A Theory of Rivalry with Endogenous Strength

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Keywords: Rivalry; Domino Effect; Avalanche Effect
JEL Classification Numbers: C72; D72; D74

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Abstract

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1 Introduction

Rivalries can take place in different forms but all of them share similar characteristics. In this paper, "*rivalry*" is defined as competition between two players or teams that comprises repeating meetings. The assumption has been that the longer a rivalry lasts, the more intense it becomes with each anticipated meeting. Although players play their best to win in a rivalry, they often share common goals. In a sports rivalry, a common goal may be to raise the general popularity and awareness of the game, whereas in a school rivalry, a common goal may be to increase the average quality of the students around the area or to attract more state or federal funding. The presence of the common goal warrants an analysis of either noncooperative or cooperative games. The current study approaches the topic with a noncooperative setting.[1]

Based on the two-period model proposed by Beviá and Corchón (2013), we consider a three-period model that is ideal for the study of rivalries. Our model deviates from Beviá and Corchón's (2013) in two important aspects. First, rather than given exogenously as in most contest models, the contestable prize is determined endogenously by the efforts of the two players in our model. In this case, if the two players dedicate more resources to the rivalry, less resources will be used for productive activities. Such an assumption is common in the conflict literature (Garfinkel and Skaperdas, 2000; Chang et al., 2007; Garfinkel and Skaperdas, 2007; Chang and Luo, 2013, 2016). For example, in a school rivalry, two schools that are involved may put more resource in time and money into recruitment of better incoming students but will work less to nurture current students and the overall cultural environment of the area.

Second, our baseline model has three periods. A three-period model allows for the analysis of not only how the relative effort levels of the two players in the later period were affected by the relative strength in the earlier period, as analyzed in Beviá and Corchón (2013), but also how relative strengths feed back to effort levels and ultimately affect the outcome of the rivalry; we not

[1] Our definition of rivalry is similar to war of attrition in the literature (Fudenberg and Tirole, 1986; Bulow and Klemperer, 1999). While one may consider rivalry as the case of war of attrition when exit is not an option, we will show in our analytical results that the expected payoffs of the two players increase over time. This contrasts with the assumption of the war of attrition game.

only study the endogeneity of relative strength but also its dynamic.

While it may be instructive to review literature on contest and conflict,[2] studies that focus on sequential election and campaigning are especially relevant (Snyder, 1989; Klumpp and Polborn, 2006). In their second setting, Klumpp and Polborn (2006) consider the possibility that one candidate is a more effective campaigner. Although the effectiveness of the campaigner is not explicitly modeled as endogenous or assumed to be a function of previous campaign efforts or electoral outcomes, it is inevitable that the effectiveness of the campaigners affects the outcome of the overall campaign, channeled through its own effect as well as campaign efforts.

This paper is organized as follows: The next section presents the setup and results of the three period model, section 3 focuses on the analysis of three effects that describe the dynamics of the model, and the conclusion is in the last section.

2 The Model

There are two Players, X and Y, and each is endowed with resource R_t in the beginning of period t. In each period, the two players allocate x_t and y_t, respectively, into fighting. We follow the literature (Garfinkel and Skaperdas, 2000; Chang and Luo, 2013) and assume that the size of the contestable prize at period t is $2R_t - x_t - y_t$.

Let p_t denote the winning probability of Player X at time t. We assume

$$p_t = \frac{\alpha_t x_t}{\alpha_t x_t + (1 - \alpha_t) y_t}, \tag{1}$$

where α_t denotes the endogenous strength of X at time t. Symmetrically, the winning probability of Player Y at time t is $1 - p_t$. According to Beviá and Corchón (2013), the relative strength at

[2]For contest models and references, see Konrad (2009). For conflict models and references, see Garfinkel and Skaperdas (2007).

time $t(> 1)$ is assumed to be a linear function of the winning probability at time $t - 1$ such that:

$$\alpha_t = f(p_{t-1}) = cp_{t-1} + d \tag{2}$$

with $f(1/2) = 1/2$, $0 < c \leq 1$, $0 \leq d \leq 1$, and $0 \leq c + d \leq 1$. Note that $f(1/2) = 1/2$ implies $d = \frac{1-c}{2}$, a condition that we assume to hold throughout this paper. Under these assumptions, the expected payoffs of the two players at time t, for a total of periods up to T, are given by

$$\pi_t^x = \sum_{i=t}^{T} \delta^{i-t} \frac{\alpha_i x_i}{\alpha_i x_i + (1 - \alpha_i) y_i} (2R_i - x_i - y_i); \tag{3a}$$

$$\pi_t^y = \sum_{i=t}^{T} \delta^{i-t} \frac{(1 - \alpha_i) y_i}{\alpha_i x_i + (1 - \alpha_i) y_i} (2R_i - x_i - y_i), \tag{3b}$$

where π_t^x and π_t^y denote, respectively, expected payoffs of Player X and Y at time t, and δ is the discount factor. Since we only consider $T = 3$, i.e., a game up to three periods, when the two players make decisions in the first period, they take into account the consequence in subsequent periods–especially how their efforts (x_t and y_t) may affect their future relative strength, winning probabilities, and payoffs. The game is solved backwardly starting with period 3.

2.1 Period 3

According to equations (3), the payoffs of the two players in period 3 are given by

$$\pi_3^x = \frac{\alpha_3 x_3}{\alpha_3 x_3 + (1 - \alpha_3) y_3} (2R_3 - x_3 - y_3);$$
$$\pi_3^y = \frac{(1 - \alpha_3) y_3}{\alpha_3 x_3 + (1 - \alpha_3) y_3} (2R_3 - x_3 - y_3).$$

The first order conditions (FOCs) of period 3 are

$$\frac{\partial \pi_3^x}{\partial x_3} = \frac{\alpha_3(1-\alpha_3)y_3}{[\alpha_3 x_3 + (1-\alpha_3)y_3]^2}(2R_3 - x_3 - y_3) - \frac{\alpha_3 x_3}{\alpha_3 x_3 + (1-\alpha_3)y_3} = 0; \tag{4a}$$

$$\frac{\partial \pi_3^y}{\partial y_3} = \frac{\alpha_3(1-\alpha_3)x_3}{[\alpha_3 x_3 + (1-\alpha_3)y_3]^2}(2R_3 - x_3 - y_3) - \frac{(1-\alpha_3) y_3}{\alpha_3 x_3 + (1-\alpha_3)y_3} = 0. \tag{4b}$$

with solutions

$$x_3 = \frac{\alpha_3 - 1 + \sqrt{\alpha_3(1-\alpha_3)}}{2\alpha_3 - 1} R_3; \tag{5a}$$

$$y_3 = \frac{\alpha_3 - \sqrt{\alpha_3(1-\alpha_3)}}{2\alpha_3 - 1} R_3. \tag{5b}$$

We note the following results from the solutions:

$$x_3 + y_3 = R_3; \tag{6a}$$

$$\frac{x_3}{y_3} = \left(\frac{1-\alpha_3}{\alpha_3}\right)^{\frac{1}{2}}; \tag{6b}$$

$$p_3 = \frac{\alpha_3\left(\frac{\alpha_3 - 1 + \sqrt{\alpha_3(1-\alpha_3)}}{2\alpha_3 - 1}\right)}{\alpha_3\left(\frac{\alpha_3 - 1 + \sqrt{\alpha_3(1-\alpha_3)}}{2\alpha_3 - 1}\right) + (1-\alpha_3)\left(\frac{\alpha_3 - \sqrt{\alpha_3(1-\alpha_3)}}{2\alpha_3 - 1}\right)} = \frac{\alpha_3 - \sqrt{\alpha_3(1-\alpha_3)}}{2\alpha_3 - 1}. \tag{6c}$$

Equation (6a) indicates that the two players (combined) expensed half of their total endowment, a common result found in the contest and conflict literature. Given (6c), equations (5) can be rewritten into

$$x_3 = (1-p_3)R_3; \tag{7a}$$

$$y_3 = p_3 R_3. \tag{7b}$$

A result to note is that the stronger player, as measured by α_3, has a winning probability (p_3) that is lower than his or her relative strength and vice versa. To see this result, the following conditions

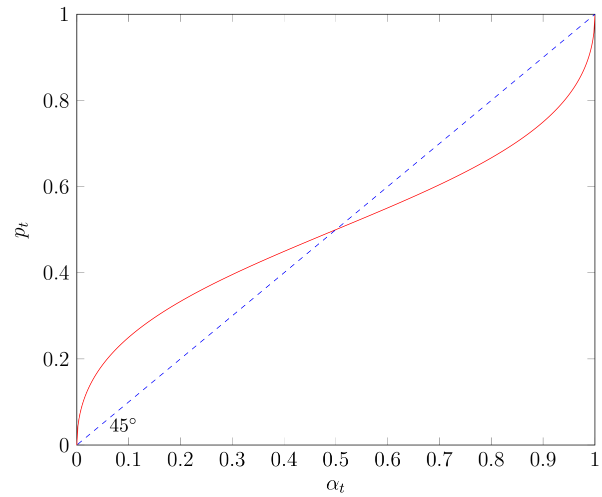

Figure 1: Plots of p_t and α_t

can be derived from a closer examination of equation (6c):

$$\begin{cases} 1/2 < p_3 < \alpha_3 \text{ iff } \alpha_3 > 1/2 \\ p_3 = \alpha_3 = 1/2 \text{ iff } \alpha_3 = 1/2 \\ 1/2 > p_3 > \alpha_3 \text{ iff } \alpha_3 < 1/2 \end{cases} .$$

The above results are also illustrated in Figure 1. In this configuration, the weaker player compensates for a disadvantage in relative strength by exerting a greater effort although the player can not reverse the adverse effect of having the innate lower strength. This result can also be seen from (6b); that the player who has higher relatively strength exerts relatively lower level of efforts (and vice versa), since relative effort allocation is a function of the reciprocal of the relative strength. As will be shown in the next few sections, similar results hold throughout the game. It should also be noted that the shape of the curve in Figure 1, i.e., first concave, then convex with the inflection point at $1/2$), between α_t and p_t will dominate the relationship between other relevant pairs of variables such as between p_t and p_{t-1} or between α_t and α_{t-1}.

2.2 Period 2

According to equations (3), the payoffs of the two players in period 2 are given by

$$\begin{aligned}
\pi_2^x &= p_2(2R_2 - x_2 - y_2) + \delta p_3 R_3 \\
&= \frac{\alpha_2 x_2}{\alpha_2 x_2 + (1-\alpha_2) y_2}(2R_2 - x_2 - y_2) + \delta \frac{\alpha_3 x_3}{\alpha_3 x_3 + (1-\alpha_3) y_3} R_3; \\
\pi_2^y &= (1-p_2)(2R_2 - x_2 - y_2) + \delta (1-p_3) R_3 \\
&= \frac{(1-\alpha_2) y_2}{\alpha_2 x_2 + (1-\alpha_2) y_2}(2R_2 - x_2 - y_2) + \delta \frac{(1-\alpha_3) y_3}{\alpha_3 x_3 + (1-\alpha_3) y_3} R_3;
\end{aligned}$$

where p_3 is a function of α_3, which is in turn a function of p_2 according to equation (2), and ultimately a function of x_2 and y_2. Plugging in the solution of period 3 from equations (5), it can be shown that

$$\frac{\partial p_3}{\partial p_2} = \frac{c}{(2d + 2cp_2 - 1)^2}\left(\frac{1}{(d + cp_2)^{\frac{1}{2}}(1 - d - cp_2)^{\frac{1}{2}}} - 1\right) > 0. \tag{8}$$

Equation (8) is positive but less than 1. A more general form of equation (8) is shown by equation (16) in Section 3.1.

From the period 2 payoffs, the FOCs are

$$\frac{\partial \pi_2^x}{\partial x_2} = \frac{\alpha_2(1-\alpha_2) y_2}{[\alpha_2 x_2 + (1-\alpha_2) y_2]^2}\left[(2R_2 - x_2 - y_2) + \delta R_3 \frac{\partial p_3}{\partial p_2}\right] - \frac{\alpha_2 x_2}{\alpha_2 x_2 + (1-\alpha_2) y_2} = 0; \tag{9a}$$

$$\frac{\partial \pi_2^y}{\partial y_2} = \frac{\alpha_2(1-\alpha_2) x_2}{[\alpha_2 x_2 + (1-\alpha_2) y_2]^2}\left[(2R_2 - x_2 - y_2) + \delta R_3 \frac{\partial p_3}{\partial p_2}\right] - \frac{(1-\alpha_2) y_2}{\alpha_2 x_2 + (1-\alpha_2) y_2} = 0. \tag{9b}$$

Equations (9) implicitly define the solutions of x_2 and y_2. Note that according to equation (8), $\frac{\partial p_3}{\partial p_2}$ is a function of p_2. Assuming $\frac{\partial p_3}{\partial p_2}$ is not a function of either x_2 or y_2 in equilibrium,[3] we solve

[3] This assumption will be verified, and shown to be true in another section.

for x_2 and y_2 from equations (9) to get

$$x_2 = \frac{\alpha_2 - 1 + \sqrt{\alpha_2(1-\alpha_2)}}{2\alpha_2 - 1}\left(R_2 + \frac{1}{2}\delta R_3 \frac{\partial p_3}{\partial p_2}\right); \tag{10a}$$

$$y_2 = \frac{\alpha_2 - \sqrt{\alpha_2(1-\alpha_2)}}{2\alpha_2 - 1}\left(R_2 + \frac{1}{2}\delta R_3 \frac{\partial p_3}{\partial p_2}\right). \tag{10b}$$

We note the following results from the above solution:

$$x_2 + y_2 = \left(R_2 + \frac{1}{2}\delta R_3 \frac{\partial p_3}{\partial p_2}\right); \tag{11a}$$

$$\frac{x_2}{y_2} = \left(\frac{1-\alpha_2}{\alpha_2}\right)^{\frac{1}{2}}; \tag{11b}$$

$$p_2 = \frac{\alpha_2 - \sqrt{\alpha_2(1-\alpha_2)}}{2\alpha_2 - 1}. \tag{11c}$$

Similar to the results from period 3, equation (11a) indicates that the total expenses of the two players are equal to half of the total endowment of the current period plus some from the next period (period 3). Equation (11b) again shows that relative effort allocation is a function of the reciprocal of the relative strength. Finally, equation (11c) confirms that in equilibrium, p_2 is a function of only α_2 but neither x_2 nor y_2. In other words, solutions in equations (10) can be expressed as functions of only α_2.

2.3 Period 1

According to equations (3), the payoffs of the two players in period 1 are given by

$$\pi_1^x = p_1(2R_1 - x_1 - y_1) + \delta p_2\left(R_2 + \frac{1}{2}\delta R_3 \frac{\partial p_3}{\partial p_2}\right) + \delta^2 p_3 R_3;$$
$$\pi_1^x = (1-p_1)(2R_1 - x_1 - y_1) + \delta(1-p_2)\left(R_2 + \frac{1}{2}\delta R_3 \frac{\partial p_3}{\partial p_2}\right) + \delta^2(1-p_3)R_3.$$

Similar to Period 2, we note that p_2 is a function of p_1 which is in turn a function of x_1 and y_1.

The FOCs in Period 1 are

$$\frac{\partial \pi_1^x}{\partial x_1} = \frac{\alpha_1(1-\alpha_1)y_1}{\left[\alpha_1 x_1 + (1-\alpha_1)y_1\right]^2}\left[(2R_1 - x_1 - y_1) + \delta R_2 \frac{\partial p_2}{\partial p_1} + \frac{1}{2}\delta^2 R_3 \frac{\partial p_3}{\partial p_2}\frac{\partial p_2}{\partial p_1}\right] - \frac{\alpha_1 x_1}{\alpha_1 x_1 + (1-\alpha_1)y_1} = 0; \tag{12a}$$

$$\frac{\partial \pi_1^y}{\partial y_1} = \frac{\alpha_1(1-\alpha_1)x_1}{\left[\alpha_1 x_1 + (1-\alpha_1)y_1\right]^2}\left[(2R_1 - x_1 - y_1) + \delta R_2 \frac{\partial p_2}{\partial p_1} + \frac{1}{2}\delta^2 R_3 \frac{\partial p_3}{\partial p_2}\frac{\partial p_2}{\partial p_1}\right] - \frac{(1-\alpha_1)y_1}{\alpha_1 x_1 + (1-\alpha_1)y_1} = 0. \tag{12b}$$

The solutions are:

$$x_1 = \frac{\alpha_1 - 1 + \sqrt{\alpha_1(1-\alpha_1)}}{2\alpha_1 - 1}\left(R_1 + \frac{1}{2}\delta R_2 \frac{\partial p_2}{\partial p_1} + \frac{1}{4}\delta^2 R_3 \frac{\partial p_3}{\partial p_2}\frac{\partial p_2}{\partial p_1}\right); \tag{13a}$$

$$y_1 = \frac{\alpha_1 - \sqrt{\alpha_1(1-\alpha_1)}}{2\alpha_1 - 1}\left(R_1 + \frac{1}{2}\delta R_2 \frac{\partial p_2}{\partial p_1} + \frac{1}{4}\delta^2 R_3 \frac{\partial p_3}{\partial p_2}\frac{\partial p_2}{\partial p_1}\right). \tag{13b}$$

Once again, we note the following results from the solutions:

$$x_1 + y_1 = \left(R_1 + \frac{1}{2}\delta R_2 \frac{\partial p_2}{\partial p_1} + \frac{1}{4}\delta^2 R_3 \frac{\partial p_3}{\partial p_2}\frac{\partial p_2}{\partial p_1}\right); \tag{14a}$$

$$\frac{x_1}{y_1} = \left(\frac{1-\alpha_1}{\alpha_1}\right)^{\frac{1}{2}}; \tag{14b}$$

$$p_1 = \frac{\alpha_1 - \sqrt{\alpha_1(1-\alpha_1)}}{2\alpha_1 - 1}. \tag{14c}$$

These results are similar to those found in period 2 and 3. In fact, the solutions to the three-period model have enough "structure" that it can easily be extended to a finite N period model. For any period i in a model with finite time horizon, without further interpretation, we note the following results:

$$
\begin{aligned}
x_i &= (1-p_i)\left\{R_i + \sum_{j=i+1}^{N}\left[\frac{1}{2^{j-1}}\left(\prod_{k=i}^{j}\frac{\partial p_{k+1}}{\partial p_k}\right)\delta^{j-1}R_j\right]\right\};\\
y_i &= p_i\left\{R_i + \sum_{j=i+1}^{N}\left[\frac{1}{2^{j-1}}\left(\prod_{k=i}^{j}\frac{\partial p_{k+1}}{\partial p_k}\right)\delta^{j-1}R_j\right]\right\};\\
x_i + y_i &= R_i + \sum_{j=i+1}^{N}\left[\frac{1}{2^{j-1}}\left(\prod_{k=i}^{j}\frac{\partial p_{k+1}}{\partial p_k}\right)\delta^{j-1}R_j\right];\\
\frac{x_i}{y_i} &= \frac{1-p_i}{p_i} = \left(\frac{1-\alpha_i}{\alpha_i}\right)^{\frac{1}{2}};\\
p_i &= \frac{\alpha_i - \sqrt{\alpha_i(1-\alpha_i)}}{2\alpha_i - 1}.
\end{aligned}
$$

While the next section will fully examine the evolutionary pattern of several variables in the model, it is worth noting that from equations (6a), (11a), and (14a) that

$$
\frac{\partial\left(x_t + y_t\right)}{\partial t} < 0,
$$

assuming $R_1 = R_2 = R_3$; the two parties collectively allocate less resources into fighting, which results in higher expected payoffs as the periods progress.

3 Dynamic Effects

This section examines three dynamic effects that are relevant to our model. We first look at the domino effect and the avalanche effect. These two effects are examined together due to their similarities in the current model. The domino effect refers to the trajectory of the winning probability p_t while the avalanche effect refers to the trajectory of the relative strength α_t. We then move onto the discouragement effect, which refers to whether the weak player has the incentive to exert more or less effort earlier rather than later.

3.1 Domino and Avalanche Effects

For the domino effect, on p_t, we find from (6c) that

$$p_t = \frac{\alpha_t - \sqrt{\alpha_t(1-\alpha_t)}}{2\alpha_t - 1}. \tag{15}$$

Because $\frac{\partial \alpha_t}{\partial p_{t-1}} = c$ according to equation (2), we get

$$\frac{\partial p_t}{\partial p_{t-1}} = -\frac{1}{2}\frac{c}{\sqrt{\alpha_t(1-\alpha_t)}}\frac{2\sqrt{\alpha_t(1-\alpha_t)}-1}{(2\alpha_t-1)^2} \geq 0, \tag{16}$$

where the inequality sign follows from $0 \leq \sqrt{\alpha_t(1-\alpha_t)} \leq \frac{1}{2}$.

Similarly, for the avalanche effect on α_t, we find from equations (2) and (6c) that

$$\alpha_{t+1} = c\left(\frac{\alpha_t - \sqrt{\alpha_t(1-\alpha_t)}}{2\alpha_t - 1}\right) + \frac{1-c}{2}, \tag{17}$$

which implies $\alpha_{t+1} = \frac{1-c}{2} \geq 0$ when $\alpha_t = 0$. Taking the derivative of α_{t+1} with respect to α_t, we have

$$\frac{\partial \alpha_{t+1}}{\partial \alpha_t} = -\frac{1}{2}\frac{c}{\sqrt{\alpha_t(1-\alpha_t)}}\frac{2\sqrt{\alpha_t(1-\alpha_t)}-1}{(2\alpha_t-1)^2} \geq 0, \tag{18}$$

where the inequality sign follows again from $0 \leq \sqrt{\alpha_t(1-\alpha_t)} \leq \frac{1}{2}$.

Equations (16) and (18) prove that the winning probability and strength of period t, p_t and α_t respectively, are increasing functions of the winning probability and strength of the previous period. We need to further examine the curvature of p_t (as a function of p_{t-1}) and α_t (as a function of α_{t-1}) in order to answer the question of whether domino and/or avalanche effects exist. Taking the second derivatives gives

$$\frac{\partial^2 p_t}{\partial p_{t-1}^2} = \frac{1}{4}\frac{1}{(2\alpha_t - 1)^3}\frac{c^2}{\alpha_t(1-\alpha_t)\sqrt{\alpha_t(1-\alpha_t)}}\left(16\alpha_t(1-\alpha_t)\sqrt{\alpha_t(1-\alpha_t)} - 12\alpha_t(1-\alpha_t) + 1\right); \tag{19a}$$

$$\frac{\partial^2 \alpha_{t+1}}{\partial \alpha_t^2} = \frac{1}{4}\frac{1}{(2\alpha_t - 1)^3}\frac{c}{\alpha_t(1-\alpha_t)\sqrt{\alpha_t(1-\alpha_t)}}\left(16\alpha_t(1-\alpha_t)\sqrt{\alpha_t(1-\alpha_t)} - 12\alpha_t(1-\alpha_t) + 1\right); \tag{19b}$$

where

$$\begin{aligned}&16\alpha_t(1-\alpha_t)\sqrt{\alpha_t(1-\alpha_t)} - 12\alpha_t(1-\alpha_t) + 1 \\ &\qquad = \left(4\sqrt{\alpha_t(1-\alpha_t)} + 1\right)\left(2\sqrt{\alpha_t(1-\alpha_t)} - 1\right)^2 \geq 0.\end{aligned}$$

As a result, the signs of $\frac{\partial^2 p_t}{\partial p_{t-1}^2}$ and $\frac{\partial^2 \alpha_{t+1}}{\partial \alpha_t^2}$ depend on the sign of $\frac{1}{(2\alpha_t-1)^3}$, which is negative *iff* $\alpha < \frac{1}{2}$ and positive *iff* $\alpha > \frac{1}{2}$. Note that in a perfectly symmetrical world, p_t and α_t, for any t, are equal to $1/2$.

To illustrate, we plot p_t against p_{t+1} (left panel) and α_t against α_{t+1} (right panel) in Figure 2 with different values of c. The curvature in the graph is derived from equations (18) and (19). As shown in Figure 2, when the value of a variable, either p or α, is less than $1/2$ in the previous period, its value in the latter period is greater than the previous period although still below $1/2$. However, when the value of a variable is greater than $1/2$ in the previous period, its value in the latter period is less than the previous period although still above $1/2$; i.e., there is neither domino nor avalanche effect in our model. In fact, both p_t and α_t exhibit convergence over time. One possible explanation for the nonexistence of these two effects is that the weaker player overcompensates with effort and ends up with a higher winning probability after falling behind in the previous period (Figure 1 in Section 2.1).

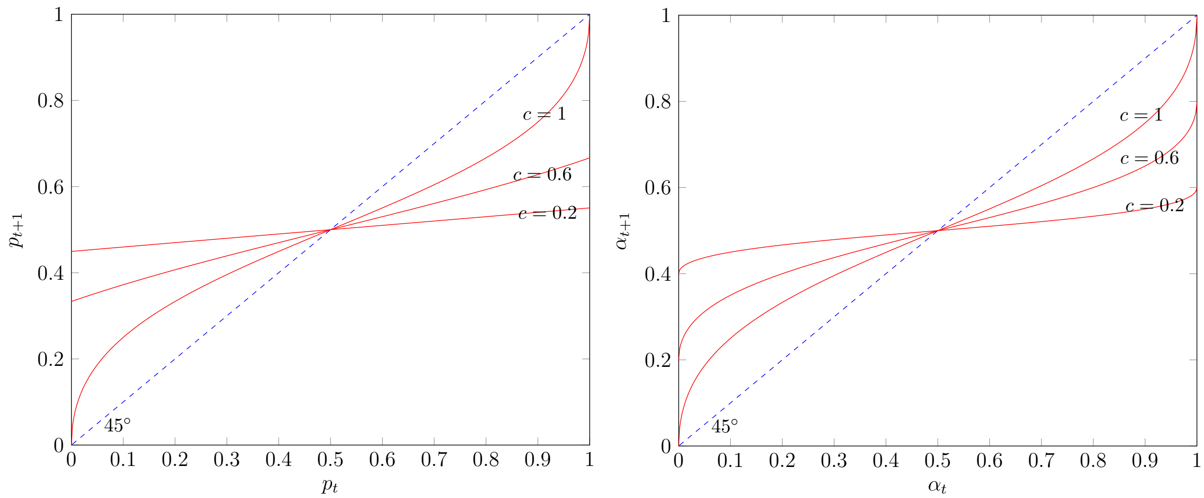

Figure 2: Plots of Domino and Avalanche Effects

3.2 The Discouragement Effect

According to the definition in Beviá and Corchón (2013), the *discouragement effect* refers to the phenomenon that the weaker player exerts less effort over time. To examine this effect, we assume $p_1 > 1/2$, i.e., with X being the stronger player. From equations (5b), (10b), and (13b), we have

$$\begin{aligned} y_3 &= p_3 R_3; \\ y_2 &= p_2 \left(R_2 + \frac{1}{2}\delta R_3 \frac{\partial p_3}{\partial p_2} \right); \\ y_1 &= p_1 \left(R_1 + \frac{1}{2}\delta R_2 \frac{\partial p_2}{\partial p_1} + \frac{1}{4}\delta^2 R_3 \frac{\partial p_3}{\partial p_2}\frac{\partial p_2}{\partial p_1} \right). \end{aligned}$$

We know from previous sections that the following results hold:

$$\frac{\partial p_3}{\partial p_2} \geq 0; \; \frac{\partial p_2}{\partial p_1} \geq 0; \; p_3 < p_2 < p_1 \text{ when } p_1 > \frac{1}{2}.$$

Assume $R_1 = R_2 = R_3$, it is straightforward to conclude that

$$y_3 < y_2 < y_1 \tag{20}$$

There exists a discouragement effect in accordance to Beviá and Corchón (2013); however, this should be interpreted with caution. As have established, when Y is the weaker player, we have $y_t > x_t$ for all t. Hence, both the weaker player's effort is declining over time, and so is that of the stronger player. Because there does not exist a domino or an avalanche effect, the two players will level off in both strength and winning probability over time. While the discouragement effect attempts to look at the change in the absolute effort levels of the weaker player, it is the relative levels that are more relevant in our model. From equations (6b), (11b), and (14b), we have

$$\frac{\partial \left(\frac{x_t}{y_t}\right)}{\partial t} < 0 \text{ when } p_1 > \frac{1}{2},$$

which indicates that the relative effort levels of the two parties will converge when Player X is the stronger player. In our multi-period model with endogenous prize and strength, despite a discouragement effect, players' relative strength and winning probability will converge toward $1/2$–the rivalry will become more balanced, but less intense, as it continues.

4 Conclusion

Rivalries could take place in the form of repeated meetings, and the outcome of each meeting relies on the circumstances of the previous one. This paper builds a three-period model for the study of rivalry with repeated meetings. Following Beviá and Corchón (2013), we assume that the relative strength of the players in a latter period depends on the results of the rivalry in the previous period. Unlike Beviá and Corchón (2013), we consider the contestable prize to be endogenous rather than exogenous, which is determined by the efforts of the players in the rivalry.

We find that the player who has a higher relative strength exerts a relatively lower level of efforts, and vice versa. This might be because the weaker player wants to compensate for his or her own disadvantage in relative strength by making more efforts to increase the winning probability. However, such endeavors will not result in the outcome such that the weaker player's winning probability exceeds that of the stronger player. As the rivalry lasts longer, we find that the players will collectively allocate less resources into fighting and will reduce their efforts from one period to the next.

By conducting a dynamic analysis, we further find that neither domino effect nor avalanche effect exits in our model. For the player whose relative strength and winning probability is lower than 0.5 in the earlier period, the values of these two variables will be higher in the latter period, and vice versa. In fact, both relative strength and winning probability are found to converge to 0.5 over time. One possible explanation for this finding is that the weaker player exerts more effort to overcome the situation of falling behind and thus yields higher winning probability, which is formally proved to be the case in the paper.

The model of our study relies on the assumption that the transition function is linear. However, this might not be true for some particular types of rivalry. It will be interesting to relax this assumption and explore the possible changes in the equilibrium outcomes.

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