Espionage and Disclosure of Cost Information in Cournot Duopoly *

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(preliminary draft, please do not circulate)

Abstract

This paper studies a firm's incentive to do espionage on its rival to learn about rival's private cost information and how does espionage and firms' ability to disclose private information affect their strategies and profits. Two firms compete in a Cournot market, each firm knows its own realized cost but Firm 1 is ignorant of Firm 2's realized cost but can do espionage to learn the cost. The result of espionage is a private noisy signal whose precision captures the intensity of espionage. Higher signal precision is associated with higher espionage cost. In equilibrium Firm 1 always does espionage and strictly benefits from espionage, irrespective of whether it is able to disclose its private information acquired or not. Firm 2, who is being spied upon, will benefit (suffer) from espionage when its own cost is lower (higher) than Firm 1's expectation before espionage. Therefore, espionage may be beneficial from the industry's prospective. Consumer surplus is also considered and under some realization of costs both the two firms and consumers benefit from espionage in expectation. When either Firm 1 or Firm 2 can disclose private information credibly and costlessly, in equilibrium there's full disclosure. Whether Firm 1 does more espionage when disclosure is possible depends on the shape of espionage cost function.

Keywords: espionage, credible disclosure, Cournot game, cost uncertainty, information acquisition

If you know the enemy and know yourself, you need not fear the result of a hundred battles. If you know yourself but not the enemy, for every victory gained you will also suffer a defeat. If you know neither the enemy nor yourself, you will succumb in every battle.

—Sun Tzu, *The Art of War* (490 B.C.)

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1 Introduction

Information gathering about rivals, or espionage, has long been a tacit routine in the business world. In 2002, Business Week reported that 90% of large companies have Competitive Intelligence (CI) stuff and that many large U.S. businesses spend more than $1 million annually on CI[1]. Moreover, 43% of American firms have suffered at least six incidents of industrial espionage[2]. Even many academic business programs incorporate seminars and courses on topics related to business intelligence into their curriculum. As the quotation from Sun Tzu seems to suggest, the more information a firm has about its rival, the more profit it can secure. Despite its importance and popularity, many fundamental questions about industrial espionage have not been addressed formally in the related literature. For example, strategically speaking, should a firm do espionage on its rival or not and if so what's the optimal level of espionage intensity? How does espionage affect firms business strategies in the industry? Can a firm that does espionage makes extra profit? Does a firm that is been spied upon necessarily suffers a loss in its profit? This paper attempts to shed some light on these questions by studying a Cournot duopoly with cost uncertainty where one firm can do espionage on the other firm to learn its private cost information.

To fix the idea, consider a Cournot market with two firms, Firm 1 and Firm 2. Each firm produces at a constant stochastic marginal cost, which captures the fluctuations of the input markets. Since the two firms' products are substitutes, they would typically use similar production technologies and inputs, so a natural assumption is that the two firms' marginal costs are positively correlated. In accordance with the business reality, each firm would know perfectly the realization of its own marginal cost before producing. I assume Firm 1 doesn't know the realized cost of Firm 2 but the realization for Firm 1's cost is commonly known. This assumption allows us to focus on one sided espionage, like almost all papers in the literature on espionage, for example Solan and Yariv (2004), Barrachina et al. (2014), Jelnov et al. (2015), and this assumption can be justified by considering Firm 2 is adopting a new technology or experimenting with a substitutable material in its production, but Firm 1 is producing with the commonly used technology or material, hence the realized cost of Firm 1 is also known by Firm 2. Espionage is model as the following:It is commonly known that Firm 1 has access to a costly espionage technology. The result of espionage is assumed to be an unbiased private noisy signal, whose precision captures the espionage intensity: With higher espionage intensity, the signal acquired is more accurate, yet Firm 1 will also induce higher expenditure. Typically the espionage cost function is strictly convex in signal accuracy or espionage intensity, since higher espionage intensity is usually associated with higher probability of law suits, fines, and increased expenditure on bribery, etc. Firm 2 is fully aware of the possibility that Firm 1 is doing espionage, but Firm 2 doesn't know the exact signal that is received by Firm 1.

Intuitively, with higher espionage intensity, Firm 1 would learn Firm 2's cost more precisely, hence its output strategy

[1]See page 75 of Nasheri (2005)
[2]See Barrachina et al. (2014)

would be more fine tuned, in other words, more close to the complete information case. This is indeed the case, as shown in this paper, with increasing espionage intensity, the equilibrium output functions converge to the equilibrium output function under complete information and with sufficiently low espionage intensity, the equilibrium converges to the no espionage case. In terms of equilibrium profit, Firm 1 always makes a higher profit in equilibrium when it can do espionage on Firm 2 to learn its cost. But whether Firm 2 benefits or suffers form espionage depends on the realization of both firms' costs: When Firm 1's cost is high and Firm 2's cost is relatively lower, Firm 2 makes higher profit in equilibrium when Firm 1 is spying; when Firm 1's cost is low and Firm 2's cost is relatively higher, the reverse holds. This result is very intuitive. With Firm 1 doing espionage, when Firm 2's cost is low, since the signal acquired by Firm 1 is unbiased, it's as if Firm 2 is conveying this low cost information to Firm 1 at the cost of Firm 1 and induces Firm 1 to produces less in expectation than under no espionage. Since the two firms' strategies are strategic complements, this allows Firm 2 to produce more and make higher profit in expectation. Therefore, from the industry's prospective, the presence of espionage may be desirable.

Besides the incentive to do espionage, another incentive we need to consider is firms' incentive to disclose their private information, suppose they could do so costlessly and credibly. Here the private information of Firm 1 is the noisy signal generated by espionage, while Firm 2's private information is its realized cost. Clearly disclosure would affect Firm 1's incentive to do espionage. For instance, in an extreme case if Firm 2 is willing to disclose its true cost to Firm 1, and such information is credible, then Firm 1 would know exactly the cost of Firm 2 and hence there's no any incentive to do espionage, so we are back to a stand Cournot game with complete information. This is indeed the equilibrium result when Firm 2 can do so. The rationale is essentially the same as that of "unrevealing" a la Milgrom (1981). Yet when Firm 1 is able to credibly and costlessly disclose its private signal to Firm 2, the effects on both firms' strategies, Firm 1's optimal choice of espionage intensity, and both firms' equilibrium profits are more subtle. Again due to "unrevealing" argument, in equilibrium we would expect full revealing, but still Firm 2's realized cost is its private information. As the paper shows, with Firm 1 fully discloses its signal, its output function becomes more sensitive to changes in its own cost and its signal, but Firm 2's equilibrium output function is less sensitive in its own cost. In terms of optimal espionage intensity, Firm 1 will still do espionage, but the optimal intensity in comparison with the one under no disclosure closely hinges on the shape of of espionage cost function: for sufficiently convex cost function, disclosing private cost would enhance Firm 1's incentive to do espionage; when the cost function is sufficiently less convex, the result reverses.

This paper adopts a linear-normal model that is common in the literature on information exchange in oligopoly: We assume the two firms's costs obey a joint normal distribution, and espionage would generate a noisy signal that is normally distributed around the true cost; the inverse demand function is assumed to be affine linear. This modeling structure allows me to show the uniqueness of equilibrium and derive explicitly the equilibrium strategy and profit of each firm.

1.1 Related Literature

This paper is directly related to several strands of literature. The literature on strategic analysis of espionage is sparse. Most of the papers along this strand of literature focus on espionage about the *action* taken by the rival. The common structure is that the first mover chooses an action that is unobservable by the second mover, who has the ability to do espionage (usually at a cost) on the first mover's action, and based on the information acquired determines an action. The first mover, when deciding on his action, has to internalize the rival's espionage behavior. Matsui (1989) studied a two-player repeated game in which there's a small probability that one or both players will be able to know rival's strategy at the beginning of the game. Here espionage is costless and the result is either perfect information or no information. In my model, I endogenize the choice of espionage precision and it is costly to do espionage. Solan and Yariv (2004) considered a general one stage competition game in which players can do costly espionage to acquire noisy information concerning rival's action. Like assumptions made in my paper, cost of espionage is strictly increasing in the accuracy of the information. They give full characterization of the set of distributions over equilibrium payoffs and show that when the game is non-degenerate, the information purchased is independent of the cost of espionage, and cost only determines whether the player should do espionage or not. In recent years, there are few applications of to real world problem, notably Barrachina et al. (2014), who analyzed the effect of industrial espionage on entry deterrence; Biran and Tauman (2008), Jelnov et al. (2015) focus on the role of intelligence in deterring weapons of mass destruction. The major difference of my paper and this strand of literature is that I focus on espionage on rival firm's *private information*, not directly on action. Since private information in my model, the Cournot competition stage in my model is a game of incomplete information.

Ho (2008) is one of the few papers that study espionage about *information*, but her focus was on the so called double-crossing phenomenon in espionage. Also some researches use the network approach to study espionage, for example Billand et al. (2010). My paper analyze quite different problem.

The second strand of literature is on information exchange in oligopoly. For this strand of literature, firms, *prior to* receiving private noisy information about their own uncertain parameters, decide simultaneously whether or not to commit to truthfully share their private information with other firms, for example, through a trade association. It turns out that the source of uncertainty (demand uncertainty or cost uncertainty), the source of information (independent values, private values, common value) and the form of competition (Cournot or Bertrand) will play a role in determining firms' equilibrium sharing strategy. A sample of literature includes Novshek and Sonnenschein (1982), Clarke (1983), Vives (1984), Gal-Or (1985), Gal-Or (1986), Li (1985), Amir et al. (2010). A brief summarization of the results in the following: (Except for Bertrand competition with cost uncertainty), unilaterally reveal all private information is a dominant strategy with independent values, with private values, or with common values and strategic complements; do not reveal any information is a dominant strategy with common value and strategic substitutes. See Section 8.3.1 of Vives (2001) for more details including welfare implications. Raith (1996) presented a general model that synthesises virtually all models along this strand of literature.

The workhorse model for almost all papers in this literature is the "linear-normal" model, in which the uncertain parameters and signals are assumed to obey normal distributions (Li (1985) is an exception, where he dropped the normality assumption but instead assumes the posterior expectations are affine linear) and the demand system is assumed to be linear. My paper adopts similar linear-normal model, but the difference is that I focus on firms' *ex post* (namely after each firm perfectly known their own realized stochastic marginal cost) incentive to do espionage and discloses information. Therefore, in this way, my paper is also related to the literature on information revelation, for instance Milgrom (1981), Matthews and Postlewaite (1985), Milgrom and Roberts (1986), Okuno-Fujiwara et al. (1990), Van Zandt and Vives (2007). The crucial assumptions are that players with private information must make a truthful report at no cost and the disclosing player's payoff is monotone is receivers' actions. They show all equilibria involve full disclosure and the receive's belief when receiving a report must be sceptical. My paper adopts similar framework, but in an environment with espionage and I focus on the influences of disclosure on firms' strategies and profits.

Finally my work is related to information acquisition in oligopoly. In Hwang (1993), Christen (2005), Jansen (2008), Ganuza and Jansen (2013) firms spend money to acquire private noisy information about their own uncertainty parameters prior to market competition, Jeitschko et al. (2015) considered a model with two-period competition and showed firms' signaling incentive would reduce their incentive to acquire information prior to competition. Contrary to this strand of literature, I focus on acquisition of rival's information and the incentive to disclose the information acquired.

The next section sets out the basic model. In section 3, we analyze the game in which both Firm 1 and Firm 2 cannot credibly and costlessly disclose their respective private information to the rival. Section 4 is devoted to the case in which only Firm 1 is able to disclose its private noisy signal acquired about Firm 2's cost as a result of espionage. Section 5 considers the other case where Firm 2 is able to disclose its private information.

2 Model Setup

Consider a Cournot market of homogeneous product with two risk neutral firms, named Firm 1 and Firm 2. Both firms are producing at constant marginal cost, $c_1,\ c_2$ for Firm 1 and Firm 2 respectively. Ex-ante, $c_1,\ c_2$ are random variables, assume they obey a joint normal distribution [3] with correlation coefficient $\rho \in [0, 1]$:[4]

$$\begin{pmatrix} c_1 \\ c_2 \end{pmatrix} \sim \mathcal{N}\left[\begin{pmatrix} \mu_c \\ \mu_c \end{pmatrix}, \begin{pmatrix} \sigma_c^2 & \rho\sigma_c^2 \\ \rho\sigma_c^2 & \sigma_c^2 \end{pmatrix}\right] \tag{1}$$

Define $\tau_c \equiv \frac{1}{\sigma_c^2}$ as the precision of the prior distribution.

Assume the realization of c_2 is Firm 2's private information while the realization of c_1 is common knowledge. [5]

The market inverse demand function is assumed to be linear and is given by the following:

$$p(q_1, q_2) = \alpha - \beta(q_1 + q_2), \quad \alpha, \beta > 0 \tag{2}$$

where $q_1, \ q_2$ denote the outputs for the two firms. $\alpha, \ \beta$ are constants.

Before market competition, it is common knowledge that Firm 1 can do espionage on Firm 2 to learn about its realized marginal cost c_2. Doing espionage is costly and we assume it will results in a noisy signal s about the realized c_2. Assume signal s is generated by

$$s = c_2 + \epsilon, \ \epsilon \sim \mathcal{N}(0, \sigma_\epsilon^2), \ \epsilon \text{ independent of } c_1, \ c_2 \tag{3}$$

Hence signal s is a conditionally unbiased estimator of c_2. Define $\tau_\epsilon \equiv \frac{1}{\sigma_\epsilon^2}$ as the precision of the signal, which essentially captures the intensity of Firm 1's espionage activity. The terminologies "signal precision" and "espionage intensity" are used interchangeably henceforth. Note that the signal s is Firm 1's private information, Firm 2 doesn't know the realized signal. However, once τ_ϵ has been determined by Firm 1, we assume Firm 2 is able to monitor the espionage intensity of the rival [6], hence the choice of τ_ϵ is common knowledge.

The espionage cost of Firm 1 is captured by a function $k(\tau_\epsilon)$, which only depends on the targeted precision τ_ϵ. $k(\cdot)$ is assumed to be continuously differentiable, strictly increasing in τ_ϵ, strictly convex, $k(0) = 0$ and that $\lim_{\tau_\epsilon \to 0} k'(\tau_\epsilon) = 0$, namely the marginal cost of espionage when signal precision approaches 0 is negligible.

[3] The normality assumption inevitably gives rise to the unpleasant fact that the support is unbounded.

[4] Since the two firms are producing homogeneous product, presumably they would use similar technology and input materials. Thereby it makes more sense to assume that the marginal costs are positively correlated. But algebraically our analysis still holds for $\rho \in [-1, 0)$.

[5] This assumption allows us to focus on one-sided espionage, like most of the papers on espionage literature (Solan and Yariv (2004), Barrachina et al. (2014), Jelnov et al. (2015) for example). This assumption is not unrealistic if we consider the possibility that Firm 2 is experimenting with some new technology (or some substitutable raw materials) in production while firm 1 is producing in the traditional methods. If that's the case, normally Firm 2 is more informed about realized c_2 than Firm 1, while realized c_1 is common knowledge since the traditional method of production is also available to Firm 2.

[6] In an environment with the possibility of espionage, it is very common that there's also *counter-espionage*, which refers to information gathered and activities conducted to protect against espionage. Monitoring rival's espionage activity is a particular aspect of counter-espionage. Hence this assumption is not unrealistic. Also, it could happen in reality that Firm 2 does *second-order espionage*, which means Firm 2 tries to figure out the information gathered by Firm 1 (namely signal s) and *third-order, fourth-order espionage* etc. One could model the result of espionage of each order as an additional unbiased noisy signal, but this only serves to adding additional noise and making the model much too complicated. In this paper, we do not consider such high-order espionage.

The objective of each firm is to maximize its expected net profit. In particular, Firm 1 has to endogenize the espionage cost when making decisions.

Given the information structure (1) and (3), it is immediate that the joint distribution of $c_2,\ c_1,\ s$ is normal. Specifically,

$$\begin{pmatrix} c_2 \\ c_1 \\ s \end{pmatrix} \sim \mathcal{N}\left[\begin{pmatrix} \mu_c \\ \mu_c \\ \mu_c \end{pmatrix}, \begin{pmatrix} \sigma_c^2 & \rho\sigma_c^2 & \sigma_c^2 \\ \rho\sigma_c^2 & \sigma_c^2 & \rho\sigma_c^2 \\ \sigma_c^2 & \rho\sigma_c^2 & \sigma_c^2 + \sigma_\epsilon^2 \end{pmatrix}\right]. \tag{4}$$

Given this joint distribution, using a well known projection theorem [7] of normal random variables (see for example DeGroot (1970), Chapter 5, Section 4), $E(c_2|c_1, s)$ can be calculated by the following:

$$E(c_2|c_1, s) = \mu_c + (\rho\sigma_c^2, \sigma_c^2)\begin{pmatrix} \sigma_c^2 & \rho\sigma_c^2 \\ \rho\sigma_c^2 & \sigma_c^2 + \sigma_\epsilon^2 \end{pmatrix}^{-1}\begin{pmatrix} c_1 - \mu_c \\ s - \mu_c \end{pmatrix}$$

For the convenience of future reference, we summarize this result as the following lemma.

Lemma 2.1 *Given the information structure (1) and (3), $E(c_2|c_1)$ is a weighted average of μ_c, c_1 and s:*

$$E(c_2|c_1, s) = \frac{(1-\rho)\tau_c}{(1-\rho^2)\tau_\epsilon + \tau_c}\mu_c + \frac{\rho\tau_c}{(1-\rho^2)\tau_\epsilon + \tau_c}c_1 + \frac{(1-\rho^2)\tau_\epsilon}{(1-\rho^2)\tau_\epsilon + \tau_c}s \tag{5}$$

It is worth noticing that this posterior expectation is affine linear in c_1 and s, which is a crucial property that gives rise to linear output functions. For details, see Proposition 1.

3 No Firm Can Credibly & Costlessly Disclose Private Information

Throughout this section, we maintain the assumption that both firms cannot credibly and costlessly disclose private information to the rival. For Firm 2, its private information is its realized constant marginal cost c_2, while the private information of Firm 1 is the signal s. This situation is in a sense the most realistic one.

[7] Let θ and s be random vectors such that $(\theta, s) \sim \mathcal{N}(\mu, \Sigma)$, where

$$\mu \equiv \begin{pmatrix} \mu_\theta \\ \mu_s \end{pmatrix}, \text{ and } \Sigma \equiv \begin{pmatrix} \Sigma_{\theta,\theta} & \Sigma_{\theta,s} \\ \Sigma_{s,\theta} & \Sigma_{s,s} \end{pmatrix},$$

are partitioned expectations and variance-covariance matrix. The conditional distribution of θ given s is normal:

$$(\theta|s) \sim \mathcal{N}(\mu_\theta + \Sigma_{\theta,s}\Sigma_{s,s}^{-1}(s - \mu_s), \Sigma_{\theta,\theta} - \Sigma_{\theta,s}\Sigma_{s,s}^{-1}\Sigma_{s,\theta}).$$

Consider Firm 2 who privately knows c_2. Normally credible disclosure of such private information is costly, since it may involve gathering and presenting to the rival solid evidences. Moreover, even when disclosure is costless, very often a firm may not be able to disclose credibly, for example, the firm may have signed confidential contracts with some suppliers or the firm is offered an under-the-table discount etc. The presence of these possibilities would render the assumption of credible & costless disclosure by Firm 2 invalid.

Similarly, from the prospective of Firm 1, since its activities may involve illegal way of gathering information, it cannot disclose such information to Firm 2, otherwise it may potentially face law suits or fines. Hence assuming the firms that does espionage cannot credibly & costlessly disclose private information is very often justifiable and realistic.

Timing. The timing of the game is following:

1. Nature draws c_1, c_2. The realization of c_1 becomes common knowledge, while realized c_2 is Firm 2's private information.

2. Firm 1, after learning c_1, chooses τ_ϵ and receives a signal s, espionage cost $k(\tau_\epsilon)$ is induced. Firm 2 cannot observed s, but Firm 2 is able to monitor the espionage intensity (namely τ_ϵ is also known by Firm 2).

3. Two firms play the Cournot game and profits are realized.

In the Cournot competition stage, the information available to Firm 1 is $c_1,\ s,\ \tau_\epsilon$. Firm 1's strategy is $(\tau_\epsilon, q_1(c_1, s))$. For Firm 2, its information in the Cournot stage is $c_1,\ c_2,\ \tau_\epsilon$, and its strategy is $q_2(c_1, c_2)$ [8].

Equilibrium. A strategy Profile $\left((\tau_\epsilon^*, q_1^*(c_1, s)), q_2^*(c_1, c_2)\right)$ is an equilibrium if

- Given τ_ϵ^*, $q_1^*(c_1, s)$, for each realized pair (c_1, c_2), $q_2^*(c_1, c_2)$ maximizes Firm 2's expected profit.
- Given τ_ϵ^*, $q_2^*(c_1, c_2)$, for each realized pair (c_1, s), $q_1^*(c_1, s)$ maximizes Firm 1's expected profit.
- Given $q_1^*(c_1, s),\ q_2^*(c_1, c_2)$, the choice of τ_ϵ^* maximizes Firm 1's expected net profit (subtract cost of espionage).

Given $c_1,\ s,\ \tau_\epsilon$ and given Firm 2 is adopting output strategy $q_2(c_1, c_2)$, Firm 1's problem in the Cournot stage is

$$\max_{q_1} \pi_1(c_1, s) \quad \Leftrightarrow \quad \max_{q_1} E^1_{c_2}\left\{[\alpha - \beta(q_1 + q_2(c_1, c_2)) - c_1]q_1 \middle| c_1, s\right\} \tag{6}$$

where $\pi_1(c_1, s)$ denotes Firm 1's expected profit given $\tau_\epsilon,\ c_1,\ s$ and that Firm 2 adopts strategy $q_2(c_1, c_2)$; "$E^1_{c_2}$" is the expectation operator that emphasizes it is Firm 1's expectation over c_2. The objective function is quadratic and concave in

[8] Since τ_ϵ has been determined prior to the Cournot competition stage and it is known by both firms, in the Cournot stage we regard τ_ϵ as a constant and suppress the dependence of $q_1,\ q_2$ on τ_ϵ

q_1, hence the first order condition is also sufficient.

$$q_1 = \frac{\alpha - c_1}{2\beta} - \frac{1}{2}E^1_{c_2}\left[q_2(c_1, c_2)|c_1, s\right] \tag{7}$$

Similarly given $c_1,\ c_2, \tau_\epsilon$ and that Firm 1 is adopting output strategy $q_1(c_1, s)$ Firm 2's problem is

$$\max_{q_2} E^2_s\left\{[\alpha - \beta(q_1(c_1, s) + q_2) - c_2]q_2\middle|c_1, c_2\right\} \tag{8}$$

from which we can derive the first order condition

$$q_2 = \frac{\alpha - c_2}{2\beta} - \frac{1}{2}E^2_s\left[q_1(c_1, s)|c_1, c_2\right] \tag{9}$$

For a given τ_ϵ, the two first order conditions (7) and (9) fully characterize the equilibrium output functions. It turns out that given the information structure and linear inverse demand function, there's a unique equilibrium pair of output functions $q_1^*(c_1, s),\ q_2^*(c_1, c_2)$ and in addition, they are all affine linear in their variables, as the following proposition illustrates:

Proposition 1 *Given τ_ϵ, for each realized $c_1,\ s,\ c_2$, there exists a unique pair of equilibrium output functions $q_1^*(c_1, s),\ q_2^*(c_1, c_2)$, which is given by the following*

$$\begin{aligned} q_1^*(c_1, s) &= \frac{\alpha}{3\beta} + \frac{4(1-\rho)\mu_c\tau_c}{3\beta[3(1-\rho^2)\tau_\epsilon + 4\tau_c]} \\ &\quad - \left[\frac{2}{3\beta} - \frac{4\rho\tau_c}{3\beta[3(1-\rho^2)\tau_\epsilon + 4\tau_c]}\right]c_1 + \frac{(1-\rho^2)\tau_\epsilon}{\beta[3(1-\rho^2)\tau_\epsilon + 4\tau_c]}s \end{aligned} \tag{10}$$

$$\begin{aligned} q_2^*(c_1, c_2) &= \frac{\alpha}{3\beta} - \frac{2(1-\rho)\mu_c\tau_c}{3\beta[3(1-\rho^2)\tau_\epsilon + 4\tau_c]} \\ &\quad - \frac{2[(1-\rho^2)\tau_\epsilon + \tau_c]}{\beta[3(1-\rho^2)\tau_\epsilon + 4\tau_c]}c_2 + \left[\frac{1}{3\beta} - \frac{2\rho\tau_c}{3\beta[3(1-\rho^2)\tau_\epsilon + 4\tau_c]}\right]c_1 \end{aligned} \tag{11}$$

Proof. The proof is done in the following two steps.

<u>STEP 1</u> Prove the equilibrium output of Firm 2 is linear in $c_1,\ c_2$, and further it can be expressed as following:

$$q_2^*(c_1, c_2) = \frac{\alpha}{3\beta} - \frac{2(1-\rho)\mu_a\tau_c}{3\beta[3(1-\rho^2)\tau_\epsilon + 4\tau_c]} - \frac{2[(1-\rho^2)\tau_\epsilon + \tau_c]}{\beta[3(1-\rho^2)\tau_\epsilon + 4\tau_c]}c_2 + \left[\frac{1}{3\beta} - \frac{2\rho\tau_c}{3\beta[3(1-\rho^2)\tau_\epsilon + 4\tau_c]}\right]c_1 \tag{12}$$

Proof. Recall the two first order conditions that characterize the equilibrium (necessary and sufficient condition for equilibrium):

$$q_1(c_1, s) = \frac{\alpha - c_1}{2\beta} - \frac{1}{2} E^1_{c_2}(q_2(c_1, c_2)|c_1, s) \quad (13)$$

$$q_2(c_1, c_2) = \frac{\alpha - c_2}{2\beta} - \frac{1}{2} E^2_s(q_1(c_1, s)|c_2) \quad (14)$$

Let's focus on (14) and plug in the expression for $q_1(c_1, s)$ in the expectation operator. We obtain

$$\begin{aligned} q_2(c_1, c_2) &= \frac{\alpha - c_2}{2\beta} - \frac{1}{2} E^2_s \left[\frac{\alpha - c_1}{2\beta} - \frac{1}{2} E^1_{c_2}(q_2(c_1, c_2)|c_1, s) \Big| c_2 \right] \\ &= \frac{\alpha - c_2}{2\beta} - \frac{\alpha - c_1}{4\beta} + \frac{1}{4} E^2_s \left[E^1_{c_2}[q_2(c_1, c_2)|c_1, s] \Big| c_2 \right] \end{aligned} \quad (15)$$

continue to plug in $q_2(c_1, c_2)$ into the inner expectation operator in (15),

$$\begin{aligned} q_2(c_1, c_2) &= \frac{\alpha - c_2}{2\beta} - \frac{\alpha - c_1}{4\beta} + \frac{1}{4} E^2_s \left[E^1_{c_2} \left[\frac{\alpha - c_2}{2\beta} - \frac{1}{2} E^2_s(q_1(c_1, s)|c_2) \Big| c_1, s \right] \Big| c_2 \right] \\ &= \frac{\alpha - c_2}{2\beta} - \frac{\alpha - c_1}{4\beta} + \frac{1}{4} E^2_s \left[\frac{\alpha - E^1_{c_2}(c_2|c_1, s)}{2\beta} - \frac{1}{2} E^1_{c_2} \left[E^2_s(q_1(c_1, s)|c_2) \Big| c_1, s \right] \Big| c_2 \right] \\ &= \frac{\alpha - c_2}{2\beta} - \frac{\alpha - c_1}{4\beta} + \frac{\alpha - E^2_s \left[E^1_{c_2}(c_2|c_1, s) \Big| c_2 \right]}{8\beta} - \frac{1}{8} E^2_s \left[E^1_{c_2} \left[E^2_s(q_1(c_1, s)|c_2) \Big| c_1, s \right] \Big| c_2 \right] \end{aligned} \quad (16)$$

continue to plug in $q_1(c_1, s)$ into the inner most expectation operator in (16), we obtain

$$\begin{aligned} q_2(c_1, c_2) &= \frac{\alpha - c_2}{2\beta} - \frac{\alpha - c_1}{4\beta} + \frac{\alpha - E^2_s \left[E^1_{c_2}(c_2|c_1, s) \Big| c_2 \right]}{8\beta} - \frac{1}{8} E^2_s \left[E^1_{c_2} \left[E^2_s \left[\frac{\alpha - c_1}{2\beta} - \frac{1}{2} E^1_{c_2}(q_2(c_1, c_2)|c_1, s) \Big| c_2 \right] \Big| c_1, s \right] \Big| c_2 \right] \\ &= \frac{\alpha - c_2}{2\beta} - \frac{\alpha - c_1}{4\beta} + \frac{\alpha - E^2_s \left[E^1_{c_2}(c_2|c_1, s) \Big| c_2 \right]}{8\beta} - \frac{\alpha - c_1}{16\beta} + \frac{1}{16} E^2_s \left[E^1_{c_2} \left[E^2_s \left[E^1_{c_2}(q_2(c_1, c_2)|c_1, s) \Big| c_2 \right] \Big| c_1, s \right] \Big| c_2 \right] \end{aligned} \quad (17)$$

continue in the same fashion we obtain

$$\begin{aligned} q_2(c_1, c_2) &= \frac{\alpha - c_2}{2\beta} - \frac{\alpha - c_1}{4\beta} + \frac{\alpha - E^2_s \left[E^1_{c_2}(c_2|c_1, s) \Big| c_2 \right]}{8\beta} - \frac{\alpha - c_1}{16\beta} + \frac{\alpha - E^2_s \left[E^1_{c_2} \left[E^2_s \left[E^1_{c_2}(c_2|c_1, s) \Big| c_2 \right] \Big| c_1, s \right] \Big| c_2 \right]}{32\beta} \\ &\quad - \frac{\alpha - c_1}{64\beta} + \cdots \end{aligned} \quad (18)$$

Let us introduce some notations to simplify the expressions. Define $\left(E^2_s E^1_{c_2}\right)^{(1)}(c_2) \equiv E^2_s \left[E^1_{c_2}(c_2|c_1, s) \Big| c_2 \right]$, $\left(E^2_s E^1_{c_2}\right)^{(2)}(c_2) \equiv E^2_s \left[E^1_{c_2} \left[E^2_s \left[E^1_{c_2}(c_2|c_1, s) \Big| c_2 \right] \Big| c_1, s \right] \Big| c_2 \right]$, and in general $\left(E^2_s E^1_{c_2}\right)^{(n)}(c_2),\ n \in \mathcal{N}_+$.

Using these new notations, (18) can be expressed as

$$\begin{aligned} q_2(c_1,c_2) &= \frac{\alpha - c_2}{2\beta} - \frac{\alpha - c_1}{2^2\beta} + \frac{\alpha - \left(E_s^2E_{c_2}^1\right)^{(1)}(c_2)}{2^3\beta} - \frac{\alpha - c_1}{2^4\beta} + \frac{\alpha - \left(E_s^2E_{c_2}^1\right)^{(2)}(c_2)}{2^5\beta} - \frac{\alpha - c_1}{2^6\beta} + \frac{\alpha - \left(E_s^2E_{c_2}^1\right)^{(3)}(c_2)}{2^7\beta} + \cdots \\ &= \sum_{i=0}^{\infty} \frac{\alpha - \left(E_s^2E_{c_2}^1\right)^{(i)}(c_2)}{2^{2i+1}\beta} - \sum_{i=1}^{\infty}\frac{\alpha - c_1}{2^{2i}\beta} \quad (19) \\ &= \left[\sum_{i=0}^{\infty}\frac{\alpha}{2^{2i+1}\beta} - \sum_{i=1}^{\infty}\frac{\alpha}{2^{2i}\beta}\right] + \sum_{i=1}^{\infty}\frac{c_1}{2^{2i}\beta} - \sum_{i=0}^{\infty}\frac{\left(E_s^2E_{c_2}^1\right)^{(i)}(c_2)}{2^{2i+1}\beta} \quad (20) \end{aligned}$$

The first two parts of (20) are easy to calculate, the last part is more difficult. Let's take a closer look at the expression $\left(E_s^2E_{c_2}^1\right)^{(n)}(c_2),\ n \in \mathcal{N}_+$.

Note we've already derived

$$E_{c_2}^1(c_2|c_1,s) = \frac{(1-\rho)\tau_c}{(1-\rho^2)\tau_\epsilon + \tau_c}\mu_c + \frac{\rho\tau_c}{(1-\rho^2)\tau_\epsilon + \tau_c}c_1 + \frac{(1-\rho^2)\tau_\epsilon}{(1-\rho^2)\tau_\epsilon + \tau_c}s \quad (21)$$

hence using the assumption that $E_s^2(s|c_2) = c_2$ (namely the signal s is conditionally unbiased), we have

$$\begin{aligned} \left(E_s^2E_{c_2}^1\right)^{(1)}(c_2) \equiv E_s^2\left[E_{c_2}^1(c_2|c_1,s)\middle|c_2\right] &= \underbrace{\frac{(1-\rho)\tau_c}{(1-\rho^2)\tau_\epsilon + \tau_c}\mu_c + \frac{\rho\tau_c}{(1-\rho^2)\tau_\epsilon + \tau_c}c_1}_{\equiv K} + \underbrace{\frac{(1-\rho^2)\tau_\epsilon}{(1-\rho^2)\tau_\epsilon + \tau_c}}_{\equiv G}c_2 \quad (22) \\ &= K + Gc_2 \quad (23) \end{aligned}$$

where K and G are known constants.

Using this result,

$$\left(E_s^2E_{c_2}^1\right)^{(2)}(c_2) = K + G\left(E_s^2E_{c_2}^1\right)^{(1)}(c_2) = K + G(K + Gc_2) = K + KG + G^2c_2$$

Hence for a general $n, n \in \mathcal{N}_+$,

$$\left(E_s^2E_{c_2}^1\right)^{(n)}(c_2) = K\sum_{i=0}^{n-1}G^i + G^nc_2 \quad (24)$$

Plug (24) into (20)

$$q_2(c_1,c_2) = \left[\sum_{i=0}^{\infty}\frac{\alpha}{2^{2i+1}\beta} - \sum_{i=1}^{\infty}\frac{\alpha}{2^{2i}\beta}\right] + \sum_{i=1}^{\infty}\frac{c_1}{2^{2i}\beta} - \sum_{i=0}^{\infty}\frac{K\sum_{j=0}^{i-1}G^i + G^ic_2}{2^{2i+1}\beta} \quad (25)$$

The last term $\sum_{i=0}^{\infty} \frac{K\sum_{j=0}^{i-1} G^i + G^i c_2}{2^{2i+1}\beta}$ can be shown to equal the following

$$\begin{aligned}
& \frac{c_2}{\beta}\left(\frac{1}{2}+\frac{G}{2^3}+\frac{G^2}{2^5}+\cdots\right)+\frac{K}{\beta}\left(\frac{1}{2^3}+\frac{1}{2^5}+\frac{1}{2^7}+\cdots\right)+\frac{KG}{\beta}\left(\frac{1}{2^5}+\frac{1}{2^7}+\frac{1}{2^9}+\cdots\right)+\cdots \\
= \; & \frac{2c_2}{\beta(4-G)}+\frac{2K}{3\beta(4-G)} \qquad (26)
\end{aligned}$$

Substitute back the expressions for K, G, we can obtain the expression for $q_2(c_1, c_2)$

$$q_2^*(c_1,c_2)=\frac{\alpha}{3\beta}-\frac{2(1-\rho)\mu_a\tau_c}{3\beta[3(1-\rho^2)\tau_\epsilon+4\tau_c]}-\frac{2[(1-\rho^2)\tau_\epsilon+\tau_c]}{\beta[3(1-\rho^2)\tau_\epsilon+4\tau_c]}c_2+\left[\frac{1}{3\beta}-\frac{2\rho\tau_c}{3\beta[3(1-\rho^2)\tau_\epsilon+4\tau_c]}\right]c_1$$

■

<u>STEP 2</u> Given that in equilibrium q_2 is linear in $c_1,\ c_2$, the equilibrium output of Firm 1 is linear in $c_1,\ s$. Furthermore, the equilibrium output function of Firm 1 is uniquely given by

$$q_1^*(c_1,s)=\frac{\alpha}{3\beta}+\frac{4(1-\rho)\mu_a\tau_c}{3\beta[3(1-\rho^2)\tau_\epsilon+4\tau_c]}-\left[\frac{2}{3\beta}-\frac{4\rho\tau_c}{3\beta[3(1-\rho^2)\tau_\epsilon+4\tau_c]}\right]c_1+\frac{(1-\rho^2)\tau_\epsilon}{\beta[3(1-\rho^2)\tau_\epsilon+4\tau_c]}s \qquad (27)$$

Proof. Using the equilibrium output function (12) derived from Step 1, plug it into the FOC of Firm 1's problem:

$$\begin{aligned}
q_1(c_1,s) &= \frac{\alpha-c_1}{2\beta}-\frac{1}{2}E^1_{c_2}(q_2^*(c_1,c_2)|c_1,s) \\
&= \frac{\alpha-c_1}{2\beta}-\frac{1}{2}\left\{\frac{\alpha}{3\beta}-\frac{2(1-\rho)\mu_a\tau_c}{3\beta[3(1-\rho^2)\tau_\epsilon+4\tau_c]}+\left[\frac{1}{3\beta}-\frac{2\rho\tau_c}{3\beta[3(1-\rho^2)\tau_\epsilon+4\tau_c]}\right]c_1-\frac{2[(1-\rho^2)\tau_\epsilon+\tau_c]}{\beta[3(1-\rho^2)\tau_\epsilon+4\tau_c]}E^1_{c_2}(c_2|c_1,s)\right\}
\end{aligned}$$

Since $E^1_{c_2}(c_2|c_1,s)$ is linear in c_1 and s, hence $q_1(c_1,s)$ is linear in c_1 and s. Plug in the expression for $E^1_{c_2}(c_2|c_1,s)$ and reducing, one obtains the equilibrium output function for Firm 1:

$$q_1^*(c_1,s)=\frac{\alpha}{3\beta}+\frac{4(1-\rho)\mu_a\tau_c}{3\beta[3(1-\rho^2)\tau_\epsilon+4\tau_c]}-\left[\frac{2}{3\beta}-\frac{4\rho\tau_c}{3\beta[3(1-\rho^2)\tau_\epsilon+4\tau_c]}\right]c_1+\frac{(1-\rho^2)\tau_\epsilon}{\beta[3(1-\rho^2)\tau_\epsilon+4\tau_c]}s \qquad (28)$$

■ Steps 1 and 2 together complete the proof. ■

Notice the equilibrium output functions are affine linear in their variables. As the proof illustrates, this linearity as well as uniqueness are consequences of normal information structure (in particular the posterior expectations are affine linear) and linear inverse demand system and that the expectation operator preserves linear properties. Firm 1's equilibrium output function q_1^* is strictly increasing in s and strictly decreasing in c_1, while q_2^* is strictly increasing in c_1 and strictly decreasing in c_2, which captures the strategic substitutability of the two firms' output strategies.

Another piece of information we can learn form this proposition is that when espionage intensity τ_ϵ increases, the slopes of both firms' output functions become steeper. In other words, $q_1^*,\ q_2^*$ are becoming increasingly sensitive to changes in each of their variables when τ_ϵ increases.

It is worthwhile to consider two related benchmark model, one in which there's perfect information, namely the realization of c_1 as well as c_2 are common knowledge. As we'll see, this benchmark is closely related to Section 5, when Firm 2 is able to costlessly and credibly disclose information about c_2 to Firm 1. For the second benchmark model, Firm 1 doesn't have any additional information about the realized c_2 other than the prior joint distribution of $c_1,\ c_2$, which is referred to as the no espionage scenario. This is also the appropriate benchmark that allows us to study how espionage affects firms' profits.

Benchmark 1. (Complete Information)

In this benchmark, it is assumed that the realization of both $c_1,\ c_2$ are common knowledge. One can easily obtain the unique equilibrium paire of output functions $q_1^\infty(c_1, c_2),\ q_2^\infty(c_1, c_2)$ (the superscript ∞ indicates infinitely precise signal, see below):

$$q_1^\infty(c_1, c_2) = \frac{\alpha}{3\beta} + \frac{c_2}{3\beta} - \frac{2c_1}{3\beta} \tag{29}$$

$$q_2^\infty(c_1, c_2) = \frac{\alpha}{3\beta} + \frac{c_2}{3\beta} - \frac{2c_1}{3\beta} \tag{30}$$

As τ_ϵ goes to infinity, the random variable s converges in probability to c_2 and $q_1^*(c_1, s),\ q_2^*(c_1, c_2)$ converge to $q_1^\infty(c_2, c_2)$, $q_2^\infty(c_1, c_2)$ respectively. This can be checked easily using (10) and (11) or more conveniently using (35) and (36).

Using the first order conditions, the equilibrium profits of the two firms under Benchmark 1 are the following:

$$\pi_i^\infty(c_1, c_2) = \beta\left[q_i^\infty(c_1, c_2)\right]^2, \quad i = 1, 2 \tag{31}$$

Benchmark 2. (No Espionage)

In this benchmark model, it is common knowledge that Firm 1 cannot do espionage (or signal precision $\tau_\epsilon = 0$). The information available to Firm 1 is therefore only c_1. Firm 2 knows $c_1,\ c_2$ but again c_2 is its private information. This is the appropriate benchmark to study the how espionage affects both firms' profits in equilibrium.

Again there exists a unique pair of equilibrium output functions $q_1^0,\ q_2^0$:

$$q_1^0(c_1) = \frac{\alpha}{3\beta} - \frac{2c_1}{3\beta} + \frac{E(c_2|c_1)}{3\beta} \tag{32}$$

$$q_2^0(c_1, c_2) = \left[\frac{\alpha}{3\beta} - \frac{1}{6\beta}(1-\rho)\mu_c\right] + \left(\frac{1}{3\beta} - \frac{\rho}{6\beta}\right)c_1 - \frac{c_2}{2\beta} = \frac{2\alpha + 2c_1 - 3c_2}{6\beta} - \frac{E(c_2|c_1)}{6\beta} \tag{33}$$

where $E(c_2|c_1) = (1-\rho)\mu_c + \rho c_1$. (Again derived by the projection theorem for normal random variables, see DeGroot (1970).) This benchmark can be regarded as the limiting case when τ_ϵ approaches 0, as one can verify easily using (10) and (11) or (35) and (36). Using the first order conditions, the equilibrium profits of the two firms under Benchmark 1 are the following:

$$\pi_1^0(c_1) = \beta\left[q_1^0(c_1)\right]^2, \quad \pi_2^0(c_1, c_2) = \beta\left[q_2^0(c_1, c_2)\right]^2 \tag{34}$$

A more useful and insightful way to write the equilibrium output functions $q_1^*(c_1, s)$ and $q_2^*(c_1, c_2)$ is following:

Corollary 1

$$q_1^*(c_1, s) = \underbrace{q_1^0(c_1)}_{\text{benchmark level}} + \underbrace{\eta^*[s - E(c_2|c_1)]}_{\text{adjustment}} \tag{35}$$

$$q_2^*(c_1, c_2) = \underbrace{q_2^0(c_1, c_2)}_{\text{benchmark level}} - \underbrace{\frac{\eta^*}{2}[c_2 - E(c_2|c_1)]}_{\text{adjustment}} \tag{36}$$

where

$$\eta^* = \frac{(1-\rho^2)\tau_\epsilon}{\beta[3(1-\rho^2)\tau_\epsilon + 4\tau_c]}, \tag{37}$$

which is positive, strictly increasing and concave in τ_ϵ.

Notice that with espionage, both firms' equilibrium outputs can be decomposed into a no-espionage benchmark level and an adjustment term. The effect of espionage is fully captured by the adjustment terms since the espionage intensity τ_ϵ only enters the coefficient η^* associated withe the adjustment terms. For Firm 1, without doing espionage its conjecture on the rival's cost would be $E(c_2|c_1)$. With espionage, a signal s is generated, the term $s - E(c_2|c_1)$ captures how Firm 1 is "surprised" by this signal. Since s is an unbiased estimator of true c_2, if s is surprisingly high (low) in the sense $s > E(c_2|c_1)$ ($s < E(c_2|c_1)$), more likely c_2 is indeed higher (lower) than expected, hence Firm 1 should produce more (less) than the no-espionage benchmark level.

Similar interpretation goes through for Firm 2 with the distinction that the adjustment term for Firm 2 depends on c_2 instead of s since s is Firm 1's private information.

Another thing to notice is when the espionage intensity of Firm 1 increases, since η^* increases, both firms would adjust their outputs more rapidly, but in comparison Firm 1's adjustment is more rapid.

The next proposition summarizes how espionage intensity, as measured by τ_ϵ, affects both firms output strategies. The proof follows directly from equations (10), (11), (35) and (36).

Proposition 2 *When espionage intensity τ_ϵ of Firm 1 increases, both $q_1^*(c_1, s)$ and $q_2^*(c_1, c_2)$ are becoming more sensitive in all their variables. For Firm 1, when $E_{c_2}^1(c_2|c_1) > s$, in equilibrium q_1^* is strictly decreasing in τ_ϵ; when $E_{c_2}^1(c_2|c_1) < s$, in equilibrium q_1^* is strictly increasing in τ_ϵ. For Firm 2, when $E_{c_2}^1(c_2|c_1) > c_2$, in equilibrium q_2^* is strictly increasing in τ_ϵ when $E_{c_2}^1(c_2|c_1) < c_2$, in equilibrium q_2^* is strictly decreasing in τ_ϵ.*

From Firm 1's prospective, if its signal s is surprisingly high in the sense that $E(c_2|c_1) < s$, then since s is conditionally unbiased, the more precise the signal is (namely higher τ_ϵ), the more confidence Firm 1 will have that c_2 is indeed high. Since q_2^* is strictly decreasing in c_2, Firm 1 will anticipate with more confidence that Firm 2 will produce less. Because the best response of Firm 1 strictly increases as q_2 decreases, Firm 1 would produce more. This explains why q_1^* is increasing in τ_ϵ when $E(c_2|c_1) < s$.

For Firm 2, given its realized cost c_2, since it knows that Firm 1's signal s is conditionally unbiased, when c_2 is surprisingly high in the sense that $c_2 > E(c_2|c_1)$, then with higher level of signal precision τ_ϵ, it is more likely that Firm 1 will receive a surprisingly high signal s (in the sense that $s > E(c_2|c_1)$). By the same argument in the previous paragraph, it's more likely that Firm 1 will produce more, so it's optimal for Firm 2 to reduce its output (again due to the best response of Firm 2 is decreasing in Firm 1's expected output).

After solving this Cournot subgame, now we go back one step and consider Firm 1's optimal choice of espionage intensity τ_ϵ.

Using the first order condition (7), given $\tau_\epsilon,\ c_1,\ s$, Firm 1's ex post equilibrium profit is

$$\pi_1^*(c_1, s) = \beta[q_1^*(c_1, s)]^2. \tag{38}$$

Plug in the result (35), Firm 1's ex ante expected profit (conditional on $c_1,\ \tau_\epsilon$) is

$$\beta E_s\left\{[q_1^*(c_1, s)]^2 \middle| c_1, \tau_\epsilon\right\} = \underbrace{\beta[q_1^0(c_1)]^2}_{\pi_1^0(c_1)} + \underbrace{B^*(\tau_\epsilon)}_{\text{benefit function}} \tag{39}$$

where

$$B^*(\tau_\epsilon) = \frac{(1-\rho^2)^3\tau_\epsilon^2 + (1-\rho^2)^2\tau_c\tau_\epsilon}{\beta\tau_c[3(1-\rho^2)\tau_\epsilon + 4\tau_c]^2} \tag{40}$$

is a strictly increasing, concave function in τ_ϵ that captures the extra benefit to Firm 1 due to espionage. The fact that the extra benefit function $B^*(\tau_\epsilon)$ is strictly increasing and positive implies that more precise information about the rival's cost is always desirable for Firm 1. It's worth noticing that this extra benefit function $B^*(\tau_\epsilon)$ doesn't involve c_1, namely irrespective of the realization of Firm 1's own cost, given the same espionage intensity level τ_ϵ, it enjoys the same amount of extra benefit from espionage in expectation. This property hinges on the quadratic-normal modeling structure.

Firm 1's objective in the first stage is to choose τ_ϵ to maximize its net expected profit:

$$\begin{aligned} &\max_{\tau_\epsilon \in [0,\infty)} E_s\left[\pi_1^*(c_1, s)|c_1)\right] - k(\tau_\epsilon) \\ \Leftrightarrow \quad &\max_{\tau_\epsilon \in [0,\infty)} \beta[q_1^0(c_1)]^2 + \underbrace{B^*(\tau_\epsilon) - k(\tau_\epsilon)}_{\text{net benefit from espionage}} \end{aligned} \tag{41}$$

The first part on the right hand side is firm 1's benchmark profit under no-espionage scenario which doesn't involve τ_ϵ. By assumption $k(\tau_\epsilon)$ is strictly convex and that we've shown $B^*(\tau_\epsilon)$ is strictly concave, hence the optimal choice τ_ϵ^* is pinned down by the first order condition:

$$MB^*(\tau_\epsilon) \equiv \frac{dB^*(\tau_\epsilon)}{d\tau_\epsilon} = k'(\tau_\epsilon) \tag{42}$$

This first order condition simply equates the marginal benefit of espionage (denoted as $MB^*(\tau_\epsilon)$) to the marginal cost $k'(\tau_\epsilon)$. In the next proposition, we show that given $q_1^*,\ q_2^*$, there's a unique optimal choice of τ_ϵ, which is always positive. Hence the entire game has a unique equilibrium.

Proposition 3 *The optimal signal precision τ_ϵ^* ($\tau_\epsilon^* > 0$) is pinned down by the unique solution to the following equation*

$$\frac{5(1-\rho^2)^3\tau_\epsilon + 4(1-\rho^2)^2\tau_c}{\beta\left[3(1-\rho^2)\tau_\epsilon + 4\tau_c\right]^3} = k'(\tau_\epsilon) \tag{43}$$

Furthermore, for the quadratic-normal model we consider, the optimal τ_ϵ doesn't depend on the realization of c_1.

Proof. First we calculate $\text{Var}(s|c_1)$. By our assumption (see equation (1)), we have

$$\begin{pmatrix} s \\ c_1 \end{pmatrix} \sim \mathcal{N}\left[\begin{pmatrix} \mu_c \\ \mu_c \end{pmatrix}, \begin{pmatrix} \sigma_c^2 + \sigma_\epsilon^2 & \rho\sigma_c^2 \\ \rho\sigma_c^2 & \sigma_c^2 \end{pmatrix}\right] \tag{44}$$

hence using the projection theorem for random variables

$$\text{Var}(s|c_1) = \sigma_c^2 + \sigma_\epsilon^2 - \rho\sigma_c^2 \cdot \frac{1}{\sigma_c^2} \cdot \rho\sigma_c^2 = (1-\rho^2)\frac{1}{\tau_c} + \frac{1}{\tau_\epsilon} \tag{45}$$

Recall the equilibrium output function of Firm 1 (35),

$$q_1^*(c_1, s) = q_1^0 + \eta^*[s - E(c_2|c_1)]$$

where q_1^0 is a known constant that doesn't involves s and τ_ϵ. With this result and recall using the first order condition we can simplify $\pi_1^*(c_1, s) = \beta[q_1^*(c_1, s)]^2$, hence conditional on c_1, Firm 1's ex ante (before receiving the signal s) expected profit is

$$\begin{aligned}
& E_s[\pi_1^*(c_1, s, \tau_\epsilon)|c_1] - k(\tau_\epsilon) \\
= \; & E_s[\beta[q_1^*(c_1, s, \tau_\epsilon)]^2|c_1] - k(\tau_\epsilon) \\
= \; & \beta\left[(q_1^0)^2 + (\eta^*)^2 \mathrm{Var}(s|c_1)\right] - k(\tau_\epsilon) \\
= \; & \beta(q_1^0)^2 + \frac{(1-\rho^2)^3\tau_\epsilon^2 + (1-\rho^2)^2\tau_c\tau_\epsilon}{\beta\tau_c[3(1-\rho^2)\tau_\epsilon + 4\tau_c]^2} - k(\tau_\epsilon)
\end{aligned}$$

The second equation follows from the fact that $E_s[s - E^1_{c_2}(c_2|c_1)|c_1] = 0$, since $s = c_2 + \epsilon$ and by assumption ϵ and $c_1,\ c_2$ are independent, and that $E_s\{[s - E_{c_2}(c_2|c_1)]^2|c_1\} = \mathrm{Var}(s|c_1)$. The last equation is obtained by by substituting the result $\mathrm{Var}(s|c_1) = (1-\rho^2)\frac{1}{\tau_c} + \frac{1}{\tau_\epsilon}$ and the expression for η^* given in (37).

One can check that

$$\frac{\partial^2 E_s[\pi_1^*(c_1, s, \tau_\epsilon)|c_1]}{\partial\tau_\epsilon^2} = \frac{-30(1-\rho^2)^4\tau_\epsilon - 16(1-\rho^2)^3\tau_c}{\beta[3(1-\rho^2)\tau_\epsilon + 4\tau_c]^4} < 0$$

On the other hand, by our assumption $k(\tau_\epsilon)$ is strictly convex, hence $-k''(\tau_\epsilon) < 0$. Therefore, Firm 1's ex ante expected net profit function is strictly concave in τ_ϵ, the optimal choice of espionage intensity τ_ϵ (if any) is pinned down by the first order condition,

$$\frac{5(1-\rho^2)^3\tau_\epsilon + 4(1-\rho^2)^2\tau_c}{\beta\left[3(1-\rho^2)\tau_\epsilon + 4\tau_c\right]^3} = k'(\tau_\epsilon)$$

The expression on the left hand side is positive, continuous, strictly decreasing in τ_ϵ (since we already see $\frac{\partial^2 E_s^2[\pi_1^*(c_1,s,\tau_\epsilon)|c_1]}{\partial\tau_\epsilon^2} < 0$) and approaches 0 when τ_ϵ goes to infinity. The expression on the right hand is continuous, strictly increasing and by our assumption $\lim_{\tau_\epsilon \to 0+} k'(\tau_\epsilon) = 0$. Hence there's a unique $\tau_\epsilon \in (0, \infty)$ that satisfies the first order condition. ■

The marginal benefit and cost function is strictly decreasing, convex, continuous and converges to 0 as τ_ϵ goes to infinity; by our assumptions on $k(\tau_\epsilon)$, $k'(\tau_\epsilon)$ is strictly increasing, $\lim_{\tau_\epsilon \to 0} k'(\tau_\epsilon) = 0$ and converges to infinity as τ_ϵ goes to infinity. Figure 1 illustrates these two functions and the unique intercept pins down the optimal espionage intensity τ_ϵ^*.

Corollary 2 *(Comparative Statics) The optimal espionage intensity τ_ϵ^* is strictly decreasing in in correlation coefficient ρ and precision of the prior belief τ_c.*

Proof. Rearrange the first order condition (43) and define

$$F(\tau_\epsilon; \rho, \tau_c) \equiv \frac{5(1-\rho^2)^3\tau_\epsilon + 4(1-\rho^2)^2\tau_c}{\beta\left[3(1-\rho^2)\tau_\epsilon + 4\tau_c\right]^3} - k'(\tau_\epsilon) = 0$$

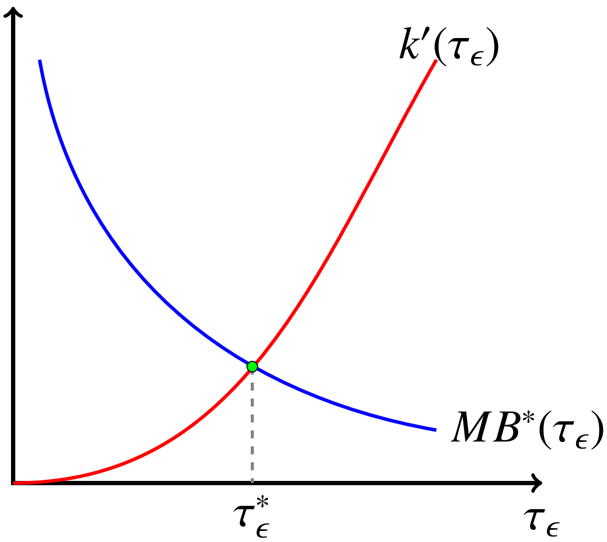

Figure 1: Optimal Choice of τ_ϵ—Graphical Illustration

Use implicit function theorem,

$$\frac{\partial \tau_\epsilon^*}{\partial \rho} = -\frac{\partial F(\tau_\epsilon;\rho,\tau_c)/\partial \rho}{\partial F(\tau_\epsilon;\rho,\tau_c)/\partial \tau_\epsilon^*}$$

One can verify easily

$$\frac{\partial F(\tau_\epsilon;\rho,\tau_c)}{\partial \rho} = \frac{-96\rho(1-\rho^2)^3\tau_c\tau_\epsilon - 64\rho(1-\rho^2)^2\tau_c^2}{\beta[3(1-\rho^2)\tau_\epsilon + 4\tau_c]^4} < 0$$

We already showed in the proof to Theorem 3 that $\partial F(\tau_\epsilon;\rho,\tau_c)/\partial \tau_\epsilon^*$ is negative (Firm 1's ex ante expected net profit is strictly concave in τ_ϵ), hence $\frac{\partial \tau_\epsilon^*}{\partial \rho} < 0$, as desired.

Similarly

$$\frac{\partial \tau_\epsilon^*}{\partial \tau_c} = -\frac{\partial F(\tau_\epsilon;\rho,\tau_c)/\partial \tau_c}{\partial F(\tau_\epsilon;\rho,\tau_c)/\partial \tau_\epsilon^*}$$

One can verify

$$\frac{\partial F(\tau_\epsilon;\rho,\tau_c)}{\partial \tau_c} = \frac{-48(1-\rho^2)^3\tau_\epsilon - 32(1-\rho^2)^2\tau_c}{\beta[3(1-\rho^2)\tau_\epsilon + 4\tau_c]^4} < 0$$

hence $\frac{\partial \tau_\epsilon^*}{\partial \tau_c} < 0$. ■

Intuitively, when Firm 1 has more precise prior information about the rival firm's marginal cost, it has less incentive to do espionage. Similarly, when the two firms marginal costs are more correlated, knowing its own realized marginal cost would provide Firm 1 more precise information about Firm 2's cost, hence reduces its incentive to do espionage.

3.1 Profit Comparison

In an environment in which it's commonly known that Firm 1 is able to do espionage, does Firm 1 make more profit in equilibrium than in an environment in which it cannot do espionage (namely Benchmark 2)? What about Firm 2? If it has been spied upon by Firm 1, will it suffer a loss in equilibrium? This subsection aims to answer these questions.

First let's focus on Firm 1. Recall under Benchmark 2 (no espionage), in equilibrium its output function is given by (32). Its expected profit conditional on c_1 is $\beta[q_1^0(c_1)]^2$.

When Firm 1 is able to do espionage on Firm 2, according to (41), Firm 1's expected equilibrium net profit is

$$\beta E_s\{[q_1^*(c_1, s)]^2 | c_1\} - k(\tau_\epsilon) = \beta[q_1^0(c_1)]^2 + B^*(\tau_\epsilon^*) - k(\tau_\epsilon^*) \tag{46}$$

where the first term on the right hand side is exactly Firm 1's expected equilibrium profit under Benchmark 2 (no espionage). Hence the profit comparison solely depends on the last two terms. The second term $B^*(\tau_\epsilon)$ is the extra benefit due to espionage (when $\tau_\epsilon = 0$, $B^*(\tau_\epsilon) = 0$), which is strictly increasing, concave and has positive slope at $\tau_\epsilon = 0$. On the other hand, by assumption $k(\tau_\epsilon)$ is convex, increasing and has slope 0 at $\tau_\epsilon = 0$. The relationship between $B^*(\tau_\epsilon)$ and $k(\tau_\epsilon)$ can be illustrated by Figure 2.

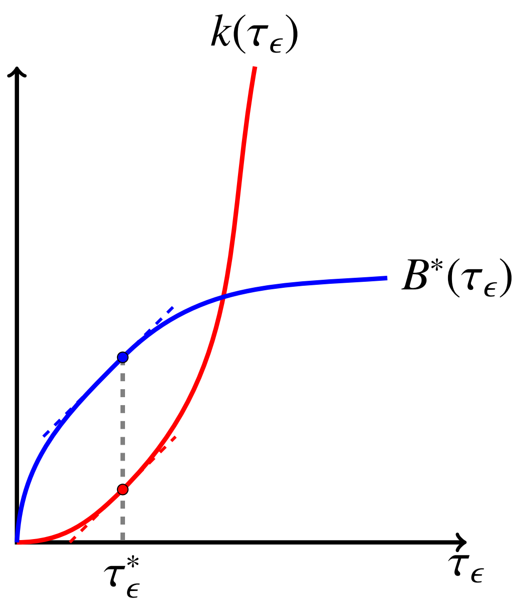

Figure 2: Extra Benefit Function $B^*(\tau_\epsilon)$ and Cost Function $k(\tau_\epsilon)$ for Firm 1

Observation $B^*(\tau_\epsilon^*) - k(\tau_\epsilon^*) > 0$. In other words, in equilibrium Firm 1 will benefit unambiguously from doing espionage on Firm 2 to learn its cost. The benefit is independent of its realized cost c_1.

Next we consider Firm 2. Note in equilibrium Firm 2's expected profit under the two scenarios are given by $\beta[q_2^0(c_1, c_2)]^2$ and $\beta[q_2^*(c_1, c_2)]^2$ respectively. Hence the profit comparison for Firm 2 depends solely on the comparison of $q_2^0(c_1, c_2)$ and $q_2^*(c_1, c_2)$. Recall from Corollary 1

$$q_2^*(c_1, c_2) = q_2^0(c_1, c_2) - \frac{\eta^*}{2}[c_2 - E(c_2|c_1)]$$

where η^* is a positive constant (given τ_ϵ). Hence $q_2^* > q_2^0$ (Firm 2's equilibrium profit is greater when Firm 1 is doing espionage) iff $c_2 < E(c_2|c_1)$. Since $E(c_2|c_1) = (1-\rho)\mu_c + \rho c_1$, this condition is equivalent to

$$c_1 > -\frac{1-\rho}{\rho}\mu_c + \frac{1}{\rho}c_2 \tag{47}$$

Firm 2's profit is higher in equilibrium when Firm 1 is doing espionage to learn c_2; for $c_1,\ c_2$ that satisfies the reverse inequality, in equilibrium Firm 2's profit is lower when Firm 1 does espionage. Figure 3 illustrates this. For realizations of $c_1,\ c_2$ in the shaded area, Firm 2 benefits from the espionage of the Firm 1; in the right half of the line, Firm 2's profit is lower if Firm 1 is able to do espionage.

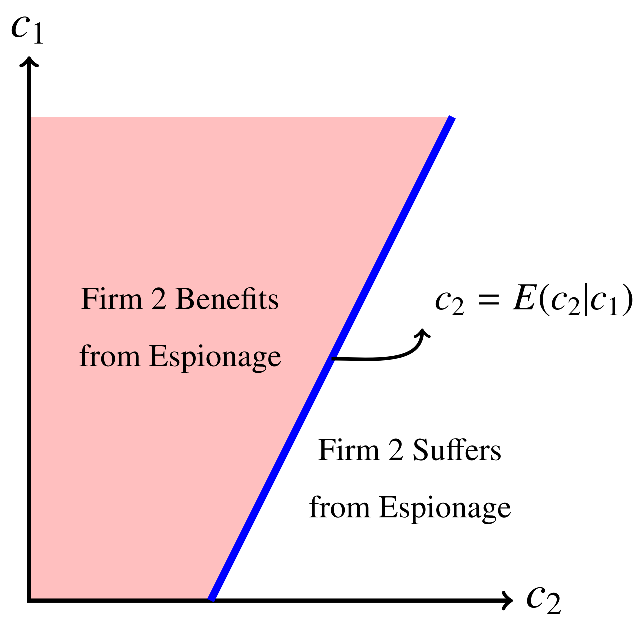

Figure 3: Profit Comparison for Firm 2

The intuition behind this result is following: When Firm 2 is relatively strong (in the sense that $c_2 < E(c_2|c_1)$), due to strategic complementarity, it would like to communicate this information to the rival so as to induce the rival to produce less, but it couldn't do so credibly and costlessly under our assumption. But when Firm 1 is doing espionage to learn c_2, since the signal s is conditionally unbiased, with higher probability that it will discover that Firm 2 is indeed strong and hence will takes a weaker action.

We summarize the findings in this subsection into the following proposition.

Proposition 4 *In equilibrium, Firm 1 always benefits from doing espionage on Firm 2 to learn c_2; Firm 2 benefits from the espionage of Firm 1* iff *$c_2 < E(c_2|c_1)$; Firm 2 suffers from the espionage of Firm 1* iff *$c_2 > E(c_2|c_1)$.*

3.2 Consumer Surplus

In this subsection we investigate how consumer surplus is affected by the espionage of Firm 1. Consider representative consumer with utility function

$$u(q) = \alpha q - \frac{\beta}{2}q^2 + m \tag{48}$$

where α, β are positive constants, q is the amount of good consumed by the representative consumer, and m is the additively separable numeraire. Suppose the market price for this good is p, the representative consumer's problem is

$$\max_{q\geq 0} \alpha q - \frac{\beta}{2}q^2 - pq \tag{49}$$

where $\alpha q - \frac{\beta}{2}q^2 - pq$ is consumer surplus. From (49) one can easily derive representative consumer's optimal choice q^d

$$q^d = \frac{\alpha - p}{\beta} \quad \Leftrightarrow \quad p = \alpha - \beta q^d \tag{50}$$

which is exactly the inverse demand function.

Market Equilibrium. A market equilibrium consists of a pair of output functions $q_1^*(c_1, s)$, $q_2^*(c_1, c_2)$, a market price p and a demand function $q^d(p)$ of the representative consumer such that:

- Given the inverse demand function $p = \alpha - \beta q$, the pair of output functions $q_1^*(c_1, s)$, $q_2^*(c_1, c_2)$ has to form a Bayes-Nash equilibrium of the Cournot subgame.
- Given the market price p, the representative consumer's demand q^d has to solve the maximization problem (49).
- The market price p has to clear the market, namely the aggregate market supply $q^*(c_1, c_2, s) \equiv q_1^*(c_1, s) + q_2^*(c_1, c_2)$ must equal to the market demand q^d.

Clearly the market equilibrium exits and is unique. We already know that $q_1^*(c_1, s)$, $q_2^*(c_1, c_2)$ is the unique Bayes-Nash equilibrium of the Cournot subgame, the market equilibrium price is simply $p^* = \alpha - \beta(q_1^*(c_1, s) + q_2^*(c_1, c_2))$, and according to (50), given $q_1^*(c_1, s) + q_2^*(c_1, c_2)$

Using the equilibrium output functions (35) and (36),

$$q^*(c_1, c_2, s) = q^0(c_1, c_2) + \eta^* \left[s - (\frac{1}{2}c_2 + \frac{1}{2}E(c_2|c_1)) \right] \tag{51}$$

where we use $q^0(c_1, c_2) = q_1^0(c_1) + q_2^0(c_1, c_2)$ to denote the aggregate equilibrium outputs under no-espionage scenario. Given c_1, c_2, s (ex post), whether aggregate output under espionage exceeds that under no espionage depends on whether the signal s is greater than the average of true c_2 and the prior expectation $E(c_2|c_1)$.

Using (50) and the market clearing condition $q^* = q^d$, ex post (given $c_1,\ c_2,\ s$) equilibrium consumer surplus is

$$CS^*(c_1, c_2, s) \equiv \alpha q - \frac{\beta}{2}q^2 - pq = \frac{\beta}{2}[q^*(c_1, c_2, s)]^2, \tag{52}$$

hence when $s > \frac{1}{2}c_2 + \frac{1}{2}E(c_2|c_1)$, consumer surplus is greater when Firm 1 does espionage; representative consumer suffers form espionage if the reverse holds.

The ex ante (given $c_1,\ c_2$) expected consumer surplus is

$$\begin{aligned} E_s[CS^*(c_1, c_2, s)|c_1, c_2] &= \frac{\beta}{2}E_s\left\{\left\{q^0(c_1, c_2) + \eta^*\left[s - \frac{1}{2}c_2 - \frac{1}{2}E(c_2|c_1)\right]\right\}^2 \middle| c_1, c_2\right\} \\ &= \frac{\beta}{2}\left\{[q^0(c_1, c_2)]^2 + 2q^0(c_1, c_2)\eta^* E_s\left[s - \frac{1}{2}c_2 - \frac{1}{2}E(c_2|c_1)\middle| c_1, c_2\right]\right. \\ &\quad \left. + (\eta^*)^2 E_s\left[[s - \frac{1}{2}c_2 - \frac{1}{2}E(c_2|c_1)]^2\middle| c_1, c_2\right]\right\} \\ &= \frac{\beta}{2}[q^0(c_1, c_2)]^2 + \frac{\beta}{2}\left\{q^0(c_1, c_2)\eta^*[c_2 - E(c_2|c_1)] + (\eta^*)^2\left[\frac{1}{\tau_\epsilon} + \frac{1}{4}[c_2 - E(c_2|c_1)]^2\right]\right\} \end{aligned} \tag{53}$$

the last equation is due to the fact that $E_s(s|c_1, c_2) = c_2$, and $E_s(s^2|c_1, c_2) = \mathrm{Var}(s|c_1, c_2) + [E(s|c_1, c_2)]^2 = 1/\tau_\epsilon + c_2^2$. The first term of (53) is the consumer surplus when there's no espionage. The second term of (53) captures the effect on consumer surplus when Firm 1 does espionage. It is evident that when $c_2 > E(c_2|c_1)$, the term in the brace of (53) is strictly positive and hence given $c_1,\ c_2$, expected consumer welfare is strictly higher when Firm 1 does espionage on Firm 2 to learn c_2. When $c_2 < E(c_2|c_1)$, the result is ambiguous since the two terms in the brace of (53) has different signs and which one dominates is indeterminate. When the difference between c_2 and $E(c_2|c_1)$ is small, since there's a positive term $(\eta^*)^2/\tau_\epsilon$ in the brace of (53), the expected consumer surplus under espionage would still be higher than under no espionage.

To summarize the finding in this subsection, and use Proposition 4, we have the following proposition.

Proposition 5 *Ex post, consumer surplus is strictly higher (lower) when Firm 1 does espionage then under no-espionage benchmark if and only if* $s > \frac{1}{2}c_2 + \frac{1}{2}E(c_2|c_1)$ *(*$s < \frac{1}{2}c_2 + \frac{1}{2}E(c_2|c_1)$*). Ex post, a sufficient (but not necessary) condition that the expected consumer surplus when Firm 1 does espionage is higher than when Firm 1 doesn't do espionage is* $c_2 > E(c_2|c_1)$*. Under this condition Firm 2's suffers from espionage but Firm 1 and consumers gain from espionage. There exist realizations of* $c_1,\ c_2$ *with* $c_2 < E(c_2|c_1)$ *such that both firms and consumers benefit from espionage in expectation.*

4 Only Firm 1 Can Credibly & Costlessly Disclose Information

In this section, we assume that Firm 1, if it decides to do espionage, then after it receives the noisy information s, it is able to disclose information about s (at least partly, to be made clear shortly). This assumption may be plausible if it is commonly known that signal s is acquired by Firm 1 using only public (or legal) informational sources. On the other hand, we maintain the assumption that Firm 2 cannot credibly and costlessly disclose information about its private information c_2.

The central questions we are interested in are the optimal disclosure policy of Firm 1 and how disclosure affects Firm 1's incentive to do espionage and the two firms' equilibrium output strategies and profits.

The timing of the game is the following:

1. Nature draws c_1, c_2. The realization of c_1 becomes common knowledge, while realized c_2 is Firm 2's private information.
2. Firm 1, after learning c_1, chooses τ_ϵ and receives a signal s. τ_ϵ is also known by Firm 2.
3. Given signal s, Firm 1 chooses a disclosure rule $\phi(s)$, the disclosed information ϕ is publicly known.
4. Firm 2, after observing the disclosed information about s, compete with Firm 1 in Cournot fashion.

Formally, following Milgrom (1981), Okuno-Fujiwara et al. (1990), a *disclosure rule* ϕ of Firm 1 is a set valued function that maps, for each signal s received, to a closed set R containing s. The requirement $s \in R$ means that Firm 1's report R must be truthful in the minimal sense. In one extreme case, the report R is a singleton, which perfectly reveals the signal s to Firm 2; another extreme case is just report the whole support, which is equivalent to revealing no information of s.

Firm 2, after receiving the report R from Firm 1, forms a belief $\lambda(s|R)$ concerning the true signal. A belief function $\lambda(\cdot)$ is said to be *consistent* with a reporting strategy ϕ if $\lambda(s|R)$ satisfies the following:

1. $\lambda(s|R) = 0$, if $s \notin R$,
2. $\lambda(s|R)$ is obtained using the Bayes rule whenever possible.

The first condition reflects the requirement that $s \in R$; the second one is the standard requirement that beliefs are consistent with the Bayesian updating.

A belief function $\lambda(\cdot)$ is called *skeptical* if for all R with $s \in R$,

$$\lambda(s|R) = \begin{cases} 1 & \text{if } s = \inf\{s'|s' \in R\} \\ 0 & \text{otherwise} \end{cases} \tag{54}$$

namely such a belief function put probability 1 on the lowest type of the reported set R.

Firm 1's information is $c_1,\ s$ and its strategy consists of a choice of espionage intensity τ_ϵ, disclosure rule $\phi(s)$, and output function $q_1(c_1, s, R)$; Firm 2's information in this game is $c_1,\ c_2,\ R$, based on its belief function $\lambda(R)$, it makes an output choice $q_2(c_1, c_2, \lambda(R))$.

Equilibrium. A strategy profile $\left(\{\tau_\epsilon^\diamond, \phi^\diamond(s), q_1^\diamond(c_1, s)\}, q_2^\diamond(c_1, c_2, R)\right)$ together with a belief function $\lambda^\diamond(R)$ of Firm 2 constitute a sequential equilibrium if

- Given $\tau_\epsilon^\diamond$, $q_1^\diamond(c_1, s)$, R, $\lambda^\diamond(R)$, for each realized c_2, $q_2^\diamond \in \arg\max_{q_2} E_s^2\left[\pi_2\left(q_1^\diamond(c_1, s), q_2, c_2\right)\Big|c_2, \lambda^\diamond(R)\right]$
- Given $\tau_\epsilon^\diamond$, s, $\phi^\diamond(s)$, $\lambda^\diamond(R)$, $q_2^\diamond(c_1, c_2, R)$, $q_1^\diamond \in \arg\max_{q_1} E_{c_2}^1\left[\pi_1\left(q_1, q_2^\diamond(c_1, c_2, R), c_1\right)\Big|c_1, s\right]$
- Given $\tau_\epsilon^\diamond$, s, $q_1^\diamond(c_1, s, R)$, $q_2^\diamond(c_1, c_2, R)$, $\lambda^\diamond(R)$, $\phi^\diamond(s)$ maximizes Firm 1's expected profit and satisfies $s \in \phi^\diamond(s)$ and that $\phi^\diamond(s)$ is closed.
- Given $\phi^\diamond(s)$, $q_1^\diamond(c_1, s)$, $q_2^\diamond(c_1, c_2, R)$, $\lambda^\diamond(R)$, the choice of τ_ϵ maximizes Firm 1's expected profit net the espionage cost $k(\tau_\epsilon)$
- Belief function $\lambda^\diamond(R)$ is consistent.

The first two conditions says that $q_1^\diamond,\ q_2^\diamond$ must be a Bayesian equilibrium for the Cournot subgame. The third and fourth equilibrium conditions says that given the subgame equilibrium strategies $q_1^\diamond$ and $q_2^\diamond$, the disclosure rule $\phi^\diamond$ and the espionage intensity $\tau_\epsilon^\diamond$ should be optimal.

Notice that by focusing on the sequential equilibria, we can rule out the non-sensible Nash equilibrium in which Firm 2 resolves to ignore Firm 1's report and Firm 1 makes only the uninformative reports, namely the whole support. At a sequential equilibrium, Firm 2 must act in its own interest and cannot resolve to ignore a report that is relevant to its optimal decision.

Before I state the next proposition, one more definition is required. We call a sequential equilibrium *fully revealing* if for any s, $\lambda^\diamond(s|\phi^\diamond(s)) = 1$, namely, Firm 2, upon receiving a report concerning s from Firm 1, can infer correctly the true signal s. The next proposition states that every sequential equilibrium involves full revelation and that Firm 2's equilibrium belief function must be skeptical.

Proposition 6 *When Firm 1 is able to disclose its private signal s credibly and costlessly, any sequential equilibrium is fully revealing with skeptical belief (for both on and off equilibrium reports).*

Proof. The proof is essentially follows Theorem 1 of Okuno-Fujiwara et al. (1990).

First notice both firms' equilibrium output functions are strictly decreasing in their own marginal costs and are strictly increasing in expected output of the rival firm, since the goods are substitutes and the output strategies are strategic substitutes. This can be seen explicitly from the first order conditions (7) and (9). Therefore, consider two beliefs $\lambda,\ \lambda'$ with λ strictly first order stochastically dominates λ'. Then under belief λ, Firm 1 can make higher profit in expectation.

Next, we show that any sequential equilibrium belief function $\lambda^{\Diamond}(R)$ must be skeptical for both on and off equilibrium reports. Consider any such sequential equilibrium. Suppose this conclusion doesn't hold. Then there's exists a report $\bar{R}$ such that $\lambda^{\Diamond}(\bar{R})$ is not degenerate at its lowest element. Let $\bar{s} \in \bar{R}$ be the largest element in $\hat{R}$ that is being assigned positive probability (if the largest element doesn't exist, consider any element that is strictly greater than the smallest of $\bar{R}$). The $\bar{s}$ type, by reporting the closed set in which $\bar{s}$ is the lowest type, can induce a belief that strictly first order stochastically dominates $\lambda^{\Diamond}(R)$. Hence type $\bar{s}$ can make a strictly higher profit than under the hypothetical equilibrium, which is a contradiction. Similarly, consider any off equilibrium report R. Suppose the equilibrium belief $\lambda^{\Diamond}(R)$ put strictly positive probability on a higher type in R, say $\hat{s}$. Then the lowest type in R is strictly better off by making this off equilibrium report R than following its equilibrium report (under which it will be regarded for sure as the lowest type in the report, which is itself). ■

Note full revelation doesn't necessarily implies that in equilibrium Firm 1's disclosure rule is $\phi^*(s) = s,\ \forall s$. Other disclosure rules satisfy as well, for example, $\phi^*(s) = [s, \infty)$. Given the equilibrium belief is sceptical, clearly we have full revelation in this case.

Given this proposition, in equilibrium Firm 2 learns perfectly Firm 1's private signal s if Firm 1 is able to disclose information about s credibly and costlessly. This will change the nature of competition between the two firms in the Cournot subgame. Now in Cournot stage, the information available to Firm 2 is $c_1,\ c_2$ and s. But as we see in the previous section in which Firm 1 cannot credibly and costlessly disclose information about s, Firm 2's strategy doesn't depend on s. A natural question to ask is how disclosure affects the competition between the two firms. Furthermore, how full disclosure affects Firm 1's incentive to do espionage? In other words, if Firm 1 is aware that it can credibly and costlessly convince the rival that through costly espionage it received signal s about its true cost c_2, will Firm 1 invest more heavily in espionage in the first place?

Next proposition gives the two firms' equilibrium output strategies in the Cournot subgame.

Proposition 7 *Given τ_ϵ, the realized $c_1,\ c_2,\ s$, and that there's full disclosure in equilibrium, there exists a unique equi-*

librium $\{q_1^\diamond(c_1, s),\ q_2^\diamond(c_1, c_2, s)\}$ *for the Cournot subgame, which are given by the following*

$$
\begin{aligned}
q_1^\diamond(c_1, s) &= \frac{\alpha}{3\beta} + \frac{(1-\rho)\mu_c\tau_c}{3\beta[(1-\rho^2)\tau_\epsilon + \tau_c]} \\
&\quad - \left[\frac{2}{3\beta} - \frac{\rho\tau_c}{3\beta[(1-\rho^2)\tau_\epsilon + \tau_c]}\right] c_1 + \frac{(1-\rho^2)\tau_\epsilon}{3\beta[(1-\rho^2)\tau_\epsilon + \tau_c]} s \qquad (55) \\
q_2^\diamond(c_1, c_2, s) &= \frac{\alpha}{3\beta} - \frac{(1-\rho)\mu_c\tau_c}{6\beta[(1-\rho^2)\tau_\epsilon + \tau_c]} \\
&\quad + \left[\frac{1}{3\beta} - \frac{\rho\tau_c}{6\beta[(1-\rho^2)\tau_\epsilon + \tau_c]}\right] c_1 - \frac{1}{2\beta} c_2 - \frac{(1-\rho^2)\tau_\epsilon}{6\beta[(1-\rho^2)\tau_\epsilon + \tau_c]} s \qquad (56)
\end{aligned}
$$

Proof. Given the full disclosure result, now Firm 2's output strategy depends on $c_1,\ c_2,\ s$. One can easily derive the first order conditions from the two firms' objective functions in the Cournot competition stage:

$$
\begin{aligned}
q_1(c_1, s) &= \frac{\alpha - c_1}{2\beta} - \frac{1}{2} E_{c_2}[q_2(c_1, c_2, s) | c_1, s] \qquad (57) \\
q_2(c_1, c_2, s) &= \frac{\alpha - c_2}{2\beta} - \frac{1}{2} q_1(c_1, s) \qquad (58)
\end{aligned}
$$

Substitute (58) into Firm 1's first order condition (57) one obtains

$$
\begin{aligned}
q_1 &= \frac{\alpha - c_1}{2\beta} - \frac{1}{2} E_{c_2}\left[\frac{\alpha - c_2}{2\beta} - \frac{1}{2} q_1 \middle| c_1, s\right] \\
&= \frac{\alpha - c_1}{2\beta} - \frac{\alpha}{4\beta} + \frac{1}{4\beta} E_{c_2}(c_2 | c_1, s) + \frac{1}{4} q_1
\end{aligned}
$$

The second equality holds since Firm 1 knows $c_1,\ s$, hence q_1 only depends on $c_1,\ s$, hence $E_{c_2}(q_1 | c_1, s) = q_1$. Plug in expression for $E_{c_2}(c_2 | c_1, s)$ (which is given by (5)) and rearrange, we can solve for the equilibrium output function $q_1^\diamond(c_1, s)$, which is exactly the one given in the proposition.

$q_2^\diamond(c_1, c_2, s)$ is obtained directly by plugging in (58) the expression for $q_1^\diamond(c_1, s)$ obtained above. ■

A more useful and insightful way to express $q_1^\diamond(c_1, s),\ q_2^\diamond(c_1, c_2, s)$ is following:

Corollary 3

$$
\begin{aligned}
q_1^\diamond(c_1, s) &= q_1^0(c_1) + \eta^\diamond[s - E(c_2|c_1)] \qquad (59) \\
q_2^\diamond(c_1, c_2, s) &= q_2^0(c_1, c_2) - \frac{\eta^\diamond}{2}[s - E(c_2|c_1)] \qquad (60)
\end{aligned}
$$

where $\eta^\diamond$ *is a positive constant given by*

$$
\eta^\diamond = \frac{(1-\rho^2)\tau_\epsilon}{3\beta[(1-\rho^2)\tau_\epsilon + \tau_c]} \qquad (61)
$$

which is positive, strictly increasing and concave in τ_ϵ.

Again as in the case in which no firm can disclose private information credibly and costlessly, the equilibrium output functions can be decomposed into two parts: a no-espionage benchmark level plus an adjustment term. The effect of espionage is fully captured by the adjustment terms. For Firm 1, the term $\eta^\diamond[s - E(c_2|c_1)]$ measures by how much Firm 1 is "surprised" by the signal s, and the coefficient $\eta^\diamond$ measures how sensitive Firm 1 reacts to this "surprise". Similar interpretation holds for $q_2^\diamond$), with the distinction that with disclosure by Firm 1, Firm 2's adjustment term is based on the difference $s - E(c_2|c_1)$ instead of on $c_2 - E(c_2|c_1)$.

Note Firm 1 is twice as sensitive to the "surprise" term as Firm 2, like in the case when no firm can disclose private information credibly and costlessly.

Now we go back one step and consider what's the optimal choice of espionage intensity of Firm 1. Using the first order condition for Firm 1's problem, Firm 1's ex post equilibrium profit can be simplified to

$$\pi_1^\diamond = \beta[q_1^\diamond(c_1, s)]^2. \tag{62}$$

Plug in the expression for $q_1^\diamond(c_1, s)$ (59), Firm 1's ex ante expected profit (conditional on $c_1,\ \tau_\epsilon$) is

$$\beta E_s\left\{[q_1^\diamond(c_1, s)]^2\middle|c_1, \tau_\epsilon\right\} = \underbrace{\beta[q_1^0(c_1)]^2}_{\pi_1^0(c_1)} + \underbrace{B^\diamond(\tau_\epsilon)}_{\text{benefit function}} \tag{63}$$

where

$$B^\diamond(\tau_\epsilon) = \frac{(1-\rho^2)^2\tau_\epsilon}{9\beta\tau_c[(1-\rho^2)\tau_\epsilon + \tau_c]} \tag{64}$$

is a positive, strictly increasing (hence more precise information about c_2 is always valuable) and concave function in τ_ϵ that captures the extra benefit to Firm 1 due to espionage.

Firm 1's objective in the first stage is to choose τ_ϵ to maximize its net expected profit:

$$\begin{aligned} &\max_{\tau_\epsilon \in [0,\infty)} E_s\left[\pi_1^\diamond(c_1, s)|c_1)\right] - k(\tau_\epsilon) \\ \Leftrightarrow \quad &\max_{\tau_\epsilon \in [0,\infty)} \beta[q_1^0(c_1)]^2 + \underbrace{B^\diamond(\tau_\epsilon) - k(\tau_\epsilon)}_{\text{net benefit from espionage}} \end{aligned} \tag{65}$$

The first part on the right hand side is firm 1's benchmark profit under no-espionage scenario which doesn't involve τ_ϵ. By assumption $k(\tau_\epsilon)$ is strictly convex and that we've shown $B^\diamond(\tau_\epsilon)$ is strictly concave, hence the optimal choice $\tau_\epsilon^\diamond$ is pinned down by the first order condition:

$$MB^{\diamond}(\tau_\epsilon) \equiv \frac{dB^{\diamond}(\tau_\epsilon)}{d\tau_\epsilon} = k'(\tau_\epsilon) \tag{66}$$

This first order condition simply equates the marginal benefit of espionage (denoted as $MB^{\diamond}(\tau_\epsilon)$) to the marginal cost $k'(\tau_\epsilon)$. In the next proposition, we show that given $q_1^{\diamond}$, $q_2^{\diamond}$, there's a unique optimal choice of τ_ϵ, which is always positive. Hence the entire game has a unique equilibrium.

Using first order conditions, the two firms' equilibrium profits are $\pi_1^{\diamond}(c_1, s) = \beta[q_1^{\diamond}(c_1, s)]^2$, $\pi_2^{\diamond}(c_1, c_2, s) = \beta[q_1^{\diamond}(c_1, c_2, s)]^2$. In the next theorem, we show that given $q_1^{\diamond}$, $q_2^{\diamond}$, there's a unique optimal choice of τ_ϵ, which is always positive. Hence the entire game has a unique equilibrium.

Proposition 8 *When only Firm 1 can credibly and costlessly disclose its private information, in equilibrium Firm 1 will do espionage on Firm 2, namely $\tau_\epsilon^{\diamond} > 0$. The optimal signal precision $\tau_\epsilon^{\diamond}$ is pinned down by the unique solution to the following equation*

$$\frac{(1-\rho^2)^2}{9\beta\left[(1-\rho^2)\tau_\epsilon + \tau_c\right]^2} = k'(\tau_\epsilon). \tag{67}$$

For the linear-normal model we consider, $\tau_\epsilon^{\diamond}$ doesn't depend on the realization of c_1.

Proof. Using Firm 1's equilibrium output function (59), Firm 1's ex ante expected net profit when espionage intensity is τ_ϵ is

$$\begin{aligned} & E_s\left[\beta[q_1^{\diamond}(c_1, s, \tau_\epsilon)]^2 | c_1\right] - k(\tau_\epsilon) \\ = \; & \beta\left\{[q_1^0(c_1)]^2 + (\eta^{\diamond})^2 \mathrm{Var}(s|c_1)\right\} - k(\tau_\epsilon) \\ = \; & \beta[q_1^0(c_1)]^2 + \frac{(1-\rho^2)^2\tau_\epsilon}{9\beta\tau_c[(1-\rho^2)\tau_\epsilon + \tau_c]} - k(\tau_\epsilon) \end{aligned}$$

The first equation follows from the fact that $E_s[s - E_{c_2}(c_2|c_1)|c_1] = 0$, since $s = c_2 + \epsilon$ and ϵ, c_1, c_2 are independent. The second equation follows by substituting the result $\mathrm{Var}(s|c_1) = (1-\rho^2)\frac{1}{\tau_c} + \frac{1}{\tau_\epsilon}$, which is obtained in the proof to Proposition 3.

One can easily check the second order condition is satisfied:

$$\frac{\partial^2\left\{E_s[\pi_1^{\diamond}(c_1, s, \tau_\epsilon)|c_1] - k(\tau_\epsilon)\right\}}{\partial\tau_\epsilon^2} = -\frac{2(1-\rho^2)}{9\beta[(1-\rho^2)\tau_\epsilon + \tau_c]^3} - k''(\tau_\epsilon) < 0$$

So the optimal choice of τ_ϵ is pinned by the first order condition:

$$\frac{(1-\rho^2)^2}{9\beta[(1-\rho^2)\tau_\epsilon + \tau_c]^2} = k'(\tau_\epsilon)$$

The expression on the left hand side is positive, continuous, strictly decreasing in τ_ϵ and approaches 0 when τ_ϵ goes to infinity. The expression on the right hand side is continuous, strictly increasing in τ_ϵ and by our assumption $\lim_{\tau_\epsilon \to 0+} k'(\tau_\epsilon) = 0$. Hence there's a unique solution $\tau_\epsilon^\Diamond \in (0, \infty)$ that satisfies the first order condition. ■

Like Corollary 2, one can obtain similar comparative statics result for $\tau_\epsilon^\Diamond$, the proof is essentially the same as Corollary 2.

Corollary 4 *(Comparative Statics) The optimal espionage intensity $\tau_\epsilon^\Diamond$ strictly decreases when c_1, c_2 are more correlated and when the Firm 1's prior knowledge about c_2 is more precise.*

The next question to study is how disclosure of s by Firm 1 affects its incentive to do espionage. We would like to know in equilibrium, which one of τ_ϵ^* and $\tau_\epsilon^\Diamond$ is greater. It turns out that this comparison depends on the shape of cost function $k(\tau_\epsilon)$ as well as the two marginal benefit functions of espionage intensity $MB^*(\tau_\epsilon)$, $MB^\Diamond(\tau_\epsilon)$.

Essentially we are comparing first order conditions (43) and (67) since τ_ϵ^* and $\tau_\epsilon^\Diamond$ are pinned down by these equations respectively. Note the right hand side of (43) and (67) are both the same marginal cost function $k'(\tau_\epsilon)$. The difference is the left hand side, namely the marginal benefit function of espionage. Let's compare the two left hand expressions for a given τ_ϵ.

$$\begin{aligned} MB^\Diamond(\tau_\epsilon) - MB^*(\tau_\epsilon) &= \frac{(1-\rho^2)^2}{9\beta[(1-\rho^2)\tau_\epsilon + \tau_c]^2} - \frac{5(1-\rho^2)^3 + 4(1-\rho^2)^2\tau_c}{\beta[3(1-\rho^2)\tau_\epsilon + 4\tau_c]^3} \\ &= \frac{(1-\rho^2)^2}{9\beta[(1-\rho^2)\tau_\epsilon + \tau_c]^2[3(1-\rho^2)\tau_\epsilon + 4\tau_c]^3} \underbrace{\left\{[(1-\rho^2)\tau_\epsilon + \tau_c][27\tau_c^2 - 18(1-\rho^2)^2\tau_\epsilon^2] + \tau_c^3\right\}}_{\equiv H(\tau_\epsilon;\tau_c,\rho)} \end{aligned} \quad (68)$$

The sign of the difference is the same as the sign of the expression in the brace $H(\tau_\epsilon; \tau_c, \rho)$. Note for any given parameters $\tau_c > 0$, $\rho < 1$, the function $H(\tau_\epsilon; \tau_c, \rho)$ is strictly decreasing. In addition, $\lim_{\tau_\epsilon \to 0+} H(\tau_\epsilon; \tau_c, \rho) = 28\tau_c^3 > 0$, $\lim_{\tau_\epsilon \to \infty} H(\tau_\epsilon; \tau_c, \rho) < 0$ and that the function $H(\tau_\epsilon; \tau_c, \rho)$ is continuous in τ_ϵ, so there's exists a unique solution to the equation $H(\tau_\epsilon; \tau_c, \rho) = 0$. Denote this unique solution as $\hat{\tau}_\epsilon(\tau_c, \rho)$. Clearly at $\hat{\tau}_\epsilon$, the two marginal benefit functions are equal; for $\tau_\epsilon < \hat{\tau}_\epsilon$, the marginal benefit function under disclosure of s exceeds that under no disclosure; for $\tau_\epsilon > \hat{\tau}_\epsilon$, the reverse holds.

Since τ_ϵ^*, $\tau_\epsilon^\Diamond$ are pinned down by the intersections of the marginal benefit curves and the marginal cost function $k'(\tau_\epsilon)$, if $k'(\tau_\epsilon)$ intersects τ_ϵ^*, $\tau_\epsilon^\Diamond$ before the threshold $\hat{\tau}_\epsilon$, then $\tau_\epsilon^* > \tau_\epsilon^\Diamond$, otherwise $\tau_\epsilon^* < \tau_\epsilon^\Diamond$. See Figure 4 for an illustration.

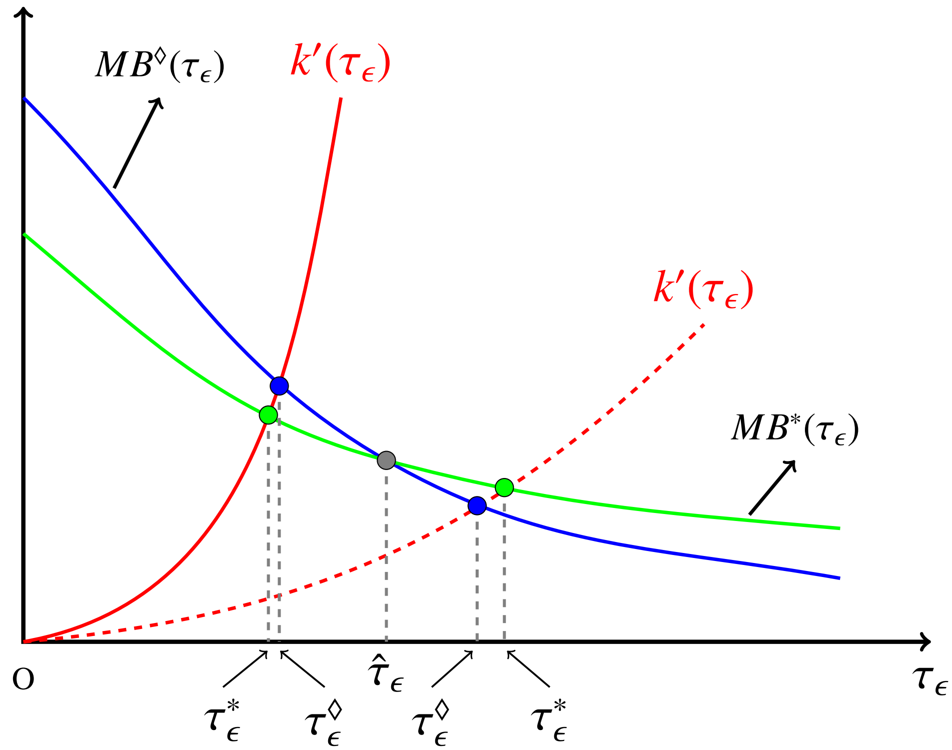

Figure 4: Comparison Between τ_ϵ^* and $\tau_\epsilon^{\lozenge}$

Hence the comparison of the optimal levels of espionage intensity is not deterministic, depending crucially on the shape of the marginal cost function $k'(\tau_\epsilon)$. The following proposition summarizes the relations between the two marginal benefit functions and the optimal choices of espionage intensity $\tau_\epsilon^*,\ \tau_\epsilon^{\lozenge}$.

Proposition 9 *There exists a unique* $\hat{\tau}_\epsilon > 0$ *such that* $MB^*(\hat{\tau}_\epsilon) = MB^{\lozenge}(\hat{\tau}_\epsilon)$. *For* $\tau_\epsilon < \hat{\tau}_\epsilon$, $MB^{\lozenge}(\tau_\epsilon) > MB^*(\tau_\epsilon)$*; for* $\tau_\epsilon > \hat{\tau}_\epsilon$, $MB^{\lozenge}(\tau_\epsilon) < MB^*(\tau_\epsilon)$. *Given* $\tau_c > 0,\ \rho < 1$, *when* $k(\tau_\epsilon)$ *is sufficiently convex,* $\tau_\epsilon^* < \tau_\epsilon^{\lozenge}$*; when* $k(\tau_\epsilon)$ *is sufficiently flat,* $\tau_\epsilon^* > \tau_\epsilon^{\lozenge}$.

The intuition is following: Recall When Firm 1 can credibly and costlessly disclose s, due to full disclosure result in equilibrium , both firms have better information of the rival's output strategy [9] Suppose the signal precision τ_ϵ is very low, consider a marginal increment based on this low level of τ_ϵ. Under disclosure of s, ?????????

4.1 Profit Comparison

We've already seen in the previous section that in equilibrium Firm 1 benefit strictly from doing espionage on Firm 2 to learn c_2; whether Firm 2 will benefit or suffer from Firm 1's espionage crucially depends on the realization of $c_1,\ c_2$: For relatively high c_1 and low c_2, Firm 2 would benefit, when c_1 is low and c_2 is high, Firm 2 would suffer a loss. In the present

[9] This has been discussed in the paragraph following Corollary 3. Essentially with disclosure the equilibrium output strategies are more sensitive to the difference $s - E(c_2|c_1)$ than under no disclosure, namely $\eta^{\lozenge}(\tau_\epsilon) > \eta^*(\tau_\epsilon)$ for fixed τ_ϵ.

scenario when Firm 1 is able to do espionage and disclose its private information s about c_2 costlessly and credibly to Firm 2, what would be the impact on the two firms' profits? This is the purpose of this subsection.

Let's first compare both firms' profits under disclosure of s with their respective profit under no-espionage benchmark (Benchmark 2). Consider Firm 1. By adopting equilibrium output strategy $q_1^\Diamond(c_1, s)$, in the proof to Theorem 8, its net expected profit is

$$\begin{aligned} & E_s\{\beta[q_1^\Diamond(c_1, s; \tau_\epsilon)]^2|c_1\} - k(\tau_\epsilon) \\ = \; & \beta[q_1^0(c_1)]^2 + \underbrace{\frac{(1-\rho^2)^2\tau_\epsilon}{9\beta\tau_c[(1-\rho^2)\tau_\epsilon + \tau_c]}}_{\equiv B^\Diamond(\tau_\epsilon)} - k(\tau_\epsilon) \end{aligned} \tag{69}$$

Recall from (34) that Firm 1's equilibrium expected profit under no-espionage benchmark is $\beta[q_1^0(c_1)]^2$, hence $B^\Diamond(\tau_\epsilon)$ is the extra benefit that Firm 1 gains in equilibrium due to espionage. The comparison between $B^\Diamond(\tau_\epsilon^\Diamond)$ and $k(\tau_\epsilon^\Diamond)$ determines whether Firm 1 will benefit from doing espionage. As in the previous section, given our assumptions on $k(\tau_\epsilon)$, we have $B^\Diamond(\tau_\epsilon^\Diamond) > k(\tau_\epsilon^\Diamond)$, hence in equilibrium, Firm 1, when able to credibly and costlessly disclose s, will strictly benefit from espionage.

Now consider Firm 2. Its ex post (given $c_1,\ c_2,\ s$) equilibrium profit is $\beta\left[q_2^\Diamond(c_1, c_2, s)\right]^2$. From the expression of $q_2^\Diamond(c_1, c_2, s)$ (see 60), $\beta\left[q_2^\Diamond(c_1, c_2, s)\right]^2 > \beta\left[q_2^0(c_1, c_2)\right]^2$ iff $s > E(c_2|c_1)$. This is similar to condition (47) when Firm 1 cannot disclose information about s, but here with full disclosure of s, the comparison hinges on s instead of on c_2. Ex ante (prior to the realization of s and conditional on $c_1,\ c_2$), Firm 2's expected profit is given by

$$\begin{aligned} & \beta E_s\left\{\left[q_2^\Diamond(c_1, c_2, s)\right]^2 \middle| c_1, c_2\right\} \\ = \; & \beta E_s\left\{\left[q_2^0(c_1, c_2) - \eta_2^\Diamond[s - E(c_2|c_2)]\right]^2 \middle| c_1, c_2\right\} \\ = \; & \beta\left[q_2^0(c_1, c_2)\right]^2 - 2\beta\eta_2^\Diamond q_2^0(c_1, c_2)[c_2 - E(c_2|c_2)] + \beta(\eta_2^\Diamond)^2\left[\frac{1}{\tau_\epsilon} + [c_2 - E(c_2|c_1)]^2\right] \end{aligned} \tag{70}$$

where the second equality is due to $E[s - E(c_2|c_1)|c_1, c_2] = c_2 - E(c_2|c_1)$ and that

$$\begin{aligned} & E_s\left\{[s - E(c_2|c_1)]^2 \middle| c_1, c_2\right\} \\ = \; & E(s^2|c_1, c_2) - 2E(s|c_1, c_2)E(c_2|c_1) + [E(c_2|c_1)]^2 \\ = \; & \text{Var}(s|c_1, c_2) + [E(s|c_1, c_2)]^2 - 2c_2E(c_2|c_1) + [E(c_2|c_1)]^2 \\ = \; & \frac{1}{\tau_\epsilon} + [c_2 - E(c_2|c_1)]^2 \end{aligned}$$

Following (70), when $c_2 < E(c_2|c_1)$, the expected profit for Firm 2 under disclosure of s by Firm 1 is greater than its equilibrium profit when Firm 1 doesn't do espionage. However, unlike the case in which Firm 1 cannot disclose information about s, the condition $c_2 < E(c_2|c_1)$ is only sufficient but not necessary.

Next we compare the two firms' profits between the two regimes. The major difficulty for this comparison is that the equilibrium choice of τ_ϵ^* and $\tau_\epsilon^\diamond$ are generally different and they are determined implicitly by equations (43) and (67), but we can utilize some results that are already available to provide us with some answer to this question.

Consider Firm 1 and fix a particular espionage intensity τ_ϵ. According to (35) and (59), the difference between these two expected profits stem from the extra benefit functions $B^*(\tau_\epsilon)$ and $B_\epsilon^\diamond$. After some algebraic manipulation, we have

$$B^*(\tau_\epsilon) - B^\diamond(\tau_\epsilon) = \frac{-6(1-\rho^2)^3\tau_\epsilon^2\tau_c - 7(1-\rho^2)^2\tau_\epsilon\tau_c^2}{9\beta[3(1-\rho^2)\tau_\epsilon + 4\tau_c]^2[(1-\rho^2)\tau_\epsilon + \tau_c]} < 0 \tag{71}$$

meaning for any fixed $\tau_\epsilon > 0$, the Firm 1's expected net profit is higher when Firm 1 is able to disclose private information s credibly and costlessly to Firm 2.

Now recall Theorem 9 that compares τ_ϵ^* with $\tau_\epsilon^\diamond$. Together with the observation above that $B^*(\tau_\epsilon) < B^\diamond(\tau_\epsilon)$, we have the following result: When $k(\tau_\epsilon)$ is sufficiently convex, Firm 1's equilibrium net profit is strictly higher when it can disclose s credibly and costlessly. For relatively flat cost function $k(\tau_\epsilon)$, it may happen that under no disclosure scenario, Firm 1's profit is higher.

Consider now Firm 2. From Theorem 4 and we know with no disclosure of s by Firm 1, Firm 2's profit is lower than with no espionage for relatively high c_2 and low c_1. We've just show earlier in this subsection that under disclosure of s, Firm 2's profit is always higher than that under no espionage. Hence a sufficient condition that under disclosure, Firm 2's profit is strictly higher than the scenario with espionage but no disclosure is that the realized c_1, c_2 satisfy $c_1 \leq -[(1-\rho)/\rho]\mu_c + (1/\rho)c_2$ (namely the unshaded region in Figure 3). For $c_1,\ c_2$ that violates this condition, we know under espionage, both with disclosure of s or not, Firm 2's profits are both higher than no espionage, but it not clear among these two scenarios, which one yields higher profit.

4.2 Consumer Surplus

Again suppose there's a representative consumer with utility function $u(q) = \alpha q - \frac{\beta}{2}q^2 + m$, as in Section 3.2. The market equilibrium is a pair of output functions $q_1^\diamond(c_1, s)$, $q_2^\diamond(c_1, c_2, s)$ a market price p and a demand function $q^d(p)$ such that given the inverse demand function the pair of output functions $q_1^\diamond(\cdot)$, $q_2^\diamond(\cdot)$ forms a Bayes-Nash equilibrium of the Cournot subgame; given price p, representative consumer chooses q^d to maximizes its utility; and that the market price p clears the market.

Let $q^\diamond(c_1, c_2, s)$ be the ex post (given $c_1,\ c_2,\ s$) aggregate market output. Similar to (52), one can show easily that ex post equilibrium consumer surplus is given by

$$CS^\diamond(c_1, c_2, s) = \frac{\beta}{2}[q^\diamond(c_1, c_2, s)]^2. \tag{72}$$

From equilibrium output functions (59) and (60),

$$q^\diamond(c_1, c_2, s) = \eta^0(c_1, c_2) + \frac{\eta^\diamond}{2}[s - E(c_2|c_1)] \tag{73}$$

Following (72) and (73), consumer surplus is strictly greater (less) than under no-espionage benchmark (which is $(\beta/2)[q^0(c_1, c_2)]^2$) ex post if and only if $s > E(c_2|c_1)$ ($s < E(c_2|c_1)$). Recall from Subsection 4.1 that ex post Firm 2's equilibrium profit is strictly lower (greater) than under no-espionage benchmark if and only if the condition $s > E(c_2|c_1)$ ($s < E(c_2|c_1)$) holds. Therefore, in the ex post sense, there's a tradeoff between Firm 2's profit and consumer surplus, while Firm 1 strictly benefit from espionage.

Ex ante (given $c_1,\ c_2$), the expected consumer surplus is

$$\begin{aligned} E_s[CS^\diamond(c_1, c_2, s)|c_1, c_2] &= \frac{\beta}{2}E_s\left\{[q^\diamond(c_1, c_2, s)]^2\middle|c_1, c_2\right\} \\ &= \frac{\beta}{2}E_s\left\{\left[q^0(c_1, c_2) + \frac{\eta^\diamond}{2}(s - E(c_2|c_1))\right]^2\middle|c_1, c_2\right\} \\ &= \frac{\beta}{2}[q^0(c_1, c_2)]^2 + \frac{\beta}{2}\left\{\eta^\diamond q^0(c_1, c_2)[c_2 - E(c_2|c_1)] + \frac{(\eta^\diamond)^2}{4}\left[\frac{1}{\tau_\epsilon} + (c_2 - E(c_2|c_1))^2\right]\right\}. \end{aligned} \tag{74}$$

The last equation follows from $E(s|c_1, c_2) = c_2$ and $E_s\{[s - E(c_2|c_1)]^2|c_1, c_2\} = 1/\tau_\epsilon + [c_2 - E(c_2|c_1)]^2$.

The first term of (74) is the consumer surplus when there's no espionage. The second term of (53) captures the effect on consumer surplus when Firm 1 does espionage. A sufficient condition that the ex ante consumer surplus when Firm 1 does espionage and is able to disclose s credibly and costlessly is strictly higher than under no-espionage benchmark is that $c_2 > E(c_2|c_1)$. But this condition is not necessary, namely there are realizations of $c_1,\ c_2$ such that $c_2 < E(c_2|c_1)$ but consumer surplus is still higher than under no-espionage benchmark. For example, when c_2 slightly less than $E(c_2|c_1)$, since there's a positive term $\frac{(\eta^\diamond)^2}{4\tau_\epsilon}$ in the bracket of (74), the expression in the bracket would be positive, hence expected consumer surplus under espionage is still higher than no-espionage benchmark. Put it in another way, ex ante, there are realizations of $c_1,\ c_2$ such that under espionage, both the two firms and the representative consumer benefit.

The following proposition summarizes the findings in this subsection.

Proposition 10 *Ex post, consumer surplus is strictly higher*

5 Only Firm 2 Can Credibly & Costlessly Disclose Private Information

to be added......

6 Extensions

In this section, we consider a few extensions.

6.1 Unobservable Espionage Intensity

In our main model, we only focused on the case in which Firm 1's espionage intensity measured by τ_ϵ is observable by Firm 2. In this extension, we consider unobservable τ_ϵ.

When both firms cannot disclose private information, Firm 1's strategy is still $(\tau_\epsilon^*, q_1^*(c_1, s))$. Firm 2, in addition to choosing a output strategy $q_2^*(c_1, c_2)$, has to form a conjecture about Firm 1's choice of τ_ϵ^*, denoted as $\bar{\tau}_\epsilon$, since the actual choice τ_ϵ^* is unobservable. The equilibrium of the game is defined as following:

Equilibrium. A strategy Profile $\left((\tau_\epsilon^*, q_1^*(c_1, s)), q_2^*(c_1, c_2)\right)$ together with a conjecture $\bar{\tau}_\epsilon$ is an equilibrium if

- Given Firm 2's conjecture $\bar{\tau}_\epsilon$, $q_1^*(c_1, s)$, for each realized pair (c_1, c_2), $q_2^*(c_1, c_2)$ maximizes Firm 2's expected profit.
- Given τ_ϵ^*, Firm 2's conjecture $\bar{\tau}_\epsilon$, $q_2^*(c_1, c_2)$, for each realized pair (c_1, s), $q_1^*(c_1, s)$ maximizes Firm 1's expected profit.
- Given $q_1^*(c_1, s)$, $q_2^*(c_1, c_2)$, $\bar{\tau}_\epsilon$, the choice of τ_ϵ^* maximizes Firm 1's expected net profit (subtract cost of espionage).
- Firm 2's conjecture $\bar{\tau}_\epsilon$ coincides with Firm 1's actual choice τ_ϵ^*.

to be completed......

7 Conclusion

to be added......

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