Extended Abstract for "Ambiguous Persuasion"

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Introduction

Imagine a government that wants to discourage a consumer from initiating a potentially harmful activity like smoking. The government could commission expert studies that demonstrate the effect smoking has on the smoker's well-being. If the potential smoker were Bayesian, he would form a probabilistic belief about the costs and benefits of smoking and decide accordingly. As a result, the consumer may refrain from smoking or he may decide to start smoking after all, based on the information revealed and his resulting belief. Kamenica and Gentzkow (2011) provide a framework to study such an environment where the government and the consumer are assumed to be Bayesian and expected utility (EU) maximizers.

Rational agents, in a broader sense, need not be expected utility maximizers. In decision theory, a well-known generalization of expected utility paradigm is ambiguity aversion. An ambiguity-averse agent might not hold a unique belief about uncertain states and display a preference for hedging.[1] Besides its appealing rational foundations, numerous experiments have supported the prevalence of ambiguity aversion in decisions under uncertainty.[2]

In this paper, we consider ambiguity-averse agents with maxmin expected utility (MEU, Gilboa and Schmeidler (1989)) and ask the following questions. If the potential smoker has maxmin expected utility (MEU, Gilboa and Schmeidler (1989)), could the government be more successful in achieving its goal? Furthermore, if not being an expected utility maximizer could be viewed as a "weakness" which is exploitable by the government, could potential smoker still benefit from his own weakness?

Apparently, the government could strictly benefit from sending an ambiguous signal if the consumer is ambiguity averse. To illustrate, suppose the consumer might experience uncertain

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[1]See Gilboa and Marinacci (2013) for a survey of the vast literature of axiomatic foundations for ambiguity-averse preferences.

[2]For instance, see Fox and Tversky (1995), Chow and Sarin (2001), Halevy (2007), Bossaerts et al. (2010), Abdellaoui et al. (2011).

utilities from smoking, $\omega \in \{0,1\}$. The cost (market price) of a pack of cigarette is c. The consumer (receiver) could choose two actions: $a = 0$ (no smoking) or $a = 1$ (smoking). The consumer has utility $u(\omega, 1) = \omega - c$ and $u(\omega, 0) = 0$. The government (sender)'s utility is $v(\omega, 1) = \omega - c - \alpha$ and $v(\omega, 0) = 0$ for some large $\alpha > 0$. Here $\alpha > 0$ describes the government's bias against smoking, for example arising from the negative externality of smoking. Assume α is very large so the government always prefers no smoking. The consumer and the government share some common prior $p(\omega = 1) = p$. Suppose $\alpha + c >> 1 > p > c > 0$, so a priori, the consumer strictly prefers smoking.

We first calculate the optimal singal government could send in the standard Bayesian persuasion framework. By Proposition 4 and 5 in Kamenica and Gentzkow (2011), the government's optimal signal π^* is a binary, labeled g (= to smoke) and b (= not to smoke), respectively. Moreover, optimal signal π^* will induce posteriors $p_g(\omega = 0) = 0, p_g(\omega = 1) = 1, p_b(\omega = 0) = 1 - c, p_b(\omega = 1) = c$. The consumer will smoke if and only if the signal is g (to smoke). At the optimal signal, the consumer will smoke with strictly positive probability $\frac{p-c}{1-c} > 0$.

The government can do strictly better by sending an ambiguous signal. In fact, it can persuade the consumer to not smoke *for sure* by sending binary signals $S = \{g, b\}$ with only ambiguous interpretation. Specifically, the signal s has ambiguous (multiple) likelihoods $\Pi = \{\pi, \pi'\}$ such that $\pi(s = g|\omega = 1) = 1$, $\pi(s = b|\omega = 0) = 1$, and $\pi'(s = g|\omega = 0) = 1$, $\pi'(s = b|\omega = 1) = 1$. For example, the government can collect research reports on the health effect of cigarret smoking from two lobbyists (say a health NGO and a tobacco company) but does not reveal the source of disclosed report. We assume the consumer has MEU preferences and performs likelihood-by-likelihood updating when the signal is observed (Epstein and Schneider, 2007). For all interior prior p, his posteriors are $P_g(\omega = 1) = \{0, 1\}$ and $P_b(\omega = 1) = \{0, 1\}$. Then no smoking is optimal upon observing either $s = g$ or $s = b$. Hence the ambiguous signal Π makes the government (sender) strictly better off than any unambiguous signal structure. More generally, the sender could benefit from an ambiguous signal, when it discourages the receiver from taking an action (smoking) that is disliked by the sender and also exposes the receiver to ambiguity.

Also observe that there is no straightforward "recommendation" signal that would generate the same equilibrium outcome as the ambiguous signal Π. Hence the "revelation principle" à la Kamenica and Gentzkow (2011) fails. To see this, suppose an equivalent direct "recommendation" mechanism exists, then it must recommend $a^* = 0$ when $s = g$ and $a^* = 0$ when $s = b$. In this case, the recommendation message is not informative and the consumer will not update her prior. Hence the consumer will smoke since $p > c$. Always recommending no smoking is not an equilibrium.

Model

We generalize Kamenica and Gentzkow's (2011) persuasion model so that agents have MEU preferences and the sender can choose ambiguous singals. Formally, let Ω be a finite state space and A be the receiver's action set. The sender and receiver have vNM utility indicies $v(a, \omega)$ and $u(a, \omega)$, respectively. They share a unique common prior p_0 over Ω. Ex-ante, the sender can commit to sending an ambiguous signal, which may have multiple likelihood distributions. An ambiguous signal is model by a tuple (S, Π), where S is the signal space and $\Pi = \{\pi_i(\cdot|\omega) \in \Delta(S)\}_{\omega \in \Omega, i \in I}$ is a set of I state-conditional likelihood distributions of signals the agents view as plausible. The prior p_0 and likelihoods Π induce multiple posteriors P_s according to likelihood-by-likelihood Bayesian updating (Epstein and Schneider, 2007), i.e.: $P_s(\omega) = \{\frac{p_0(\omega)\pi(s|\omega)}{\sum_{\omega'} p_0(\omega')\pi(s|\omega')} : \pi \in \Pi\}$ for all ω. We use an analogous concept of Sender preferred subgame perfect equilibrium. Conditional on singal s, the receiver will choose an optimal action $\hat{a}(P_s)$ that maximizes his conditional MEU $\min_{p_s \in P_s} E_{p_s}[u(a, \omega)]$ and is sender-preferred if there is a tie; anticipating $\hat{a}(P_s)$, the sender chooses an optimal ambiguous signal Π that maximizes her (ex-ante) MEU $\min_{\pi \in \Pi} E_{p_0, \pi}[v(\hat{a}(P_s), \omega)]$.

Allowing for ambiguity imposes significant challenge to the tractability of the general problem. In particular, the equivalence result (Proposition 1) that greatly simplifies the problem in Kamenica and Gentzkow (2011) fails. This is demonstrated by failure of "revelation principle" in the illustrating example. Moreover, their "concavification" technique of characterizing the optimal signal no longer applies.

Literature

Our paper belongs to a recent theoretical literature exploring the role of ambiguity-averse preferences in games and mechanisms (Bose et al., 2006; Lopomo et al., 2011; Wolitzky, 2015; Lopomo et al., 2014). The construction of the endogenous ambiguous signal in our lead example is related to that in Bose and Renou (2014); while they consider a mechanism design problem.[3] Two recent papers are most related to our work. Lipnowski and Mathevet (2015) consider a model of Bayesian persuasion where the receiver has psychological preferences, where beliefs directly enter the receiver's payoff function. They demonstrate that the sender might not want to fully reveal his information to the receiver even if there is no intrinsic conflict of interest between them. They demonstrate that the revelation principle might fail to hold (in Section 5 of their paper), when the receiver's preferences depend on beliefs in a way that violates expected utility maximization.[4] Kellner and Le Quement (2015) introduce ambiguous messages into the cheap-talk framework of Crawford and Sobel (1982). They demonstrate that the possibility of ambiguous messages, coupled with ambiguity aversion of the receiver, may improve the communication between the sender and the receiver. Here we consider a persuasion framework of Kamenica and Gentzkow (2011).

[3]Bose and Renou (2014) consider mechanism design with a mediated communication stage, and the key insight is the designer can cleverly use ambiguous communication to implement a wider set of social choice functions.

[4]There is, however, no explicit foundation of such preferences.

References

Abdellaoui, M., A. Baillon, L. Placido, and P. P. Wakker (2011). The rich domain of uncertainty: Source functions and their experimental implementation. *American Economic Review 101*(2), 695–723.

Bose, S., E. Ozdenoren, and A. Pape (2006, December). Optimal auctions with ambiguity. *Theoretical Economics 1*(4), 411–438.

Bose, S. and L. Renou (2014). Mechanism design with ambiguous communication devices. *Econometrica 82*(5), 1853–1872.

Bossaerts, P., P. Ghirardato, S. Guarnaschelli, and W. R. Zame (2010). Ambiguity in asset markets: Theory and experiment. *Review of Financial Studies 23*(4), 1325–1359.

Camerer, C. and M. Weber (1992). Recent developments in modeling preferences: Uncertainty and ambiguity. *Journal of Risk and Uncertainty 5*(4), 325–70.

Chow, C. C. and R. K. Sarin (2001). Comparative ignorance and the Ellsberg paradox. *Journal of Risk and Uncertainty 22*(2), 129–139.

Crawford, V. and J. Sobel (1982). Strategic information transmission. *Econometrica 50*(6), 1431–51.

Epstein, L. G. and M. Schneider (2007). Learning under ambiguity. *Review of Economic Studies 74*(4), 1275–1303.

Epstein, L. G. and M. Schneider (2010). Ambiguity and asset markets. *Annual Review of Financial Economics 2*(1), 315–346.

Fox, C. R. and A. Tversky (1995). Ambiguity aversion and comparative ignorance. *The Quarterly Journal of Economics 110*(3), 585–603.

Gilboa, I. and M. Marinacci (2013). Ambiguity and the bayesian paradigm. In *Advances in Economics and Econometrics: Theory and Applications, Tenth World Congress of the Econometric Society.*

Gilboa, I. and D. Schmeidler (1989). Maxmin expected utility with non-unique prior. *Journal of Mathematical Economics 18*(2), 141–153.

Halevy, Y. (2007). Ellsberg revisited: An experimental study. *Econometrica 75*(2), 503–536.

Kamenica, E. and M. Gentzkow (2011). Bayesian persuasion. *American Economic Review 101*(6), 2590–2615.

Kellner, C. and M. T. Le Quement (2015). Endogenous ambiguity in cheap talk. B*onn working paper*.

Lipnowski, E. and L. Mathevet (2015). Disclosure to a psychological audience. NYU Working Paper.

Lopomo, G., L. Rigotti, and C. Shannon (2011). Knightian uncertainty and moral hazard. *Journal of Economic Theory 146*(3), 1148 – 1172. Incompleteness and Uncertainty in Economics.

Lopomo, G., L. Rigotti, and C. Shannon (2014). Uncertainty in mechanism design. Working Paper.

Mukerji, S. and J.-M. Tallon (2004). An overview of economic applications of david schmeidler's models of decision making under uncertainty. In *Uncertainty in Economic Theory,* (I. Gilboa, ed.), New York: Routledge.

Wolitzky, A. (2015). Mechanism design with maxmin agents: Theory and an application to bilateral trade. *Theoretical Economics (Forthcoming)*.