The Attack and Defense of Weakest-Link Networks

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Abstract

In a two-player game of attack and defense of a weakest-link network of targets, the attacker's objective is to successfully attack at least one target and the defender's objective is to defend all targets. We experimentally test two theoretical models that differ with regards to the contest success function (CSF) that is used to model the conflict at each target (specifically, the lottery and auction CSF), and which result in qualitatively different patterns of equilibrium behavior. We find some support for the comparative statics predictions of both models. Consistent with the theoretical predictions, under both the lottery and auction CSF, as the attacker's valuation increases, the average resource expenditure, the probability of winning, and the average payoff increase for the attacker and decrease for the defender. Also, consistent with equilibrium behavior under the auction CSF, attackers utilize a stochastic "guerrilla warfare" strategy, which involves randomly attacking at most a single target and allocating a random level of force to that target. However, under the lottery CSF, instead of the theoretical prediction of a "complete coverage" strategy, which involves attacking all targets, we find that attackers use the "guerrilla warfare" strategy and attack only one target.

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1. Introduction

In many network applications, such as cyber-security, electrical power grids, or oil pipeline systems, the failure of any individual component in the network may be sufficient to disable the entire network or to create a terrorist "spectacular." In the case of a system of dikes on the perimeter of an island, Hirshleifer (1983) coined the term weakest-link to describe this particular type of intra-network complementarity among components.[1] In addition to networks with physically linked components, political considerations may create a situation in which physically disjoint components are connected by a form of weakest-link complementarity in preferences. For example, a single terrorist spectacular may allow a terrorist organization to influence its target audience and cause anti-terrorism policies to be seen as a failure.[2]

Defending a weakest-link network against a potential attack can be modeled as a multi-battle contest in which a risk neutral attacker and defender simultaneously allocate a resource (forces) at constant unit cost (normalized to one) across the set of targets. Because the destruction of a single target renders the entire network inoperable, the attacker wins and receives a prize if he successfully attacks at least one target. Conversely, the defender wins and receives a prize if he successfully defends all targets. For each player, the probability of winning any given target is determined by the levels of the resource that the players allocate to that target and the contest success function (CSF) that maps the two players' resource allocations into their respective probabilities of winning. We experimentally examine two theoretical models that

[1] A number of environments can be described using the weakest-link structure. Kremer (1993) develops a theoretical model in which the performance of an organization depends mainly on the weakest-link. Moore et al. (2009) argue that "fixing online crime is hard because Internet security is often weakest-link". Milgrom (2007) considers an example of a package auction in which the objective of some bidders may be to obtain only one good (weakest-link), if goods are perfect substitutes.

[2] As stated in the *Joint House-Senate Intelligence Inquiry into September 11*, 2001 (US Congress, 2002), terrorists need to be successful only once to kill Americans and demonstrate the inherent vulnerabilities they face.

analyze strategic behavior in this attack and defense game using two different CSFs to model the conflict at each target.

Clark and Konrad (2007) theoretically analyze the game of attack and defense under the assumption that the conflict at each target takes the form of the lottery CSF (i.e., the probability of winning a target equals the ratio of a player's resource allocation to the sum of the two players' resource allocations to that target). In this setup, equilibrium is in pure strategies and the main theoretical prediction is that the attacker uses a "complete coverage" pure-strategy which involves attacking all targets with a strictly positive level of force, and the defender should defend all targets.[3] The attack and defense of a weakest-link network is also a special case of the model examined in Kovenock and Roberson (2015) where the conflict at each target takes the form of the auction CSF (i.e., the player with the greater resource allocation to a target wins that target with certainty).[4] With the auction CSF equilibrium is in mixed strategies, and in sharp contrast to the "complete coverage" pure-strategy, in any mixed strategy equilibrium the attacker utilizes a stochastic "guerrilla warfare" strategy, which involves randomly attacking at most a single target, where each target is equally likely to be the one that is attacked, and allocating a random level of force to that target. Conversely, the defender stochastically covers all of the targets, allocating a random level of force to each of the targets.[5]

The differences in the theoretical predictions of these two models are driven by the different CSFs used to model the conflict at each target. In particular, the auction and lottery

[3] For the attacker, this prediction holds for all parameter configurations. For the defender, this prediction holds if the ratio of the attacker's valuation of success to the defender's valuation of success is below a certain threshold.
[4] Kovenock and Roberson (2015) examine the problem of attack and defense of network connectivity in which there exist multiple sets of nodes in the network (known as cutsets) which if destroyed would disconnect the network. A weakest-link network corresponds to the special case in which each minimal cutset consists of a single node. See also the closely related papers Dziubiński and Goyal (2013a, 2013b).
[5] For almost all configurations of the players' valuations of winning, one of the two players drops out with positive probability by allocating zero resources to each target, with the identity of the dropout determined by a measure of asymmetry in the conflict that takes into account both the ratio of the players' valuations and the number of targets.

CSFs are two special cases of the general ratio-form contest success function $x_A^r/(x_A^r + x_D^r)$ where x_A and x_D are the attacker's and defender's allocations of force and the parameter $r > 0$ is inversely related to the level of noise, or randomness, in the determination of the winner of the conflict (i.e., a low value of r implies a large amount of noise and a high value of r implies a small amount of noise). In the lottery CSF $r = 1$, and the auction CSF corresponds to the limiting case where $r = \infty$. That is, the lottery CSF represents a situation in which the outcome of the conflict at each target has a relatively high amount of noise, and the auction CSF represents a situation in which there is no noise (i.e., the player that allocates the higher level of force wins with certainty). However, as noted − in the case of a single contest with linear costs − by Baye et al. (1994) and Alcalde and Dahm (2010), for all r greater than 2 there exist equilibria that are payoff equivalent to the $r = \infty$ case whenever $r > 2$. Thus, the case of the auction CSF with $r = \infty$ is a relevant theoretical benchmark for all levels of noise $r > 2$.

In the attack and defense of a weakest-link network, the nature of the noise in the CSF is a key determinant of equilibrium behavior. Note that, regardless of the CSF, beyond the first successful attack the attacker's marginal value for an additional successful attack is zero. Thus, if an attacker allocates strictly positive forces to more than one target in equilibrium, then it must be the case that the increase in the probability of winning at least one target outweighs the additional cost of attacking more than one target. With the auction CSF, there is no exogenous noise in the component conflicts but equilibrium is in mixed strategies, and, thus, each player faces a form of endogenous (strategic) noise in the outcome at each target. Conversely, with the lottery CSF, equilibrium is in pure strategies and so there is no endogenous (strategic) noise, but the lottery CSF introduces a relatively high level of exogenous noise in the component conflicts. In Clark and Konrad (2007) the exogenous noise for each component contest is independently distributed, and they provide an equilibrium in which all targets are attacked. In contrast, Kovenock and Roberson (2015) show that in all equilibria of the game with the auction CSF the defender uses a mixed strategy joint distribution that results in a correlation structure of endogenous noise that makes all multiple target attacks payoff dominated by a single target attack. In this paper, we complete the characterization of equilibrium in the lottery CSF version of the game by showing that equilibrium is unique and test the implications of these two models in a laboratory experiment with a two-by-two design that investigates the impact of the CSF (lottery versus auction) and the relative valuation of the attacker's prize (low versus high) on behavior of attackers and defenders.

At the aggregate level, the results of our experiment provide support for the comparative statics predictions with regard to a change in the attacker's valuation. Consistent with predictions, under the lottery and auction CSF, as the attacker's valuation increases, the attacker's resource expenditure increases and the defender's expenditure decreases. As a result, the attacker's probability of winning and the average payoff also increase. However, contrary to predictions, under both CSFs, both players' resource expenditures exceed their respective theoretical predictions, which is common in contest experiments (Dechenaux et al., 2015).

A more novel finding of our study is about the behavior of attackers and defenders under the two alternative CSFs. The results of our experiment support the theoretical prediction that, under the auction CSF, attackers use a stochastic "guerrilla warfare" strategy which involves randomly attacking a single target (ignoring the remaining targets) and allocating a random level of force to that target, and defenders use a stochastic "complete coverage" strategy which involves allocating a strictly positive level of force to each target. In contrast, under the lottery CSF, instead of the pure-strategy Nash equilibrium "complete coverage" strategy, the expenditures of both the attackers and defenders are distributed over the entire strategy space. In fact, under the lottery CSF, attackers utilize, almost 45% of the time, a “guerrilla warfare” strategy of attacking only a single target, instead of using a “complete coverage” strategy, which is observed less than 30% of the time.

The rest of the paper is organized as follows. In Section 2, we provide a brief review of the multi-battle contest literature. Section 3 presents a theoretical model of the game of attack and defense. Section 4 describes the experimental design, procedures and hypotheses. Section 5 reports the results of our experiment and Section 6 concludes.

2. Literature Review

Multi-battle contests provide a theoretical framework that is applicable to a host of complex economic, information, military, and political environments in which players strategically allocate resources among multiple dimensions such as, market segments, projects, battles, or electoral districts.[6] Most of the existing theoretical work focuses on the case of symmetric objectives. For example, in a common formulation of the Colonel Blotto game, originating with Borel (1921), each player maximizes the expected number of battles won. However, in many relevant applications such as cyber-security, terrorist attacks, and military battles, objectives are asymmetric. For example, many relevant applications such as cyber-security and terrorism, the attacker's objective is often to successfully attack only one target (weakest-link) and the defender's objective is to successfully defend all targets (Sandler and

[6] For a survey see Kovenock and Roberson (2012). Recent work on multi-battle/Blotto-type games includes extensions such as: asymmetric players (Roberson, 2006; Hart, 2008; Weinstein, 2012; Dziubiński, 2013; Macdonell and Mastronardi, 2015), non-constant-sum variations (Szentes and Rosenthal, 2003; Kvasov, 2007; Hortala-Vallve and Llorente-Saguer, 2010, 2012; Roberson and Kvasov, 2012), alternative definitions of success (Golman and Page, 2009; Tang et al., 2010; Rinott et al., 2012), and political economy applications (Laslier, 2002; Laslier and Picard, 2002; Roberson, 2008; Thomas, 2012).

Enders, 2004). This is exactly the objective asymmetry that arises in the two models of weakest-link networks with differing CSFs that we test experimentally (Clark and Konrad, 2007; Kovenock and Roberson, 2015).

To the best of our knowledge, our study is the first to examine behavior in the game of attack and defense of weakest-link networks, using both the lottery CSF and the auction CSF. While most of the existing experimental studies focus on single-battle contests, there is a growing interest in multi-battle contests. For a comprehensive review of the experimental literature on contests see Dechenaux et al. (2015). Experimental studies on multi-battle contests have examined how different factors such as budget constraint (Avrahami and Kareev, 2009; Arad and Rubinstein, 2012), temporal structure (Deck and Sheremeta, 2012, 2015; Mago et al., 2013; Mago and Sheremeta, 2014; Irfanoglu et al., 2015), information (Horta-Vallve and Llorente-Saguer, 2010), contest success function (Chowdhury et al., 2013), asymmetry in resources and battlefields (Arad, 2012; Holt et al., 2015; Montero et al., 2015) impact individual behavior in contests. Most of these studies find support for the comparative statics predictions, but often report significant over-expenditure of resources (also known as overbidding or over-dissipation) relative to the Nash equilibrium prediction (see the review by Dechenaux et al., 2015).

Consistent with the previous studies, we find significant over-expenditure relative to the Nash equilibrium predictions under both the lottery CSF and the auction CSF. However, our most surprising result is that the theoretical prediction of attackers using a "guerrilla warfare" strategy under the auction CSF is also observed under the lottery CSF. This is surprising because, as mentioned above, almost all multi-battle contest experiments in the literature find strong qualitative support for the theoretical predictions even if the precise quantitative predictions are refuted. A potential explanation why attackers use a "guerrilla warfare" strategy under the lottery CSF is that subjects may find it natural to concentrate resources on just one target since one successful attack is enough to win. Such a heuristic strategy also explains why individual behavior is so close to the theoretical predictions under the auction CSF.

3. The Game of Attack and Defense

Consider the following multi-battle contest involving the attack and defense of a weakest-link network. Two risk neutral players, an attacker A and a defender D, simultaneously allocate their respective resources across n targets. The probability that each player wins target i depends on the players' allocations of a one-dimensional resource to the target, x_A^i and x_D^i for A and D respectively. The players' resource expenditures are mapped into their respective probabilities of winning by the contest success function (CSF). One prominent contest success function is the general ratio-form, or Tullock, CSF (Tullock, 1980). According to this CSF, player D wins target i with probability:

$$p_D^i(x_A^i, x_D^i) = \begin{cases} \frac{(x_D^i)^r}{(x_A^i)^r + (x_D^i)^r} & \text{if } x_A^i + x_D^i > 0 \\ 1 & \text{otherwise} \end{cases}, \tag{1}$$

and player A wins target i with probability $1 - p_D^i(x_A^i, x_D^i)$. If $r = 1$ (the lottery CSF), then each player's probability of winning the target equals the ratio of that player's resource expenditure to the sum of both of the players' resource expenditures. If $r = \infty$ (the auction CSF), then the player that allocates the higher level of resources wins the target with certainty.[7]

[7] In the case that the players allocate the same level of the resource to a target, it is assumed that the defender wins the target. However, a range of tie-breaking rules yields similar results. A detailed description of the theoretical model can be found in Clark and Konrad (2007) for the lottery CSF and Kovenock and Roberson (2015) for the auction CSF.

The attacker and the defender have asymmetric objectives.[8] The defender's objective is to successfully defend all n targets in the network, in which case he receives a "prize" of value v_D. Therefore, the expected payoff of D is equal to his probability of winning all targets times the prize value, minus the sum of all his resource expenditures across all of the targets:

$$E(\pi_D) = \left(\prod_{i=1}^{n} p_D^i\left(x_A^i, x_D^i\right)\right)v_D - \sum_{i=1}^{n} x_D^i. \qquad (2)$$

The attacker's objective is to successfully attack at least one target, in which case he receives a prize of value v_A. The expected payoff of A is equal to his probability of winning at least one target (which is 1 minus the probability that D wins all targets) times the prize value, minus the sum of all his resource expenditures:

$$E(\pi_A) = \left(1 - \prod_{i=1}^{n} p_D^i\left(x_A^i, x_D^i\right)\right)v_A - \sum_{i=1}^{n} x_A^i. \qquad (3)$$

The nature of equilibrium in this game depends on the parameter r. Clark and Konrad (2007) characterize a Nash equilibrium for the lottery CSF $(r = 1)$. We complete the characterization of equilibrium by showing that the equilibrium is unique (see the Appendix A).

***Proposition* 1:**

(i) If $v_D \geq (n-1)v_A$, then there exists a unique Nash equilibrium, which is in pure strategies. In equilibrium, player A allocates $x_A^* = \frac{v_A^2 v_D^n}{(v_A+v_D)^{n+1}}$ to every target and player D allocates $x_D^* = \frac{v_A v_D^{n+1}}{(v_A+v_D)^{n+1}}$ to every target.

(ii) If $v_D < (n-1)v_A$, then there exists a unique Nash equilibrium, which is in mixed strategies. In equilibrium, player A allocates $x_A^* = \frac{(n-1)^{n-1}}{n^{n+1}} v_D$ to each target and player D

[8] See also Milgrom (2007) who considers an example of a package auction in which the objective of some bidders may be to obtain all the goods (best-shot objective), if goods are perfect complements, while for other bidders the objective is to obtain only one good (weakest-link objective), if goods are perfect substitutes.

randomizes by allocating $x_D^* = \frac{(n-1)^n}{n^{n+1}}$ to every target with probability $q^* = \frac{v_D}{(n-1)v_A}$ and $x_D^* = 0$ to every target with the probability $1 - q^*$.

Proposition 1 can be summarized as follows. If the ratio of the defender's valuation to the attacker's valuation exceeds a threshold, $v_D \geq (n-1)v_A$, then it is worthwhile for the defender to play a pure strategy that allocates the same level of resources to each target and defend all of the targets with probability one. However, if $v_D < (n-1)v_A$, the defender earns a zero expected payoff in equilibrium and the probability that the defender engages in the conflict by allocating the same positive level of resources to each target is only $q^* = \frac{v_D}{(n-1)v_A}$. For this range of values, the defender "surrenders" with strictly positive probability, allocating a zero level of the resource to every target. In contrast, for all parameter configurations the attacker plays a pure strategy. Although the attacker's objective is to win at least one target, due to the decreasing returns to expenditure exhibited by the lottery CSF, the optimal strategy is actually to attack each and every target with an identical strictly positive level of resources.

Kovenock and Roberson (2015) characterize properties of the set of Nash equilibria for the game of attack and defense over a weakest-link network for the case of the auction CSF ($r = \infty$). They show that all equilibria are in mixed strategies, where in this case a mixed strategy is an *n*-variate joint distribution function. That paper completely characterizes the set of equilibrium payoffs and univariate marginal distributions, which are unique for all parameter configurations. These results are summarized as follows:

***Proposition* 2:**

(i) If $v_D \geq nv_A$, then with probability $1 - \frac{nv_A}{v_D}$ player A allocates 0 to every target. With the remaining probability, $\frac{nv_A}{v_D}$, player A randomly attacks a single target with a resource allocation drawn from a uniform distribution over the interval $[0, v_A]$. To each and every target, player D allocates a random level of the resource drawn from a uniform distribution over the interval $[0, v_A]$. The players' sets of equilibrium univariate marginal distribution functions are unique, and for each target j are given by: $F_A^j(x_A^j) = 1 - \frac{v_A}{v_D} + \frac{x_A^j}{v_D}$ and $F_D^j(x_D^j) = \frac{x_D^j}{v_A}$, respectively, over the interval $[0, v_A]$.

(ii) If $v_D < nv_A$, player A randomly attacks a single target with a resource allocation drawn from a uniform distribution over the interval $[0, \frac{v_D}{n}]$. With probability $1 - \frac{v_D}{nv_A}$ player D allocates 0 to every target. With the remaining probability, $\frac{v_D}{nv_A}$, player D allocates to each target a random level of resources drawn from a uniform distribution over the interval $[0, \frac{v_D}{n}]$. The players' sets of equilibrium univariate marginal distribution functions for every target are unique, and for each target j are given by: $F_A^j(x_A^j) = 1 - \frac{1}{n} + \frac{x_A^j}{v_D}$ and $F_D^j(x_D^j) = 1 - \frac{v_D}{nv_A} + \frac{x_D^j}{v_A}$, respectively, over the interval $[0, \frac{v_D}{n}]$.

It is important to note that, although there are multiple equilibria in this game, there exists a unique set of equilibrium univariate marginal distribution functions. Kovenock and Roberson (2015) also show that the equilibrium joint distribution functions exhibit several distinctive properties. For example, in all equilibria of the auction CSF game, the attacker optimally allocates a strictly positive amount of resources to at most one target (each target being chosen with positive probability) while the defender optimally allocates a strictly positive amount of resources to either all targets or no target. This particular property provides a striking contrast with the equilibrium in the lottery CSF game (see *Proposition 1*) in which the attacker allocates a strictly positive amount of resources to every target.

4. Experimental Design, Procedures, and Hypotheses

4.1. Experimental Design

Table 1 summarizes the experimental design. We employ a two-by-two design, by varying the CSF (*Lottery* versus *Auction*) and the relative valuation of the attacker's prize (*Low* versus *High*). In all four treatments there are four targets and two players (attacker and defender). The experimental instructions, shown in the Appendix B, used a context neutral language. For example, the attacker and the defender were called participant 1 and participant 2, while the targets were called boxes.

In the *Lottery-Low* and *Lottery-High* treatments the probability that a player wins a given target is equal to the ratio of that player's allocation of resources to the target (tokens) to the sum of both players' allocations to that target. The defender's valuation of defending all targets is $v_D = 200$ experimental francs. The attacker's valuation of successfully attacking at least one target is $v_A = 40$ francs in the *Lottery-Low* treatment and $v_A = 80$ francs in the *Lottery-High* treatment.[9] For the parameter configuration in the *Lottery-Low* treatment, Proposition 1 part (i) applies and in the pure-strategy equilibrium the attacker allocates 3.2 tokens to each target and the defender allocates 16.1 tokens to each target. For the parameter configuration in the *Lottery-High* treatment, Proposition 1 part (ii) applies and in equilibrium the attacker allocates 5.3 tokens to each target and the defender allocates 15.8 tokens to every target with probability 0.83 and 0 tokens to every target with probability 0.17.

[9] We chose 4 targets and valuation ratios of 200/40 for the *Lottery-Low* and *Auction-Low* treatments to ensure that the theoretical predictions are derived from Proposition 1 (i) and Proposition 2 (i). Similarly, we chose 4 targets and valuation ratios of 200/80 for the *Lottery-High* and *Auction-High* treatments to match Proposition 1 (ii) and Proposition 2 (ii). The 4 targets and the ratio of the attacker's valuation to the defender's valuation were also chosen to ensure that the allocation problem for the subjects is non-trivial and so that both the attacker and the defender had substantial chances of winning some targets.

The only difference in the *Auction-Low* and *Auction-High* treatments is that the winner of each target is determined by the auction CSF rather than the lottery CSF. That is, the player who allocates the higher level of resources to a target wins that target with certainty. In both the *Auction-Low* and *Auction-High* treatments, there are no pure-strategy equilibria. From Proposition 2 part (i), in any mixed-strategy equilibrium of the *Auction-Low* treatment the attacker attacks no targets with probability 0.2 and, with probability 0.8, chooses exactly one target to attack at random and stochastically allocates, according to a uniform distribution, between 0 and 40 tokens to that target. The defender randomizes according to a joint distribution function that, in addition to satisfying the necessary properties for equilibrium given by Kovenock and Roberson (2015), stochastically allocates between 0 and 40 tokens to each target according to a uniform marginal distribution. In the *Auction-High* treatment, Proposition 2 part (ii) applies. In equilibrium the attacker randomly chooses one of the targets to attack and stochastically allocates between 0 and 50 tokens to that target according to a uniform distribution. The defender employs a strategy in which, with probability 0.375 he engages in no defensive efforts and, with probability 0.625, the defender allocates a stochastic number of tokens, uniformly distributed between 0 and 50, to each target.

4.2. Procedures

The experiment was conducted at the Vernon Smith Experimental Economics Laboratory. The computerized experimental sessions were run using z-Tree (Fischbacher, 2007). A total of 96 subjects participated in eight sessions, summarized in Table 2. All subjects were Purdue University undergraduate students who participated in only one session of this study.

Some students had participated in other economics experiments that were unrelated to this research.

Each experimental session had 12 subjects and proceeded in two parts, corresponding to the lottery and auction treatments.[10] Each subject played for 20 periods in the *Lottery-Low* (*Auction-Low*) treatment and 20 periods in the *Lottery-High* (*Auction-High*) treatment. The sequence was varied so that half the sessions had the *Lottery-High* (*Auction-High*) treatment first, and half had the *Lottery-Low* (*Auction-Low*) treatment first.

In the first period of each treatment subjects were randomly and anonymously assigned as attacker or defender (participant 1 or participant 2). All subjects remained in the same role assignment for the first 10 periods and then changed their assignment for the last 10 periods.[11] Subjects of opposite assignments were randomly re-paired each period to form a new two-player group. Each period, subjects were asked to choose how many tokens to allocate to 4 targets (boxes). All subjects could allocate to each target any number of tokens between 0 and their valuation. The total number of tokens could not exceed the subject's valuation. All subjects were informed that regardless of who wins, they would have to forfeit all tokens allocated to each target. After all subjects made their allocations, the computer displayed the following information: allocations of the attacker, allocations of the defender, which targets they won, and individual earnings for the period. In the *Lottery-High* and *Lottery-Low* treatments, the winner was chosen according to the simple lottery rule, independently across targets. In the *Auction-*

[10] Subjects also made 15 choices in simple lotteries, similar to Holt and Laury (2002), at the beginning of the experiment. These were used to elicit their risk aversion preferences, and subjects were paid for one randomly selected choice. We did not find any interesting patterns or correlations between risk attitudes and behavior in weakest-link contests. So, we omit any discussion of this issue from the article.

[11] The main reason for using role switching is to avoid any social preferences, i.e., subjects who were assigned as disadvantaged attackers knew that they would also play the role of the advantaged defenders. The role switching also induces better learning, since subjects have an opportunity to learn strategies in the game in both roles.

High and *Auction-Low* treatments, the player who allocated more tokens to a particular target was chosen as the winner of that target.[12]

After completing all 40 decision periods (two treatments), 4 periods were randomly selected for payment (2 periods for each treatment). The sum of the total earnings for these 4 periods was exchanged at the rate of 26 tokens = $1. Additionally, all players received a participation fee of $20 to cover potential losses. On average, subjects earned $25 each, ranging from $11 to $36, and this was paid in cash. Each experimental session lasted about 80 minutes.

4.3. Hypotheses

Our experiment tests five hypotheses motivated by the theoretical predictions. The first hypothesis addresses the comparative static properties of equilibrium in terms of a change in the attacker's valuation.[13] The next two describe predictions concerning individual behavior of the attacker and defender in the *Lottery-Low* and *Lottery-High* treatments. The final two hypotheses describe predictions concerning individual behavior of the attacker and defender in the *Auction-High* and *Auction-High* treatments.

Hypothesis 1: Under the lottery and auction CSF, as the attacker's valuation increases from 40 to 80, the average resource allocation, the probability of winning, and the average payoff increase for the attacker and decrease for the defender.

Hypothesis 2: In the *Lottery-Low* and *Lottery-High* treatments the attacker uses a "complete coverage" strategy, which involves allocating a strictly positive and identical level of the resource across all targets.

[12] When both players allocated the same amount to a given target, the computer always chose the defender as the winner of that target.

[13] Although the comparative statics results are framed in terms of a change in the attacker's valuation, due to invariance of preferences with respect to affine transformations of utility, the theoretical benchmark would also apply to a decrease in the unit cost of resource expenditure of the attacker.

Hypothesis 3: In the *Lottery-Low* treatment the defender uses a "complete coverage" strategy. In the *Lottery-High* treatment the defender allocates a strictly positive and identical level of the resource across all targets with positive probability, and a zero level of the resource with the remaining probability.

Hypothesis 4: In the *Auction-Low* and *Auction-High* treatments the attacker uses a stochastic "guerrilla warfare" strategy, which involves allocating a random level of the resource to at most one target.

Hypothesis 5: In the *Auction-Low* and *Auction-High* treatments the defender uses a stochastic "complete coverage" strategy, which involves allocating random levels of the resource to all of the targets. In the *Auction-Low* treatment these random allocations are positive with probability one. In the *Auction-High* treatment the defender also allocates a zero level of the resource with positive probability.

5. Results

5.1. Aggregate Behavior

The findings are presented as a series of results corresponding to the five hypotheses provided in the previous section. Table 3 summarizes the average allocation of tokens, the probability of winning, and the average payoff by the attacker and the defender in each treatment. Consistent with *Hypothesis 1*, when the attacker's valuation increases from 40 to 80, the average allocation of tokens by the attacker increases from 4.4 to 7.8 under the lottery CSF, and it increases from 4.4 to 7.7 under the auction CSF. The average allocation of tokens by the defender decreases from 24.4 to 15.8 under the auction CSF, but not under the lottery CSF (19.4 versus 19.3). To support these conclusions we estimate panel regressions, reported in the top panel of Table 4, where the dependent variable is allocation to a target and the independent variables are a treatment dummy-variable (*High*), a period trend (*Period*), and a constant (*Constant*). The models includes a random effects error structure, with the individual subject as the random effect, to account for the multiple allocation decisions made by individual subjects over the course of the experiment. The standard errors are clustered at the session level to account for session effects. The treatment dummy-variable is significant in all regressions (p-values < 0.01), except the one where we compare the behavior of the defender in the *Lottery-High* and *Lottery-Low* treatments.

Also, consistent with *Hypothesis 1*, the attacker's probability of winning in the *Lottery-High* treatment (0.68) is higher than his probability of winning in the *Lottery-Low* treatment (0.51), and the probability of winning in *Auction-High* (0.68) is higher than the probability of winning in *Auction-Low* (0.33). These differences are significant (all p-values < 0.01) based on the estimation of random effects probit models, reported in the middle panel of Table 4, where the dependent variable is whether the player won or not, and the independent variables are a treatment dummy-variable (*High*), a period trend (*Period*), and a constant (*Constant*). Consequently, the defender's probability of winning decreases as the attacker's valuation increases.

Finally, consistent with *Hypothesis 1,* as the attacker's valuation increases from 40 to 80, the attacker's payoff increases and the defender's payoff decreases. From Table 3 it is clear that the defender's (attacker's) payoff in the *Lottery-Low* and *Auction-Low* treatments is higher (lower) than in the *Lottery-High* and *Auction-High* treatments. These differences are significant as indicated in the bottom panel of Table 4.

Result 1: Consistent with predictions (*Hypothesis 1*), under the lottery and auction CSF, as the attacker's valuation increases, the average allocation of tokens, the probability of winning, and the average payoff increase for the attacker and decrease for the defender.

These findings support the comparative static prediction that the attacker's advantage over the defender increases as the attacker's valuation increases. Although the qualitative predictions of the theory are supported by our experiment, there is significant over-expenditure of resources by both player types in all treatments. In the *Lottery-Low* treatment, the attacker allocates on average 4.4 tokens, instead of the predicted 3.2, and in the *Lottery-High* treatment, the attacker allocates 7.8 tokens, instead of 5.3. The relative magnitude of over-expenditure by the defender is similar: 19.4 tokens instead of 16.1 and 19.3 tokens instead of 13.1. The range of average over-expenditure is 21%-47%. Over-expenditure is also observed in the *Auction-High* and *Auction-Low* treatments, however, the magnitude is around 10%-22%.[14] As a result of significant over-expenditure, in all treatments both player types receive lower payoffs than predicted (see Table 3).

Significant over-expenditure in our experiment is consistent with previous experimental findings on all-pay auctions and lottery contests (Davis and Reilly, 1998; Potters et al., 1998; Gneezy and Smorodinsky, 2006; Sheremeta and Zhang, 2010; Price and Sheremeta, 2011; 2015). Suggested explanations for over-expenditure include bounded rationality (Sheremeta, 2011; Chowdhury et al., 2014), utility of winning (Sheremeta, 2010; Cason et al., 2012, 2015), other-regarding preferences (Fonseca, 2009; Mago et al., 2015), judgmental biases (Shupp et al.,

[14] A standard Wald test, conducted on estimates of panel regression models, rejects the hypothesis that the average expenditures under the lottery CSF are equal to the predicted theoretical values in Table 3 (all p-values < 0.05). Under the auction CSF we can reject the null hypothesis only for the defender (p-value < 0.05).

2013), and impulsive behavior (Sheremeta, 2015).[15] The same arguments can be made to explain over-expenditure in our experiment.

5.2. Behavior of Attackers under the Lottery CSF

Next we look at the individual behavior of attackers in the *Lottery-Low* and *Lottery-High* treatments. Theory predicts that under the lottery CSF, the attacker should allocate a uniform level of tokens to each target. Nevertheless, contrary to *Hypothesis 2*, tokens are distributed over the entire strategy space. Figure 1 displays, by treatment and player type, the cumulative distribution functions of tokens to a given target (all targets are treated the same). Instead of placing a mass point at 3.2 in the *Lottery-Low* treatment and 5.3 in the *Lottery-High* treatment, the attacker's resources are distributed between 0 and 50.

Another inconsistency with theoretical predictions stated in *Hypothesis 2* is that, instead of allocating a strictly positive amount of tokens to each target, the attacker places a significant mass point at 0 (see Figure 1). Table 5 shows the strategies used by subjects in the *Lottery-Low* and *Lottery-High* treatments. Most commonly the attacker uses a "guerrilla warfare" strategy by attacking a single target and ignoring the remaining targets (44% in the *Lottery-Low* treatment and 46% in the *Lottery-High* treatment). The equilibrium strategy of "complete overage" is used only 24% of time in the *Lottery-Low* treatment and 32% in the *Lottery-High* treatment.

Result 2: Contrary to predictions (*Hypothesis 2*), in the *Lottery-Low* and *Lottery-High* treatments, the attacker's resources are distributed over the entire strategy space. Instead of using the "complete coverage" strategy, the attacker uses the "guerrilla warfare" strategy by attacking a single target and ignoring the remaining targets.

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[15] For a detailed review of possible explanations for the over-expenditure phenomenon see Sheremeta (2013, 2015).

The fact that the attacker's resources are distributed over the entire strategy space is consistent with previous experimental studies documenting high variance of individual expenditures in lottery contests (Davis and Reilly, 1998; Potters et al., 1998; Chowdhury et al., 2014). Several explanations have been offered based on the probabilistic nature of lottery contests and bounded rationality (Chowdhury et al., 2013), both of which could explain the pattern of data observed in our experiment.

A more interesting and novel finding of our study is the use of a "guerrilla warfare" strategy by the attacker. This kind of behavior cannot be explained by a different equilibrium because, as we show in the Appendix A, the equilibrium in the attack and defense game under the lottery CSF is unique. Also, it is unlikely that attackers adjust their strategy given the suboptimal behavior of defenders because, as we discuss later, defenders behave more in line with theoretical predictions by using a "complete coverage" strategy. More importantly, an allocation of zero to one of the targets is a fairly costly deviation for the attacker because for $x_D^i > 0$ the derivative of $p_A^i\left(x_A^i, x_D^i\right)$ is infinite at $x_A^i = 0$. Therefore, even if defenders behave completely irrational, the attacker should never leave any targets intact.

A likely explanation why attackers use a "guerrilla warfare" strategy is that subjects may find it natural to concentrate resources on the necessary number of targets needed for victory (one in our case). Although such a strategy is not optimal, it is an appealing focal point (Schelling, 1960). It has been well documented in the experimental literature that subjects naturally gravitate towards focal points even when it is not necessarily in their best interest (Roth, 1985; Crawford et al., 2008). Such a heuristic strategy can also explain why individual behavior is so close to the theoretical predictions under the auction CSF (as we discuss below).

5.3. Behavior of Defenders under the Lottery CSF

Although the attacker's behavior under the lottery CSF is inconsistent with the theoretical predictions stated in *Hypothesis 2*, there is some evidence concerning the behavior of the defender that supports *Hypothesis 3*. In particular, theory predicts that in the *Lottery-Low* treatment the defender always allocates a strictly positive and identical level of the resource to each target. Table 5, showing the strategies used by subjects, indicates that the defender covers all targets in the *Lottery-Low* treatment 92% of the time, supporting *Hypothesis 3*. In the *Lottery-High* treatment, theory predicts that the defender covers all of the targets with probability 0.83 and none of the targets with probability 0.17. Consistent with this prediction, the data indicate that the defender covers all of the targets in the *Lottery-High* treatment 84% of the time and none of the targets 12% of the time. However, contrary to *Hypothesis 3*, instead of allocating an identical level of tokens across all targets, the defender's resources are distributed between 0 and 50 (see Figure 1).

Result 3: Consistent with predictions (*Hypothesis 3*), in the *Lottery-Low* and *Lottery-High* treatments, the defender uses a "complete coverage" strategy by defending all targets. Contrary to predictions, instead of allocating the same amount of tokens to each target, resources are distributed on the entire strategy space.

As in the case of attackers, the distribution of resources over the entire strategy space by defenders is consistent with previous experimental findings, and could be explained by the probabilistic nature of lottery contests and bounded rationality (Chowdhury et al., 2013).

5.4. Behavior of Attackers under the Auction CSF

Next we look at individual behavior under the auction CSF and begin with the behavior of attackers. Theory predicts that in the *Auction-Low* and *Auction-High* treatments, the attacker should employ a stochastic "guerrilla warfare" strategy, which involves allocating a random level of the resource to at most one target. Figure 2 displays the cumulative distribution functions of the resource allocations and it indicates that, in the aggregate, the attacker's behavior is consistent with this theoretical prediction.[16] The stochastic "guerrilla warfare" strategy is characterized by a significant mass point at 0 for the attacker, which is very close to the predicted value (0.75 versus 0.80 in the *Auction-Low* treatment and 0.67 versus 0.75 in the *Auction-High* treatment).[17]

Moreover, the individual data also show substantial support for the theoretically optimal behavior of the attacker. Table 6 shows the strategies used by subjects in the *Auction-Low* and *Auction-High* treatments. Theory predicts that under the auction CSF the attacker should allocate tokens either to one target or no targets at all. From Table 6, we can see that this happens 89% of the time in the *Auction-Low* treatment and 81% of the time in the *Auction-High* treatment. These findings provide substantial support for *Hypothesis 4*.

Result 4: Consistent with predictions (*Hypothesis 4*), in the *Auction-Low* and *Auction-High* treatments, the attacker uses a stochastic "guerrilla warfare" strategy, which involves allocating a random level of the resource to at most one target.

[16] We combined the distribution of tokens to each of the 4 targets into one target, since marginal distributions to each target are identical across targets.

[17] In calculating the empirical mass points at 0 (Figures 1 and 2), we use an allocation of less than 1 token as an approximation of 0. This approximation is used because the tie-breaking rule favors defenders, and therefore it may encourage attackers to place a very small allocation in some targets in order to reduce the tie-breaking disadvantage. However, even if we use only 0 allocations to compute mass points at 0, we still get results that are close to the theoretical predictions (in the *Auction-Low* and *Auction-High* treatments, for example, the mass points at 0 for the attackers are 0.6 and 0.5).

5.5. Behavior of Defenders under the Auction CSF

The behavior of the defender is also consistent with the theoretical predictions stated in *Hypothesis 5*. In particular, theory predicts that in the *Auction-Low* treatment the defender uses a stochastic "complete coverage" strategy that allocates a strictly positive level of resources to each target with probability one. The data indicate that the defender covers all of the targets 87% of the time (see Table 6). Moreover, consistent with the theoretical predictions, in the *Auction-Low* treatment the defender's resources are uniformly distributed between 0 and 40 (see Figure 2). Similarly, in the *Auction-High* treatment the behavior of the defender is consistent with the theoretical predictions. Theory predicts that the defender should employ a strategy in which, with probability 0.375 he engages in no defensive efforts and, with probability 0.625, the defender allocates a stochastic number of tokens, uniformly distributed between 0 and 50. The data indicate that the defender covers all four targets 62% of the time, three targets 2%, two targets 2%, one target 4%, and zero targets 30% of the time (see Table 6).[18] Moreover, the defender's resources are uniformly distributed between 0 and 50 (see Figure 2).

Result 5: Consistent with predictions (*Hypothesis 5*), in the *Auction-Low* and *Auction-High* treatments, the defender uses a stochastic "complete coverage" strategy, which involves allocating random levels of the resource to all of the targets.

6. Conclusions

This study experimentally investigates individual behavior in a game of attack and defense of a weakest-link network. The attacker's objective is to successfully attack at least one

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[18] The fact that the defender allocates 0 resources to all four targets 30% of the time may be a simple consequence of a behavioral spillover. As we mentioned previously, all subjects changed their role assignment (either as a defender or as an attacker) after 10 periods of the experiment, which might have triggered the defender to behave as the attacker during periods 11-20. Nevertheless, when we analyze the data only for the first 10 periods, we find a similar behavioral pattern for the defenders.

target and the defender's objective is to successfully defend all targets. We apply two benchmark contest success functions: the auction CSF and the lottery CSF. The results of our experiment indicate that under both CSFs both players' resource expenditures exceed their respective theoretical predictions. However, behavior appears to conform to the comparative statics of Nash equilibrium for the parameters chosen: as the attacker's valuation of success increases, the attacker's expenditure increases and the defender's expenditure decreases.

One of the most interesting findings is that the auction CSF's theoretical prediction that the attacker uses a "guerrilla warfare" strategy and the defender uses a "complete coverage" strategy is observed under both the auction and lottery CSFs. This is inconsistent with Nash equilibrium behavior under the lottery CSF. However, such behavior is consistent with a simple heuristic strategy of focusing only on the necessary number of targets needed for victory (one in our case).

Our study contributes to several areas of research. First, our study contributes to the growing experimental literature on multi-battle contests (Dechenaux et al., 2015). While most existing studies focus on multi-battle contests with symmetric objectives (Avrahami and Kareev, 2009; Chowdhury et al., 2013; Montero et al., 2015), we examine behavior in the game of attack and defense of weakest-link networks, in which attackers and defenders have asymmetric objectives. Also, we examine behavior under both the lottery CSF and the auction CSF. Unlike Chowdhury et al. (2013), who study the original Colonel Blotto game and find substantial support for comparative statics predictions under both CSFs, we find important deviation from the theory under the lottery CSF. We hypothesize that this is mainly because subjects use a heuristic strategy of focusing only on the necessary number of targets needed for victory.

Our study also contributes to a large literature on overbidding in contests (Sheremeta, 2013, 2015). Although all of the existing studies document significant over-expenditure in single-battle contests and all-pay auctions, we find that over-expenditure is also an important phenomenon in multi-battle contests. It would be interesting, therefore, to examine if overbidding in single-battle and over-expenditure in multi-battle contests are driven by the same phenomenon. This is a question that we leave for the future research.

Finally, the current study contributes to the rapidly developing literature on defense against terrorism.[19] The two recent studies by Clark and Konrad (2007) and Kovenock and Roberson (2015) provide 'fully' strategic formal analyses of a simultaneous-move game of attack and defense that allows for an endogenous number of targets to be attacked,[20] and for the conflict at each target to be a non-trivial contest with endogenous choice of attack and defense effort.[21] The behavior of the attackers in our experiment provides an alternative explanation for the empirical finding that "periods of high terrorism" seem to be relatively infrequent (Enders, 2007). The common explanation of such a phenomenon is that terrorists face a resource constraint, and therefore they cannot constantly attack all of the targets. Our experiment provides evidence that infrequent "periods of high terrorism" may simply be the result of asymmetric objectives and strategic interactions between the attackers and defenders within a weakest-link type of contest.

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[19] Cadigan and Schmitt (2010) experimentally study terrorism and defense in the context of a two stage entry deterrence game, where in the first stage the government chooses defense expenditures and in the second stage terrorist groups decide whether to attack targets or not.

[20] Our focus on allowing both the defender and attacker to allocate endogenous levels of the resource to an endogenous number of targets contrasts with the reliability-theoretic approach to attack and defense which implicitly assumes that the attacker's allocation of the resource is fixed and is (almost everywhere) allocated to a single target. See for example Bier et al. (2007) and Powell (2007a, 2007b).

[21] Our focus on allowing for the conflict at each target to be a non-trivial contest with endogenous choice of attack and defense effort contrasts with works such as Dziubiński and Goyal (2013a, b) which allow for a much more general network structure, but assume that defense is a zero-one decision and defense is perfect in that a defended node survives an attack with probability one.

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Table 1: Experimental Parameters and Theoretical Predictions

Treatment	Player	Value	Average Allocation	Expected Payoff	Probability of Winning
Lottery-Low	*A*	40	3.2	7.8	0.52
	D	200	16.1	32.2	0.48
Lottery-High	*A*	80	5.3	37.8	0.74
	D	200	13.1	0.0	0.26
Auction-Low	*A*	40	4.0	0.0	0.40
	D	200	20.0	40.0	0.60
Auction-High	*A*	80	6.3	30.0	0.69
	D	200	15.6	0.0	0.31

Average allocation in the *Auction-Low* and *Auction-High* treatments are calculated based on equilibrium mixed strategies.

Table 2: Experimental Sessions

Session Number	Design	Matching Protocol	Participants per Session	Periods per Treatment
1-2	*Lottery-Low* → *Lottery-High*	Strangers	12	20
3-4	*Lottery-High* → *Lottery-Low*	Strangers	12	20
5-6	*Auction-Low* → *Auction-High*	Strangers	12	20
7-8	*Auction-High* → *Auction-Low*	Strangers	12	20

Table 3: Average Allocation, Probability of Winning, and Payoff by Treatment

Treatment	Player	Value	Average Allocation		Probability of Winning		Expected Payoff	
			Predicted	Actual	Predicted	Actual	Predicted	Actual
Lottery-Low	*Attacker*	40	3.2	4.4 (2.5)	0.52	0.51 (0.50)	7.8	2.7 (18.6)
	Defender	200	16.1	19.4 (10.7)	0.48	0.49 (0.50)	32.2	20.6 (98.4)
Lottery-High	*Attacker*	80	5.3	7.8 (4.3)	0.74	0.68 (0.47)	37.8	23.6 (37.3)
	Defender	200	13.1	19.3 (13.0)	0.26	0.32 (0.47)	0.0	-14.1 (90.1)
Auction-Low	*Attacker*	40	4.0	4.4 (3.5)	0.40	0.33 (0.47)	0.0	-4.5 (16.8)
	Defender	200	20.0	24.4 (12.8)	0.60	0.67 (0.47)	40.0	36.2 (82.1)
Auction-High	*Attacker*	80	6.3	7.7 (4.6)	0.69	0.68 (0.47)	30.0	23.2 (33.5)
	Defender	200	15.6	15.8 (15.2)	0.31	0.32 (0.47)	0.0	1.7 (85.6)

Standard deviation in parentheses.

Table 4: Panel Estimation Testing *Hypothesis 1*

Treatments	*Lottery-Low* and *Lottery-High*		*Auction-Low* and *Auction-High*	
Player	*Attacker*	*Defender*	*Attacker*	*Defender*
Dependent variable	*Average Allocation*			
High	3.36***	-0.08	3.26***	-8.57***
[1 if high value]	(1.27)	(1.97)	(0.45)	(3.29)
Period	1.16**	4.86**	1.21**	6.48***
[inverse period trend]	(0.57)	(2.00)	(0.58)	(2.07)
Constant	4.19***	18.55***	4.22***	23.22***
	(0.44)	(1.87)	(0.48)	(1.65)
Dependent variable	*Probability of Winning*			
High	0.47***	-0.48***	0.92***	-0.95***
[1 if high value]	(0.13)	(0.13)	(0.11)	(0.11)
Period	-0.20	0.20	-0.30	0.35
[inverse period trend]	(0.22)	(0.23)	(0.27)	(0.28)
Constant	0.06	-0.06	-0.40***	0.40***
	(0.07)	(0.07)	(0.05)	(0.05)
Dependent variable	*Expected Payoff*			
High	20.91***	-34.69**	27.70***	-34.48***
[1 if high value]	(2.22)	(17.22)	(2.00)	(10.60)
Period	-12.53***	-4.30	-9.07***	3.44
[inverse period trend]	(3.14)	(13.73)	(2.37)	(10.29)
Constant	4.98***	21.41*	-2.87	35.58***
	(0.84)	(11.49)	(1.85)	(6.45)
Observations	960	960	960	960

* significant at 10%, ** significant at 5%, *** significant at 1%. All models include a random effects error structure, with the individual subject as the random effect, to account for the multiple decisions made by the subject over the course of the experiment. The standard errors are clustered at the session level to account for session effects.

Table 5: Strategies Used in the *Lottery-Low* and *Lottery-High* Treatments

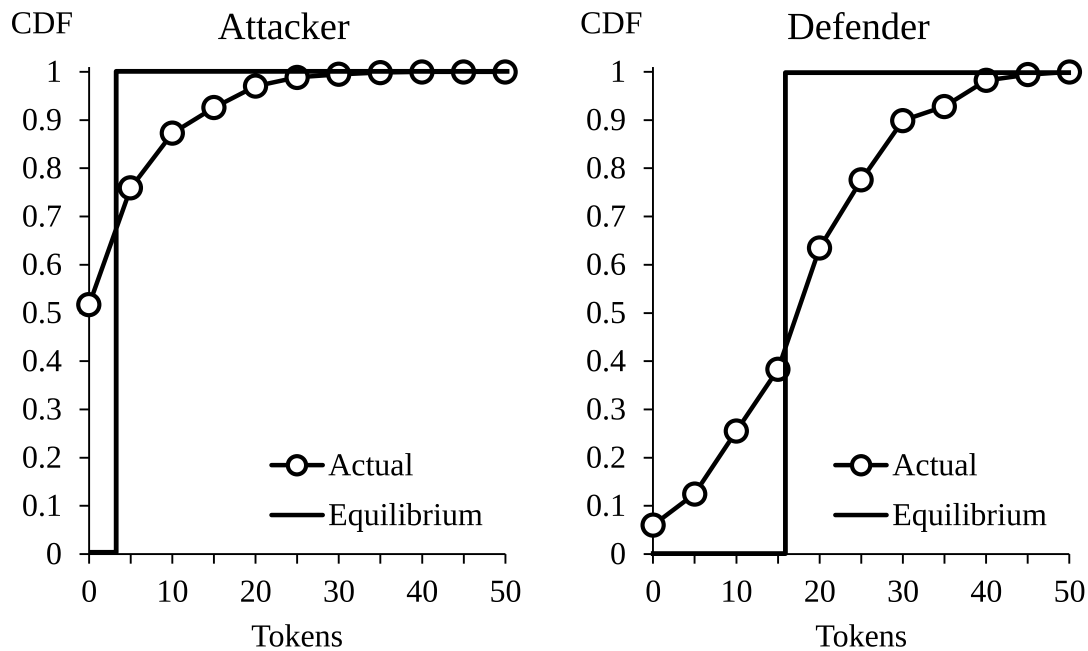
Treatment	Player	Frequency of Allocating Tokens to				
		0 Targets	1 Target	2 Targets	3 Targets	4 Targets
Lottery-Low	*Attacker*	0.10	0.44	0.14	0.08	0.24
	Defender	0.05	0.01	0.01	0.01	0.92
Lottery-High	*Attacker*	0.05	0.46	0.13	0.04	0.32
	Defender	0.12	0.01	0.01	0.02	0.84

Table 6: Strategies Used in the *Auction-Low* and *Auction-High* Treatments

Treatment	Player	Frequency of Allocating Tokens to				
		0 Targets	1 Target	2 Targets	3 Targets	4 Targets
Auction-Low	*Attacker*	0.28	0.61	0.04	0.01	0.06
	Defender	0.06	0.02	0.02	0.03	0.87
Auction-High	*Attacker*	0.11	0.70	0.05	0.02	0.12
	Defender	0.30	0.04	0.02	0.02	0.62

Figure 1: CDF of Tokens in the *Lottery-Low* and *Lottery-High* Treatments

The *Lotter-Low* treatment



The *Lotter-High* treatment

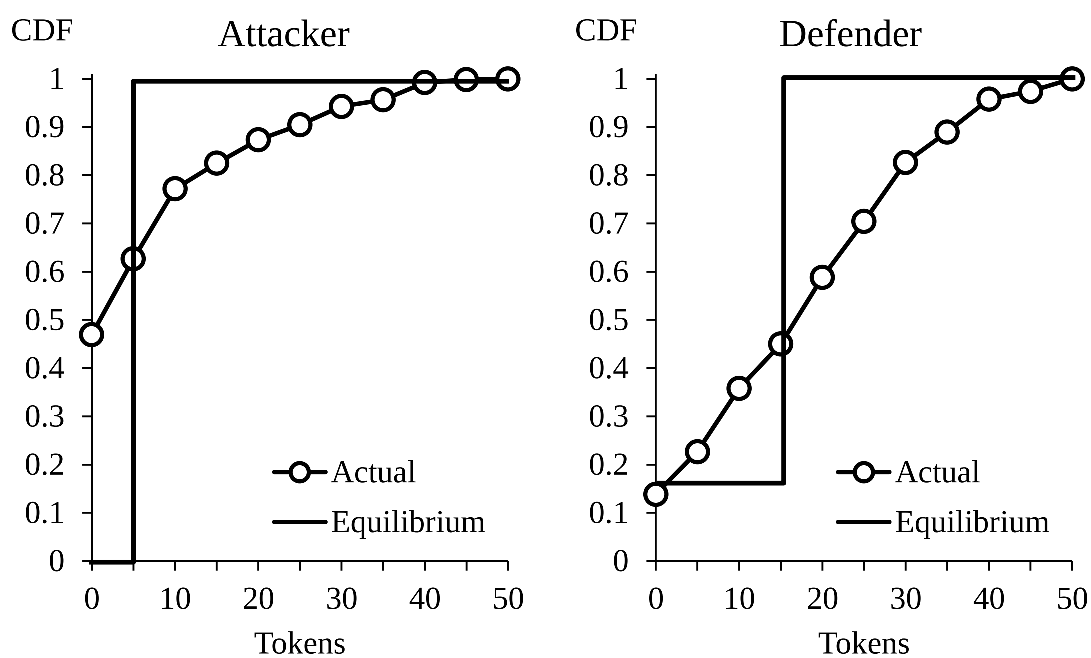
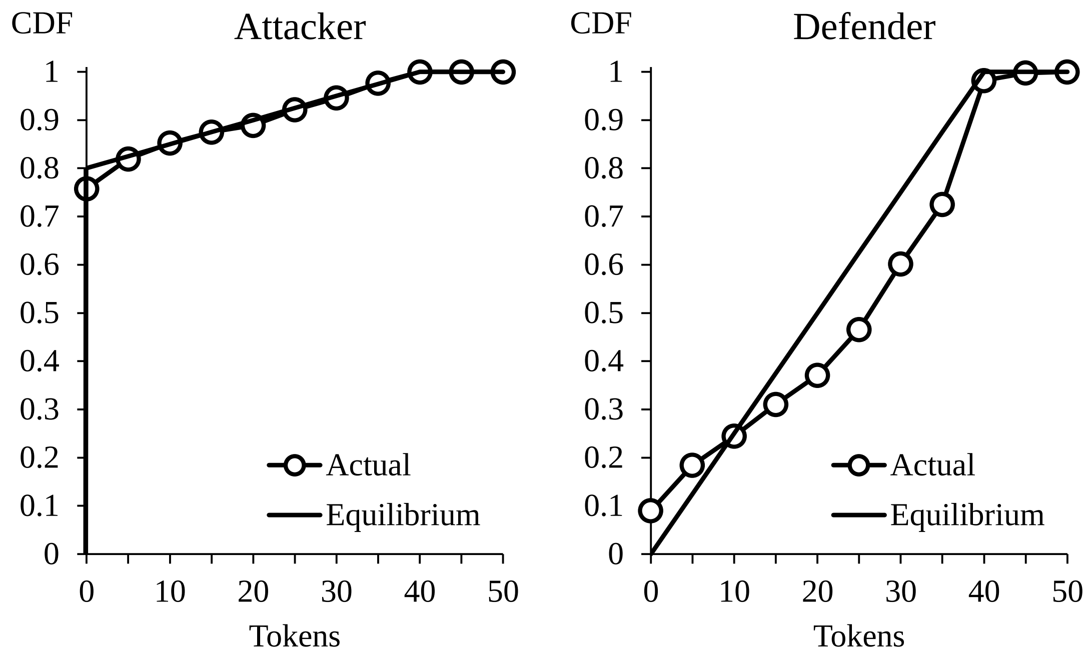
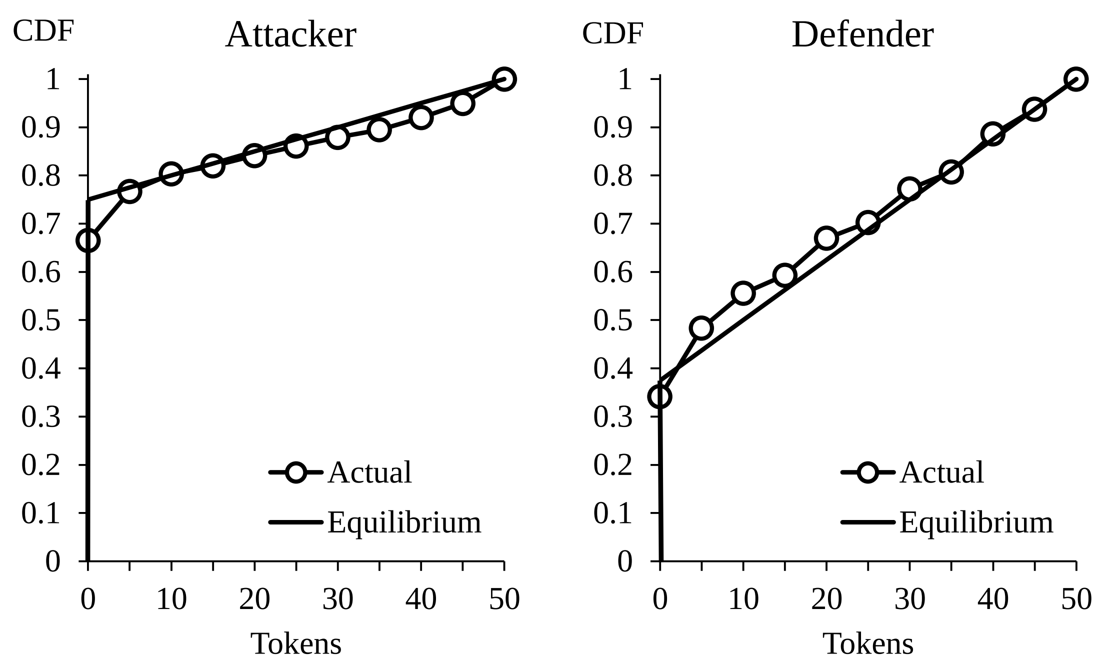


Figure 2: CDF of Tokens in the *Auction-Low* and *Auction-High* Treatments

The *Auction-Low* treatment



The *Auction-High* treatment



Appendix A – Uniqueness of Equilibrium in Lottery CSF Game

Klumpp and Polborn (2006) establish that there exists a unique equilibrium in a two-player simultaneous move game with a set of lottery CSF contests in which each player's objective is to win a majority of the component contests. In the following proof we extend that uniqueness argument for the case in which the players have a symmetric majority objective to the case in which the players have asymmetric attack and defense objectives.

***Definition 1*:** Player j (A or D) plays a *uniform strategy* if he chooses $x \in \mathbb{R}_+$ according to some cumulative distribution function F_j and sets $x_j^i = x$ for all $i \in \{1, \dots, n\}$. An equilibrium in which both players choose uniform strategies is called a *uniform equilibrium*.

We now show that every equilibrium is a uniform equilibrium and that there exists a unique uniform equilibrium. The proof involves three steps. First, we show that every pure-strategy equilibrium is a (degenerate) uniform equilibrium. Second, we show that for $v_D < (n-1)v_A$ equilibrium is in mixed strategies, every mixed-strategy equilibrium is a (nondegenerate) uniform equilibrium, and there exists a unique uniform equilibrium. Third, we show that for $v_D \geq (n-1)v_A$ there exists no mixed-strategy equilibrium and that there exists a unique (degenerate) uniform equilibrium.

We begin the proof of the first step by noting that there exists no pure-strategy equilibrium in which for some target i $x_D^i = 0$ and/or $x_A^i = 0$. By way of contradiction, suppose that there exists a pure-strategy equilibrium in which $x_D^i = 0$ for some target i. For this to be an equilibrium, player A must be playing a best response, but if $x_D^i = 0$, then player A's best-response is undefined. Hence, we have a contradiction. Conversely, if $x_A^i = 0$ for some target i, then player D's best-response is $x_D^i = 0$. But as argued above if $x_D^i = 0$, then $x_A^i = 0$ is not part of a best-response because player A's best-response is undefined. Thus, in any pure-strategy equilibrium, $x_A^i > 0$ and $x_D^i > 0$ for all i.

Next, consider an arbitrary (i.e., possibly non-uniform) pure-strategy equilibrium $\{x_A^i, x_D^i\}_{i=1}^n$ with $x_A^i > 0$ and $x_D^i > 0$ for all i. For each target i, player D's necessary first-order condition is

$$v_D \left(\prod_{i' \neq i} \frac{x_D^{i'}}{x_D^{i'} + x_A^{i'}} \right) \left(\frac{x_A^i}{\left(x_D^i + x_A^i\right)^2} \right) - 1 = 0 \tag{A1}$$

and similarly, for player A,

$$v_A \left(\prod_{i' \neq i} \frac{x_D^{i'}}{x_D^{i'} + x_A^{i'}} \right) \left(\frac{x_D^i}{\left(x_D^i + x_A^i\right)^2} \right) - 1 = 0 \tag{A2}$$

It follows from (A1) and (A2) that in any pure-strategy equilibrium

$$x_D^i = \frac{x_A^i v_D}{v_A} \tag{A3}$$

for all i, where $x_A^i > 0$ and $x_D^i > 0$. Inserting (A3) into (A2) and solving for x_D^i and x_A^i, yields

$$x_D^i = \frac{v_A v_D^{n+1}}{(v_A + v_D)^{n+1}} \text{ and } x_A^i = \frac{v_A^2 v_D^n}{(v_A + v_D)^{n+1}} \tag{A4}$$

for all i. Thus, all pure-strategy equilibria are uniform equilibria, and this completes the proof of part 1.

For the proof of part 2, note that player D's expected payoff under the (A4) strategy profile is

$$v_D \left(\frac{v_D}{v_D + v_A} \right)^n - n v_A \left(\frac{v_D}{v_D + v_A} \right) \geq 0 \tag{A5}$$

Where the inequality in (A5) holds if and only if $v_D \geq (n-1) v_A$. If $v_D < (n-1) v_A$, then there exists no pure-strategy equilibrium. As noted in Clark and Konrad (2007) player A's expected payoff under the (A4) strategy profile is strictly positive for all v_D and v_A satisfying $v_D \geq (n-1) v_A$.

In the proof that follows we will make use of a revised statement of *Lemma 1* of Clark and Konrad (2007).

Lemma 1 (Clark and Konrad, 2007): If player $-j$ plays a uniform pure or mixed strategy P_{-j} in which there exists no contest i with $x_{-j}^{i} = 0$ with certainty, then each of player j's pure-strategy best responses to P_{-j} is a uniform strategy.

The proof of this revised statement of *Lemma 1* follows directly from the proof of *Lemma 1* in Clark and Konrad (2007), when $E_{P_{-j}}\left(\prod_{i=1}^{n} p_D^i\left(x_A^i, x_D^i\right)\right) \in (0,1)$ or when player j's best-response involves setting $x_j^i = 0$ for all i, which is a uniform strategy. Note that if P_A is a mixed strategy in which there exists no contest i with $x_A^i = 0$ with certainty, then (i) $E_{P_A}\left(\prod_{i=1}^{n} p_D^i\left(x_A^i, x_D^i\right)\right) \in (0,1)$ if and only if $x_D^i > 0$ for all i and, as a result, (ii) player D has no pure-strategy best response in which there exist targets i and i', with $i \neq i'$, such that $x_D^i > 0$ and $x_D^{i'} = 0$. Similarly, $E_{P_D}\left(\prod_{i=1}^{n} p_D^i\left(x_A^i, x_D^i\right)\right) \in (0,1)$ if there exists a contest i with $x_A^i > 0$. Because (i) any player *D* pure-strategy best response to P_A, satisfying the conditions in Lemma 1, must involve either $x_D^i > 0$ for all i or $x_D^i = 0$ for all i and (ii) any player *A* pure-strategy best response to P_D, satisfying the conditions in Lemma 1, must involve either $x_A^i > 0$ for some target i or $x_A^i = 0$ for all i, this revised statement of *Lemma 1* follows directly.

For $v_D < (n-1)v_A$ and by way of contradiction, suppose that there exists a second (possibly non-uniform) mixed-strategy equilibrium (P_A, P_D) that is distinct from the equilibrium in case (ii) of *Proposition 1*, where F_D^* is used to denote player *D*'s equilibrium uniform strategy. Let ι_A^i be an indicator function that takes a value of one in the event that player *A* wins the contest at target i. Because these are both equilibria we know that neither player has a payoff increasing deviation:

$$1-\Pr\left(\max_{i\in\{1,\dots,n\}} \iota_A^i = 1|P_A,P_D\right) - \frac{\sum_{i=1}^n E_{P_D}\left(x_D^i\right)}{v_D} \geq 1-\Pr\left(\max_{i\in\{1,\dots,n\}} \iota_A^i = 1|P_A,F_D^*\right) - \frac{\sum_{i=1}^n E_{F_D^*}(x_D^i)}{v_D} \quad \text{(A6)}$$

$$1-\Pr\left(\max_{i\in\{1,\dots,n\}} \iota_A^i = 1|x_A^*,F_D^*\right) - \frac{\sum_{i=1}^n E_{F_D^*}\left(x_D^i\right)}{v_D} \geq 1-\Pr\left(\max_{i\in\{1,\dots,n\}} \iota_A^i = 1|x_A^*,P_D\right) - \frac{\sum_{i=1}^n E_{P_D}(x_D^i)}{v_D} \quad \text{(A7)}$$

$$\Pr\left(\max_{i\in\{1,\dots,n\}} \iota_A^i = 1|P_A,P_D\right) - \frac{\sum_{i=1}^n E_{P_A}(x_D^i)}{v_A} \geq \Pr\left(\max_{i\in\{1,\dots,n\}} \iota_A^i = 1|x_A^*,P_D\right) - \frac{nx_A^*}{v_A} \quad \text{(A8)}$$

$$\Pr\left(\max_{i\in\{1,\dots,n\}} \iota_A^i = 1|x_A^*,F_D^*\right) - \frac{nx_A^*}{v_A} \geq \Pr\left(\max_{i\in\{1,\dots,n\}} \iota_A^i = 1|P_A,F_D^*\right) - \frac{\sum_{i=1}^n E_{P_A}(x_D^i)}{v_A} \quad \text{(A9)}$$

Taking the sum of (A6)-(A9), we have

$$2-\frac{\sum_{i=1}^n E_{P_D}\left(x_D^i\right)}{v_D}-\frac{\sum_{i=1}^n E_{F_D^*}\left(x_D^i\right)}{v_D}-\frac{\sum_{i=1}^n E_{P_A}\left(x_D^i\right)}{v_A}-\frac{nx_A^*}{v_A} \geq 2-\frac{\sum_{i=1}^n E_{P_D}\left(x_D^i\right)}{v_D}-\frac{\sum_{i=1}^n E_{F_D^*}(x_D^i)}{v_D}-\frac{\sum_{i=1}^n E_{P_A}(x_D^i)}{v_A}-\frac{nx_A^*}{v_A} \quad \text{(A5)}$$

which implies that (A6)-(A9) all hold with equality. Thus, the two equilibria (P_A, P_D) and (x_A^*, F_D^*) are interchangeable, which implies that P_D is a best-response to x_A^* and that P_A is a best response to F_D^*. Because, x_A^* and F_D^* are uniform strategies, it follows from *Lemma 1* that P_A and P_D are uniform strategies. To complete the proof of part 2, note that as shown in Clark and Konrad (2007) given the restriction to uniform strategies x_A^* is the unique best response to F_D^*, and given that player A is using the uniform pure-strategy x_A^*, player D is indifferent between the two uniform pure-strategies in the support of F_D^* with all other uniform pure strategies being strictly payoff dominated. Furthermore, player D's randomization between these two points is also uniquely determined by the interchangeability of equilibria. This completes the proof of the uniqueness of equilibrium for $v_D < (n-1)v_A$.

The part 3 proof that over $v_D \geq (n-1)v_A$ equilibrium is unique follows from a similar interchangeability of equilibrium argument, and is, thus, omitted. This completes the proof of the uniqueness of equilibrium in the lottery CSF specification of the attacker-defender game.

Appendix B – The *Lottery-Low* and *Lottery-High* treatments

GENERAL INSTRUCTIONS

This is an experiment in the economics of strategic decision making. Various research agencies have provided funds for this research. The instructions are simple. If you follow them closely and make careful decisions, you can earn an appreciable amount of money.

The experiment will proceed in three parts. Each part contains decision problems that require you to make a series of economic choices which determine your total earnings. The currency used in Part 1 of the experiment is U.S. Dollars. The currency used in Parts 2 and 3 of the experiment is francs. Francs will be converted to U.S. Dollars at a rate of _26_ francs to _1_ dollar. You have already received a **$20.00** participation fee. At the end of today's experiment, you will be paid in private and in cash. **12** participants are in today's experiment.

It is very important that you remain silent and do not look at other people's work. If you have any questions, or need assistance of any kind, please raise your hand and an experimenter will come to you. If you talk, laugh, exclaim out loud, etc., you will be asked to leave and you will not be paid. We expect and appreciate your cooperation. At this time we proceed to Part 1 of the experiment.

INSTRUCTIONS FOR PART 1

In this part of the experiment you will be asked to make a series of choices in decision problems. How much you receive will depend partly on **chance** and partly on the **choices** you make. The decision problems are not designed to test you. What we want to know is what choices you would make in them. The only right answer is what you really would choose.

For each line in the table in the next page, please state whether you prefer option A or option B. Notice that there are a total of **15 lines** in the table but just **one line** will be randomly selected for payment. Each line is equally likely to be chosen, so you should pay equal attention to the choice you make in every line. After you have completed all your choices a token will be randomly drawn out of a bingo cage containing tokens numbered from **1 to 15**. The token number determines which line is going to be paid.

Your earnings for the selected line depend on which option you chose: If you chose option A in that line, you will receive **$1**. If you chose option B in that line, you will receive either **$3** or **$0**. To determine your earnings in the case you chose option B there will be second random draw. A token will be randomly drawn out of the bingo cage now containing twenty tokens numbered from **1 to 20**. The token number is then compared with the numbers in the line selected (see the table). If the token number shows up in the left column you earn $3. If the token number shows up in the right column you earn $0.

Are there any questions?

Decision no.	**Option A**	**Option B**		Please choose A or B
1	**$1**	**$3** never	**$0** if 1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20	
2	**$1**	**$3** if 1 comes out of the bingo cage	**$0** if 2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20	
3	**$1**	**$3** if 1 or 2	**$0** if 3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20	
4	**$1**	**$3** if 1,2,3	**$0** if 4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20	
5	**$1**	**$3** if 1,2,3,4,	**$0** if 5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20	
6	**$1**	**$3** if 1,2,3,4,5	**$0** if 6,7,8,9,10,11,12,13,14,15,16,17,18,19,20	
7	**$1**	**$3** if 1,2,3,4,5,6	**$0** if 7,8,9,10,11,12,13,14,15,16,17,18,19,20	
8	**$1**	**$3** if 1,2,3,4,5,6,7	**$0** if 8,9,10,11,12,13,14,15,16,17,18,19,20	
9	**$1**	**$3** if 1,2,3,4,5,6,7,8	**$0** if 9,10,11,12,13,14,15,16,17,18,19,20	
10	**$1**	**$3** if 1,2,3,4,5,6,7,8,9	**$0** if 10,11,12,13,14,15,16,17,18,19,20	
11	**$1**	**$3** if 1,2, 3,4,5,6,7,8,9,10	**$0** if 11,12,13,14,15,16,17,18,19,20	
12	**$1**	**$3** if 1,2, 3,4,5,6,7,8,9,10,11	**$0** if 12,13,14,15,16,17,18,19,20	
13	**$1**	**$3** if 1,2, 3,4,5,6,7,8,9,10,11,12	**$0** if 13,14,15,16,17,18,19,20	
14	**$1**	**$3** if 1,2, 3,4,5,6,7,8,9,10,11,12,13	**$0** if 14,15,16,17,18,19,20	
15	**$1**	**$3** if 1,2, 3,4,5,6,7,8,9,10,11,12,13,14	**$0** if 15,16,17,18,19,20	

INSTRUCTIONS FOR PART 2

The second part of the experiment consists of **20 decision-making periods**. At the beginning of the first period, you will be randomly assigned either as **participant 1** or as **participant 2**. You will stay in the same role assignment for the **first 10 periods** and then change your role assignment for the **last 10 periods** of the experiment. Each period you will be randomly re-paired with another participant of opposite assignment to form **a two-person group**. So, if you are participant 1, each period you will be randomly re-paired with another participant 2. If you are participant 2, each period you will be randomly re-paired with another participant 1.

Each period, both participants will choose how many tokens to allocate to **4 boxes** in order to receive a reward. Each token costs **1 franc**. The reward is worth **200 francs** to participant 1 and **40 francs** to participant 2. An example of a decision screen is shown below.

Participant 1 can allocate any number of tokens between **0** and **200** (including 0.1 decimal points) to each box. The total number of tokens in all boxes cannot exceed **200**. Similarly, participant 2 can allocate any number of tokens between **0** and **40** (including 0.1 decimal points). The total number of tokens in all boxes cannot exceed **40**.

The more tokens you allocate to a particular box, the more likely you are to win that box. The more tokens the other participant allocates to the same box, the less likely you are to win that box. Specifically, for **each token** you allocate to a particular box you will receive **10 lottery tickets**. At the end of each period the computer **draws randomly** one ticket among all the tickets purchased by you and the other participant in your group. The owner of the drawn ticket wins. Thus, your chance of winning a particular box is given by the number of tokens you allocate to that box divided by the total number of tokens you and the other participant allocate to that box.

$$\text{Chance of winning a box} = \frac{\text{Number of tokens you allocate to that box}}{\text{Number of tokens you allocate + Number of tokens the other participant allocates to that box}}$$

In case both participants allocate zero to the same box, the computer will randomly chose a winner of that box. Therefore, each participant has the same chance of winning the box.

Example of the Random Draw

This is a hypothetical example used to illustrate how the computer makes a random draw. Let's say participant 1 and participant 2 allocate their tokens to the 4 boxes in the following way.

Box	Participant 1	Participant 2	Chance of winning the box for Participant 1	Chance of winning the box for Participant 2
1	20.2	15	20.2/(20.2+15) = 0.57	15/(20.2+15) = 0.43
2	18.5	15	18.5/(18.5+15) = 0.55	15/(18.5+15) = 0.45
3	25	0	25/(25+0) = 1.00	0/(25+0) = 0.00
4	40	5	40/(40+5) = 0.89	5/(40+5) = 0.11
Total	103.7	35		

Participant 1 allocates 20.2 tokens to box 1, 18.5 tokens to box 2, 25 tokens box 3, and 40 tokens to box 4 (a total of 103.7 tokens). Participant 2 allocates 15 tokens to box 1, 15 tokens to box 2, 0 tokens to box 3, and 5 tokens to box 4 (a total of 35 tokens). Therefore, the computer will assign lottery tickets to participant 1 and to participant 2 according to their allocation of tokens.

For example, in box 1, the computer will assign 202 lottery tickets to participant 1 and 150 lottery tickets to participant 2. Then the computer will randomly draw one lottery ticket out of 352 (202+150). As you can see, participant 1 has a higher chance of winning box 1: 20.2/(20.2+15) = 0.57. Participant 2 has lower chance of winning box 1: 15/(20.2+15) = 0.43.

Similarly, in box 3, the computer will assign 250 lottery tickets to participant 1 and 0 lottery tickets to participant 2. Then the computer will randomly draw one lottery ticket out of 250 (250+0). As you can see, participant 2 has no chance of winning box 3: 0/(25+0) = 0.0. Therefore, participant 1 will win box 3 for sure: 25/(25+0) = 1.0.

YOUR EARNINGS

After both participants allocate their tokens and press the OK button, the computer will make a random draw for each box separately and independently. The random draws made by the computer will decide which boxes you win. Then the computer will assign a reward either to participant 1 or participant 2. **The computer will assign a reward to participant 1 only if participant 1 wins all 4 boxes. Otherwise, the computer will assign the reward to participant 2.** The reward is worth **200** francs to participant 1 and **40** francs to participant 2. Regardless of who receives the reward, both participants will have to pay for the tokens they allocated to the 4 boxes (each token costs 1 franc). Thus, the period earnings will be calculated in the following way:

If participant 1 receives the reward:

Participant 1's earnings = 200 – Tokens allocated to 4 boxes

Participant 2's earnings = 0 – Tokens allocated to 4 boxes

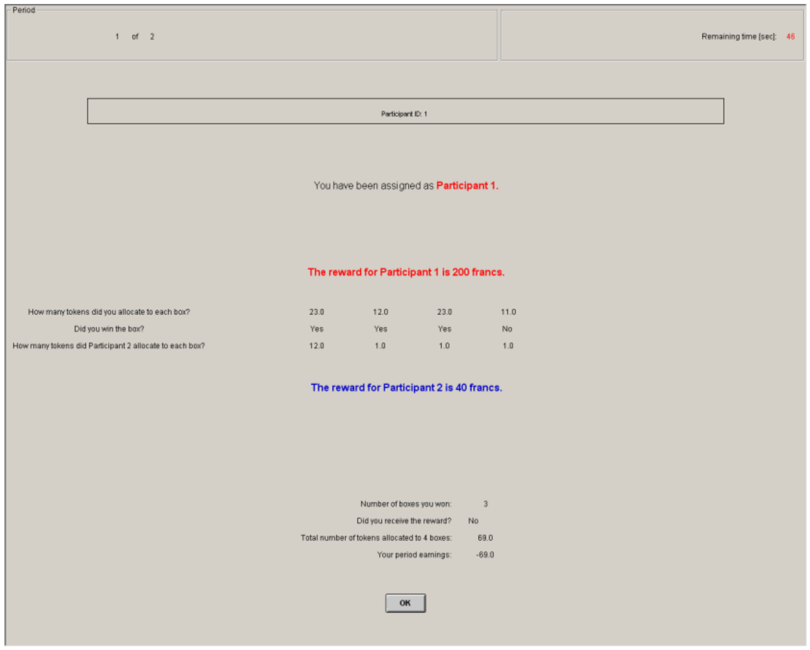
If participant 2 receives the reward:

Participant 1's earnings = 0 – Tokens allocated to 4 boxes

Participant 2's earnings = 40 – Tokens allocated to 4 boxes

Remember you have already received a **$20.00** participation fee (equivalent to **520 francs**). Depending on the outcome in a given period, you may receive either positive or negative earnings. At the end of the experiment we will randomly select 1 out of the first 10 periods and 1 out of the last 10 periods of the experiment for actual payment. You will sum the total earnings for these two periods and convert them to a U.S. dollar payment. If the earnings are negative, we will subtract them from your participation fee. If the earnings are positive, we will add them to your participation fee.

At the end of each period, the allocation of your tokens, the allocation of the other participant's tokens, which boxes you win, whether you received the reward or not, and your period earnings are reported on the outcome screen as shown below. Once the outcome screen is displayed you should record your results for the period on your **Personal Record Sheet** under the appropriate heading.



IMPORTANT NOTES

At the beginning of the first period, you will be randomly assigned either as participant 1 or as participant 2. You will stay in the same role assignment for the first 10 periods and then change your role assignment for the last 10 periods of the experiment. Each period you will be randomly re-paired with another participant of opposite assignment to form a two-person group. So, if you are participant 1, each period you will be randomly re-paired with another participant 2. If you are participant 2, each period you will be randomly re-paired with another participant 1.

Both participants will choose how many tokens to allocate to 4 boxes. After both participants allocate their tokens, the computer will make a random draw for each box separately and independently. You can never guarantee that you will win a particular box. However, by increasing your allocation to that box, you can increase your chance of winning that box. The computer will assign a reward to participant 1 only if participant 1 wins all 4 boxes. Otherwise, the computer will assign the reward to participant 2. Regardless of who receives the reward, both participants will have to pay for the tokens they allocated to 4 boxes. At the end of the experiment we will randomly select 1 out of the first 10 periods and 1 out of the last 10 periods of the experiment for actual payment. You will sum the total earnings for these two periods and convert them to a U.S. dollar payment.

INSTRUCTIONS FOR PART 3

The third part of the experiment consists of **20 decision-making periods**. The rules for Part 3 are exactly the same as the rules for Part 2. As in Part 2, at the beginning of the first period, you will be randomly assigned either as **participant 1** or as **participant 2**. You will stay in the same role assignment for the **first 10 periods** and then change your role assignment for the **last 10 periods** of the experiment. Each period you will be randomly re-paired with another participant of opposite assignment to form **a two-person group**. So, if you are participant 1, each period you will be randomly re-paired with another participant 2. If you are participant 2, each period you will be randomly re-paired with another participant 1.

Each period, both participants will choose how many tokens to allocate to **4 boxes** in order to receive a reward. Each token costs **1 franc**. The only difference from Part 2 is that in Part 3 the reward is worth **200 francs** to participant 1 and **80 francs** (instead of 40 francs) to participant 2. Participant 1 can allocate any number of tokens between **0** and **200** (including 0.1 decimal points) to each box. The total number of tokens in all boxes cannot exceed **200**. Similarly, participant 2 can allocate any number of tokens between **0** and **80** (including 0.1 decimal points). The total number of tokens in all boxes cannot exceed **80**.

After both participants allocate their tokens and press the OK button, the computer will make a random draw for each box separately and independently. The random draws made by the computer will decide which boxes you win. Then the computer will assign a reward either to participant 1 or participant 2. **The computer will assign a reward to participant 1 only if participant 1 wins all 4 boxes. Otherwise, the computer will assign the reward to participant 2.** The reward is worth **200** francs to participant 1 and **80** francs to participant 2. Regardless of who receives the reward, both participants will have to pay for the tokens they allocated to the 4 boxes (each token costs 1 franc).

At the end of each period, the allocation of your tokens, the allocation of the other participant's tokens, which boxes you win, whether you received the reward or not, and your period earnings are reported on the outcome screen. Once the outcome screen is displayed you should record your results for the period on your **Personal Record Sheet** under the appropriate heading. At the end of the experiment we will randomly select 1 out of the first 10 periods and 1 out of the last 10 periods of the experiment for actual payment. You will sum the total earnings for these two periods and convert them to a U.S. dollar payment.