Risk Attitudes and Heterogeneity in Simultaneous and Sequential Contests

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Abstract: We analyze a class of rent-seeking contests in which players are heterogeneous in both risk preferences and production technology. We find that there exists a unique Nash equilibrium whenever each player's absolute measure of risk aversion is constant and marginal productivity is non-increasing in rent-seeking investment. If the number of risk-loving players is large enough, the aggregate investment in equilibrium will exceed the rent and all risk-neutral and risk-averse players will exit the contest. In a standard Tullock contest with two players and homogeneous technology, the player who is less downside-risk-averse is the favorite to win the rent. In a sequential contest, if the first mover is less (more) downside-risk-averse and the second mover is risk-averse (risk-loving), the first mover will be the favorite (underdog) in the contest.

Keywords: Risk attitude, Downside risk, Simultaneous contest, Sequential contest, Rent dissipation

1 Introduction

Tullock (1980) introduces a seminal framework to analyze winner-take-all contests in which each player competes with one another to win a given prize. The winning probability of each player is determined by his irreversible investment relative to the total investment by all players participating in the contest. In this paper, we study a class of rent-seeking contests with heterogeneity in players' risk preferences and production technology. To the best of our knowledge, this paper is the first to show that there is a unique equilibrium in rent-seeking contests where some players are risk-averse and some are risk-loving. We prove that there exists a unique Nash equilibrium whenever each player's absolute measure of risk aversion (or risk lovingness) is constant and marginal productivity is non-increasing in rent-seeking investment. Specifically, we generalize the CARA (constant absolute risk aversion) utility functions by allowing the risk parameter of each player to exhibit decreasing or increasing marginal utility of income.

In addition to proving existence and uniqueness of equilibrium given heterogeneous players, we expand the literature of rent-seeking contests in three directions. First, we theoretically show that, under Expected Utility Theory, rent over-dissipation or under-dissipation may occur (i.e., total rent-seeking investment by all participants may or may not exceed the rent) in equilibrium of a standard Tullock contest depending on each player's risk parameter. While rent over-dissipation seems to be a more important issue to economists and policy makers since it implies excess social waste, both rent over-dissipation and under-dissipation are empirically supported in laboratory settings.[1] Assuming linear utility functions, Tullock (1980) uses numerical examples to show that rent may be over-dissipated given a class of convex rent-seeking productions. However, Baye, Kovenock, and De Vries (1994 and 1999) argue that pure-strategy equilibrium does not exist in those cases and thus rent over-dissipation is not possible under risk neutrality.[2] Contrasting Tullock's theoretic approach to explaining rent over-dissipation, we allow some players to be risk-loving while

[1]See Houser and Stratmann (2012) and Sheremeta (2013) for thorough reviews of experimental findings.

[2]See also Cornes and Hartley (2005).

assuming linear technology. We find that if the number of risk-loving players is larger than a threshold, rent over-dissipation will occur in such an equilibrium. Moreover, in an equilibrium where rent is over-dissipated, all risk-neutral and risk-averse players will not participate in the contest. We also find that if there are less than four risk-loving players in a contest, rent will be under-dissipated in equilibrium.

Second, while a prior research focus is on how risk aversion affects rent-seeking investment in equilibrium[3], we show that, in fact, attitudes toward downside risk play a more significant role. Konrad and Schlesinger (1997) show that in general there is no monotonic relationship between a player's measure of risk aversion and his rent-seeking investment in equilibrium, that is, in a standard Tullock contest with two risk-averse players, the more-risk-averse player may or may not invest more than the less-risk-averse one. However, in a special case where the two risk-averse players have CARA utility functions, Cornes and Hartley (2003) find that the less-risk-averse player will invest more. In this paper, we generalize Cornes and Hartley's (2003) model by allowing for convex CARA functions and find that Cornes and Hartley's prediction may not hold if at least one of the players is risk-loving. Specifically, we find that the player who is less downside-risk-averse will invest more in equilibrium and become the favorite to win the rent, regardless of whether each player is risk-averse or risk-loving.

Third, this paper is the first to analyze risk attitudes in sequential rent-seeking games. Since the seminal analysis of preemptive investment in contests by Dixit (1987), most theoretical development in the literature has been on rent-seeking technology (i.e., contest success functions) and asymmetry in reward and information[4] while players' risk attitudes have been ignored. By allowing for nonlinear utility functions, we can derive results contrasting Dixit's (1987) well-known prediction that there is no incentive to move first in a standard Tullock contest since equilibrium outcomes in simultaneous and sequential contests are identical. We show that even when both players have identical attitudes toward

[3]See Hillman and Katz (1984), Skaperdas and Gan (1995), Konrad and Schlesinger (1997), Cornes and Hartley (2003, 2012), and Treich (2010).

[4]See, Baik and Shogren (1992), Baye and Shin (1999), Morgan (2003), and Morgan and Várdy (2007).

risk and downside risk, the first-mover (second-mover) has an advantage if both players are risk-averse (risk-loving). In a sequential contest with heterogeneous players, we find that each player's attitudes toward risk and downside risk together determine whether a first-mover or second-mover advantage arises in equilibrium. In particular, given the same linear rent-seeking technology, if the first mover is less (more) downside-risk-averse and the second mover is risk-averse (risk-loving), then the first mover will be the favorite (underdog) in the contest.

Even though risk aversion may seem to be a standard presumption of human behavior, we cannot deny that some decision makers especially in the laboratory may behave as if they are risk-loving. Focusing on pay-to-bid auctions, Platt, Price, and Tappen (2013) find that allowing for risk lovingness has the biggest impact in explaining bidding behavior and suggest that pay-to-bid auction is a mild form of gambling. For some experimental subjects, a rent-seeking contest may be a form of gambling too. Our framework for analyzing a rent-seeking contest is very flexible since we allow each individual to have a different degree of risk-aversion or risk lovingness. For future research, it should be worthwhile to revisit various experimental data sets from rent-seeking games and analyze players' attitudes towards risk and downside risk. While recent developments in the literature seem to suggest that different equilibrium concepts (Gneezy and Smorodinsky, 2006, and Lim, Matros, and Turocy, 2014) or psychological factors (Sheremeta, 2013) can explain rent over-dissipation in the laboratory, we propose a possible explanation under the canonical expected utility model.

This paper is organized as follows. In Section 2, we prove existence and uniqueness of equilibrium and derive sufficient conditions for rent under-dissipation and over-dissipation in equilibrium. We analyze the relationship between each player's risk attitude and equilibrium investment in Section 3. In Section 4, we derive equilibria in sequential contests and also sufficient conditions for first- and second-mover advantages. We conclude in Section 5.

2 Existence and Uniqueness of Equilibrium

Consider an n-player contest for a fixed prize R with $n \geq 2$ and $R > 0$. Player i, for $i = 1, ..., n$, has an initial wealth $I_i \geq R$ and a utility function with the following functional form.

Assumption 1. *Player i's utility function is given by*

$$u_i(w) = \begin{cases} \alpha_i e^{\alpha_i w} & \text{if } \alpha_i \neq 0 \\ w & \text{if } \alpha_i = 0 \end{cases} \tag{1}$$

for all w.

Player i's utility function is strictly increasing and the corresponding Arrow-Pratt absolute measure of risk aversion is $-\alpha_i$ for any level of wealth. Thus each utility function is a generalized CARA function which allows for risk-averse, risk-neutral, and risk-loving attitudes. We may refer to a player with the absolute measure of risk aversion of $-\alpha_i$ as a type α_i player who is risk-averse, risk-neutral, or risk-loving if α_i is less than, equal to, or greater than zero, respectively. Each player can invest $x_i \geq 0$ in the contest and his corresponding probability of winning the prize is given by

$$p_i = \frac{f_i(x_i)}{\sum_{j=1}^{n} f_j(x_j)}. \tag{2}$$

We assume that the production function f_i satisfies the following assumption for $i = 1, ..., n$.

Assumption 2. *$f_i(x_i)$ is a twice differentiable function with $f_i(0) = 0, f_i'(x_i) \in (0, \infty)$, and $f_i''(x_i) \leq 0$ for all $x_i \geq 0$.*

The probability of winning the rent is zero if player i does not invest in the contest. Given any level of investment by all other players, player i's probability of winning is strictly increasing in his own investment. Moreover, player i's production function is concave since its second derivative is less than or equal to zero. Following Jindapon and Whaley (2015), we let $y_i = f_i(x_i)$ be player i's output level and $Y_{-i} = \sum_{j \neq i} y_j$ be the sum of all other players' output. Since f_i is bijective, we can define the cost function $g_i = f_i^{-1}$ to be player i's investment cost of output y_i. According to Assumption 2, we know that $g_i(0) = 0$, $g_i'(y_i) > 0$, and $g_i''(y_i) \geq 0$ for all $y_i \geq 0$. Therefore, we can rewrite the probability of winning in (2) as

$$p_i = \frac{y_i}{y_i + Y_{-i}} \tag{3}$$

and say that player i chooses an optimal output y_i, rather than an investment x_i, to maximize his expected utility

$$U_i(y_i|Y_{-i}) = \frac{y_i}{y_i + Y_{-i}} u_i(I_i + R - g_i(y_i)) + \frac{Y_{-i}}{y_i + Y_{-i}} u_i(I_i - g_i(y_i)). \tag{4}$$

Note that $U_i(0|0)$ is undefined yet it is not relevant since given $Y_{-i} = 0$, player i's optimal choice of y_i to maximize (4) is strictly positive. In this paper, we use Cornes and Hartley's (2003 and 2005) concept of share functions to derive an equilibrium. Given his optimal production, denoted by y_i^*, we derive player i's share, i.e., the probability of winning corresponding to his best response as a function of total production, from $s_i(Y) = y_i^*/Y$ where $Y = y_i^* + Y_{-i}$. We find that if Assumptions 1 and 2 hold, a share function for player i given $Y > 0$ always exists and it is unique.

Lemma 1. *Define*

$$\kappa_i = \begin{cases} \frac{e^{\alpha_i R} - 1}{\alpha_i g_i'(0)} & \text{if } \alpha_i \neq 0 \\ \frac{R}{g_i'(0)} & \text{if } \alpha_i = 0. \end{cases} \tag{5}$$

Suppose that Assumptions 1 and 2 hold. There exists a unique share function $s_i(Y)$ for $Y < \kappa_i$ such that

$$g_i'(s_i(Y)Y)Y = \begin{cases} \frac{(1-s_i(Y))(e^{\alpha_i R}-1)}{(s_i(Y)(e^{\alpha_i R}-1)+1)\alpha_i} & \text{if } \alpha_i \neq 0 \\ (1 - s_i(Y))R & \text{if } \alpha_i = 0. \end{cases} \tag{6}$$

If $Y \geq \kappa_i$, then $s_i(Y) = 0$.

Proof. See Appendix A. □

Lemma 1 suggests that player i will not participate in the contest if the total production Y is greater than his threshold κ_i. Otherwise, he will choose the optimal level of individual production so that his probability of winning satisfies (6). The participation threshold has the following properties.

Remark 1. *Properties of κ_i given any α_i:*

1. $\kappa_i > 0$.

2. *κ_i is continuous in α_i.*

3. *κ_i is strictly increasing in α_i.*

Proof. Parts 1 and 2 follow immediately from the definition of κ_i in (5). This definition implies

$$\frac{d\kappa_i}{d\alpha_i} = \begin{cases} \frac{\alpha_i R e^{\alpha_i R} - e^{\alpha_i R} + 1}{\alpha_i^2 g_i'(0)} & \text{if } \alpha_i \neq 0 \\ \frac{R^2}{2g_i'(0)} & \text{if } \alpha_i = 0. \end{cases} \tag{7}$$

Thus, to prove part 3, we only have to show that $\alpha_i R e^{\alpha_i R} - e^{\alpha_i R} + 1 > 0$ given any $\alpha_i \neq 0$. Define $H(z) = e^z - z$. We have $H'(z) = e^z - 1$ and $H''(z) = e^z$. We find that $H(z)$ is a strictly convex function with the minimum value of 1 at $z = 0$. So $e^z - z > 1$ for all $z \neq 0$. Letting $z = -\alpha_i R$ where $\alpha_i \neq 0$ yields $e^{-\alpha_i R} + \alpha_i R > 1$. By multiplying both sides of this inequality by $e^{\alpha_i R}$, we have $1 + \alpha_i R e^{\alpha_i R} > e^{\alpha_i R}$, that is, $\alpha_i R e^{\alpha_i R} - e^{\alpha_i R} + 1 > 0$ given any $\alpha_i \neq 0$. □

Given $Y \in (0, \kappa_i)$, we cannot explicitly solve the share function $s_i(Y)$ defined in (6) without a specific functional form of g_i. Nonetheless, we can derive some general properties of $s_i(Y)$ as follows.

Remark 2. *Properties of $s_i(Y)$ given $Y \in (0, \kappa_i)$:*

1. *$s_i(Y)$ is continuous in Y.*

2. *$s_i(Y)$ is strictly decreasing in Y.*

3. $\lim_{Y \to 0} s_i(Y) = 1$ *and* $\lim_{Y \to \kappa_i} s_i(Y) = 0$.

Proof. Given $Y \in (0, \kappa_i)$ and the definition of κ_i in (5), player i's share, s_i, implicitly defined by (6) can be written as

$$\frac{g_i'(s_iY)Y}{g_i'(0)\kappa_i} = \frac{1-s_i}{1-s_i+s_ie^{\alpha_iR}} \tag{8}$$

and it is continuous in Y.

An increase in Y will cause the left-hand side of (8) to rise because g_i is convex. To keep both sides equal, s_i must fall. A decrease in s_i will lower the left-hand side and also increase the right-hand side of (8).

As $Y \to 0$, the left-hand side of (8) approaches zero. To maintain the equality, the right-hand side must converge to zero and therefore $s_i \to 1$. As $Y \to \kappa_i$, the left-hand side of (8) approaches $\frac{g_i'(s_iY)}{g_i'(0)}$ which is greater than or equal to 1 because g_i is convex. Since the right-hand side never exceeds 1, to maintain the equality, both sides must converge to 1 and therefore $s_i \to 0$. $\square$

Given the properties of individual share in Remark 2, we find that $\sum_{i=1}^n s_i(Y) \to n$ as $Y \to 0$ and $\sum_{i=1}^n s_i(Y) \to 0$ as $Y \to \max\{\kappa_1, ..., \kappa_n\}$. Since each player's share function is continuous and strictly decreasing in Y, the sum of all shares possesses the same properties. Thus, there exists a unique value of Y such that $\sum_{i=1}^n s_i(Y) = 1$, i.e., a Nash equilibrium. We call this value Y^e and it follows that each player's output corresponding to the aggregate output Y^e is $y_i^e = s_i(Y^e)Y^e$. We can derive each player's equilibrium rent-seeking investment from $x_i^e = g_i(y_i^e)$ and call the total investment in equilibrium X^e which is equal to $\sum_{i=1}^n x_i^e$. We formally state this finding as follows.

Proposition 1. *Consider a simultaneous contest with n heterogeneous players. If Assumptions 1 and 2 hold, there exists a unique Nash equilibrium for the contest. In such an equilibrium, player i invests $x_i^e = g_i(s_i(Y^e)Y^e)$ where Y^e satisfies $\sum_{i=1}^n s_i(Y^e) = 1$.*

Proof. This result follows immediately from Lemma 1 and Remark 2. $\square$

Consider the following example:

Example 1: Let $R = 1$ and $n = 3$ with $\alpha_1 = -1$, $\alpha_2 = 0$, and $\alpha_3 = 2$. Assume that $f_i(x_i) = x_i$, that is, $g_i(y_i) = y_i$, for all i. According to (5), we find $\kappa_1 = 0.632$, $\kappa_2 = 1$, and $\kappa_3 = 3.195$. Figure 1 illustrates each player's share function. The only value of Y that makes the sum of all shares equal to 1 is $0.579 = Y^e$ (see the vertical line in Figure 1). Each player's share function implies that the probabilities of winning the rent for players 1, 2, and 3 are 0.200, 0.421, and 0.379, respectively. It follows that $x_1^e = 0.115$, $x_2^e = 0.244$, and $x_3^e = 0.220$.

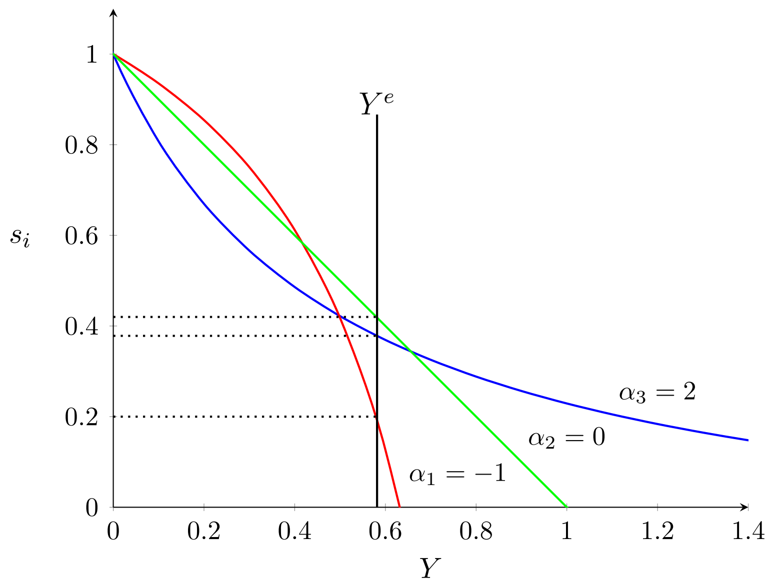


Figure 1: Share functions in Example 1

Proposition 1 is a generalization of the main finding in Cornes and Hartley (2003) where all players are risk-averse. Given $\alpha_i < 0$ for all i, Cornes and Hartley show that if all players have the same production function, there is a monotonic relationship between a player's type and his investment in equilibrium. Specifically, $\alpha_i < \alpha_j$ implies $x_i^e < x_j^e$.[5] This result however does not hold when some players are risk-loving as allowed in our model. Unlike contests with only risk-averse players, we cannot say that x_i^e is increasing in α_i as demonstrated above. In Example 1, player 2 invests more than player 3 in equilibrium even

[5]Since Cornes and Hartley (2003) use the conventional CARA functional form, i.e., $u_i(w) = -e^{-\alpha_i w}$, they state this result as "$\alpha_i > \alpha_j$ implies $x_i^e < x_j^e$."

though $\alpha_2 < \alpha_3$. However, Remark 1 suggests that player i's participation threshold, κ_i is increasing with α_i. If production functions are identical, players who do not participate in equilibrium must have lower values of α_i than those who participate in equilibrium. We formally state this finding as follows.

Proposition 2. *Consider a simultaneous contest where Assumptions 1 and 2 hold. Let $\alpha_1 \leq ... \leq \alpha_n$. If production functions are identical for all players and there exists player k such that $x_k^e = 0$, then $x_i^e = 0$ for all $i = 1, ..., k-1$.*

Proof. Given identical production functions and $\alpha_1 \leq ... \leq \alpha_n$, we know from Remark 1 that $\kappa_1 \leq ... \leq \kappa_n$. If $x_k^e = 0$, then $s_k(Y^e) = 0$ and we know from Lemma 1 that $Y^e \geq \kappa_k$. It follows that $Y^e \geq \kappa_i$ and hence $x_i^e = 0$ for all $i = 1, ..., k-1$. □

We find in Example 1 that $s_3(Y)$ is strictly positive for some $Y > 1$ because $\kappa_3 > 1$ (see Figure 1). It follows that if there are many players whose preferences are the same as player 3, it is possible that Y^e, which is equal to X^e in this example, will be greater than R. Specifically, since there exists $Y \in (R, \kappa_3)$ such that $s_3(Y) \in (0, 1)$, there exist $n > 1$ and $Y \in (R, \kappa_3)$ such that $ns_3(Y) \geq 1$ and thus rent over-dissipation will occur in equilibrium. Consider the following example:

Example 2: Let $R = 1$ and $n = 7$ with $\alpha_1 = -1$, $\alpha_2 = 0$, and $\alpha_3 = ... = \alpha_7 = 2$. Assume that $f_i(x_i) = x_i$, that is, $g_i(y_i) = y_i$, for all i. We find that the value of Y such that the sum of all shares equals one is 1.122. Since both κ_1 and κ_2 are smaller than 1.122, players 1 and 2 do not participate in the contest. Each of players 3 to 7 invests 0.224 and, therefore, has an equal chance of winning the rent.

In this example, there are five risk-loving players and the total investment exceeds the rent because of these players' aggressive strategies. In this section we derive sufficient and necessary conditions for rent over-dissipation in a contest. For intuitive exposition in subsequent findings, we assume hereafter that all players have the same production function given by $f_i(x_i) = x_i$ as in Tullock's standard contest.[6] It follows that player i's

[6] All of the following results also hold for $f_i(x_i) = ax_i$ where $a > 0$. We let $a = 1$ as in Tullock's

participation threshold and share function in (5) and (6) can be written as

$$\kappa_i = \begin{cases} \frac{e^{\alpha_i R}-1}{\alpha_i} & \text{if } \alpha_i \neq 0 \\ R & \text{if } \alpha_i = 0 \end{cases} \tag{9}$$

and

$$s_i(Y) = \begin{cases} \frac{\kappa_i - Y}{\kappa_i - Y + Ye^{\alpha_i R}} & \text{if } Y < \kappa_i \\ 0 & \text{if } Y \geq \kappa_i \end{cases} \tag{10}$$

respectively. Given (9) and (10), we can state the following results.

Remark 3. *Properties of κ_i and $s_i(Y)$ given $f_i(x_i) = x_i$ and $Y \in (0, \kappa_i)$:*

1. $\kappa_i \gtreqless R$ *if and only if* $\alpha_i \gtreqless 0$.

2. $\frac{\partial^2 s_i(Y)}{\partial Y^2} \gtreqless 0$ *if and only if* $\alpha_i \gtreqless 0$.

Proof. In the proof of Remark 1 we find that $H(z) = e^z - z > 1$ for all $z \neq 0$. Letting $z = \alpha_i R$ yields $e^{\alpha_i R} - 1 > \alpha_i R$ whenever $\alpha_i \neq 0$. We obtain the result in part 1 by dividing both sides by α_i. Given (10), we find $\frac{\partial^2 s_i(Y)}{\partial Y^2} = \frac{2\alpha_i(1+\alpha_i\kappa_i)}{\kappa_i(1+\alpha_i Y)^3}$ and the result in part 2 follows. □

Using Part 1 of Remark 3, we provide a necessary condition for rent over-dissipation, that is, if rent over-dissipation occurred in equilibrium, all risk-averse and risk-neutral players would not invest in the contest.

Proposition 3. *Consider a simultaneous contest where Assumption 1 holds and $f_i(x_i) = x_i$ for $i = 1, ..., n$. All risk-averse and risk-neutral players do not participate in equilibrium in which rent over-dissipation occurs.*

Proof. Given $f_i(x_i) = x_i$ for all i, we find that $Y^e = X^e$. It follows from Remark 3 that if player i is not risk-loving, then $\kappa_i \leq R$. If $Y^e = X^e > R \geq \kappa_i$, then $s_i(Y^e)$=0 and therefore $x_i^e = 0$. □

standard contest without loss of generality.

Figure 1 suggests that the more participating risk-lovers in a contest, the more likely rent over-dissipation will occur in equilibrium. How many risk-loving players do we need to guarantee rent over-dissipation in equilibrium? It is obvious that this number depends on types (α_i) of those risk-loving players. To answer this question, we consider a special case where there are n homogeneous players in the contest. Since $\alpha_i = \alpha$ for all $i = 1, ..., n$, then $\kappa_i = \kappa$ for all $i = 1, ..., n$. By solving for the total investment in equilibrium from $s_i(Y) = \frac{1}{n}$, we find

$$X^e = \frac{(n-1)\kappa}{n + \alpha\kappa} = \begin{cases} \frac{(n-1)(e^{\alpha R}-1)}{\alpha(n+e^{\alpha R}-1)} & \text{if } \alpha \neq 0 \\ \frac{(n-1)R}{n} & \text{if } \alpha = 0. \end{cases} \tag{11}$$

According to (11), we find that $X^e > R$ if and only if

$$(n-1)(\kappa - R) > Re^{\alpha R} \tag{12}$$

and we can state the following results.

Proposition 4. *Consider a simultaneous contest where Assumption 1 holds with $\alpha_i = \alpha$ and $f_i(x_i) = x_i$ for $i = 1, ..., n$.*

1. *Rent over-dissipation occurs in equilibrium if and only if $\alpha > 0$ and $n > 1 + \frac{\alpha R e^{\alpha R}}{e^{\alpha R} - \alpha R - 1}$.*

2. *Rent over-dissipation will never occur in equilibrium if $n \leq 4$.*

Proof. Consider (12). For the inequality to hold, κ must be greater than R. According to Remark 3 part 1, we need $\alpha > 0$. Thus rent over-dissipation will not occur in equilibrium when $\alpha \leq 0$. Solving for n from (12) given $\kappa > R$ and the definition of κ in (9) yields $n > 1 + J(z)$ where $J(z) = \frac{ze^z}{e^z - z - 1}$ and $z = \alpha R$. Given $z > 0$, $J(z)$ is a strictly convex function with the minimum value of 3.35 at $z = 1.79$. If $n \leq 4$, then we have $n < 1 + J(\alpha R)$ for any $\alpha > 0$, and rent over-dissipation will not occur in equilibrium. □

Under the assumption that all players are identical, if either the players are not risk-loving or the number of players is not greater than 4, then rent over-dissipation will not occur in equilibrium. Rent over-dissipation occurs if and only if the players are risk-loving and the number of players is large enough. Note that the minimum number of players to guarantee rent over-dissipation is not monotonic in α. For example, given $R = 1$, when $\alpha = 0.5$, 1, and 4, we need at least 7, 5, and 6 players, respectively, for rent over-dissipation to occur in equilibrium.

3 Risk Attitudes in Two-Player Contests

In the previous section, we show that aggregate investment in equilibrium will be larger than the rent if the number of risk-loving players is large enough. The reason is that, given linear technology, a risk-loving player has a participation threshold, κ_i, that is larger than R. Specifically, we find that this threshold is increasing in α_i. As the aggregate investment increases, some players drop out from the contest by choosing to invest nothing and less-risk-averse players remain participating. However this does not imply that in equilibrium a less-risk-averse player will always invest more than a more-risk-averse player (See Example 1). In this section, we focus on contests with two players and show that in fact it is the players' downside risk attitudes that determine how much each player invests in equilibrium.

First we consider a contest with two homogeneous players with $\alpha_1 = \alpha_2 = \alpha$. By substituting $n = 2$ in (11), we find that player i's investment in equilibrium is given by

$$x_i^e = \begin{cases} \frac{(e^{\alpha R}-1)}{2\alpha(e^{\alpha R}+1)} & \text{if } \alpha \neq 0 \\ \frac{R}{4} & \text{if } \alpha = 0 \end{cases} \tag{13}$$

for $i = 1, 2$. It follows immediately that x_i^e is strictly decreasing in $|\alpha|$. To understand the intuition of this result, we solve player i's optimization problem using the objective function in (4) without the specific functional form. We find that each player, in symmetric equilibrium, chooses x_i^e that solves

$$\frac{1}{4x_i^e}(u(I - x_i^e + R) - u(I - x_i^e)) = \frac{1}{2}(u'(I - x_i^e + R) + u'(I - x_i^e)) \tag{14}$$

which can be written as

$$x_i^e = \frac{R}{4}\left[\frac{\frac{1}{R}\int\limits_{I-x_i^e}^{I-x_i^e+R} u'(y)dy}{\frac{1}{2}(u'(I-x_i^e+R)+u'(I-x_i^e))}\right]. \tag{15}$$

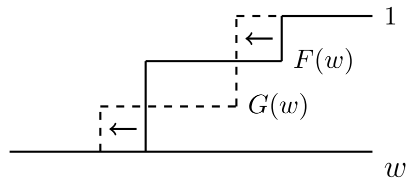
Applying second-order Taylor approximation of u' around $\bar{w} = I - x_i^e + \frac{R}{2}$, the midpoint of the relevant wealth domain $[I - x_i^e, I - x_i^e + R]$, in (15) yields

$$x_i^e \approx \frac{R}{4}\left[\frac{1 + D(\bar{w})\frac{R^2}{24}}{1 + D(\bar{w})\frac{R^2}{8}}\right] \tag{16}$$

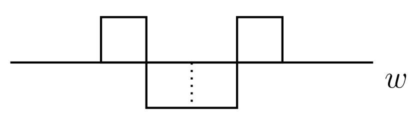
where $D(\bar{w}) = \frac{u'''(\bar{w})}{u'(\bar{w})}$ is Modica and Scarsini's (2005) local measure of downside risk aversion evaluated at $\bar{w}$.[7] Given (16), we find that x_i^e is decreasing in $D(\bar{w})$. For the functional form in Assumption 1, $D(w) = \alpha^2$ for all w. Therefore as $|\alpha|$ increases, both players become more downside-risk-averse and optimally reduce their investment in equilibrium.

Why does an increase in rent-seeking investment seem to be an adverse move for downside-risk-averse players? To answer this question we refer to Menezes, Geiss, and Tressler's (1980) definition of an increase in downside risk: given common support (a, b), $G(w)$ has more downside risk than $F(w)$ if and only if (i) $E_G = E_F$; (ii) $\int_a^b \int_a^y (G(w) - F(w))dw\, dy = 0$; (iii) $\int_a^z \int_a^y (G(w) - F(w))dw\, dy \geq 0$ for all z in $[a, b]$ and > 0 for some z in (a, b). Let F and G be a player's distribution functions of his final wealth before and after an increase in rent-seeking investment respectively. In Figure 2 (a) we assume that the increase in rent-seeking investment is small and it occurs around a symmetric equilibrium where the probability of winning for each player is 0.5. Furthermore, there is no change in expected final wealth so there is no first-moment benefit from the increase in investment. The difference in the two distributions is drawn in Figure 2 (b). Note that $G(w) - F(w)$ is consisted of a mean-preserving spread and a mean-preserving contraction of equal size on the left and the right of the vertical dashed line. In addition, $\int_a^y (G(w) - F(w))dw$ and $\int_a^z \int_a^y (G(w) - F(w))dw\, dy$ are drawn in Figure 2 (c) and (d) respectively.

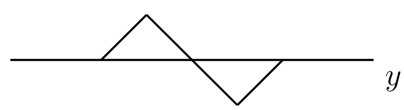
[7]See discussions of downside-risk aversion in Jindapon and Neilson (2007) and Crainich and Eeckhoudt (2008).



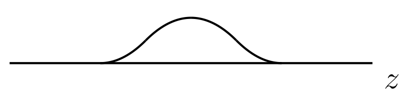
(a) $F(w)$ and $G(w)$



(b) $G(w) - F(w)$



(c) $\int_a^y (G(w) - F(w))dw$



(d) $\int_a^z \int_a^y (G(w) - F(w))dw\, dy$

Figure 2: An increase in rent-seeking investment increases downside risk

We can see from Figure 2 that the increase in rent-seeking investment increases downside risk according to Menezes, Geiss, and Tressler's (1980) definition and hence skews the distribution to the left. While the third moment of the distribution has changed, there is no second-moment effect.[8] Thus each player's measure of risk aversion in the sense of Arrow-Pratt does not play a role in (16). This is consistent with (13) where we find that each player's equilibrium investment depends on the magnitude of α, not its sign. Given the functional form in Assumption 1, player i is downside-risk-averse whenever $\alpha \neq 0$. Otherwise he is downside-risk-neutral. Thus, in standard Tullock contests with two homogeneous players of type α, each player's equilibrium investment is largest when $\alpha = 0$ and decreases as $|\alpha|$ increases.

Next, we analyze a standard Tullock contest with two heterogeneous players. Given $f_i(x_i) = x_i$ for $i = 1, 2$, player i's share function can be written as (10) and it follows that $\frac{\partial^2 s_i(Y)}{\partial Y^2} \gtreqless 0$ if and only if $\alpha_i \gtreqless 0$. That is, $s_i(Y)$ is strictly concave (convex) when player i is risk-averse (risk-loving). See examples in Figure 3. We also find that within the set of possible values of Y^e, i.e., the interval $(0, \min\{\kappa_1, \kappa_2\})$, if $\alpha_1 \neq \alpha_2$, the two share functions cross only once. In the following result, we call the value of Y where $s_1(Y)$ and $s_2(Y)$ cross ψ. Knowing ψ will help us determine who is the favorite and who is the underdog in equilibrium based on α_1 and α_2.

Lemma 2. *Consider share functions $s_i(Y)$ given $f_i(x_i) = x_i$ for $i = 1, 2$ and $Y \in (0, \min\{\kappa_1, \kappa_2\})$. Define ψ as Y such that $s_1(Y) = s_2(Y)$. If $\alpha_1 \neq \alpha_2$, then ψ uniquely exists and has the following properties:*

1. *If $\alpha_1 = -\alpha_2$, then $s_1(\psi) = s_2(\psi) = \frac{1}{2}$.*

2. *If $|\alpha_1| < |\alpha_2|$ and*

[8] It is possible that an increase in rent-seeking investment also affects the first- and second- moments of the distribution. A downside-risk-averse player will invest more if the benefits from the first- and second-order effects outweigh the increase in downside risk. See Chiu (2005) for trade-off analysis between (second-order) risk and downside risk.

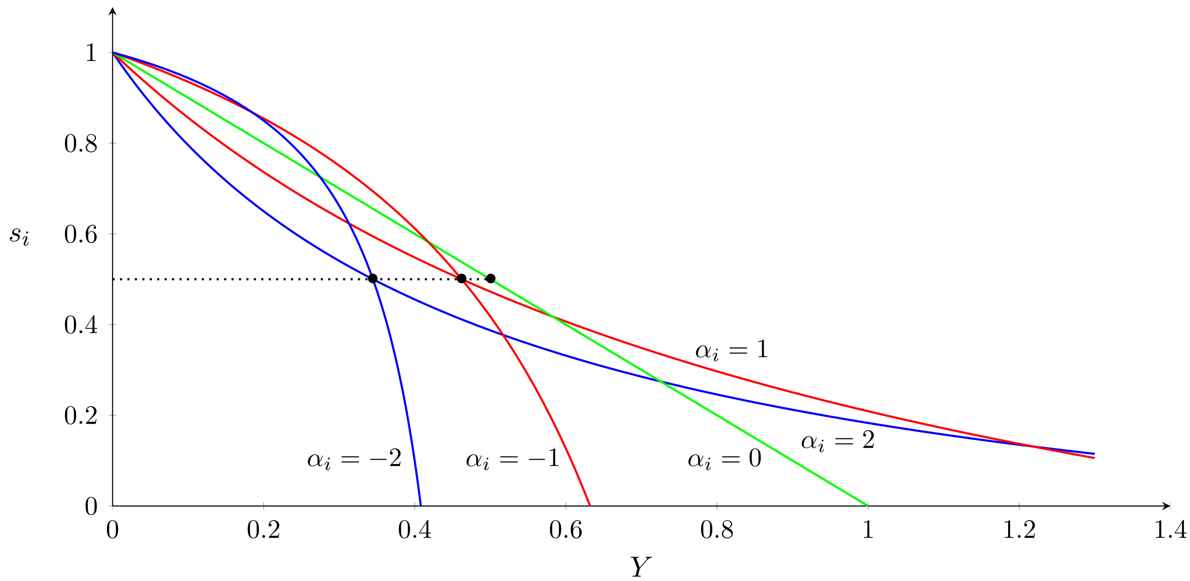


Figure 3: Share functions in standard Tullock contest given $R = 1$ and $\alpha_i = -2, -1, 0, 1, 2$

(a) *$\alpha_2 < 0$ (i.e., $0 \geq \alpha_1 > \alpha_2$ or $-\alpha_2 > \alpha_1 \geq 0 > \alpha_2$), then $s_1(\psi) = s_2(\psi) > \frac{1}{2}$, $s_1(Y) < s_2(Y)$ for $Y < \psi$, and $s_1(Y) > s_2(Y)$ for $Y > \psi$.*

(b) *$\alpha_2 > 0$ (i.e., $0 \leq \alpha_1 < \alpha_2$ or $-\alpha_2 < \alpha_1 \leq 0 < \alpha_2$), then $s_1(\psi) = s_2(\psi) < \frac{1}{2}$, $s_1(Y) > s_2(Y)$ for $Y < \psi$, and $s_1(Y) < s_2(Y)$ for $Y > \psi$.*

Proof. See Appendix B. □

Whenever $\alpha_1 = -\alpha_2$, we find that both players' share functions cross at $s_1(Y) = s_2(Y) = \frac{1}{2}$ and it follows that the corresponding value of Y constitutes an equilibrium. Thus, as long as $|\alpha_1| = |\alpha_2|$, both players invest the same amount in equilibrium and there is no favorite or underdog in the contest.

Proposition 5. *Consider a simultaneous contest with two players where Assumption 1 holds and $f_i(x_i) = x_i$ for $i = 1, 2$.*

1. *If $|\alpha_1| = |\alpha_2|$, then $x_1^e = x_2^e$.*

2. *If $|\alpha_1| < |\alpha_2|$, then $x_1^e > x_2^e$.*

Proof. Part 1 follows immediately from Lemma 2 part 1. For part 2, consider 2 cases: (a) $\alpha_2 < 0$ and (b) $\alpha_2 > 0$. In case (a), Lemma 2 part 2 (a) implies (i) $Y^e > \psi$ because $s_1(\psi)+s_2(\psi) > 1$ and (ii) $s_1(Y^e) > s_2(Y^e)$ because $s_1(Y) > s_2(Y)$ given $Y > \psi$. In case (b), Lemma 2 part 2 (b) implies (i) $Y^e < \psi$ because $s_1(\psi) + s_2(\psi) < 1$ and (ii) $s_1(Y^e) > s_2(Y^e)$ because $s_1(Y) > s_2(Y)$ given $Y < \psi$. In either case, we have $x_1^e > x_2^e$.

□

4 Sequential Contests

In this section we analyze two-player sequential contests where we let player 1 choose her rent-seeking investment first and allow player 2 to observe player 1's investment before choosing his investment. We show that a subgame-perfect equilibrium exists if player 1 is not too risk-averse and that both order of moves and player types determine the favorite and underdog in a sequential contest.

Since the share function approach is not compatible with sequential contests, we solve the problem backward as conventionally done in a Stackelberg game. Specifically, we derive player 2's best response function and then solve player 1's optimization problem given player 2's best response. For $i = 1, 2$ we define player i's expected utility given the other player's investment, denoted by x_{-i}, as

$$V_i(x_1, x_2) = \left(\frac{x_i}{x_1 + x_2}\right) u_i(I_i + R - x_i) + \left(\frac{x_{-i}}{x_1 + x_2}\right) u_i(I_i - x_i) \tag{17}$$

where u_i has the functional form given in Assumption 1.

In period 2, taking x_1 as given, we derive the first-order condition for player 2's interior solution by setting $\Phi_2(x_1, x_2) := \frac{\partial V_2}{\partial x_2} = 0$. Given $\alpha_2 \neq 0$, we have

$$\Phi_2(x_1, x_2) = \frac{\alpha_2 e^{\alpha_2(I_2 - x_2)}}{(x_1 + x_2)^2} \left[x_1(e^{\alpha_2 R} - 1) - \alpha_2 x_2(x_1 + x_2)(e^{\alpha_2 R} - 1) - \alpha_2(x_1 + x_2)^2\right] = 0. \tag{18}$$

The second-order condition, $\frac{\partial^2 V_2}{\partial x_2^2}<0$, is satisfied as derived in the proof of Lemma 1. By setting the sum in the brackets of (18) equal to zero, we can derive player 2's best response from the following quadratic function:

$$e^{\alpha_2 R}x_2^2 + (e^{\alpha_2 R}+1)x_1x_2 + (x_1^2 - \kappa_2 x_1) = 0 \tag{19}$$

where $\kappa_2 = \frac{e^{\alpha_2 R}-1}{\alpha_2}$. Thus, player 2's best-response function is the plausible solution of (19) which can be written as

$$x_2^*(x_1) = \begin{cases} \frac{-(e^{\alpha_2 R}+1)x_1 + \sqrt{(e^{\alpha_2 R}-1)^2x_1^2 + 4\kappa_2 e^{\alpha_2 R}x_1}}{2e^{\alpha_2 R}} & \text{if } x_1 < \kappa_2 \\ 0 & \text{if } x_1 \geq \kappa_2. \end{cases} \tag{20}$$

For the risk-neutral case, we can derive player 2's best response function and find it to be a special case of (20) with $\alpha_2 = 0$. Thus, we can say that (20) is player 2's best response function for any α_2.

Remark 4. *Player 2's best-response function $x_2^*(x_1)$ given in (20) has the following properties:*

1. *There exists a unique value of $\mu \in (0, \kappa_2)$ such that the best response function is strictly increasing in x_1 for $x_1 \in (0, \mu)$ and strictly decreasing in x_1 for $x_1 \in (\mu, \kappa_2)$.*
2. *There exists a unique value of $\phi \in (0, \kappa_2)$ such that $x_2^*(\phi) = \phi$. Moreover, $\phi \gtreqless \mu$ if and only if $\alpha_2 \lesseqgtr 0$.*

Proof. For $x_1 < \kappa_2$, the slope of player 2's best-response function is given by

$$\frac{dx_2^*}{dx_1} = \frac{1}{2e^{\alpha_2 R}}\left[\frac{(e^{\alpha_2 R}-1)^2x_1 + 2\kappa_2 e^{\alpha_2 R}}{\sqrt{(e^{\alpha_2 R}-1)^2x_1^2 + 4\kappa_2 e^{\alpha_2 R}x_1}} - (e^{\alpha_2 R}+1)\right]. \tag{21}$$

The value inside the brackets of (21) is zero when

$$\alpha_2^2\kappa_2 x_1^2 + 4e^{\alpha_2 R}x_1 - \kappa_2 e^{\alpha_2 R} = 0. \tag{22}$$

Thus, the plausible root of the above quadratic function can be written as

$$\mu = \begin{cases} \frac{-2e^{\alpha_2 R} + \sqrt{4e^{2\alpha_2 R} + \alpha_2^2\kappa_2^2 e^{\alpha_2 R}}}{\alpha_2^2\kappa_2} & \text{if } \alpha_2 \neq 0 \\ \frac{R}{4} & \text{if } \alpha_2 = 0. \end{cases}$$

Setting $x_2^*(x_1) = x_1$ yields

$$\phi = \begin{cases} \frac{\kappa_2}{2(e^{\alpha_2 R}+1)} & \text{if } \alpha_2 \neq 0 \\ \frac{R}{4} & \text{if } \alpha_2 = 0. \end{cases} \tag{23}$$

It follows that $\phi \gtreqless \mu$ if and only if $\alpha_2 \lesseqgtr 0$. □

The above remark suggests that player 2's best response function is a hump-shaped curve and μ is the value of x_1 that induces the highest investment by player 2. If we plot x_2^* on x_1, we will find that the peak of the best-response function is located to the left (right) of the 45-degree line if and only if player 2 is risk-averse (risk-loving). See for example where we assume $\alpha_2 < 0$ in Figure 4 below. This result will help us identify a subgame-perfect equilibrium for a sequential contest.

Anticipating player 2's best response in period 2, player 1 chooses her investment x_1 to maximize $V_1(x_1, x_2^*(x_1))$ in period 1. Note that player 1 will never choose $x_1 > \kappa_2$ since her investment of κ_2 is large enough to keep player 2 from participating in the contest. For an interior solution, player 1's first-order condition $\Phi_1(x_1) := \frac{dV_1}{dx_1} = 0$ given $\alpha_1 \neq 0$ can be written as

$$\Phi_1(x_1) = -\alpha_1^2[p(x_1)e^{\alpha_1(I_1+R-x_1)} + (1-p(x_1))e^{\alpha_1(I_1-x_1)}] + \alpha_1[e^{\alpha_1(I_1+R-x_1)} - e^{\alpha_1(I_1-x_1)}]p'(x_1) = 0 \tag{24}$$

where $p(x_1) := \frac{x_1}{x_1 + x_2^*(x_1)}$. In the proof of Proposition 6, we show that $p'(x_1) > 0$ and $p''(x_1) < 0$ for all $x_1 \in (0, \kappa_2)$. The second-order condition is satisfied if

$$\begin{aligned} \frac{d\Phi_1(x_1)}{dx_1} &= \alpha_1^3[p(x_1)e^{\alpha_1(I_1+R-x_1)} + (1-p(x_1))e^{\alpha_1(I_1-x_1)}] \\ &\quad - 2\alpha_1^2[e^{\alpha_1(I_1+R-x_1)} - e^{\alpha_1(I_1-x_1)}]p'(x_1) + \alpha_1[e^{\alpha_1(I_1+R-x_1)} - e^{\alpha_1(I_1-x_1)}]p''(x_1) \\ &= -\alpha_1\Phi_1(x_1) + \alpha_1[e^{\alpha_1(I_1+R-x_1)} - e^{\alpha_1(I_1-x_1)}][p''(x_1) - \alpha_1 p'(x_1)] \end{aligned} \tag{25}$$

is negative. Since $\Phi_1(x_1) = 0$ and $\alpha_1[e^{\alpha_1(I_1+R-x_1)} - e^{\alpha_1(I_1-x_1)}] > 0$, it follows from (25) that player 1's second-order condition is satisfied if $p''(x_1) - \alpha_1 p'(x_1) < 0$, that is, if $\alpha_1 > \hat{\alpha}$ where $\hat{\alpha} := \frac{p''(x_1)}{p'(x_1)} < 0$. If $\alpha_1 = 0$, the first-order condition for player 1 is given by

$$Rp'(x_1) - 1 = 0 \tag{26}$$

and the second-order condition is satisfied because $p''(x_1) < 0$. Thus we can say that player 1's problem has a unique solution if α_1 is greater than a threshold. Since this threshold is negative, we can say that a subgame-perfect equilibrium of the game uniquely exists if player 1 is not too risk-averse.

Proposition 6. *Consider a sequential contest with two players where Assumption 1 holds and $f_i(x_i) = x_i$ for $i = 1, 2$. There exists a threshold $\hat{\alpha} < 0$ such that a subgame-perfect equilibrium uniquely exists whenever $\alpha_1 > \hat{\alpha}$.*

Proof. See Appendix C. □

In Proposition 6, we show that a subgame-perfect equilibrium exists for some α_1 and any α_2. Specifically, α_1 must be larger than a threshold which has a negative value. It is well known in the literature that such an equilibrium exists if the two players are risk-neutral, i.e., $\alpha_1 = \alpha_2 = 0$. Dixit (1987) solves the standard Tullock contest with two risk-neutral players and show that outcomes in simultaneous and sequential contests are identical. There is neither first-mover advantage nor disadvantage so there is no incentive for preemptive investment. In the rest of this section, we show that Dixit's theoretical prediction will not hold if at least one of the players is not risk-neutral. Following Dixit (1987), as player 1 chooses x_1 to maximize $V_1(x_1, x_2^*(x_1))$, we have

$$\frac{dV_1}{dx_1} = \frac{\partial V_1}{\partial x_1} + \frac{\partial V_1}{\partial x_2} \cdot \frac{dx_2^*}{dx_1}. \tag{27}$$

In equilibrium of simultaneous contest where $x_1 = x_1^e$, we know that $\frac{\partial V_1}{\partial x_1}|_{x_1 = x_1^e} = 0$. Since $\frac{\partial V_1}{\partial x_2} < 0$ for any x_1, player 1 will increase (decrease) her investment in the sequential contest from x_1^e if $\frac{dx_2^*}{dx_1}|_{x_1 = x_1^e}$ is negative (positive). Since $\frac{dx_2^*}{dx_1} = -\frac{\partial \Phi_2 / \partial x_1}{\partial \Phi_2 / \partial x_2}$, the slope of player 2's best-response function and $\frac{\partial \Phi_2}{\partial x_1}$ have the same sign. Given $\Phi_2(x_1, x_2)$ in (18), we have

$$\frac{\partial \Phi_2}{\partial x_1} = \left[\frac{\alpha_2 (e^{\alpha_2 (I_2 + R - x_2)} - e^{\alpha_2 (I_2 - x_2)})}{(x_1 + x_2)^3} \right] [\alpha_2 x_2 (x_1 + x_2) + (x_2 - x_1)]. \tag{28}$$

Since the first bracketed term is positive, the sign of the right-hand side of (28) depends on the second bracketed term. That is, at a Nash equilibrium in simultaneous contest,

$\frac{dx_2^*}{dx_1} \gtreqless 0$ if and only if

$$\alpha_2 x_2^e (x_1^e + x_2^e) + (x_2^e - x_1^e) \gtreqless 0. \quad (29)$$

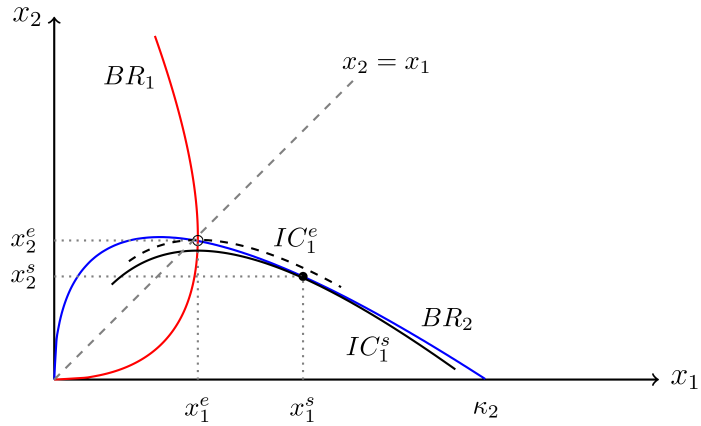
It follows from Proposition 5 that $x_2^e - x_1^e \gtreqless 0$ whenever $|\alpha_1| \gtreqless |\alpha_2|$. Hence, we can state the following results where the favorite and underdog in sequential contests can be determined from each player's type.

Proposition 7. *Consider a sequential contest with two players where Assumption 1 holds and $f_i(x_i) = x_i$ for $i = 1, 2$. Assume $\alpha_1 > \hat{\alpha}$ so that a subgame-perfect equilibrium exists. Let x_i^s denote player i's investment in such an equilibrium.*

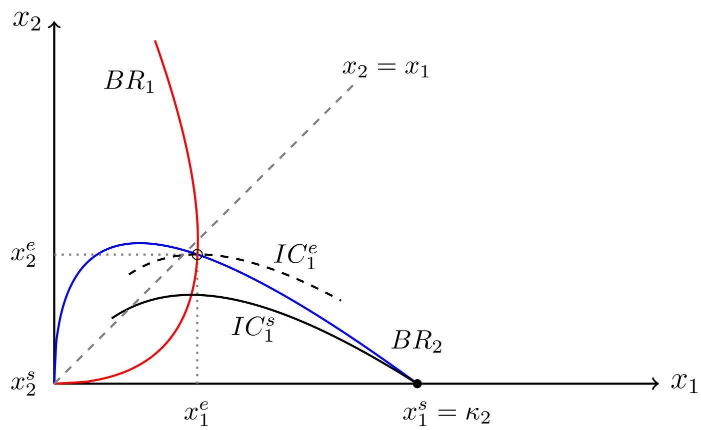
1. *If $\alpha_2 < 0$ and $|\alpha_1| \leq |\alpha_2|$, then $x_1^s > x_2^s$.*
2. *If $\alpha_2 > 0$ and $|\alpha_1| \geq |\alpha_2|$, then $x_1^s < x_2^s$.*
3. *If $\alpha_2 = 0$, then $x_1^s \leq x_2^s$.*

Proof. All these results follow immediately from (27), (29), Remark 4, and Proposition 5. □

The first result of Proposition 7 can be illustrated in panel (a) of Figure 4 where we let $R = 1$, $\alpha_1 = 0$, and $\alpha_2 = -0.6$. Since $|\alpha_1| < |\alpha_2|$, the Nash equilibrium (x_1^e, x_2^e) of simultaneous contest lies below the 45 degree line as predicted by Proposition 5. In a sequential contest, player 2's best response depicted by BR_2 becomes a constraint for player 1's optimization problem. Specifically, player 1 can maximize her expected utility $V_1(x_1, x_2)$ by choosing a point on BR_2 that is on the lowest indifference curve since a lower indifference curve corresponds to a higher value of $V_1(x_1, x_2)$. The dashed curve IC_1^e represents player 1's indifference curve corresponding to the simultaneous contest equilibrium. Since the slope of this indifference curve is zero at (x_1^e, x_2^e) while the slope of BR_2 at the same point is negative, player 1 can increase her expected utility by moving along BR_2 to the right. Player 1 will optimally choose x_1^s as her investment in period 1 and player 2 will respond by investing x_2^s in period 2. Note that the best attainable indifference curve is labeled IC_1^s and (x_1^s, x_2^s) is the tangent point of IC_1^s and BR_2.



(a) $\alpha_1 = 0$ and $\alpha_2 = -0.6$



(b) $\alpha_1 = 0$ and $\alpha_2 = -1$

Figure 4: Subgame-perfect equilibria in which player 2 participates (in panel (a)) and does not participate (in panel (b))

An interior solution in a sequential contest does not always exist. As player 2 becomes more risk-averse, α_2 decreases and so does κ_2. When α_2 is low enough, it will be impossible to find a tangent point on BR_2 such that $x_1 \in [0, \kappa_2]$. For example, consider the case where $\alpha_1 = 0$ and $\alpha_2 = -1$ as illustrated in panel (b) of Figure 4. Using (x_1^e, x_2^e) as the starting point, player 1 can increase her expected utility by moving her investment rightward along BR_2 all the way to κ_2 yet the slopes of her indifference curve and player 2's best-response function are not equalized. Choosing $x_1 = \kappa_2$ stops player 2 from competing in the rent-seeking contest and maximizes player 1's expected utility. At this corner solution, player 1 wins the rent with probability one and player 2 will not participate in the contest. How low does α_2 have to be for player 1 to be able to drive player 2 out of a contest? This threshold value depends on both R and α_1. In proposition 8 below, we derive the maximum type of player 2 that yields a corner solution in equilibrium when player 1 is risk-neutral.

Proposition 8. *Consider a sequential contest with two players where Assumption 1 holds and $f_i(x_i) = x_i$ for $i = 1, 2$. If $\alpha_1 = 0$ and $\alpha_2 \leq -\frac{4}{5R}$, then $x_1^s = \kappa_2$ and $x_2^s = 0$.*

Proof. Given $\alpha_1 = 0$, player 1's first-order condition in (26) suggests that a corner solution where $x_1^s = \kappa_2$ will be obtained if

$$Rp'(\kappa_2) - 1 \geq 0. \tag{30}$$

This condition is equivalent to

$$e^{2\alpha_2 R} - \alpha_2 R - 1 \geq 0 \tag{31}$$

which is implied by $\alpha_2 R \leq -\frac{4}{5}$. □

Proposition 8 suggests that if $\alpha_2 \leq -\frac{4}{5R}$, player 1 only needs to invest κ_2 to secure the rent. In particular, player 1 will win the rent with probability one and her profit from the contest will be $R - \kappa_2 > 0$.[9] Since κ_2 is strictly increasing in α_2, we can say that as player 2 becomes more risk-averse, the amount of rent-seeking investment in period 1 needed to keep player 2 out of the contest will be smaller. Thus, player 1's profit rises as player 2 becomes more risk-averse.

Our results in this section provide new insights on first-mover advantage and disadvantage in a sequential contest. Contrasting Dixit's (1987) seminal result, even when the two players are identical and no one has an advantage on rent-seeking technology, we show that there is an advantage of going first or last depending on the players' risk attitudes. Consider a two-player sequential contest with $\alpha_1 = \alpha_2 = \alpha$ which is a special case of parts 1 and 2 of Proposition 7. We can say that if α is negative, i.e., both players are risk-averse, then the first player will be the favorite to win the contest. On the other hand, if α is positive, then the second player will be the favorite to win the contest.

[9] We show in Remark 3 that $\kappa_2 < R$ if and only if $\alpha_2 < 0$.

5 Conclusion

In this paper, we analyze a class of rent-seeking contests in which players are heterogeneous in both risk preferences and production technology. We find that there exists a unique Nash equilibrium whenever each player's absolute measure of risk aversion is constant and marginal productivity is non-increasing in rent-seeking investment. If production functions are linear and the total number of risk-loving players is large enough, all risk-averse players will be crowded out from the contest and rent over-dissipation occurs in equilibrium. On the other hand, if the total number of risk-loving players is less than four, rent over-dissipation will never occur.

We also analyze the effects of players' risk attitudes on rent-seeking investment in a standard Tullock contest with two players. If both players move simultaneously the player who is less downside-risk-averse will be the favorite to win the rent. If the players move sequentially, the first mover will be the favorite (underdog) whenever the first mover is less (more) downside-risk-averse and the second mover is risk-averse (risk-loving). As a special case of this result, even when both players have the same risk preference, the first mover will be the favorite (underdog) if both players are risk-averse (risk-loving).

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Appendix

A Proof of Lemma 1

We write player i's optimization problem, for $\alpha_i \neq 0$, as

$$\max_{y_i} U_i(y_i|Y_{-i}) = \frac{y_i}{y_i + Y_{-i}}\alpha_i e^{\alpha_i[I_i+R-g_i(y_i)]} + \frac{Y_{-i}}{y_i + Y_{-i}}\alpha_i e^{\alpha_i[I_i-g_i(y_i)]}. \tag{32}$$

The first-order condition is given by

$$\frac{dU_i}{dy_i} = \frac{1}{Y^2}\alpha_i e^{\alpha_i[I_i-g_i(y_i)]}[(Y-y_i)(e^{\alpha_i R}-1) - \alpha_i g_i'(y_i)y_i Y(e^{\alpha_i R}-1) - \alpha_i g_i'(y_i)Y^2] = 0. \tag{33}$$

Let $\hat{y}_i$ be a critical value of y that satisfies (33). We find that $\frac{d^2U_i}{dy_i^2} < 0$ whenever $y_i = \hat{y}_i$ so we know that $\hat{y}_i$ is unique for a given Y_{-i}. Since $u_i'(w) > 0$, we know that $\hat{x}_i := g_i(\hat{y}_i) < R$ so that first-order stochastic dominance is not violated. Given $R \leq I_i$, we can assure that each player is not constrained by his budget. Thus the only case for corner solution is when $\hat{y}_i < 0$ and player i's best response, y_i^*, is 0. It follows that , given $y_i = 0$, $\frac{dU_i}{dy_i} \leq 0$ if and only if $Y_{-i} \geq \kappa_i$ where

$$\kappa_i = \frac{e^{\alpha_i R}-1}{\alpha_i g_i'(0)}. \tag{34}$$

Therefore, $y_i^* = \hat{y}_i$ if $Y_{-i} < \kappa_i$, otherwise $y_i^* = 0$. Given the definition of share function $s_i(Y) := \frac{y_i^*}{(y_i^*+Y_{-i})}$ and the first-order condition in (33), we find that, if $Y_{-i} < \kappa_i$, player i chooses y_i^* such that $s_i(Y)$ satisfies

$$\frac{(1-s_i(Y))(e^{\alpha_i R}-1)}{[s_i(Y)(e^{\alpha_i R}-1)+1]\alpha_i} - g_i'(s_i(Y)Y)Y = 0. \tag{35}$$

Define $\lambda(s_i)$ as the left-hand side of (35). We find that $\lambda(1) < 0$ and $\lambda(0) > 0$ whenever $Y < \kappa_i$. Since $\lambda'(s_i) < 0$, there exists a unique value of $s_i > 0$ such that (35) holds if $Y < \kappa_i$. If $Y_{-i} \geq \kappa_i$, player i chooses $y_i^* = 0$. It follows that $Y = Y_{-i}$ and hence $s_i(Y) = 0$ if $Y \geq \kappa_i$.

When $\alpha_i = 0$, the optimization problem and the corresponding first-order condition are much simpler. Following similar arguments, we find that player i chooses y_i^* such that $s_i(Y)$ satisfies

$$(1-s_i(Y))R - g_i'(s_i(Y)Y)Y = 0 \tag{36}$$

if $Y < \frac{R}{g_i'(0)}$, otherwise $s_i(Y) = 0$.

B Proof of Lemma 2

Part 1: Let Y_i be $Y \in (0, \kappa_i)$ such that $s_i(Y) = \frac{1}{2}$ for $i = 1, 2$. Given (9) and (10), we find $Y_i = \frac{e^{\alpha_i R} - 1}{\alpha_i (e^{\alpha_i R} + 1)}$ when $\alpha_i \neq 0$. If $\alpha_1 = -\alpha_2$, then we have

$$Y_1 = \frac{e^{-\alpha_2 R} - 1}{-\alpha_2 (e^{-\alpha_2 R} + 1)} = \frac{1 - e^{\alpha_2 R}}{-\alpha_2 (1 + e^{\alpha_2 R})} = \frac{e^{\alpha_2 R} - 1}{\alpha_2 (e^{\alpha_2 R} + 1)} = Y_2. \tag{37}$$

Thus, there exists ψ such that $s_1(\psi) = s_2(\psi) = \frac{1}{2}$ whenever $\alpha_1 = -\alpha_2$.

Part 2: Now we show that ψ uniquely exists for $\alpha_1 \neq \alpha_2$. Given $s_1(\psi) = s_2(\psi)$, then we can derive

$$\psi = \frac{\kappa_1 e^{\alpha_2 R} - \kappa_2 e^{\alpha_1 R}}{e^{\alpha_2 R} - e^{\alpha_1 R}}. \tag{38}$$

where κ_i is given by (9). Next we define

$$\omega_i = \begin{cases} \frac{\kappa_i - R}{\alpha_i R} & \text{if } \alpha_i \neq 0 \\ \frac{R}{2} & \text{if } \alpha_i = 0. \end{cases} \tag{39}$$

It follows that (i) $\omega_i \in (0, \kappa_i)$, (ii) $\frac{\partial \omega_i}{\partial \alpha_i} > 0$ for any α_i, and (iii) given $\alpha_1 < \alpha_2$, then $\omega_1 < \psi < \omega_2$. Considering the share function in (10), we find that

$$\frac{\partial s_i(Y)}{\partial \alpha_i} = \frac{(Y - \omega_i) Y R e^{\alpha_i R}}{[\kappa_i (1 + \alpha_i Y)]^2}. \tag{40}$$

Hence, (iv) $\frac{\partial s_i(Y)}{\partial \alpha_i} \gtreqless 0$ if and only if $Y \gtreqless \omega_i$. Properties (ii) and (iv) imply that there is a pivot point $(\omega_i, s_i(\omega_i))$ such that, as α_i marginally increases, (a) s_i rotates counterclockwise around the pivot point and (b) this pivot point travels rightward along s_i. Since $s_i(Y)$ is strictly decreasing in Y, both (a) and (b) suggest that, given $\alpha_1 < \alpha_2$, $s_1(Y)$ and $s_2(Y)$ cross only once at $Y = \psi \in (\omega_1, \omega_2)$, $s_1(Y) > s_2(Y)$ given $Y \in (0, \psi)$, and $s_1(Y) < s_2(Y)$ given $Y \in (\psi, \kappa_2)$.

Part 3: Here we show whether $s_i(\psi)$ is greater or less than $\frac{1}{2}$. Since $\omega_i < \kappa_i$, we have

$$s_i(\omega_i) = \frac{\kappa_i - \omega_i}{\kappa_i - \omega_i + \omega_i e^{\alpha_i R}} = \frac{\alpha_i R e^{\alpha_i R} - e^{\alpha_i R} + 1}{(e^{\alpha_i R} - 1)^2} \tag{41}$$

for $\alpha_i \neq 0$. It follows that

$$s_i(\omega_i) - \frac{1}{2} = \frac{2\alpha_i R e^{\alpha_i R} - e^{2\alpha_i R} + 1}{2(e^{\alpha_i R} - 1)^2} \gtreqless 0 \tag{42}$$

if and only if $\alpha_i \lesseqgtr 0$. Given $\alpha_i = 0$, we have $s_i(Y) = \frac{R-Y}{R}$ so that $s_i(\omega_i) = \frac{1}{2}$. Here we consider 4 cases:

Case 1: $0 \geq \alpha_1 > \alpha_2$. We have $s_1(\omega_1) \geq \frac{1}{2}$ and $s_2(\omega_2) > \frac{1}{2}$. Since $\omega_2 < \psi < \omega_1$, then $s_1(\psi) = s_2(\psi) > \frac{1}{2}$.

Case 2: $0 \leq \alpha_1 < \alpha_2$. We have $s_1(\omega_1) \leq \frac{1}{2}$ and $s_2(\omega_2) < \frac{1}{2}$. Since $\omega_1 < \psi < \omega_2$, then $s_1(\psi) = s_2(\psi) < \frac{1}{2}$.

For the following cases, we define ψ_{ij} as the value of Y such that $s_i(Y) = s_j(Y)$ for $i, j = 1, 2, 3$.

Case 3: $-\alpha_2 > \alpha_1 \geq 0 > \alpha_2$. If we let $\alpha_3 = -\alpha_2$, then we know that $s_3(\psi_{23}) = \frac{1}{2} > s_3(\psi_{13})$. Since s_3 is strictly decreasing, then $\psi_{23} < \psi_{13}$. Since $\alpha_3 > \alpha_1$, then $s_1(Y) > s_3(Y)$ for all $Y \in (0, \psi_{13})$. It follows that $\psi_{12} < \psi_{23}$ and $s_1(\psi_{12}) = s_2(\psi_{12}) > \frac{1}{2}$.

Case 4: $-\alpha_2 < \alpha_1 \leq 0 < \alpha_2$. If we let $\alpha_3 = -\alpha_2$, then we know that $s_3(\psi_{23}) = \frac{1}{2} < s_3(\psi_{13})$. Since s_3 is strictly decreasing, then $\psi_{23} > \psi_{13}$. Since $\alpha_3 < \alpha_1$, then $s_1(Y) > s_3(Y)$ for all $Y \in (\psi_{13}, \kappa_1)$. It follows that $\psi_{12} > \psi_{23}$ and $s_1(\psi_{12}) = s_2(\psi_{12}) < \frac{1}{2}$.

C Proof of Proposition 6

Given $x_2^*(x_1)$ in (20), we rewrite

$$p(x_1) = \frac{A}{\alpha_2 + \sqrt{\alpha_2^2 + \frac{2A}{x_1}}} \tag{43}$$

where $A := \frac{2\alpha_2 e^{\alpha_2 R}}{e^{\alpha_2 R} - 1} = \frac{2e^{\alpha_2 R}}{\kappa_2} > 0$ for all α_2. We find that

$$p'(x_1) = \frac{A^2}{x_1^2 \sqrt{\alpha_2^2 + \frac{2A}{x_1}} \left(\alpha_2 + \sqrt{\alpha_2^2 + \frac{2A}{x_1}}\right)^2} > 0. \tag{44}$$

Let $B := \alpha_2^2 + \frac{2A}{x_1} > 0$. The denominator in (44) can be written as C^2, where $C := \alpha_2 x_1 B^{\frac{1}{4}} + x_1 B^{\frac{3}{4}} > 0$. Thus, we have

$$p''(x_1) = -\frac{2A^2}{C^3} \frac{dC}{dx_1}. \tag{45}$$

We find that

$$\frac{dC}{dx_1} = (\alpha_2 + B^{\frac{1}{2}})B^{\frac{1}{4}} - \frac{A(\alpha_2 + 3B^{\frac{1}{2}})}{2B^{\frac{3}{4}}x_1}. \tag{46}$$

Thus $\frac{dC}{dx_1} > 0$ if and only if

$$D := \frac{(\alpha_2 + \sqrt{B})Bx_1}{(\alpha_2 + 3\sqrt{B})} - \frac{e^{\alpha_2 R}}{\kappa_2} > 0. \tag{47}$$

If $\alpha_2 \geq 0$, then (47) always holds. If $\alpha_2 < 0$, to show that (47) holds we need to restrict the possible values of x_1 to be in the interval $(0, \kappa_2)$, the set of all possible interior solutions for player 1. Note that player 1 will not choose $x_1 > \kappa_2$ because $x_1 = \kappa_2$ is large enough to drive player 2 out from the contest. We find that (i) $\frac{\partial D}{\partial R} < 0$ and (ii) $\lim_{R\to\infty} D = 0$ because $\lim_{R\to\infty} A = 0$ for all $\alpha_2 < 0$. These two properties of D imply that $D > 0$ for all $\alpha_2 < 0$. Thus we can conclude that $\frac{dC}{dx_1} > 0$ and $p''(x_1) < 0$ for all α_2. It follows that the second-order condition for player 1 always holds as long as $\alpha_1 > \hat{\alpha}$ where $\hat{\alpha} = \frac{p''(x_1)}{p'(x_1)}$.