Mitigating Matching Externalities Via The "Old Boys' Club" *

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Abstract

This paper introduces a dynamic matching mechanism in which a persistent contracting relationship - an "old boys' club" - occurs in *ex post* subgame perfect Nash equilibrium when the high school in the club is sufficiently patient. Matching occurs in two stages: first, contracting between the college and a high school; second, running a Vickrey auction in the simplified post-contracting admissions market. The mechanism provides the second best total surplus among several mechanisms in a repeated college admissions market in which externalities preclude solutions using standard mechanism and market design techniques. An "old boys' club" emerges between one college and a sufficiently patient single high school as a consequence of contract enforcement rather than *ex ante* bias on the part of the college. Members of the club benefit at the expense of the non-contracted high school.

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1 Introduction

This paper shows how a persistent contracting relationship emerges in a repeated college admissions market from a randomly initiated one-shot contract offered by an unbiased expected utility maximizing college. Faced with an admissions problem that cannot be addressed by an efficient, incentive compatible market design, a college offers a contract to a randomly selected high school; they agree that the college will admit some number of the high school's most promising students immediately. The high school agrees to this contract because it is happy to guarantee admission for as many of its students as possible in advance of the general admissions market. The college offers this contract for two reasons: first, it too is happy to guarantee itself sufficiently good students in advance of the general admissions market; second, contracting away some its slots reduces the complexity of its general admissions market design problem. The contracted high school will not renege on the contract in the sense of sending subpar students to fill the contracted slots because the college learns from the general admissions market whether the contracted high school reneged. The college credibly promises to reward the contracted high school's compliance with a future contract and credibly threatens to punish the contracted high school's noncompliance by contracting with some other high school in one or more subsequent rounds of admissions.

An obvious objection grounded in a human sense of equity arises: the high school holding the contract is at a distinct advantage since students admitted via the contracted slots avoid the need to compete in the general admissions market. Moreover, the method of contract enforcement guarantees that if the initially contracted high school is sufficiently patient, it will honor the contract in every round of admissions, guaranteeing that its contracting relationship will persist *ad infinitum*. This paper assumes that the high schools are *ex ante* identical and the college is *ex ante* unbiased in the sense that it offers the initial contract at random. Nevertheless, the lifetime welfare loss to a non-contracted high school is large. In the context of college admissions in the United States, intertwined with historical cultural biases that favor of certain demographics over others, it leaves the uncomfortable conclusion that merely mandating equal consideration of all applicants is insufficient to dismantle these long standing relationships, hereinafter referred to as "old boys' clubs."

Matching markets in which there is no mechanism that is both efficient and incentive compatible abound; indeed this type of market is the rule rather than the exception. The main results in this vein are due to Vickrey [10], who showed that there is no auction in which both buyers and sellers reveal their private values truthfully; Roth [8], who showed that there is no matching mechanism in which participants on both sides of the market reveal their preferences truthfully; and Jehiel and Moldovanu [6], who showed that in multi-unit auctions with sufficient interdependence there is in general no efficient, incentive compatible auction mechanism. The lack of a mechanism that is both efficient and incentive compatible does not mitigate the need of participants in these markets to form matches by some means, however imperfect the method or outcome may be. A college faced with this problem nonetheless needs to admit a freshman class every year: educating students is its *raison d'être*. It is therefore natural to study inefficient but incentive compatible mechanisms to understand both how these mechanisms perform and how their inefficiencies are distributed. Both the initial contract and its persistence are completely understandable from the classical economic standpoint of selfish, expected utility maximizing agents. However, despite *ex ante* equal expected lifetime payoffs for the high schools, as soon as the initial contract is signed, one high school benefits from being a member of the "old boys' club" in every single subsequent round while the other is left out forever. The college benefits from the "old boys' club" regardless of which high school belongs to the club; however, once established, the college needs to maintain the contracting relationship in order to keep the good students coming from the contracted high school.

There are two main lenses through which to view these results. One option is in the tradition of Chatterjee and Samuelson's work on bilateral trade [2]. That work characterizes how frequently a natural trading mechanism is *ex post* inefficient, and how costly that inefficiency is to the trading partners. This paper considers a matching mechanism that is *ex post* inefficient and shows that it outperforms other candidate mechanisms. A second point of view sees this work as the generalization of Ausubel et. al.'s work on demand reduction in auctions [1] to matching environments. The college knows that it wants to admit a valedictorian, but it does not know which one. In expectation, it can admit one valedictorian at no cost to itself.

From the high schools' perspective, if the college were auctioning both slots, the high school whose valedictorian was admitted should reduce demand and have her valedictorian admitted for free. The contracting stage of this paper's admissions game is analogous to the contracted high school reducing demand; the subsequent Vickrey auction is analogous to the auction for the remaining unit. Unlike a pure auction environment, the matching game must repeat since demand reduction is accomplished by contract rather than by the auction itself.

2 Model

There is one college C and two high schools H_1 and H_2. In each round of admissions, C has two slots for incoming freshmen and each high school has two graduating seniors. Each period, the college faces the problem of choosing two of the four graduating seniors to fill its freshman class. The college and two high schools repeat this admissions process infinitely many times, with common discount factor $\delta \in (0, 1)$. The college seeks to maximize the total lifetime payoff of its matriculated students, while each high school seeks to maximize the total lifetime payoff of its graduates.

Students at the high schools have types independently drawn from a publicly known, common density $f(\cdot)$ with support $[0, a] \subseteq [0, \infty)$ and corresponding distribution $F(\cdot)$. Assume that $f(\cdot)$ is sufficiently well behaved that every integral of interest in this paper converges. Each high school knows the type of each of its two students but not the types of the students at the other high school; the college does not know the types of any students. If students of types θ_1 and θ_2 matriculate to C in some period, student θ_1 receives payoff $U(\theta_1, \theta_2)$ and student θ_2 receives payoff $U(\theta_2, \theta_1)$, where $U(\cdot, \cdot)$ is non-negative increasing in both arguments, strictly supermodular, bounded and integrable on $[0, a]^4$. A student who does not go to college receives outside option 0. Herein lies C's market design impossibility: strict supermodularity in $U(\cdot, \cdot)$ implies sufficient payoff interdependence to preclude a mechanism that is both efficient and incentive compatible [6].

The college maximizes the lifetime payoff of its matriculated students; each high school maximizes the lifetime payoff of its graduates.

Let $s_{1n} \geq s_{2n}$ denote the types of H_1's period n good and bad students respectively; let $t_{1n} \geq t_{2n}$ denote the types of H_2's period n good and bad students respectively. Thus, $(s_{1n}, s_{2n}, t_{1n}, t_{2n})$ has joint density $4f(s_{1n})f(s_{2n})f(t_{1n})f(t_{2n})$ at its support is $\{(x, y, z, w) \in [0, a]^4 | x \geq y \text{ and } z \geq w\}$.

Abusing notation, I use a student's type as her identifier. Let X_{in} denote the ith best student and her type (across both high schools) in period n.

3 Contracting Mechanism

This section considers the novel combined contracting and auction admissions mechanism of this paper. In the contracting regime, C offers a randomly chosen high school a contract for its good student and runs a Vickrey auction to assign its remaining slot. If C offers H_i the contract in period n, H_i always accepts since it prefers to guarantee a berth for at least one of its students. The subsequent Vickrey auction allows C to identify and admit the best of the remaining three students as well as to discern before period $n+1$ whether the contracted high school honored the period n contract (sent its good student on the contract) or reneged (sent its bad student). Let q_n be the probability that C contracts with H_1 in period n. Period n of this game has the extensive form shown in Figure 1.

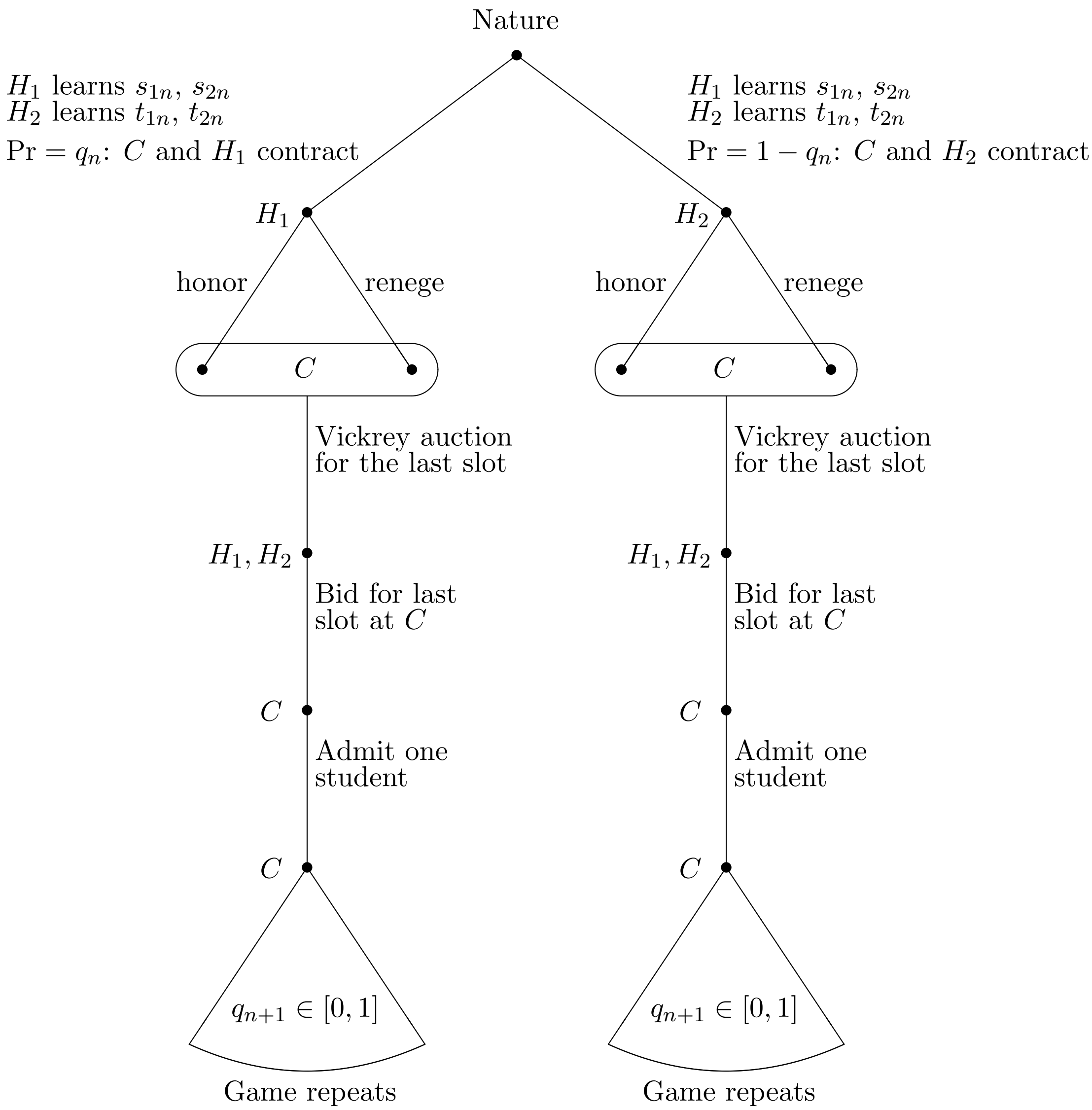

Figure 1

I show that there is an *ex post* subgame perfect Nash equilibrium in this game in which C and a sufficiently patient high school contract in every period, and the contracted high school honors the contract every time.

Theorem 3.1. *Consider the following strategies in the contracting game:*

1. *C chooses probability $q_0 \in (0, 1)$ with which it offers H_1 the contract in period* 0
2. *C offers H_1 the contract in period n with probability q_n*
3. *If offered the contract in period n, H_i accepts and honors the contract by sending C its good student immediately to fill the contracted slot*

4. *C runs a Vickrey auction to assign the remaining slot and admits the student with the higher report in the auction regardless of high school of origin*
5. *C discerns from the reports in the Vickrey auction whether the contracted high school played "honor" or "renege" and chooses q_{n+1} according to*

$$q_{n+1} = \begin{cases} 1 & \text{if} \quad H_1 \text{ played "honor" in period } n \text{ or } H_2 \text{ played "renege" in period } n \\ 0 & \text{if} \quad H_1 \text{ played "renege" in period } n \text{ or } H_2 \text{ played "honor" in period } n \end{cases}$$

These strategies form an ex post *subgame perfect Nash equilibrium in the contracting game whenever*

$$\delta \geq \frac{S}{1+S}$$

where

$$S = \sup \left\{ \frac{A(s_{20}, s_{10}) - A(s_{10}, s_{20})}{E[A(s_{1k}, s_{2k})] - B} \middle| (s_{10}, s_{20}) \in [0, a] \times [0, a] \text{ and } s_{10} \geq s_{20} \right\}$$

$$A(s_{10}, s_{20}) = \Pr(s_{20} \geq t_{10})[U(s_{10}, s_{20}) + U(s_{20}, s_{10})]$$

$$B = \frac{5}{6} E[U(s_{1k}, t_{1k}) | s_{1k} \geq t_{2k}]$$

Proof. See Appendix 6. □

The college's updating rule for q_{n+1} serves to enforce the contract and improves total welfare, as well as making the college and the contracted high school individually better off. This enforcement and social improvement comes at the expense of the high school that is not offered the contract in period 0. The college C and the period 0 contracted high school thus form an "old boys' club" in the sense that contracting in period 0 ensures that they contract forever; the high school with no contract in period 0 is shut out from the benefits of this relationship. In each period the contracting game is *ex post* efficient with probability 5/6 since it selects the *ex post* efficient match unless the top two overall students are from the non-contracted high school.

4 Performance

Having identified the *ex post* subgame perfect Nash equilibrium in the contracting game, I now compare its performance to two other incentive compatible matching mechanisms and to the socially optimal outcome under full information. This paper will consider the following options in addition to the contracting game of Section 3.

1. random admission in every period
2. C admits the valedictorians from H_1 and H_2 respectively

After describing each method, agents' equilibrium behavior, and expected surplus, this section will show that contracting is most likely to be *ex post* efficient, followed by admitting the valedictorians, and that random matching fares worst. Finally, I will consider several examples with explicit closed forms for $f(\cdot)$ and $U(\cdot,\cdot)$ to give some idea of the potential attractiveness of contracting relative to the other options.

4.1 Alternative Matching Mechanisms

4.1.1 Socially Optimal Matching

Under full information, positive assortative matching is the unique socially optimal and $C-$optimal outcome [9]. C admits the top two of the four students, regardless of their high school(s) of origin. Lifetime *ex ante* expected payoffs are

$$EC^{\text{opt}} = \sum_{n=0}^{\infty} \delta^n E[U(X_{1n}, X_{2n}) + U(X_{2n}, X_{1n})]$$

$$EH_1^{\text{opt}} = \frac{1}{2} \sum_{n=0}^{\infty} \delta^n E[U(X_{1n}, X_{2n}) + U(X_{2n}, X_{1n})]$$

$$EH_2^{\text{opt}} = \frac{1}{2} \sum_{n=0}^{\infty} \delta^n E[U(X_{1n}, X_{2n}) + U(X_{2n}, X_{1n})]$$

The high schools' respective *ex ante* expected payoffs are equal since the probabilities that a given high school has zero, one, or two of the best two students are identical across high schools. Full efficiency always occurs under full information.

4.1.2 Random Matching

If C abandons all efforts at extracting students' information it avoids any incentive problems at considerable loss of efficiency. Let w and z denote the randomly chosen students. Lifetime *ex ante* expected payoffs are

$$
\begin{aligned}
EC^{\text{random}} &= \sum_{n=0}^{\infty} \delta^n E[U(w,z) + U(z,w)] \\
EH_1^{\text{random}} &= \frac{1}{2}\sum_{n=0}^{\infty} \delta^n E[U(w,z) + U(z,w)] \\
EH_2^{\text{random}} &= \frac{1}{2}\sum_{n=0}^{\infty} \delta^n E[U(w,z) + U(z,w)]
\end{aligned}
$$

This method is *ex post* efficient in period n when C randomly chooses the top two students overall in that period; this event occurs with probability 1/6. The high schools' respective *ex ante* expected payoffs are equal since the probabilities that C chooses zero, one, or two of a high school's students are equal across high schools.

4.1.3 Admit The Valedictorians

An intermediate option for C is to ask each high school to identify its good student in each period, and admit the good student from each high school. With probability 1, it is strictly dominant for each high school to reveal its valedictorian truthfully since the high school's payoff increases in its admitted student's type.[1] Expected payoffs under this regime are

$$
\begin{aligned}
EC^{\text{Val}} &= \sum_{n=0}^{\infty} \delta^n E[U(s_{1n}, t_{1n}) + U(t_{1n}, s_{1n})] \\
EH_1^{\text{Val}} &= \sum_{n=0}^{\infty} \delta^n E[U(s_{1n}, t_{1n})] \\
EH_2^{\text{Val}} &= \sum_{n=0}^{\infty} \delta^n E[U(t_{1n}, s_{1n})]
\end{aligned}
$$

[1] If H_i's students are identical, truthful revelation is only weakly dominant, but this event occurs with probability 0.

This method is *ex post* efficient in period n so long as the two valedictorians are the top two students overall in that period; this event occurs with probability 2/3.

4.2 Performance

The following tables compares performance and one period expected surplus of the three matching mechanisms to each other and to the socially optimal outcome under full information. In the case of the contracting game, assume that H_1 randomly wins the period 0 contract; its one period payoff in each of the tables is the expected payoff after making the contract but before learning students' types.

Mechanism	Pr *ex post* efficient	EU_C	EU_{H_1}	EU_{H_2}
Social Optimum	1	1	$\frac{1}{2}$	$\frac{1}{2}$
Contracting	5/6	$\frac{98}{108}$	$\frac{58}{108}$	$\frac{40}{108}$
Valedictorians	2/3	$\frac{96}{108}$	$\frac{48}{108}$	$\frac{48}{108}$
Random Matching	1/2	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$

Types i.i.d. $U[0,1]$; $U(x,y) = xy$. *EU* is one period *ex ante* expectation.

Mechanism	Pr *ex post* efficient	EU_C	EU_{H_1}	EU_{H_2}
Social Optimum	1	$\frac{386}{72\lambda^2}$	$\frac{193}{72\lambda^2}$	$\frac{193}{72\lambda^2}$
Contracting	5/6	$\frac{1003}{216\lambda^2}$	$\frac{598}{216\lambda^2}$	$\frac{405}{216\lambda^2}$
Valedictorians	2/3	$\frac{18}{4\lambda^2}$	$\frac{9}{4\lambda^2}$	$\frac{9}{4\lambda^2}$
Random Matching	1/2	$\frac{2}{\lambda^2}$	$\frac{1}{\lambda^2}$	$\frac{1}{\lambda^2}$

Types i.i.d. $f(x) = \lambda e^{-\lambda x}$; $U(x,y) = xy$. *EU* is one period *ex ante* expectation.

In the first case, the contracting game performs 2.1% better than admitting the valedictorians; in the second case, the contracting game is 3.2% better. However, in the case of the exponential distribution, the benefit to the contracted high school is much larger than in the case of the uniform distribution.

5 Conclusion

Existing impossibility results show that there are many mechanism design problems lacking an efficient solution. Consequentially, there is relatively little development of mechanisms designed for such situations, despite their common occurrence. When such mechanisms do exist, they rely on assumptions (often reasonable) that successful deceit is difficult and therefore rare; therefore, one may simply ignore the possibility of strategic misrepresentation. I consider an alternative, in which a relationship is used as a partial substitute for direct information revelation.

This paper introduces a mechanism in a repeated college admissions game which uses a contracting stage to admit a good student and simplify the information elicitation problem in the post-contract admissions market. The college enforces the contract in each period with the promise of maintaining the contracting relationship in the next period. The mechanism efficient or closer to it more often than other plausible incentive compatible mechanisms and thus improves total surplus. However, the gains of this matching approach are distributed inequitably, and repeatedly so. The college and the persistently contracted high school both receive higher surplus in the contracting game than in the "admit the valedictorians" mechanism; however, the non-contracted high school's surplus drops significantly. The persistent contracting relationship shows that even when a relationship is initiated at random, once established an "old boys' club" consistently exploits non-members for its own benefit. The college and contracted high school enrich themselves at considerable cost to the non-contracted high school. Future work should produce a general result about performance this type of hybrid contracting / direct revelation mechanism in matching markets.

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6 Appendix

6.1 Joint Densities

The joint density of $(s_{1n}, s_{2n}, t_{1n}, t_{2n})$ is given by

$$\frac{f(s_{1n})f(s_{2n})f(t_{1n})f(t_{2n})}{\int_0^a \int_0^{s_{1n}} \int_0^a \int_0^{t_{1n}} dF(t_{2n})dF(t_{1n})dF(s_{2n})dF(s_{1n})} = 4f(s_{1n})f(s_{2n})f(t_{1n})f(t_{2n})$$

6.2 Proof of Theorem 3.1

Proof. Without loss of generality, suppose that H_1 wins the period 0 contract. Given s_{10} and s_{20}, the lifetime expected payoff of honoring the period 0 contract is

$$\begin{aligned}
&\Pr(s_{20} \geq t_{10})[U(s_{10}, s_{20}) + U(s_{20}, s_{10})] + \Pr(t_{10} \geq s_{20})E[U(s_{10}, t_{10})|t_{10} \geq s_{20}] \\
&\quad + \sum_{k=1}^{\infty} \delta^k \Bigg\{ \Pr(s_{2k} \geq t_{1k})E[U(s_{1k}, s_{2k}) + U(s_{2k}, s_{1k})|s_{2k} \geq t_{1k}] \\
&\quad + \Pr(t_{1k} \geq s_{2k})E[U(s_{1k}, t_{1k})|t_{1k} \geq s_{2k}] \Bigg\}
\end{aligned}$$

Given s_{10} and s_{20}, the lifetime expected payoff of reneging on the period 0 contract is

$$\begin{aligned}
&\Pr(s_{10} \geq t_{10})[U(s_{10}, s_{20}) + U(s_{20}, s_{10})] + \Pr(t_{10} \geq s_{10})E[U(s_{20}, t_{10})|t_{10} \geq s_{10}] \\
&+ \sum_{k=1}^{\infty} \delta^k \Bigg\{ \Pr(s_{1k} \geq t_{2k})E[U(s_{1k}, t_{1k})|s_{1k} \geq t_{2k}] \Bigg\}
\end{aligned}$$

Therefore the strategies form an *ex post* subgame perfect Nash equilibrium if for all realizations of s_{10} and s_{20}

$$\begin{aligned}
&\Pr(s_{20} \geq t_{10})[U(s_{10}, s_{20}) + U(s_{20}, s_{10})] + \Pr(t_{10} \geq s_{20})E[U(s_{10}, t_{10})|t_{10} \geq s_{20}] \\
&\quad + \sum_{k=1}^{\infty} \delta^k \Bigg\{ \Pr(s_{2k} \geq t_{1k})E[U(s_{1k}, s_{2k}) + U(s_{2k}, s_{1k})|s_{2k} \geq t_{1k}] \\
&\quad + \Pr(t_{1k} \geq s_{2k})E[U(s_{1k}, t_{1k})|t_{1k} \geq s_{2k}] \Bigg\} \\
&\geq \Pr(s_{10} \geq t_{10})[U(s_{10}, s_{20}) + U(s_{20}, s_{10})] + \Pr(t_{10} \geq s_{10})E[U(s_{20}, t_{10})|t_{10} \geq s_{10}] \\
&\quad + \sum_{k=1}^{\infty} \delta^k \Bigg\{ \Pr(s_{1k} \geq t_{2k})E[U(s_{1k}, t_{1k})|s_{1k} \geq t_{2k}] \Bigg\}
\end{aligned}$$

Since student types are independently and identically distributed this inequality simplifies to:

$$\begin{aligned}
&F(s_{20})^2[U(s_{10}, s_{20}) + U(s_{20}, s_{10})] + (1 - F(s_{20})^2)E[U(s_{10}, t_{10})|t_{10} \geq s_{20}] \\
&+ \sum_{k=1}^{\infty} \delta^k \left\{ \frac{1}{6}E[U(s_{1k}, s_{2k}) + U(s_{2k}, s_{1k})|s_{2k} \geq t_{1k}] + \frac{5}{6}E[U(s_{1k}, t_{1k})|t_{1k} \geq s_{2k}] \right\} \\
&\geq F(s_{10})^2[U(s_{10}, s_{20}) + U(s_{20}, s_{10})] + (1 - F(s_{10})^2)E[U(s_{20}, t_{10})|t_{10} \geq s_{10}] \\
&+ \sum_{k=1}^{\infty} \delta^k \left\{ \frac{5}{6}E[U(s_{1k}, t_{1k})|s_{1k} \geq t_{2k}] \right\}
\end{aligned}$$

Let

$$A(s_{10}, s_{20}) = \Pr(s_{20} \geq t_{10})[U(s_{10}, s_{20}) + U(s_{20}, s_{10})] + \Pr(t_{10} \geq s_{20})E[U(s_{10}, t_{10})|t_{10} \geq s_{20}]$$

and

$$B = \frac{5}{6}E[U(s_{1k}, t_{1k})|s_{1k} \geq t_{2k}]$$

Note that B is a constant, and $E[A(s_{1k}, s_{2k})] > B$. Therefore

$$\frac{\delta}{1-\delta} \geq \frac{A(s_{20}, s_{10}) - A(s_{10}, s_{20})}{E[A(s_{1k}, s_{2k})] - B}$$

The assumptions about the existence of all moments of interest and the boundedness of $U(\cdot, \cdot)$ imply that

$$0 \leq \sup \left\{ \frac{A(s_{20}, s_{10}) - A(s_{10}, s_{20})}{E[A(s_{1k}, s_{2k})] - B} \middle| (s_{10}, s_{20}) \in [0, a] \times [0, a] \text{ and } s_{10} \geq s_{20} \right\} < \infty$$

Let

$$S = \sup \left\{ \frac{A(s_{20}, s_{10}) - A(s_{10}, s_{20})}{E[A(s_{1k}, s_{2k})] - B} \middle| (s_{10}, s_{20}) \in [0, a] \times [0, a] \text{ and } s_{10} \geq s_{20} \right\}$$

The critical $\overline{\delta}$ such that the strategies form an *ex post* subgame perfect Nash equilibrium is therefore

$$\overline{\delta} = \frac{S}{1+S}$$

□

6.3 Mathematica code

This code computes the values in the tables in Section 4.

```
(*Expected payoffs when student types are uniformly distributed*)
U := #1*#2 &
f := 1 &

(*Full Info - college's EU. Each high school gets half.*)
Integrate[24*2*s1*s2*f[s1]*f[s2]*f[t1]*f[t2]
, {s1, 0, 1}, {s2, 0, s1}, {t1, 0, s2}, {t2, 0, t1}]

(*Random - college's EU. Each high school gets half.*)
Integrate[2*s1*s2*f[s1]*f[s2], {s1, 0, 1}, {s2, 0, 1}]

(*Valedictorians - college's EU. Each high school gets half.*)

Integrate[4*2*s1*t1*f[s1]*f[s2]*f[t1]*f[t2]
, {s1, 0, 1}, {s2, 0, s1}, {t1, 0, 1}, {t2, 0, t1}]

(*Contracting - H1 wins the period 1 contract. College gets*)
(1/6)*Integrate[24*2*s1*s2*f[s1]*f[s2]*f[t1]*f[t2]
, {s1, 0, 1}, {s2, 0, s1}, {t1, 0, s2}, {t2, 0, t1}]
+ (5/6)*Integrate[4*2*s1*t1*f[s1]*f[s2]*f[t1]*f[t2]
```

```
, {s1, 0, 1}, {s2, 0, s1}, {t1, 0, 1}, {t2, 0, t1}]

(*Contracting - H1 gets*)
(1/6)*Integrate[24*2*s1*s2*f[s1]*f[s2]*f[t1]*f[t2]
, {s1, 0, 1}, {s2, 0, s1}, {t1, 0, s2}, {t2, 0, t1}]
+(5/6)*Integrate[4*s1*t1*f[s1]*f[s2]*f[t1]*f[t2]
, {s1, 0, 1}, {s2, 0, s1}, {t1, 0, 1}, {t2, 0, t1}]

(*Contracting - H2 gets*)
(5/6)*Integrate[4*s1*t1*f[s1]*f[s2]*f[t1]*f[t2]
, {s1, 0, 1}, {s2, 0, s1}, {t1, 0, 1}, {t2, 0, t1}]

(*Expected payoffs when student types are exponentially distributed*)
U := #1 * #2 &
f := 1*Exp[-1 #] &

(*Full Info - college's EU. Each high school gets half.*)
Integrate[24*2*s1*s2*f[s1]*f[s2]*f[t1]*f[t2]
, {s1, 0, Infinity}, {s2, 0, s1}, {t1, 0, s2}, {t2, 0, t1}]

(*Random - college's EU. Each high school gets half.*)
Integrate[2*s1*s2*f[s1]*f[s2], {s1, 0, Infinity}, {s2, 0, Infinity}]

(*Valedictorians - ollege's EU. Each high school gets half.*)
Integrate[4*2*s1*t1*f[s1]*f[s2]*f[t1]*f[t2]
, {s1, 0, Infinity}, {s2, 0, s1}, {t1, 0, Infinity}, {t2, 0, t1}]

(*Contracting - H1 wins the period 1 contract. College gets*)
(1/6)*Integrate[24*2*s1*s2*f[s1]*f[s2]*f[t1]*f[t2]
```

```
, {s1, 0, Infinity}, {s2, 0, s1}, {t1, 0, s2}, {t2, 0, t1}]
+ (5/6)*Integrate[4*2*s1*t1*f[s1]*f[s2]*f[t1]*f[t2]
, {s1, 0, Infinity}, {s2, 0, s1}, {t1, 0, Infinity}, {t2, 0, t1}]

(*Contracting - H1 gets*)
(1/6)*Integrate[24*2*s1*s2*f[s1]*f[s2]*f[t1]*f[t2]
, {s1, 0, Infinity}, {s2, 0, s1}, {t1, 0, s2}, {t2, 0, t1}]
+ (5/6)*Integrate[4*s1*t1*f[s1]*f[s2]*f[t1]*f[t2]
, {s1, 0, Infinity}, {s2, 0, s1}, {t1, 0, Infinity}, {t2, 0, t1}]

(*Contracting - H2 gets*)
(5/6)*Integrate[4*s1*t1*f[s1]*f[s2]*f[t1]*f[t2]
, {s1, 0, Infinity}, {s2, 0, s1}, {t1, 0, Infinity}, {t2, 0, t1}]
```