Entry in quota-managed industries: A global game with placement uncertainty

Sunanda Roy,[*] Rajesh Singh,[†] Quinn Weninger,[‡] and Keith Evans[§]

Revised: April 15, 2016

Preliminary, for comments only

Abstract

We present a model of firm entry in an industry that is managed with a cap-and-trade quota regulation. Firms are heterogeneous in their individual productivities; each knows its own productivity but is uncertain about where it ranks within the set of potential entrants in the firm population. Entry is modeled as a simultaneous move game with incomplete information. Under an industry wide quota, the entry payoff is high if average productivity among the set of entrants, active firms, is low. In this case, the quota price is low and the return to vested capital is high. The opposite holds when the average productivity among the set of active firms is high. We derive a threshold entry strategy which separates active and inactive firms. We show that placement uncertainty in general increases entry relative to a full information benchmark. Additional comparative statics and efficiency implications are provided. We extend our model to consider placement overconfidence, whereby a firm believes it ranks higher on the productivity continuum than is objectively warranted. We show that this form of overconfidence exacerbates the over-entry problem. Our results explain investment/divestment patterns in overcapitalized industries adopting quota regulations, commercial fisheries in particular. The results can also explain excess entry by overconfident entrepreneurs who believe the failure rate for their firm will be far less than the industry average.

JEL Classification: L10, C72, Q58
Keywords: Entry and exit, quota managed industry, asymmetric information, global game.

[*]Department of Economics, 483 Heady Hall, Iowa State University, Ames IA 50011-1070; Phone 515-294-4197; Email: `sunanda@iastate.edu`

[†](Corresponding Author) Associate Professor, Department of Economics, 281 Heady Hall, Iowa State University, Ames IA 50011-1070; Phone 515-294-5213; Email: `rsingh@iastate.edu`

[‡]Professor, Department of Economics, 471 Heady Hall, Iowa State University, Ames IA 50011-1070; Phone 515-294-8976; Email: `sunanda@iastate.edu`

[§]Assistant Professor, School of Economics, University of Maine, Orono, ME 04469-5782; Phone 207-581-3178; Email: `keith-evans@maine.edu`

1 Introduction

Quota or cap-and-trade regulation is an increasingly common response to overproduction in industries that generate negative production externalities.[1] A quota on industry output or on an unwanted byproduct of output in the case of pollution prices the externality and introduces a new cost for participating firms that is over and above the capital and factor input costs that otherwise govern firm investment and production decisions. Firms operating in quota-managed industries must hold shares of an aggregate quota to be in compliance with the regulation. If the quota binds, a quota unit used in production by one firm implies one-for-one decrease in quota units used by rival producer(s). The regulation thus provides a mechanism to align production capacity with a socially preferred target output, i.e., a mechanism to reverse the tragedy of the commons.

Can a tradable permit regulation, however, lead to excess entry, and in particular, excess entry of inefficient firms? In other words, while keeping the total production fixed at the socially preferred target, do conditions exist for which cap-and-trade regulations encourage production inefficiency through excess entry and inefficient sharing of capped production responsibilities? Our paper addresses this question.

Our work is inspired by commercial fisheries which, over time, have attracted excess and redundant investment in fishing capital due to the well-known commons problem (Gordon, 1954; Smith, 1969). In response to the inevitable outcome where "too many fishermen chase too few fish" managers have resorted to quota regulations which allocate harvest permits gratis to participating fishermen and allow market forces to remove the excess harvesting capacity that has accumulated under the previous regulatory regime; a process referred to as *rationalization*.[2]

In our model, excess entry and production inefficiency arise from *placement uncertainty*, that is, we assume potential industry entrants are uncertain about where they rank along a productivity continuum. Firms are ex-ante heterogenous; firm managers know their own productivity but not the productivity of their rivals. Equilibrium quota prices are determined by the number and productivity of entering firms. In our model, the decision to commit or not to commit capital must be based on the firm manager's belief about his/her productivity rank or placement in an entrant population. A mistake, in particular an underestimate of the level of post entry competition for the fixed production quota, could mean higher than expected quota costs and a lower than required return to the entry investment.

We study a two-stage entry game in a competitive market environment. In the first stage, atomistic firms choose whether or not to commit a unit of capital to the industry. They base their decision on private signals of their own cost type. Based on their private information, they must form beliefs about the productivity distribution of the entrant population. In the second stage, firms who have entered submit net quota demands to a Walrasian auctioneer. The auctioneer announces the equilibrium quota price equating the quota demand of entrants with the fixed supply. Quota

[1] In the natural resource and environmental regulation realm, cap-and-trade regulations are commonly used to control overfishing and to reduce pollution emissions to socially preferred levels.

[2] Prior to the introduction of quota management, regulators have typically relied on input restrictions, e.g., vessel limits, gear restrictions, and seasonal closures, to limit total industry harvest to sustainable levels. Input control regulation, in the presence of concurrent technological advance, often fails to prevent the build up of harvest capacity. At the point that quota-regulations are implemented, fleet capacity may largely exceed target sustainable catch levels, prompting substantial fleet downsizing, or *rationalization* (see Grafton et al., 2000, Committee to Review Individual Fishing Quotas, 1999 for discussion and evidence of overcapitalization in U.S. and world fisheries.

prices determine firm-specific shares of the total quota and consequently capital rents. The payoff to entry is therefore determined by the set of firms that choose to enter the industry in the first stage. Further, the payoff interdependency among firms operates through the equilibrium quota price and the payoff function is decreasing in the set of active firms. The equilibrium that we study is thus a (rational expectations) Bayesian Nash equilibrium of a market game with strategic substitution. We establish the existence of a symmetric equilibrium in pure *switching strategies* of the following nature: an individual firm manager commits capital to the industry if his/her cost efficiency is below an endogenously determined threshold; the firm manager allocates capital to an alternative use, otherwise.

We contribute to multiple literatures. First, our paper contributes to the literature on global games with strategic substitutes. While the theoretical and applied literature on global games with strategic complementarities is substantial (Carlson and Van Damme, 1993; Morris and Shin, 1998, 2003, 2005; Frankel and Pauzner, 2000; Frankel, Morris and Pauzner, 2003) much less work deals with strategic substitutability (Karp, Lee and Mason 2007; Harrison and Jara-Moroni, 2013; Morris and Shin, 2009). The research gap is partly caused by the fact that games of strategic substitutes, in general, are inherently harder to solve as a Nash equilibrium in pure strategies may not exist. Karp, Lee and Mason (2007) show the existence of a (switching) pure strategy Bayesian Nash equilibrium in a game that combines strategic complementarity and substitution. In their model, a players' payoff is an exogenously given function of the actions of rival players and has a simple parametric form. The players' payoff function in our model is endogenous, operating through the equilibrium quota price determination. Thus a main contribution of our paper in the context of the global games literature is to show existence of a unique pure strategy Nash equilibrium in a game of strategic substitutes with endogenous payoff functions (Propositions 2 and 3).

A comparison of the Bayesian equilibrium with the full information equilibrium yields our main result (Proposition 4 / Theorem 1). We show that, under specific parametric configurations, more cost inefficient firms enter (commit their capital to the quota-regulated industry) under incomplete information about their relative productivity than under the full information case, in an expected sense.[3] The result holds for specific parametric configurations only and is hence non-obvious.

Our model provides a natural framework to study the role of overconfidence, specifically over-placement bias, on industry structure and performance. Behavioral economists and psychologists have observed that individuals often over-estimate their ability to complete complex tasks and tend to rank their abilities higher than is warranted by objective reality. A sizable empirical and experimental literature suggests that this form of overconfidence can lead to over-investment, and failed business ventures (e.g., Camerer and Lovallo, 1999; Koellinger et al., 2007). We extend our model of placement uncertainty to study the implications of over-placement bias on entry and performance in quota-managed industries. We show that the equilibrium threshold declines and more firms enter under over-placement bias, i.e., overconfidence causes firm managers to overestimate their relative productivity and therefore perceive the competition for the fixed quota to be less ex ante than it is ex post. This results in more entry, and lower average industry productivity than in the unbiased baseline. Over-placement bias reinforces the effects of incomplete information, causing even more entry and lower productivity relative to the full information benchmark (Proposition 6).

We also compare and contrast placement uncertainty with pure cost uncertainty, a scenario that is more widely assumed in the standard incomplete information literature. Under the latter scenario,

[3]The parametric form of the firms' cost functions assumed in the paper (quadratic) help us keep the model tractable is standard in the literature.

firms are assumed to be ex-ante identical. They do not observe their own costs and by extension also do not observe the average of firm population costs (as the two are equal). Firms receive noisy signals about their own costs (by extension, that of the population average). We show that on the issue of excessive entry, placement uncertainty is actually a mitigating factor. Fewer cost inefficient firms commit their capital to the industry when they are certain about their own costs but uncertain about their rank, than when they are certain about their rank, i.e., the same as all other firms, but uncertain about their costs (proposition 7). The paper provides the necessary intuition.

Our work is closely related to the recent and fast growing literature on welfare analysis in economies with incomplete private information. This literature highlights the dual nature of prices - as conveyors of information and as determinants of resource allocation. Moreover, various types of inefficiencies - aggregative or distributional - are traced to externalities arising from this dual role (see e.g. Morris and Shin, 2002, 2005; Angeletos and Pavan, 2007, 2009; Amador and Weil, 2010; Vives, 1993, 1997, 2013). The literature primarily focuses on the relative values of private and public information (as conveyed by prices) in a framework of ex-ante (but not ex-post, since they receive different signals) identical agents facing common shocks. The set of agents participating in the market is assumed to be constant - all agents are active. The focus of our paper is the agent's decision itself of whether or not to enter and compete for a fixed quota. The set of active firms is thus endogenous. Within the context of this decision problem, we identify and model a new source of strategic uncertainty that is associated with agent heterogeneity - placement uncertainty and overconfidence bias. To the best of our knowledge such an attempt is new to this literature.

The traditional literature on firm dynamics under complete and incomplete information is vast. Jovanovic (1982) studies entry (and exit) in a setting where firms are uncertain about their own productivity and learn by doing/producing once entry has occurred. We reverse the information set up; we envision a relatively mature, perhaps overgrown industry in which firms are aware of their own productivity. The quota regulation, by pricing an externality, forces managers to carefully assess their productivity placement in order to determine whether or not they can remain active. Our model can therefore be interpreted as one of strategic exit, although exit occurs for reasons that have not been considered in the literature.[4]

It should be pointed out that, although the specific framework adopted in the paper is that of a quota-managed industry, the results apply more generally. The endogenously determined quota price may be replaced by an endogenously determined goods price derived under inelastic product demand. This replacement does not fundamentally change the model. The market forces that are driving the main results are not specific to quota-based industries, in any way. The assumption of a constant price helps keep the model simple and the main results of excess entry finds ready support in quota based industries.

[4]Ghemawat and Nalebuff (1985, 1990) present a model strategic exit in an exogenously declining industry. In their setting industry capacity must be reduced in order to restore profitability. The question is which of the incumbent firms will either exit or reduce its capacity first. Firms play a war of attrition game. The authors derive unique subgame-perfect equilibria in which the larger of the competing firms either exits, in the all or nothing version of the model (Ghemawat and Nalebuff, 1985), or reduces its productive capacity first (in the general version, Ghemawat and Nalebuff, 1990). Larger firms suffer greater losses than small firms as demand declines in the industry, and thus a somewhat counterintuitive result where smaller firms exit last. Fudenberg and Tirole (1986) introduce incomplete information into a dynamic duopoly competition game. As in the model of this paper, firms in Fudenberg and Tirole (1986) know their own costs but must choose when if ever to exit based on their beliefs about rival costs. Over time, active firms become more pessimistic about the cost of their rival. In long run equilibrium, an industry shake out occurs with high cost firms exiting and low cost firms remaining active.

The paper is organized as follows. The next section presents the model. Section 3 derives the equilibrium quota price, entry behavior and examines market performance under a benchmark full information scenario. Section 4 presents the results under incomplete information. Section 5 compares quota prices, industry structure and market performance under full and incomplete information. Section 6 examines the effects of placement bias. Finally, Section 7 compares placement uncertainty with common cost uncertainty.

2 The model

We assume there is a continuum of firms of unit mass in the population. Each firm is endowed with a unit of physical capital which can be used to produce a valued consumer product. We denote the set of all firms as S.

The productivity of capital varies due to differences in the managerial ability of the owner, i.e., the firm manager. Productivity differences manifest as variation in variable costs of production. We let θ_i denote an inverse productivity parameter for firm i; larger values correspond to higher costs. Let $c(q|\theta_i)$ denote variable cost where q is individual firm production. Variable costs are assumed increasing and strictly convex in q. To simplify the analysis that follows $c(.)$ is assumed to take the following form:

$$c(q|\theta_i) = \theta_i q + \frac{1}{2}\lambda q^2.$$

In the sequel, we will often refer to θ_i as simply the productivity or cost efficiency of firm i.

An industry-wide production quota limits aggregate production to the level, Q in each period; $w_i \geq 0$ will denote an initial quota endowment for firm i.[5]

The opportunity cost of allocating capital to the quota-managed industry is its earning potential in a next highest valued use. We denote this per-period capital cost as $\delta_i \geq 0$ for firm i. We will simplify the analysis, and assume capital costs are common for all firms, i.e., $\delta_i = \delta, \ \forall i \in S$.

We consider a representative production period. Suppose Q has been allocated *gratis* to some subset of S.[6] We focus attention on two key decisions for firm managers, hereafter, just *firms*. The first is a decision to forego δ and commit the firm's capital to the quota-managed industry. This decision is made by all $i \in S$ at the beginning of the production period. The set of firms who allocate their capital to the quota-managed industry are hereafter referred to as *active* firms. We denote the set of entrants or active firms, $A \subseteq S$.

A second decision involves quota trading in a post entry permit lease market. Quota has no value outside the quota-managed industry, or outside of the fixed production period. We assume all

[5]We imagine an industry where a quota regulation has been adopted to correct overproduction due to a negative externality. Examples include, fisheries and polluting industries. Our analysis will characterize equilibrium industry structure and corresponding costs of production over a range of aggregate quota quantities, Q. Once industry costs are determined, the value of Q that maximizes social welfare is determined by equating associated marginal benefits and marginal costs. We do not specify a benefit function in our model and therefore do not derive the optimal Q in this paper.

[6]The analysis will soon show that the initial allocation of Q is irrelevant for our results as long as a frictionless quota trading market exists.

quota holders participate in the quota lease market.

All firms announce a net trade schedule to a market maker. This schedule determines the amount of quota the firm is willing to lease for all possible lease prices. The market maker organizes schedules of active firms and determines the market clearing lease price, r. At this price all active firms are required to transact according to their reported net trade schedule (see Maleug and Yates, 2009 for a similar construction).

We use v_i to denote the net quota leased. If v_i is positive (negative) i is a net buyer (seller) of quota. Because one cannot sell more quota than is held, leasing is constrained by $w_i + v_i \geq 0$.

The cost efficiency θ_i is private information throughout. We will consider two information scenarios for individual firms. In the first, firms know the average cost efficiency in the population. The full information scenario will provide a benchmark for comparing the effects of incomplete information. Under the second scenario, incomplete information, each firm knows its own productivity value but is uncertain about the productivity of others. Firms share a common belief about the productivity distribution in S. Specifically, θ_i is assumed to be uniformly distributed over an interval $[\theta-\varepsilon, \theta+\varepsilon]$, where θ will denote the mean cost parameter for all $i \in S$, and $\varepsilon > 0$. Henceforth we describe θ as the population (over S) mean cost efficiency.

The timing of the game is the following. In the first stage, each firm (player) $i \in S$ allocates its capital either to the quota-managed industry or to its next highest-valued use. In other words, firms decide whether or not to belong to $A \subset S$. The set A is thus endogenous. Simultaneously, all firms in S submit net quota lease schedules to the market maker. The market maker then (re)allocates quota to all the active firms, that is to all $i \in A$. In the second stage, active firms produce output that is no larger than their quota allocation with the goal of maximizing variable profit, the latter also being the rent to the capital that is committed in the quota-managed industry.

We first examine the equilibrium outcome in the quota lease market.

2.1 Leasing behavior and capital rent

Each active firm $i \in A$ chooses v_i to maximize capital rent plus permit trading receipts:

$$\pi_i = \max_{v_i} \left[p(w_i + v_i) - rv_i - \theta_i(w_i + v_i) - \frac{1}{2}\lambda(w_i + v_i)^2 \right] \tag{1}$$

where p denotes the fixed output price.

The Lagrangian for the maximization problem is,

$$L = p(w_i + v_i) - rv_i - \theta_i(w_i + v_i) - \frac{1}{2}\lambda(w_i + v_i)^2 - \mu(-w_i - v_i),$$

where μ is the Lagrange multiplier associated with the constraint on feasible quota sales. As v_i can be of any sign in equilibrium, the Kuhn-Tucker necessary conditions are:

$$p - r - \theta_i - \lambda(w_i + v_i) + \mu = 0, \tag{2a}$$

$$w_i + v_i \geq 0, \quad \mu[w_i + v_i] = 0 \tag{2b}$$

$$\mu \geq 0. \tag{2c}$$

From (2a) and (2b) we derive an expression for net quota demand for firm i:

$$v_i = \begin{cases} \frac{1}{\lambda}[p - r - \theta_i] - w_i & \text{for } i \in A \\ -w_i & \text{for } i \notin A. \end{cases} \tag{3}$$

For an active firm, quota demand is increasing in $p - r$, and decreasing in the cost parameter, θ_i, and in the quota endowment, w_i. Notice that $p - r$ is a post-entry or short run virtual supply price (Neary and Roberts, 1980). A quota-unconstrained firm facing output price $p - r$ would produce $q_i = w_i + v_i$ as determined in 3 in order to maximize its variable profit. The equilibrium quota price r will determine the share of total industry revenue that is paid to allocated capital versus the share that flows to the fixed quota Q, which we next show.

Net lease schedules from all quota holders are combined to determine the market clearing quota lease price. Carrying out this derivation obtains,

$$r = \max\{p - \frac{\int_{i \in A} \theta_i d\theta_i}{A} - \frac{\lambda Q}{A}, 0\}. \tag{4}$$

The equilibrium lease price is zero if the net aggregate demand, $\int_{i \in A} v_i + \int_{i \notin A} v_i$ is strictly negative. When positive, the market clearing quota trading price equals the average marginal profit, as follows. Given the form of the cost function, the marginal cost of firm i producing q_i units is $MC_i = \theta_i + \lambda q_i$. Averaging the marginal cost across active firms yields $\frac{\int_{i \in A} \theta_i d\theta_i}{A} + \frac{\lambda Q}{A}$. Thus equilibrium r, when positive, equals the average marginal profit among the set of active firms.

The equilibrium lease price increases, one-for-one, with the output price. For a given set of active firms A, the lease price is lower when Q is larger and when the marginal costs of production is higher. That is, the higher the average over cost efficiency parameters θ_i's and λ, the lower is the equilibrium quota price. Holding $\int_{i \in A} \theta_i$ fixed, we see that a larger A increases r. This reflects the fact that with fixed Q, a larger A means lower production per firm and higher marginal profit (lower marginal cost) under our decreasing returns technology.

Combining (3) and (4) determines the quantity for each active firm as a function of relative cost efficiency (depending on whether equilibrium permit price is positive or zero):

$$q_i = w_i + v_i = \begin{cases} \frac{1}{\lambda}\left(\bar{\theta}(A) - \theta_i + \lambda \bar{Q}(A)\right) & \text{for } r > 0 \\ \frac{1}{\lambda}(p - \theta_i) & \text{for } r = 0 \end{cases}$$

where $\bar{\theta}(A) = \int_{i \in A} \theta_i d\theta_i / A$ is the average cost efficiency among the set of active firms and $\bar{Q}(A) = Q/A$ is average production. We describe $\bar{\theta}(A)$, henceforth, as the mean cost efficiency amongst the active, as opposed to θ which is the population mean. $\bar{\theta}(A)$, like A is thus endogenously determined. Quantity produced by firm i is thus increasing in the firm's relative cost efficiency, $\bar{\theta}(A) - \theta_i$. q_i also increases with the total available quota relative to the active production capacity, as measured by $\bar{Q}(A)$. Very importantly, the equilibrium does not depend on the initial quota holdings.[7]

[7] This allows us to identify the factors that affect the decision to be a buyer/seller of quota in the lease market. Rewriting the equation for quota demand, we find

$$v_i = \frac{1}{\lambda}\left(\bar{\theta}(A) - \theta_i\right) + \lambda \bar{Q}(A) - w_i,$$

Let $\pi(\theta_i|r)$ denote the variable profit or capital rent for firm i, conditional on quota price r. From (3) we are able to rewrite this expression as,

$$\pi(\theta_i|r) = \frac{1}{2\lambda}\left(p - \theta_i - r\right)^2 + rw_i \tag{5}$$

Sometimes it may be useful to express the capital rent for an active firm in terms of relative cost efficiency and the set A. When $r > 0$, we have,

$$\pi(\theta_i|A) = \frac{1}{2\lambda}\left(\bar{\theta}(A) - \theta_i + \lambda\bar{Q}(A)\right)^2 + \left(p - \bar{\theta}(A) - \lambda\bar{Q}(A)\right) w_i. \tag{6}$$

The expression above further highlights the dependence of capital rent in the quota-managed industry on the set of active firms, A.

2.2 Capital allocation

The decision to allocate capital to the industry is determined by,

$$\pi(\theta_i|r) = \begin{cases} \frac{1}{2\lambda}\left(p - \theta_i - r\right)^2 + rw_i & \text{if } i \in A \\ \delta + rw_i & \text{if } i \notin A \end{cases} \tag{7}$$

The capital allocation decision depends on the value of r that is realized in the second stage. In this paper, the Nash equilibria that we study are also rational expectations equilibria. A firm correctly anticipates the equilibrium quota price (function) and incorporates it into the entry decision. Thus the Nash equilibria are also sub-game perfect.

Further, note that payment to the initial quota endowment w_i is collected regardless of where the firm's capital is employed. That is, in our model, which assumes frictionless quota trades, the capital allocation decision is independent of the individual quota endowments, w_i.[8]

3 Entry and market equilibrium under full information

In a context of cost heterogeneity, a natural type of strategy to study is the "threshold" or "switching" strategy of a firm. Under such a strategy, a firm commits capital to the quota-managed

Then the decision to be a net buyer of quota depends on a comparison of:

1. Relative cost efficiency, $\bar{\theta}(A) - \theta_i$: $\frac{\partial v_i}{\partial(\bar{\theta}(A)-\theta_i)} = \frac{1}{\lambda} > 0$
2. Average production relative to private endowment, $\bar{Q}(A) - w_i$: $\frac{\partial v_i}{\partial(\bar{Q}(A)-w_i)} = 1$

The relative important of cost efficiency and the output share to endowment depends on the slope of the marginal cost curve λ. If $\lambda \in (0, 1)$ (the marginal cost curve is flat) then the relative cost efficiency is the driving force in the decision. If, however, $\lambda > 1$ (the marginal cost curve is steep) then the relative endowment of quota is the driving force.

[8] The conditions under which an initial quota allocation will impact efficiency (in a post-trade equilibrium) in cap-and-trade markets has been extensively studied the environmental economics literature. Montgomery (1972) shows that quota market efficiency is independent of the initial quota allocations, when quota trade is frictionless. The initial quota allocation will affect market performance in the presence of transactions costs (Stavins, 1995) or market power (Hahn, 1984; Maleug and Yates, 2009). Our finding the initial quota allocations have no effect on capital investment in quota-managed industries has, to our knowledge, not appeared in earlier literature.

industry if its cost efficiency parameter θ_i is less than or equal to an *endogenously determined* threshold, denoted θ^*. The firms employs its capital in the outside alternative if $\theta_i > \theta^*$. Let $\sigma(\theta_i)$ denote the probability that a firm with cost parameter θ_i enters the quota-managed industry. We explore the existence of a pure strategy Nash equilibrium such that,

$$\sigma(\theta_i) = \begin{cases} 1, & \text{if } \theta_i \leq \theta^* \\ 0, & \text{if } \theta_i > \theta^* \end{cases}$$

The equilibrium existence is studied under alternative information structures. In this section, we assume full or complete information of the mean cost efficiency parameter θ for all players.

Since θ_i is uniformly distributed over $[\theta - \varepsilon, \theta + \varepsilon]$, under an equilibrium threshold θ^* (if it exists) and for a given θ, the proportion of active firms in the population, $\alpha(\theta, \theta^*)$, have the expression

$$\alpha(\theta, \theta^*) = \begin{cases} 0, & \text{if } \theta^* < \theta - \varepsilon, \text{ or } A = \emptyset \\ \frac{1}{2\varepsilon}\int_{\theta-\varepsilon}^{\theta^*} d\theta_i = \frac{\theta^* - (\theta - \varepsilon)}{2\varepsilon}, & \text{if } \theta - \varepsilon \leq \theta^* \leq \theta + \varepsilon, \text{ or } A \subset S \\ 1, & \text{if } \theta + \varepsilon < \theta^*, \text{ or } A = S \end{cases} \tag{8}$$

The mean cost efficiency amongst active firms has the form,

$$\bar{\theta}(A) = \bar{\theta}(\theta, \theta^*) = \begin{cases} 0, & \text{if } \theta^* < \theta - \varepsilon, \quad \text{as } A = \emptyset \\ \frac{\frac{1}{2\varepsilon}\int_{\theta-\varepsilon}^{\theta^*} \theta_i d\theta_i}{\alpha(\theta, \theta^*)} = \frac{\theta^* + \theta - \varepsilon}{2}, & \text{if } \theta - \varepsilon \leq \theta^* \leq \theta + \varepsilon, \\ \theta, & \text{if } \theta + \varepsilon < \theta^*, \quad \text{as } A = S \end{cases} \tag{9}$$

Then, (4), (8) and (9) may be used to express the equilibrium quota price as a function of the given θ and the endogenous θ^*, as follows.

$$r(\theta, \theta^*) = \begin{cases} \max\{p - \theta - \lambda Q, 0\}, & \text{for } \theta < \theta^* - \varepsilon, \text{ since } A = S \\ \max\{p - \frac{\theta^* + \theta - \varepsilon}{2} - \frac{2\varepsilon\lambda Q}{\theta^* - (\theta - \varepsilon)}, 0\}, & \text{for } \theta^* - \varepsilon \leq \theta \leq \theta^* + \varepsilon, \text{ as } A \subset S \\ 0, & \text{for } \theta^* + \varepsilon < \theta, \text{ since } A = \emptyset \end{cases} \tag{10}$$

For a given θ^*, the equilibrium quota price is decreasing in θ. There are two ways through which θ influences r. First, for a given θ^*, a rise in θ decreases the proportion of active firms, $\alpha(\theta, \theta^*)$. This raises the marginal cost of production under the decreasing returns technology with constant Q. Second, a higher value of θ implies that the mean cost efficiency of the active set, $\bar{\theta}(\theta, \theta^*)$, is also higher.

It is also useful to note the relationship between r and θ^* for a given value of θ. So long as $\theta < \theta^* - \varepsilon$, an increase in θ^* has no impact because the set A is unchanged, being equal to S. When $\theta^* - \varepsilon \leq \theta \leq \theta^* + \varepsilon$, for a given θ, a rise in θ^* enlarges the active set bringing in those with higher θ_is. This increases the mean cost efficiency of the active set, $\bar{\theta}(A)$, and exerts a downward pressure on the quota price r. However, under a decreasing returns technology, a larger active set also implies a lower marginal cost as each active firm produces less of the fixed quota and this in turn places an upward pressure on the quota price.

Proposition 1 below shows the existence of a Nash equlibrium in pure threshold strategies, when θ is observable and therefore, firms have complete information about it. Under this scenario, the equilibrium threshold θ^* has a closed form. The form depends on the parametric configuration involving the parameters,λ, Q, δ and ε.

Proposition 1 *1. When $\delta < \frac{1}{2\lambda}(\lambda Q - \varepsilon)^2$, there exists a pure strategy Nash equilibrium under complete information of θ. The equilibrium strategy of firm i is given by,*

$$\sigma(\theta_i) = \begin{cases} 1, & \text{if } \theta_i \leq \theta^* = p - \sqrt{2\lambda\delta} \\ 0, & \text{if } \theta_i > \theta^* \end{cases}$$

2. When $\delta \geq \frac{1}{2\lambda}(\lambda Q - \varepsilon)^2$, there exists a pure strategy Nash equilibrium under complete information of θ. the equilibrium strategy of firm i given by,

$$\sigma(\theta_i) = \begin{cases} 1, & \text{if } \theta_i \leq \theta^*(\theta) \\ & \text{where } \theta^*(\theta) = \begin{cases} (\theta - \varepsilon) + \sqrt{2\lambda\delta + 4\varepsilon\lambda Q} - \sqrt{2\lambda\delta} & \text{for } \theta \leq \hat{\theta} \\ p - \sqrt{2\lambda\delta} & \text{for } \theta > \hat{\theta} \end{cases} \\ 0, & \text{if } \theta_i > \theta^*(\theta) \end{cases}$$

where $\hat{\theta} = p + \varepsilon - \sqrt{2\lambda\delta + 4\varepsilon\lambda Q}$.

Under both types of equilibria, an active firm produces a strictly positive quantity.

PROOF: SEE APPENDIX I.

Proposition 1 provides several useful insights:

REMARK 1. When $\delta < \frac{1}{2\lambda}(\lambda Q - \varepsilon)^2$, the equilibrium threshold is independent of θ and information of the population mean cost efficiency does not influence the individual entry decision. The reason for this seemingly unintuitive equilibrium feature is as follows. Under this scenario, there is a range of θ values, namely, $\theta \leq p - \varepsilon - \sqrt{2\lambda\delta}$ for which *all* firms find it profitable to employ their capital in the quota-managed industry. This is because the outside capital rent δ is too low. When $A = S$, the equilibrium virtual price $p - r$ equals $\theta + \lambda Q$. Recall that for the highest cost firm in S $\theta_i = \theta + \varepsilon$. Equilibrium profit for the highest cost firm is $\frac{1}{2\lambda}(\theta + \lambda Q - \theta - \varepsilon)^2 = \frac{1}{2\lambda}(\lambda Q - \varepsilon)^2$ which is greater than δ. Values of $\theta \geq p - \varepsilon - \sqrt{2\lambda\delta}$ have no effect on individual profitability because (as the Proof shows in detail) the equilibrium permit price attains its minimum value of zero in the full participation region of θ. Thus for firms who continue to be active for $\theta \geq p - \varepsilon - \sqrt{2\lambda\delta}$, the virtual price, $p - r$, is always at its highest level, p. For $\theta \lesseqgtr p - \varepsilon - \sqrt{2\lambda\delta}$, entry decisions are therefore guided purely by the individual θ_is. In sum, the first type of perfect information equilibrium thus applies for low capital costs.

REMARK 2. When $\delta > \frac{1}{2\lambda}(\lambda Q - \varepsilon)^2$ - that is, when δ is higher than some critical value - for no value of θ is it profitable for *all* firms to be simultaneuosly active. This scenario is most interesting because under these conditions, a firm chooses a threshold depending on the observed value of θ. θ^* is a linear and increasing function of θ, implying that as the population mean cost parameter θ increases (mean cost efficiency decreases), more high cost types are encouraged to enter.

Note, moreover, that even for very low values of the population mean θ, some firms on the upper end of the distribution always decide to stay out. The reason is the direct decreasing relationship between r and θ for a given θ^* - thus as θ decreases, r increases. Further as θ decreases, $\theta^*(\theta)$ decreases too and casts an increasing influence on r. Although the net effect of a decrease in θ^* on r is thus ambiguous (see earlier discussion) the combined effect of a lower θ and θ^* is such that some firms on the upper end of the distribution always find the equilibrium quota price and their individual cost θ_i to be too high for entry to be profitable.

Remark 3. Which of the two full information equilibria of Proposition 1 is more likely to occur, is an empirical question. There is one feature of our model, however, which while simplifying the analysis also understates the likelihood of the second type of equilibrium. It is the feature that all firms face a uniform capital cost. A non-uniform δ could make it more likely for some firms not to commit even with low individual θ_i.

Remark 4. The full information equilibria highlight the strategic role played by the virtual price function $p - r$ in this game. p is assumed exogenously determined. Because of our assumption of a continuum of agents, an individual firm has a negligible impact on the quota price. However, in making his/her entry decision, an individual firm always takes into consideration the effect of others' participation on r and thus on the firm's capital rent. Thus the full information model is akin to a Cournot model with a continuum of agents. Furthermore, the virtual price $p - r$, rather than p per se, plays the important role in this game.

Remark 5. Note that although under both full information equilibria, the form of the threshold function is identical for a part that applies when $r = 0$, namely $\theta^* = p - \sqrt{2\lambda\delta}$, the threshold values are certain to be different because of different δ values for the same λ and p values. For fixed p and λ, as the value of δ under scenario (2) is certainly greater than the value of δ under scenario (1), $\theta^* = p - \sqrt{2\lambda\delta}$ is certainly lower under scenario (2) than under scenario (1).

4 Entry and market equilibrium under incomplete information with no bias

In the incomplete information version of the game, at stage one, Nature chooses the population mean cost efficiency, θ, which is unobserved by the individual firms. Firm i' knows its own cost parameter θ_i. The uniform distribution of the θ_is around the unobserved mean θ (with noise, ε) is common knowledge. Consistent with the standard global games framework, we assume θ to have a *prior* (improper) uniform distribution over the real space $\mathbf{R}$, from which Nature picks a value. As is well known, however, the prior distribution of θ plays no part in the equilibrium, eventually.

Each firm forms a *posterior* on the distribution of the unknown population mean cost efficiency, θ, given the private signal it has received about its own cost efficiency, θ_i. In this section, we assume that every firm believes itself to be the population average. In other words, a firm with cost efficiency θ_i, believes that θ is posteriori uniformly distributed over $[\theta_i - \varepsilon, \theta_i + \varepsilon]$. In particular, firm i believes that $E(\theta) = \theta_i$.

Based on its private signal, θ_i, firm i believes that the signal received by any other firm j is symmetrically (but not uniformly) distributed over $[\theta_i - 2\varepsilon, \theta_i + 2\varepsilon]$. Thus, the interval $[\theta_i - 2\varepsilon, \theta_i + 2\varepsilon]$ provides, a posteriori (according to firm i), the range of possible cost parameters of the other firms. Note however, the although the posterior distribution of θ_j is symmetric on $[\theta_i - 2\varepsilon, \theta_i + 2\varepsilon]$, it is not uniform over this range. The posterior distribution of the θ_j plays an important role in defining the expected variable profit function of firm i under incomplete information.

As we proceed to discuss the equilibrium threshold strategy under incomplete information, the issue of *post-entry* capital idleness becomes important. When the population mean cost efficiency θ is unknown, the capital allocation decision must be based on *expected* capital rent. Under uncertainty, a firm may potentially find its value of θ_i to be too high relative to the *ex-post* virtual price $p - r(\theta, \theta^*)$, so that it is optimal to keep the capital (it has already committed) idle and not produce at all. We need to rule out this possibility through a parametric restriction under which post-entry production and thus capital rent is strictly positive for firms that have chosen to be active at the first stage. A sufficient condition that guarantees ex-post production and variable profits are strictly positive is

Assumption 1 (No-idleness condition) $\lambda Q > \varepsilon$.

Appendix 10.1 shows why this condition is sufficient. The restriction in particular implies that an equilibrium without ex-post idleness of capital may not exist for any size of the noise parameter, ε.

4.1 Equilibrium threshold strategy

Assume all firms follow a threshold strategy with threshold value θ^*. The expected capital rent for a θ_i-type firm depends on the quota price and A, and takes one of three forms:

$$\begin{array}{lll} a. & \big(r(\theta,\theta^*) > 0,\ A = S\big): & \pi(\theta_i) = \frac{1}{2\varepsilon}\int_{\theta_i-\varepsilon}^{\theta_i+\varepsilon} \frac{1}{2\lambda}(\theta + \lambda Q - \theta_i)^2 d\theta \\ b. & \big(r(\theta,\theta^*) > 0,\ A \subset S\big): & \pi(\theta_i) = \frac{1}{2\varepsilon}\int_{\theta_i-\varepsilon}^{\theta_i+\varepsilon} \frac{1}{2\lambda}\left(\frac{\theta^*+\theta-\varepsilon}{2} + \frac{2\varepsilon\lambda Q}{\theta^*-(\theta-\varepsilon)} - \theta_i\right)^2 d\theta \\ c. & \big(r(\theta,\theta^*) = 0\big): & \pi(\theta_i) = \frac{1}{2\varepsilon}\int_{\theta_i-\varepsilon}^{\theta_i+\varepsilon} \frac{1}{2\lambda}(p - \theta_i)^2 d\theta \end{array} \tag{11}$$

We seek a symmetric pure strategy Bayesian Nash equilibrium under which all firms adopt the identical threshold strategy with threshold value θ^*. Such a θ^* has the following property. If a firm receives a private signal $\theta_i = \theta^*$, then its expected capital rent is equal to the return from the alternative, δ. Thus to determine θ^*, we first derive the expected capital rent for a $\theta_i = \theta^*$-type firm.

For the $\theta_i = \theta^*$ firm, the posterior distribution for unknown θ is uniform on $[\theta^*-\varepsilon, \theta^*+\varepsilon]$. Moreover, the probability that all firms are active is zero, for the following reason. For such a firm, the lowest value that θ can attain is $\theta^* - \varepsilon$. In this case, the firm is also the highest cost firm in S and $A = S$. Since, we are dealing with a continuous distribution, the probability of such an event is zero. For all other possible values of θ, some firms will have cost parameters higher than θ^* and hence will not be active. Therefore, for a firm with $\theta_i = \theta^*$, depending on whether $r(\theta, \theta^*)$ is strictly positive or zero, the expected capital rent follows form $b.$ or $c.$ in equation (11) or some combination of the two.

Analogous to the full information scenario, the characteristics of the incomplete information equilibrium threshold θ^* depend on the parametric configuration involving the parameters, δ, λ, Q and ε. Depending on this configuration, we have either an equilibrium with (1) $r(\theta, \theta^*) > 0$ for some (or all) $\theta \in [\theta^* - \varepsilon, \theta^* + \varepsilon]$ or (2) with $r(\theta, \theta^*) = 0$ for all $\theta \in [\theta^* - \varepsilon, \theta^* + \varepsilon]$.

A necessary condition for an equilibrium with $r(\theta, \theta^*) > 0$ for some (or all) $\theta \in [\theta^* - \varepsilon, \theta^* + \varepsilon]$ is that, $r(\theta^* - \varepsilon, \theta^*) > 0$. Substituting $\theta = \theta^* - \varepsilon$ into the (10), the necessary condition is,

$$r(\theta^* - \varepsilon, \theta^*) = p - (\theta^* - \varepsilon) - \lambda Q > 0, \Longrightarrow p - \theta^* > \lambda Q - \varepsilon \tag{12}$$

Similarly, a necessary condition for an equilibrium with $r(\theta, \theta^*) = 0$ for all $\theta \in [\theta^* - \varepsilon, \theta^* + \varepsilon]$ is that, $r(\theta^* - \varepsilon, \theta^*) = 0$. From the equilibrium quota price function (10), the necessary condition reduces to,

$$p - (\theta^* - \varepsilon) - \lambda Q \leq 0, \Longrightarrow p - \theta^* \leq \lambda Q - \varepsilon \tag{13}$$

Conditions (12) and (13) thus divide up the parameter space into two zones. When $r(\theta^* - \varepsilon, \theta^*) = 0$, the expected profit of the firm with realization $\theta_i = \theta^*$ is given by,

$$\frac{1}{2\varepsilon} \int_{\theta^*-\varepsilon}^{\theta^*+\varepsilon} \frac{1}{2\lambda}(p - \theta^*)^2 d\theta = \frac{1}{2\lambda}(p - \theta^*)^2 = \delta$$

Then from condition (13), $\delta = \frac{1}{2\lambda}(p - \theta^*)^2 \leq \frac{1}{2\lambda}(\lambda Q - \varepsilon)^2$.

Thus the necessary parametric condition for an equilibrium with $r(\theta, \theta^*) = 0$ for all $\theta \in [\theta^* - \varepsilon, \theta^* + \varepsilon]$ is that $\delta \leq \frac{1}{2\lambda}(\lambda Q - \varepsilon)^2$. Similarly, $\delta > \frac{1}{2\lambda}(\lambda Q - \varepsilon)^2$ is a necessary condition for the existence of an equilibrium θ^* with $r(\theta^* - \varepsilon, \theta^*) > 0$ for some $\theta \in [\theta^* - \varepsilon, \theta^* + \varepsilon]$.

The incomplete information equilibrium is therefore differently characterized for two different regions in which δ may lie: (1) $\delta \in \left(\frac{1}{2\lambda}(\lambda Q - \varepsilon)^2, \frac{(\lambda Q)^2}{2\lambda} + \frac{\varepsilon^2}{6\lambda}\right]$, and (2) $\delta \in \left[0, \frac{1}{2\lambda}(\lambda Q - \varepsilon)^2\right]$.

4.2 $\delta \in \left(\frac{1}{2\lambda}(\lambda Q - \varepsilon)^2, \frac{(\lambda Q)^2}{2\lambda} + \frac{\varepsilon^2}{6\lambda}\right]$

For any given θ^*, the quota price function, $r(\theta, \theta^*) = p - \frac{\theta^* + \theta - \varepsilon}{2} - \frac{2\varepsilon\lambda Q}{\theta^* - (\theta - \varepsilon)}$ attains zero at a value of $\theta < \theta^* + \varepsilon$, since p is finite. Given θ^*, the roots of $p - \frac{\theta^* + \theta - \varepsilon}{2} - \frac{2\varepsilon\lambda Q}{\theta^* - (\theta - \varepsilon)} = 0$ are given by $\hat{\theta}(\theta^*) = (p + \varepsilon) \pm \sqrt{(p - \theta^*)^2 + 4\varepsilon\lambda Q}$. The restriction $\hat{\theta}(\theta^*) < \theta^* + \varepsilon$ implies that only the root $\hat{\theta}(\theta^*) = (p + \varepsilon) - \sqrt{(p - \theta^*)^2 + 4\varepsilon\lambda Q}$ need be considered. Moreover by condition (12), $\hat{\theta}(\theta^*) > \theta^* - \varepsilon$, implying $\theta^* - \varepsilon < \hat{\theta}(\theta^*) < \theta^* + \varepsilon$, for a given θ^*.

Thus the expected profit of a firm with $\theta_i = \theta^*$ is given by,

$$\frac{1}{2\varepsilon} \int_{\theta^*-\varepsilon}^{\hat{\theta}(\theta^*)} \frac{1}{2\lambda}\left(\frac{\theta^* + \theta - \varepsilon}{2} + \frac{2\varepsilon\lambda Q}{\theta^* - (\theta - \varepsilon)} - \theta^*\right)^2 d\theta + \frac{1}{2\varepsilon} \int_{\hat{\theta}(\theta^*)}^{\theta^*+\varepsilon} \frac{1}{2\lambda}(p - \theta^*)^2 d\theta)$$

The following Proposition characterizes the incomplete information equilibrium for this region.

Proposition 2 *A unique pure strategy Bayesian Nash equilibrium in switching strategies exists for every* $\delta \in \left(\frac{1}{2\lambda}(\lambda Q - \varepsilon)^2, \frac{(\lambda Q)^2}{2\lambda} + \frac{\varepsilon^2}{6\lambda}\right]$. *The equilibrium strategy has the form*

$$\sigma(\theta_i) = \begin{cases} 1, & \text{if } \theta_i \leq \theta^*(\delta) \\ 0, & \text{if } \theta_i > \theta^*(\delta) \end{cases}$$

where $\theta^*(\delta)$ solves the following equations:

$$\frac{1}{4\lambda\varepsilon}[\int_{\theta^*-\varepsilon}^{\hat{\theta}(\theta^*)}(\frac{\theta^*+\theta-\varepsilon}{2}+\frac{2\varepsilon\lambda Q}{\theta^*-(\theta-\varepsilon)}-\theta^*)^2 d\theta+\int_{\hat{\theta}(\theta^*)}^{\theta^*+\varepsilon}(p-\theta^*)^2 d\theta)]=\delta \quad (14)$$

$$p-(\theta^*-\varepsilon)-\lambda Q>0 \quad (15)$$

Proof: See Appendix II.

4.3 $\delta \in \left[0, \frac{1}{2\lambda}(\lambda Q-\varepsilon)^2\right]$

The following Proposition characterizes the incomplete information equilibrium for this region.

Proposition 3 *A unique pure strategy Bayesian Nash equilibrium in switching strategies exists for every $\delta \in \left[0, \frac{1}{2\lambda}(\lambda Q-\varepsilon)^2\right]$. The equilibrium strategy has the form*

$$\sigma(\theta_i) = \begin{cases} 1, & \text{if } \theta_i \leq \theta^*(\delta) \\ 0, & \text{if } \theta_i > \theta^*(\delta) \end{cases}$$

where $\theta^*(\delta)$ solves the following equations:

$$\frac{1}{4\lambda\varepsilon}[\int_{\theta^*-\varepsilon}^{\theta^*+\varepsilon}(p-\theta^*)^2=\delta \quad (16)$$

$$p-(\theta^*-\varepsilon)-\lambda Q \leq 0 \quad (17)$$

Proof: See Appendix II.

4.4 Incomplete information equilibrium with no bias

Remark 6: Propositions 1, 2, and 3 show that uncertainty regarding firms' efficiency rank matters for capital investment, only when $\delta \geq \frac{1}{2\lambda}(\lambda Q-\varepsilon)^2$. Under the alternative scenario, $\delta \in \left[0, \frac{1}{2\lambda}(\lambda Q-\varepsilon)^2\right)$, the equilibrium threshold θ^* is identical under full and incomplete information, implying that for a given θ, the set of active firms is identical under both information structures. This is because, the equilibrium quota price $r(\theta, \theta^*)$ - through which the θ influences the determination of θ^* - is zero, whether θ is observed or unobserved.

Thus placement uncertainty does not always matter for entry decision. It matters when the opportunity cost of capital or return from its best alternative use, δ, is higher than a critical level. A high enough δ provides a justification for the less cost efficient firm to sell off their quotas to more cost efficient firms.

Remark 7: The present section characterizes the incomplete information equilibria under the an assumed belief on the part of each firm with regards to its rank in the population distribution, namely each firm i regards itself as the population mean. Many studies suggest however that this may not be the most prevalent attitude amongst potential entrants in the face of placement uncertainty. Section 6 extends the characterization of the equilibria to the situation when firms are over-confident about their efficiencies and regard themselves as above average.

5 Active mass: incomplete vis-à-vis complete information

In this section we discuss the effect of placement uncertainty on the mass of active firms in an industry. The mass of entrants also serves as a surrogate for total amount of capital invested in the industry.

Conventional wisdom would lead us to expect too many cost inefficient entrants into the industry if potential entrants know their own cost efficiency but not the population average efficiency, compared to a situation when they know both. In terms of our model, we should expect the mass of active firms A to be larger if θ is unobserved compared to a situation when it is not. In this section, we investigate whether this conventional wisdom is true.

The mass of active firms, A, is given by:

$$\alpha(\theta, \theta^*) = \begin{cases} 0, & \text{if } \theta^* < \theta - \varepsilon, \quad \text{none active} \\ \frac{1}{2\varepsilon} \int_{\theta-\varepsilon}^{\theta^*} d\theta_i = \frac{\theta^* - (\theta - \varepsilon)}{2\varepsilon}, & \text{if } \theta - \varepsilon \le \theta^* \le \theta + \varepsilon, \quad \text{some active} \\ 1, & \text{if } \theta + \varepsilon < \theta^*, \quad \text{all active} \end{cases} \tag{18}$$

In what follows, we use $\left\{\theta^{*f}, \alpha^f\right\}$ and $\left\{\theta^{*^i}, \alpha^i\right\}$ to differentiate the thresholds and active masses of firms under full and incomplete information, respecitively. Furthermore, note that these thresholds and active masses are functions of several parameters, in particular, the parameter of special interest, δ.

Remark 6 points out that θ^{*f} and θ^{*i} are identical when $\delta \in \left[0, \frac{1}{2\lambda}(\lambda Q - \varepsilon)^2\right]$ and differ for, $\delta \in \left[\frac{1}{2\lambda}\left(\lambda Q - \varepsilon\right)^2, \frac{\lambda Q^2}{2} + \frac{\varepsilon^2}{6\lambda}\right]$. For the purpose of this section, we therefore only consider values of δ in the latter region.

Under both information structure, the realized mass of active firms thus depends on (1) the entry threshold, θ^* and (2) the realization of the populaton mean, θ. Under incomplete information, the entry threshold is independent of θ. Under full information, it is a linear and incresing function of θ. Thus, θ^{*f} may be greater, less or equal to θ^{*i}, depending on the realization of θ. The active mass under the two information structures, α^f and α^i, can therefore be compared only in an expected sense. In other words, the comparables for our study are the magnitudes, $E(\alpha^f)$ and $E(\alpha^i)$. As both expectations depend on the actual distribution of θ, we need to address this issue first.

We assumed in the introduction that θ is a priori (improper) uniformly distributed on **R** and that this is common knowledge. The equilibrium threshold, θ^{*i}, however does not depend on the nature of the prior distribution of θ and the assumption is more of a modelling convenience standard in the literature. It is possible to dispense with this assumption. The assumption that is critical for the determination of θ^{*i} is that the uniform distribution of the θ_i around the unknown θ is common knowledge.

For a meaningful comparative study of the active masses, it is important to assume that θ has a bounded distribution, with θ^l as the lower bound and θ^h, the upper bound. More specifically, as we explain towards the end of section 5.2 below, the comparative static results become trivial without a lower bound on the actual θ. Furthermore, from a purely realistic point of view, such bounds exist.

It is nevertheless important to find a way to reconcile the two different views - the view in section 4 and the present one - about the actual distribution of θ. To this end, we may assume that the actual distribution, $\theta \in [\theta^l, \theta^h]$ is known only to the modeller. The agents continue to believe, without implication for equilibrium behavior, that θ is unbounded on **R**.

It will be shown below that the active mass, $\alpha = 0$ for $\theta \geq p - \sqrt{2\delta\lambda} + \varepsilon$, with or without placement uncertainty. We may therfore, without loss of generality, assume that $\theta^h = p + \varepsilon - \sqrt{2\delta\lambda}$.

By contrast, there is no unique natural choice for θ^l. We therefore need to lay out an admissible *range* of values for θ^l. As θ is the intercept of the marginal cost of a firm, a value of $\theta^l = \varepsilon$, implies that the resulting distribution of θ_i's is given by, $\theta_i \sim U[0, 2\varepsilon]$. In other words, when $\theta^l = \varepsilon$, the lowest cost firm in the distribution has a zero intercept. We may assume, without loss of generality, that firms do not have negative marginal costs and therefore, θ^l can be as low as ε.

We may assume the highest admissible value for θ^l to be $\theta^{*i} - \varepsilon$, for a given θ^*, as the incomplete information equilibrium is defined for $\theta \in [\theta^{*i} - \varepsilon, \theta^{*i} + \varepsilon]$.

Our comparative study is therefore based on the following distribution for θ: $\theta \sim U[\theta^l, \theta^h]$, where $\theta^h = p + \varepsilon - \sqrt{2\delta\varepsilon}$ and $\theta^l \in [\varepsilon, \theta^{*i} - \varepsilon]$ for a given θ^*. Thus the assumed distribution of θ has a support that may be as wide as $[\varepsilon, p + \varepsilon - \sqrt{2\delta\varepsilon}]$ or as narrow as $[\theta^* - \varepsilon, p + \varepsilon - \sqrt{2\delta\varepsilon}]$ for a given θ^*.

Fix a δ and a θ^l. Following Proposition 1, the active mass under full information has the form,

$$\alpha^f(\delta) = \begin{cases} \frac{\sqrt{2\delta\lambda + 4\varepsilon\lambda Q} - \sqrt{2\delta\lambda}}{2\varepsilon} < 1, \text{ for } \theta \leq \hat{\theta} \\ \frac{(p - \sqrt{2\delta\lambda}) - (\theta + \varepsilon)}{2\varepsilon}, \quad \theta \in \left[\hat{\theta}, p + \varepsilon - \sqrt{2\delta\lambda}\right] \\ 0, \quad \theta > p - \sqrt{2\delta\lambda} + \varepsilon. \end{cases} \tag{19}$$

where, it was shown previously, $\hat{\theta}(\delta) = p + \varepsilon - \sqrt{2\lambda\delta + 4\varepsilon\lambda Q}$.

As $\theta^{*i}(\delta)$, does not have a closed form, an analogous expression for $\alpha^i(\delta)$ may not be found. It is nevertheless possible to describe the difference bewteen the two active masses, $(\alpha^i(\delta) - \alpha^f(\delta))$, analytically.

The function, $(\alpha^i(\delta) - \alpha^f(\delta))$ turns out to be discontinuos but linear in θ and consists of different segments. The form differs slightly depending on whether (1) $\hat{\theta}(\delta) \leq \theta^{*i}(\delta)+\epsilon$ or (2) $\theta^{*i}(\delta)+\epsilon \leq \hat{\theta}(\delta)$ is true. However, as we explain below, the main result of the paper (Proposition 4) does not depend upon which inequality holds.

Details of both forms are discussed in Appendix III. For the purposes of presenting and explaning our main result, however, the function is illustrated for a specific value of δ and assuming condition (1) holds.

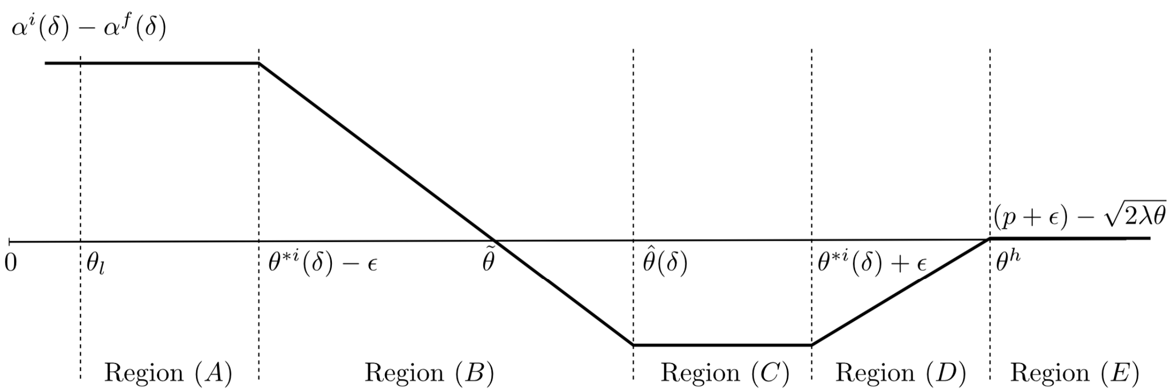

Figure 1: $(\alpha^i(\delta) - \alpha^f(\delta))$: Case 1

The following features of the function, $\alpha^i - \alpha^f$, are worth noting. First, the function takes on positive as well as negative values depending on θ. Our interest is in the net area under the curve which represents the *expected* difference in the active masses. However, there is no obvious reason why the positive area under the curve must be greater than the negative area. The value of the function over the different zones (A) - (D), and the position of the cardinal points, $\theta^{*i} - \varepsilon$, $\hat{\theta}$, $\theta^{*i} + \varepsilon$, $\theta^h = p - \sqrt{2\delta\lambda} + \varepsilon$ and the point $\tilde{\theta}$ at which the function crosses the zero line, all depend in a complicated way on the parameters of the model, in particular, on the parameter of interest, δ. Thus, there is no obvious reason for the conventional wisdom about excessive entry under incomplete information to be true.

Second, assuming no upper bound on θ or having θ unbounded above, does not materially affect the net area under this function because the function has a value of zero for all $\theta > \theta^h = p - \sqrt{2\delta\lambda} + \varepsilon$. On the other hand, assuming no lower bound on θ increases the positive area under the function indefinitely. Thus conventional wisdom holds trivially true in the special case when the lower bound θ^l is sufficiently low or does not exist. Such cases, moreover, may also be less interesting from a realistic point of view. Thus the results of this section draw their main appeal from the bounds placed on θ, in particular, the lower bound on θ.

The net area under the function, $\alpha^i - \alpha^f$, represents the expected difference in active masses. It is straightforward to show that for a given θ^l and δ, this area is given by,

$$E(\alpha^i - \alpha^f) = (\theta^{*i}(\delta) - \theta^l) + \lambda Q - (p + \varepsilon - \theta^l)\left(\frac{\sqrt{2\delta\lambda + 4\varepsilon\lambda Q} - \sqrt{2\delta\lambda}}{2\varepsilon}\right) \tag{20}$$

Appendix III provides details of all calculations.

Since the main result of this section deals with the relationship between $E(\alpha^i(\delta) - \alpha^f(\delta))$ and δ, we begin by discussing how the function $\alpha^i(\delta) - \alpha^f(\delta)$ is affected by changes in δ. The height of the function at the point $\theta^{*i}(\delta) - \varepsilon$ is given by $\left(1 - \frac{\sqrt{2\delta\lambda+4\varepsilon\lambda Q} - \sqrt{2\delta\lambda}}{2\varepsilon}\right)$, a magnitude that increases with δ. The equilibrium values of $\theta^{*i}(\delta)$ and $\hat{\theta}(\delta)$ are both inversely related to δ, implying that all the cardinal points of the function, $\theta^{*i}(\delta) - \varepsilon$, $\theta^{*i}(\delta) + \varepsilon$, $\hat{\theta}(\delta)$, $(p + \varepsilon - \sqrt{2\delta\lambda})$ and $\tilde{\theta}$ shift left, closer towards the fixed θ^l. The height of the function at $\hat{\theta}(\delta)$ is given by $\frac{\theta^{*i}(\delta)+(\sqrt{2\lambda\delta}-p}{2\varepsilon}$ under Case 1. It is not clear how this height varies with δ as the first two terms in the numerator are affected in opposite ways. The same height under Case 2 has a different expression that is clearly increasing in δ. The function $\alpha^i(\delta) - \alpha^f(\delta)$ is thus affected in complex ways by changes in δ.

The expression (20) is the simplified form of the sum of the following two components,

$$\begin{aligned} E(\alpha^i - \alpha^f) &= (\theta^{*i}(\delta) - \varepsilon - \theta^l)\left(1 - \frac{\sqrt{2\delta\lambda + 4\varepsilon\lambda Q} - \sqrt{2\delta\lambda}}{2\varepsilon}\right) \\ &\quad + \left(\lambda Q + \varepsilon - (p - \theta^{*i}(\delta) + 2\varepsilon)\left(\frac{\sqrt{2\delta\lambda + 4\varepsilon\lambda Q} - \sqrt{2\delta\lambda}}{2\varepsilon}\right)\right) \end{aligned}$$

The first component represents the positive area supported by $\theta \in [\theta^l, \theta^{*i}(\delta) - \varepsilon]$ and the second component represents the net positive area with support $\theta \in [\theta^{*i}(\delta) - \varepsilon, \theta^h]$. It is clear from the previous discussions that there is no obvious reason why the sum of the two components must be postive for any given value of δ and θ^l. We conjecture however that for sufficiently low values of δ and appropriate values of θ^l, the sum is positive - implying that for these values of δ and θ^l, the active mass under incomplete information is bigger than that under full information, in an expected sense. Proposition 4 below shows that the conjecture is correct.

Proposition 4 *Denote* $\frac{1}{2\lambda}(\lambda Q - \varepsilon)^2 = \delta^l$. *For every* $\theta^l \in \left[\varepsilon, \theta^{*i}(\delta^l) - \varepsilon\right]$, *there exists an interval* $(\delta^l, \hat{\delta}(\theta^l))$, *such that if* $\delta \in (\delta^l, \hat{\delta}(\theta^l))$, $E(\alpha^i - \alpha^f) > 0$.

Proof: The right side of expression (20) is 0 for all $\theta^l \in [\varepsilon, \theta^{*i} - \varepsilon]$ if $\delta = \delta^l$ (See Appendix III for details). The derivative of expression (20) at $\delta = \delta^l$ is given by (see Appendix III for details),

$$\frac{\partial E(\alpha^i - \alpha^f)}{\partial \delta} = \frac{\lambda}{\lambda Q - \varepsilon}\left(\frac{p + \varepsilon - \theta^l}{\lambda Q + \varepsilon} - 1\right) \tag{21}$$

The right side of equation (21) is strictly positive iff $\theta^l < p - \lambda Q$ and equal to zero if $\theta^l = p - \lambda Q$. For $\delta = \delta^l$ and $\theta^l = \theta^{*i}(\delta^l) - \varepsilon$, it can be shown that $\theta^l = \theta^{*i}(\delta^l) - \varepsilon = p - \lambda Q$. Hence the derivative is zero for $\theta^l = \theta^{*i}(\delta^l) - \varepsilon$. As, $\frac{\partial^2 E(\alpha^i - \alpha^f)}{\partial \theta^l \partial \delta} < 0$, for $\theta^l < \theta^{*i}(\delta^l) - \varepsilon = p - \lambda Q$, the derivative is strictly positive.

Hence for every $\theta^l \in \left[\varepsilon, \theta^{*i}(\delta^l) - \varepsilon\right]$, there exist values of $\delta > \delta^l$ but sufficiently close to it for which $E(\alpha^i - \alpha^f) > 0$. Hence, for every $\theta^l \in \left[\varepsilon, \theta^{*i}(\delta^l) - \varepsilon\right]$, there exists an interval $(\delta^l, \hat{\delta}(\theta^l))$, such that if $\delta \in (\delta^l, \hat{\delta}(\theta^l))$, $E(\alpha^i - \alpha^f) > 0$. Note, that the interval depends on θ^l. Further, as $\theta^{*i}(\delta)$ is decreasing in δ, $\theta^{*i}(\delta) - \varepsilon < \theta^{*i}(\delta^l) - \varepsilon$ for $\delta > \delta^l$ and the range of admissible values of θ^l for any such interval is a strict subset of $\left[\varepsilon, \theta^{*i}(\delta^l) - \varepsilon\right]$. This completes our proof. Δ.

Remark 8: Proposition 4 shows that over entry of firms or over investment of capital is possible under incomplete information about the average cost efficiency, for values of δ within a range, namely $\delta \in (\delta^l, \hat{\delta}(\theta^l))$. If the return on the alternative use of the capital, δ, is very low - that is less than a critical level, δ^l - information or the lack of it, about the mean cost efficiency, does not matter for entry decisions. A firm's entry decision is based on its own cost efficiency, θ_i, only as the expected permit price - the instrument through which the set of active firms affect an individual firm's entry decision - is zero. Similarly, when δ is above a certain value, lack of information about average cost efficiency actually deters entry. With full information, certain high cost frims are encouraged to enter. Thus conventional wisdom about over-entry under incomplete information is true ony for δ values within a specific range.

6 Placement Bias

In this section, we study how the incomplete information equilibrium changes when we relax the assumption that an agent unsure of his relative rank in a population, assumes that he is the average. Instead, we assume that all agents are over-confident about their abilities and assume that they have above average skills. In other words, an agent observing his cost parameter θ_i, believes that the unobserved mean cost parameter, θ lies in the range, $[\theta_i + \beta - \varepsilon, \theta_i + \beta + \varepsilon]$ where $\beta > 0$. Specifically, the agent believes that $\theta \sim U[\theta_i + \beta - \varepsilon, \theta_i + \beta + \varepsilon]$, implying $E(\theta) = \theta_i + \beta > \theta_i$. Recall that under the previous scenario, in the absence of over-confidence, an agent believes that $E(\theta) = \theta_i$.

The parameter β provides a measure of the agent's over-confidence and is henceforth, variously described as *placement bias*, *confidence bias* or *bias* for short. We assume that β is uniform across all agents. We assume further that although an agent is unaware that his belief about himself is the result of a bias (he believes that he is truly superior) he knows that all other agents have a bias. In other words, we assume that β is common knowledge.

Recall that a fundamental assumption of the model is that the distribution of the individual θ_i's around the unobserved θ is common knowledge - that is, $\theta_i \sim U[\theta - \varepsilon, \theta + \varepsilon]$ is common knowledge. The two assumptions of common knowledge are consistent with each other only if $\beta \leq 2\varepsilon$, for the following reason.

If $\theta_i \sim U[\theta - \varepsilon, \theta + \varepsilon]$ is common knowledge, all agents know that their individual θ_i cannot be less than $\theta - \varepsilon$. Since for $\beta > 0$, the minimum $\theta = \theta_i + \beta - \varepsilon$, the former implies that all agents know that θ_i cannot be less than $\theta_i + \beta - 2\varepsilon$ or $\beta - 2\varepsilon$ cannot be strictly positive. In other words, $\beta \leq 2\varepsilon$. Thus any arbitrarily large measure of over-confidence is not consistent with common knowledge of the distribution of the individual θ_is.

Further note that when $\beta = \varepsilon$, an agent with cost parameter θ_i, believes that $\theta \sim U[\theta_i, \theta_i + 2\varepsilon]$, implying $E(\theta) = \theta_i + \varepsilon$ or $\theta_i = E(\theta - \varepsilon)$. In other words, he believes that he has the lowest cost parameter in a population that is uniformly distributed around an unknown mean. Whereas, if $\beta > \varepsilon$, $E(\theta - \varepsilon) = \theta_i + \beta - \varepsilon > \theta_i$ - the agent believes himself to be an outlier. Moreover, as θ_i is arbitrary, all agents believe themselves to be outliers. This is a very special and we think, an unlikely scenario. Therefore, although not crucial for the existence results, we shall implicitly assume that $\beta \leq \varepsilon$.

As before, we explore the existence of a threshold θ_i, denoted μ^* (to differentiate from the no-bias equilibrium θ^*), such that firms commit capital if $\theta_i \leq \mu^*$ and do not commit, otherwise. Note that with over-confidence bias $\beta > 0$, the expressions for the proportion of active firms, the mean cost efficiency of active firms and the equilibrium quota price have the same form as before - namely, expressions (8), (9) and (10) respectively - except that, θ^* is replaced by μ^*. Upon learning its cost parameter $\theta_i = \mu^*$, a firm believes that $\theta \sim U[\mu^* + \beta - \varepsilon, \mu^* + \beta + \varepsilon]$. Equilibrium μ^* is thus characterized by,

$$\frac{1}{2\varepsilon} \int_{\mu^*+\beta-\varepsilon}^{\mu^*+\beta+\varepsilon} \pi(\theta, \mu^*) d\theta = \delta$$

Notably, equilibrium μ^* depends in general not only on δ but on the parameter β as well, that is $\mu^* = \mu^*(\delta, \beta)$.

As in the case of $\beta = 0$, we explore two types of equilibrium - (1) an equilibrium characterized by positive permit price for some values of $\theta \in [\mu^* + \beta - \varepsilon, \mu^* + \beta + \varepsilon]$ and (2) an equilibrium characterized by zero permit price for all values of $\theta \in [\mu^* + \beta - \varepsilon, \mu^* + \beta + \varepsilon]$.

We begin by noting that the permit price function $r(\theta, \mu^*)$ has the same functional form irrespective of the value of β. In particular, $r(\theta, \mu^*)$ is influenced by β only through μ^*.

Hence $r(\theta, \mu^*) = 0$ for all $\theta \in [\mu^* + \beta - \varepsilon, \mu^* + \beta + \varepsilon]$, iff

$$r(\mu^* + \beta - \varepsilon, \mu^*) = 0$$

which on substitution is equivalent to the condition

$$p - \frac{2\mu^* + \beta - 2\varepsilon}{2} - \frac{2\varepsilon\lambda Q}{2\varepsilon - \beta} \leq 0$$

$$\implies p - \mu^* \leq \frac{2\varepsilon\lambda Q}{2\varepsilon - \beta} - \frac{2\varepsilon - \beta}{2}$$

When $r(\theta, \mu^*) = 0$ for all $\theta \in [\mu^* + \beta - \varepsilon, \mu^* + \beta + \varepsilon]$, the equilibrium μ^* is given by the solution of

$$\frac{1}{2\varepsilon}\int_{\mu^*+\beta-\varepsilon}^{\mu^*+\beta+\varepsilon} \frac{1}{2\lambda}(p-\mu^*)^2 d\theta = \delta$$

$$\Longrightarrow \mu^* = p - \sqrt{2\lambda\delta}$$

Substituting the expression for μ^* in the previous expression, we obtain the following necessary condition on the parameters that must be satisfied for the type (2) equilibrium.

$$\delta \leq \frac{1}{2\lambda}\left(\frac{2\varepsilon\lambda Q}{2\varepsilon - \beta} - \frac{2\varepsilon - \beta}{2}\right)^2$$

Thus as in the case of $\beta = 0$, we identify two regions in which δ may lie, each region supporting a different type of equilibrium.

6.1 Equilibrium with placement bias

Proposition 5 1. *Suppose* $\delta \in (\frac{1}{2\lambda}(\frac{2\varepsilon\lambda Q}{2\varepsilon-\beta} - \frac{2\varepsilon-\beta}{2})^2, \frac{(\lambda Q+\beta)^2}{2\lambda} + \frac{\varepsilon^2}{6\lambda}]$. *A unique pure strategy Bayesian Nash equilibrium in switching strategies exist. The equilibrium threshold* μ^* *such that a firm commits if* $\theta_i \leq \mu^*$ *and does not commit otherwise, is the unique solution of the following equations:*

$$\frac{1}{4\lambda\varepsilon}\left[\int_{\mu^*+\beta-\varepsilon}^{\hat{\theta}(\mu^*)} \left(\frac{\mu^*+\theta-\varepsilon}{2} + \frac{2\varepsilon\lambda Q}{\mu^*-(\theta-\varepsilon)} - \mu^*\right)^2 d\theta + \int_{\hat{\theta}(\mu^*)}^{\mu^*+\beta+\varepsilon} (p-\mu^*)^2 d\theta)\right] = \delta \tag{22}$$

$$p - \mu^* > \left(\frac{2\varepsilon\lambda Q}{2\varepsilon-\beta} - \frac{2\varepsilon-\beta}{2}\right) \tag{23}$$

where $\hat{\theta}(\mu^*) = (p+\varepsilon) - \sqrt{(p-\mu^*)^2 + 4\varepsilon\lambda Q}$. *The equilibrium permit price,* $r(\theta, \mu^*)$*, is strictly positive for some values of* $\theta \in [\mu^* + \beta - \varepsilon, \mu^* + \beta + \varepsilon]$.

2. *Suppose* $\delta \leq (\frac{1}{2\lambda}(\frac{2\varepsilon\lambda Q}{2\varepsilon-\beta} - \frac{2\varepsilon-\beta}{2})^2$. *A unique pure strategy Bayesian Nash equilibrium in switching strategies exist. The equilibrium* μ^**, such that a firm commits if* $\theta_i \leq \mu^*$ *and does not commit otherwise, is the unique solution of the following equation:*

$$\frac{1}{4\lambda\varepsilon}\left[\int_{\mu^*+\beta-\varepsilon}^{\mu^*+\beta+\varepsilon} (p-\mu^*)^2 = \delta \right. \tag{24}$$

Equilibrium $\mu^* = p - \sqrt{2\lambda\delta}$.

Proof: See Appendix IV.

Remark 9: The equilibrium has the feature that the permit price is zero for all values of $\theta \in U[\mu^* + \beta - \varepsilon, \mu^* + \beta + \varepsilon]$. Moreover, the equilibrium μ^* does *not* depend on β. The firms choose the same threshold, whether or not they have biased beliefs about themselves.

Remark 10: The proofs of both Propositions are very similar to the corresponding no-bias cases, with minor differences in expression. The major difference with the no-bias case lies in the fact that existence is guaranteed for a different range for δ. Specifically, $\frac{1}{2\lambda}(\frac{2\varepsilon\lambda Q}{2\varepsilon-\beta} - \frac{2\varepsilon-\beta}{2})^2 < \delta < \frac{\lambda Q^2}{2} + \frac{\varepsilon^2}{6\lambda}$, where $\frac{1}{2\lambda}(\frac{2\varepsilon\lambda Q}{2\varepsilon-\beta} - \frac{2\varepsilon-\beta}{2})^2 > \frac{1}{2\lambda}(\lambda Q - \varepsilon)^2$ and $\frac{(\lambda Q+\beta)^2}{2\lambda} + \frac{\varepsilon^2}{6\lambda} > \frac{(\lambda Q)^2}{2\lambda} + \frac{\varepsilon^2}{6\lambda}$, the previous upper bound for δ. Thus, both upper and lower bounds for the range of δ values are higher than in the previous case.

6.2 Active mass under incomplete information with bias

In this subsection we show that the problem of excessive entry is aggravated if firms are over-confident about their cost efficiencies.

Note that $\theta^{*i} = \mu^*(\delta, 0)$. For the above claim to be true, it therefore suffices to show that $\mu^*(\delta,\beta) > \theta^{*i}$ for $\beta > 0$ and for values of $\delta \in (\frac{1}{2\lambda}(\frac{2\varepsilon\lambda Q}{2\varepsilon-\beta} - \frac{2\varepsilon-\beta}{2})^2, \frac{(\lambda Q+\beta)^2}{2\lambda} + \frac{\varepsilon^2}{6\lambda}]$, as in this situation, Proposition 4 holds a fortiori. The following proposition establishes our claim.

Proposition 6 *$\mu^*(\delta,\beta) > \theta^{*i}$ for $\beta > 0$ for values of $\delta \in (\frac{1}{2\lambda}(\frac{2\varepsilon\lambda Q}{2\varepsilon-\beta} - \frac{2\varepsilon-\beta}{2})^2, \frac{(\lambda Q+\beta)^2}{2\lambda} + \frac{\varepsilon^2}{6\lambda}]$.*

Proof: $\mu^*(\delta,\beta)$ is determined by the solution of the following equation, for a given β and δ

$$\pi(k,\beta) = \frac{1}{2\varepsilon}\int_{k+\beta-\varepsilon}^{\hat{\theta}(k)} \frac{1}{2\lambda}\left(\frac{k+\theta-\varepsilon}{2} + \frac{2\varepsilon\lambda Q}{k-(\theta-\varepsilon)} - k\right)^2 d\theta + \frac{1}{2\varepsilon}\int_{\hat{\theta}(k)}^{k+\beta+\varepsilon} \frac{1}{2\lambda}(p-k)^2 d\theta = \delta$$

Note that the expression within brackets in the first term is increasing and convex in θ. Hence, as the limits of the integrals are increasing in β, $\pi(k,\beta)$ is increasing in β. Hence, the solution is increasing in β. Δ

7 Placement uncertainty vis-á-vis common cost uncertainty

7.1 Common cost uncertainty

Many standard models often assume that firms face uncertainty about their own costs as well as about the costs of the rival firms. It is useful to compare and contrast placement uncertainty with this more common notion of cost uncertainty studied in the literature.

Consider a scenario of a continuum of firms with identical cost function, $c(q) = \theta q + \frac{1}{2}\lambda q^2$, where the common θ parameter is unobserved and uniformly distributed over $\mathbf{R}$. As before, each firm receives a signal θ_i and believes that θ is a posteriori uniformly distributed over $[\theta_i - \varepsilon, \theta_i + \varepsilon]$. Under this scenario, firms have cost uncertainty about themselves and others but no placement uncertainty as all firms are identical.

Following equation (4), the equilibrium permit price function now turns out to be

$$r = \max\{p - \theta - \frac{\lambda Q}{A}, 0\}. \tag{25}$$

where A is the endogenously determined mass of active firms. Note in particular that as all firms have identical θ, the mean value of the parameter over the active set is also θ. Under a Bayesian Nash equilibrium with all firms playing the same switching strategy with switch point, θ^{*c}, the active mass is given by equation (8). The equilibrium permit price function thus translates into,

$$r(\theta, \theta^{*c}) = \begin{cases} \max\{p - \theta - \lambda Q, 0\}, & \text{for } \theta < \theta^{*c} - \varepsilon, \text{ since } A = S \\ \max\{p - \theta - \frac{2\varepsilon\lambda Q}{\theta^{*c} - (\theta - \varepsilon)}, 0\}, & \text{for } \theta^{*c} - \varepsilon \leq \theta \leq \theta^{*c} + \varepsilon, \text{ as } A \subset S \\ 0, & \text{for } \theta^{*c} + \varepsilon < \theta, \text{ since } A = \emptyset \end{cases} \tag{26}$$

Moreover, with common costs, the firm's state contingent profit function has the simpler form given by,

$$\frac{1}{2\lambda}\left(p - r(\theta, \theta^{*c}) - \theta\right)^2 = \frac{1}{2\lambda}\left(\frac{\lambda Q}{A}\right)^2$$

when equilibrium permit price is positive and where A is given by (8). When $r(\theta, \theta^{*c}) = 0$, the state profit function is given by $\frac{1}{2\lambda}(p - \theta)^2$.

Then equilibrium threshold θ^{*c} is determined from

$$\frac{1}{4\lambda\varepsilon}\left(\int_{\theta^{*c}-\varepsilon}^{\bar{\theta}^{*c}} \left(\frac{2\lambda\varepsilon Q}{\theta^{*c} - \theta + \varepsilon}\right)^2 d\theta + \int_{\bar{\theta}^{*c}}^{\theta^{*c}+\varepsilon} (p - \theta)^2\, d\theta\right) = \delta \tag{27}$$

where $\bar{\theta}^{*c}$ is the value of θ that yields a zero permit price for a given θ^{*c}. A set of steps similar to the ones laid out in Section 4.2 shows that

$$\bar{\theta}^{*c} = \bar{\theta}\left(\theta^*\right) = \frac{p + \theta^{*c} + \varepsilon - \sqrt{\left(p - (\theta^{*c} + \varepsilon)\right)^2 + 8\varepsilon\lambda Q}}{2}$$

It is straightforward to show that the solution, θ^{*c}, to equation (27) is unique and an equilibrium for $\delta \in [\frac{(\lambda Q)^2}{2\lambda} - \varepsilon Q + \frac{2\varepsilon^2}{3\lambda}, \frac{(\lambda Q)^2}{2\lambda}]$. Note that as, $[\frac{(\lambda Q)^2}{2\lambda} - \varepsilon Q + \frac{2\varepsilon^2}{3\lambda}, \frac{(\lambda Q)^2}{2\lambda}] \subset [\frac{1}{2\lambda}(\lambda Q - \varepsilon)^2, \frac{(\lambda Q)^2}{2\lambda} + \frac{\varepsilon^2}{6\lambda}]$, the existence of a unique switching strategy Bayesian Nash equilibrium can be shown to exist for a smaller range of δ values.

7.2 Placement vs common cost uncertainty

It is natural to ask how the thresholds θ^{*c} and θ^{*i} are related, specifically, which of the two is higher. An answer to this question may help explain whether placement or common cost uncertainty is a bigger contributor to overcapitalisation.

Equation (26) provides the first important insight by identifying exactly what their main difference is. A comparision of (10) and (26) reveals that the two functions are the same when the entire population of firms is active. When however $A \subset S$, it is easy to check that for any given threshold, k, and population mean, θ, the equilibrium permit price with identical costs is less than the equilibrium permit price under cost heterogeneity. To see why, denote by $r^c(k, \theta)$, the equilibrium permit price for a given k and θ. Denote by $r(k, \theta)$, the equilibrium permit price under cost heterogeneity. Their respective forms have been shown as,

$$\begin{aligned} r^c(k, \theta) &= p - \theta - \frac{2\varepsilon\lambda Q}{k - (\theta - \varepsilon)} \\ r(k, \theta) &= p - \frac{k + \theta - \varepsilon}{2} - \frac{2\varepsilon\lambda Q}{k - (\theta - \varepsilon)} \end{aligned}$$

Note that as $k - \varepsilon \leq \theta \leq k + \varepsilon$ under both models, for any k, $\frac{k-\varepsilon+\theta}{2} \leq \theta$. Hence $r^c(k, \theta) \leq r(k, \theta)$ for any given k and population mean, θ. The intuition is as follows. With heterogeneity, for a given k, the mean cost parameter (mean θ_i) over the active set is less than when the firms have identical θ_i s. As the quasi rent from harvest is higher for firms with lower θ_i s, the permit demand by these firms are higher than the permit demand by the higher end firms, causing the equilibrium price to be higher.

Furthermore, for a given k, both functions are decreasing in θ but $r^c(k, \theta)$ has a steeper slope than $r(k, \theta)$. These two observations combined prove that for a given k, $\bar{\theta}(k) < \hat{\theta}(k)$. In other words, equilibrium permit price drops to zero at a lower value of θ under common costs. (It can also be shown that $\bar{\theta}(k) < k$, although a similar statement cannot be made about $\hat{\theta}(k)$.)

Thus, for $\theta \leq k$, the state contingent profit functions under common costs is greater than that under heterogenous costs. That is,

$$\frac{1}{2\lambda}(p - r^c(k, \theta) - \theta)^2 > \frac{1}{2\lambda}(p - r(k, \theta) - k)^2$$

In particular all of these observations combine to show that for a given k, and provided $\hat{\theta}(k) \leq k$, the expected profits under the two scenarios, over a specific range of θ, are related in the following way:

$$\frac{1}{4\lambda\varepsilon}\left(\int_{k-\varepsilon}^{\bar{\theta}(k)} \left(\frac{2\lambda\varepsilon Q}{k - \theta + \varepsilon}\right)^2 d\theta + \int_{\bar{\theta}(k)}^{k} (p - \theta)^2 \, d\theta\right) \geq$$

$$\frac{1}{4\lambda\varepsilon}\left(\int_{k-\varepsilon}^{\bar{\theta}(k)} \left(\frac{2\lambda\varepsilon Q}{k - \theta + \varepsilon} - \frac{k - \theta + \varepsilon}{2}\right)^2 d\theta + \int_{\bar{\theta}(k)}^{\hat{\theta}(k)} \left(\frac{2\lambda\varepsilon Q}{k - \theta + \varepsilon} - \frac{k - \theta + \varepsilon}{2}\right)^2 d\theta + \int_{\hat{\theta}(k)}^{k} (p - k)^2 d\theta\right)$$

However, as

$$\int_{k}^{k+\varepsilon}(p-\theta)^2 d\theta < \int_{k}^{k+\varepsilon}(p-k)^2 d\theta$$

we cannot compare the relative magnitude of profits under common and heterogenous costs in an expected sense, for any given k. The next Proposition shows however that such a comparison is possible for values of k close to $p-\lambda Q+\varepsilon$. This in turn implies that $\theta^{*c} > \theta^{*i}$ for values of δ close to $\frac{(\lambda Q)^2}{2\lambda} - \varepsilon Q + \frac{2\varepsilon^2}{3\lambda}$.

Proposition 7 *There exists an open neighborhood* $(\frac{(\lambda Q)^2}{2\lambda} - \varepsilon Q + \frac{2\varepsilon^2}{3\lambda}, \hat{\delta})$, *such that if* $\delta \in (\frac{(\lambda Q)^2}{2\lambda} - \varepsilon Q + \frac{2\varepsilon^2}{3\lambda}, \hat{\delta})$, $\theta^{*c}(\delta) > \theta^{*i}(\delta)$

PROOF: It is straightforward to check that when $k = p-\lambda Q+\varepsilon$, the corresponding $\bar{\theta}(k) = p-\lambda Q = k-\varepsilon = \hat{\theta}(k)$. Hence, the expected profits under common and heterogenous costs are,

$$\pi^c(k) = \frac{1}{4\lambda\varepsilon}\int_{k-\varepsilon}^{k+\varepsilon}(p-\theta)^2 d\theta, \quad \text{and,} \quad \pi(k) = \frac{1}{4\lambda\varepsilon}\int_{k-\varepsilon}^{k+\varepsilon}(p-k)^2 d\theta$$

By Jensen's inequality,

$$\frac{1}{4\lambda\varepsilon}\int_{k-\varepsilon}^{k+\varepsilon}(p-\theta)^2 d\theta > \frac{1}{2\lambda}(p-k)^2 = \frac{1}{4\lambda\varepsilon}\int_{k-\varepsilon}^{k+\varepsilon}(p-k)^2 d\theta$$

Moreover, at $k = p-\lambda Q+\varepsilon$, $\pi(k) < \pi^c(k) = \frac{(\lambda Q)^2}{2\lambda} - \varepsilon Q + \frac{2\varepsilon^2}{3\lambda}$, the greatest lower bound of δ. Since the first inequality is strict and both $\pi^c(k)$ and $\pi(k)$ are (strictly) monotonically dcreasing and continuous, there exists a neighborhood, $(p-\lambda Q+\varepsilon, \hat{k}]$, such that for $k \in (p-\lambda Q+\varepsilon, \hat{k}]$, $\pi^c(k) \geq \pi(k)$. Let $\pi^c(\hat{k}) = \hat{\delta}$ and the proposition follows.

8 References

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9 Appendix I

9.1 Equilbrium under complete information: Proof of Proposition 1

Under complete information about θ, the profit function for a firm with cost parameter θ_i assumes two different forms:

$$\pi(\theta_i,\theta) = \begin{cases} \frac{1}{2\lambda}(p - r(\theta,\theta^*) - \theta_i)^2, & \text{if for some } \ \theta \in [\theta_i - \varepsilon, \theta_i + \varepsilon], r(\theta,\theta^*) > 0 \\ \frac{1}{2\lambda}(p - \theta_i)^2, & \text{if for all } \ \theta \in [\theta_i - \varepsilon, \theta_i + \varepsilon], r(\theta,\theta^*) = 0 \end{cases}$$

As the equilibrium threshold θ^* is given by the solutions of $\pi(\theta^*,\theta) = \delta$, there are two different types of solutions to consider: (1) the solution of $\frac{1}{2\lambda}(p-\theta^*)^2 = \delta$ which applies when $r(\theta,\theta^*) = 0$ for all $\theta \in [\theta^* - \varepsilon, \theta^* + \varepsilon]$ and (2) the solution of $\frac{1}{2\lambda}(p - r(\theta,\theta^*) - \theta^*)^2 = \delta$ which applies when $r(\theta,\theta^*) > 0$ for some $\theta \in [\theta^* - \varepsilon, \theta^* + \varepsilon]$.

9.1.1 $\delta < \frac{1}{2\lambda}(\lambda Q - \varepsilon)^2$

$\frac{1}{2\lambda}(p-\theta^*)^2 = \delta$ yields $\theta^* = p - \sqrt{2\lambda\delta}$ as the unique solution, if $\delta > 0$. For such a θ^* to be an equlibrium, it must be the case that $r(\theta,\theta^*) = 0$ for $\theta \in [\theta^* - \varepsilon, \theta^* + \varepsilon]$. This implies that $r(\theta,\theta^*)$ attains a value of 0, at a value of $\theta < \theta^* - \varepsilon$, that is at a value of θ at which all firms are active and $A = S$. Given the form of the permit price function that applies when $A = S$, the value of θ at which this price falls to 0, is given by, $\hat{\theta} = p - \lambda Q$.

Hence $\theta^* = p - \sqrt{2\lambda\delta}$ is an equilibrium if the following conditions hold.

1. $\frac{1}{2\lambda}(\theta + \lambda Q - \theta_i)^2 > \delta$ for $\theta < p - \lambda Q$ and $\theta_i \in [\theta - \varepsilon, \theta + \varepsilon]$.
2. $\frac{1}{2\lambda}(p - \theta_i)^2 > \delta$ for $\theta \geq p - \lambda Q$, $\theta_i \in [\theta - \varepsilon, \theta + \varepsilon]$ and $\theta_i \leq p - \sqrt{2\lambda\delta}$.
3. $\frac{1}{2\lambda}(p - \theta_i)^2 < \delta$ for $\theta \geq p - \lambda Q$, $\theta_i \in [\theta - \varepsilon, \theta + \varepsilon]$ and $\theta_i > p - \sqrt{2\lambda\delta}$.

Condition (1) implies, $\frac{1}{2\lambda}(\theta + \lambda Q - \theta - \varepsilon)^2 = \frac{1}{2\lambda}(\lambda Q - \varepsilon)^2 > \delta$ which is true under the given parametric configuration. Since $\frac{1}{2\lambda}(p - \theta_i)^2$ is monotone decreasing in θ_i and has a zero at $\theta_i = p - \sqrt{2\lambda\delta}$, (2) and (3) are true.

Furthermore as $p - \theta^* > 0$, for all active firms with $\theta_i \leq \theta^*$, $p - \theta_i > 0$, implying every active firm produces strictly positive quantity in equilibrium.

9.1.2 $\delta \geq \frac{1}{2\lambda}(\lambda Q - \varepsilon)^2$

When, $r(\theta, \theta^*) > 0$ for some $\theta \in [\theta^* - \varepsilon, \theta^* + \varepsilon]$, the threshold θ^* is determined by $\frac{1}{2\lambda}\left(\frac{\theta^* + \theta - \varepsilon}{2} - \theta^* + \frac{2\varepsilon\lambda Q}{\theta^* - (\theta - \varepsilon)}\right)^2 = \delta$ which yields

$$\theta^*(\theta) = (\theta - \varepsilon) \pm \sqrt{2\delta\lambda + 4\varepsilon\lambda Q} - \sqrt{2\delta\lambda}$$

Note that the form of the profit function applies only if $\theta - \varepsilon \leq \theta^*(\theta) \leq \theta + \varepsilon$. The first inequality implies that the relevant solution for θ^* is

$$\theta^*(\theta) = (\theta - \varepsilon) + \sqrt{2\delta\lambda + 4\varepsilon\lambda Q} - \sqrt{2\delta\lambda}$$

The second inequality implies $(\theta - \varepsilon) + \sqrt{2\lambda\delta + 4\varepsilon\lambda Q} - \sqrt{2\lambda\delta} \leq \theta + \varepsilon$, which in turn implies $\delta \geq \frac{1}{2\lambda}(\lambda Q - \varepsilon)^2$, upon simplification and is satisfied by the parametric configuration assumed.

For the form of $\theta^*(\theta)$ under consideration, the admissible form for $r(\theta, \theta^*)$ is,

$$r(\theta, \theta^*) = \begin{cases} p + \varepsilon - \frac{\sqrt{2\lambda\delta + 4\varepsilon\lambda Q} - \sqrt{2\lambda\delta}}{2} - \frac{2\varepsilon\lambda Q}{\sqrt{2\lambda\delta + 4\varepsilon\lambda Q} - \sqrt{2\lambda\delta}} - \theta & \text{for } \theta < \hat{\theta} \\ 0, & \text{for } \theta > \hat{\theta} \end{cases}$$

where,

$$\begin{aligned} \hat{\theta} &= p + \varepsilon - \frac{\sqrt{2\lambda\delta + 4\varepsilon\lambda Q} - \sqrt{2\lambda\delta}}{2} - \frac{2\varepsilon\lambda Q}{\sqrt{2\lambda\delta + 4\varepsilon\lambda Q} - \sqrt{2\lambda\delta}} \\ &= p + \varepsilon - \sqrt{2\lambda\delta + 4\varepsilon\lambda Q} \end{aligned}$$

upon simplification.

Hence the admissible threshold function is

$$\theta^*(\theta) = \begin{cases} (\theta - \varepsilon) + \sqrt{2\delta\lambda + 4\varepsilon\lambda Q} - \sqrt{2\delta\lambda} & \text{for} \quad \theta < \hat{\theta} \\ p - \sqrt{2\lambda\delta} & \text{for} \quad \theta \geq \hat{\theta} \end{cases}$$

Note that as the condition $\theta^*(\theta) < \theta + \varepsilon$ implies that it is never profitable for all firms to be active simultaneously, in an equilibrium, the following must hold,

1. For $\theta < \hat{\theta}$

$$\begin{array}{ll} \frac{1}{2\lambda}\left(\frac{\theta^*(\theta)+\theta-\varepsilon}{2} + \frac{2\varepsilon\lambda Q}{\theta^*(\theta)-(\theta-\varepsilon)} - \theta_i\right)^2 > \delta & \text{for} \quad \theta_i \leq \theta^*(\theta) \\ \frac{1}{2\lambda}\left(\frac{\theta^*(\theta)+\theta-\varepsilon}{2} + \frac{2\varepsilon\lambda Q}{\theta^*(\theta)-(\theta-\varepsilon)} - \theta_i\right)^2 < \delta & \text{for} \quad \theta_i > \theta^*(\theta) \end{array}$$

2. For $\theta > \hat{\theta}$

$$\begin{array}{ll} \frac{1}{2\lambda}(p-\theta_i)^2 > \delta & \text{for} \quad \theta_i \leq \theta^*(\theta) \\ \frac{1}{2\lambda}(p-\theta_i)^2 < \delta & \text{for} \quad \theta_i > \theta^*(\theta) \end{array}$$

These conditions are satisfied because the profit functions are monotone decreasing in θ_i and has zeros at $\theta^*(\theta)$ for the relevant regions. Finally as, since $p - r(\theta, \theta^*) - \theta^* > 0$, all active firms with $\theta_i \leq \theta^*$ earn strictly positive variable profits, $p - r(\theta, \theta^*) - \theta_i$, implying that every active firm produces strictly positive quantity in equilibrium.

10 Appendix II

10.1 Equilibrium under incomplete information - condition for strictly positive production

First we want to ensure that $p - r(\theta, \theta^*) - \theta_i > 0$ for the case when all firms are active - that is for all $\theta < \theta^* - \varepsilon$ and $\theta_i \in [\theta - \varepsilon, \theta + \varepsilon]$. Under this situation,

$$p - r(\theta, \theta^*) - \theta_i = \theta + \lambda Q - \theta_i$$

It suffices to ensure that the required condition holds for the highest cost firm $\theta + \varepsilon$. Thus, we want $\theta + \lambda Q - \theta - \varepsilon > 0$ which yields $\lambda Q > \varepsilon$, the first part of Assumption 1.

We next want to ensure that $p - r(\theta, \theta^*) - \theta_i > 0$ for $\theta^* - \varepsilon \leq \theta \leq \theta^* + \varepsilon$ and $\theta_i \in [\theta - \varepsilon, \theta + \varepsilon]$. Thus we want

$$\begin{array}{ll} \frac{\theta^*+\theta-\varepsilon}{2} + \frac{2\varepsilon}{\theta^*-(\theta-\varepsilon)} - \theta^* > 0 & \text{for } \; r > 0 \\ p - \theta^* > 0 & \text{for } \; r = 0 \end{array}$$

The first inequality reduces to the requirement, $\theta^* + \varepsilon - \theta) < \pm 2\sqrt{\varepsilon\lambda Q}$ on simplification. As we are looking at the case when $\theta \leq \theta^* + \varepsilon$, the requirement reduces to $\theta > \theta^* + \varepsilon - 2\sqrt{\varepsilon\lambda Q}$.

Note that $\lambda Q > \varepsilon$ implies $\sqrt{\varepsilon\lambda Q} > \varepsilon$ which in turn implies that $\theta^*+\varepsilon-2\sqrt{\varepsilon\lambda Q} < \theta^*+\varepsilon-2\varepsilon = \theta^*-\varepsilon$.

Since $\theta \geq \theta^*-\varepsilon$, the first inequality is automatically satisfied. Since under $\lambda Q > \varepsilon$, $p-r(\theta,\theta^*)-\theta^* > 0$ for $r > 0$, it is automatically true that $p - \theta^* > 0$ when $r = 0$.

10.2 Proof of Proposition 2:

There are multiple steps through which this Proposition will be proved.

Step 1.: Note that

$$\frac{\lambda Q^2}{2} + \frac{\varepsilon^2}{6\lambda} > \frac{1}{2\lambda}\left(\lambda Q - \varepsilon\right)^2 = \frac{\lambda Q^2}{2} + \frac{\varepsilon^2}{2\lambda} - \varepsilon Q$$
$$\Leftrightarrow \lambda Q > \frac{\varepsilon}{3}$$

which is satisfied by Assumption 1. Hence the interval $(\frac{1}{2\lambda}(\lambda Q-\varepsilon)^2, \frac{\lambda Q^2}{2} + \frac{\varepsilon^2}{6\lambda}]$ is non-empty.

Step II. We next show that there is a unique solution θ^* that satisfy (14) and (15).

Consider the function,

$$\pi(k) = \frac{1}{2\varepsilon}\int_{k-\varepsilon}^{\hat{\theta}(k)} \frac{1}{2\lambda}\left(\frac{k+\theta-\varepsilon}{2} + \frac{2\varepsilon\lambda Q}{k-(\theta-\varepsilon)} - k\right)^2 d\theta + \frac{1}{2\varepsilon}\int_{\hat{\theta}(k)}^{k+\varepsilon} \frac{1}{2\lambda}(p-k)^2 d\theta$$

where $\hat{\theta}(k) = p + \varepsilon - \sqrt{\left(p-k\right)^2 + 4\varepsilon\lambda Q}$. On simplification,

$$\pi(k) = \frac{1}{4\lambda\varepsilon}\int_{k-\varepsilon}^{\hat{\theta}(k)} \left(\frac{4\left(\varepsilon\lambda Q\right)^2}{\left(\theta-(k+\varepsilon)\right)^2} + \frac{\left(\theta-(k+\varepsilon)\right)^2}{4} - 2\varepsilon\lambda Q\right) d\theta$$
$$+ \frac{1}{4\lambda\varepsilon}\left(p-k\right)^2\left[(k-p) + \sqrt{\left(p-k\right)^2 + 4\varepsilon\lambda Q}\right]$$

A change of variable $z = \theta - (k+\varepsilon)$ allows us to evaluate the first integral. With this change of variable, the upper and lower limits of the integration are, respectively, $\hat{\theta}\left(k\right) - k - \varepsilon = (p-k) - \sqrt{\left(p-k\right)^2 + 4\varepsilon\lambda Q}$ and -2ε. Evaluating the integral using the new variable and then substituting the new variable back and simplifying, we have,

$$\pi(k) = \frac{1}{4\lambda\varepsilon}\left[\begin{array}{c} -\frac{4(\varepsilon\lambda Q)^2}{(p-k)-\sqrt{(p-k)^2+4\varepsilon\lambda Q}} - \frac{4(\varepsilon\lambda Q)^2}{2\varepsilon} + \frac{\left((p-k)-\sqrt{(p-k)^2+4\varepsilon\lambda Q}\right)^3}{12} \\ +\frac{8\varepsilon^3}{12} - 2\varepsilon\lambda Q\left[(p-k) - \sqrt{(p-k)^2+4\varepsilon\lambda Q} + 2\varepsilon\right] \\ -\left[(p-k)-\sqrt{(p-k)^2+4\varepsilon\lambda Q}\right](p-k)^2 \end{array}\right] \tag{28}$$

We try to show next that $\frac{d\pi(k)}{dk} < 0$. A second change of variable helps us to do that. Define

$$x \equiv \sqrt{(p-k)^2 + 4\varepsilon\lambda Q} - (p-k) > 0$$

and note that

$$\frac{dx}{dk} = 1 - \frac{p-k}{\sqrt{(p-k)^2 + 4\varepsilon\lambda Q}} > 0$$

Further note that $(p-k)^2 = \frac{x^2}{4} + \frac{(2\varepsilon\lambda Q)^2}{x^2} - 2\varepsilon\lambda Q$.

With the second change of variable, $\pi(k)$ can be rewritten as

$$\pi(k, I_k) = \frac{1}{4\lambda\varepsilon}\left[\begin{array}{c} -\frac{4(\varepsilon\lambda Q)^2}{2\varepsilon} + \frac{8\varepsilon^3}{12} - 2\varepsilon\lambda Q(2\varepsilon) \\ +\frac{x^3}{6} + \frac{8(\varepsilon\lambda Q)^2}{x} \end{array}\right]$$

Thus, whether $\pi(k)$ is increasing or decreasing in k depends on whether it decreases or increases in x.

$$\begin{aligned} \frac{d\pi}{dx} &= \frac{1}{4\lambda\varepsilon}\left[\frac{x^2}{2} - \frac{8(\varepsilon\lambda Q)^2}{x^2}\right] \\ &= \frac{1}{4\lambda\varepsilon}\left(\frac{x}{\sqrt{2}} + \frac{2\sqrt{2}\varepsilon\lambda Q}{x}\right)\left(\frac{x}{\sqrt{2}} - \frac{2\sqrt{2}\varepsilon\lambda Q}{x}\right) \end{aligned}$$

Thus the sign of $\frac{d\pi(k)}{dk}$ depends on the sign of $\left(\frac{x}{\sqrt{2}} - \frac{2\sqrt{2}\varepsilon\lambda Q}{x}\right)$. Substituting the expression for x back and simplifying, it is straightforward to check that so long as $(p-k) > 0$ (true for values of k we are interested in), $\left(\frac{x}{\sqrt{2}} - \frac{2\sqrt{2}\varepsilon\lambda Q}{x}\right) < 0$.

Thus $\frac{d\pi(k)}{dk} < 0$.

Hence, if the function $\pi(k)$ can be shown to be greater than δ for some k and less than δ for some k, it has a unique intersection with δ. We proceed to show it as follows.

Note that, for any given k, the following inequality is true.

$$\pi(k) \leq \frac{1}{2\varepsilon}\int_{k-\varepsilon}^{k+\varepsilon} \frac{1}{2\lambda}\left(p-k\right)^2 d\theta = \frac{1}{2\lambda}(p-k)^2 \tag{29}$$

It is straightforward to verify that for $k = p - (\lambda Q - \varepsilon)$, $\frac{1}{2\lambda}(p-k)^2 = \frac{1}{2\lambda}\left(\lambda Q - \varepsilon\right)^2$. Hence, $\pi(k) \leq \frac{1}{2\lambda}\left(\lambda Q - \varepsilon\right)^2 < \delta$ for some value of k.

Next, note that for any given k, the following inequality is true, so long as $\theta + \lambda Q \leq p$.

$$\frac{1}{2\varepsilon}\int_{k-\varepsilon}^{k+\varepsilon} \frac{1}{2\lambda}\left(\theta + \lambda Q - k\right)^2 d\theta \leq \pi(k) \tag{30}$$

The inequality is true by the following arguments. At $\theta = k-\varepsilon$, the expression $\left(\frac{k+\theta-\varepsilon}{2} + \frac{2\varepsilon\lambda Q}{k-(\theta-\varepsilon)}\right) = \theta+\lambda Q$. Both functions are increasing in θ, but the slope of $\theta+\lambda Q$ is 1. The slope of $\left(\frac{k+\theta-\varepsilon}{2} + \frac{2\varepsilon\lambda Q}{k-(\theta-\varepsilon)}\right)$ is given by $\left(\frac{1}{2} + \frac{2\varepsilon\lambda Q}{(k-(\theta-\varepsilon))^2}\right)$. As $\frac{2\varepsilon\lambda Q}{(k-(\theta-\varepsilon))^2} > \frac{4\varepsilon^2}{(k-(\theta-\varepsilon))^2}$ by Assumption 1 and $k-(\theta-\varepsilon) \leq 2\varepsilon$, the ratio $\frac{2\varepsilon\lambda Q}{(k-(\theta-\varepsilon))^2} > 1$ and hence $(\theta+\lambda Q - k) \leq \left(\frac{k+\theta-\varepsilon}{2} + \frac{2\varepsilon\lambda Q}{k-(\theta-\varepsilon)} - k\right)$ for $\theta \in [k-\varepsilon, k+\varepsilon]$, for any given k. Finally note that for any given k, the following is true, as long as $\theta + \lambda Q \leq p$.

$$(\theta+\lambda Q - k) \leq \left(\frac{k+\theta-\varepsilon}{2} + \frac{2\varepsilon\lambda Q}{k-(\theta-\varepsilon)} - k\right) \leq (p-k)$$

Hence, inequality (30) is true for $\theta + \lambda Q \leq p$.

Therefore, choose $k = p - \lambda Q - \varepsilon$. At $k = p - \lambda Q - \varepsilon$,

$$\pi(k) \geq \frac{1}{2\varepsilon}\int_{k-\varepsilon}^{k+\varepsilon} \frac{1}{2\lambda}\left(\theta+\lambda Q - k\right)^2 d\theta = \frac{1}{12\lambda\varepsilon}\left[(\lambda Q+\varepsilon)^3 - \lambda Q - \varepsilon)^3\right] = \frac{\lambda Q^2}{2} + \frac{\varepsilon^2}{6\lambda} \geq \delta$$

Thus the function $\pi(k)$ is greater than δ for some k and less than δ for some k. It therefore has a unique intersection $k = \theta^*$ with δ.

Although θ^* does not have a closed form, equations (29) and (30) are useful because they provide upper and lower bounds within which θ^* lie. Specifically, the above steps show us that, $p - \lambda Q - \varepsilon \leq \theta^*(\delta) \leq p - \lambda Q + \varepsilon$.

Step III. We next show that the switching strategy with the threshold θ^* is an equilibrium. The proof for this part involves showing that for any firm of type θ_i, $\pi(\theta_i, \theta^*) > \delta$ for $\theta_i < \theta^*$ and $\pi(\theta_i, \theta^*) < \delta$ for $\theta_i > \theta^*$. As a first step, we need to characterize the function $\pi(\theta_i, \theta^*)$ for different zones in which θ_i may lie, given the solution θ^*.

The following list characterizes and explains the individual profit function $\pi(\theta_i, \theta^*)$, for each zone in which θ_i may lie. In each case the condition that the profit function must satisfy for θ^* to be an equilibrium, is also provided.

1. $\theta_i < \theta^* - 2\varepsilon$

$$\pi(\theta_i, \theta^*) = \frac{1}{4\lambda\varepsilon} \int_{\theta_i - \varepsilon}^{\theta_i + \varepsilon} (\theta + \lambda Q - \theta_i)^2 \, d\theta > \delta$$

   $\theta_i < \theta^* - 2\varepsilon \implies \theta_i + 2\varepsilon < \theta^*$. Hence, from the perspective of the θ_i-type, all possible types for any $\theta \in [\theta_i - \varepsilon, \theta_i - \varepsilon]$ are below the threshold θ^*. Thus, all firms are active for any $\theta \in [\theta_i - \varepsilon, \theta_i - \varepsilon]$, resulting in the above profit function. The expected profit from harvest for the θ_i type must be clearly higher than δ and the type will enter.

2. $\theta^* - 2\varepsilon < \theta_i < \hat{\theta}(\theta^*) - \varepsilon < \theta^*$.

$$\pi(\theta_i, \theta^*) = \frac{1}{4\lambda\varepsilon} \int_{\theta_i - \varepsilon}^{\theta^* - \varepsilon} (\theta - \theta_i + \lambda Q)^2 \, d\theta + \frac{1}{4\lambda\varepsilon} \int_{\theta^* - \varepsilon}^{\theta_i + \varepsilon} \left(\frac{\theta^* + \theta - \varepsilon}{2} - \theta_i + \lambda \frac{2\varepsilon Q}{\theta^* - (\theta - \varepsilon)} \right)^2 d\theta > \delta$$

   Under this case, not all firms are active for any $\theta \in [\theta_i - \varepsilon, \theta_i - \varepsilon]$. $\theta^* - 2\varepsilon < \theta_i \implies \theta^* < \theta_i + 2\varepsilon$. Hence if the actual $\theta = \theta_i + \varepsilon$, clearly some of the highest cost types will not enter. The expected profit function therefore has two parts depending on the values that θ can assume. Once again as $\theta_i < theta^*$, the expected profit must be greater than δ.

3. $\hat{\theta}(\theta^*) - \varepsilon < \theta_i < \theta^*$

$$\begin{aligned} \pi(\theta_i, \theta^*) = & \frac{1}{4\lambda\varepsilon} \int_{\theta_i - \varepsilon}^{\theta^* - \varepsilon} (\theta - \theta_i + \lambda Q)^2 \, d\theta + \frac{1}{4\lambda\varepsilon} \int_{\theta^* - \varepsilon}^{\hat{\theta}(\theta^*)} \left(\frac{\theta^* + \theta - \varepsilon}{2} - \theta_i + \lambda \frac{2\varepsilon Q}{\theta^* - (\theta - \varepsilon)} \right)^2 d\theta \\ & + \frac{1}{4\lambda\varepsilon} \int_{\hat{\theta}(\theta^*)}^{\theta_i + \varepsilon} (p - \theta_i)^2 \, d\theta > \delta \end{aligned}$$

   Similar arguments as above explain the first two components of the function. The third component is explained by the fact that if the actual $\theta > \hat{\theta}(\theta^*)$, the permit price falls to zero, assuming all other firms adopt the threshold of θ^*.

4. $\theta^* < \theta_i < \hat{\theta}(\theta^*) + \varepsilon < \theta^* + 2\varepsilon$

$$\pi(\theta_i, \theta^*) = \frac{1}{4\lambda\varepsilon} \int_{\theta_i - \varepsilon}^{\hat{\theta}(\theta^*)} \left(\frac{\theta^* + \theta - \varepsilon}{2} - \theta_i + \lambda \frac{2\varepsilon Q}{\theta^* - (\theta - \varepsilon)} \right)^2 d\theta + \frac{1}{4\lambda\varepsilon} \int_{\hat{\theta}(\theta^*)}^{\theta_i + \varepsilon} (p - \theta_i)^2 \, d\theta < \delta$$

   The quota price is positive for some values of θ and zero for others, accounting for the two components. As $\theta_i > \theta^*$, production must be less profitable for the θ_i-type, compared to the outside option.

5. $\hat{\theta}(\theta^*) + \varepsilon < \theta_i < \theta^* + 2\varepsilon$

$$\pi(\theta_i, \theta^*) = \frac{1}{4\lambda\varepsilon} \int_{\theta_i - \varepsilon}^{\theta_i + \varepsilon} (p - \theta_i)^2 \, d\theta < \delta$$

Finally, it can be checked that $\pi(\theta_i, \theta^*)$ is continuous in θ_i.

To prove the rest of the proposition, we need to show that the required inequality involving $\pi(\theta_i, \theta^*)$ and δ for each zone is satisfied.

1. When $\theta_i < \theta^* - 2\varepsilon$,

$$\pi(\theta_i, \theta^*) = \frac{1}{4\lambda\varepsilon} \int_{\theta_i - \varepsilon}^{\theta_i + \varepsilon} (\theta + \lambda Q - \theta_i)^2 \, d\theta = \frac{1}{12\lambda\varepsilon} \left[(\lambda Q + \varepsilon)^3 - \lambda Q - \varepsilon)^3 \right] = \frac{\lambda Q^2}{2} + \frac{\varepsilon^2}{6\lambda} > \delta$$

   and the first inequality is satisfied.

2. When $\theta^* - 2\varepsilon < \theta_i < \hat{\theta}(\theta^*) - \varepsilon < \theta^*$, the argument immediately following equation (**??**) shows that for any θ_i and given θ^*,

$$\pi(\theta_i, \theta^*) = \frac{1}{4\lambda\varepsilon} \int_{\theta_i - \varepsilon}^{\theta^* - \varepsilon} (\theta - \theta_i + \lambda Q)^2 \, d\theta + \frac{1}{4\lambda\varepsilon} \int_{\theta^* - \varepsilon}^{\theta_i + \varepsilon} \left(\frac{\theta^* + \theta - \varepsilon}{2} - \theta_i + \lambda \frac{2\varepsilon Q}{\theta^* - (\theta - \varepsilon)} \right)^2 d\theta$$

$$\geq \frac{1}{4\lambda\varepsilon} \int_{\theta_i - \varepsilon}^{\theta_i + \varepsilon} (\theta + \lambda Q - \theta_i)^2 \, d\theta > \delta$$

   Thus the second inequality is also true.

3. Note that $\pi(\theta_i, \theta^*)$ is not only continuous but differentiable (in fact, twice differentiable) over the interval $\hat{\theta}(\theta^*) - \varepsilon < \theta_i < \theta^*$ as well. Because of continuity and our arguments in the previous case, at $\theta_i = \hat{\theta}(\theta^*) - \varepsilon$, $\pi(\theta_i, \theta^*) > \delta$. Moreover, as $\theta_i \longrightarrow \theta^*$, $\pi(\theta_i, \theta^*) \longrightarrow \pi(\theta^*) = \delta$. **Step II** of this proof shows that $\pi(\theta^*)$ is declining at $\theta_i = \theta^*$. Hence the slopes of the two functions must also converge as $\theta_i \longrightarrow \theta^*$ and in particular $\pi(\theta_i, \theta^*)$ must be declining at $\theta_i = \theta^*$. These statements taken together imply that $\pi(\theta_i, \theta^*)$ must have at least one stationary point that is a maximum in this interval.

   We therefore check the roots of the derivative of $\pi(\theta_i, \theta^*)$ with respect to θ_i.

   The derivative of the first term of $\pi(\theta_i, \theta^*)$ with respect to θ_i is given by (using Leibnitz's rule),

$$\begin{aligned} &\frac{-1}{2\lambda\varepsilon} \int_{\theta_i - \varepsilon}^{\theta^* - \varepsilon} (\theta - \theta_i + \lambda Q) \, d\theta - (\lambda Q - \varepsilon)^2 \\ &= -(\theta^* - \theta_i + \lambda Q - \varepsilon)^2 + (\lambda Q - \varepsilon)^2 - (\lambda Q - \varepsilon)^2 \\ &= -(\theta^* - \theta_i + \lambda Q - \varepsilon)^2 \end{aligned}$$

   The derivative of the second term is given by,

$$\begin{aligned} &\frac{-1}{2\lambda\varepsilon} \int_{\theta^* - \varepsilon}^{\hat{\theta}(\theta^*)} \left(\frac{\theta^* + \theta - \varepsilon}{2} - \theta_i + \lambda \frac{2\varepsilon Q}{\theta^* - (\theta - \varepsilon)} \right) d\theta \\ &= 2(\theta^* - \varepsilon)^2 - \frac{\left(\theta^* + \hat{\theta} - \varepsilon\right)^2}{2} + 4\varepsilon\lambda Q \left(\ln\left[\theta^* + \varepsilon - \hat{\theta}\right] - \ln[2\varepsilon] \right) + 2\theta_i \left(\hat{\theta}(\theta^*) - (\theta^* - \varepsilon) \right) \end{aligned}$$

The derivative of the third term is given by

$$\frac{1}{4\lambda\varepsilon}\left[3\left(p-\theta_i\right)^2-2\left(p-\theta_i\right)\sqrt{\left(p-\theta^*\right)^2+4\varepsilon\lambda Q}\right]$$

These terms of the derivatives can be combined to get

$$\frac{1}{2\lambda\varepsilon}\left[\theta_i^2-2\left(p-\frac{\lambda Q+\varepsilon}{2}\right)\theta_i+\Omega\left[\theta^*\right]\right] \tag{31}$$

where

$$\Omega\left[\theta^*\right]\equiv\left(\theta^*-\varepsilon\right)^2-\frac{\left(\hat{\theta}\left(\theta^*\right)+\theta^*-\varepsilon\right)^2}{4}+2\varepsilon\lambda Q\log\left[1-\frac{\hat{\theta}\left(\theta^*\right)-\left(\theta^*-\varepsilon\right)}{2\varepsilon}\right]$$
$$+\frac{p^2-\left(\lambda Q+\theta^*-\varepsilon\right)^2}{2}+p\left(\hat{\theta}\left(\theta^*\right)-\varepsilon\right)$$

Derivative (31) has two roots given by

$$\theta_{1,2}^R\left(\theta^*\right)\equiv p-\frac{\lambda Q+\varepsilon}{2}\pm\sqrt{\left(p-\frac{\lambda Q+\varepsilon}{2}\right)^2-\Omega\left[\theta^*\right]},$$

Note that both roots cannot be less than θ^* because of the following contradiction.

$$\theta_1^R=\theta_{1,2}^R\left(\theta^*\right)\equiv p-\frac{\lambda Q+\varepsilon}{2}-\sqrt{\left(p-\frac{\lambda Q+\varepsilon}{2}\right)^2-\Omega\left[\theta^*\right]}<\theta^*\Longrightarrow 2\left(p-\frac{\lambda Q+\varepsilon}{2}\right)\theta^*-\theta^{*2}>\Omega\left[\theta^*\right]$$

$$\theta_2^R=\theta_{1,2}^R\left(\theta^*\right)\equiv p-\frac{\lambda Q+\varepsilon}{2}+\sqrt{\left(p-\frac{\lambda Q+\varepsilon}{2}\right)^2-\Omega\left[\theta^*\right]}<\theta^*\Longrightarrow 2\left(p-\frac{\lambda Q+\varepsilon}{2}\right)\theta^*-\theta^{*2}<\Omega\left[\theta^*\right]$$

but $\theta_2^R>\theta_1^R$.

As one of the roots must be less than θ^* for $\pi(\theta_i,\theta^*)$ to be declining at $\theta_i=\theta^*$, $\theta_1^R<\theta^*$. Thus $\pi(\theta_i,\theta^*)$ has only one stationary point in the interval which is a maximum and assumes a value of δ at $\theta_i=\theta^*$. Hence $\pi(\theta_i,\theta^*)>\delta$ over the interval.

4. For $\theta^*<\theta_i<\hat{\theta}(\theta^*)+\varepsilon<\theta^*+2\varepsilon$, as in the previous case, the function $\pi(\theta_i,\theta^*)$ is twice differentiable. The derivative of the profit funtcion is $\frac{1}{4\lambda\varepsilon}$ times the expression,

$$\left(p-\theta_i\right)^2-\left[\begin{array}{c}\left(\frac{\theta^*-\varepsilon+\theta_i-\varepsilon}{2}-\theta_i+\lambda\frac{2\varepsilon Q}{\theta^*+\varepsilon-(\theta_i-\varepsilon)}\right)^2\\+2\left(\int_{\theta_i-\varepsilon}^{\hat{\theta}(\theta^*)}\left(\frac{\theta^*+\theta-\varepsilon}{2}-\theta_i+\frac{2\lambda\varepsilon Q}{\theta^*-(\theta-\varepsilon)}\right)d\theta+2\int_{\hat{\theta}(\theta^*)}^{\theta_i+\varepsilon}\left(p-\theta_i\right)d\theta\right)\end{array}\right] \tag{32}$$

where the term within square brackets is positive. In particular, note that

$$\left(\frac{\theta^* - \varepsilon + \theta_i - \varepsilon}{2} - \theta_i + \lambda \frac{2\varepsilon Q}{\theta^* + \varepsilon - (\theta_i - \varepsilon)} > 0\right) \quad \text{for} \quad \theta \in [\theta_i - \varepsilon, \hat{\theta}(\theta^*)]$$

a property that we use in the very next step.

The second order cross partial derivative, $\frac{\partial^2 \pi(\theta_i,\theta^*)}{\partial \theta^* \partial \theta_i}$, is given by $\frac{1}{4\lambda\varepsilon}$ times the expression,

$$-\left[\begin{array}{c} \left(\frac{\theta^*-\varepsilon+\theta_i-\varepsilon}{2} - \theta_i + \lambda\frac{2\varepsilon Q}{\theta^*+\varepsilon-(\theta_i-\varepsilon)}\right)\left(1 - \frac{4\varepsilon\lambda Q}{(\theta^*+2\varepsilon-\theta_i)^2}\right) \\ \int_{\theta_i-\varepsilon}^{\hat{\theta}(\theta^*)} \left(1 - \frac{4\varepsilon\lambda Q}{(\theta^*+\varepsilon-\theta)^2}\right) d\theta \end{array}\right]$$

As $\lambda Q > \varepsilon$, $4\varepsilon\lambda Q > 4\varepsilon^2 \geq (\theta^* + 2\varepsilon - \theta_i)^2$, the first term within square brackets is negative. Similarly, as $\lambda Q > \varepsilon$, $4\varepsilon\lambda Q > 4\varepsilon^2 \geq (\theta^* + \varepsilon - \theta)^2$, the second term within square brackets is also negative. Hence $\frac{\partial^2 \pi(\theta_i,\theta^*)}{\partial \theta^* \partial \theta_i} \geq 0$ for this region, and the slope of $\pi(\theta_i, \theta^*)$ has the single crossing property. In particular this implies that if for a given θ^* and some θ_i such that $\theta^* = \theta_i$, the slope of $\pi(\theta_i, \theta^*)$ is negative, then the slope cannot be positive for higher values of θ_i for which $\theta^* < \theta_i$.

Finally, it can be checked that for the (Step II) solution θ^*, $\theta_i = \theta^*$, the slope of the region 4 and the region 3 profit functions are identical and negative. Hence, the required inequality is satisfied.

5. For, $\hat{\theta}(\theta^*) + \varepsilon < \theta_i < \theta^* + 2\varepsilon$, as the profit function equals

$$\pi(\theta_i, \theta^*) = \frac{1}{4\lambda\varepsilon} \int_{\theta_i-\varepsilon}^{\theta_i+\varepsilon} (p - \theta_i)^2 \, d\theta = \frac{1}{4\lambda\varepsilon} (p - \theta_i)^2$$

the derivative is given by

$$-\frac{1}{2\lambda} (p - \theta_i)$$

which is negative for admissible values of p. This proves the proposition.

10.3 Proof of Proposition 3

As under the previous scenario, the Proposition will be proved through multiple steps.

Step I. We first show that there is a unique solution θ^* that satisfy (16) and (17).

This is straightforward, as the unique solution of

$$\frac{1}{2\lambda} (p - \theta^*)^2 = \delta$$

that satisfies the condition $\delta \leq \frac{1}{2\lambda}(\lambda Q - \varepsilon)^2$ and hence condition (17), is $\theta^* = p - \sqrt{2\lambda\delta}$.

Step II We show that the solution $\theta^* = p - \sqrt{2\lambda\delta}$ is an equilibrium. As we are looking for an equilibrium under which $r(\theta, \theta^*) = 0$ for all $\theta \in [\theta^* - \varepsilon, \theta^* + \varepsilon]$, at the value of θ at which the equilibrium permit price falls to zero, all firms are active. Hence the value of θ at which the equilibrium permit price equals zero is given by $p-\theta-\lambda Q=0$. Denote by $\hat{\theta}=p-\lambda Q$, this value and note that it is independent of θ^*.

The individual profit function takes on three different forms depending on three zones in which θ_i may lie. We discuss these forms and the condition that each form must satisfy for θ^* to be equilibrium below. We also simultaneously show that these conditions are met.

1. For $\theta_i < p-\sqrt{2\lambda\delta}-2\varepsilon$, all firms participate for all possible values of θ. Hence individual profit function must satisfy the condition,

$$\pi(\theta_i,\theta^*)=\frac{1}{4\lambda\varepsilon}\int_{\theta_i-\varepsilon}^{\theta_i+\varepsilon}(\theta-\theta_i+\lambda Q)^2\,d\theta>\delta$$

Note that

$$\begin{aligned}\pi(\theta_i,\theta^*)&=\frac{1}{4\lambda\varepsilon}\int_{\theta_i-\varepsilon}^{\theta_i+\varepsilon}(\theta-\theta_i+\lambda Q)^2\,d\theta\\&=\frac{1}{12\lambda\varepsilon}\left[(\lambda Q+\varepsilon)^3-(\lambda Q-\varepsilon)^3\right]\\&=\frac{1}{12\lambda\varepsilon}(2\varepsilon)\left((\lambda Q)^2-\varepsilon^2+2(\lambda Q)^2+2\varepsilon^2\right)\\&=\frac{\lambda Q^2}{2}+\frac{\varepsilon^2}{6\lambda}>\frac{1}{2\lambda}(\lambda Q-\varepsilon)^2\geq\delta\end{aligned}$$

Hence the condition is satisfied.

2. For $p-\sqrt{2\lambda\delta}-2\varepsilon\leq\theta_i<p-\lambda Q+\varepsilon\leq\theta^*$, depending upon the value of θ, we may either have all firms active and the permit price is strictly positive, or else permit price is zero. Hence the profit function and the required condition are

$$\pi(\theta_i,\theta^*)=\frac{1}{4\lambda\varepsilon}\int_{\theta_i-\varepsilon}^{p-\lambda Q}(\theta-\theta_i+\lambda Q)^2\,d\theta+\frac{1}{4\lambda\varepsilon}\int_{p-\lambda Q}^{\theta_i+\varepsilon}(p-\theta_i)^2\,d\theta>\delta$$

Note that,

$$\begin{aligned}\pi(\theta_i,\theta^*)&=\frac{1}{4\lambda\varepsilon}\int_{\theta_i-\varepsilon}^{p-\lambda Q}(\theta-\theta_i+\lambda Q)^2\,d\theta+\frac{1}{4\lambda\varepsilon}\int_{p-\lambda Q}^{\theta_i+\varepsilon}(p-\theta_i)^2\,d\theta\\&=\frac{1}{4\lambda\varepsilon}\left[\frac{(p-\theta_i)^3}{3}-\frac{(\lambda Q-\varepsilon)^3}{3}+(p-\theta_i)^2(\theta_i+\varepsilon-p+\lambda Q)\right]\end{aligned}$$

The above equals $\frac{\lambda Q^2}{2}+\frac{\varepsilon^2}{6\lambda}$ when $\theta_i=p-\lambda Q-\varepsilon$, and $\frac{1}{2\lambda}(\lambda Q-\varepsilon)^2\geq\delta$ when $\theta_i=p-\lambda Q+\varepsilon$, The derivative of $\pi(\theta_i,I_{\theta^*})$ with respect to θ_i is $\frac{1}{4\lambda\varepsilon}$ times

$$
\begin{aligned}
&-(\lambda Q-\varepsilon)^2+(p-\theta_i)^2-2\int_{\theta_i-\varepsilon}^{p-\lambda Q}(\theta-\theta_i+\lambda Q)\,d\theta-2\int_{p-\lambda Q}^{\theta_i+\varepsilon}(p-\theta_i)\,d\theta\\
&=-(\lambda Q-\varepsilon)^2+(p-\theta_i)^2-(p-\theta_i)^2+(\lambda Q-\varepsilon)^2-2\int_{p-\lambda Q}^{\theta_i+\varepsilon}(p-\theta_i)\,d\theta\\
&=-2\int_{p-\lambda Q}^{\theta_i+\varepsilon}(p-\theta_i)\,d\theta<0
\end{aligned}
$$

Hence, it is strictly declining in θ_i through the range under consideration and therefore the condition is satisfied.

3. When $\theta_i > p - \lambda Q + \varepsilon \geq \theta^*$, we must have

$$\pi(\theta_i, \theta^*) = \frac{1}{2\varepsilon}\int_{\theta_i-\varepsilon}^{\theta_i+\varepsilon}\frac{1}{2\lambda}(p-\theta_i)^2 < \delta$$

Since $\frac{1}{2\varepsilon}\int_{\theta_i-\varepsilon}^{\theta_i+\varepsilon}\frac{1}{2\lambda}(p-\theta_i)^2 = \frac{1}{2\lambda}(p-\theta_i)^2$, and the latter function is strictly declining in θ_i, the condition is satisfied, because at $\theta_i = \theta^*$, $\frac{1}{2\lambda}(p-\theta_i)^2 = \delta$.

Thus the Proposition is proved.

11 Appendix III

The function is characterized below for two different cases - (1) $\hat{\theta}(\delta) \leq \theta^{*i}(\delta) + \epsilon$ which holds when $2\lambda\delta < (p - \theta^{*i}(\delta))^2 \leq 2\lambda\delta + 4\varepsilon\lambda Q$ and (2) $\theta^{*i}(\delta) + \epsilon \leq \hat{\theta}(\delta)$ which holds when $2\lambda\delta < 2\lambda\delta + 4\varepsilon\lambda Q < (p - \theta^{*i}(\delta))^2$.

11.1 Case 1: $\hat{\theta}(\delta) \leq \theta^{*i}(\delta) + \varepsilon$

The function, $(\alpha^i(\delta) - \alpha^f(\delta))$, has the following form,

A. For $\theta^l \leq \theta \leq \theta^{*i} - \varepsilon$, as $\alpha^i(\delta) = 1$,

$$(\alpha^i(\delta) - \alpha^f(\delta)) = 1 - \frac{\sqrt{2\delta\lambda + 4\varepsilon\lambda Q} - \sqrt{2\delta\lambda}}{2\varepsilon} \tag{33}$$

which is constant in θ.

B. For $\theta^{*i} - \varepsilon \leq \theta \leq \hat{\theta}(\delta) \leq \theta^{*i} + \varepsilon$,

$$(\alpha^i(\delta) - \alpha^f(\delta)) = \frac{\theta^{*i} - (\theta - \varepsilon)}{2\varepsilon} - \frac{\sqrt{2\delta\lambda + 4\varepsilon\lambda Q} - \sqrt{2\delta\lambda}}{2\varepsilon} \tag{34}$$

which is linear and decreasing in θ.

C. For $\hat{\theta}(\delta) \leq \theta \leq \theta^{*i} + \varepsilon$,

$$(\alpha^i(\delta) - \alpha^f(\delta)) = \frac{\theta^{*i} - (p - \sqrt{2\lambda\delta})}{2\varepsilon} \tag{35}$$

after due simplification and is a negative constant in θ.

D. For $\theta \leq \theta^{*i} + \varepsilon \leq \theta \leq \theta^h$, as $\alpha^i(\delta) = 0$,

$$(\alpha^i(\delta) - \alpha^f(\delta)) = \frac{(\theta - \varepsilon) - (p - \sqrt{2\lambda\delta})}{2\varepsilon} \tag{36}$$

after due simplification and is linear and increasing in θ.

E. For $\theta > \theta^h$, the active mass is zero under both incomplete and full information. Hence, $(\alpha^i(\delta) - \alpha^f(\delta)) = 0$

Figure 1 plots the difference, $\alpha^i - \alpha^f$ for Case 1, for a given value of δ and θ^l.

11.2 Case 2: $\theta^{*i}(\delta) + \varepsilon \leq \hat{\theta}(\delta)$

The function, $(\alpha^i(\delta) - \alpha^f(\delta))$, has the following form,

A. For $\theta^l \leq \theta \leq \theta^{*i} - \varepsilon$, as $\alpha^i(\delta) = 1$,

$$(\alpha^i(\delta) - \alpha^f(\delta)) = 1 - \frac{\sqrt{2\delta\lambda + 4\varepsilon\lambda Q} - \sqrt{2\delta\lambda}}{2\varepsilon} \tag{37}$$

which is constant in θ.

B. For $\theta^{*i} - \varepsilon \leq \theta \leq \theta^{*i} + \varepsilon \leq \hat{\theta}(\delta)$,

$$(\alpha^i(\delta) - \alpha^f(\delta)) = \frac{\theta^{*i} - (\theta - \varepsilon)}{2\varepsilon} - \frac{\sqrt{2\delta\lambda + 4\varepsilon\lambda Q} - \sqrt{2\delta\lambda}}{2\varepsilon} \tag{38}$$

which is linear and decreasing in θ.

C. For $\theta^{*i} + \varepsilon \leq \theta \leq \hat{\theta}(\delta)$, as $\alpha^i(\delta) = 0$,

$$(\alpha^i(\delta) - \alpha^f(\delta)) = -\left(\frac{\sqrt{2\delta\lambda + 4\varepsilon\lambda Q} - \sqrt{2\delta\lambda}}{2\varepsilon}\right) \tag{39}$$

which is negative and constant in θ.

D. For $\hat{\theta}(\delta) \leq \theta \leq \theta^h$,

$$(\alpha^i(\delta) - \alpha^f(\delta)) = \frac{(\theta - \varepsilon) - (p - \sqrt{2\lambda\delta})}{2\varepsilon} \tag{40}$$

which is linear and increasing in θ.

E. For $\theta > \theta^h$, as before, $(\alpha^i(\delta) - \alpha^f(\delta)) = 0$

Figure 2 plots the difference, $\alpha^i - \alpha^f$ for Case 1, for a given value of δ and θ^l.

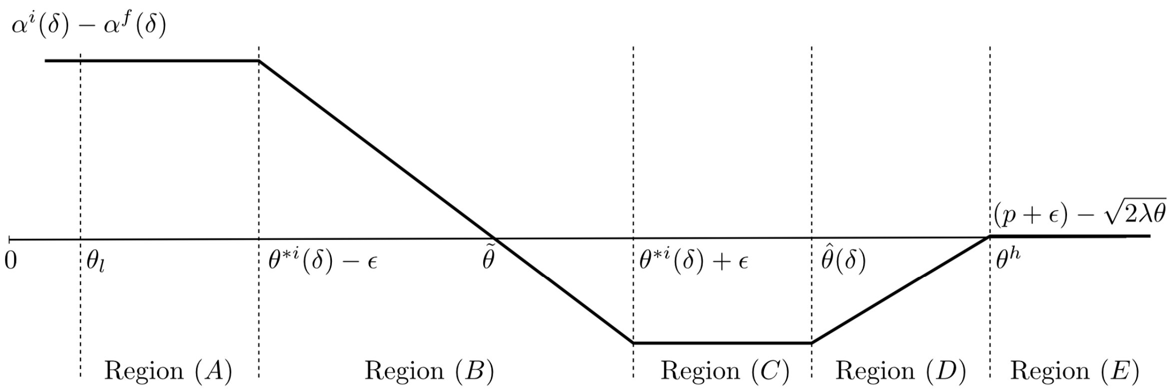

Figure 2: $(\alpha^i(\delta) - \alpha^f(\delta))$: Case 2

Thus under both cases, the function has the same form over the first two and the last zones.

We first derive the expression for the expected difference in active mass, $E(\alpha^i - \alpha^f)$, for the Case 5.1. Note that this expected difference is the definite integral of the function, $(\alpha^i - \alpha^f)$, over the domain $[\theta^l, \theta^h]$. As depicted in the figures, the function attains zero at some value of $\theta = \tilde{\theta} \in [\theta^{*i}(\delta) - \varepsilon, \hat{\theta}(\delta)]$. Using the functional form for this domain, we obtain,

$$\tilde{\theta} = \hat{\theta} - (p - \sqrt{2\lambda\delta} - \theta^{*i}(\delta))$$

The height of the function at $\theta = \theta^{*i}(\delta) - \varepsilon$ is $1 - \left(\frac{\sqrt{2\delta\lambda + 4\varepsilon\lambda Q} - \sqrt{2\delta\lambda}}{2\varepsilon}\right)$ and the height at $\theta = \hat{\theta}$ is $\frac{(p - \sqrt{2\lambda\delta} - \theta^{*i}(\delta))}{2\varepsilon}$.

It is straightforward to show that the negative area enclosed by the function is given by,

$$\left(p - \sqrt{2\lambda\delta} - \theta^{*i}(\delta)\right)\left(\frac{\sqrt{2\delta\lambda + 4\varepsilon\lambda Q} - \sqrt{2\delta\lambda}}{2\varepsilon}\right)$$

Similarly, the postive area supported by $\tilde{\theta} - (\theta^{*i}(\delta) - \varepsilon)$ is,

$$\left(\frac{(2\varepsilon - (\sqrt{2\delta\lambda + 4\varepsilon\lambda Q} - \sqrt{2\delta\lambda}))^2}{4\varepsilon}\right)$$

And the positive area supported by $((\theta^{*i}(\delta) - \varepsilon) - \theta^l)$ is

$$\left((\theta^{*i}(\delta)-\varepsilon)-\theta^l\right)\left(1-\left(\frac{\sqrt{2\delta\lambda+4\varepsilon\lambda Q}-\sqrt{2\delta\lambda}}{2\varepsilon}\right)\right)$$

After due simplification, the net area has the required expression.

Following the same steps, it can be shown that the expression has the same form for Case 5.2.

Since $\delta^l = \frac{(\lambda Q-\varepsilon)^2}{2\lambda}$, for $\delta = \delta^l$, $\theta^{*i}(\delta) = p - \sqrt{2\lambda\delta}$ and $\sqrt{2\delta\lambda+4\varepsilon\lambda Q} - \sqrt{2\delta\lambda} = 2\varepsilon$. Upon substitution, the statements in Proposition 4 follows.

12 Appendix IV

12.1 Proof of Proposition 5:

As in the case of $\beta = 0$, the Proposition will be proved through multiple steps.

Step 1.: To show that the interval $(\frac{1}{2\lambda}(\frac{2\varepsilon\lambda Q}{2\varepsilon-\beta} - \frac{2\varepsilon-\beta}{2})^2, \frac{(\lambda Q+\beta)^2}{2\lambda} + \frac{\varepsilon^2}{6\lambda}]$ is non-empty under appropriate restrictions on the parameters, it suffices to show that the following inequality is true for appropriate restrictions on the parameters.

$$\left(\frac{2\varepsilon\lambda Q}{2\varepsilon-\beta} - \frac{2\varepsilon-\beta}{2}\right)^2 < (\lambda Q+\beta)^2$$

Above inequality implies

$$\left(\frac{2\varepsilon\lambda Q}{2\varepsilon-\beta} - \frac{2\varepsilon-\beta}{2}\right) < (\lambda Q+\beta)$$

which on simplification turns out to be

$$\lambda Q < \frac{(2\varepsilon-\beta)(2\varepsilon-\beta)}{2\beta}$$

Thus for any value of $\beta \in (0, 2\varepsilon]$, in particular for any value of $\beta \in (0, \varepsilon]$, there exists an upper bound on λQ, for which the above inequality is satisfied and the desired interval is non-empty. For example, when $\beta = \varepsilon$, the desired interval is non-empty if $\lambda Q < \frac{3}{2}\varepsilon$.

Step II. We next show that there is a unique solution μ^* that satisfy (22) and (23).

Consider the function,

$$\pi(k) = \frac{1}{2\varepsilon}\int_{k+\beta-\varepsilon}^{\hat{\theta}(k)} \frac{1}{2\lambda}\left(\frac{k+\theta-\varepsilon}{2} + \frac{2\varepsilon\lambda Q}{k-(\theta-\varepsilon)} - k\right)^2 d\theta + \frac{1}{2\varepsilon}\int_{\hat{\theta}(k)}^{k+\beta+\varepsilon} \frac{1}{2\lambda}(p-k)^2 d\theta$$

where $\hat{\theta}(k) = p + \varepsilon - \sqrt{(p-k)^2 + 4\varepsilon\lambda Q}$. On simplification,

$$\pi(k) = \frac{1}{4\lambda\varepsilon} \int_{k+\beta-\varepsilon}^{\hat{\theta}(k)} \left(\frac{4\left(\varepsilon\lambda Q\right)^2}{\left(\theta - (k+\varepsilon)\right)^2} + \frac{\left(\theta - (k+\varepsilon)\right)^2}{4} - 2\varepsilon\lambda Q \right) d\theta$$
$$+ \frac{1}{4\lambda\varepsilon} (p-k)^2 \left[(k+\beta-p) + \sqrt{(p-k)^2 + 4\varepsilon\lambda Q} \right]$$

As before, a change of variable $z = \theta - (k+\varepsilon)$ allows us to evaluate the first integral. With this change of variable, the upper and lower limits of the integration are, respectively, $\hat{\theta}(k) - k - \varepsilon = (p-k) - \sqrt{(p-k)^2 + 4\varepsilon\lambda Q}$ and $\beta - 2\varepsilon$. Evaluating the integral using the new variable and then substituting the new variable back and simplifying, we have,

$$\pi(k) = \frac{1}{4\lambda\varepsilon} \left[\begin{array}{c} -\frac{4(\varepsilon\lambda Q)^2}{(p-k)-\sqrt{(p-k)^2+4\varepsilon\lambda Q}} - \frac{4(\varepsilon\lambda Q)^2}{2\varepsilon} + \frac{\left((p-k)-\sqrt{(p-k)^2+4\varepsilon\lambda Q}\right)^3}{12} \\ +\frac{(2\varepsilon-\beta)^3}{12} - 2\varepsilon\lambda Q \left[(p-k) - \sqrt{(p-k)^2 + 4\varepsilon\lambda Q} + 2\varepsilon - \beta \right] \\ - \left[(p-k) - \beta - \sqrt{(p-k)^2 + 4\varepsilon\lambda Q} \right] (p-k)^2 \end{array} \right] \tag{41}$$

We try to show next that $\frac{d\pi(k)}{dk} < 0$. A second change of variable helps us to do that. Define

$$x \equiv \sqrt{(p-k)^2 + 4\varepsilon\lambda Q} - (p-k) > 0$$

and note that

$$\frac{dx}{dk} = 1 - \frac{p-k}{\sqrt{(p-k)^2 + 4\varepsilon\lambda Q}} > 0$$

Further note that $(p-k)^2 = \frac{x^2}{4} + \frac{(2\varepsilon\lambda Q)^2}{x^2} - 2\varepsilon\lambda Q$.

With the second change of variable, $\pi(k)$ can be rewritten as

$$\pi(k) = \frac{1}{4\lambda\varepsilon} \left[\begin{array}{c} -\frac{4(\varepsilon\lambda Q)^2}{2\varepsilon-\beta} + \frac{(2\varepsilon-\beta)^3}{12} - 2\varepsilon\lambda Q\,(2\varepsilon-\beta) \\ +\frac{x^3}{6} + \frac{8(\varepsilon\lambda Q)^2}{x} + \beta(\frac{x}{2} - \frac{2\varepsilon\lambda Q}{x})^2 \end{array} \right]$$

Thus, whether $\pi(k)$ is increasing or decreasing in k depends on whether it decreases or increases in x.

$$\frac{d\pi}{dx} = \frac{1}{4\lambda\varepsilon} \left[\frac{x^2}{2} - \frac{8(\varepsilon\lambda Q)^2}{x^2} \right] + \beta \left(\frac{x}{2} - \frac{2\varepsilon\lambda Q}{x} \right) \left(\frac{1}{2} + \frac{2\varepsilon\lambda Q}{x} \right)$$
$$= \frac{1}{4\lambda\varepsilon} \left(\frac{x}{\sqrt{2}} + \frac{2\sqrt{2}\varepsilon\lambda Q}{x} \right) \left(\frac{x}{\sqrt{2}} - \frac{2\sqrt{2}\varepsilon\lambda Q}{x} \right) + \beta \left(\frac{x}{2} - \frac{2\varepsilon\lambda Q}{x} \right) \left(\frac{1}{2} + \frac{2\varepsilon\lambda Q}{x} \right)$$

Thus the sign of $\frac{d\pi(k)}{dk}$ depends on the signs of the terms, $\left(\frac{x}{\sqrt{2}} - \frac{2\sqrt{2}\varepsilon\lambda Q}{x}\right)$ and $\left(\frac{x}{2} - \frac{2\varepsilon\lambda Q}{x}\right)$.

Substituting the expression for x back and simplifying, it is straightforward to check that so long as $(p-k) > 0$ (true for values of k we are interested in),

$$\left(\frac{x}{\sqrt{2}} - \frac{2\sqrt{2}\varepsilon\lambda Q}{x}\right) = -\sqrt{2}(p-k) < 0$$

and

$$\left(\frac{x}{2} - \frac{2\varepsilon\lambda Q}{x}\right) = -(p-k) < 0$$

Thus $\frac{d\pi(k)}{dk} < 0$.

We next need to show that the function $\pi(k)$ is greater than δ for some k and less than δ for some k.

As before, for any given k, the following inequality is true.

$$\pi(k) \leq \frac{1}{2\varepsilon}\int_{k+\beta-\varepsilon}^{k+\beta+\varepsilon} \frac{1}{2\lambda}(p-k)^2\, d\theta = \frac{1}{2\lambda}(p-k)^2 \tag{42}$$

Moreover, for $k = p - (\lambda Q - \varepsilon)$, $\pi(k) \leq \frac{1}{2\lambda}(p-k)^2 = \frac{1}{2\lambda}(\lambda Q - \varepsilon)^2$. Since, $\frac{1}{2\lambda}(\lambda Q - \varepsilon)^2 < (\frac{1}{2\lambda}(\frac{2\varepsilon\lambda Q}{2\varepsilon-\beta} - \frac{2\varepsilon-\beta}{2})^2 < \delta$ for $0 < \beta \leq 2\varepsilon$, $\pi(k) < \delta$ for some value of k.

Similarly, for any given k, the following inequality is true for $\theta \leq p - \lambda Q$.

$$\frac{1}{2\varepsilon}\int_{k+\beta-\varepsilon}^{k+\beta+\varepsilon} \frac{1}{2\lambda}(\theta + \lambda Q - k)^2\, d\theta \leq \pi(k) \tag{43}$$

The inequality is true by the following arguments. At $\theta = k + \beta - \varepsilon$, the expressions within integral signs have following values:

$$\theta + \lambda Q - k = \lambda Q + \beta - \varepsilon$$

$$\left(\frac{k+\theta-\varepsilon}{2} + \frac{2\varepsilon\lambda Q}{k-(\theta-\varepsilon)} - k\right) = \left(\frac{2\varepsilon\lambda Q}{2\varepsilon-\beta} - \frac{2\varepsilon-\beta}{2}\right)$$

Note that,

$$\left(\frac{2\varepsilon\lambda Q}{2\varepsilon-\beta} - \frac{2\varepsilon-\beta}{2}\right) - (\lambda Q + \beta - \varepsilon) = \frac{\beta}{2}\left(\frac{2\varepsilon\lambda Q}{2\varepsilon-\beta} - 1\right) > 0,$$

since $\frac{2\varepsilon\lambda Q}{2\varepsilon-\beta} > 1$.

Hence, for $\theta = k + \beta - \varepsilon$,

$$\theta + \lambda Q - k < \left(\frac{k + \theta - \varepsilon}{2} + \frac{2\varepsilon\lambda Q}{k - (\theta - \varepsilon)}\right)$$

Both functions are increasing in θ, but the slope of $\theta + \lambda Q$ is 1 and following the same steps as in Proposition 2, we can show that the slope of the RHS expression is greater than 1.

Hence, for any given k and $\theta \leq p - \lambda Q$, inequality (43) is true.

Thus, for $k = p - \lambda Q - \varepsilon$,

$$\pi(k) \geq \frac{1}{2\varepsilon}\int_{k-\varepsilon}^{k+\varepsilon} \frac{1}{2\lambda}(\theta + \lambda Q - k)^2 \, d\theta = \frac{1}{12\lambda\varepsilon}\left[(\lambda Q + \beta + \varepsilon)^3 - (\lambda Q + \beta - \varepsilon)^3\right] = \frac{(\lambda Q + \beta)^2}{2\lambda} + \frac{\varepsilon^2}{6\lambda} \geq \delta$$

Hence $\pi(k)$ has a unique intersection $k = \mu^*$ with δ.

Step III. We next show that the switching strategy with the threshold μ^* is an equilibrium. As the proof is very similar to the proof in Proposition 2, we shorten or skip altogether the parts that are identical and focus only on those where there are differences. As before, we need to show that for any firm of type θ_i, $\pi(\theta_i, \mu^*) > \delta$ for $\theta_i < \mu^*$ and $\pi(\theta_i, \mu^*) < \delta$ for $\theta_i > \mu^*$.

The following list characterizes the individual profit function $\pi(\theta_i, \mu^*)$, for each zone in which θ_i may lie and provides the condition that the profit function must satisfy, for θ^* to be an equilibrium, in each of these zones. The rational for the form of the profit function for each zone is identical to that for the no-bias ($\beta = 0$) case and is therefore omitted. The only exception is zone 4 below which is new to $\beta > 0$.

1. $\theta_i < \mu^* - \beta - 2\varepsilon$

$$\pi(\theta_i, \mu^*) = \frac{1}{4\lambda\varepsilon}\int_{\theta_i+\beta-\varepsilon}^{\theta_i+\beta+\varepsilon} (\theta + \lambda Q - \theta_i)^2 \, d\theta > \delta$$

2. $\mu^* - \beta - 2\varepsilon < \theta_i < \hat{\theta}(\mu^*) - \beta - \varepsilon < \mu^* - \beta$.

$$\pi(\theta_i, \mu^*) = \frac{1}{4\lambda\varepsilon}\left[\int_{\theta_i+\beta-\varepsilon}^{\mu^*-\varepsilon} (\theta - \theta_i + \lambda Q)^2 \, d\theta + \int_{\mu^*-\varepsilon}^{\theta_i+\beta+\varepsilon} \left(\frac{\mu^* + \theta - \varepsilon}{2} - \theta_i + \frac{2\varepsilon\lambda Q}{\mu^* - (\theta - \varepsilon)}\right)^2 d\theta\right] > \delta$$

3. $\hat{\theta}(\mu^*) - \beta - \varepsilon < \theta_i < \mu^* - \beta$

$$\pi(\theta_i, \mu^*) = \frac{1}{4\lambda\varepsilon}\left[\int_{\theta_i+\beta-\varepsilon}^{\mu^*-\varepsilon} (\theta - \theta_i + \lambda Q)^2\, d\theta + \int_{\mu^*-\varepsilon}^{\hat{\theta}(\mu^*)} \left(\frac{\mu^* + \theta - \varepsilon}{2} - \theta_i + \frac{2\varepsilon\lambda Q}{\mu^* - (\theta - \varepsilon)}\right)^2 d\theta \right.$$
$$\left. + \int_{\hat{\theta}(\mu^*)}^{\theta_i+\beta+\varepsilon} (p - \theta_i)^2\, d\theta\right] > \delta$$

4. $\mu^* - \beta < \theta_i < \mu^*$.

$$\pi(\theta_i, \mu^*) = \int_{\theta_i+\beta-\varepsilon}^{\hat{\theta}(\mu^*)} \left(\frac{\mu^* + \theta - \varepsilon}{2} - \theta_i + \frac{2\varepsilon\lambda Q}{\mu^* - (\theta - \varepsilon)}\right)^2 d\theta + \int_{\hat{\theta}(\mu^*)}^{\theta_i+\beta+\varepsilon} (p - \theta_i)^2\, d\theta > \delta$$

   Since $\theta \in [\theta_i + \beta - \varepsilon, \theta_i + \beta + \varepsilon]$, $\mu^* - \beta < \theta_i \Longrightarrow \mu * - \varepsilon < \theta$ for all possible values of θ. Hence, it is never the case that all firms are active and this explains the form of the profit function.

5. $\mu^* < \theta_i < \hat{\theta}(\mu^*) - \beta + \varepsilon$

$$\pi(\theta_i, \mu^*) = \int_{\theta_i+\beta-\varepsilon}^{\hat{\theta}(\mu^*)} \left(\frac{\mu^* + \theta - \varepsilon}{2} - \theta_i + \frac{2\varepsilon\lambda Q}{\mu^* - (\theta - \varepsilon)}\right)^2 d\theta + \int_{\hat{\theta}(\mu^*)}^{\theta_i+\beta+\varepsilon} (p - \theta_i)^2\, d\theta < \delta$$

6. $\hat{\theta}(\mu^*) - \beta + \varepsilon < \theta_i$

$$\pi(\theta_i, \mu^*) = \frac{1}{4\lambda\varepsilon}\int_{\theta_i+\beta-\varepsilon}^{\theta_i+\beta+\varepsilon} (p - \theta_i)^2\, d\theta < \delta$$

To prove the rest of the proposition, we need to show that the required inequality involving $\pi(\theta_i, \mu^*)$ and δ for each zone is satisfied.

1. For $\theta_i < \mu^* - 2\varepsilon$,

$$\pi(\theta_i, \mu^*) = \frac{1}{4\lambda\varepsilon}\int_{\theta_i+\beta-\varepsilon}^{\theta_i+\beta+\varepsilon} (\theta + \lambda Q - \theta_i)^2\, d\theta = \frac{1}{12\lambda\varepsilon}\left[(\lambda Q + \beta + \varepsilon)^3 - (\lambda Q + \beta - \varepsilon)^3\right] = \frac{(\lambda Q + \beta)^2}{2\lambda} + \frac{\varepsilon^2}{6\lambda} \geq \delta$$

   and the first inequality is satisfied.

2. The proof for this region is identical to the one for the case $\beta = 0$ and is hence ommitted.

3. We shall verify the inequalities for the next three regions together.

   Using the same arguments as in Step III of Proposition 2, we note that the slopes of the two functions, $\pi(\theta_i, \mu^*)$ and $\pi(\mu^*)$ must converge as $\theta_i \longrightarrow \mu^*$ and in particular $\pi(\theta_i, \mu^*)$ must be declining at $\theta_i = \mu^*$. These statements taken together imply that $\pi(\theta_i, \mu^*)$ must have at least one stationary point that is a maximum in the interval, $[\hat{\theta}(\mu^*) - \beta - \varepsilon, \mu^*]$ which includes the region $[\hat{\theta}(\mu^*) - \beta - \varepsilon, \mu^* - \beta]$.

We therefore check the roots of the derivative of $\pi(\theta_i, \mu^*)$ with respect to θ_i.

The derivatives of the first and the second term of $\pi(\theta_i, \mu^*)$ with respect to θ_i are the same as the derivatives of the first and second terms of $\pi(\theta_i, \theta^*)$ in Proposition 2.

The derivative of the third term is given by

$$\frac{1}{4\lambda\varepsilon}\left[3\left(p-\theta_i\right)^2 - 2\left(p-\theta_i\right)\left(\sqrt{\left(p-\theta^*\right)^2 + 4\varepsilon\lambda Q} + \beta\right)\right]$$

which is more conveniently written as,

$$\frac{1}{4\lambda\varepsilon}\left[3\left(p-\theta_i\right)^2 - 2\left(p-\theta_i\right)\left(p + \varepsilon - \hat{\theta}(\mu^*) + \beta\right)\right]$$

As before, these terms can be combined to get

$$\frac{d\pi(\theta_i, \mu^*)}{d\theta_i} = \frac{1}{2\lambda\varepsilon}\left[\theta_i^2 - 2\left(p - \frac{\lambda Q + \varepsilon + \beta}{2}\right)\theta_i + \Omega\left[\mu^*, \beta\right]\right] \tag{44}$$

where

$$\Omega\left[\mu^*, \beta\right] \equiv \left(\mu^* - \varepsilon\right)^2 - \frac{\left(\hat{\theta}\left(\mu^*\right) + \mu^* - \varepsilon\right)^2}{4} + 2\varepsilon\lambda Q \log\left[1 - \frac{\hat{\theta}\left(\mu^*\right) - \left(\mu^* - \varepsilon\right)}{2\varepsilon}\right]$$
$$+ \frac{p^2 - \left(\lambda Q + \mu^* - \varepsilon\right)^2}{2} + p\left(\hat{\theta}\left(\mu^*\right) - \varepsilon - \beta\right)$$

Derivative (44) has two roots given by

$$\theta^R_{1,2}\left(\mu^*\right) \equiv \left(p - \frac{\lambda Q + \varepsilon + \beta}{2}\right) \pm \sqrt{\left(p - \frac{\lambda Q + \varepsilon + \beta}{2}\right)^2 - \Omega\left[\mu^*, \beta\right]},$$

Using the same steps as in the case of $\beta = 0$ in Proposition 2, we show that both roots cannot be less than $\mu^* - \beta$ because of a contradiction.

We next show that $\frac{d\pi(\theta_i,\mu^*)}{d\theta_i} < 0$ for $\theta_i \in [\mu^* - \beta, \hat{\theta}(\mu^*) - \beta + \varepsilon]$. If the last statement is true, then the necessary maxima of $\pi(\theta_i, \mu^*)$ lies in the interval, $[\hat{\theta}(\mu^*) - \beta - \varepsilon, \mu^* - \beta]$ and is unique.

As the form of the function, $\pi(\theta_i, \mu^*)$, is identical over the sub-intervals $[\mu^* - \beta, \mu^*]$ and $[\mu^*, \hat{\theta}(\mu^*) - \beta + \varepsilon]$, the arguments put forth for the function, $\pi(\theta_i, \theta^*)$ for region 4 in Proposition 2 apply and $\frac{d\pi(\theta_i,\mu^*)}{d\theta_i} < 0$ for the entire interval $\theta_i \in [\mu^* - \beta, \hat{\theta}(\mu^*) - \beta + \varepsilon]$.

Thus $\pi(\theta_i, \mu^*)$ has a unique maxima in $[\hat{\theta}(\mu^*) - \beta - \varepsilon, \mu^* - \beta]$. Hence all the three required inequalities for regions 3, 4 and 5 are satisfied.

6. The required inequality follows from the same argument provided for region 5 in Proposition 2.

12.2 Proof of Proposition 6

We need to show that the solution $\mu^* = p - \sqrt{2\lambda\delta}$ is an equilibrium.

As we are looking for an equilibrium under which $r(\theta, \mu^*) = 0$ for all $\theta \in [\mu^* + \beta - \varepsilon, \mu^* + \beta + \varepsilon]$, at the value of θ at which the equilibrium permit price falls to zero, all firms are active. Hence the value of θ at which the equilibrium permit price equals zero is given by $\hat{\theta} = p - \lambda Q < \mu^* + \beta - \varepsilon$.

The individual profit function takes on three different forms depending on three zones in which θ_i may lie. We discuss these forms and the condition that each form must satisfy for μ^* to be equilibrium. We also simultaneously show that these conditions are met.

1. For $\theta_i < \mu^* - \beta - 2\varepsilon$, we require

$$\pi(\theta_i, \mu^*) = \frac{1}{4\lambda\varepsilon} \int_{\theta_i+\beta-\varepsilon}^{\theta_i+\beta+\varepsilon} (\theta - \theta_i + \lambda Q)^2 \, d\theta > \delta$$

   The condition has been shown to be satisfied in Step III of the previous Proposition 5.

2. For $\mu^* - \beta - 2\varepsilon \leq \theta_i \leq \mu^* - \beta$, the profit function and the required condition are

$$\pi(\theta_i, \mu^*) = \frac{1}{4\lambda\varepsilon} \int_{\theta_i+\beta-\varepsilon}^{p-\lambda Q} (\theta - \theta_i + \lambda Q)^2 \, d\theta + \frac{1}{4\lambda\varepsilon} \int_{p-\lambda Q}^{\theta_i+\beta+\varepsilon} (p - \theta_i)^2 \, d\theta > \delta$$

   Upon simplification,

$$\pi(\theta_i, \mu^*) = \frac{1}{4\lambda\varepsilon} \left[\frac{(p-\theta_i)^3}{3} - \frac{(\lambda Q + \beta - \varepsilon)^3}{3} + (p - \theta_i)^2 (\theta_i + \beta + \varepsilon - p + \lambda Q) \right]$$

   The expression equals $\frac{(\lambda Q+\beta)^2}{2\lambda} + \frac{\varepsilon^2}{6\lambda}$ when $\theta_i + \beta + \varepsilon = p - \lambda Q$, implying that the profit function is continuous at this value of θ_i.

   It is easy to show that the derivative of the function with respect to θ_i is negative. Hence, $\pi(\theta_i, \mu^*)$ is strictly declining in θ_i through the range under consideration. Since $\pi(\theta_i, \mu^*) \longrightarrow \delta$ as $\theta_i \longrightarrow \mu^*$, the condition is satisfied.

3. When$\mu^* - \beta < \theta_i < \mu^*$, we must have

$$\pi(\theta_i, \mu^*) = \frac{1}{2\varepsilon} \int_{\theta_i+\beta-\varepsilon}^{\theta_i+\beta+\varepsilon} \frac{1}{2\lambda} (p - \theta_i)^2 \geq \delta$$

   Since $\frac{1}{2\varepsilon} \int_{\theta_i+\beta-\varepsilon}^{\theta_i+\beta+\varepsilon} \frac{1}{2\lambda} (p-\theta_i)^2 = \frac{1}{2\lambda} (p - \theta_i)^2$, and the latter function is strictly declining in θ_i, the condition is satisfied, because at $\theta_i = \mu^*$, $\frac{1}{2\lambda} (p - \theta_i)^2 = \delta$.

4. When $\mu^* < \theta_i$, we must have

$$\pi(\theta_i, \mu^*) = \frac{1}{2\varepsilon} \int_{\theta_i+\beta-\varepsilon}^{\theta_i+\beta+\varepsilon} \frac{1}{2\lambda} (p - \theta_i)^2 < \delta$$

By the same arguments as in the previous step, the condition is satisfied.