here; the benefit from investment for a firm within a collective reputation depends on the type of the other firm.

Like in the individual reputation, a competent firm under a collective reputation has an incentive to invest in the focal period $t = 0$ because its good outcome will allow a better reputation of the brand in the next T periods, during which consumers will pay a premium. However, unlike in the case of the individual reputation, consumers' belief about the group quality always remains uncertain, even under special signal structures with $\pi_L = 0$ or $\pi_H = 1$. Technically, it also implies that the cutoff level \hat{c} is very hard to compute, as none of the terms for the price premium vanishes. So, instead of computing the exact cutoff, we compute its lower bound and show that it is still greater than the exact cutoff for an individual brand, \hat{c}^{ind} .

First, suppose $\pi_L = 0$. Before choosing a lower bound, we need to identify the history that minimizes the benefit from investment and the type of the other firm. Since this is an environment that a brand can build up reputation relatively easily with a good outcome,

the best possible history with all good outcomes $\mathbf{h}_{old} = G^T$ must provide the lowest benefit. Also, new outcomes that replace older memories after some periods are likely to maintain the clean sheet if the other firm is also competent. It is in fact straightforward to show that for a large enough μ , $\mathbf{h}_{old} = G^T$ and $\theta = C$ together provide \hat{c} .

Now to find a lower bound for \hat{c} , we sum over a subset of all premiums. Note in equation 12 that the expected price premium in period $t = k$ depends on the realization of future outcomes, $\mathbf{h}^{1:k}$. We will focus on events that only good outcomes are realized, and treat other expressions as zero. Such an event occurs with a probability π_H^k since both firms of the group are competent. Conditional on this event, the price premium enjoyed by the firm is $p(G^T) - p(BG^{T-1})$. The one bad outcome in the latter term is due to the decision not to invest in period $t = 0$. Otherwise, the firm keeps the clean sheet under this event.

Second, suppose $\pi_H = 1$. This is an environment where a brand's reputation can fall as a consequence of bad outcomes. Therefore, a firm of a collective brand is most discouraged from investing when it has produced many bad outcomes. It is also straightforward to show that $\mathbf{h}_{old} = B^T$ and $\theta = I$ together provides \hat{c} for small enough μ .

Similar to the previous case, we focus on the event that the brand only produces bad outcomes in the future, which happens with probability $(1 - \frac{\pi_H + \pi_L}{2})^k$ as there is one of each type in the firm. Conditional on this event, the firm's history would be GB^{T-1} if invested in $t = 0$, and B^T if not. Therefore, the expected price premium in this event in period $t = k$ is $p(GB^{T-1}) - p(B^T)$.

We summarize the analysis on collective reputation and the characterization of lower bounds in the next lemma:

Lemma 8. *1. Under the environment of quality control ($\pi_L = 0$), if μ is large enough, a competent firm of a collective brand faces the lowest benefit from investment if it has produced all good outcomes in the remembered history ($\mathbf{h}_{old} = G^T$) and the other firm is also competent ($\theta = C$). A lower bound for the cutoff level, $\hat{c}_{(\pi_L=0)}$, is obtained by focusing on the event in which only good outcomes are realized ($\mathbf{h}^{1:k} = G^k$).*

2. Under the structure of quality control ($\pi_H = 1$), if μ is small enough, a competent firm of a collective brand faces the lowest benefit from investment if it has produced all bad outcomes in the remembered history ($\mathbf{h}_{old} = B^T$) and the other firm is incompetent ($\theta = I$). A lower bound for the cutoff level, $\hat{c}_{(\pi_H=1)}$, is obtained by focusing on the event in which only bad outcomes are realized ($\mathbf{h}^{1:k} = B^k$).⁶

4.3 Comparing Individual and Collective Brands

Comparing the cutoff level for an individual brand, \hat{c}^{ind} , and the corresponding lower bound for a collective brand, we find the following.

Proposition 3. *Under the environment of exclusive technology ($\pi_L = 0$), if μ is large enough, we can find a large enough π_H such that $\hat{c}_{(\pi_L=0)} > \hat{c}_{(\pi_L=0)}^{ind}$. Also, under the environment of quality control ($\pi_H = 1$), if μ is small enough and δ not too large, we can find small enough π_L such that $\hat{c}_{(\pi_H=1)} > \hat{c}_{(\pi_H=1)}^{ind}$.*

Proposition 3 implies that the true cutoff for collective reputation can be greater than the cutoff for individual reputation. Therefore, for any arbitrarily finite memory of consumers, the reputational equilibrium can be better sustained for a collective brand. The short-run incentive to exploit the reputation the brand has built in fact becomes stronger with a longer history, which worsens the commitment problem for an individual brand. Although the longer history also hurts incentives for a collective brand, too, but to a much less extent.

5 Endogenous Brand Formation

So far, we have assumed exogenous brand formation. However, in many realistic examples, whether to be part of a collective brand or to build an individual brand is a firm's decision. In this section, we examine a firm's branding incentives and investigate robustness of our

⁶For the expressions for and $\hat{c}_{(\pi_L=0)}$ $\hat{c}_{(\pi_H=1)}$, see the Appendix.

main results under endogenous branding. For simplicity, we restrict our attention to the two-period memory.

In order to sustain the result that competent firms sometimes find it optimal to group with another firm (competent, or incompetent) to gain more commitment power for investment, we must compare the payoffs to the firm for each branding decision. We are only interested in the parameter regions specified in ?? and ?? where the reputational equilibrium exists for collective brands, but not for individual ones. Then, we need to define the payoffs and in particular what kind of strategies would a firm play as an individual firm if reputational equilibrium is not attainable.

We consider a two-stage game to address these issues. In the first stage, the firm decides whether to group with another firm of a known type. Then, in the second stage, the firm receives a payoff equivalent to an average period-profit under the best feasible stationary (pure-strategy) equilibrium. For example, in the region of our focus, if the firm decides to form a group, the firm receives the average period-profit from the reputational equilibrium. Otherwise, it builds a brand individually and receives the average profit from the second-best equilibrium.

We investigate all equilibria in the case of individual reputation to identify the second best equilibrium next to the reputational equilibrium. A Markovian strategy depends on the payoff-relevant state, which in our model is the most recent outcome the brand produced, $h_1 \in \mathcal{H} = \{G, \emptyset, B\}$. Therefore, there can be eight possible equilibria, each of which is specified by a subset of $\{G, \emptyset, B\}$. Then, as the observed history evolves according to a Markov chain, we can compute the steady-state probability distribution over the set. Given the distribution and the equilibrium strategy, we can compute the average period-profit for each equilibrium.

To define a competent firm's equilibrium strategy more formally, it is mapping $\sigma^{\mathcal{S}} : \mathcal{H} \rightarrow \{0, 1\}$, where $\mathcal{S} \subset \mathcal{H}$ prescribes the outcomes after which the firm invests, i.e. $\sigma^{\mathcal{S}}(h_1) = 1$ if

and only if $h_1 \in \mathcal{S}$. Then, the equilibrium average flow profit is:

$$\begin{aligned} \Pi^{\mathcal{S}} = & \underbrace{\sum_{h_1 \in \mathcal{S}, h_2 \in \mathcal{H}} \Pr_C^{\mathcal{S}}(h_2 h_1) \cdot (\hat{\mu}^{\mathcal{S}}(h_2 h_1) \cdot \pi_H + (1 - \hat{\mu}^{\mathcal{S}}(h_2 h_1)) \cdot \pi_L - c)}_{\text{invest}} \\ & + \underbrace{\sum_{h_1 \notin \mathcal{S}, h_2 \in \mathcal{H}} \Pr_C^{\mathcal{S}}(h_2 h_1) \cdot \pi_L}_{\text{not invest}}. \end{aligned}$$

Given an equilibrium \mathcal{S} and two-period memory $\mathbf{h} = h_2 h_1$, consumers rationally expects the firm to invest in quality if and only if it is a competent type and a right time to invest prescribed by the equilibrium, i.e. $h_1 \in \mathcal{S}$. Let $\Pr_C^{\mathcal{S}}(h_2 h_1)$ denote a competent firm's stationary distribution over all possible 2-period history (\mathcal{H}^2), and $\hat{\mu}^{\mathcal{S}}(h_2 h_1)$ the consumers' posterior belief that the firm is competent upon observing the history ($h_2 h_1$). By Bayes' rule, $\hat{\mu}^{\mathcal{S}}(h_2 h_1) = \frac{\mu \cdot \Pr_C^{\mathcal{S}}(h_2 h_1)}{\mu \cdot \Pr_C^{\mathcal{S}}(h_2 h_1) + (1 - \mu) \cdot \Pr_I^{\mathcal{S}}(h_2 h_1)}$. Then, the first line of the equation above is the expected payoff from investment, where the firm is paid the price $\hat{\mu}^{\mathcal{S}} \cdot \pi_H + (1 - \hat{\mu}^{\mathcal{S}}) \cdot \pi_L$ and incurs the investment cost c . The second line is the expected payoff when it does not invest in quality and consumers pay the minimum price.

Since we can compute the expected payoff from all equilibria, we need to examine which ones exist. We already know that by assumption the reputational equilibrium does not, while the "bad" equilibrium in which the firm never invests always exists. There are six equilibria remaining: $\mathcal{S} = \{G, \emptyset\}, \{G\}, \{\emptyset, B\}, \{B\}, \{G, B\}$, and $\{\emptyset\}$.

In all of these equilibria, the firm sometimes invests and other times not. This implies that an equilibrium exists if and only if the investment cost is neither too small nor too large, i.e. $\underline{c}^{\mathcal{S}} < c < \bar{c}^{\mathcal{S}}$. In order for this condition to coincide with the region of our focus, the interval $(\underline{c}^{\mathcal{S}}, \bar{c}^{\mathcal{S}})$ must contain zero in two limits: $\pi_L = 0, \mu = 1$ and $\pi_H = 1, \mu = 0$.

We are still working on this section. We have been able to see through simulation (and feel very confident that we can show) that for the limit as π_L approaches 0 and μ 1, the reputational equilibrium and the bad equilibrium are the only ones that exist. It's less clear

for the other limit, and we may have more equilibria that coexist in that region.

Therefore, with our analysis thus far, we can state the following result.

Proposition 4. *Under exclusive technology ($\pi_L = 0$) and consumers' prior belief (μ) is close to 1, then a competent firm gains a greater profits by building a collective brand with an incompetent firm than that as an stand-alone firm.*

Proposition 4 shows that under exclusive technology, the commitment power that a firm gains through branding collectively can be large enough that it may find it optimal to group with an incompetent firm. Since the second best equilibrium for an individual brand is the worst equilibrium in which the firm never invests, the firm would gain substantially by forming a collective brand.

It remains to show that a similar result holds for the quality control case.

6 Conclusion

In this paper, we have examined models of collective and individual reputation. In particular, we have found that a collective brand sustains the good equilibrium better if either π_L is low and μ is high, or π_H is high and μ is low. We also explored a firm's endogenous branding decision and found a competent firm sometimes obtains a greater profit by grouping with an incompetent firm than it would alone. These two results together highlight the benefit of collective reputation. For firms facing a moral hazard problem, where competent firms alone cannot commit to invest always, collective brands may provide an additional commitment power to investment. And the commitment allows trust between firms and consumers and greater profits for firms to arise.

While our main contribution is in providing an explanation for why collective brands may overcome free-riding incentives and sustain good reputation, it is important to map our conditions for results onto practical examples. The prior μ would be largely determined by ex-ante beliefs about the quality across markets, industries and economies. And signal

strengths π_L and π_H indicates how integral no investment or investment is for failure or success of a product, respectively. Therefore, $\pi_L \approx 0$ would be relevant to products that entail very complicated or advanced manufacturing process where lack of investment will surely fail. On the other hand, $\pi_H = 1$ would be more applicable to products with simpler manufacturing process with few unexpected obstacles so that an investment always leads to a success. Therefore, our results would imply that collective brands would work well for reputable wine appellations in France or Italy and country of origin for Japanese or German electronics. On the other hand, a high quality electronics manufacturer in an economy with low prior would prefer establishing an independent brand and dissociate from the economy. We have witnessed such trends earlier with elite Japanese firms, more recently with South Korean companies, and now with other high quality firms from China and other developing economies.

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A Appendix: Proofs

Proof. [Proof of Lemma 1] The posterior beliefs about the quality of the product after observing history h are given by

$$\begin{aligned} \hat{\mu}^{\text{ind}}(GG) &= \frac{\mu\pi_H^2}{\mu\pi_H^2 + (1-\mu)\pi_L^2}, & \hat{\mu}^{\text{ind}}(GB) &= \hat{\mu}^{\text{ind}}(BG) = \\ & & & \frac{\mu\pi_H(1-\pi_H)}{\mu\pi_H(1-\pi_H) + (1-\mu)\pi_L(1-\pi_L)}, \\ \hat{\mu}^{\text{ind}}(BB) &= \frac{\mu(1-\pi_H)^2}{\mu(1-\pi_H)^2 + (1-\mu)(1-\pi_L)^2}, & \hat{\mu}^{\text{ind}}(G\emptyset) &= \hat{\mu}^{\text{ind}}(\emptyset G) = \frac{\mu\pi_H}{\mu\pi_H + (1-\mu)\pi_L} \\ & & \hat{\mu}^{\text{ind}}(\emptyset\emptyset) &= \mu, & \hat{\mu}^{\text{ind}}(B\emptyset) &= \hat{\mu}^{\text{ind}}(\emptyset B) = \frac{\mu(1-\pi_H)}{\mu(1-\pi_H) + (1-\mu)(1-\pi_L)}. \end{aligned}$$

Investment by the competent firm can only be sustained if her incentive compatibility constraint is satisfied after every possible history. The firm invests after a history $(h, 1)$ if and only if $V^{\text{ind}}(h, 1) \geq \tilde{V}^{\text{ind}}(h, 1)$ which is equivalent to

$$c \leq \hat{c}^{\text{ind}}(h_1) \equiv \frac{\delta(\pi_H - \pi_L)}{2} \cdot \underbrace{(V^{\text{ind}}(h_1G, 1) - V^{\text{ind}}(h_1B, 1) + V^{\text{ind}}(h_1G, \emptyset) - V^{\text{ind}}(h_1B, \emptyset))}_{\equiv \hat{d}^{\text{ind}}(h_1)}.$$

Note that $\hat{d}^{\text{ind}}(h_1)$ can potentially depend on c . Using $V(h, \emptyset) = \frac{\delta}{2}(V(h_1\emptyset, 1) + V(h_1\emptyset, \emptyset))$ we can calculate

$$\begin{aligned} V^{\text{ind}}(h_{-1}G, 1) - V^{\text{ind}}(h_{-1}B, 1) &= p^{\text{ind}}(h_{-1}G) - p^{\text{ind}}(h_{-1}B) \\ &+ \frac{\delta}{2}\pi_H \underbrace{(V^{\text{ind}}(GG, 1) - V^{\text{ind}}(BG, 1))}_{=p^{\text{ind}}(GG) - p^{\text{ind}}(BG)} + \underbrace{(V^{\text{ind}}(GG, \emptyset) - V^{\text{ind}}(BG, \emptyset))}_{=0} \\ &+ \frac{\delta}{2}(1-\pi_H) \underbrace{(V^{\text{ind}}(GB, 1) - V^{\text{ind}}(BB, 1))}_{=p^{\text{ind}}(GB) - p^{\text{ind}}(BB)} + \underbrace{(V^{\text{ind}}(GB, \emptyset) - V^{\text{ind}}(BB, \emptyset))}_{=0}. \end{aligned}$$

Similarly, $V^{\text{ind}}(h_{-1}G, \emptyset) - V^{\text{ind}}(h_{-1}B, \emptyset) = \frac{\delta}{2}(p^{\text{ind}}(G\emptyset) - p^{\text{ind}}(B\emptyset))$. □

Proof. [Proof of Lemma 2] First, note that

$$\begin{aligned}
p^{\text{ind}}(GG) - p^{\text{ind}}(GB) &= (\pi_H - \pi_L) \left(\frac{\mu\pi_H^2}{\mu\pi_H^2 + (1-\mu)\pi_L^2} - \frac{\mu\pi_H(1-\pi_H)}{\mu\pi_H(1-\pi_H) + (1-\mu)\pi_L(1-\pi_L)} \right) \\
&= \mu\pi_H(\pi_H - \pi_L) \left(\frac{\pi_H}{\mu\pi_H^2 + (1-\mu)\pi_L^2} - \frac{1-\pi_H}{\mu\pi_H(1-\pi_H) + (1-\mu)\pi_L(1-\pi_L)} \right) \\
&= \frac{\mu(1-\mu)\pi_H\pi_L(\pi_H - \pi_L)^2}{\Pr(GG) \cdot \Pr(GB)},
\end{aligned}$$

and

$$\begin{aligned}
p^{\text{ind}}(GB) - p^{\text{ind}}(BB) &= (\pi_H - \pi_L) \left(\frac{\mu\pi_H(1-\pi_H)}{\mu\pi_H(1-\pi_H) + (1-\mu)\pi_L(1-\pi_L)} - \frac{\mu(1-\pi_H)^2}{\mu(1-\pi_H)^2 + (1-\mu)(1-\pi_L)^2} \right) \\
&= \mu(1-\pi_H)(\pi_H - \pi_L) \left(\frac{\pi_H}{\mu\pi_H(1-\pi_H) + (1-\mu)\pi_L(1-\pi_L)} - \frac{1-\pi_H}{\mu(1-\pi_H)^2 + (1-\mu)(1-\pi_L)^2} \right) \\
&= \frac{\mu(1-\mu)(1-\pi_H)(1-\pi_L)(\pi_H - \pi_L)^2}{\Pr(GB) \cdot \Pr(BB)},
\end{aligned}$$

and

$$\begin{aligned}
p^{\text{ind}}(G\emptyset) - p^{\text{ind}}(B\emptyset) &= (\pi_H - \pi_L) \left(\frac{\mu\pi_H}{\mu\pi_H + (1-\mu)\pi_L} - \frac{\mu(1-\pi_H)}{\mu(1-\pi_H) + (1-\mu)(1-\pi_L)} \right) \\
&= \frac{\mu(1-\mu)(\pi_H - \pi_L)^2}{\Pr(G) \cdot \Pr(B)} \geq \min\{p^{\text{ind}}(GG) - p^{\text{ind}}(GB), p^{\text{ind}}(GB) - p^{\text{ind}}(BB)\}.
\end{aligned}$$

Hence, history $h_{-1} = G$ provides the binding constraint if and only if $\frac{\pi_H\pi_L}{\Pr(GG)\Pr(GB)} \leq \frac{(1-\pi_H)(1-\pi_L)}{\Pr(GB)\Pr(BB)}$,

which holds if and only if

$$\begin{aligned}
&\Pr(BB) \cdot \pi_H\pi_L \leq \Pr(GG) \cdot (1-\pi_H)(1-\pi_L) \\
&\Leftrightarrow \pi_H\pi_L(\mu(1-\pi_H)^2 + (1-\mu)(1-\pi_L)^2) \leq (1-\pi_H)(1-\pi_L)(\mu\pi_H^2 + (1-\mu)\pi_L^2) \\
&\Leftrightarrow \mu\pi_H(1-\pi_H) \geq (1-\mu)\pi_L(1-\pi_L)
\end{aligned}$$

This inequality holds if and only if $\mu \geq \bar{\mu} \equiv \frac{\pi_L(1-\pi_L)}{\pi_H(1-\pi_H) + \pi_L(1-\pi_L)}$. \square

Proof. [Proof of Lemma 3] The **RE** exists if and only if $V(h, d; \theta) \geq \tilde{V}(h, d; \theta)$. A competent firm

invests after a history $(h, 1)$ if and only if

$$c \leq \hat{c}(h_{-1}) \equiv \delta \cdot \frac{\pi_H - \pi_L}{2} \cdot \underbrace{(V(h_{-1}G, 1; \theta) - V(h_{-1}B, 1; \theta) + V(h_{-1}G, \emptyset; \theta) - V(h_{-1}B, \emptyset; \theta))}_{\equiv \hat{d}(h_{-1}; \theta)}.$$

First, note that for all $q_1, q_2, x \in \{G, B\}$, we have that $V(q_1x, 1) - V(q_2x, 1) = p(q_1x) - p(q_2x)$ and $V(q_1x, \emptyset; \theta) - V(q_2x, \emptyset; \theta) = 0$. Using this, we can calculate

$$\begin{aligned} V(h_{-1}G, 1; \theta) - V(h_{-1}B, 1; \theta) &= p(h_{-1}G) - p(h_{-1}B) \\ &+ \frac{\delta\pi_H}{2}(V(GG, 1; \theta) - V(BG, 1; \theta)) + \frac{\delta(1 - \pi_H)}{2}(V(GB, 1; \theta) - V(BB, 1; \theta)) \\ &+ \frac{\delta\pi_H}{2}(V(GG, 0; \theta) - V(BG, 0; \theta)) + \frac{\delta(1 - \pi_H)}{2}(V(GB, 0; \theta) - V(BB, 0; \theta)) \\ &= p(h_{-1}G) - p(h_{-1}B) + \frac{\delta\pi_H}{2}(p(GG) - p(BG)) + \frac{\delta(1 - \pi_H)}{2}(p(GB) - p(BB)) \end{aligned}$$

Likewise,

$$\begin{aligned} V(h_{-1}G, \emptyset; \theta) - V(h_{-1}B, \emptyset; \theta) &= \frac{\delta\pi(\theta)}{2}(V(GG, 1; \theta) - V(BG, 1; \theta)) + \frac{\delta(1 - \pi(\theta))}{2}(V(GB, 1; \theta) - V(BB, 1; \theta)) \\ &+ \frac{\delta\pi(\theta)}{2}(V(GG, 0; \theta) - V(BG, 0; \theta)) + \frac{\delta(1 - \pi(\theta))}{2}(V(GB, 0; \theta) - V(BB, 0; \theta)) \\ &= \frac{\delta\pi(\theta)}{2}(p(GG) - p(BG)) + \frac{\delta(1 - \pi(\theta))}{2}(p(GB) - p(BB)) \end{aligned}$$

where $\pi(\theta) = \pi_L$ if $\theta = I$ and π_H if $\theta = C$. □

Proof. [Proof of Lemma 4] First note that

$$\begin{aligned}
\hat{\eta}_{CC}(GG) &= \frac{\mu^2 \pi_H^2}{\mu^2 \pi_H^2 + 2\mu(1-\mu) \left(\frac{1}{4} \pi_H^2 + \frac{1}{2} \pi_H \pi_L + \frac{1}{4} \pi_L^2 \right) + (1-\mu)^2 \pi_L^2}, \\
\hat{\eta}_{CI}(GG) &= \hat{\eta}_{IC}(GG) \\
&= \frac{\mu(1-\mu) \left(\frac{1}{4} \pi_H^2 + \frac{1}{2} \pi_H \pi_L + \frac{1}{4} \pi_L^2 \right)}{\mu^2 \pi_H^2 + 2\mu(1-\mu) \left(\frac{1}{4} \pi_H^2 + \frac{1}{2} \pi_H \pi_L + \frac{1}{4} \pi_L^2 \right) + (1-\mu)^2 \pi_L^2}, \\
\hat{\eta}_{II}(GG) &= 1 - \hat{\mu}_{CC}(GG) - 2\hat{\eta}_{CI}(GG), \\
\hat{\eta}_{CC}(GB) &= \frac{\mu^2 \pi_H (1 - \pi_H)}{\mu^2 \pi_H (1 - \pi_H) + 2\mu(1-\mu) \frac{1}{4} (\pi_H (1 - \pi_H) + \pi_H (1 - \pi_L) + \pi_L (1 - \pi_H) + \pi_L (1 - \pi_L)) + (1-\mu)^2 \pi_L (1 - \pi_L)}, \\
\hat{\eta}_{CI}(GB) &= \hat{\eta}_{IC}(GB) \\
&= \frac{\mu(1-\mu) \frac{1}{4} (\pi_H (1 - \pi_H) + \pi_H (1 - \pi_L) + \pi_L (1 - \pi_H) + \pi_L (1 - \pi_L))}{\mu^2 \pi_H (1 - \pi_H) + 2\mu(1-\mu) \frac{1}{4} (\pi_H (1 - \pi_H) + \pi_H (1 - \pi_L) + \pi_L (1 - \pi_H) + \pi_L (1 - \pi_L)) + (1-\mu)^2 \pi_L (1 - \pi_L)}, \\
\hat{\eta}_{II}(GB) &= 1 - \hat{\eta}_{CC}(GB) - 2\hat{\eta}_{CI}(GB), \\
\hat{\eta}_{CC}(BB) &= \frac{\mu^2 (1 - \pi_H)^2}{\mu^2 (1 - \pi_H)^2 + 2\mu(1-\mu) \left(\frac{1}{4} (1 - \pi_H)^2 + \frac{1}{2} (1 - \pi_H)(1 - \pi_L) + \frac{1}{4} (1 - \pi_L)^2 \right) + (1-\mu)^2 (1 - \pi_L)^2}, \\
\hat{\eta}_{CI}(BB) &= \hat{\eta}_{IC}(BB) \\
&= \frac{\mu(1-\mu) \left(\frac{1}{4} (1 - \pi_H)^2 + \frac{1}{2} (1 - \pi_H)(1 - \pi_L) + \frac{1}{4} (1 - \pi_L)^2 \right)}{\mu^2 (1 - \pi_H)^2 + 2\mu(1-\mu) \left(\frac{1}{4} (1 - \pi_H)^2 + \frac{1}{2} (1 - \pi_H)(1 - \pi_L) + \frac{1}{4} (1 - \pi_L)^2 \right) + (1-\mu)^2 (1 - \pi_L)^2}, \\
\hat{\eta}_{II}(BB) &= 1 - \hat{\eta}_{CC}(BB) - 2\hat{\eta}_{CI}(BB).
\end{aligned}$$

$$\begin{aligned}
p(GG) - p(GB) &= (\pi_H - \pi_L) \left(\frac{\mu^2 \pi_H^2 + \frac{1}{4} \mu(1-\mu)(\pi_H + \pi_L)^2}{\Pr(GG)} - \frac{\mu^2 \pi_H (1 - \pi_H) + \frac{1}{4} \mu(1-\mu)(\pi_H + \pi_L)(2 - \pi_H - \pi_L)}{\Pr(GB)} \right) \\
&= \frac{\mu(1-\mu)(\pi_H - \pi_L)^2 \left(\mu^2 (\pi_H - \pi_L)^2 + 2\mu(\pi_H - \pi_L)\pi_L + \pi_L(\pi_H + \pi_L) \right)}{4 \cdot \Pr(GG) \cdot \Pr(GB)} \\
&\xrightarrow{\pi_L \rightarrow 0} \frac{(1-\mu)\mu\pi_H}{(1+\mu)(1-\pi_H + 1 - \mu\pi_H)} \\
p(GB) - p(BB) &= (\pi_H - \pi_L) \left(\frac{\mu^2 \pi_H (1 - \pi_H) + \frac{1}{4} \mu(1-\mu)(\pi_H + \pi_L)(2 - \pi_H - \pi_L)}{\Pr(GB)} - \frac{\mu^2 (1 - \pi_H)^2 + \frac{1}{4} \mu(1-\mu) \left((1 - \pi_H) + (1 - \pi_L) \right)^2}{\Pr(BB)} \right) \\
&= \frac{\mu(1-\mu)(\pi_H - \pi_L)^2 \left(\mu^2 (\pi_H - \pi_L)^2 - 2\mu(\pi_H - \pi_L)(1 - \pi_L) + (1 - \pi_L)(2 - \pi_H - \pi_L) \right)}{4 \cdot \Pr(GB) \cdot \Pr(BB)} \\
&\xrightarrow{\pi_L \rightarrow 0} \frac{(1-\mu)\pi_H \left((1 - \mu\pi_H)^2 + 1 - \pi_H \right)}{\left((1 - \mu\pi_H)^2 + \mu(1 - \pi_H)^2 + 1 - \mu \right) (1 - \pi_H + 1 - \mu\pi_H)}
\end{aligned}$$

$p(GG) - p(GB) \leq p(GB) - p(BB)$ if and only if

$$\begin{aligned}
\frac{\mu^2 (\pi_H - \pi_L)^2 + 2\mu(\pi_H - \pi_L)\pi_L + \pi_L(\pi_H + \pi_L)}{\Pr(GG)} &\leq \frac{\mu^2 (\pi_H - \pi_L)^2 - 2\mu(\pi_H - \pi_L)(1 - \pi_L) + (1 - \pi_L)(2 - \pi_H - \pi_L)}{\Pr(BB)} \\
&= \frac{\mu^2 (\pi_H - \pi_L)^2 + 2\mu(\pi_H - \pi_L)\pi_L + \pi_L(\pi_H + \pi_L) - (2\mu - 1)(\pi_H - \pi_L) + 2(1 - \pi_H - \pi_L)}{\Pr(GG) - \mu(\pi_H - \pi_L)\pi_L + \frac{1}{2}}
\end{aligned}$$

Therefore, above condition can be re-written as $\frac{A}{C} \leq \frac{A+B}{C+D}$, where $A = \mu^2 (\pi_H - \pi_L)^2 + 2\mu(\pi_H - \pi_L)\pi_L + \pi_L(\pi_H + \pi_L)$, $B = -(2\mu - 1)(\pi_H - \pi_L) + 2(1 - \pi_H - \pi_L)$, $C = \Pr(GG)$, and $D =$

$-\mu(\pi_H - \pi_L)\pi_L + \frac{1}{2}$. This holds if and only if $AD \leq BC \Leftrightarrow$

$$\begin{aligned} (\pi_H - \pi_L) (2\mu^3(\pi_H - \pi_L)^2 - 3\mu^2(\pi_H - \pi_L)^2 + \mu(2 - \pi_H - \pi_L)(\pi_H + \pi_L) - 2(1 - \pi_L)\pi_L) &\geq 0 \\ f(\mu, \pi_H, \pi_L) \triangleq \mu^2(2\mu - 3)(\pi_H - \pi_L)^2 + \mu(2 - \pi_H - \pi_L)(\pi_H + \pi_L) - 2(1 - \pi_L)\pi_L &\geq 0. \end{aligned}$$

Note that if $\mu = 1$, the LHS is equivalent to $2(1 - \pi_H)\pi_H \geq 0$ and if $\mu = 0$, $-2(1 - \pi_L)\pi_L \leq 0$. Therefore, it vanishes at least once for some value of μ between 0 and 1. The question is whether it can vanish more than once. To see when f is increasing in μ

$$\frac{\partial f}{\partial \mu} = \mu(\mu - 1) + \frac{(\pi_H + \pi_L)(2 - \pi_H - \pi_L)}{6(\pi_H - \pi_L)^2} > 0 \Leftrightarrow (1 - \mu)\mu < \frac{(\pi_H + \pi_L)(2 - \pi_H - \pi_L)}{6(\pi_H - \pi_L)^2}$$

First, if $\pi_H - \pi_L$ is small, RHS becomes large and the condition holds always, so f crosses 0 at a single point. It is when $\pi_H - \pi_L$ is substantially large that the condition holds for small and large values of μ . Then, though $f(0, \pi_H, \pi_L) < 0$, it increases in μ for μ close to 0, at which point f may cross 0 for the first time. Then, for intermediate values of μ , f decreases and may cross 0 one more time. Then, lastly f increases for large values of μ and cross 0 again.

Although we do not fully identify necessary and sufficient conditions for $f \geq 0$, under symmetric signals with $\pi_L = 1 - \pi_H < \frac{1}{2}$, $f(\mu, \pi_H, 1 - \pi_H) \geq 0$ if and only if $\frac{1}{2} < \pi_H \leq \frac{3+\sqrt{6}}{6}$, $\frac{1}{2} \leq \mu \leq 1$ or $\frac{3+\sqrt{6}}{6} < \pi_H < 1$ and $\mu \in [\frac{1}{2} - \frac{\sqrt{1-12\pi_H+12\pi_H^2}}{2}, \frac{1}{2}] \cup [\frac{1}{2} + \frac{\sqrt{1-12\pi_H+12\pi_H^2}}{2}, 1]$. This exactly coincides with the patten described above. \square

Proof. [Proof of Proposition 1] First, note that it follows from Lemma 2 that in the limit ($\pi_L = 0$), $\hat{c}^{\text{ind}} = \hat{c}^{\text{ind}}(G)$ for all parameters. Similarly, it follows from Lemma 4 that $\hat{c} = \hat{c}(G; C)$ for $\pi_L = 0$ and high and low values of μ .

Moreover, under individual reputation,

$$\begin{aligned} \lim_{\pi_L \rightarrow 0} \hat{d}^{\text{ind}}(G) &= \lim_{\pi_L \rightarrow 0} \left(1 + \frac{\delta\pi_H}{2}\right) \underbrace{(p(GG) - p(GB))}_{\rightarrow 0} + \frac{\delta(1 - \pi_H)}{2} (p(GB) - p(BB)) + \frac{\delta}{2} (p(G\emptyset) - p(B\emptyset)) \\ &= \delta\pi_H(1 - \mu) \left(\frac{(1 - \pi_H)}{2} \frac{1}{1 - \mu\pi_H(2 - \pi_H)} + \frac{1}{2} \frac{1}{1 - \mu\pi_H} \right). \end{aligned}$$

For the case of collective reputation,

$$\begin{aligned}
\lim_{\pi_L \rightarrow 0} \hat{d}(G; C) &= \lim_{\pi_L \rightarrow 0} \left(1 + \frac{\delta\pi_H}{2}\right) (p(GG) - p(GB)) + \frac{\delta(1 - \pi_H)}{2} (p(GB) - p(BB)) \\
&= (1 - \mu)\pi_H \cdot \left[\left(1 + \frac{\delta\pi_H}{2}\right) \frac{(1 - \mu)\mu\pi_H}{(1 + \mu)(2 - (1 + \mu)\pi_H)} \right. \\
&\quad \left. + \frac{\delta(1 - \pi_H)}{2} \frac{(1 - \mu\pi_H)^2 + 1 - \pi_H}{((1 - \mu\pi_H)^2 + \mu(1 - \pi_H)^2 + 1 - \mu)(2 - (1 + \mu)\pi_H)} \right]
\end{aligned}$$

First, to compare the two cutoffs for μ close to 1, note that

$$\lim_{\mu \rightarrow 1} \lim_{\pi_L \rightarrow 0} \frac{(1 - \pi_H)}{2} \frac{1}{1 - \mu\pi_H(2 - \pi_H)} + \frac{1}{2} \frac{1}{1 - \mu\pi_H} = \frac{1}{1 - \pi_H}$$

and

$$\begin{aligned}
\lim_{\mu \rightarrow 1} \lim_{\pi_L \rightarrow 0} \frac{(1 + \frac{\delta\pi_H}{2})\mu}{(1 + \mu)(2 - (1 + \mu)\pi_H)} + \frac{\delta(1 - \pi_H)}{2} \frac{((1 - \mu\pi_H)^2 + 1 - \pi_H)}{((1 - \mu\pi_H)^2 + \mu(1 - \pi_H)^2 + 1 - \mu)(2 - (1 + \mu)\pi_H)} \\
= \left(1 + \frac{\delta\pi_H}{2}\right) \frac{1}{2(2 - 2\pi_H)} + \frac{\delta(1 - \pi_H)}{2} \frac{((1 - \pi_H)^2 + 1 - \pi_H)}{((1 - \pi_H)^2 + (1 - \pi_H)^2)(2 - 2\pi_H)} \\
= \frac{1 + \delta}{4(1 - \pi_H)}.
\end{aligned}$$

Since $\frac{1 + \delta}{4(1 - \pi_H)} \geq \frac{\delta}{1 - \pi_H}$ if and only if $\delta < \frac{1}{3}$, for sufficiently small μ collective brands are better whenever $\delta < \frac{1}{3}$.

Similarly, to compare the two cutoffs for μ close to 0, note that

$$\lim_{\mu \rightarrow 0} \lim_{\pi_L \rightarrow 0} (1 - \mu) \frac{(1 - \pi_H)}{2} \frac{1}{1 - \mu\pi_H(2 - \pi_H)} + \frac{1}{2} \frac{1}{1 - \mu\pi_H} = \frac{2 - \pi_H}{2}$$

and

$$\begin{aligned}
\lim_{\mu \rightarrow 0} \lim_{\pi_L \rightarrow 0} \frac{(1 + \frac{\delta\pi_H}{2})\mu}{(1 + \mu)(2 - (1 + \mu)\pi_H)} + \frac{\delta(1 - \pi_H)}{2} \frac{((1 - \mu\pi_H)^2 + 1 - \pi_H)}{((1 - \mu\pi_H)^2 + \mu(1 - \pi_H)^2 + 1 - \mu)(2 - (1 + \mu)\pi_H)} \\
= \frac{\delta(1 - \pi_H)}{4}.
\end{aligned}$$

Since $\delta \frac{2 - \pi_H}{2} \geq \frac{\delta(1 - \pi_H)}{4}$ for all $\pi_H \in [0, 1]$, individual reputation building is better for sufficiently low priors. \square

Proof. [Proof of Proposition 2] The proof is analogous to the proof of Proposition 1. It follows from Lemma 2 that in the limit $\pi_H \rightarrow 1$, $\hat{c}^{\text{ind}} = \hat{c}^{\text{ind}}(B)$ for all parameters. Similarly, it follows from Lemma 4 that $\hat{c} = \hat{c}(B; I)$ for high and low values of μ as $\pi_H \rightarrow 1$.

Under individual reputation

$$\begin{aligned} \lim_{\pi_H \rightarrow 1} \hat{d}^{\text{ind}}(B) &= \lim_{\pi_H \rightarrow 1} \frac{\delta}{2} (p^{\text{ind}}(GG) - p^{\text{ind}}(GB)) + (p^{\text{ind}}(GB) - p^{\text{ind}}(BB)) + \frac{\delta}{2} (p^{\text{ind}}(G\emptyset) - p^{\text{ind}}(B\emptyset)) \\ &= \frac{\delta}{2} \left(\frac{\mu(1 - \pi_L)}{\mu + (1 - \mu)\pi_L^2} + \frac{\mu(1 - \pi_L)}{\mu + (1 - \mu)\pi_L} \right) \end{aligned}$$

and under collective reputation

$$\begin{aligned} \lim_{\pi_H \rightarrow 1} \hat{d}(B; I) &= \lim_{\pi_H \rightarrow 1} (p(BG) - p(BB)) + \frac{\delta}{2} \cdot ((1 + \pi_L) \cdot (p(GG) - p(GB)) + (1 - \pi_L)(p(GB) - p(BB))) \\ &= \frac{\mu(1 - \pi_L)}{2} \cdot \frac{-2(1 + \delta)\mu^3(1 - \pi_L)^2 + 2\pi_L(\delta + 2\pi_L + 3\delta\pi_L) + \mu(2 + \delta + 4(1 + \delta)\pi_L - (10 + 9\delta)\pi_L^2) - 2\mu^2(1 - \pi_L)(4\pi_L + \delta(-1 + 3\pi_L))}{(2 - \mu)(\mu(1 - \pi_L) + 2\pi_L)(\mu(1 + \mu) + 2(1 - \mu)\mu\pi_L + (2 - \mu)(1 - \mu)\pi_L^2)} \end{aligned}$$

To compare the two cutoffs for μ close to 0, note that

$$\begin{aligned} \lim_{\mu \rightarrow 0} & \frac{\delta}{\mu + (1 - \mu)\pi_L^2} + \frac{\delta}{\mu + (1 - \mu)\pi_L} - \\ & \frac{-2(1 + \delta)\mu^3(1 - \pi_L)^2 + 2\pi_L(\delta + 2\pi_L + 3\delta\pi_L) + \mu(2 + \delta + 4(1 + \delta)\pi_L - (10 + 9\delta)\pi_L^2) - 2\mu^2(1 - \pi_L)(4\pi_L + \delta(-1 + 3\pi_L))}{2(2 - \mu)(\mu(1 - \pi_L) + 2\pi_L)(\mu(1 + \mu) + 2(1 - \mu)\mu\pi_L + (2 - \mu)(1 - \mu)\pi_L^2)} \\ &= \frac{\delta}{\pi_L^2} + \frac{\delta}{\pi_L} - \frac{\delta + 2\pi_L + 3\delta\pi_L}{4\pi_L^2} > 0. \end{aligned}$$

Thus, for μ close to 0, $\hat{d}^{\text{ind}}(B) < \hat{d}(B; I)$ whenever $\delta < \frac{2\pi_L}{3 + \pi_L}$.

To compare the two cutoffs for μ close to 1, note that

$$\begin{aligned} \lim_{\mu \rightarrow 1} & \frac{1}{\mu + (1 - \mu)\pi_L^2} + \frac{1}{\mu + (1 - \mu)\pi_L} - \\ & \frac{-2(1 + \delta)\mu^3(1 - \pi_L)^2 + 2\pi_L(\delta + 2\pi_L + 3\delta\pi_L) + \mu(2 + \delta + 4(1 + \delta)\pi_L - (10 + 9\delta)\pi_L^2) - 2\mu^2(1 - \pi_L)(4\pi_L + \delta(-1 + 3\pi_L))}{2(2 - \mu)(\mu(1 - \pi_L) + 2\pi_L)(\mu(1 + \mu) + 2(1 - \mu)\mu\pi_L + (2 - \mu)(1 - \mu)\pi_L^2)} \\ &= 2 - \frac{1}{4}\delta(1 + \pi_L) > 0. \end{aligned}$$

Thus, for μ close to 1, $\hat{d}^{\text{ind}}(B) > \hat{d}(B; I)$. □

Proof. [Proof of Proposition 4] To compute the stationary distribution over histories, define the probability the firm will produce a good signal after a history $h \in \mathcal{H}$ by $\pi^{\mathcal{S}}(h) = \sigma^{\mathcal{S}}(h) \cdot \pi_H + (1 -$

$\sigma^S(h) \cdot \pi_L$. Since the firm invests in quality if and only if $h \in \mathcal{S}$, $\pi^S(h) = \pi_H$ if and only if $h \in \mathcal{S}$, and π_L otherwise.

$$\begin{aligned} \Pr_C^S(GG) &= \frac{\pi^S(B) + \pi^S(\emptyset)}{2(2 + \pi^S(B) - \pi^S(G))} \cdot \frac{\pi^S(G)}{2}, \quad \Pr_C^S(GB) = \frac{\pi^S(B) + \pi^S(\emptyset)}{2(2 + \pi^S(B) - \pi^S(G))} \cdot \frac{1 - \pi^S(G)}{2} \\ \Pr_C^S(G\emptyset) &= \frac{\pi^S(B) + \pi^S(\emptyset)}{2(2 + \pi^S(B) - \pi^S(G))} \cdot \frac{1}{2}, \quad \Pr_C^S(BG) = \left(\frac{1}{2} - \frac{\pi^S(B) + \pi^S(\emptyset)}{2(2 + \pi^S(B) - \pi^S(G))} \right) \cdot \frac{\pi^S(B)}{2} \\ \Pr_C^S(BB) &= \left(\frac{1}{2} - \frac{\pi^S(B) + \pi^S(\emptyset)}{2(2 + \pi^S(B) - \pi^S(G))} \right) \cdot \frac{1 - \pi^S(B)}{2}, \quad \Pr_C^S(B\emptyset) = \left(\frac{1}{2} - \frac{\pi^S(B) + \pi^S(\emptyset)}{2(2 + \pi^S(B) - \pi^S(G))} \right) \cdot \frac{1}{2} \\ \Pr_C^S(\emptyset G) &= \frac{1}{2} \cdot \frac{\pi^S(\emptyset)}{2}, \quad \Pr_C^S(\emptyset B) = \frac{1}{2} \cdot \frac{1 - \pi^S(\emptyset)}{2}, \quad \Pr_C^S(\emptyset\emptyset) = \frac{1}{2} \cdot \frac{1}{2}, \end{aligned}$$

Since an incompetent firm cannot invest in quality, plugging in $\mathcal{S} = \emptyset$ to probabilities above, $\Pr_C^S(h)$, for all histories, we obtain

$$\begin{aligned} \Pr_I^S(GG) &= \frac{\pi_L^2}{4}, \quad \Pr_I^S(GB) = \Pr_I^S(BG) = \frac{\pi_L(1 - \pi_L)}{4} \\ \Pr_I^S(G\emptyset) &= \Pr_I^S(\emptyset G) = \frac{\pi_L}{4}, \quad \Pr_I^S(B\emptyset) = \Pr_I^S(\emptyset B) = \frac{1 - \pi_L}{4} \\ \Pr_I^S(BB) &= \frac{(1 - \pi_L)^2}{4}, \quad \Pr_I^S(\emptyset\emptyset) = \frac{1}{2} \cdot \frac{1}{2}. \end{aligned}$$

□

Proof. [Proof of Lemma 5] So far, we have denoted a firm's value function of the firm by $V(\cdot)$, the present-discounted profit once the current customer visits the firm. We find it useful to introduce a notation for a value function prior to the customer's assignment, $W(\cdot)$, which we define:

$$W(\mathbf{h}^T) \equiv \underbrace{\frac{1}{2}(p(\mathbf{h}^T) - c)}_{\text{expected current period}} + \delta \left(\underbrace{\frac{\pi_H}{2} \cdot W(\mathbf{h}^{T-1}G) + \frac{1 - \pi_H}{2} \cdot W(\mathbf{h}^{T-1}B)}_{\text{future profit if visited this period}} + \underbrace{\frac{1}{2} \cdot W(\mathbf{h}^{T-1}\emptyset)}_{\text{otherwise}} \right).$$

In the current period, the firm's expected profit is the probability of having the customer visit, $\frac{1}{2}$, multiplied by the profit margin, $p(\mathbf{h}^T) - c$. The firm can be under three different information sets, each of which generate different profits accordingly. With probability $\frac{\pi_H}{2}$ the firm is visited today and produces a good history, bringing about a stream of profits summarized by $W(\mathbf{h}^{T-1}G)$. With

probability $\frac{1-\pi_H}{2}$, the firm produces a bad history and generates a profit of $W(\mathbf{h}^{T-1}B)$. Lastly, with probability a half, the firm is not chosen today and therefore generates an empty signal. Note that $W(\cdot)$ is the firm's expected present-discounted payoff before the consumer's visit is determined.

If the firm is visited, the firm makes the investment decision by comparing expected payoffs from investment and no investment, denoted by V^* and \hat{V} , respectively.

$$\begin{aligned} V^*(\mathbf{h}^T) &= p(\mathbf{h}^T) - c + \delta(\pi_H \cdot W(\mathbf{h}^{T-1}G) + (1 - \pi_H) \cdot W(\mathbf{h}^{T-1}B)), \\ \hat{V}(\mathbf{h}^T) &= p(\mathbf{h}^T) + \delta(\pi_L \cdot W(\mathbf{h}^{T-1}G) + (1 - \pi_L) \cdot W(\mathbf{h}^{T-1}B)). \end{aligned}$$

The **RE** exists iff $V^*(\mathbf{h}^T) \geq \hat{V}(\mathbf{h}^T)$, which holds if and only if

$$c \leq \hat{c}^{\text{ind}} \equiv \delta(\pi_H - \pi_L) \cdot \min_{\mathbf{h}^{T-1} \in \{G, \emptyset, B\}^{T-1}} \Delta W(\mathbf{h}^{T-1}), \quad (13)$$

where $\Delta W(\mathbf{h}^{T-1}) := W(\mathbf{h}^{T-1}G) - W(\mathbf{h}^{T-1}B)$ defines reputational benefit to be realized in the future generated by today's investment decision, conditional on the relevant history \mathbf{h}^{T-1} . The expression for \hat{c}^{ind} is intuitive. The benefit from an investment in the current period comes from an increase in the future payoff, which is increasing in the discount factor δ and the informativeness of different signals, $\Delta\pi := \pi_H - \pi_L$. $\Delta W(\mathbf{h}^{T-1})$ summarizes the future reputational benefit through continued sum of price premiums that the decision to investment creates.

As in the analysis with two-period memory, we focus on parameter regions with small values of π_L by taking a limit $\pi_L \rightarrow 0$. Then, one good history fully reveals that the firm is competent. We now compute $\Delta W(\cdot)$ for a general model with a finite history length $t \geq 3$ and identify the binding

constraint to characterize c^{ind} . First,

$$\begin{aligned}
W(\mathbf{h}^{T-1}G) &= \underbrace{\frac{1}{2} \sum_{k=0}^{T-1} \delta^k \sum_{\mathbf{f} \in \mathcal{H}^k} \Pr(\mathbf{f})(p(\mathbf{h}^{T-k-1}G\mathbf{f}) - c)}_{\text{First } T-1 \text{ Periods}} + \underbrace{\frac{1}{2} \sum_{j=0}^{\infty} \delta^{T+j} \sum_{\mathbf{g} \in \mathcal{H}^T} \Pr(\mathbf{g})(p(\mathbf{g}) - c)}_{\text{After } T \text{ Periods}} \\
&= \frac{1}{2} \sum_{k=0}^{T-1} \delta^k \left(\sum_{i+j+l=k} \left(\frac{\pi_H}{2}\right)^i \left(\frac{1-\pi_H}{2}\right)^j \left(\frac{1}{2}\right)^l \left(\sum_{\mathbf{G}(\mathbf{f})=i, \mathbf{B}(\mathbf{f})=j} p(\mathbf{h}^{T-1-k}G\mathbf{f}) \right) - c \right) \\
&\quad + \frac{1}{2} \delta^T \sum_{k=0}^{\infty} \delta^k \left(\sum_{i+j+l=T} \left(\frac{\pi_H}{2}\right)^i \left(\frac{1-\pi_H}{2}\right)^j \left(\frac{1}{2}\right)^l \left(\sum_{\mathbf{G}(\mathbf{f})=i, \mathbf{B}(\mathbf{f})=j} p(\mathbf{f}) \right) - c \right).
\end{aligned}$$

Likewise,

$$\begin{aligned}
W(\mathbf{h}^{T-1}B) &= \frac{1}{2} \sum_{k=0}^{T-1} \delta^k \left(\sum_{i+j+l=k} \left(\frac{\pi_H}{2}\right)^i \left(\frac{1-\pi_H}{2}\right)^j \left(\frac{1}{2}\right)^l \left(\sum_{\mathbf{G}(\mathbf{f})=i, \mathbf{B}(\mathbf{f})=j} p(\mathbf{h}^{T-1-k}B\mathbf{f}) \right) - c \right) \\
&\quad + \frac{1}{2} \delta^T \sum_{k=0}^{\infty} \delta^k \left(\sum_{i+j+l=T} \left(\frac{\pi_H}{2}\right)^i \left(\frac{1-\pi_H}{2}\right)^j \left(\frac{1}{2}\right)^l \left(\sum_{\mathbf{G}(\mathbf{f})=i, \mathbf{B}(\mathbf{f})=j} p(\mathbf{f}) \right) - c \right).
\end{aligned}$$

Therefore, subtracting the two gives

$$\begin{aligned}
\Delta W(\mathbf{h}^{T-1}) &= \frac{1}{2} \cdot \sum_{k=0}^{T-1} \delta^k \sum_{\mathbf{f} \in \mathcal{H}^k} \Pr(\mathbf{f})(p(\mathbf{h}^{T-k-1}G\mathbf{f}) - p(\mathbf{h}^{T-k-1}B\mathbf{f})) \\
&= \frac{1}{2} \cdot \sum_{k=0}^{T-1} \delta^k \left(\sum_{i+j+l=k} \left(\frac{\pi_H}{2}\right)^i \left(\frac{1-\pi_H}{2}\right)^j \left(\frac{1}{2}\right)^l \left(\sum_{\mathbf{G}(\mathbf{f})=i, \mathbf{B}(\mathbf{f})=j} (p(\mathbf{h}^{T-1-k}G\mathbf{f}) - p(\mathbf{h}^{T-1-k}B\mathbf{f})) \right) \right) \tag{15}
\end{aligned}$$

□

Proof. [Proof of Lemma ??] We simplify the expression above by computing the price difference, $p(\mathbf{h}^{T-1-k}G\mathbf{f}) - p(\mathbf{h}^{T-1-k}B\mathbf{f})$. With $\pi_L \rightarrow 0$, $p(\mathbf{h}) = \pi_H \hat{\mu}(\mathbf{h})$ for any history \mathbf{h} . Therefore, we must find $\pi_H(\hat{\mu}(\mathbf{h}^{T-1-k}G\mathbf{f}) - \hat{\mu}(\mathbf{h}^{T-1-k}B\mathbf{f}))$ for any $\mathbf{f} \in \mathcal{H}^k$. Because a good signal fully reveals that a firm is competent, $\Delta \hat{\mu}(\mathbf{h}^{T-1}, k, \mathbf{f}) \equiv \hat{\mu}(\mathbf{h}^{T-1-k}G\mathbf{f}) - \hat{\mu}(\mathbf{h}^{T-1-k}B\mathbf{f}) = 0$ if and only if $\mathbf{G}(\mathbf{h}^{T-1-k}B\mathbf{f}) \geq 1$. Since our current goal is to find a history \mathbf{h}^{T-1} that minimizes $\Delta W(\cdot)$, and given that $\Delta \hat{\mu}(\mathbf{h}^{T-1}, k, \mathbf{f}) \geq 0$ always, it is clear that we want $\Delta \hat{\mu} = 0$ for as many \mathbf{f} as possible. Therefore we require $h_1 = G$, where h_1 is the most recent history of the firm's history at the time of the investment decision ($\mathbf{h}^T = h_T h_{T-1} \dots h_2 h_1$). Then, $\Delta \hat{\mu} = 0$ for all \mathbf{f} for all $k = 0, 1, \dots, T-2$. However, when $k = T-1$, T periods have passed after the investment, and the entire history \mathbf{h}^T is

forgotten and $\Delta\hat{\mu}(\emptyset, T-1, \mathbf{f}) = \hat{\mu}(G\mathbf{f}) - \hat{\mu}(B\mathbf{f})$, where $\mathbf{f} \in \mathcal{H}^{T-1}$. Again, this vanishes if and only if $\mathbf{G}(\mathbf{f}) \geq 1$. Therefore, $\Delta\hat{\mu}(\cdot)$ is positive if and only if T periods have passed, and none of the new history generated was good.

$$\lim_{\pi_L \rightarrow 0} \Delta W(\mathbf{h}^{T-1}) = \frac{\pi_H}{2} \cdot \delta^{T-1} \left(\sum_{j=0}^{T-1} \binom{T-1}{j} \left(\frac{1-\pi_H}{2}\right)^j \left(\frac{1}{2}\right)^{T-1-j} \cdot \lim_{\pi_L \rightarrow 0} (\hat{\mu}(GB^j\emptyset^{T-1-j}) - \hat{\mu}(B^{j+1}\emptyset^{T-1-j})) \right).$$

$$\hat{\mu}(GB^j\emptyset^{T-1-j}) = 1 \text{ because a good history causes a full revelation, and } \hat{\mu}(B^{j+1}\emptyset^{T-1-j}) = \frac{\mu(1-\pi_H)^{j+1}}{\mu(1-\pi_H)^{j+1} + 1 - \mu}.$$

Therefore,

$$\lim_{\pi_L \rightarrow 0} \Delta W(\mathbf{h}^{T-1}) = \frac{\pi_H(1-\mu)}{2^T} \cdot \delta^{T-1} \left(\sum_{j=0}^{T-1} \binom{T-1}{j} \frac{(1-\pi_H)^j}{\mu(1-\pi_H)^{j+1} + (1-\mu)} \right),$$

and consequently,

$$\lim_{\pi_L \rightarrow 0} \hat{c}^{\text{ind}} = \frac{\delta^T \pi_H^2 (1-\mu)}{2^T} \cdot \left(\sum_{j=0}^{T-1} \binom{T-1}{j} \cdot \frac{(1-\pi_H)^j}{\mu(1-\pi_H)^{j+1} + (1-\mu)} \right).$$

□

Proof. [Proof of Lemma ??]

$$\lim_{\pi_H \rightarrow 1} \Delta W(\mathbf{h}^{T-1}) = \lim_{\pi_H \rightarrow 1} \frac{\delta^{T-1}(\pi_H - \pi_L)}{2} \left(\sum_{j=0}^{T-1} \binom{T-1}{j} \left(\frac{\pi_H}{2}\right)^j \left(\frac{1}{2}\right)^{T-1-j} (\hat{\mu}(G^{j+1}\emptyset^{T-1-j}) - \hat{\mu}(BG^j\emptyset^{T-1-j})) \right).$$

$$\hat{\mu}(BG^j\emptyset^{T-1-j}) = 0 \text{ because a good history causes a full revelation, and } \hat{\mu}(G^{j+1}\emptyset^{T-1-j}) = \frac{\mu}{\mu + (1-\mu)\pi_L^{j+1}}.$$

Therefore,

$$\lim_{\pi_H \rightarrow 1} \Delta W(\mathbf{h}^{T-1}) = \frac{\delta^{T-1}(1-\pi_L)\mu}{2^T} \left(\sum_{j=0}^{T-1} \binom{T-1}{j} \frac{1}{\mu + (1-\mu)\pi_L^{j+1}} \right),$$

and consequently,

$$\lim_{\pi_H \rightarrow 1} \hat{c}^{\text{ind}} = \frac{\delta^T(1-\pi_L)^2\mu}{2^T} \cdot \left(\sum_{j=0}^{T-1} \binom{T-1}{j} \frac{1}{\mu + (1-\mu)\pi_L^{j+1}} \right).$$

□

Proof. [Proof of Lemma ??] Let $\Delta\hat{\eta}(i) := \hat{\eta}(\mathbf{g}_2) - \hat{\eta}(\mathbf{g}_1)$ where $\mathbf{G}(\mathbf{g}_2) = \mathbf{G}(\mathbf{g}_1) + 1 = i + 1$. That is, $\Delta\hat{\eta}(i)$ denotes the difference in two posteriors where \mathbf{g}_1 has i good outcomes, one less than \mathbf{g}_2 does. Then,

$$\begin{aligned}
& \sum_{\mathbf{f} \in \{G, B\}^k} \Pr(\mathbf{f}, \theta) \cdot (\hat{\eta}(\mathbf{h}^{T-k-1} \mathbf{G} \mathbf{f}) - \hat{\eta}(\mathbf{h}^{T-k-1} \mathbf{B} \mathbf{f})) \\
& \geq \pi_H^k (\hat{\eta}(G^T) - \hat{\eta}(BG^{T-1})) \\
& = \pi_H^k \left(\frac{\mu \pi_H^T + (1 - \mu) \cdot \left(\frac{\pi_H}{2}\right)^T}{\mu \pi_H^T + 2(1 - \mu) \cdot \left(\frac{\pi_H}{2}\right)^T} - \frac{\mu \pi_H^{T-1} (1 - \pi_H) + (1 - \mu) \cdot \left(\frac{\pi_H}{2}\right)^{T-1} \left(1 - \frac{\pi_H}{2}\right)}{\mu \pi_H^{T-1} (1 - \pi_H) + 2(1 - \mu) \cdot \left(\frac{\pi_H}{2}\right)^{T-1} \left(1 - \frac{\pi_H}{2}\right)} \right) \\
& = \pi_H^k \left(\frac{A \pi_H + B \left(\frac{\pi_H}{2}\right)}{A \pi_H + 2B \left(\frac{\pi_H}{2}\right)} - \frac{A(1 - \pi_H) + B \left(1 - \frac{\pi_H}{2}\right)}{A(1 - \pi_H) + 2B \left(1 - \frac{\pi_H}{2}\right)} \right) \\
& = \pi_H^k \cdot \frac{AB \pi_H}{2 \left(A \pi_H + 2B \left(\frac{\pi_H}{2}\right)\right) \left(A(1 - \pi_H) + 2B \left(1 - \frac{\pi_H}{2}\right)\right)} \\
& = \pi_H^k \cdot \frac{\mu(1 - \mu) \frac{\pi_H^{2(T-1)}}{2^{T-1}}}{2 \left(\mu \pi_H^{T-1} + (1 - \mu) \left(\frac{\pi_H}{2}\right)^{T-1}\right) \left(\mu \pi_H^{T-1} (1 - \pi_H) + 2(1 - \mu) \left(\frac{\pi_H}{2}\right)^{T-1} \left(1 - \frac{\pi_H}{2}\right)\right)} \\
& = \frac{\pi_H^k \mu(1 - \mu)}{2^T \left(\mu + \frac{1-\mu}{2^{T-1}}\right) \left(\mu(1 - \pi_H) + \frac{1-\mu}{2^{T-1}} (2 - \pi_H)\right)}.
\end{aligned}$$

where $A = \mu \pi_H^{T-1}$ and $B = (1 - \mu) \left(\frac{\pi_H}{2}\right)^{T-1}$. Therefore, a lower bound for

$$\Delta W(\mathbf{h}^{T-1}) \geq \underline{\Delta W(\mathbf{h}^{T-1})} := \frac{1 - \delta^T \pi_H^T}{1 - \delta \pi_H} \cdot \frac{\mu(1 - \mu) \pi_H}{2^{T+1} \left(\mu + \frac{1-\mu}{2^{T-1}}\right) \left(\mu(1 - \pi_H) + \frac{1-\mu}{2^{T-1}} (2 - \pi_H)\right)},$$

and $\hat{c} \geq \delta \pi_H \cdot \underline{\Delta W(\mathbf{h}^{T-1})}$. □

Proof. [Proof of Proposition ??]

$$\frac{1 - \delta^T \pi_H^T}{1 - \delta \pi_H} \cdot \frac{\delta \mu(1 - \mu) \pi_H^2}{2^{T+1} \left(\mu + \frac{1-\mu}{2^{T-1}}\right) \left(\mu(1 - \pi_H) + \frac{1-\mu}{2^{T-1}} (2 - \pi_H)\right)} > \frac{\delta^T \pi_H^2 (1 - \mu)}{2^T} \sum_{j=0}^{T-1} \binom{T-1}{j} \cdot \frac{(1 - \pi_H)^j}{\mu(1 - \pi_H)^{j+1} + (1 - \mu)},$$

Now we plug $\pi_H \rightarrow 1$. RHS becomes $\frac{\delta^T}{2^T}$ and LHS $\frac{1-\delta^T}{1-\delta} \cdot \frac{2^{T-2}\delta\mu}{(2^T-2)\mu+2}$.

$$\begin{aligned} \frac{1-\delta^T}{1-\delta} \cdot \frac{2^{T-2}\delta\mu}{(2^T-2)\mu+2} &> \frac{\delta^T}{2^T} \\ \frac{(1-\delta^T)2^{T-2}}{(1-\delta)\delta^{T-1}} &> \frac{(2^T-2)\mu+2}{2^T\mu} = 1 + \frac{2(1-\mu)}{2^T\mu}. \end{aligned}$$

LHS is decreasing in δ , while RHS is decreasing in μ . Since we are considering a large μ and $T \geq 3$, RHS is at most $1 + \frac{1}{4} = \frac{5}{4}$. RHS is equivalent to $2^{T-2}(1 + \frac{1}{\delta} + \dots + \frac{1}{\delta^{T-1}}) > 2$. Therefore, for all values of δ and $T \geq 3$, the condition holds. \square

Proof. [Proof of Proposition ??] Now we find $\lim_{\pi_H \rightarrow 1} (\hat{\eta}(GB^{T-1}) - \hat{\eta}(B^T))$

$$\begin{aligned} &= \frac{\mu \cdot \left(\frac{1+\pi_L}{2}\right) \left(\frac{1-\pi_L}{2}\right)^{T-1}}{2\mu \cdot \left(\frac{1+\pi_L}{2}\right) \left(\frac{1-\pi_L}{2}\right)^{T-1} + (1-\mu)\pi_L(1-\pi_L)^{T-1}} - \frac{\mu \cdot \left(\frac{1-\pi_L}{2}\right)^T}{2\mu \cdot \left(\frac{1-\pi_L}{2}\right)^T + (1-\mu)(1-\pi_L)^T} \\ &= \frac{\mu(1+\pi_L)}{2\mu(1+\pi_L) + 2^T(1-\mu)\pi_L} - \frac{\mu}{2\mu + 2^T(1-\mu)} \\ &= \frac{2^T\mu(1-\mu)}{(2\mu(1+\pi_L) + 2^T(1-\mu)\pi_L)(2\mu + 2^T(1-\mu))}. \end{aligned}$$

Therefore,

$$\begin{aligned} \lim_{\pi_H \rightarrow 1} \underline{\Delta W}(B^{T-1}; I) &= \left(\frac{1-\pi_L}{2}\right) \cdot \sum_{k=0}^{T-1} \delta^k \left(\frac{1-\pi_L}{2}\right)^k \frac{2^T\mu(1-\mu)}{(2\mu(1+\pi_L) + 2^T(1-\mu)\pi_L)(2\mu + 2^T(1-\mu))} \\ &= \left(\frac{1-\pi_L}{2}\right) \cdot \frac{1 - \left(\frac{(1-\pi_L)\delta}{2}\right)^T}{1 - \frac{(1-\pi_L)\delta}{2}} \cdot \frac{2^T\mu(1-\mu)}{(2\mu(1+\pi_L) + 2^T(1-\mu)\pi_L)(2\mu + 2^T(1-\mu))}. \end{aligned}$$

Now, we compare $\lim_{\pi_H \rightarrow 1} \underline{c}(B^{T-1}; I)$ and $\lim_{\pi_H \rightarrow 1} \hat{c}^{\text{ind}}$ to find a sufficient condition for $\hat{c} > \hat{c}^{\text{ind}}$ for a large π_H . The comparison is still complicated, and we take the limit $\pi_L \rightarrow 0$. From equation 11, \hat{c}^{ind} converges to δ^T , while \underline{c} converges to $\frac{\delta}{4} \cdot \frac{1 - (\frac{\delta}{2})^T}{1 - \frac{\delta}{2}} \cdot \frac{2^T(1-\mu)}{(2\mu + 2^T(1-\mu))}$. Then, $\lim_{\pi_L \rightarrow 0} \lim_{\pi_H \rightarrow 1} \underline{c}(B^{T-1}; I) > \lim_{\pi_L \rightarrow 0} \lim_{\pi_H \rightarrow 1} \hat{c}$ if and only if

$$\begin{aligned} \frac{\delta}{4} \cdot \frac{1 - (\frac{\delta}{2})^T}{1 - \frac{\delta}{2}} \cdot \frac{2^T(1-\mu)}{(2\mu + 2^T(1-\mu))} &\geq \delta^T \\ \Leftrightarrow \frac{1}{4} \cdot \frac{1 - (\frac{\delta}{2})^T}{(1 - \frac{\delta}{2})\delta^{T-1}} &\geq 1 + \frac{2\mu}{2^T(1-\mu)}. \end{aligned}$$

RHS is increasing in μ , while LHS is decreasing in δ . Since LHS is less than 1 if $\delta = 1$, which then is always less than RHS, the condition holds for δ not too large. \square