Conflict without an Apparent Cause[1]

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Abstract: A game-theoretic model of repeated interaction between two potential adversaries is analyzed to illustrate how conflict can arise from rational decision-makers endogenously processing information, without any exogenous changes to the fundamentals of the environment. This occurs as a result of a convergence of beliefs about the true state of the world by the two players. During each period, each adversary must decide to either stage an attack or not. Conflict ensues if either player chooses to initiate an attack. Choosing to not stage an attack in a given period reveals information to a player's rival. Thus, over time, beliefs about the true state of the world converge. Depending upon the true state of the world, we can ultimately have either of the two adversaries initiating an attack (either with or without regret) after an arbitrarily long period of tranquility. When this happens, it is as if conflict has suddenly arisen without any apparent cause or impetus. Alternatively (again, depending upon the true state of the world), we could possibly have beliefs converge to a point where neither adversary wants to initiate conflict.

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1. Introduction

This paper illustrates how conflict between two parties can arise as a result of rational decision-makers *endogenously* processing information – that is, without any exogenous changes to the fundamentals of the environment. Consequently, an agent may choose to initiate a conflict for no apparent reason: without instigation or provocation; without observing a change in the behavior of the rival party; without new information about the type of the rival.

The forces at play in the present model are identical to those identified by Hart and Tauman (2004), where a market environment between two potential traders is examined.[2] Agent behavior eventually changes as a result of the gradual processing of information and updating of beliefs. In a market setting, trade will occur when one party wants to be a buyer and one party wants to be a seller. A "crash" will occur when both want to be sellers; a "bubble" will occur when both want to be buyers. A market setup is fairly symmetric, in that each party wants to buy when they believe the value is high and sell when they believe the value is low. Transactions occur when agents have different beliefs, but the market collapses once beliefs converge.

In the game-theoretic model of conflict presented and analyzed below, there are two agents who are opposed to one another ("Adversary A," denoted as A, and "Adversary B," denoted as B). Each wants to act (i.e., initiate an attack) if and only if he believes that the probability that he possesses the upper-hand is sufficiently high. Thus, as beliefs converge, it may result in one agent choosing to attack after not doing so in previous periods (but it could also potentially result in a stable state of indefinite stability). For this model (depending upon the true state of the world), we can ultimately have: (i) B eventually initiating an attack without regret; (ii) B eventually initiating an attack with regret; (iii) A eventually initiating an attack

[2] The work of Hart and Tauman builds upon Geanakoplos and Polemarchakis (1982).

without regret; (iv) *A* eventually initiating an attack with regret; or (v) neither agent choosing to initiate an attack (indefinite stability).

Numerous scholars within the field of international relations have broadly examined the causes of conflict. Levy (1998) provides an extensive overview of the literature, assessing multiple explanations including balance of power theories, economic interdependence and war, domestic coalition theories, and decision-making under risk and uncertainty. Academics such as Bueno De Mesquita (1985) and Fearon (1995) have put forth ideas that treat the decision to engage in conflict as a rational choice.[3] Such formulations are directly compatible with traditional economic and game-theoretic analyses of behavior. Consistent with this rationality based approach, van Evera (1998) argues that conflict is more likely to arise when parties view conquest to be easy or low cost. Also in this vein, Ohlson (2008) argues that, "people take to arms because they have *Reasons* in the form of grievances and goals, they have *Resources* in the form of capabilities and opportunities, and they have *Resolve* because they perceive of no alternative to violence in order to achieve their goals" (page 134). From here it follows that an adversary would be more inclined to initiate conflict if one of these factors were to change exogenously. For example, if a rival country enters a time of political turmoil, leading to an economic downturn and corresponding decline in military capacity, an adversary might very well choose to launch an opportunistic attack. Identifying such a change in external circumstances as the cause of conflict is often quite natural and consistent with a view of conflict as a rational decision. But, the main point of the present study is that an easily identifiable external cause of a conflict might not be present, even when a rational agent chooses to initiate conflict. Instead, it can be the case that an agent chooses to rationally initiate a conflict (after multiple periods of

[3] In contrast, Levy (1983) highlights how misperceptions – about adversary's capabilities, adversary's intentions, or third party interests – can essentially serve as the root cause of conflict.

peace) for no apparent reason, as a consequence of refining his beliefs about the true state of the world. If this occurs, then any attempt to identify the changed external factor which caused the conflict would be a search in vain.

2. Framework of Example

Consider a simple game in which there are two parties to a potential conflict: “Adversary A” (denoted as A) and “Adversary B” (denoted as B). These two parties could be two sovereign nations, a sovereign nation and a terrorist organization,[4] or, more generally, any two parties that could possibly be engaged in conflict with one another. In each period, each of these players must decide to either attack or wait (i.e., initiate or not initiate conflict). If either party chooses attack, conflict ensues in the present period. If both parties choose wait, no conflict ensues in the present period and the decision process repeats itself in a subsequent period. Assume that each player makes this decision to maximize his expected payoff, taking into account the costs of engaging in conflict, coupled with the expected gain from and probability of “winning” the conflict.

Let the continuous interval $\Omega = [0,1]$ denote the set of possible states of the world. Each state of the world is characterized as one of two environments: “favorable to A” (f_A) or “favorable to B” (f_B). Engaging in a conflict costs A resources of $C_A = 6$ and costs B resources of $C_B = 3$, regardless of the outcome. Separate from these costs, if the true environment is f_A, then A expects to gain and B expects to lose $V(f_A) = 12$ from a conflict. If instead the true environment is f_B, then (again, separate from these costs) B expects to gain and A expects to lose

[4] For an overview of the literature analyzing terrorism using tools of game theory, see Sandler and Arce M. (2003).

$V(f_B) = 8$ from a conflict. Assuming a baseline payoff of zero from no conflict, A will want to attack if and only if he believes that the probability of f_B, denoted q, is such that:

$$(1-q)(12-6)+q(-8-6) \geq 0$$

$$6(1-p)-14q \geq 0$$

$$q \leq \tfrac{3}{10}.$$

Similarly, B will want to attack if and only if he believes q is such that:

$$q(8-3)+(1-q)(-12-3) \geq 0$$

$$5q-15(1-q) \geq 0$$

$$q \geq \tfrac{3}{4}.$$

Suppose that the prior probability assessments of A and B are such that they both accurately believe that each point on the interval $\Omega = [0,1]$ is equally likely. The private information of each agent can be summarized by a partition of the state space. Suppose, as illustrated in Figure 1, that Adversary A's partitions are:

$$A_1 = [0,.2],\ A_2 = (.2,.4],\ A_3 = (.4,.6],\ A_4 = (.6,.8], \text{ and } A_5 = (.8,1],$$

and that Adversary B's partitions are:

$$B_1 = [0,.3],\ B_2 = (.3,.5],\ B_3 = (.5,.7], \text{ and } B_4 = (.7,1].$$

To understand the interpretation of these information partitions, suppose that the true state of the world is $\omega = .45$. In this case, A knows that the true state of the world is $\omega \in A_3 = (.4,.6]$, but cannot distinguish between the points within this subset. Similarly, B knows that the true state of the world is $\omega \in B_2 = (.3,.5]$, but cannot distinguish between the points within this subset.

Assume that

$$f_B = [0,.18] \cup [.28,.38] \cup [.48,.58] \cup [.9,1],$$

so that

$$f_A = [0,1] \backslash f_B = f_B^C = (.18, .28) \cup (.38, .48) \cup (.58, .9).$$

3. Analysis of Example

By way of considering different actual states of the world and analyzing the behavior of the agents in order to see how their interaction with each other will ultimately play out, we aim to show that after multiple initial periods in which both choose wait, we can then have: (i) B choose to initiate an attack without regret (i.e., when the true environment is f_B); (ii) B choose to initiate an attack with regret (i.e., when the true environment is f_A); (iii) A choose to initiate an attack without regret (i.e., when the true environment is f_A); (iv) A choose to initiate an attack with regret (i.e., when the true environment is f_B); or (v) perpetual stability (with neither agent ever choosing to attack in any period).

When analyzing agent behavior in each period, it is critical to correctly determine the probability assessment of each agent over each possible environment (f_A and f_B) given the agent's knowledge regarding what states of the world are possible. To properly do so, it is important to recognize what information is common knowledge between the two agents at any point in time. Let $p = 1,2,3, \ldots$ denote the different periods of interaction between the agents, and let Ω^p denote the set of information that is common knowledge between the two agents at the start of Period p (before they choose to either wait or attack in Period p).

Because of the way in which the information partitions overlap one another, regardless of the true state of the world in Period 1 we have $\Omega^1 = \Omega$. That is, at the start it is *not* common knowledge that any particular value of $\omega \in [0,1]$ did *not occur*. For example, again suppose the true state of the world is $\omega = .45$. A knows $\omega \in A_3 = (.4, .6]$, and B knows $\omega \in B_2 = (.3, .5]$. Thus, while it is *mutual knowledge* that the true state of the world is neither $\omega \leq .3$ nor $\omega > .6$, these facts are not *common knowledge*. As defined by Aumann (1976), something is common knowledge between person A and person B if and only if: A knows it; B knows it; A knows that B knows it; B knows that A knows it; A knows that B knows that A knows it; *ad infinitum*.

To understand why neither $\omega \leq .3$ nor $\omega > .6$ are common knowledge in Period 1, let $K_A S$ denote the smallest subset S for which it can be stated that "A knows that the true state of the world is within S." Likewise, let $K_A K_B S$ denote the smallest subset S for which it can be stated that "A knows that B knows that the true state of the world is within S," and so on.[5] Given the information structure as summarized by Figure 1:

$$K_A S = A_3,$$

$$K_A K_B S = B_2 \cup B_3,$$

$$K_A K_B K_A S = A_2 \cup A_3 \cup A_4, \text{ and}$$

$$K_A K_B K_A K_B S = B_1 \cup B_2 \cup B_3 \cup B_4 = \Omega.$$

That is, when the true state of the world is $\omega = .4$, it is never the case that A knows that B knows that A knows that B knows that the true state of the world is *not* any particular value of $\omega \in [0,1]$.[6]

3.1. Example in which B ultimately chooses to initiate an attack

Suppose the true state of the world is $\omega \in A_1 = [0, .2]$ – that is, somewhere on the interval between $\omega = 0$ and $\omega = .2$. At the start of Period 1, A knows $\omega \in A_1 = [0, .2]$, and B knows $\omega \in B_1 = [0, .3]$. Consequently, A computes the probability of the true environment being f_B to be $P_A(f_B | A_1 \cap \Omega^1) = \frac{.18}{.2} = \frac{9}{10}$, and B computes the probability of the true environment being f_B

[5] This hierarchical description of layers of knowledge was formalized by Aumann (1999).

[6] Similarly, $K_B K_A K_B K_A K_B S = B_1 \cup B_2 \cup B_3 \cup B_4 = \Omega$. More generally, regardless of the actual true state of the world, we quickly reach a point where both $K_A K_B \ldots S = \Omega$ and $K_B K_A \ldots S = \Omega$.

to be $P_B(f_B|B_1 \cap \Omega^1) = \frac{.18+.02}{.3} = \frac{2}{3}$. The agents base their actions in Period 1 (and in each subsequent period) upon these computed probabilities over this interval which they know to be the true state of the world (denoted by $\widetilde{A_1}$ and $\widetilde{B_1}$, as in Figure 1). Thus, A will choose to wait (i.e., not attack) since $P_A(f_B|A_1 \cap \Omega^1) = \frac{9}{10} > \frac{3}{10}$, and B will choose to wait (i.e., not attack) since $P_B(f_B|B_1 \cap \Omega^1) = \frac{2}{3} < \frac{3}{4}$.

But to comprehend how information is updated over time, it is necessary to compute each agent's perceived probability of f_B in each distinct information set (in order for agents to be able to infer how their rival would have behaved in other states of the world). To this end, in Period 1: $P_A(f_B|A_2 \cap \Omega^1) = \frac{.1}{.2} = \frac{1}{2}$, $P_A(f_B|A_3 \cap \Omega^1) = \frac{.1}{.2} = \frac{1}{2}$, $P_A(f_B|A_4 \cap \Omega^1) = \frac{0}{.2} = 0$, and $P_A(f_B|A_5 \cap \Omega^1) = \frac{.1}{.2} = \frac{1}{2}$ for A; and $P_B(f_B|B_2 \cap \Omega^1) = \frac{.1}{.2} = \frac{1}{2}$, $P_B(f_B|B_3 \cap \Omega^1) = \frac{.08}{.2} = \frac{2}{5}$, and $P_B(f_B|B_4 \cap \Omega^1) = \frac{.1}{.3} = \frac{1}{3}$ for B. These probabilities are summarized in the top row of Table 1.

At the start of Period 2, it is common knowledge that neither agent chose to attack in Period 1. As a consequence, it is common knowledge that the true state of the world is not within A_4, since if it were then A would have chosen to attack in Period 1. This leads to the new common knowledge information set being $\Omega^2 = [0, .6] \cup (.8,1]$. For this refinement of common knowledge: $A_4 \cap \Omega^2 = \{\emptyset\}$, $B_3 \cap \Omega^2 \subset B_3 \cap \Omega^1$, and $B_4 \cap \Omega^2 \subset B_4 \cap \Omega^1$. Consequently, $P_A(f_B|A_4 \cap \Omega^2)$ would be meaningless to compute. Further,

$$P_B(f_B|B_3 \cap \Omega^2) = \frac{.08}{.1} = \frac{4}{5} \neq \frac{2}{5} = P_B(f_B|B_3 \cap \Omega^1) \text{ and}$$

$$P_B(f_B|B_4 \cap \Omega^2) = \frac{.1}{.2} = \frac{1}{2} \neq \frac{1}{3} = P_B(f_B|B_4 \cap \Omega^1).$$

Each of the other six assessed probabilities are effectively unchanged from Period 1. These probabilities are summarized in the second row of Table 1. Again, A will choose to wait since $P_A(f_B|A_1 \cap \Omega^2) = \frac{9}{10} > \frac{3}{10}$, and B will choose to wait since $P_B(f_B|B_1 \cap \Omega^2) = \frac{2}{3} < \frac{3}{4}$.

Now, at the start of Period 3, it is common knowledge that neither agent chose to attack in Period 2. Thus, it is common knowledge that the true state of the world is not within B_3, since if it were then B would have chosen to attack in Period 2. The common knowledge information set becomes $\Omega^3 = [0,.5] \cup (.8,1]$. For this refined information set, $P_B(f_B|B_3 \cap \Omega^3)$ would be meaningless to compute and $P_A(f_B|A_3 \cap \Omega^3) = \frac{.02}{.1} = \frac{1}{5} \neq \frac{1}{2} = P_A(f_B|A_3 \cap \Omega^2)$. The other assessed probabilities are effectively unchanged from Period 2. These probabilities are summarized in the third row of Table 1. Again, A will wait since $P_A(f_B|A_1 \cap \Omega^3) = \frac{9}{10} > \frac{3}{10}$, and B will wait since $P_B(f_B|B_1 \cap \Omega^3) = \frac{2}{3} < \frac{3}{4}$.

At the start of Period 4 it is now common knowledge that the true state of the world is not within A_3, since if it were then A would have chosen to attack in Period 3. The common knowledge information set is now further refined to $\Omega^4 = [0,.4] \cup (.8,1]$. Consequently, $P_B(f_B|B_2 \cap \Omega^4) = \frac{.08}{.1} = \frac{4}{5} \neq \frac{1}{2} = P_B(f_B|B_2 \cap \Omega^3)$. Again, neither A nor B will attack since $P_A(f_B|A_1 \cap \Omega^4) = \frac{9}{10} > \frac{3}{10}$ and $P_B(f_B|B_1 \cap \Omega^4) = \frac{2}{3} < \frac{3}{4}$.

At the start of Period 5 it is common knowledge that the true state of the world is not within B_2 (since if it were, B would have chosen to attack in Period 4). Thus, $\Omega^5 = [0,.3] \cup (.8,1]$ and $P_A(f_B|A_2 \cap \Omega^5) = \frac{.02}{.1} = \frac{1}{5} \neq \frac{1}{2} = P_B(f_B|A_2 \cap \Omega^4)$. Again, both agents choose to wait.

After A chooses to wait in Period 5 it becomes common knowledge that the true state of the world is not within A_2. Accordingly, $\Omega^6 = [0,.2] \cup (.8,1]$. But this refinement alters B's perception of the true state of the world in a meaningful way. Agent B now knows that the true state of the world is *not* in $A_2 \cap B_1 = (.2,.3]$, but rather must be in $A_1 \cap B_1 = [0,.2]$. As a result, B computes the probability of the true environment being f_B to be $P_B(f_B|B_1 \cap \Omega^6) = \frac{.18}{.2} = \frac{9}{10}$. Since $P_B(f_B|B_1 \cap \Omega^6) = \frac{9}{10} > \frac{3}{4}$, agent B will choose to attack in Period 6. Thus, conflict is initiated in Period 6 after five initial periods of calm. This conflict results without any observed impetus – there was no new event or apparent provocation. Rather, the stimulus was simply the updating of information (i.e., the refinement of the common knowledge information set) undertaken by the agents after observing the actions chosen by their rival.

When B chooses to initiate the attack, the information set partitions of the two agents have converged and both agents agree that the true state of the world is $\omega \in A_1 = [0, .2]$. However, within this range both f_B and f_A are possible, the former being true for $\omega \in [0, .18]$ and the latter being true for $\omega \in (.18, .2]$. Thus, this example illustrates how (i) B could choose to initiate an attack without regret (i.e., if the true environment is f_B) and (ii) B could choose to initiate an attack with regret (i.e., if the true environment is f_A).

3.2. Example in which A chooses to initiate an attack

Instead suppose the true state of the world is $\omega \in (.4, .5]$ – that is, somewhere on the interval between $\omega = .4$ and $\omega = .5$. At the start of Period 1, A knows $\omega \in A_3 = (.4, .6]$, and B knows $\omega \in B_2 = (.3, .5]$. Consequently, the agents base their actions upon the computed probabilities $P_A(f_B|A_3 \cap \Omega^1) = \frac{1}{2}$ and $P_B(f_B|B_2 \cap \Omega^1) = \frac{1}{2}$. Both agents choose to wait (i.e., not attack) since $P_A(f_B|A_3 \cap \Omega^1) = \frac{1}{2} > \frac{3}{10}$ and $P_B(f_B|B_2 \cap \Omega^1) = \frac{1}{2} < \frac{3}{4}$. Since in Period 1 we have $\Omega^1 = \Omega$, all nine of the computed probabilities (i.e., five for A and four for B) are equal in value to what they were in the example from subsection 3.1, as reported in the top row of Table 2.

As a first step, we again have that at the start of Period 2 – after both agents observe that no attack occurred in Period 1 – it becomes common knowledge that the true state of the world is *not* in A_4. Thus, $\Omega^2 = [0, .6] \cup (.8,1]$. Again, $P_A(f_B|A_4 \cap \Omega^2)$ would be meaningless to compute, and $P_B(f_B|B_3 \cap \Omega^2) = \frac{.08}{.1} = \frac{4}{5} \neq \frac{2}{5} = P_B(f_B|B_3 \cap \Omega^1)$. Both agents again choose to wait, since $P_A(f_B|A_3 \cap \Omega^2) = \frac{1}{2} > \frac{3}{10}$ and $P_B(f_B|B_2 \cap \Omega^2) = \frac{1}{2} < \frac{3}{4}$.

After observing no attack in Period 2, in Period 3 we have $\Omega^3 = [0, .5] \cup (.8,1]$. As reported in Table 3, this results in $P_A(f_B|A_3 \cap \Omega^3) = \frac{1}{5} < \frac{3}{10}$, for which A will choose to attack. Similar to the example in subsection 3.1, conflict is initiated in Period 3 after multiple initial periods of calm.

In contrast to the first example, when the true state of the world is $\omega \in (.4, .5]$, conflict is initiated before the information set partitions of the agents have converged. When A chooses to attack, he knows that the true state of the world is $\omega \in (.4, .5]$, but B more broadly believes that the true state of the world could be anywhere in the larger set $\omega \in (.3, .5]$. Nonetheless, in this example both f_B and f_A are possible, the former being true for $\omega \in [.48, .5]$ and the latter being true for $\omega \in (.4, .48)$. This example illustrates how (iii) A could choose to initiate an attack without regret (i.e., if the true environment is f_A) and (iv) A could choose to initiate an attack with regret (i.e., if the true environment is f_B).

3.3. Example in which neither agent ever chooses to initiate an attack

Finally, suppose the true state of the world is $\omega \in A_5 = (.8,1]$ – that is, somewhere on the interval between $\omega = .8$ and $\omega = 1$. At the start of Period 1, A knows $\omega \in A_5 = (.8,1]$, and B knows $\omega \in B_4 = (.7,1]$. Since in Period 1 we have $\Omega^1 = \Omega$, all nine of the computed probabilities are again equal in value to what they were in the two previous examples presented in subsections 3.1 and 3.2, as reported in the top row of Table 3. In Period 1 both agents choose to not attack since $P_A(f_B|A_5 \cap \Omega^1) = \frac{1}{2} > \frac{3}{10}$ and $P_B(f_B|B_4 \cap \Omega^1) = \frac{1}{3} < \frac{3}{4}$. Moreover, through the end of the fifth period the observed behaviors and the evolution of the common knowledge information sets are identical to the example analyzed in subsection 3.1 (as reported in Table 1, and replicated in the top five rows of Table 3).

In Period 6 the computed probabilities are again identical to those from the example in subsection 3.1, but, since A knows the true state of the world to be $\omega \in A_5 = (.8,1]$ and B knows the true state of the world to be $\omega \in B_4 \cap \Omega^6 = (.8,1]$, they base their actions upon $P_A(f_B|A_5 \cap \Omega^6) = \frac{1}{2}$ and $P_B(f_B|B_4 \cap \Omega^6) = \frac{1}{2}$ respectively and both choose to not attack.

In Period 7 the common knowledge information set now becomes $\Omega^7 = (.8,1]$, and the two relevant probabilities are $P_A(f_B|A_5 \cap \Omega^7) = \frac{1}{2}$ and $P_B(f_B|B_4 \cap \Omega^7) = \frac{1}{2}$ (for which both agents choose to not attack). At this point, no current or future actions lead to any further refinement of the common knowledge information set or the subsequent assessed probabilities. That is, beliefs and assessed probabilities have converged and are equal to $\Omega^t = (.8,1]$, $P_A(f_B|A_5 \cap \Omega^t) = \frac{1}{2}$, and $P_B(f_B|B_4 \cap \Omega^t) = \frac{1}{2}$ for all $t = 7, 8, 9, \ldots$. Consequently, neither agent ever chooses to attack in any future period. This illustrates a situation in which we realize (v) indefinite stability, in which neither agent ever chooses to initiate an attack.

Note that an outcome of this nature could not be realized in the model of market trade analyzed by Hart and Tauman. This is because within their model the two agents had a common probability cutoff upon which behavior was based. Once a sufficient number of periods transpire, agents beliefs converge and both parties will either agree that the better action is buy or that the better action is sell. Eventually the agents agree and want to behave in the same manner, causing the market to collapse (with either a crash or a bubble occurring).

In contrast, in the model of conflict presented here, the interests of an individual agent are diametrically opposed to those of their rival. An agent only wants to “act” (i.e., initiate an attack) when he believes that the probability with which he possesses the upper-hand is sufficiently high. Given the negative constant sum nature of the payoffs realized if either agent initiates an attack (as described in Section 2, the gain/loss of $V(f_A)$ or $V(f_B)$ sums to zero across the two players, so that the positive valued costs result in a combined loss of $C_A + C_B = 9$ when conflict occurs) it is clearly possible for the behavioral cutoffs for the two agents to differ from one another. That is, the probability of f_B below which A will want to initiate an attack can be strictly less than the probability of f_B above which B will want to initiate an attack. If this is the case and the true state of the world is such that beliefs converge to the point where the common assessed probability of f_B is between these two cutoffs, then the agents can realize the harmonious outcome of indefinite stability.

4. Generalization of Example

The aim of this section is to show how the example presented in Section 2 and analyzed in Section 3 can be generalized so that conflict is initiated by either party after any arbitrarily large number of periods of tranquility. Suppose A has a total of $X \geq 4$ information sets. Let $A_1 = [0, .2]$ and $A_X = (.8, 1]$. Divide the interval from .2 to .8 into $X - 2$ information sets of equal length, so that $A_j = \left(.2 + \frac{.6(j-2)}{X-2}, .2 + \frac{.6(j-1)}{X-2}\right]$ for $j = 2, \ldots, X-1$. Each intermediate segment has length of $\frac{.6}{X-2}$, with a midpoint of $.2 + \frac{.6j - .9}{X-2}$ for $j = 3, \ldots, X-1$. Suppose B has a total of $X - 1$ information sets. Let $B_1 = \left[0, .2 + \frac{.3}{X-2}\right]$, $B_{X-1} = \left[.2 + \frac{.6(X-1) - .9}{X-2}, 1\right]$, and $B_j = \left[.2 + \frac{.6j - .9}{X-2}, .2 + \frac{.6(j+1) - .9}{X-2}\right]$ for $j = 2, \ldots, X-2$. Further define the following $X - 1$ intervals: $f_{1,B} = \left[0, .15 + \frac{.09}{X-2}\right]$, $f_{X-1,B} = [.9, 1]$, and $f_{j,B} = \left[.2 + \frac{.6j - .96}{X-2}, .2 + \frac{.6j - .66}{X-2}\right]$ for $j = 2, \ldots, X-2$. Let $f_B = f_{1,B} \cup f_{2,B} \cup \ldots \cup f_{X-2,B} \cup f_{X-1,B}$, and let $f_A = [0,1] \backslash f_B = f_B^C$. Note that $A_{X-1} \cup f_B =$

$\{\emptyset\}$. Recognize that the example presented in Section 2 and analyzed in Section 3 is a special case of this environment with $X = 5$.

Observe that for each information set B_j, $j = 2, \ldots, X-2$, the lower boundary is the midpoint of A_j and the upper boundary is the midpoint of A_{j+1}. Consequently, because of the manner in which the information sets of the two agents overlap one another, regardless of the true state of the world in Period 1 we have $\Omega^1 = \Omega$. Table 4 summarizes each agent's perceived probability of f_B in each distinct information when $\Omega^1 = \Omega$ (which is true at the start of Period 1, no matter the true state of the world).

To see that B might choose to initiate conflict after an arbitrarily large number of periods of tranquility, consider $\omega \in A_1 = [0, .2]$. At the start of Period 1, A knows $\omega \in A_1$ and B knows $\omega \in B_1$. Thus, $P_A(f_B|A_1 \cap \Omega^1) = \frac{3}{4} + \frac{.45}{X-2} > \frac{3}{4}$ and $P_B(f_B|B_1 \cap \Omega^1) = \frac{.15(X-2)+.15}{.2(X-2)+.3} < \frac{3}{4}$. Neither agent chooses to attack. For each period up through Period X we have $B_1 \subset \Omega^P$, so that the relevant probability upon which each agent bases his behavior remains unchanged. However, after A chooses to not attack in Period X, $\Omega^P = A_1 \cup A_X$. Consequently, $P_B(f_B|B_1 \cap \Omega^{X+1}) = \frac{3}{4} + \frac{.45}{X-2} > \frac{3}{4}$, prompting B to attack in Period $X+1$. Since X can be arbitrarily large, this example illustrates how B might eventually choose to stage an attack after arbitrarily many periods of tranquility. Recognize that since both f_A and f_B are possible within $A_1 = [0, .2]$, we have that B may either regret or not regret initiating this attack.

Next, to see that A might choose to initiate conflict after an arbitrarily large number of periods of tranquility, consider $\omega \in A_2 \cap B_1 = \left(.2, .2 + \frac{.3}{X-2}\right]$. At the start of Period 1, A knows $\omega \in A_2$ and B knows $\omega \in B_1$. Thus, $P_A(f_B|A_2 \cap \Omega^1) = \frac{1}{2}$ and $P_B(f_B|B_1 \cap \Omega^1) = \frac{.15(X-2)+.15}{.2(X-2)+.3} < \frac{3}{4}$. Neither agent chooses to attack. Up through Period $X-1$ we have $A_2 \cup B_1 \subset \Omega^P$, so that the relevant probability upon which each agent bases his behavior remains unchanged. However, after B chooses to not attack in Period $X-1$, $\Omega^P = B_1 \cup A_X$. Accordingly, $P_A(f_B|A_2 \cap \Omega^X) = \frac{1}{5}$, prompting A to attack in Period X. Since X can be arbitrarily large, this example illustrates how A might eventually choose to stage an attack after arbitrarily many periods of tranquility. Again, since both f_B and f_A are possible within $\omega \in A_2 \cap B_1 = (.2, .2 + \frac{.3}{X-2}]$, we have that A may either regret or not regret initiating this attack.

Finally, to see that this general example can lead to indefinite stability, consider $\omega \in A_X = (.8,1]$. In Period 1, A knows $\omega \in A_X$ and B knows $\omega \in B_{X-1}$. Consequently, $P_A(f_B|A_X \cap \Omega^1) = \frac{1}{2}$ and $P_B(f_B|B_{X-1} \cap \Omega^1) = \frac{X-2}{2X-1} < \frac{3}{4}$. For these probabilities, neither agent chooses to attack. After A chooses not to attack in Period 1, B knows $\omega \in A_X$ so that $P_B(f_B|B_{X-1} \cap \Omega^2) = \frac{1}{2}$. More precisely, $P_A(f_B|A_X \cap \Omega^t) = P_B(f_B|B_{X-1} \cap \Omega^t) = \frac{1}{2}$ for every $t = 2,3, \ldots$. After Period 1, $\Omega^P = A_X$. The common knowledge information set is not further reduced, and each agent chooses to not attack in every future period. Indefinite stability results.

5. Concluding Remarks

The model presented and analyzed here illustrates how it is possible for an agent to rationally choose to initiate conflict without instigation or provocation, without observing a change in the behavior of his rival, or without acquiring any new information about the type of his rival. Rather, conflict may result when rational decision-makers *endogenously* process information – that is, without any exogenous changes to the fundamentals of the environment.

The forces at play are identical to those identified by Hart and Tauman (2004) in a trading environment. But, the model itself and the ultimate results are qualitatively different. In the present model the state space is continuous, not discrete. Further, because of the negative constant sum nature of the payoffs realized when an attack occurs, the probabilities which serve as behavioral cutoffs for the two agents can differ in value from one another. Consequently, it is possible for the system to reach an equilibrium in which an attack is never initiated. The parallel outcome in Hart and Tauman would be for the market to never collapse (that is, never realize a crash or a bubble), but this cannot occur since the traders are symmetric and have the same common cutoff dictating whether they want to buy or sell.

In the present model, as beliefs converge, we ultimately have: (i) B eventually initiating an attack without regret; (ii) B eventually initiating an attack with regret; (iii) A eventually initiating an attack without regret; (iv) A eventually initiating an attack with regret; or (v) neither agent choosing to initiate an attack (indefinite stability). It is important to note that we can realize all five of these different outcome for a single example, depending upon what the actual true state of the world happens to be. However, this does not imply that the indefinite avoidance of conflict can ever be expected, even if the true state of the world at the onset is $\omega \in A_5 = (.8,1]$. After all, overtime there clearly could be exogenous changes to the system (e.g., changes of types, changes of cost/benefits which lead to changes in cutoff probabilities, changes in fundamental information partitions) which lead to rational agents choosing to initiate an attack.

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FIGURE 1

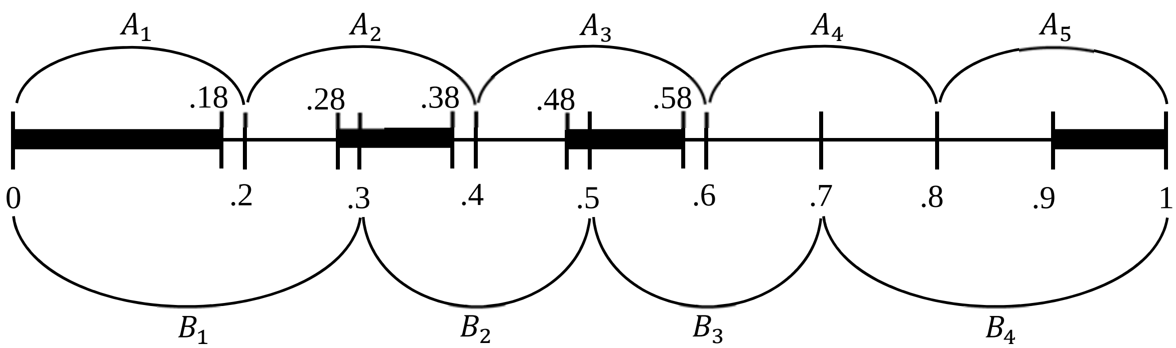

TABLE 1

Period	Ω^p	$\widetilde{A}_1$	A_2	A_3	A_4	A_5	Action	$\widetilde{B}_1$	B_2	B_3	B_4	Action
		Adversary A						Adversary B				
1	[0,1]	$\frac{9}{10}$	$\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{1}{2}$	wait	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{2}{5}$	$\frac{1}{3}$	wait
2	$[0,.6] \cup (.8,1]$	$\frac{9}{10}$	$\frac{1}{2}$	$\frac{1}{2}$		$\frac{1}{2}$	wait	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{4}{5}$	$\frac{1}{2}$	wait
3	$[0,.5] \cup (.8,1]$	$\frac{9}{10}$	$\frac{1}{2}$	$\frac{1}{5}$		$\frac{1}{2}$	wait	$\frac{2}{3}$	$\frac{1}{2}$		$\frac{1}{2}$	wait
4	$[0,.4] \cup (.8,1]$	$\frac{9}{10}$	$\frac{1}{2}$			$\frac{1}{2}$	wait	$\frac{2}{3}$	$\frac{4}{5}$		$\frac{1}{2}$	wait
5	$[0,.3] \cup (.8,1]$	$\frac{9}{10}$	$\frac{1}{5}$			$\frac{1}{2}$	wait	$\frac{2}{3}$			$\frac{1}{2}$	wait
6	$[0,.2] \cup (.8,1]$	$\frac{9}{10}$				$\frac{1}{2}$	wait	$\frac{9}{10}$			$\frac{1}{2}$	attack

TABLE 2

Period	Ω^p	A_1	A_2	$\widetilde{A}_3$	A_4	A_5	Action	B_1	$\widetilde{B}_2$	B_3	B_4	Action
		Adversary A						Adversary B				
1	[0,1]	$\frac{9}{10}$	$\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{1}{2}$	wait	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{2}{5}$	$\frac{1}{3}$	wait
2	$[0,.6] \cup (.8,1]$	$\frac{9}{10}$	$\frac{1}{2}$	$\frac{1}{2}$		$\frac{1}{2}$	wait	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{4}{5}$	$\frac{1}{2}$	wait
3	$[0,.5] \cup (.8,1]$	$\frac{9}{10}$	$\frac{1}{2}$	$\frac{1}{5}$		$\frac{1}{2}$	attack	$\frac{2}{3}$	$\frac{1}{2}$		$\frac{1}{2}$	wait

TABLE 3

Period	Ω^p	Adversary A						Adversary B				
		A_1	A_2	A_3	A_4	$\overbrace{A_5}$	Action	B_1	B_2	B_3	$\overbrace{B_4}$	Action
1	$[0,1]$	$\frac{9}{10}$	$\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{1}{2}$	wait	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{2}{5}$	$\frac{1}{3}$	wait
2	$[0,.6] \cup (.8,1]$	$\frac{9}{10}$	$\frac{1}{2}$	$\frac{1}{2}$		$\frac{1}{2}$	wait	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{4}{5}$	$\frac{1}{2}$	wait
3	$[0,.5] \cup (.8,1]$	$\frac{9}{10}$	$\frac{1}{2}$	$\frac{1}{5}$		$\frac{1}{2}$	wait	$\frac{2}{3}$	$\frac{1}{2}$		$\frac{1}{2}$	wait
4	$[0,.4] \cup (.8,1]$	$\frac{9}{10}$	$\frac{1}{2}$			$\frac{1}{2}$	wait	$\frac{2}{3}$	$\frac{4}{5}$		$\frac{1}{2}$	wait
5	$[0,.3] \cup (.8,1]$	$\frac{9}{10}$	$\frac{1}{5}$			$\frac{1}{2}$	wait	$\frac{2}{3}$			$\frac{1}{2}$	wait
6	$[0,.2] \cup (.8,1]$	$\frac{4}{5}$				$\frac{1}{2}$	wait	$\frac{4}{5}$			$\frac{1}{2}$	wait
7	$(.8,1]$					$\frac{1}{2}$	wait				$\frac{1}{2}$	wait
⋮	⋮					⋮	⋮				⋮	⋮

TABLE 4

Period	Ω^p	Adversary A				Adversary B			
		A_1	A_j	A_{X-1}	A_X	B_1	B_j	B_{X-2}	B_{X-1}
1	$[0,1]$	$\frac{3}{4}+\frac{.45}{X-2}$	$\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{.15(X-2)+.15}{.2(X-2)+.3}$	$\frac{1}{2}$	$\frac{2}{5}$	$\frac{X-2}{2X-1}$
			$j = 2, \dots X-2$				$j = 2, \dots X-3$		