Social Polarization: A Network Approach

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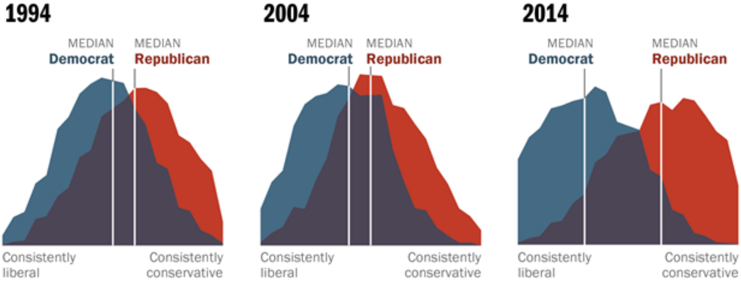
Polarization growing in the last 2 decades

Democrats and Republicans More Ideologically Divided than in the Past

Distribution of Democrats and Republicans on a 10-item scale of political values



Source: 2014 Political Polarization in the American Public

Notes: Ideological consistency based on a scale of 10 political values questions (see Appendix A). The blue area in this chart represents the ideological distribution of Democrats; the red area of Republicans. The overlap of these two distributions is shaded purple. Republicans include Republican-leaning independents; Democrats include Democratic-leaning independents (see Appendix B).

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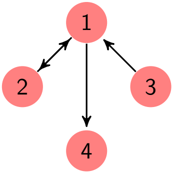
This paper

- **Objective**: understand what are the main drivers of Polarization dynamics
- **Key Ingredients**:
  - Individuals are connected through a Network and exchange information
  - Receive private signals (Bayesian) but also incorporate friends' opinions (non-bayesian)
  - Presence of "fanatics" prevents Society to learn the truth and might create cycles of polarization
- **Innovation**: simulate large number of random networks to decompose the importance of their characteristics in driving polarization (homophily, density, clustering, etc)

Basic Structure: Social Network

- Finite and fixed set of agents $N = \{1, 2, \ldots, n\}$
- Connectivity among these agents at every time t is described by a *directed graph* $\mathbf{G}^t = (N, \mathbf{g}^t)$



$$\mathbf{g}^t = \begin{pmatrix} 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

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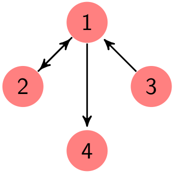

$$\tilde{\mathbf{g}}^t = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Basic Structure: Network Motion

- Sequence of time, $t = 1, \ldots, T$
- For all $t \geq 1$, we associate a *clock* to every directed link of the form (i,j) in the initial adjacency matrix $\mathbf{g}^0$
- Ticking: i.i.d. samples from a Bernoulli with fixed and common parameter $p \in [0, 1]$

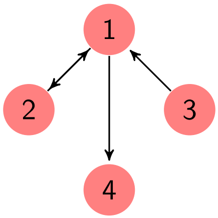

Figure: $\mathbf{g}^0$

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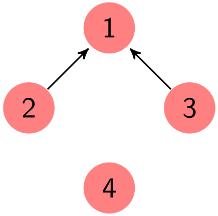

Figure: $\mathbf{g}^1$

Basic Structure: Network Motion

- Draws: $n \times n$ matrix $\mathbf{c}^t$, with regular elements $c_{ij}^t \in \{0, 1\}$ and $c_{ii}^t = 1$
- Graph Law of Motion: $\mathbf{g}^t = \mathbf{g}^0 \circ \mathbf{c}^t$

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  2. the unemployment rate in the next year

Basic Structure: Utility Maximization Problem

- For each opinion $y_{i,t}$ and signal $s_{i,t}$, agent's expected utility is:

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- The distribution of opinions is denoted by the mass function $f(y)$

Example: Worldviews and Opinions

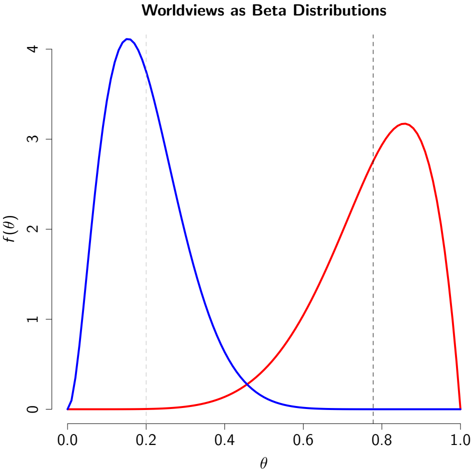


Figure: Agent Blue: $\alpha = 3, \beta = 12$; Agent Red: $\alpha = 7, \beta = 2$

Basic Structure: Timeline of Events

- $t = 0$
  - Network $\mathbf{g}^0$ is randomly formed (explain later)
  - Nature draw parameters vectors (α_0, β_0)
  - Initial Opinions vector y_0 is formed
- $t > 0$
  - **Morning**: Agents receive signals, Update (Bayesian)
  - **Afternoon**: Meet Friends, Update (Non-Bayesian)
  - **Night**: Revise opinion
- Agents are partially Bayesians: they are influenced by people in their network (DeGrootian)
- Parameter λ measure how Bayesian a Society is

Basic Structure: Belief Update

- Departure from:
  - Epstein, Noor and Sandroni (2010)
  - Jadbabaie, Molavi, Sandroni and Tahbaz-Salehi (2012)
- Update Rule

$$\alpha_{t+1} = \left[\lambda \mathbb{I} + (\mathbb{I} - \lambda)\tilde{g}^{t+1}\right]\left(\alpha_t + \mathbf{s}_{t+1}\right)$$

$$\beta_{t+1} = \left[\lambda \mathbb{I} + (\mathbb{I} - \lambda)\tilde{g}^{t+1}\right]\left(\beta_t + \mathbf{1} - \mathbf{s}_{t+1}\right)$$

- Special cases:
  - Bayes: $\lambda = 1$
  - DeGroot: $\lambda = 0$

Definition 1 (Fanatic Agents)

Fanatic Agents are characterized by disregarding information both from private signals and friends

- Parameters:
  - Type 0: $\alpha = 0$ and $\beta = \beta^{max}$
  - Type 1: $\beta = 0$ and $\alpha = \alpha^{max}$

Definition 2 (Opinion Consensus)

A group $C \subseteq N = \{1, 2, \ldots, n\}$ reaches a consensus for any initial distribution of parameters (α_0, β_0) if

$$|\operatorname*{plim}_{t\to\infty} y_{i,t} - \operatorname*{plim}_{t\to\infty} y_{j,t}| < \epsilon$$

Definition 3 (Information Aggregation)

Information Aggregation is a measure of how close agents' opinions are to the true state of nature θ^*.

We say that society aggregates information if

$$\max_i \left| \operatorname*{plim}_{t\to\infty} y_{i,t} - \theta^* \right| < \epsilon$$

Following: *Esteban and Ray* (**1994, 2004**)

Definition 4 (Social Polarization)

Social Polarization P is a measure that aggregates both **Identification** and **Alienation** across citizens:

$$P_t^a(f) = \frac{1}{2} \sum_i \sum_{j \neq i} f(y_{i,t})^{1+a} \, f(y_{j,t}) \, |\tilde{y}_{i,t} - \tilde{y}_{j,t}|$$

where $a \in [0.25, 1]$ and $\tilde{y}_{i,t}$ is the opinion of agent i normalized by the average of society's opinion, denoted by $\tilde{y}_{i,t} = \frac{y_{i,t}}{\frac{1}{n}\sum_{i \in N} y_{i,t}}$, for all $i \in G$

Part 1: Asymptotic Analysis

Lemma 1

Social Polarization converges in probability to zero if agents reach Consensus.

Part 1: Asymptotic Analysis

Proposition 1

Information Aggregation implies lack of Social Polarization. The converse is not necessarily true.

Part 1: Asymptotic Analysis

Proposition 2

If the Social Network $G^0 = (N, g^0)$ is strongly connected and aperiodic, then even when the edges are not activated every period and using this particular rule on Non-Bayesian Learning, Social Polarization still converges to zero as $t \to \infty$.

Part 2: Simulation-based exercise

- Limiting properties of Polarization are hard to ascertain analytically, then Simulation is a good tool for examining the importance of each characteristic.
- Simulate a large number of networks with different characteristics (clock, bayesian, proportion of fanatics, centrality of fanatics, etc...) and analyze their effects on:
  1. Degree of Polarization (Average, Maximum)
  2. Speed of Convergence
  3. Dynamics (Cycles)
- Regression and Decomposition: $Y = g(X\beta) + \epsilon$

Part 2: Simulation-based exercise

Two particular examples

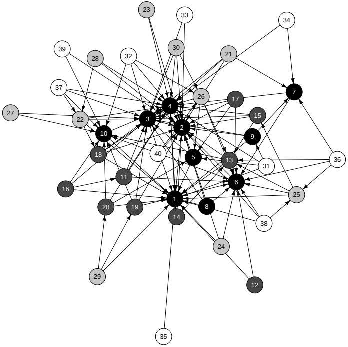


Figure: Barabasi-Albert
$n = 40$, Power$= 1$,
Out Dist $= (0.01, 0.04, 0.10, 0.25, 0.60)$

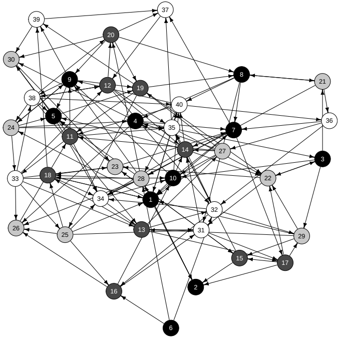
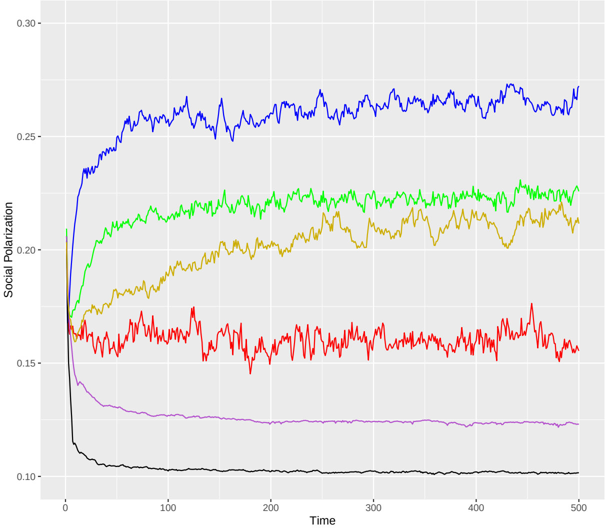


Figure: Erdos-Renyi
$n = 40$, $p = 10\%$

Polarization: Different Levels and Cycles



Initial Decomposition

	Average Polarization
	OLS
Proportion of F0	0.6336***
Average In-Degree F0	0.0011**
Proportion of F0 x Average In-Degree F0	0.0016*
Clock	0.0062
Bayes	−0.0155**
Homophily	−0.0282
Clustering	−0.1252***
Diameter	0.0025***
Initial Polarization	1.4920***
Initial Polarization (squared)	−1.9425***
Constant	−0.1287***
Note:	*p<0.1; **p<0.05; ***p<0.01

Next Steps

1. Run more simulations (tighten standard errors)
2. Barabasi-Albert (Preferential attachment)
3. Clock: other stochastic process (more or less persistence)
4. Splitting economies with cycles from those that do not exhibit them for regressions
5. Comparative statics (only move one parameter)
6. Key-player analysis: How to reduce Polarization
7. Related paper: Cheap-Talk