Structures of Freedom and Rationality
On Theory of Choice
Abstract

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Rationality and irrationality, as the dual aspects of economic behaviors, have been two of fundamental paradigms in the evolution of economic theory, game theory, and other behavioral science. It is one of natural issues if there is a unified platform, as the theory of choice, in which both rationality and irrationality could be examined together. In this thesis, the theory of choice is explored in following two directions: the proposition of two alternative paradigms in which an economic being evaluates its options through employing a series of space transformations or behaves within its choice structure in terms of fixed points, and the examination of the structure of the economic behavior in which an economic being pursues as much the equality of its all opportunities, called freedom, with the perspective of the alternative paradigm.

I. Rationality: the Paradigms of Choice and its Structure

In this section, first we develop two paradigms for the theory of choice. Then, we examine the structure of choice with integration of rationality and irrationality.

1.1. The paradigm of choice through employing a series of space transformation.

For an economic being and its finite set of pure options S, following the notions of mixed strategies and correlated equilibria in game theory, we introduce its **set of statistical options**, or **option set** that consists of

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all probability distributions on S, denoted as Δ^s, where Δ^s is a standard simplex. If $U \subset S$, Δ^u is a face of Δ^s. The evaluation system of an economic being has is a *s-tuple* evaluation map $T = \{T^i\}^s$ that transforms its option set Δ^s to its evaluation image $T(\Delta^s)$ in the same Euclidean space where its standard simplex Δ^s is embedded.

Theorem 1. *An economic being $\{\Delta^s, T\}$ is (von Neumann and Morgenstern) rational if and only if its evaluation map T is affine map or its evaluation image $T(\Delta^s)$ is a simplex, i.e., $T : \Delta^s \to \Delta^s_T$.*

In this paradigm, an rational being $\{\Delta^s, T\}$ evaluates its options through employing the intersection of its evaluation image $T(\Delta^s)$ and a family of hyperplane that are parallel to its standard simplex Δ^s. Their intersection is equal to its von Neumann and Morgenstern expected utility $E(p, T) = p \cdot T(S) = \sum p(s^i)h(s^i)$.

1.2. The paradigm of choice through applying the theory of fixed point

A *s-tuple* function f from Δ^s to itself is **rational** if $f = \frac{T}{E(p,T)}$ and T is affine map. Following the Brouwer fixed-point theorem, a continuous map f and $f : \Delta^s \to \Delta^s$, its set of fixed points $F(S) = \{p \in \Delta^s | f(p) = p\}$ is not empty. $p \in F(S)$ is an **optimal fixed-point** if $E(p, T) \geq E(q, T)$, $\forall q \in F(S)$.

Theorem 2. *For a rational being $\{\Delta^s, T\}$, a option $p \in \Delta^s$ is its rational choice if and only if p is an optimal fixed-point of its rational map f.*

Theorem 3. ***[Structure of Fixed Points]***

The fixed-point set of a rational map from a simplex Δ^s to itself is a subset of its cluster, $F \subset \partial\Delta^s$. For $U \subset S$, if $T^i(s^i) = \alpha \times s^i$, $\forall s^i \in U$ and α is a constant,$T(\Delta^u) = \Delta^u_\alpha \in F$.

1.3. The choice structure with integration of rationality and irrationality

With the paradigm of theory of choice, let's explore the complexity of economic behaviors. For an rational being $\{\Delta^s, T\}$, its rational choice is its best optimal one, which is the face of Δ^s_T, denoted as $\Delta^1_T \in \partial\Delta^s$; its second rational choice is its second best optimal one Δ^s_T, or is its best optimal at the image without rational face, Δ^{s-1}_T. Its **sub-rational choices** includes all *n th*-rational choices which are its *n th* best. The last choice is also called as its **anti-rational choice**. For a rational being, its **choice structure** consists of all its choices and its order.

Theorem 4. *For a rational being $\{\Delta^s, T\}$, a option $p \in \Delta^s$ is its choice if and only if p is a fixed point of its rational map f.*

Theorem 5. ***[Structure of Rational Choice and Spectrum of Rationality]*** *For a rational being $\{\Delta^s, T\}$, its structures of choice and fixed-point coincide. The structure of rational choice could be represented by a discreet spectrum in terms of utility, called the spectrum of rationality, if its preference is not trivial; otherwise, it is a point.*

II. Freedom: its Structure and Duality

2.1. Concept and measures of freedom

An economic being is **free or in freedom** if its choice tends to as much the equality of its all option or a uniform distribution on its pure options (Liu (2015)). Following the Kullback-Leibler divergence, freedom could be measured in terms of entropy. For an economic being, its free evaluation system T is a combination of freedom measure Q and rationality measure H by a rational degree λ, i.e., $T = Q + \lambda H = \{T^i\}^s$, where $T^i = -p(s^i)ln(p(s^i)) + \lambda h(s^i)p(s^i)$. In general, its evaluation imagine, $T(\Delta^s)$, is not a simplex in its original Euclidean space.

2.2. The choice structure with freedom

For an economic being in freedom, its set of choices is its set of fixed points of $f = \frac{T}{E(p,T)}$ where T is its free evaluation system.

Theorem 6. ***[Structure of Free Choice, Spectrum of Freedom]*** *If $\lambda \neq \infty$, there is unique its optimal choice with freedom, which is a fixed point. The optimal free choice is out of its rational set of fixed points. And its spectrum of freedom has one more point than its spectrum of rationality.*

2.3. The duality between freedom and rationality

For an economic being $\{\Delta^s, T\}$, its structure of choice is its structure of fixed points: set of fixed points and its order. Two economic beings are duality if and only if they have the same structure of choices.

Theorem 7. ***Duality of Freedom and Rationality*** *For a free being, there exists its rational duality, i.e., a rational being who has the same structure of choice; and vice versa.*

If an economic being is perfect free, i.e., $\lambda = 0$, its unique choice is its equality of option, the uniform distribution on its pure options.

For a perfect free being, its rational duality is uniquely the origin of choice, i.e, 0-dimensional simplex.

Employing the paradigm developed in this thesis, we examine the concept of freedom of choice, which measures sets of opportunities. For a pair of sets of options, $A = \{x\}$ and $B = \{x, y\}$, where x, y are two options, and $y \neq x$, according to the duality theorem of freedom and rationality, we could show that B is more free than A.

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