Inspection game with Partial Inspections

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An inspection game is a mathematical model for the game between two players, where one of them (the inspectee / potential violator) has enough potential to violate certain legal acts. The other player (the inspector) tries to verify the inspectee's adherence to those legal acts by carrying out inspections over certain period of times.

M. Dresher (1962) has studied the case where the inspectee can commit at most one violation during m periods of times, while the number of inspections is limited to a fixed number n, [3].

Dresher supposed that if the inspector inspects when the inspectee violates, the violation would be detected with probability $P = 1$. In each stage, both players know how many inspections and time periods are left. So, if n≥m then the inspectee will not violate as he knows that it will be detected for certain. Because of lack of budget usually n<m. Dresher has found the value and optimal strategies for these cases. Beside Dresher's paper, inspections have also been analysed in several studies; such as in the work by V. J. Baston et al. (1991), B. von Stengel (1991) and M. J. Canty et al. (2000), [1][4][2].

In our study about inspection games, we assume that the inspector may run some "partial inspections" where the probability of detection is not equal to one. In contrast to the partial inspection, the inspections with the probability of detection equal to one would be called "full inspection".

The assumption of partial inspection is reasonable as in many real world scenarios where full inspection can be too costly or time consuming. Hence, instead of a full inspection the inspector may run some partial inspections at lower costs or effort. Hence, by conducting a partial check the probability of

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detection would be P, which is not necessarily equal to one. One of the most famous forms of partial inspections applies to airplane safety checks. The full safety check of the plane typically takes more than 2 days which is a long and costly ground time for the planes, making partial inspections a more favourable proposition. However, it is critical that effective inspections guarantee the safety of the flight. We model the situation as a non-cooperative zero sum game and describe the value and optimal strategies of the game using recursive formulae. In particular, we compare the value of the game for inspection games with full and partial inspections only and hence determine the opportunity costs for using these technologies. In a number of cases we provide closed form solutions for the values of the game and the optimal strategies.

In our study initially, we analyse the inspection game with just partial inspections. By providing a formula for calculating the value of this game, we can compare the inspection game with partial game with the game introduced by Dresher (1962).

In the second stage, we investigate the case when the inspector can choose between full inspection, partial inspection and no control in each period of time. We show that in equilibrium the inspector always mix between full inspections and no control. It means that as long as the opportunity for a full inspection exists, the inspector never starts his sequential inspections with a partial inspection. We also provide a way to calculate the values; which can provide a tool to compare the inspection game with mixture of partial and full inspection with the classical results of inspection game with just full inspections.

Refrences

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