Tournaments and the Optimal Organizational Structure *

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Abstract

This paper develops a theoretical framework that incorporate the destructive effects of relative incentive pay on cooperation in order to generate a more complete theory of the optimal organizational structure, which is a decision consisting of authority allocation and a choice between vertical or horizontal communication. I study how well information will be generated and utilized by either a centralized or a decentralized setting. One of the main results suggest that in high productivity environments, the introduction of a limited liability constraint will favor decentralization and can even increase the agents' compensation, which is at odds with the literature. Among other things, I also argue that regions with tighter labor markets are more likely to favor a centralized organizational structure and managers are less likely to micro-manage their employees in high productivity environments.

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1 Introduction

1.1 Motivation

The predominance of promotion tournaments to fill higher-ranked positions in firms is well documented. In a database that followed 600 firms for over 8 years, Bognanno (2001)[7] observed that roughly 80% of executives were promoted from within instead of being filled with outside hires. Similarly, using a data set of a large firm employing over 100,000 people, Lazear (1992)[24] reported that 99.7% of workers where in positions that provided opportunities for promotions.

Given the near universal presence of promotions, a theory that attempts to map the underlying characteristics of a firm to an optimal organizational structure would be more complete if it incorporated the impacts of these promotion tournaments. As Lazear (1989)[23] argued and Drago and Garvey (1998)[16] and Chan, Li and Pierce (2014)[8] documented, tournaments and other forms of individual incentive pay can have some negative effects on cooperation among employees. However, while the motivational effect of tournaments can be approximated by output contingent wage contracts, its deleterious impacts on cooperation between the participants are typically absent in most models of the optimal organizational structure. The absence of these negative effects in models can bias results, especially if these effects are only present in only one specific organizational structure like decentralization. For example, a large part of the literature on optimal organizational structure models disagreements between players with a bias parameter that remains fixed regardless of the structure of the firm.

One form these negative effects of relative performance evaluation can take is to cut the flow of information inside a firm. If an exchange of information can increase the profitability of a firm, then discouraging this exchange will harm a firm's expected profitability. Firms and their agents constantly acquire information about their environment to react to changing market conditions. This is essential to any decision maker who has to deal with uncertainty. But as sources of information tend to be dispersed within a firm (see Radner [1993][30]), information flow can be crucial in getting information to the appropriate person. As many papers on the optimal organizational structure tend to focus on information acquisition (see Aghion and Tirole [1997][1], Angelucci [2015][4] and Gibbons, Holden and Powell [2012][18] among others) and exchange (see Mookherjee and Tsumugari [2014][27] and Dessein, Garicano and Gertner [2010][14] among others), evaluating the influence of promotion tournaments on these two issues could be a crucial step towards a more powerful theory, which is the fundamental contribution of this paper.

To do so, I develop a framework with one principal and two agents. The two agents acquire information concerning the states of nature at a cost and may communicate this information through a nonfalsifiable message to the player with the relevant authority. This decision maker then makes a production decision where this advice is a valuable input. Agents are motivated to acquire this costly information by the prizes of a tournament. I assume that ordinal performance measures are easy to contract upon while contracts based on absolute performance are infeasible. This justifies the use of tournaments as the compensation structure.

Under centralization, the authority over the production decision is retained by the principal while it is delegated to the agents under decentralization. In a centralized setting, agents communicate with the principal but this vertical communication is subject to noise. In a decentralized setting, communication is horizontal, that is, agents communicate with each other. However, an agent will be unwilling to share any information since the receiver of his message will be his tournament rival. This withholding of information represents the repercussion of a misalignment of preferences specific to decentralization.

A centralized tournament will be different from a decentralized tournament since agents will have different responsibilities in both structures. Subsequently, this leads to two sets of incentives which yields two different effort levels. These contrasting efforts as well as the vertical communication noise and the decentralized inhibiting of communication will be the main components that the principal will have to take into account when determining the optimal organizational structure.

The communication noise between the principal and the agents and the noise in the production process itself lead us to several observations. First, I uncover a fundamental trade-off between production noise and communication noise. While larger communication noise will be correlated with decentralized setting, larger production noise will actually be correlated with centralization. I also find that even when communication between agents and the principal is perfect, decentralization can still be preferred to centralization. This is explained by the different incentives provided by both structures: a decentralized setting may provide better incentives to exert efforts than a centralized setting.

When I allow the principal to set the prizes himself, I create an interesting counterfactual in subsection 3.1 where the principal is allowed to use negative prizes. This allows me to identify the impact of introducing the limited liability constraint on the optimal prizes and the optimal organizational structure. I discover that in high productivity settings, imposing a minimum boundary on the agents' compensation strictly favors decentralization and can even motivate the principal to increase the prizes, which is at odds with the literature on limited liability constraints. I also argue that a sufficient condition for decentralization to unambiguously dominate centralization is for production to be sufficiently low. In contrast, when there is no communication noise between the principal and the agents, decentralization can still dominate centralization. Furthermore, I establish a link between a region's labor market and the preferred organizational structure firms will prefer. I show that firms operating in a tighter labor market will be more likely to prefer centralization.

I eventually relax the assumption that communication noise between the agents and the principal is exogenous. If I allow the principal to choose the level of communication noise between himself and the agents (vertical communication), this adds a new channel through which an increase in the value of information benefits centralization. Furthermore, the productivity of an environment will actually influence the optimal communication noise: if the principal is working in a high productivity environment, then he is less likely to micro-manage his agents through exhaustive meetings or oversight.

This article is structured as follows. I first review the existing literature in section 1.2. Then, I introduce the model and some basic results in section 2. The main results put forth in this the paper are explained in section 3 and 4. Finally, the paper is concluded in section 5.

1.2 Literature Review

An important part of the literature focuses on the importance of coordination and adaption in the decision to centralize authority. Alonso, Dessein and Matouschek (2008)[2] model centralization as a structure that optimizes coordination in contrast to decentralization which optimizes adaptation. They find that even when coordination is very important, decentralization can still dominate centralization because of information distortion. In contrast, Choe and Ishiguro (2012)[10] argue that as long as the principal is sufficiently competent relative to the agents, centralization always dominates if coordination is sufficiently important. Rantakari (2008)[32] argues for a non-monotonic relation between coordination and the optimal organizational structure which is argued to vary from decentralization to centralization back to decentralization as the importance of coordination ranges from insignificant to critical. In a one principal and one agent setting, Dessein (2002)[12] focuses on another trade-off and argues centralization dominates decentralization if the misalignment of preferences between the agents is sufficiently large (as does Rantakari (2008)[32] for a one principal two agents setting). In contrast, my paper instead emphasizes the importance of cooperation in the decision to centralize or decentralize by asserting that a lack of cooperation is a flaw of decentralization.

The study of the optimal organizational structure is not restricted to a simple debate between centralization versus decentralization. For instance, Rantakari (2012)[33] asks how much managerial supervision should accompany delegation if the latter dominates centralization. In a setting where both the employee and the manager generate ideas at a cost, the author argues that the answer to his question is determined by the relative cost structure of generating ideas: the more qualified the employee is relative to the manager at generating ideas and projects, the less managerial supervision will accompany delegation. If the employee is too incompetent, the manager does everything by herself (centralization). Without directly referring to centralization or decentralization, Marino and Zabojnik (2004)[26] study the question of which structure provides the best incentives to the agents. They argue that M-form firm, where divisions compete for resources, is optimal for large firms because these internal tournaments provide better incentives than a U-form firms, which motivate their employees through a simple profit sharing scheme.

Various other subjects can be linked to the centralization versus decentralization question. An interesting illustration of this Swank and Visser (2015)[37], who wish to study the impact of career concerns on the optimal organizational structure. The authors create a setting where agents' utility are partially determined by their perceived reputation. They distinguish between local markets, where an agent's reputation is based on his own action, and global markets, where an agent's reputation is based on the action taken by him and some other agents, and find that centralization strictly dominates decentralization in the case of global markets. To relate the question of delegation to competition, Alonso, Dessein and Matouschek (2015)[3] develop a framework where revenues are based on uncertain consumer demands, traditionally left out of the optimal organizational structure literature. They find that if the price sensitivities of consumers increase (due to increased competition), centralization should dominate decentralization[1].

[1]Many other papers relate the question of the optimal organizational structure to different and interesting phenomenons. For example, this question is associated with the intensity of human/physical capital by Rajan and Zingales (2001)[31], with liquidity constraints by Zabojnik (2002)[38], with the informativeness of price mechanism by Gibbons, Holden and Powell (2012), with rational inattention by Dessein, Galeotti and Santos (forthcoming)[13], with dynamic and timing issues by Li, Matouschek and Powell (forthcoming)[25] and Grenadier, Malenko and Malenko (forthcoming)[19] and is studied through a mechanism design approach by Bester (2009)[5] and Mookherjee and Tsumugari

One contribution of this paper is that it endogenizes the conflict between the two agents through a tournament structure. However, this paper is not the first to do so. In Ozbas (2005)[29], managers are empire builders and always prefer to acquire additional resources, which leads to a misalignment of preferences between the managers and the corporate headquarters. Ozbas (2005)[29] introduces some team production and makes the managers' compensation dependent on the firm's overall profits, which allows for an endogenous bias. In a similar spirit, Friebel and Raith (2010)[17] and Rantakari (2013)[34] assume that the organization can choose the extent to which managers' bonuses are based on their own division performance versus the second division's performance, thereby endogenizing the alignment of preferences between the managers and the firm. Similar to Holmstrom and Ricart I Costa (1986)[21], Hirata (2015)[20] assumes the agent's future wage is dependent on the market's assessment of his ability, thereby giving the agent an incentive to always choose to invest in a project (regardless of its quality) and inflate output in order to send the best possible signal about his abilities to the market. This allows Hirata (2015)[20] to micro-founds the agent's misalignment of preferences with the principal through career concerns.

This paper instead chooses to assume an almost omnipresent form of compensation structure, promotion tournaments, and develops a misalignment of preferences between the players based on the deleterious effect this competition has on cooperation. A small subset of the literature on optimal organizational structure tries to incorporate the negative impact of competition between employees. Ozbas (2005)[29] and Stein (2002)[36] model this as a competition between managers for a fixed amount of resources. Managers have to report the profitability of their projects and these contests motivate the managers to amplify the quality of their projects. Inderst and Klein (2007)[22] argue that the deleterious impact of competition is to exacerbate the managers' tendencies to undertake suboptimal investment. In contrast to these papers, Friebel and Raith (2010)[17] detect a positive motivational effect following a merger of two firms who must now compete for resources, as does Rantakari (2012)[**?**] when employees compete for the authority over project selection. This paper consolidates both the positive and negative effects of tournaments: agents are motivated to exert efforts by the tournament prize but are also led to withhold valuable information from each others.

A large part of the literature models communication as cheap talk [2]. The popularity of cheap talk in the optimal organizational structure can mostly

(2014)[27].

[2]See, among others, Friebel and Raith (2010), Swank and Visser (2015), Alonso, Dessein and Matouschek (2008, 2015), Rantakari (2008, 2013) and Grenadier, Malenko and Malenko (forthcoming)[19]

be explained by the tractable way in which it models communication between a sender and a receiver. An additional and attractive feature of cheap talk is that it endogeneizes the communication entropy between the different players. This entropy is related to how closely aligned are the preferences of both players: the larger the preference bias between the two players is, the less valuable the information transmitted during the communication stage will be (see Crawford and Sobel [1982][11]). An important feature of this paper is that messages exchanged between players are assumed to be hard or nonfalsifiable. This is done for tractability and conceptual reasons. First, in the context of this model, the most tractable form of communication is hard information. The fact that the communication strategy is binary (to disclose or not) greatly simplifies the analysis and the comparison between organizational structures. Second, within the context of a firm or some other organization, communication is rarely cheap. Information exchanges are usually done through some formal and costly mediums such as business reports or market analyses. Even in the context of informal conversation, one party's reputation can be at stakes[3]. Third, if one assumes cheap talk, then messages between players cannot be contracted upon. However, if a group of employees' sole purpose is to produce various market analysis and a bonus is to be handed out based on an ordinal performance measure, it would be logical to assume that this performance measure consist of the quality of the various market analyses, thereby assuming messages contingent contracts are feasible. For these reasons, this paper assumes a hard information structure like in Che and Kartik (2009)[9] and Dessein and Santos (2006)[15].

2 Preliminaries

2.1 Model Framework

This is a model with three risk-neutral players: one principal and two agents. The model has two possible organizational structures. The first one allocates the authority of the production decisions to the agents and communication is horizontal, meaning agents communicate with each other. This is referred to as decentralization. The second allocates the authority of the production

[3]There exists a conceptual difference between business units and individual agents and one component of this difference can be the cost of lying. Ex-post punishment for having wrongfully led a company in one direction will most likely be harsher for an individual than a business unit, given the cost employment termination for one employee versus the costs of restructuring an entire firm. This might suggest that, within the context of a firm, cheap talk might be a better critical assumption for business units and hard information a better one for individual agents.

decisions to the principal and communication is vertical, meaning agents communicate with the principal. This is referred to as centralization.

The production output $u(a, \omega, \epsilon) = u_1(\omega_1, a_1) + \epsilon_1 + u_2(\omega_2, a_2) + \epsilon_2$ depends on the costless action $a = (a_1, a_2)$ taken by the players with the relevant authority, with $a_i \in A$ where A is the action space. It also depends on the state of nature $\omega = (\omega_1, \omega_2) \in \Omega$, where $\Omega = (\Omega_1, \Omega_2)$ is a finite two dimensional space from which the state of nature is drawn. Finally, it also depends on the random errors $\epsilon_2 - \epsilon_1 \sim U[-L, L]$ where $0 < L < \infty$. All three players have the same prior beliefs over ω and its cumulative and density probability distributions are respectively denoted by $F()$ and $f()$. I also assume that

$$\arg\max_{a_i \in A_i} u_i(\omega_i, a_i)$$

is strictly increasing with ω_i.

To get the high production output in dimension i, information about ω_i needs to be acquired by the agents. The information acquisition process is costly and can result in one of two outcomes, a failure or a success. If the information gathering process has been a failure, the agent learns nothing and the optimal costless action will be

$$\hat{a}_i \equiv \arg\max_{a_i \in A_i} \int_{\Omega_i} u_i(\omega_i, a_i) dF(\omega_i),$$

resulting in a production output denoted by $\hat{u}_i = \int_{\Omega_i} u_i(\omega_i, \hat{a}_i) dF(\omega_i)$. If the information gathering process has been successful, the agent learns $\omega = (\omega_1, \omega_2)$ and the optimal costless action is simply:

$$\bar{a}_i(\omega_i) = \arg\max_{a_i \in A_i} u(\omega_i, a_i)$$

resulting in $\bar{u}_i = u_i(\omega_i, \bar{a}_i(\omega_i))$. Furthermore, I assume that $\hat{u}_i(\omega_i, \hat{a}_i) = \hat{u}_{-i}(\omega_{-i}, \hat{a}_{-i}) = \hat{u}$ and $\bar{u}_i(\omega_i, \bar{a}_i(\omega_i)) = \bar{u}_{-i}(\omega_{-i}, \bar{a}_{-i}(\omega_{-i})) = \bar{u}$ for any $\omega_i \in \Omega_i$. These assumptions yield a production process that is, in ex-ante expectation, binary: $u_i \in \{\hat{u}_i, \bar{u}_i\}$ with $\hat{u}_i < \bar{u}_i$ for all $i \in \{1, 2\}$.

Agent i exerts effort $e_i \in [0, 1]$ at cost $c(e_i) = \theta \frac{e_i^2}{2}$ in order to acquire information. Henceforth, $0 < \theta < \infty$ will be referred to as the cost parameter of the agents. Agent i learns $\omega = (\omega_1, \omega_2)$ with probability e_i and learns nothing with probability $(1 - e_i)$. These assumptions imply that the agents are equally efficient at learning the state of nature in either dimensions. Furthermore, when an agent learns ω, he may decide to share this information with another player. In decentralized setting, that information can only be shared with the other agent (horizontal communication) whereas in a centralized setting, that information can be shared with the principal (vertical communication). Denote by $m_i = \{0, 1\}$ the decision to communicate, with $m_i = 0$ meaning agent i refused to communicate ω and $m_i = 1$ the opposite.

At the end of the game, two prizes, W^U and W^L with $W^U > W^L$, are handed out to the agents. Assume for now that the set of these prizes is the same in each organizational structure and is exogenous. These assumptions will be relaxed in section 3. To determine the winner, a centralized tournament will be carried out in a centralized structure while a decentralized tournament will take place in a decentralized structure. The prizes are the agents' sole motivation to acquire information. Furthermore, I assume that only relative performance measures are feasible, which justify our focus on a tournament setting. To simplify the notation, denote by $e^g = (e_1^g, e_2^g)$ and $m^g = (m_1^g, m_2^g)$ the strategies adopted by the agents in an organizational structure $g \in \{C, D\}$.

Agent i's payoff function is

$$Q_i^g(e^g, m^g)W^U + [1 - Q_i^g(e^g, m^g)]W^L - \theta \frac{(e_i^g)^2}{2}$$

and the principal's expected profits are

$$\sum_{i=1}^{2} \{B_i^g(e^g, m^g)\bar{u}_i + [1 - B_i^g(e^g, m^g)]\hat{u}_i\} - W^U - W^L$$

where the notation $B_i^g(e^g, m^g)$ represents the probability that the decision maker responsible for action a_i in g will learn ω_i and $Q_i^g(e^g, m^g)$ represents the probability of agent i winning the tournament in g.

The messages sent by the agents are assumed to be nonfalsifiable. Therefore, the only communication decision the agent faces is whether or not to disclose the information he acquired. As I have explained in the literature review, since cheap talk is neither a critical nor a simplifying assumption in this framework, I therefore resort to using verifiable (hard) information.

The timing of the game is as follows. First, the principal chooses whether to centralize or decentralize the decision process. Second, agents observe the organizational structure simultaneously, chose the level of effort to exert in learning the state of nature. Third, the state of nature is potentially revealed to the agents. The fourth step is the communication stage. In a decentralized setting, each agent decides whether to disclose or not their information to the other agent. In a centralized setting, agents communicate with the principal. Then, production decisions are made by the players with the relevant authority. Finally, the production outputs are realized and the winner of the tournament is determined.

As it was briefly stated, the first step of the game involves the principal choosing between centralization and decentralization. Each structure can be considered as a subgame of the overall game. The next two subsections describe the optimal strategies of the agents and the resulting payoffs for all three players for each subgame.

2.2 Decentralization

In a decentralized organizational structure, agents communicate with each other and agent i has the authority over a_i, the output decision in dimension i. During the decentralized communication stage, if agent i learns what ω is, agent i has the choice between sharing this information with agent -i or keeping it to himself. In decentralization, there is no communication between the agents and the principal. In essence, the notation $m_i = 1$ indicates that agent i shared ω and $m_i = 0$ otherwise. Examples of information exchanges are sharing details about clients, the behavior of competitors, production conditions, investment opportunities, tips for solving production problems, the costs of introducing new features, etc.

In a decentralized setting, agent i is the winner if the output for which he is responsible is larger than the other one: $u_i(a_i, \omega_i)+\epsilon_i > u_{-i}(a_{-i}, \omega_{-i})+\epsilon_{-i}$. The agent solves the following problem in a decentralized setting

$$\max_{e_i \in [0,1], m_i \in \{0,1\}} E(V_i^D) = E[Q_i^D(e^D, m^D)]W^U + \{1 - E[Q_i^D(e^D, m^D)]\}W^L - \theta\frac{e_i^2}{2}$$

where $E[Q_i^D(e^D, m^D)]$ is the expected probability of winning the decentralized tournament and W^U and W^L are the prizes awarded to the winner and loser respectively. To simplify the notation, I will use $Q_i^D(e, m)$ instead of $Q_i^D(e^D, m^D)$. The probability of agent i winning is:

$$Q_i^D(e, m) = Prob[u_i(\omega_i, a_i) + \epsilon_i > u_{-i}(\omega_{-i}, a_{-i}) + \epsilon_{-i}]$$

$$\Leftrightarrow Q_i^D(e, m) = Prob[u_i(\omega_i, a_i) - u_{-i}(\omega_{-i}, a_{-i}) > \epsilon_{-i} - \epsilon_i].$$

Since $\epsilon_{-i} - \epsilon_i \sim U[-L, L]$, it can be seen that:

$$\Leftrightarrow Q_i^D(e, m) = \begin{cases} \frac{u_i(\omega_i, a_i) - u_{-i}(\omega_{-i}, a_{-i}) + L}{2L} & \text{if } u_i(\omega_i, a_i) - u_{-i}(\omega_{-i}, a_{-i}) < L \\ 1 & \text{if } u_i(\omega_i, a_i) - u_{-i}(\omega_{-i}, a_{-i}) > L. \end{cases}$$

For expositional purposes, it will henceforth be implicitly assumed that probabilities are automatically set to 1 when the numerator exceeds the denominator. Denote by $P_i(e,m)$ the probability that agent i has learn $\omega = (\omega_1, \omega_2)$ given the effort and message vectors. This can be done through the information acquisition process or by receiving a message from the other agent. The expected probability of agent i winning is therefore:

$$E[Q_i^D(e,m)] = \frac{P_i(e,m)(\bar{u}-\hat{u}) + \hat{u} - P_{-i}(e,m)(\bar{u}-\hat{u}) - \hat{u} + L}{2L}$$

$$\Leftrightarrow E[Q_i^D(e,m)] = \frac{[P_i(e,m) - P_{-i}(e,m)](\bar{u}-\hat{u}) + L}{2L}.$$

Since $P_i(e,m) = e_i + m_{-i}[(1-e_i)e_{-i}]$, $P_i(e,m) - P_{-i}(e,m)$ can be rewritten as

$$P_i(e,m) - P_{-i}(e,m) = e_i - e_{-i} - m_i e_i(1-e_{-i}) + m_{-i}e_{-i}(1-e_i)$$

which implies

$$E[Q_i^D(e,m)] = \frac{[e_i - e_{-i} - m_i e_i(1-e_{-i}) + m_{-i}e_{-i}(1-e_i)](\bar{u}-\hat{u}) + L}{2L}.$$

The agent's problem becomes

$$\max_{e_i\in[0,1], m_i\in\{0,1\}} \{\frac{[e_i - e_{-i} - m_i e_i(1-e_{-i}) + m_{-i}e_{-i}(1-e_i)]\Delta u + L}{2L}\}\Delta W + W^L - \theta\frac{e_i^2}{2}$$

where $\Delta W = W^U - W^L$ and $\Delta u = \bar{u} - \hat{u}$. Focusing on a symmetric equilibrium, the first order conditions to this problem are

$$\frac{\Delta W \Delta u}{2L\theta} = e_i = e_D^* \qquad m_i^* = m_D^* = 0.$$

The double derivative of $E(V_i^D)$ with respect to e_i is clearly negative so e_D^* is a global maximum[4]. Additionally, if $\Delta W \Delta u > 2L\theta$, then the agent simply chooses $e_D^* = 1$. From the point of view of the principal, decentralization yields an expected return of:

[4]In the cases where $L \to 0$ or $\theta \to 0$, the restriction $e_D^* \leq 1$ prevents the introduction of some nonconvexity in the maximand that would have disrupted the pure strategy solution (see Nalebuff and Stiglitz [1983][28].). As for the cases where $L \to \infty$ and/or $\theta \to \infty$, they are ruled out by the finiteness of both L and θ

$$\Pi^D = e_D^* 2\Delta u + 2\hat{u} - W^U - W^L \tag{1}$$

$$\Leftrightarrow \Pi^D = \frac{\Delta W(\Delta u)^2}{L\theta} + 2\hat{u} - W^U - W^L.$$

As it will be seen, both organizational structure will generate an inefficient use of efforts. There can either be a **duplication** of efforts, where one effort level is redundant and results in excessive cost, or a **wastefulness** of efforts, where the agents' effort are underutilized by the decision maker(s) which results in a suboptimal revenue. In a decentralized setting, if both agents are informed, there is a duplication of efforts since one effort level is redundant. If only one agent (say agent i) is informed but withholds his information during the communication stage, then there is a wastefulness of efforts since agent i's effort is not being properly utilized: information concerning state ω_{-i} will be squandered. With probability $2e_D^*(1-e_D^*)$, one effort level is partially wasted which results in a loss of Δu. With probability $(e_D^*)^2$, efforts will be duplicated resulting in a loss of $\frac{\theta(e_D^*)^2}{2}$.

2.3 Centralization

In a centralized organizational structure, agents communicate with the principal who subsequently makes both production decisions. Therefore, agents are no longer in competition for a prize with the receiver of their message and will share all information acquired, so $m_i = 1$ for both agents in a centralized setting. However, communication with the principal is subject to errors. This may be due to multiple things, like the principal having a limited amount of time for understanding the messages sent by the agents, having limited knowledge of the production process, language barriers, faulty communication equipment or needlessly long business reports.

With probability q, all messages sent by both agents are fully received by the principal. With probability 1 - q, none of the messages sent by the agents are received by the principal.

The agents now have only two tasks, which is to acquire information and send it to the principal. Since agents have different tasks compared to a decentralized setting, the winner of the tournament will be determined by who communicated the most information. For example, assuming messages were received by the principal, if he receives ω by agent 1 and nothing by agent 2, then the winner of the tournament will be agent 1 and vice-versa. If both agents send the same amount of information, the winner is determined by the flip of a coin. If messages were not successfully received by the principal, the winner is also determined by the flip of a coin.

The probabilities of both agents acquiring the same amount of information, of agent i acquiring more information than the other and of agent i acquiring less information are respectively

$$\bar{P}^C(e) = (1 - e_i)(1 - e_{-i}) + e_i e_{-i} = 1 - e_i - e_{-i} + 2e_i e_{-i}$$

$$P_i^C(e) = e_i(1 - e_{-i})$$

$$[1 - P_i^C(e) - \bar{P}^C(e)] = e_{-i}(1 - e_i).$$

The probability of agent i winning the tournament adds up to:

$$Q_i^C(e) = q(P_i^C + \frac{\bar{P}^C}{2}) + \frac{(1-q)}{2} = q[e_i - e_i e_{-i} + \frac{1}{2} - \frac{e_{-i}}{2} - \frac{e_i}{2} + e_i e_{-i}] + \frac{(1-q)}{2}$$

$$\Leftrightarrow Q_i^C(e) = q(\frac{e_i}{2} - \frac{e_{-i}}{2}) + \frac{1}{2}$$

Given all this, agent i's problem can be written as :

$$\max_{e_i \in [0,1]} \quad E(V_i^C) = Q_i^C(e)\Delta W + W^L - \theta \frac{e_i^2}{2}.$$

Solving this problem gives us the optimal effort function of agent i:

$$e_i = \frac{q\Delta W}{2\theta} = e_C^*.$$

Centralization returns an expected profit to the principal of:

$$\Pi^C = \sum_{i=1}^{2}((1-q)\hat{u} + q\{[(e_C^*)^2 + 2(1-e_C^*)e_C^*]\Delta u + \hat{u}\}) - W^L - W^U$$

$$\Leftrightarrow \Pi^C = 2\hat{u} - W^L - W^U + q[2e_C^* - (e_C^*)^2]2\Delta u \tag{2}$$

$$\Leftrightarrow \Pi^C = 2\hat{u} - W^L - W^U + q[\frac{q\Delta W}{\theta} - \frac{(q\Delta W)^2}{4\theta^2}]2\Delta u$$

Lemma 1: Assuming $e_D^* = e_C^*$, then decentralization is more likely duplicate efforts than centralization.

Lemma 2: When prizes are exogenous, centralization will offer better incentives to exert effort than decentralization if and only if $q > \frac{\Delta u}{L}$.

Lemma 3: Assuming $e_D^* = e_C^*$, then centralization is more likely to waste efforts than decentralization if and only if the communication noise if sufficiently high ($q \leq \frac{e^2}{2e-e^2}$).

Lemma 2 is useful for pointing out that while both organizational structure differ in how they provide decision makers with information, they also differ in how they motivate agents to exert to effort. So the trade-off between centralization and decentralization is not just about the information and communication structure but also about the incentives provided by both settings. Lemma 2 also argues that the difference in the incentives provided by both settings is solely determined by the communication noise, which negatively affects e_C^*, the production noise, which negatively affects e_D^*, and the value of information which positively affects e_D^*. Furthermore, it also suggests that even when communication is perfect, a decentralized setting could still provide the agents with more powerful incentives if $\Delta u > L$.

Whereas Lemma 2 is straightforward, Lemma 1 is slightly more complicated. Akin to a decentralized setting, duplication of efforts will occur when both agents are informed, which happens with probability $(e_C^*)^2$. However, an additional condition for one effort to be redundant is that the messages have been received by the principal. Therefore, the probability that efforts are duplicated in a centralized setting is $q(e_C^*)^2$, resulting in a cost of $\frac{\theta(e_C^*)^2}{2}$. Since $q \leq 1$, this proves Lemma 1. In contrast, wastefulness of efforts is not related to any withholding of information but to the communication noise between the agents. With probability $(1-q)[2e_C^* - (e_C^*)^2]$, some efforts will be wasted in a centralized setting. It is then straightforward to show that Lemma 3 holds when $e_D^* = e_C^*$. It should also be noted that if one agent becomes informed and messages are successfully received by the principal, there is no inefficient use of efforts which is an importance difference with decentralization which always has some inefficient use of efforts.

In summary, it can be seen that centralization provides the decision maker, the principal, with two signals that are subject to communication noise. In contrast, decentralization provides the decision makers, the agents, with one signal without the noise. Ignoring the difference in effort levels, one can thereby interpret the decentralization versus centralization question as a preference between providing the decision maker with two ex-ante (weakly) inferior signals versus providing the decision makers with only one signal of superior quality.

2.4 Perfect Bayesian Equilibrium

Denote by G the decision of the principal to choose between centralization and decentralization with $G \in \{D, C\}$, by ω^* the realized state of nature and by $f_i^*(\omega)$ the updated belief for player $i \in \{1, 2, P\}$. Furthermore, the type of a player is defined by their private information, which is $t \in \mathcal{T} \equiv \{\text{Uninformed}\} \cup \Omega$. In other words, a player can either be uninformed or know what the state of nature is and thereby have a type defined by $\omega \in \Omega$.

In a decentralized setting, a strategy for agent i of type t is a set of probability distributions $\sigma_i^D(\cdot|t)$ over his production decision a_i, his communication decision m_i^D and his effort level e_i^D. In a centralized setting, a strategy for agent i of type t is a set of probability distributions $\sigma_i^C(\cdot|t)$ over his communication decision m_i^D and his effort level e_i^C. The principal (of type t) strategy consists of his decision to choose between centralization and decentralization $G \in \{D, C\}$ as well as a set of probability distributions $\sigma_P(\cdot|t)$ over the production decisions $a = (a_1, a_2)$ in the case of centralization.

Given the prior beliefs $f(\omega)$, a Perfect Bayesian Equilibrium will consist of the strategy profile $\sigma^* = (\sigma_1^{*D}, \sigma_2^{*D}, \sigma_1^{*C}, \sigma_2^{*C}, \sigma_P^*, G^*)$ and all three players' posterior beliefs $f_i^*(\omega)$ for $i \in \{1, 2, P\}$ such that:

-$\forall$ t, $\sigma_i^{*g}(\cdot|t)$ maximizes $E(V_i^g)$ $\forall$ $i \in \{1, 2\}$ and $g \in \{D, C\}$,
-$\forall$ t, $\sigma_P^*(\cdot|t)$ maximizes Π^C,
-$G^* = D$ if $\Pi^D > \Pi^C$ or $G^* = C$ otherwise,
-the beliefs $f(\omega)$ are updated using Bayes' rule[5].

2.5 The Productivity of an Organizational Structure

Before moving on to the main results of this paper, I need to introduce the following definitions, which will be helpful in deciphering the results put forth by this paper.

Definition: The productivity of decentralization is defined as $z^D = e_D^*$.

Definition: The productivity of centralization is defined as $z^C = q[2e_C^* - (e_C^*)^2]$.

[5]If, for any reasons, player i learns the state of nature ω^*, his belief is updated to

$$f_i^*(\omega) = \begin{cases} 1 & \text{if } \omega^* = \omega \\ 0 & \text{otherwise.} \end{cases}$$

If player i does not learn ω^*, his posterior belief is simply $f_i^*(\omega) = f(\omega)$.

These are based on Π^D and Π^C (equations 1 and 2), with z^g defined simply as the terms next to $2\Delta u$ in their respective expressions. Both types of productivities reflect by how much the value of information Δu will be discounted by the various flaws of each organizational structure. Figure 1 is helpful in illustrating the relation between an organizational productivity and the corresponding agents' efforts. It is obvious that decentralized productivity is linear in the decentralized effort. However, while centralized productivity is also increasing in centralized effort, these marginal increases are decreasing with centralized effort.

To explain this, one must observe that the returns to the principal from an agent getting informed in a centralized setting is either 0 or $2q\Delta u$. This is in stark contrast to the returns in a decentralized setting which is always Δu. This is due to the communication and decision structure of both settings. Indeed, in a decentralized setting, the returns to the principal from one agent becoming informed are independent of the information his rival possesses. The agents do not communicate with each other and take production decisions separately. In a centralized setting, the returns to the principal from one agent becoming informed are dependent of what happens to the other agent: if both agents are informed, then one of the informative messages serves no purpose. The higher e_C^* is, the more likely it is that both agents will discover ω and that the returns on information (from the principal's perspective) will be zero for one of these messages. This explains why z^c has diminishing marginal returns with respect to e_C^*.

2.6 The Motivational Impact of Prizes

The next set of results is important because it highlights the effect that an increase (or a decrease) of the tournament's prizes will have on the agents' incentives to exert effort and subsequently, the principal's profits. Lemma 4 is essential in understanding the following section where the principal will optimally choose the winner's and the loser's prizes.

Lemma 4

i) In a decentralized setting, the benefit of increasing ΔW is proportional to e_D^*.

ii) In a centralized setting, the benefit of increasing the spread of prizes ΔW is maximized at $e_C^* = \frac{1}{2}$ and minimized at either $e_C^* = 0$ or $e_C^* = 1$.

Proof: See the appendix for this and all remaining proofs.

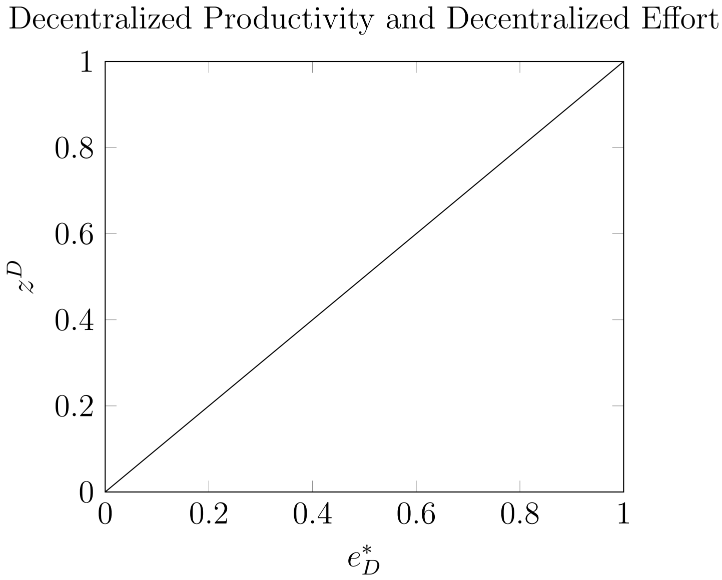

Figure 1a

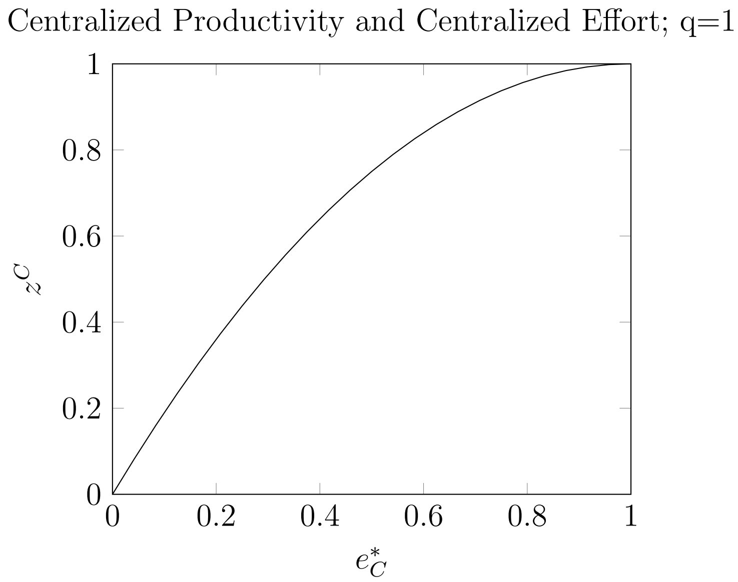

Figure 1b

Recall from Figure 1a that decentralized productivity is linear in the optimal decentralized effort. Based on this linear function, one can infer that additional motivation to exert effort (increasing ΔW) has a larger impact on the decentralized productivity z^D when the decentralized incentives to exert effort are high (this is reflected by a large e^*_D). The opposite also holds: when e^*_D is low, agents have little incentive to exert efforts and increasing their motivation to exert effort will have a negligible impact on decentralized productivity. This is result i).

The relation between centralized productivity z^C and the spread of prizes has two components, which can be observed in the following equations:

$$-\frac{\partial \Pi^C}{\partial \Delta W} = -2\Delta u \frac{\partial z^C}{\partial \Delta W}$$

$$\Leftrightarrow -\frac{\partial \Pi^C}{\partial \Delta W} = 2\Delta u [\frac{\partial z^C}{\partial e^*_C}][-\frac{\partial e^*_C}{\partial \Delta W}] \tag{3}$$

$$\Leftrightarrow -\frac{\partial \Pi^C}{\partial \Delta W} = 2\Delta u [2q(1-e^*_C)][\frac{e^*_C}{\Delta W}] \tag{4}$$

First, increasing the spread of prizes has a larger (smaller) impact on productivity when the centralized incentives to exert effort are high (small) which is reflected by a high (low) e^*_C. This is the motivational effect and is represented by the second set of brackets in equations 3 and 4. However, a second and opposite force is also in play: by looking at Figure 1b, it is obvious that increasing e^*_C has a smaller impact on z^C when e^*_C is large. This is caused by the decreasing returns of centralized efforts on centralized productivity. This is the production effect and is represented by the first set of brackets in equations 3 and 4 : it is inversely related to e^*_C. Combining these two effects leads to result ii).

Lemma 4 is interesting because it argues that increasing the winner's prize (and therefore ΔW) has little effect on revenues if the agents' incentives to exert effort are low. This holds for both organizational structure. This suggests that while a hike in ΔW might encourage the agents to exert more efforts, this would only have marginal effects on profits in low wages environment.

3 Endogenous Prizes

The optimal organizational structure will be the one that maximizes the principal's expected profit. Given that the principal reacts optimally to exogenous parameters, there will exist a mapping from these variables to the optimal organizational structure.

As a remainder, (1-q) represents the communication noise between the agents and the principal, L is referred to as the production noise, Δu is the value of information, ΔW is the difference between the winner's prize W^U and the loser's prize W^L and θ is the agents' cost parameter. Some basic results and interesting observations are summarized in proposition 1 below.

Even though wages are sometimes dictated by the market, the assumption of a set of fully exogenous prizes is a fairly strong one. The next subsections are devoted to partially relaxing that assumption by stating that the principal can optimally choose the tournament prizes for each organizational structure constrained only by an individual rationality constraint. The timing of the game would remain the same except that the principal would set the prizes for both organizational structures and then choose between centralization and decentralization. These optimal choices for the principal are simply to be added to the solution concept. I then study the impact of introducing a limited liability constraint which prevents the use of negative prizes. As it will be shown, the limited liability constraint creates a friction which prevents, for a set of parameters, the internalization by the principal of the cost of the agents' efforts.

3.1 Endogenous Prizes without a Limited Liability Constraint

3.1.1 Centralization

The centralized problem without a limited liability constraint is

$$\max_{W^U,W^L} 2q\Delta u[\frac{q}{\theta}(W^U - W^L) - \frac{q^2}{4\theta^2}(W^U - W^L)^2] + 2\hat{u} - W^U - W^L$$

$$W^U + W^L \geq \frac{q^2}{4\theta}(W^U - W^L)^2. \qquad \text{PC}$$

The steps leading to this solution have been placed in appendix. The optimal prizes are $W^L = \frac{2\Delta u^2\theta q^2 - 4q\Delta u^2\theta - 2\Delta u\theta^2}{(2q\Delta u+\theta)^2}$ and $W^U = \frac{2q^2\Delta u^2\theta + 2\Delta u\theta^2 + 4q\Delta u^2\theta}{(2q\Delta u+\theta)^2}$ and they yield a centralized profit of

$$\Pi^C_{no\ LL} = \frac{4q^2\Delta u^2}{2q\Delta u + \theta} + 2\hat{u}.$$

3.1.2 Decentralization

The decentralized problem without limited liability constraint is

$$\max_{W^U,W^L} 2\Delta u(\frac{\Delta u \Delta W}{2L\theta}) + 2\hat{u} - (W^U + W^L)$$

$$W^U + W^L \geq \frac{\Delta u^2 \Delta W^2}{4L^2\theta}. \qquad \text{PC}$$

The steps leading to this solution have once again been placed in appendix. Due to the presence of corner solutions, the optimal prizes are slightly more complicated. If $\Delta u \leq \theta$, the optimal prizes are $W^L = \frac{\Delta u^2 - 2L\theta}{2\theta}$ and $W^U = \frac{\Delta u^2 + 2L\theta}{2\theta}$. If $\Delta u \geq \theta$, the optimal prizes are $W^L = \frac{(\Delta u - 2L)\theta}{2\Delta u}$ and $W^U = \frac{(\Delta u + 2L)\theta}{2\Delta u}$. These solutions yield a decentralized profit function of

$$\Pi^D_{no\ LL} = \begin{cases} \frac{\Delta u^2}{\theta} + 2\hat{u} & \text{if } \Delta u \leq \theta \\ 2\Delta u + 2\hat{u} - \theta & \text{if } \Delta u \geq \theta. \end{cases}$$

Lemma 5: In an endogenous prize setting with no limited liability constraint, if $q > q^*(\equiv \frac{\Delta u}{4\theta} - \frac{\sqrt{\Delta u^2 + 4\theta^2}}{4\theta})$, then centralization strictly dominates decentralization.

This result is interesting because it highlights that if communication between the principal and the agents is sufficiently effective (assuming $q^* < 1$), centralization will strictly dominate decentralization. As it will be seen in proposition 1-iii, this result no longer holds when the principal is force to compose with limited liability constraints. This suggest that the ability to set any penalty on the tournament's loser seems to favor (at least partially) centralization.

3.2 Endogenous Prizes with a Limited Liability Constraint

The notion that the principal can set negative prizes is a very strong assumption that will be deal with in subsection 3.2. By introducing limited liability constraints, I force the principal to set (weakly) positive prizes so that the agents never receive a negative prize. While there exists scenarios where employees can receive negative compensation, like a bank teller being liable for one of her mistakes, these examples are fairly marginal so it is with little loss of generality that I impose the limited liability constraints. In fact, given the predominance of regulations and laws imposing some bound on contractual penalties, the limited liability constraint is actually an important critical assumption in reflecting the features of the modern firm.

3.2.1 Centralization

The centralized problem with a limited liability constraint is

$$\max_{W^U,W^L} 2q\Delta u[\frac{q}{\theta}(W^U - W^L) - \frac{q^2}{4\theta^2}(W^U - W^L)^2] + 2\hat{u} - W^U - W^L$$

$$0 \geq \frac{q^2}{4\theta}(W^U - W^L)^2 - W^U - W^L \qquad \text{PC}$$

$$W^L \geq 0. \qquad \text{LL}$$

Solving this problem is quite intractable so the steps leading to the solutions have been placed in the appendix. The only possible solutions are $W^L = 0$ and

$$W^U = \begin{cases} 0 & \text{if } \Delta u \leq \frac{\theta}{2q^2} \\ \frac{\theta(2q^2\Delta u - \theta)}{q^3 \Delta u} & \text{if } \Delta u \geq \frac{\theta}{2q^2}. \end{cases}$$

In both cases, the participation constraint (PC) is satisfied and $e_C^* < 1$. However, when $\Delta u \geq \frac{\theta}{2q^2}$, the participation constraint no longer binds, meaning the principal does not incorporate the cost of the agents' efforts. This yields a centralized profit of

$$\Pi^C = \begin{cases} 2\hat{u} \equiv \Pi_0^C & \text{if } \Delta u \leq \frac{\theta}{2q^2} \\ \frac{(2q^2\Delta u - \theta)^2}{2q^3\Delta u} + 2\hat{u} \equiv \Pi_1^C & \text{if } \Delta u \geq \frac{\theta}{2q^2}. \end{cases}$$

3.2.2 Decentralization

The decentralized problem with a limited liability constraint is

$$\max_{W^U,W^L} 2\Delta u(\frac{\Delta u \Delta W}{2L\theta}) + 2\hat{u} - W^U - W^L$$

subject to

$$0 \geq \frac{\Delta u^2 \Delta W^2}{4L^2\theta} - W^U - W^L \qquad \text{PC}$$

$$W^L \geq 0. \qquad \text{LL}$$

Once again, since the process of solving this Lagrangian is long and yields little intuition, it has been placed in the appendix. The solutions to this problem are fairly complicated due to the presence of corner solutions.

If $\theta \leq 2L$, then only corner solutions are possible ($e_D^* \in \{0, 1\}$) and the solutions are

$$\Pi^D = \begin{cases} 2\hat{u} \equiv \Pi_0^D & \text{when } \Delta u \leq \sqrt{L\theta} \\ 2\hat{u} + 2\Delta u - \frac{2L\theta}{\Delta u} \equiv \Pi_1^D & \text{when } \Delta u \in [\sqrt{L\theta}, 2L] \\ 2\hat{u} + 2\Delta u - \theta \equiv \Pi_4^D & \text{when } \Delta u \geq 2L. \end{cases}$$

If $\theta \in [2L, 4L]$, then all five possible solutions are feasible:

$$\Pi^D = \begin{cases} 2\hat{u} \equiv \Pi_0^D & \text{when } \Delta u \leq \sqrt{L\theta} \\ 2\hat{u} + 2\Delta u - \frac{2L\theta}{\Delta u} \equiv \Pi_1^D & \text{when } \Delta u \in [\sqrt{L\theta}, 2L] \\ 2\hat{u} + \frac{4L(\Delta u^2 - L\theta)}{\Delta u^2} \equiv \Pi_2^D & \text{when } \Delta u \in [2L, \sqrt{2L\theta}] \\ 2\hat{u} + \frac{\Delta u^2}{\theta} \equiv \Pi_3^D & \text{when } \Delta u \in [\sqrt{2L\theta}, \theta] \\ 2\hat{u} + 2\Delta u - \theta \equiv \Pi_4^D & \text{when } \Delta u \geq \theta. \end{cases}$$

If $\theta \geq 4L$, then the solutions are

$$\Pi^D = \begin{cases} 2\hat{u} \equiv \Pi_0^D & \text{when } \Delta u \leq \sqrt{L\theta} \\ 2\hat{u} + \frac{4L(\Delta u^2 - L\theta)}{\Delta u^2} \equiv \Pi_2^D & \text{when } \Delta u \in [\sqrt{L\theta}, \sqrt{2L\theta}] \\ 2\hat{u} + \frac{\Delta u^2}{\theta} \equiv \Pi_3^D & \text{when } \Delta u \in [\sqrt{2L\theta}, \theta] \\ 2\hat{u} + 2\Delta u - \theta \equiv \Pi_4^D & \text{when } \Delta u \geq \theta. \end{cases}$$

Lemma 6: If the communication noise is above a certain threshold ($q \leq \sqrt{\frac{\theta}{2\Delta u}}$), then decentralization always (weakly) dominates centralization.

Lemma 7: When $\Delta u \geq \min\{\sqrt{2L\theta}, \theta\}$, decentralized profits is independent of the production noise L.

Lemma 8: $\Delta u > \max\{2L, \sqrt{2L\theta}\}$ is a necessary and sufficient condition to ensure that W^L in a decentralized setting with a limited liability constraint is strictly positive.

3.3 Optimal Organizational Structure

Denote by $\Delta\Pi_{ij} = \Pi_i^C - \Pi_j^D$. With this notation, I am now ready to present the main results of this paper.

Proposition 1

i) Decreasing communication noise (increasing q) favors centralization.
ii) Increasing the production noise (L) favors centralization.
iii Even when communication between the agents and the principal is perfect (q=1), decentralization can still dominate centralization.

The intuition for part i) is fairly straightforward. A reduction in communication noise (an increase in q) has two positive effects in a centralized setting. First, it motivates agents to exert additional effort since they are more likely to be judged based on the content of their message than blind luck. Second, it increases the efficiency of the decision making process by increasing the odds of the decision maker becoming informed. In contrast, a decrease in communication noise has no impact on decentralization. For those three reasons, a reduction in communication noise creates a centralizing force inside this model, which has also been empirically documented by Bloom, Garicano, Sadun and Van Reenen (2014)[6].

Similarly to part i), behind result ii) lies a disparity in how each organizational structure motivates the two agents to exert effort. In a decentralized setting, an increase in the production noise L diminishes the returns to acquiring information. This is driven by the increased importance of the error term in the expectation of the probability of winning the decentralized tournament. Subsequently, the benefits of acquiring information, are diminished, leading the agents to reduce their efforts. However, in a centralized setting, this production noise has no impact on the agents' efforts. The agent simply attempts to acquire information and, if successful, pass on that information to the principal. In no way does production noise impact the agent's chance of winning a centralized tournament. The centralized profits are therefore unaffected while the decentralized profits are depressed, which explains result ii). The first two parts of proposition 1 embodies the fundamental trade-off of this model between production noise and communication noise. A large production noise L and a small communication noise (high q) favors centralization and vice-versa.

However, even when communication is perfect (q = 1), it is possible that $\Pi^D > \Pi^C(q = 1)$. This indicates that communication being perfect is not a sufficient condition for centralization to dominate decentralization since it imposes no restriction on the parameters determining the profitability of a decentralized structure. Indeed, it is possible that decentralization motivates agents to exert more effort than a centralized setting even when communication is perfect.

The main takeaway from proposition 1-iii is that not only do both organizational structures differ in how they utilize information but they also differ in how they motivate agents to exert effort[6], as suggested by Lemma 2. The withholding of information during horizontal communication is the main disadvantage in how decentralization utilizes information. In contrast, the problem with how centralization utilizes information is the communication noise during vertical communication. Proposition 1-iii is a key result because it demonstrates that even if centralization utilizes information perfectly, it is still insufficient to guarantee the superiority of centralization with respect to decentralization. If the incentives to exert effort are sufficiently high in a decentralized setting, that alone can be a sufficient condition for decentralization to dominate centralization regardless of communication noise.

Proposition 2:

i) The introduction of the limited liability constraint depresses both centralized and decentralized profits if and only if the limited liability constraint binds.

ii) When the limited liability constraint is imposed, the principal will induce the agents to exert no effort in either setting if the value of information is too low ($\Delta u \leq \frac{\theta}{2q^2}$ in centralization and $\Delta u \leq \sqrt{L\theta}$ in decentralization).

iii) In centralization, the introduction of the limited liability constraint causes the principal to reduce the spread of the prizes.

iv) In decentralization, the introduction of the limited liability constraint can cause the principal to maintain or even increase the spread of the prizes.

v) Suboptimal efforts are induced by the principal whenever the limited liability constraint binds.

vi) If the value of information is sufficiently high ($\Delta u \geq \max\{2L, \sqrt{2L\theta}\}$), then the introduction of the limited liability constraint strictly favors decentralization.

Regardless of the organizational structure, the imposition of a limited liability constraint on the principal depresses his profits only when this con-

[6]This is similar to the point made by Rantakari (2013): a centralized and decentralized structure will differ not only in the quality of information each structure generates but also in the value of information, which he defines as how well each structure uses this information.

straint binds. This should come as no surprise since one way to look at the limited liability constraint is to view it as a regulation that expands the agents' bargaining power[7]. When the limited liability constraint does not bind, then it is essentially meaningless and has no impact on the principal's profits. An interesting result to note is that the limited liability constraint either binds or induces zero effort level in centralization whereas it will not bind in a decentralized setting if the value of information is sufficiently high.

As for proposition 2-ii, this result is somewhat standard in the principal-agent literature with limited liability constraint. Indeed, Sappington (1983)[35] argues that for the lowest states of nature (which he refers to as "productivity parameters"), it is optimal for the principal to induce zero effort by the agent. In this paper, the different levels for the value of information Δu are somewhat akin to Sappington's state of nature. It is the productivity of the different states of nature to which Sappington (1983) is referring to when suggesting the lowest (least productive) states of nature will push the principal to induce no effort by the agents. From the principal's perspective, the value of information can be seen as productivity parameters associated with the agents' efforts, so proposition 2-ii can be seen as a confirmation of this classic result.

Proposition 2-iii and iv are to be analyzed jointly. The fact that the agents' prizes (or compensation) decreases following the introduction of a limited liability constraint is another standard result in the literature. However, I believe proposition 2-iv to be novel. First, one must observe that the participation constraint is essentially an upper bound on the spread of prizes ΔW. Then, when the limited liability constraint binds, it implies that the loser's prize is zero. The combination of a binding participation constraint and a binding limited liability constraint therefore imply an upper bound for the winner's prize W^U. Furthermore, once the limited liability constraint binds, the principal has the possibility of extracting all of the agent surplus by making the participation bind through an increase in W^U. It is optimal for the principal to implement this increase if the marginal returns on W^U remain higher than its marginal cost.

In a decentralized setting, the marginal returns for an increase in W^U are proportional to e_D^* (see Lemma 4-i). This is due to the information structure, the communication strategies as well as the allocation of authority. And if $\Delta u \in [\sqrt{L\theta}, \sqrt{2L\theta}]$ and $e_D^* < 1$, these marginal returns to increasing W^U are higher than the marginal cost. So in this situation, the introduction of a *binding* limited liability constraint will set a ceiling for W^U which it will be

[7]Sappington (1983)[35] lists the guarantee of a subsistence level of well-being for each member of society as an explanation for the popularity of limited liability clauses.

optimal for the principal to reach. In contrast, in a centralized setting, the marginal returns of W^U might actually decrease in e_C^* (see Lemma 4-ii). As it was explained in section 2.5, the odds of one of the agents' efforts being redundant increases with centralized effort. The benefits of increasing W^U is derived solely from motivational purposes (inducing the agent to exert additional effort). But due to the presence of decreasing returns in effort levels, it is not optimal for the principal to increase the winner's prize such that the participation constraint binds. In fact, following the inefficiencies introduced by the limited liability constraint, the principal actually finds it optimal to reduce W^U in order to trim down his cost in a centralized setting.

Proposition 2-v follows directly from proposition 2-iii-iv. Given that prizes are modified when the limited liability constraint binds, the effort levels are necessarily affected and subsequently deviated from a situation with no limited liability constraint. This is similar to a result put forth by Sappington (1983. He argues that the limited liability constraint is associated with a contract that induces the agent to exert inefficient amount of effort for intermediate productivity parameters whereas I argue the inefficient level of effort materializes whenever the limited liability constraint holds.

Finally, proposition 2-vi suggests that the limited liability constraint strictly favors decentralization when the value of information is sufficiently high. This follows directly from lemma and proposition 2-i. It can be observed that the loser's prize is always negative in a centralized setting with no limited liability constraint whereas it will be negative in a decentralized setting with no limited liability constraint only when the value information is too low. Since a binding limited liability constraint depresses profits, proposition 2-vi follows directly from those two results.

Proposition 2-vi provides theoretical consequences of the use of laws regulating maximum penalties on the optimal organizational structure. This is important because it suggest that limitations on the maximum penalty an employee can occur for a relatively poor performance can be a decentralizing force when employees are sufficiently productive. Section 3.1 serves as a sort of counterfactual that argues that if employees could be penalized for poor performance with limits on these penalties, centralization would be more likely to dominate decentralization for high productivity settings.

Lemma 9: $\frac{\partial \Delta \Pi_{ij}}{\partial \Delta u} \leq 0$ for all $i \in \{0,1\}$ and $j \in \{0,2,3,4\}$ but $\frac{\partial \Delta \Pi_{11}}{\partial \Delta u} \lesseqgtr 0$

Proposition 3
i) If $\frac{\theta}{2q^2} < \sqrt{L\theta}$, then $\Pi^C > \Pi^D$ for all $\Delta u \in (\frac{\theta}{2q^2}, \sqrt{L\theta})$
ii) If $\frac{\theta}{2q^2} > \sqrt{L\theta}$, then $\Pi^D > \Pi^C$ for all $\Delta u \in (\sqrt{L\theta}, \frac{\theta}{2q^2})$

iii) If $\theta \geq 4L$, then decentralization always (weakly) dominates centralization.

Proposition 3 further highlights a result already presented in proposition 2-v: if the value of information is low enough, the principal would then prefer to let the decision maker(s) use their common prior instead of paying out prizes in order to capture the productivity benefits of Δu. This is fairly intuitive: if the benefits of entering into the information acquisition process are too low and lower bounds on prizes are too high, the principal would decide to remain out of the information acquisition process by setting the prizes to zero.

A notable feature is that both organizational structures have different thresholds of Δu below which the principal stays out of the information acquisition process. This embodies the different characteristics of both settings. While the cost parameter of the agents (θ) is an input into both thresholds, the decentralized threshold is also a function of production noise whereas the centralized threshold is a function of communication noise. This can be seen as another application of the fundamental trade-off between communication and production noise.

This is important because it affects the set of parameters for which one structure can unambiguously dominate the other. For instance, suppose production noise is sufficiently high relative to the communication noise ($\frac{\theta}{2q^2} < \sqrt{L\theta}$). If that were the case, there would exist a set of values for $\Delta u \in [\frac{\theta}{2q^2}, \sqrt{L\theta}]$ such that the principal would enter the information acquisition process expecting a positive return in a centralized setting. In contrast, he would refuse to enter the same process in a decentralized setting and content himself with $2\hat{u}$. In this specific scenario, centralization would strictly dominate decentralization (proposition 3-i). Conversely, if $\frac{\theta}{2q^2} > \sqrt{L\theta}$, the inverse scenario would hold such that decentralization strictly dominates centralization for $\Delta u \in [\sqrt{L\theta}, \frac{\theta}{2q^2}]$.

Finally, if the production noise is sufficiently low, then decentralization always (weakly) dominates centralization. This is simply a combination of proposition 3-ii and Lemma 9. It can be observed that $\theta \geq 4L$ implies $\frac{\theta}{2q^2} \geq \sqrt{L\theta}$, which ensures that $\Pi^D > \Pi^C$ for all $\Delta u \in [\sqrt{L\theta}, \frac{\theta}{2q^2}]$. Then, it can also be observed that $\theta \geq 4L$ is sufficient to ensure Π_1^D is not feasible, which by Lemma 9 ensures that $\frac{\partial \Delta \Pi_{ij}}{\partial \Delta u} \leq 0$ for all $i \in \{0, 1\}$ and $j \in \{0, 2, 3, 4\}$. This guarantees that $\Pi^D > \Pi^C$ for all $\Delta u \geq \sqrt{L\theta}$. One way to look at proposition 3-iii is to realize that there are two kinds of demotivating factors: the general kind, which is the cost parameter, and the specific kind, which is the production noise for decentralization and the communication noise for centralization. Proposition 3-iii argues that when the general demotivating factor θ is sufficiently important relative to the decentralized specific factor L, then it is enough to guarantee that centralization will be weakly dominated by decentralization.

Up until now, I have assumed, for tractability purposes, that the reservation utility was zero. For the sake of the next result, I will drop this simplifying assumption and denote by R the reservation utility of the agents, which I assume is the same for both agents.

Proposition 4: If the value information is sufficiently high ($\Delta u \geq \max\{2L, \sqrt{2L\theta}\}$), then an increase in the reservation utility (R) starting from R=0 makes centralization more likely to dominate decentralization.

This is an interesting result because it relates the "tightness" of the labor market to the optimal organizational structure. A higher reservation utility embodies a larger bargaining power for the agents, which can be represented by a tighter labor market where there are more jobs than workers. Proposition 4 argues that in sectors where employees are relatively more productive (high Δu), a city with a tighter labor market is more likely to have centralized firms than a city with a slack labor market. This essentially happens because it is never optimal for the principal to set a binding participation constraint in a centralized setting. This is due to the various factors already discussed in proposition 2-iii. A consequence of this slack participation constraint is that it shields the principal's profits from any moderate variation in the agents' reservation utility (starting from R=0). In contrast, when the value of information is sufficiently high, the principal finds it optimal to make the participation constraint bind, so he is fully affected by any variation of the reservation utility.

One might try to extrapolate and argue that during periods of economic expansion, centralization should be theoretically favored than decentralization on the basis of proposition 4 since the labor market should be tighter. However, during periods of economic expansion, the value of information, which can also be viewed as as a productivity parameters of the agents, is bound to go up. By Lemma 9, this creates an opposite force that favors decentralization. It is therefore difficult to argue for a definitive result linking periods of economic expansions/recessions to an optimal organizational structure because of the presence of multiple parameter variations with contradictory repercussions on the centralization versus decentralization question.

4 Endogenous Communication Noise

The communication noise between the principal and agents has been assumed to be exogenous. With some exogenous probability 1-q, none of the messages sent by the agents are received by the principal. This was meant to model the various communication problem that can arise when an employee discloses some data to his superior. However, in many situations, the principal has some control over the communication noise between his employees and himself. For instance, the principal can also choose to spend more time and efforts into acquiring information and understanding the agents' messages, like in Dessein, Galeotti and Santos (forthcoming)[13]. A company can also choose to adapt an intranet technology at some cost, which allows for an easier transfer of data on sales, forecasts of market conditions, etc [8]. Regardless of the mechanisms, allowing the principal to choose the level of communication noise at some cost can lead to a more powerful theory since this additional optimal choice adds a new channel of influence for each exogenous variable.

For expositional purposes, I will revert to the simplifying assumption that the prizes are exogenously determined. The timing of the game would have to be slightly modified to incorporate this new decision. After his decision to centralize but before the agents take their decisions to exert effort, the principal would choose q at some cost s(q). If the principal chooses decentralization, then the timing remains the same as before. This would lead to the following problem for the principal in a centralized setting:

$$\max_q 2q\Delta u\{2e_C^*(q) - [e_C^*(q)]^2\} + 2\hat{u} - W^U - W^L - s(q)$$

FOC(q) :

$$2\Delta u\{2e_C^*(q) - [e_C^*(q)]^2\} + 2q\Delta u\{2\frac{e_C^*(q)}{q} - 2[\frac{e_C^*(q)^2}{q}]\} - s'(q) = 0 \qquad (5)$$

$$\Leftrightarrow 2\Delta u e_C^*[4 - 3e_C^*] - s'(q) = 0$$

Assume, for tractability purposes, that $s(q) = \theta^P \frac{q^2}{2}$. With $e_C^* = \frac{q\Delta W}{2\theta}$, this yields:

$$\Leftrightarrow \frac{q\Delta u\Delta W}{\theta}(4 - \frac{3q\Delta W}{2\theta}) - \theta^P q = 0$$

[8]For more details, see Bloom et al. (2014)

$$\Leftrightarrow \frac{4\Delta u \Delta W - \theta\theta^P}{\theta} q - [\frac{3\Delta u (\Delta W)^2}{2\theta^2}] q^2 = 0$$

Besides q=0, the other optimal solution is

$$q^* = \begin{cases} 0 & \text{if } \frac{\theta\theta^P}{\Delta W \Delta u} \leq 4 \\ \frac{\theta}{\Delta W}(\frac{8}{3}) - \frac{\theta^2}{\Delta W^2}\frac{2\theta^P}{3\Delta u} & \text{if } \frac{\theta}{\Delta W}(\frac{8}{3}) - \frac{\theta^2}{\Delta W^2}\frac{2\theta^P}{3\Delta u} \in [0,1] \\ 1 & \text{if } 3\Delta W^2 \Delta u + 2\theta^2\theta^P \geq 8\theta \Delta W \Delta u. \end{cases}$$

Proposition 5

i) $\frac{\partial q^*}{\partial \theta^P} \leq 0$; $\frac{\partial q^*}{\partial \Delta u} \geq 0$

ii) There exists a $\frac{\tilde{\theta}}{\Delta \tilde{W}}$ such that:
-$\frac{\partial q^*}{\partial \Delta W} \leq 0$; $-\frac{\partial q^*}{\partial \theta} \leq 0$ for all $\frac{\Delta W}{\theta} > \frac{\Delta \tilde{W}}{\tilde{\theta}}$
-$\frac{\partial q^*}{\partial \Delta W} \geq 0$; $-\frac{\partial q^*}{\partial \theta} \geq 0$ for all $\frac{\Delta W}{\theta} < \frac{\Delta \tilde{W}}{\tilde{\theta}}$

The result that $\frac{\partial q^*}{\partial \theta^P} < 0$ is fairly straightforward: the harder it is for the principal to decrease the communication noise between himself and the agents, the more costly increasing q will become, resulting in an lesser q^*. Since a growing communication noise still favors decentralization, it is then obvious that a higher θ^P (a less competent principal) will strictly favor decentralization through a reduced q^*. As for $\frac{\partial q^*}{\partial \Delta u} > 0$, this comparative statics is also straightforward: the more value information has, the more resources the principal is willing to expand to ensure its appropriate use. This suggests that the more valuable a business report will be, the more time a manager will spend in trying to decipher it.

In order to understand proposition 5-ii), a more thorough understanding of q^* is needed. Reducing the communication noise (increasing q) has two positive effects. First, it leads to an increased chance of producing the extra Δu. For a given level of centralized effort, the principal has a higher chance of receiving the agents' messages than before. This is the communication benefit and is represented by the first term on the left hand side of equation 5: it always increases in e_C^* but has decreasing marginal returns to e_C^*.

The second positive impact is a motivational benefit and is represented by the second term on the left hand side of equation 5. When communication with the principal is subject to less communication noise, the benefits of acquiring information are increased from an agent's perspective. Agents are more likely to be judged based on the content of their message, which is a function of their information acquisition effort, rather than blind luck. This alters their optimal strategy and leads them to exert more effort. However, the productivity benefit also incorporates the manner in which these increased efforts are utilized. As Lemma 4 made it clear, increasing the centralized effort has the highest impact when $e_C^* = 1/2$ and its lowest impact when $e_C^* = 0$ or $e_C^* = 1$. Finally, increasing q has a cost, which is $\theta^P \frac{q^2}{2}$.

This decomposition exposes an interesting phenomenon: once $e_c^* \geq 0.75$, the sum of the communication and the production benefits of q^* will be diminished following an increase in ΔW or a decrease in θ. When further combined with the cost of q^*, this suggest a threshold of $\frac{\Delta \hat{W}}{\hat{\theta}}$ below which the principal spends more time in meetings or conference calls (raises q^*) following a rise in either $\frac{1}{\theta}$ or ΔW and above which the opposite occurs.

One implication of proposition 5 is that a centralized firm will tend react differently to a rise in either ΔW or $\frac{1}{\theta}$ depending on the quality of their employees. If promotion bonuses are high and employees are highly qualified ($\frac{\Delta W}{\theta} \geq \frac{\Delta \hat{W}}{\hat{\theta}}$), then the manager of a centralized firms should decide to spend less time in meetings or conference calls following a positive shock to bonuses or on-the-job training. This phenomenon is related to the shrinking productivity benefits of q^* and its increasing costs. This is due to the increasing odds of one effort being duplicated. Inversely, when employees are unskilled and poorly paid in a centralized setting, then the opposite result would hold given the low probability of efforts being duplicated. A manager should actually decide to spend more time meeting with his employees and going through their various reports in order to harvest these additional efforts.

5 Conclusion

In this paper, I study the influence of tournaments and the limited liability constraint on the optimal organizational structure. I showed that tournaments will harm cooperation between agents in a decentralized setting by inhibiting communication among rivals. I identify a fundamental trade-off between production noise, which harms decentralization, and communication noise, which harms centralization, during the decision to choose between centralization and decentralization. I then showed that while some classical results in the limited liability constraint literature continue to hold in this framework, I show that it can be optimal for the principal to increase the tournament prizes when faced with limited liability constraint. This is due to the different relation between an organizational structure's productivity and the agents' efforts. It has also been argued that the introduction of a limited liability constraint can strictly favor decentralization if the value of information (or the environment's productivity) is sufficiently high. This result is derived from the fact that a limited liability constraint will always depress profits in a centralized setting but will have no impact in a sufficiently productive decentralized setting. I then link the optimal organizational structure to the labor market in which the firm operates and argue that a tighter labor market will make it more likely that a firm will prefer a centralized setting. I finally show that the principal's incentives to micro-manage his employee will be a function of the quality of his employees, with the incentives to micro-manage being at their highest for employees whose productivities are average.

6 Appendix

6.1 Centralized Lagrangian without a Limited Liability Constraint

It is obvious that the participation constraint should clearly hold with equality. Substituting it into the principal's objective function yields

$$\max_{W^U, W^L} \frac{2q^2\Delta u}{\theta}(\Delta W - \frac{q\Delta W^2}{4\theta}) + 2\hat{u} - \frac{q^2\Delta W^2}{4\theta}$$

with a first order condition of

$$\frac{2q^2\Delta u}{\theta}(1 - \frac{q\Delta W}{2\theta}) - \frac{q^2\Delta W}{2\theta} = 0. \qquad \text{FOC}(W^U)$$

$$\Leftrightarrow \Delta W = \frac{4\Delta u\theta}{2q\Delta u + \theta} \tag{6}$$

I can rewrite the participation constraint (which holds with equality) as

$$2W^L + \Delta W = \frac{q^2\Delta W^2}{4\theta}. \tag{7}$$

By plugging in 6 into 7, I would get

$$W^L = \frac{2\Delta u^2\theta q^2 - 4q\Delta u^2\theta - 2\Delta u\theta^2}{(2q\Delta u + \theta)^2} \tag{8}$$

which is clearly negative. This results in

$$W^U = \frac{2q^2\Delta u^2\theta + 2\Delta u\theta^2 + 4q\Delta u^2\theta}{(2q\Delta u + \theta)^2}.$$

These are the solutions for the centralized problem without limited liability constraint for any parameters.

6.2 Decentralized Lagrangian without a Limited Liability Constraint

It is obvious that the participation constraint should hold with equality. After plugging the binding PC into the principal's objective function, the problem becomes

$$\max_{W^U, W^L} \frac{\Delta u^2 \Delta W}{L\theta} + 2\hat{u} - \frac{\Delta u^2 \Delta W^2}{4L^2\theta}$$

which yields a first order condition of

$$\frac{\Delta u^2}{L\theta} = \frac{\Delta u^2 \Delta W}{2L^2\theta} \qquad \text{FOC}(W^U)$$

$$\Leftrightarrow \Delta W = 2L$$

By plugging $\Delta W = 2L$ into the binding participation constraint, I get

$$2W^L + 2L = \theta \frac{\Delta u^2}{4L^2\theta^2} 4L^2$$

$$\Leftrightarrow W^L = \frac{\Delta u^2 - 2L\theta}{2\theta}$$

which results in

$$W^U = \frac{\Delta u^2 + 2L\theta}{2\theta}.$$

This results in a profit function of

$$\Pi^D = \frac{\Delta u^2}{\theta} + 2\hat{u}$$

If $\Delta u > \theta$, the principal is faced with a situation where $\frac{\Delta u \Delta W}{2L\theta} > 1$ but $e_D^* = 1$. When $\Delta u > \theta$, it therefore becomes optimal for the principal to change the spread of prizes to $\Delta W = \frac{2L\theta}{\Delta u}$ so that $e_D^* = 1$ and maintain a binding participation constraint

$$\frac{1}{2}(W^U W^L) = \frac{\theta}{2}(\frac{\Delta u \Delta W}{2L2})$$

$$\Leftrightarrow 2w^L + \Delta W = \theta\frac{\Delta u \Delta W}{2L\theta}$$

$$\Leftrightarrow W^L = \frac{\theta(\Delta u - 2L)}{2\Delta u}$$

which implies $W^U = \frac{\theta(\Delta u+2L)}{2\Delta u}$. The profits become

$$\Pi^D = 2\Delta u + 2\hat{u} - \theta$$

which is larger than $2\Delta u + 2\hat{u} - \frac{\Delta u^2}{\theta}$ when $\Delta u > \theta$. This results in a decentralized profit function without a limited liability constraint of

$$\Pi^D_{no\ LL} = \begin{cases} \frac{\Delta u^2}{\theta} + 2\hat{u} & \text{if } \Delta u \leq \theta \\ 2\Delta u + 2\hat{u} - \theta & \text{if } \Delta u \geq \theta. \end{cases}$$

6.3 Centralized Lagrangian with Limited Liability Constraint

Step 1: To be able to use the Kuhn-Tucker conditions, I must simply show that these three conditions hold.

Step 1-a: First, I must show that $f(W) = 2q\Delta u[\frac{q\Delta W}{\theta} - \frac{q^2\Delta W^2}{4\theta^2}] + 2\hat{u} - W^U - W^L$ is concave. For W^U, I have to show:

$$\frac{2q^2\Delta u}{\theta}[\lambda W^U + (1-\lambda)\hat{W}^U - W^L - \frac{q[\lambda W^U + (1-\lambda)\hat{W}^U - W^L]^2}{4\theta}]$$
$$+ 2\hat{u} - \lambda W^U - (1-\lambda)\hat{W}^U - W^L$$
$$\overset{?}{\geq} \lambda(\frac{2q^2\Delta u}{\theta})[W^U - W^L - \frac{q(W^U - W^L)^2}{4\theta}] + \lambda(2\hat{u} - W^U - W^L)$$
$$+ (1-\lambda)(\frac{2q^2\Delta u}{\theta})[\hat{W}^U - W^L - \frac{q(\hat{W}^U - W^L)^2}{4\theta}] + (1-\lambda)(2\hat{u} - \hat{W}^U - W^L)$$

$$\Leftrightarrow -[\lambda^2(W^U)^2 + \lambda(1-\lambda)W^U\hat{W}^U - \lambda W^U W^L + \lambda(1-\lambda)W^U\hat{W}^U + (1-\lambda)^2(\hat{W}^U)^2$$
$$- (1-\lambda)\hat{W}^U W^L - \lambda W^U W^L - (1-\lambda)\hat{W}^U W^L + (W^L)^2] \overset{?}{\geq}$$
$$-[\lambda(W^U)^2 - 2\lambda W^U W^L + \lambda(W^L)^2] - [(1-\lambda)(\hat{W}^U)^2 - 2(1-\lambda)\lambda\hat{W}^U W^L + (1-\lambda)(W^L)^2]$$

$$\Leftrightarrow \lambda(W^U)^2(\lambda-1) + 2\lambda(1-\lambda)W^U\hat{W}^U + (1-\lambda)(\hat{W}^U)^2(-\lambda) \overset{?}{\leq} 0$$

$$\Leftrightarrow 0 \overset{?}{\leq} (\hat{W}^U - W^U)^2$$

which always hold. As for W^L, I have to show

$$\frac{2q^2\Delta u}{\theta}[W^U - \lambda W^L - (1-\lambda)\hat{W}^L - \frac{q[W^U - \lambda W^L - (1-\lambda)\hat{W}^L]^2}{4\theta}]$$
$$+ 2\hat{u} - W^U - \lambda W^L - (1-\lambda)\hat{W}^L$$
$$\overset{?}{\geq} \lambda(\frac{2q^2\Delta u}{\theta})[W^U - W^L - \frac{q(W^U - W^L)^2}{4\theta}] + \lambda(2\hat{u} - W^U - W^L)$$
$$+ (1-\lambda)(\frac{2q^2\Delta u}{\theta})[W^U - \hat{W}^L - \frac{q(W^U - \hat{W}^L)^2}{4\theta}] + (1-\lambda)(2\hat{u} - W^U - \hat{W}^L)$$

$$\Leftrightarrow -[(W^U)^2 - \lambda W^U W^L - (1-\lambda)W^U\hat{W}^L - \lambda W^U W^L + \lambda^2(W^L)^2$$
$$\lambda(1-\lambda)W^L\hat{W}^L - (1-\lambda)W^U\hat{W}^L + \lambda(1-\lambda)W^L\hat{W}^L + (1-\lambda)^2(\hat{W}^L)^2]$$
$$\overset{?}{\geq} -\lambda[(W^U)^2 - 2W^U W^L + (W^L)^2] - (1-\lambda)[(W^U)^2 - 2W^U\hat{W}^L + (\hat{W}^L)^2]$$

which simplifies into

$$\Leftrightarrow 0 \overset{?}{\leq} (W^L - \hat{W}^L)^2$$

which always hold. This means $2q\Delta u[\frac{q\Delta W}{\theta} - \frac{q^2\Delta W^2}{4\theta^2}] + 2\hat{u} - W^U - W^L$ is concave in both of its arguments.

Step 1-b:

Now, I define the PC constraint as $g(W) = \frac{q^2(W^U - W^L)^2}{4\theta} - W^U - W^L$ with $W = (W^L, W^U)$. I now show that g(W) is convex in W^U:

$$\frac{q^2[\lambda W^U + (1-\lambda)\hat{W}^U - W^L]^2}{4\theta} - \lambda W^U - (1-\lambda)\hat{W}^U - W^L \overset{?}{\leq}$$
$$\frac{\lambda q^2(W^U - W^L)^2}{4\theta} - \lambda W^U - \lambda W^L + \frac{(1-\lambda)q^2(\hat{W}^U - W^L)^2}{4\theta} - (1-\lambda)\hat{W}^U - (1-\lambda)W^L$$

$$\Leftrightarrow \lambda^2(W^U)^2+\lambda(1-\lambda)W^U\hat{W}^U-\lambda W^UW^L+\lambda(1-\lambda)W^U\hat{W}^U+(1-\lambda)^2(\hat{W}^U)^2$$
$$-(1-\lambda)\hat{W}^UW^L-\lambda W^UW^L-(1-\lambda)\hat{W}^UW^L+(W^L)^2 \overset{?}{\leq} \lambda(W^U)^2-2\lambda W^UW^L$$
$$+\lambda(W^L)^2+(1-\lambda)(\hat{W}^U)^2-2(1-\lambda)\hat{W}^UW^L+(1-\lambda)(W^L)^2$$

$$\Leftrightarrow 0 \overset{?}{\leq} (W^U-\hat{W}^U)^2$$

which always holds. I now have to show that g(W) is convex in W^L:

$$\frac{q^2[W^U-\lambda W^L-(1-\lambda)(\hat{W}^L)]^2}{4\theta}-W^U-\lambda W^L-(1-\lambda)\hat{W}^L \overset{?}{\leq} \frac{\lambda q^2(W^U-W^L)^2}{4\theta}$$
$$-\lambda W^U-\lambda W^L+\frac{(1-\lambda)q^2(W^U-\hat{W}^L)^2}{4\theta}-(1-\lambda)W^U-(1-\lambda)\hat{W}^L$$

$$\Leftrightarrow (W^U)^2-\lambda W^UW^L-(1-\lambda)W^U\hat{W}^L-\lambda W^UW^L+\lambda^2(W^L)^2+\lambda(1-\lambda)W^L\hat{W}^L$$
$$-(1-\lambda)W^U\hat{W}^L+\lambda(1-\lambda)W^L\hat{W}^L+(1-\lambda)^2(\hat{W}^L)^2 \overset{?}{\leq} \lambda(W^U)^2-2\lambda W^UW^L$$
$$+\lambda(W^L)^2+(1-\lambda)(W^U)^2-2(1-\lambda)W^U\hat{W}^L+(1-\lambda)(\hat{W}^L)^2$$

$$\Leftrightarrow 0 \overset{?}{\leq} (W^L-\hat{W}^L)^2$$

which always holds. Therefore, the PC constraint $g(W)=\frac{q^2(W^U-W^L)^2}{4\theta}-W^U-W^L$ is convex. The remaining limited liability constraint (LL) is linear and therefore convex.

<u>Step 1-c</u>:

Thirdly, since the constraints of centralized problem are convex, we also need to ensure the existence of a set of parameters such that

$$W^U+W^L>\frac{q^2}{4\theta}(W^U-W^L)^2$$

and

$$W^L>0$$

hold in order to use the Kuhn-Tucker conditions. Since I have not impose any restrictions on the parameters such that the above inequalities cannot hold, the third conditions is satisfied.

Step 2: I can now justify using the Kuhn-Tucker conditions to solve for the centralized problem with the limited liability constraint.

The Lagrangian of this problem is

$$\mathcal{L} = \frac{2q^2\Delta u}{\theta}[W^U - W^l - \frac{q}{4\theta}(W^U - W^L)^2] + 2\hat{u} - W^U - W^L \\ - \lambda[\frac{q^2(W^U - W^L)^2}{4\theta} - W^U - W^L] + \mu W^L.$$

The Kuhn-Tucker conditions are

$$\frac{\partial \mathcal{L}}{\partial W^U} = 0$$

$$\Leftrightarrow \frac{2q^2\Delta u}{\theta}[1 - \frac{q}{2\theta}(W^U - W^L)] - 1 - \lambda[\frac{q^2}{2\theta}(W^U - W^L) - 1] = 0 \quad (9)$$

$$\frac{\partial \mathcal{L}}{\partial W^L} = 0$$

$$\Leftrightarrow \frac{2q^2\Delta u}{\theta}[-1 + \frac{q}{2\theta}(W^U - W^L)] - 1 - \lambda[-\frac{q^2}{2\theta}(W^U - W^L) - 1] + \mu = 0 \quad (10)$$

$$\lambda[\frac{q^2(W^U - W^L)^2}{4\theta} - W^U - W^L] = 0 \quad (11)$$

$$\mu W^L = 0 \quad (12)$$

as well as $W^U + W^L \geq \frac{q^2(W^U - W^L)^2}{4\theta}$, $W^L \geq 0$, $\lambda \geq 0$ and $\mu \geq 0$.

Case $\lambda = 0$: By 9, this yields

$$\frac{2q^2\Delta u}{\theta}[1 - \frac{q}{2\theta}(W^U - W^L)] = 1$$

$$\Leftrightarrow \frac{\theta(2q^2\Delta u - \theta)}{q^3\Delta u} = \Delta W. \quad (13)$$

I then put 13 into 10 to get

$$\frac{2q^2\Delta u}{\theta}[\frac{q}{2\theta}\frac{\theta(2q^2\Delta u-\theta)}{q^3\Delta u}-1]-1+\mu=0$$

which simplifies into

$$\Leftrightarrow \mu = 2.$$

The fact that $\mu > 0$ implies $W^L = 0$ by 12 which then implies by 13

$$W^U = \frac{\theta(2q^2\Delta u-\theta)}{q^3\Delta u}.$$

So one possible **solution** is $W^U = \frac{\theta(2q^2\Delta u-\theta)}{q^3\Delta u}$, $W^L = 0$, $\lambda = 0$ and $\mu = 2$. Obviously, this solution only holds when $2q^2\Delta u > \theta$ since $W^U \geq 0$ is a necessary condition for PC to hold when $W^L = 0$.

By looking at the Kuhn-Tucker conditions, it can be observed that $W^U = W^L = 0$ is a possible solution. Indeed, if $W^U = W^L = 0$, then 9 implies

$$\frac{2q^2\Delta u}{\theta} - 1 + \lambda = 0$$

$$\lambda = \frac{\theta - 2q^2\Delta u}{\theta}.$$

The Kuhn-Tucker condition $\lambda \geq 0$ requires $\theta \geq 2q^2\Delta u$, which complements the previous solution. Finally, 10 implies

$$\frac{-2q^2\Delta u}{\theta} - 1 + (\frac{\theta - 2q^2\Delta u}{\theta}) + \mu = 0$$

$$\mu = \frac{4q^2\Delta u}{\theta}.$$

So another **solution** is $W^L = 0$, $W^U = 0$, $\lambda = \frac{\theta-2q^2\Delta u}{\theta}$ and $\mu = \frac{4q^2\Delta u}{\theta}$.

Case $\lambda > 0$: By condition 11, this implies

$$W^U + W^L = \frac{q^2}{4\theta}(W^U)^2 - \frac{q^2}{2\theta}W^U W^L + \frac{q^2}{4\theta}(W^L)^2$$

$$\Leftrightarrow 0 = (W^U)^2 - [\frac{2(2\theta + q^2 W^L)}{q^2}]W^U + (W^L)^2 - \frac{4\theta}{q^2}W^L.$$

This yields

$$W^U = \frac{2q^2W^L + 4\theta}{2q^2}$$

$$\pm \frac{1}{2}\sqrt{\frac{4q^4(W^L)^2 + 16q^2\theta W^L + 16\theta^2}{q^4} - 4(W^L)^2 + \frac{16\theta W^L}{q^2}}$$

$$\Leftrightarrow W^U = \frac{2q^2W^L + 4\theta}{2q^2} \pm \frac{1}{2}\sqrt{\frac{32q^2\theta W^L + 16\theta^2}{q^4}}$$

$$\Leftrightarrow W^U = W^L + \frac{2\theta}{q^2} \pm \frac{2}{q^2}\sqrt{\theta(2q^2W^L + \theta)}. \tag{14}$$

Subcase $\mu = 0$:

Condition 9 implies

$$\frac{2q^2\Delta u\theta - q^3\Delta W\Delta u - \theta^2}{\theta^2} = \lambda(\frac{q^2\Delta W - 2\theta}{2\theta})$$

$$\Leftrightarrow \lambda = \frac{2(2q^2\Delta u\theta - q^3\Delta W\Delta u - \theta^2)}{\theta(q^2\Delta W - 2\theta)}$$

and condition 10 implies

$$\lambda(\frac{q^2\Delta W + 2\theta}{2\theta}) = \frac{2q^2\theta\Delta u - q^3\Delta u\Delta W + \theta^2}{\theta^2}$$

$$\Leftrightarrow \lambda = \frac{2(2q^2\Delta u\theta - q^3\Delta W\Delta u + \theta^2)}{\theta(q^2\Delta W + 2\theta)}.$$

Combining both modified conditions, I get

$$\frac{2(2q^2\Delta u\theta - q^3\Delta W\Delta u - \theta^2)}{\theta(q^2\Delta W - 2\theta)} = \frac{2(2q^2\Delta u\theta - q^3\Delta W\Delta u + \theta^2)}{\theta(q^2\Delta W + 2\theta)}$$

$$\Leftrightarrow 4\theta(2q^2\Delta u\theta - q^3\Delta u\Delta W) = 2q^2\theta^2\Delta W$$

$$\Leftrightarrow W^U = W^L + \frac{4\Delta u\theta}{\theta + 2q\Delta u}. \tag{15}$$

When I combine 14(+) and 15, I get

$$\frac{4\Delta u\theta}{\theta + 2q\Delta u} = \frac{2\theta}{q^2} + \frac{2}{q^2}\sqrt{\theta(2q^2W^L + \theta)}$$

$$\Leftrightarrow \frac{2q^2\Delta u\theta}{\theta + 2q\Delta u} - \theta = \sqrt{\theta(2q^2W^L + \theta)}$$

$$\Leftrightarrow \frac{4q^4\Delta u^2\theta^2}{(\theta + 2q\Delta u)^2} - \frac{4q^2\Delta u\theta^2}{\theta + 2q\Delta u} + \theta^2 = \theta(2q^2W^L + \theta)$$

$$\Leftrightarrow W^L = \frac{2q^2\Delta u^2\theta}{(\theta + 2q\Delta u)^2} - \frac{2\Delta u\theta}{\theta + 2q\Delta u}.$$

However, I need $W^L \geq 0$, which is equivalent to

$$2q^2\Delta u^2\theta \geq 2\Delta u\theta(\theta + 2q\Delta u)$$

$$\Leftrightarrow q\Delta u(q-2) \geq \theta$$

which does not hold. Repeating the procedure with 14 (-) yields the same outcome. Therefore, a solution with $\lambda > 0$ and $\mu = 0$ is not feasible.

Subcase $\mu > 0$

This implies $W^L = 0$. By 11, it also implies $W^U = \frac{4\theta}{q^2}$ or $W^U = 0$. The case for $W^U = W^L = 0$ has already been study so it will be ignored.

From 9, this implies

$$\frac{2q^2\Delta u}{\theta} - \frac{q^3\Delta u}{\theta^2}\frac{4\theta}{q^2} - 1 - \lambda = 0$$

$$\Leftrightarrow \lambda = \frac{2q\Delta u(q-2)}{\theta} - 1$$

which is a contradiction with $\lambda \geq 0$. Therefore, a solution with $\lambda \geq 0$ and $\mu \geq 0$ is not feasible.

6.4 Decentralized Lagrangian with a Limited Liability Constraint

Step 1: Once again, to use the Kuhn-Tucker conditions, I must show that three conditions hold. First, the objective function has to be concave. Since $\frac{\Delta u^2 \Delta W}{L\theta} + 2\hat{u} - W^U - W^L$ is linear, it is also concave. Second, the constraint $\frac{\Delta u \Delta W^2}{4L^2\theta} - W^U - W^L$ is convex in W^U if and only if

$$\frac{\Delta u^2}{4L^2\theta}[\lambda W^U + (1-\lambda)\hat{W}^U - W^L]^2 - \lambda W^U - (1-\lambda)\hat{W}^U - W^L$$
$$\overset{?}{\leq} \frac{\lambda \Delta u^2 (W^U - W^L)^2}{4L^2\theta} - \lambda W^U - \lambda W^L + \frac{(1-\lambda)\Delta u^2(\hat{W}^U - W^L)^2}{4L^2\theta} - (1-\lambda)\hat{W}^U - (1-\lambda)W^L \qquad \lambda \in [0,1]$$

$$\Leftrightarrow 0 \overset{?}{\leq} \lambda(1-\lambda)(W^U)^2 + \lambda(1-\lambda)(\hat{W}^U)^2 - 2\lambda(1-\lambda)W^U\hat{W}^U$$

$$\Leftrightarrow 0 \overset{?}{\leq} (W^U - \hat{W}^U)^2$$

which always hold. It also needs to be convex in W^L:

$$\frac{\Delta u^2}{4L^2\theta}[W^U - \lambda W^L - (1-\lambda)\hat{W}^L]^2 - W^U - \lambda W^L - (1-\lambda)\hat{W}^L$$
$$\overset{?}{\leq} \frac{\lambda \Delta u^2 (W^U - W^L)^2}{4L^2\theta} - \lambda W^U - \lambda W^L + \frac{(1-\lambda)\Delta u^2(W^U - \hat{W}^L)^2}{4L^2\theta} - (1-\lambda)W^U - (1-\lambda)\hat{W}^L \qquad \lambda \in [0,1]$$

$$\Leftrightarrow 0 \overset{?}{\leq} \lambda(1-\lambda)(W^L)^2 + \lambda(1-\lambda)(\hat{W}^L)^2 - 2\lambda(1-\lambda)W^L\hat{W}^L$$

$$\Leftrightarrow 0 \overset{?}{\leq} (W^L - \hat{W}^L)^2$$

which always hold. Finally, I need to ensure the existence of a set of parameters such that

$$0 > \frac{\Delta u^2 \Delta W^2}{4L^2\theta} - W^U - W^L$$

and $W^L > 0$ hold. Since I have not imposed any restrictions on these parameters, then the above inequalities can hold.

Step 2: The Lagrangian of this problem is

$$\mathcal{L} = \frac{\Delta u^2 \Delta W}{L\theta} + 2\hat{u} - W^U - W^L - \lambda(\frac{\Delta u^2 \Delta W^2}{4L^2\theta} - W^U - W^L) + \mu W^L.$$

I can then use the Kuhn-Tucker conditions

$$\frac{\partial \mathcal{L}}{\partial W^U} = 0$$

$$\Leftrightarrow \frac{\Delta u^2}{L\theta} - 1 - \lambda(\frac{\Delta u^2 \Delta W}{2L^2\theta} - 1) = 0 \tag{16}$$

$$\frac{\partial \mathcal{L}}{\partial W^L} = 0$$

$$\Leftrightarrow -\frac{\Delta u^2}{L\theta} - 1 - \lambda(-\frac{\Delta u^2 \Delta W}{2L^2\theta} - 1) + \mu = 0 \tag{17}$$

$$\lambda[\frac{\Delta u^2 \Delta W^2}{4L^2\theta} - W^U - W^L] = 0 \tag{18}$$

$$\mu W^L = 0 \tag{19}$$

as well as $\lambda \geq 0$, $\mu \geq 0$, $W^U + W^L \geq \frac{\Delta u^2 \Delta W^2}{4L^2\theta}$ and $W^L \geq 0$ to find a solution to the decentralized problem.

Case $\lambda = 0$: It can be seen that 16 implies $\frac{\Delta u^2}{L\theta} - 1 = 0$. This clearly indicates the presence of a corner solutions. If $\Delta u \geq \sqrt{L\theta}$, then W^U should be as high as possible. If $\Delta u \leq \sqrt{L\theta}$, then W^U should be zero.

Subcase $\mu = 0$: By 17, this implies $-\frac{\Delta u^2}{L\theta} - 1$ which also indicates a corner solutions suggesting W^L should be as low as possible.

Since W^U needs to be at a maximum and W^L at a minimum, I will set ΔW such that $e_D^* = 1$. This yields $\Delta W = \frac{2L\theta}{\Delta u}$. The minimum value W^L can have is provided by the participation constraint:

$$W^L + W^U \geq \frac{\Delta u^2}{4L^2\theta} \Delta W^2$$

$$\Leftrightarrow 2W^L + \Delta W \geq \frac{\Delta u^2}{4L^2\theta} \Delta W^2$$

$$\Leftrightarrow 2W^L + \frac{2L\theta}{\Delta u} \geq \frac{\Delta u^2}{4L^2\theta} (\frac{2L\theta}{\Delta u})^2$$

$$\Leftrightarrow W^L \geq \frac{\theta(\Delta u - 2L)}{2\Delta u}.$$

Given the corner solution of 17 when $\lambda = \mu = 0$, it can be concluded that $W^L = \frac{\theta(\Delta u - 2L)}{2\Delta u}$. Along with $\Delta W = \frac{2L\theta}{\Delta u}$, this implies $W^U = \frac{\theta(\Delta u + 2L)}{2\Delta u}$. Since the constraint $W^L \geq 0$ always has to hold, this solution is only possible if $\Delta u \geq 2L$ and $\Delta u \geq \sqrt{L\theta}$.

A possible **solution** is therefore $\lambda = \mu = 0$, $W^L = \frac{\theta(\Delta u - 2L)}{2\Delta u}$ and $W^U = \frac{\theta(\Delta u + 2L)}{2\Delta u}$ when $\Delta u \geq 2L$.

Subcase $\mu > 0$: By 17, this implies that $\mu = 1 + \frac{\Delta u^2}{L\theta}$. Since $\mu > 0$, 19 requires $W^L = 0$. This implies $W^U = \Delta W$. If $\Delta u \geq \sqrt{L\theta}$, then $W^U = \Delta W$ needs to be as high as possible. Since the benefits of ΔW comes from motivating the agent, the upper boundary for ΔW is set by $e_D^* = 1$ which results in $W^U = \frac{2L\theta}{\Delta u}$.

Since the participation constraint always has to hold,

$$\frac{2L\theta}{\Delta u} \geq \frac{\Delta u^2}{4L^2\theta}(\frac{4L^2\theta^2}{\Delta u^2})$$

$$\Leftrightarrow 2L \geq \Delta u$$

has to hold. Therefore, a another possible **solution** is $\lambda = 0$, $\mu = 1 + \frac{\Delta u^2}{L\theta}$, $W^L = 0$ and $W^U = \frac{2L\theta}{\Delta u}$ if $\Delta u \in [\sqrt{L\theta}, 2L]$.

<u>Case $\lambda > 0$</u>: This case and 18 implies $\frac{\Delta u^2 \Delta W^2}{4L^2\theta} = W^U + W^L$.

Subcase $\mu > 0$: By 19, this implies $W^L = 0$ and, by 18, $W^U = \frac{4L^2\theta}{\Delta u^2}$.

By 16, this implies

$$\frac{\Delta u^2 - L\theta}{L\theta} = \lambda.$$

By 17, this implies

$$\mu + \lambda 3 = \frac{\Delta u^2}{L\theta}.$$

This implies

$$\mu = \frac{2(2L\theta - \Delta u^2)}{L\theta}.$$

In order for the conditions $\lambda \geq 0$ and $\mu \geq 0$ to be satisfied simultaneously, $\Delta \in [\sqrt{L\theta}, \sqrt{2L\theta}]$ is required. Therefore, it can be seen that, if $\Delta u^2 \in [L\theta, 2L\theta]$, a **solution** is $W^U = \frac{4L^2\theta}{\Delta u^2}$, $W^L = 0$, $\lambda = \frac{\Delta u^2 - L\theta}{L\theta}$ and $\mu = \frac{2(2L\theta - \Delta u^2)}{L\theta}$.

Subcase $\mu = 0$: This implies $W^L \geq 0$.

If $\mu = 0$ and $W^L = 0$, then $W^U = \frac{4L^2\theta}{\Delta u^2}$. By 16 and 17, $\Delta u = \sqrt{2L\theta}$. But this is simply part of the previous solution.

If $\mu = 0$ and $W^L > 0$, then 18 implies

$$\frac{\Delta u^2[(W^U)^2 - 2W^U W^L + (W^L)^2]}{4L^2\theta} = W^U + W^L$$

$$\Leftrightarrow \Delta u^2 (W^U)^2 - 2\Delta u^2 W^U W^L + \Delta u^2 (W^L)^2 = 4L^2\theta W^U + 4L^2\theta W^L$$

$$\Leftrightarrow (W^U)^2 - W^U(\frac{2\Delta u^2 W^L + 4L^2\theta}{\Delta u^2}) + (W^L)^2 - \frac{4L^2\theta}{\Delta u^2} W^L = 0$$

This yields

$$\begin{aligned} W^U = & \frac{2\Delta u^2 W^L + 4L^2\theta}{2\Delta u^2} \\ & \pm \frac{1}{2}\sqrt{\frac{4\Delta u^4 (W^L)^2 + 16\Delta u^2 L^2\theta W^L + 16L^4\theta^2}{\Delta u^4} - 4(W^L)^2 + \frac{16L^2\theta W^L}{\Delta u^2}} \end{aligned}$$

$$\Leftrightarrow W^U = \frac{2\Delta u^2 W^L + 4L^2\theta}{2\Delta u^2} \pm \frac{\sqrt{32\Delta u^2 L^2\theta W^L + 16L^4\theta^2}}{2\Delta u^2}$$

$$\Leftrightarrow W^U = \frac{2\Delta u^2 W^L + 4L^2\theta}{2\Delta u^2} \pm 4L\frac{\sqrt{2\Delta u^2\theta W^L + L^2\theta^2}}{2\Delta u^2}. \tag{20}$$

Also, 16 implies

$$\frac{\Delta u^2}{L\theta} - 1 - \lambda(\frac{\Delta u^2 \Delta W - 2L^2\theta}{2L^2\theta}) = 0$$

$$\Leftrightarrow \lambda = \frac{2L(\Delta u^2 - L\theta)}{\Delta u^2 \Delta W - 2L^2\theta} \tag{21}$$

and 17 implies

$$\lambda(\frac{\Delta u^2 \Delta W + 2L^2\theta}{2L^2\theta}) = \frac{\Delta u^2}{L\theta} + 1$$

$$\Leftrightarrow \lambda = \frac{2L(\Delta u^2 + L\theta)}{\Delta u^2 \Delta W + 2L^2\theta}. \quad (22)$$

Equations 21 and 22 imply

$$\frac{2L(\Delta u^2 + L\theta)}{\Delta u^2 \Delta W + 2L^2\theta} = \frac{2L(\Delta u^2 - L\theta)}{\Delta u^2 \Delta W - 2L^2\theta}$$

$$\Leftrightarrow \Delta u^4 \Delta W - \Delta u^2 2L^2\theta + L\theta\Delta u^2 \Delta W - 2L^3\theta^2$$
$$= \Delta u^4 \Delta W + \Delta u^2 2L^2\theta - L\theta\Delta u^2 \Delta W - 2L^3\theta^2$$

$$\Leftrightarrow 2L\theta\Delta u^2\Delta W = 4L^2\theta\Delta u^2$$

$$\Leftrightarrow \Delta W = 2L$$

$$W^U = W^L + 2L. \quad (23)$$

I can then combine equation 20 (with +) and 23 to get

$$W^L = W^L + \frac{2L^2\theta}{\Delta u^2} + \frac{2L\sqrt{2\Delta u^2\theta W^L + L^2\theta^2}}{\Delta u^2} - 2L$$

$$\Leftrightarrow (\Delta u^2 - L\theta)^2 = 2\Delta u^2\theta W^L + L^2\theta^2$$

$$\Leftrightarrow W^L = \frac{\Delta u^2 - 2L\theta}{2\theta}$$

which implies

$$W^U = \frac{\Delta u^2 + 2L\theta}{2\theta}.$$

These prizes lead to

$$\lambda = \frac{(\Delta u^2 + L\theta)2L}{\Delta u^2 2L + 2L^2\theta} \Leftrightarrow \lambda = 1.$$

It can then be verified that a combination of 20 (with -) and 23 yield the same solution. If $\Delta u \geq \sqrt{2L\theta}$, then the **solution** is $W^L = \frac{\Delta u^2 - 2L\theta}{2\theta}$, $W^U = \frac{\Delta u^2 + 2L\theta}{2\theta}$, $\mu = 0$ and $\lambda = 1$.

It can also be observed that $W^L = W^L = 0$ satisfies both 18 and 19. $W^U = W^L = 0$ and 16 implies

$$\frac{\Delta u^2}{L\theta} - 1 + \lambda = 0 \Leftrightarrow \lambda = \frac{L\theta - \Delta u^2}{L\theta}.$$

By 17, it can also be seen that

$$-\frac{\Delta u^2}{L\theta} - 1 + (\frac{L\theta - \Delta u^2}{L\theta}) + \mu = 0$$

$$\Leftrightarrow \mu = \frac{2\Delta u^2}{L\theta}.$$

Therefore, if $\Delta u \leq \sqrt{L\theta}$, then $W^L = W^U = 0$, $\mu = \frac{2\Delta u^2}{L\theta}$ and $\lambda = \frac{L\theta - \Delta u^2}{L\theta}$ is a **solution**.

The following Lemmas are necessary in order to decipher which solution dominates for a specific set of parameters.

Lemma 10: The set of wages $W^L = 0$ and $W^U = \frac{4L^2\theta}{\Delta u^2}$ are more profitable than $W^L = 0$ and $W^U = \frac{2L\theta}{\Delta u}$ if and only if $\Delta u \geq 2L$.

Lemma 11: The set of wages $W^L = \frac{\Delta u^2 - 2L\theta}{2\theta}$ and $W^U = \frac{\Delta u^2 + 2L\theta}{2\theta}$ are more profitable than $W^L = \frac{(\Delta u - 2L)\theta}{2\Delta u}$ and $W^U = \frac{(\Delta u + 2L)\theta}{2\Delta u}$ if and only if $\Delta u \leq \theta$.

Lemma 12: If $\Delta u \geq \theta$ and $\Delta u \geq \sqrt{2L\theta}$ hold, then the set of wages $W^L = 0$ and $W^U = \frac{2L\theta}{\Delta u}$ is more profitable than $W^L = \frac{\Delta u^2 - 2L\theta}{2\theta}$ and $W^U = \frac{\Delta u^2 + 2L\theta}{2\theta}$.

Finally, this Lagrangian results in five different solutions. Focusing strictly on the solutions with $\lambda > 0$, it can be seen that three solutions arise for three different intervals: the solution $W^U = W^L = 0$ for $\Delta u \leq \sqrt{L\theta}$, the solution $W^L = 0$ and $W^U = \frac{4L^2\theta}{\Delta u^2}$ for $\Delta u \in [\sqrt{L\theta}, \sqrt{2L\theta}]$ and the solution $W^L = \frac{\Delta u^2 - 2L\theta}{2\theta}$ and $W^U = \frac{\Delta u^2 + 2L\theta}{2\theta}$ for $\Delta u \geq \sqrt{2L\theta}$. How the solutions with $\lambda = 0$ fit in here depend on where $2L$ goes insides these three intervals.

If $2L \geq \sqrt{2L\theta} \Leftrightarrow \theta \leq 2L$, then the solution $W^L = 0$ and $W^U = \frac{4L^2\theta}{\Delta u^2}$ is dominated by $W^L = 0$ and $W^U = \frac{2L\theta}{\Delta u}$ for all $\Delta u \in [\sqrt{L\theta}, \sqrt{2L\theta}]$ by Lemma 10. Furthermore, for any $\Delta u \in [\sqrt{2L\theta}, 2L]$, the solution $W^L = 0$ and $W^U = \frac{2L\theta}{\Delta u}$ dominates $W^L = \frac{\Delta u^2 - 2L\theta}{2\theta}$ and $W^U = \frac{\Delta u^2 + 2L\theta}{2\theta}$ by Lemma 12. Lastly, when $\Delta u \geq 2L$ which implies $\Delta u \geq \theta$, then the solution $W^L = \frac{(\Delta u - 2L)\theta}{2\Delta u}$ and

$W^U = \frac{(\Delta u+2L)\theta}{2\Delta u}$ dominates $W^L = \frac{\Delta u^2-2L\theta}{2\theta}$ and $W^U = \frac{\Delta u^2+2L\theta}{2\theta}$ by Lemma 11 and $W^L = 0$ and $W^U = \frac{2L\theta}{\Delta u}$ is no longer feasible. The solutions are then

$$W^L = \begin{cases} 0 & \text{when } \Delta u \leq \sqrt{L\theta} \\ 0 & \text{when } \Delta u \in [\sqrt{L\theta}, 2L] \\ \frac{(\Delta u-2L)\theta}{2\Delta u} & \text{when } \Delta u \geq 2L \end{cases}$$

$$W^U = \begin{cases} 0 & \text{when } \Delta u \leq \sqrt{L\theta} \\ \frac{2L\theta}{\Delta u} & \text{when } \Delta \in [\sqrt{L\theta}, 2L] \\ \frac{(\Delta u+2L)\theta}{2\Delta u} & \text{when } \Delta u \geq 2L \end{cases}$$

and

$$\Pi^D = \begin{cases} 2\hat{u} \equiv \Pi_0^D & \text{when } \Delta u \leq \sqrt{L\theta} \\ 2\Delta u + 2\hat{u} - \frac{2L\theta}{\Delta u} \equiv \Pi_1^D & \text{when } \Delta u \in [\sqrt{L\theta}, 2L] \\ 2\Delta u - \theta + 2\hat{u} \equiv \Pi_4^D & \text{when } \Delta u \geq 2L. \end{cases}$$

If $2L \leq \sqrt{L\theta} \Leftrightarrow \theta \geq 4L$, the solution $W^L = 0$ and $W^U = \frac{4L^2\theta}{\Delta u^2}$ dominate $W^L = 0$ and $W^U = \frac{2L\theta}{\Delta u}$ by Lemma 10 for all $\Delta u \in [\sqrt{L\theta}, \sqrt{2L\theta}]$. Also, the solution $W^L = \frac{\Delta u^2-2L\theta}{2\theta}$ and $W^U = \frac{\Delta u^2+2L\theta}{2\theta}$ is guaranteed to exist by Lemma 11 since the interval $[\sqrt{2L\theta}, \theta]$ is non-empty by assumption. The solutions would then be

$$W^L = \begin{cases} 0 & \text{when } \Delta u \leq \sqrt{L\theta} \\ 0 & \text{when } \Delta u \in [\sqrt{L\theta}, \sqrt{2L\theta}] \\ \frac{\Delta u^2-2L\theta}{2\theta} & \text{when } \Delta u \in [\sqrt{2L\theta}, \theta] \\ \frac{(\Delta u-2L)\theta}{2\Delta u} & \text{when } \Delta u \geq \theta \end{cases}$$

$$W^U = \begin{cases} 0 & \text{when } \Delta u \leq \sqrt{L\theta} \\ \frac{4L^2\theta}{\Delta u^2} & \text{when } \Delta u \in [\sqrt{L\theta}, \sqrt{2L\theta}] \\ \frac{\Delta u^2+2L\theta}{2\theta} & \text{when } \Delta u \in [\sqrt{2L\theta}, \theta] \\ \frac{(\Delta u+2L)\theta}{2\Delta u} & \text{when } \Delta u \geq \theta \end{cases}$$

and

$$\Pi^D = \begin{cases} 2\hat{u} \equiv \Pi_0^D & \text{when } \Delta u \leq \sqrt{L\theta} \\ \frac{4L(\Delta u^2-L\theta)}{\Delta u^2} + 2\hat{u} \equiv \Pi_2^D & \text{when } \Delta u \in [\sqrt{L\theta}, \sqrt{2L\theta}] \\ \frac{\Delta u^2}{\theta} + 2\hat{u} \equiv \Pi_3^D & \text{when } \Delta u \in [\sqrt{2L\theta}, \theta] \\ 2\Delta u - \theta + 2\hat{u} \equiv \Pi_4^D & \text{when } \Delta u \geq \theta. \end{cases}$$

If $2L \leq \sqrt{2L\theta}$ and $2L \geq \sqrt{L\theta}$ simultaneously hold, which is the equivalent of $\theta \in [2L, 4L]$, then all five solutions are guaranteed to exist and dominate within their own intervals. The solutions would then be

$$W^L = \begin{cases} 0 & \text{when } \Delta u \leq \sqrt{L\theta} \\ 0 & \text{when } \Delta u \in [\sqrt{L\theta}, 2L] \\ 0 & \text{when } \Delta u \in [2L, \sqrt{2L\theta}] \\ \frac{\Delta u^2 - 2L\theta}{2\theta} & \text{when } \Delta u \in [\sqrt{2L\theta}, \theta] \\ \frac{(\Delta u - 2L)\theta}{2\Delta u} & \text{when } \Delta u \geq \theta \end{cases}$$

$$W^U = \begin{cases} 0 & \text{when } \Delta u \leq \sqrt{L\theta} \\ \frac{2L\theta}{\Delta u} & \text{when } \Delta \in [\sqrt{L\theta}, 2L] \\ \frac{4L^2\theta}{\Delta u^2} & \text{when } \Delta u \in [2L, \sqrt{2L\theta}] \\ \frac{\Delta u^2 + 2L\theta}{2\theta} & \text{when } \Delta u \in [\sqrt{2L\theta}, \theta] \\ \frac{(\Delta u + 2L)\theta}{2\Delta u} & \text{when } \Delta u \geq \theta \end{cases}$$

and

$$\Pi^D = \begin{cases} 2\hat{u} \equiv \Pi^D_0 & \text{when } \Delta u \leq \sqrt{L\theta} \\ 2\Delta u + 2\hat{u} - \frac{2L\theta}{\Delta u} \equiv \Pi^D_1 & \text{when } \Delta u \in [\sqrt{L\theta}, 2L] \\ \frac{4L(\Delta u^2 - L\theta)}{\Delta u^2} + 2\hat{u} \equiv \Pi^D_2 & \text{when } \Delta u \in [2L, \sqrt{2L\theta}] \\ \frac{\Delta u^2}{\theta} + 2\hat{u} \equiv \Pi^D_3 & \text{when } \Delta u \in [\sqrt{2L\theta}, \theta] \\ 2\Delta u - \theta + 2\hat{u} \equiv \Pi^D_4 & \text{when } \Delta u \geq \theta. \end{cases}$$

6.5 Proof of Propositions and Lemmas

Proof of Lemma 4: The proof is explained in the main text.

Proof of Lemma 5: First, it must be observed that

$$\frac{4q^2\Delta u^2}{2q\Delta u+\theta} \geq \frac{\Delta u^2}{\theta}$$

$$\Leftrightarrow 4q^2\theta - 2q\Delta u - \theta \geq 0$$

$$\Leftrightarrow q^2 - (\frac{\Delta u}{2\theta})q - \frac{1}{4} \geq 0 \tag{24}$$

is a sufficient condition for centralization to dominate decentralization when $\Delta u \leq \theta$. Inequality 24 would bind if

$$q = \frac{1}{2}(\frac{\Delta u}{2\theta} \overset{+}{-} \sqrt{\frac{\Delta u^2}{4\theta^2}+1})$$

$$\Leftrightarrow q = \frac{\Delta u}{4\theta} \overset{+}{-} \frac{\sqrt{\Delta u^2+4\theta^2}}{4\theta}.$$

Since $\frac{\Delta u}{4\theta} - \frac{\sqrt{\Delta u^2+4\theta^2}}{4\theta}$ is negative, I take $q^* \equiv \frac{\Delta u}{4\theta} + \frac{\sqrt{\Delta u^2+4\theta^2}}{4\theta}$. For any $q > q^*$, centralization would dominate decentralization for all $\Delta u \leq \theta$.

The second step is to show that $\frac{4q^2\Delta u^2}{2q\Delta u+\theta} \geq 2\Delta u - \theta$ whenever $q > q^*$ and $\Delta u \geq \theta$. Putting $q = q^*$ into $\frac{4q^2\Delta u^2}{2q\Delta u+\theta} \overset{?}{\geq} 2\Delta u - \theta$ results in

$$4\Delta u^2(\frac{\Delta u^2}{16\theta^2} + \frac{2\Delta u\sqrt{\Delta u^2+4\theta^2}}{16\theta^2} + \frac{\Delta u^2+4\theta^2}{16\theta^2}) \overset{?}{\geq} (2\Delta u - \theta)[2\Delta u(\frac{\Delta u}{4\theta} + \frac{\sqrt{\Delta u^2+4\theta^2}}{4\theta}) + \theta]$$

$$\Leftrightarrow \frac{\Delta u^4}{4\theta^2} + \frac{\Delta u^3\sqrt{\Delta u^2+4\theta^2}}{2\theta^2} + \frac{\Delta u^4}{4\theta^2} + \Delta u^2 \overset{?}{\geq} \frac{2\Delta u^3}{\theta} + \frac{2\Delta u^2\sqrt{\Delta u^2+4\theta^2}}{\theta} + 2\Delta u\theta - \Delta u^2 - \Delta u\sqrt{\Delta u^2+4\theta^2} - \theta^2$$

$$\Leftrightarrow \frac{\Delta u^4}{2\theta^2} + \Delta u^2 + (\Delta u - \theta)^2 - \frac{2\Delta u^3}{\theta} + \Delta u\sqrt{\Delta u^2+4\theta^2}(\frac{\Delta u^2}{2\theta^2} - \frac{2\Delta u}{\theta} + 1) \overset{?}{\geq} 0$$

$$\Leftrightarrow \frac{\Delta u^2 + \Delta u\sqrt{\Delta u^2+4\theta^2}}{2} + (\Delta u^2 + \Delta u\sqrt{\Delta u^2+4\theta^2})(\frac{(\Delta u-\theta)^2}{2\theta^2}) + (\Delta u - \theta)^2 \overset{?}{\geq} 0$$

which always hold, so the lemma is proven.

QED

Proof of Lemma 6-8: These follow directly from the solutions to the centralized and decentralized problems with limited liability.

Proof of Proposition 1

i) An increase in production noise L has the following effects:

$$\frac{\partial \Delta\Pi_{12}}{\partial L} = -4 + \frac{8L\theta}{\Delta u^2} = 4(-1 + \frac{2L\theta}{\Delta u^2}).$$

Since $\Delta u \leq \sqrt{2L\theta}$ for $\Delta\Pi_{12}$, then $\frac{\partial \Delta\Pi_{12}}{\partial L} \geq 0$. Furthermore, it can also be seen that

$$\frac{\partial \Delta\Pi_{11}}{\partial L} = -\frac{2\theta}{\Delta u}$$

is negative. Since $\frac{\partial \Pi_0^C}{\partial L} = \frac{\partial \Pi_0^D}{\partial L} = \frac{\partial \Pi_3^D}{\partial L} = \frac{\partial \Pi_4^D}{\partial L} = 0$ and $\frac{\partial \Delta\Pi_{11}}{\partial L} = \frac{2L\theta}{\Delta u}$, part i) holds.

ii) A reduction in communication noise (an increase in q) will strictly favor centralization if and only if $\Delta u \geq \frac{\theta}{2q^2}$:

$$\frac{\partial \Delta\Pi_{1j}}{\partial q} = 2\Delta u + \frac{2\theta}{q^2} - \frac{3\theta^2}{2q^4\Delta u} \qquad \forall\ j \in \{0,1,2,3,4\}$$

It is obvious that $\frac{\partial^2 \Delta\Pi_{1j}}{\partial q \partial \Delta u} > 0$ for all $j \in \{0,1,2,3,4\}$. Therefore, I impose the lowest possible value for Δu allowed by the restriction $\Delta u > \frac{\theta}{2q^2}$ in order to see if $\frac{\partial \Delta\Pi_{1j}}{\partial q}$ for all $j \in \{0,1,2,3,4\}$ would still hold:

$$\frac{\partial \Delta\Pi_{1j}(\Delta u = \frac{\theta}{2q^2})}{\partial q} = 2(\frac{\theta}{2q^2}) + \frac{2\theta}{q^2} - \frac{3\theta^2}{2q^4}(\frac{2q^2}{\theta})$$

$$\Leftrightarrow \frac{\partial \Delta\Pi_{1j}(\Delta u = \frac{\theta}{2q^2})}{\partial q} = 0 \qquad \forall\ j \in \{0,1,2,3,4\}. \tag{25}$$

Given 25 and $\frac{\partial^2 \Delta\Pi_{1j}}{\partial q \partial \Delta u} > 0$ for all $j \in \{0,1,2,3,4\}$, I can conclude that $\frac{\partial \Delta\Pi_{1j}}{\partial q} \geq 0$ for all $j \in \{0,1,2,3,4\}$.

As for part iii), all I need is an example where $\Pi_j^D > \Pi^C(q=1)$. Let's compare Π_4^D and $\Pi^C(q=1)$:

$$2\Delta u - \theta + 2\hat{u} \overset{?}{>} \frac{4\Delta u^2 - 4\Delta u\theta + \theta^2}{2\delta u}$$

$$\Leftrightarrow (4\Delta u - \theta - 1)\theta \overset{?}{>} 0.$$

Since no parameter restrictions prevent the above from happening, this proves part iii).

QED

Proof of Proposition 2-i: I start by proving the limited liability constraint depresses centralized profits. The first step in the proof is to show that $\Pi^C_{no\ LL}(\Delta u = \frac{\theta}{2q^2}) > \Pi^C_{with\ LL}(\Delta u = \frac{\theta}{2q^2})$. I ask

$$\frac{4q^2(\theta^2/4q^4)}{2q(\theta/2q^2) + \theta} \overset{?}{\geq} 2q(\frac{\theta}{2q^2}) - \frac{2\theta}{q} + \frac{\theta^2}{2q^2}(\frac{2q^2}{\theta})$$

$$\Leftrightarrow \frac{\theta^2}{q^2}(\frac{q}{\theta + q\theta}) \overset{?}{\geq} \frac{\theta - 2\theta - q\theta}{q}$$

$$\Leftrightarrow \frac{1}{1+q} \overset{?}{\geq} 1 - 2 + q$$

$$\Leftrightarrow 1 \overset{?}{\geq} -(1-q)(1+q)$$

which obviously holds. This proves the first step.

The second step is to show that

$$\frac{\partial \Pi^C_{no\ LL}}{\partial \Delta u} > \frac{\partial \Pi^C_{with\ LL}}{\partial \Delta u}$$

$$\Leftrightarrow \frac{8q^2\Delta u(2q\Delta u + \theta) - 4q^2\Delta u^2(2q)}{(2q\Delta u + \theta)^2} > 2q - \frac{\theta^2}{2q^3\Delta u^2}$$

$$\Leftrightarrow \frac{8q^2\Delta u(q\Delta u + \theta)}{(2q\Delta u + \theta)^2} > \frac{4q^4\Delta u^2 - \theta^2}{2q^3\Delta u^2}$$

$$\Leftrightarrow 16q^6\Delta u^4 + 16q^5\Delta u^3\theta > 16q^6\Delta u^4 + 16q^5\Delta u^3\theta + 4q^4\Delta u^2\theta^2 - 4q^2\Delta u^2\theta^2 - 4q\Delta u\theta^2 - \theta^3$$

$$\Leftrightarrow 0 > 4q^2\Delta u^2\theta^2(q^2 - 1) - 4q\Delta u\theta^2 - \theta^3$$

which clearly hold. Therefore, since $\Pi^C_{no\ LL}(\Delta u = \frac{\theta}{2q^2}) > \Pi^C_{with\ LL}(\Delta u = \frac{\theta}{2q^2})$ and $\frac{\partial \Pi^C_{no\ LL}}{\partial \Delta u} > \frac{\partial \Pi^C_{with\ LL}}{\partial \Delta u}$ both hold, I can conclude that $\Pi^C_{no\ LL} > \Pi^C_{with\ LL}$ for all $\Delta u > \frac{\theta}{2q^2}$ and that the introduction of a limited liability constraint depresses profits.

I now show that the limited liability constraint depresses decentralized profits. Since it is obvious that both $\frac{\partial \Pi^D_1}{\partial \Delta u} > 0$ and $\frac{\partial \Pi^D_2}{\partial \Delta u}$ hold, it is sufficient to simply show $\Pi^D_1(\Delta u = \sqrt{L\theta}) \leq \Pi^D_2(\Delta u = \sqrt{L\theta})$ and $\Pi^D_1(\Delta u = \sqrt{2L\theta}) \leq \Pi^D_2(\Delta u = \sqrt{2L\theta})$.

First, it can be seen that $\Pi^D_1(\Delta u = \sqrt{L\theta}) \overset{?}{\leq} \Pi^D_2(\Delta u = \sqrt{L\theta})$ is equivalent to

$$\Leftrightarrow 4L - \frac{4L^2\theta}{L\theta} \overset{?}{\leq} \frac{L\theta}{\theta}$$

$$\Leftrightarrow 0 \overset{?}{\leq} L$$

which obviously holds.

Second, $\Pi^D_1(\Delta u = \sqrt{2L\theta}) \overset{?}{\leq} \Pi^D_2(\Delta u = \sqrt{2L\theta})$ is equivalent to

$$\Leftrightarrow 4L - \frac{4L^2\theta}{2L\theta} \overset{?}{\leq} \frac{2L\theta}{\theta}$$

$$\Leftrightarrow 2L \overset{?}{\leq} 2L$$

which also obviously hold.

QED

Proof of Lemma 9: I compute

$$\frac{\partial \Delta \Pi_{12}}{\partial \Delta u} = 2q - \frac{\theta^2}{2q^3 \Delta u^2} - \frac{8L^2\theta}{\Delta u^3}.$$

Clearly, $\frac{\partial \Delta \Pi_{12}}{\partial \Delta u}$ increases with Δu and q. To check if $\frac{\partial \Delta \Pi_{12}}{\partial \Delta u} > 0$ is possible, I set q=1 and $\Delta u = \sqrt{2L\theta}$ to get

$$\frac{\partial \Delta \Pi_{12}}{\partial \Delta u} = 2 - \frac{\theta}{4L} - \frac{4L}{\sqrt{2L\theta}}.$$

For any positive combination of positive values for θ and L, it can be seen that $\frac{\partial \Delta \Pi_{12}}{\partial \Delta u} < 0$.

Then, it can be seen that

$$\frac{\partial \Delta \Pi_{13}}{\partial \Delta u} = 2q - \frac{\theta^2}{2q^3 \Delta u^2} - \frac{2\Delta u}{\theta}.$$

To check if $\frac{\partial \Delta \Pi_{13}}{\partial \Delta u} \geq 0$ is possible, I set q=1 and ask $\frac{\partial \Delta \Pi_{13}}{\partial \Delta u} \overset{?}{\geq} 0$, I get

$$\Leftrightarrow 4 \geq \frac{\theta^2}{\Delta u^2} + \frac{4\Delta u}{\theta}$$

which never holds for positive values for θ and L.
I then compute and ask $\frac{\partial \Delta \Pi_{14}}{\partial \Delta u} \overset{?}{\leq} 0$

$$\Leftrightarrow -\frac{\theta^2}{2q^3 \Delta u^2} \leq 2(1-q)$$

which always holds.
Finally, I compute

$$\frac{\partial \Delta \Pi_{11}}{\partial \Delta u} = 2q - \frac{\theta^2}{2q^3 \Delta u^2} - 2 + \frac{2L\theta}{\Delta u^2}$$

and observe that if $q = \sqrt{\frac{\theta}{2\Delta u}}$, then $\frac{\partial \Delta \Pi_{11}}{\partial \Delta u}$ becomes $-2 + \frac{2L\theta}{\Delta u}$ which is clearly negative since $\Delta u \geq \sqrt{L\theta}$ is a necessary assumption for Π_1^D to exist. However, if I suppose $q = 1$, I get

$$\frac{\theta}{\Delta u^2}(\frac{4L - \theta}{2}).$$

For Π_1^D to exist, $2L \geq \sqrt{L\theta} \Leftrightarrow 4L \geq \theta$ is necessary, so the above is clearly positive. Therefore, $\frac{\partial \Delta \Pi_{11}}{\partial \Delta u}$ can be either positive or negative.

QED

Proof of Proposition 3: This follows directly from the solutions to the centralized and decentralized problem with a limited liability constraint.

Proof of Proposition 4: Instead of simply assuming that the reservation utility is zero, I will now assume it is embodied by the variable R and solve the decentralized problem again. The decentralized Lagrangian becomes

$$\mathcal{L} = \frac{\Delta u^2 \Delta W}{L\theta} + 2\hat{u} - W^U - W^L - \lambda(\frac{\Delta u^2 \Delta W^2}{4L^2\theta} - W^U - W^L + R) + \mu W^L$$

and the Kuhn-Tucker conditions are

$$\frac{\partial \mathcal{L}}{\partial W^U} = 0 \Leftrightarrow \frac{\Delta^2}{L\theta} - 1 - \lambda(\frac{\Delta u^2 \Delta W}{2L^2\theta} - 1) = 0 \tag{26}$$

$$\frac{\partial \mathcal{L}}{\partial W^L} = 0 \Leftrightarrow -\frac{\Delta^2}{L\theta} - 1 - \lambda(-\frac{\Delta u^2 \Delta W}{2L^2\theta} - 1) + \mu = 0 \tag{27}$$

$$\lambda(\frac{\Delta u^2 \Delta W^2}{4L^2\theta} - W^U - W^L + R) = 0 \tag{28}$$

$$\mu W^L = 0 \tag{29}$$

Case $\lambda = 0$: This, 26 and 27 imply

$$\frac{\Delta u^2}{L\theta} - 1 = 0$$

and

$$-\frac{\Delta u^2}{L\theta} - 1 + \mu = 0.$$

Subcase: $\mu = 0$

If $\Delta u^2 > L\theta$, ΔW should be set at a maximum. These also imply W^L should be set at a minimum, which is provided by the participation constraint:

$$W^L + W^U \geq \frac{\Delta u^2}{4L^2\theta}(W^U - W^L)^2 + R$$

$$\Leftrightarrow 2W^L \geq (\frac{\Delta u^2}{4L^2\theta})\Delta W^2 - \Delta W + R. \tag{30}$$

ΔW is set so that $e_D^* = 1$ which implies $\Delta W = \frac{2L\theta}{\Delta u}$. Put this into 30, I get

$$W^L \geq \frac{\theta}{2} - \frac{L\theta}{\Delta u} + \frac{R}{2}.$$

Since W^L has to be set a minimum, I set $W^L = \frac{\theta}{2} - \frac{L\theta}{\Delta u} + \frac{R}{2}$. This also implies

$$W^U - \frac{\theta}{2} + \frac{L\theta}{\Delta u} - \frac{R}{2} = \frac{2L\theta}{\Delta u}$$

$$\Leftrightarrow W^U = \frac{L\theta}{\Delta u} + \frac{\theta}{2} + \frac{R}{2}.$$

So, if $\frac{\theta+R}{2} \geq \frac{L\theta}{\Delta u} \Leftrightarrow \Delta u \geq \frac{2L\theta}{\theta+R}$ (this ensures $W^L \geq 0$) and $\Delta u \geq \sqrt{L\theta}$, then a **solution** is $W^L = \frac{\theta}{2} - \frac{L\theta}{\Delta u} + \frac{R}{2}$, $W^U = \frac{L\theta}{\Delta u} + \frac{\theta}{2} + \frac{R}{2}$ and $\lambda = \mu = 0$.

Subcase $\mu > 0$: This and 29 imply $W^L = 0$. If $\Delta u \geq \sqrt{L\theta}$, then $\Delta W = W^U$ needs to be set at a maximum, which is $\Delta W = W^U = \frac{2L\theta}{\Delta u}$.

In order for the participation constraint to hold, I need:

$$\frac{2L\theta}{\Delta u} \geq (\frac{\Delta u^2}{4L^2\theta})(\frac{4L^2\theta^2}{\Delta u^2}) + R$$

$$\Leftrightarrow \frac{2L\theta}{\theta + R} \geq \Delta u.$$

So if $\Delta u \geq \sqrt{L\theta}$ and $\frac{2L\theta}{\theta+R} \geq \Delta u$ hold, then a **solution** is $W^L = 0$, $W^U = \frac{2L\theta}{\Delta u}$, $\mu = \frac{\Delta u^2+L\theta}{L\theta}$ and $\lambda = 0$.

Case $\lambda > 0$: This directly implies that the participation constraint must hold:

$$\frac{\Delta u^2 \Delta W^2}{4L^2\theta} - W^U - W^L + R = 0. \tag{31}$$

Subcase $\mu > 0$: This implies $W^L = 0$ which along with 31 implies

$$\frac{\Delta u^2 (W^U)^2}{4L^2\theta} - W^U + R = 0$$

$$(W^U)^2 - (\frac{4L^2\theta}{\Delta u^2})W^U + (\frac{4L^2\theta}{\Delta u^2})R = 0. \tag{32}$$

Using the quadratic formula to solve the above equation, I get:

$$W^U = (\frac{4L^2\theta}{\Delta u^2} \pm \sqrt{\frac{16L^4\theta^2}{\Delta u^4} - \frac{16L^2\theta R}{\Delta u^2}})\frac{1}{2}$$

$$\Leftrightarrow W^U = (\frac{4L^2\theta}{\Delta u^2} \pm \frac{4L}{\Delta u^2}\sqrt{L^2\theta^2 - \theta\Delta u^2 R})\frac{1}{2}$$

$$\Leftrightarrow W^U = \frac{2L}{\Delta u^2}(L\theta \pm \sqrt{L^2\theta^2 - \theta\Delta u^2 R}).$$

In order for 32 to have a solution, I need $\Delta u \leq L\sqrt{\frac{\theta}{R}}$ (this ensures the discriminant is positive). Now I compute the value of λ using W^U and 26:

$$\frac{\Delta u^2}{L\theta} - 1 - \lambda[\frac{\Delta u^2}{2L^2\theta}(\frac{2L}{\Delta u^2}(L\theta \pm \sqrt{L^2\theta^2 - \theta\Delta u^2 R})) - 1] = 0$$

$$\Leftrightarrow \frac{\Delta u^2 - L\theta}{L\theta} = \lambda[\pm\frac{\sqrt{L^2\theta^2 - \theta\Delta u^2 R}}{L\theta}]$$

$$\Leftrightarrow \pm\frac{(\Delta u^2 - L\theta)}{\sqrt{L^2\theta^2 - \theta\Delta u^2 R}} = \lambda.$$

Since $\lambda \geq 0$ has to hold, then

$$\frac{(\Delta u^2 - L\theta)}{\sqrt{L^2\theta^2 - \theta\Delta u^2 R}} = \lambda.$$

I also need to compute μ using W^U, λ and 27:

$$-\frac{\Delta u^2}{L\theta} - 1 + \lambda[\frac{\Delta u^2}{2L^2\theta}(\frac{2L}{\Delta u^2}(L\theta + \sqrt{L^2\theta^2 - \theta\Delta u^2 R})) + 1] + \mu = 0$$

$$\Leftrightarrow \mu = \frac{\Delta u^2 + L\theta}{L\theta} - (\frac{\Delta u^2 - L\theta}{\sqrt{L^2\theta^2 - \theta\Delta u^2 R}})(\frac{2L\theta + \sqrt{L^2\theta^2 - \theta\Delta u^2 R}}{L\theta})$$

$$\mu = \frac{2(\sqrt{L^2\theta^2 - \theta\Delta u^2 R} + L\theta - \Delta u^2)}{\sqrt{L^2\theta^2 - \theta\Delta u^2 R}}.$$

To ensure that μ is positive, it must be that

$$\Delta u^2 \leq L\theta + \sqrt{L^2\theta^2 - \theta\Delta u^2 R}$$

$$\Leftrightarrow \Delta u \leq \sqrt{2L\theta - \theta R}.$$

So for $W^L = 0$, $W^U = \frac{2L}{\Delta u^2}(L\theta + \sqrt{L^2\theta^2 - \theta\Delta u^2 R})$, $\lambda = \frac{\Delta u^2 - L\theta}{\sqrt{L^2\theta^2 - \theta\Delta u^2 R}}$ and $\mu = \frac{2(\sqrt{L^2\theta^2 - \theta\Delta u^2 R} + L\theta - \Delta u^2)}{\sqrt{L^2\theta^2 - \theta\Delta u^2 R}}$ to be a **solution**, $\Delta u \leq L\sqrt{\frac{\theta}{R}}$ and $\Delta u \in [\sqrt{L\theta}, \sqrt{2L\theta - \theta R}]$ must hold.

Subcase $\mu = 0$: I can now use 26 and 27 to solve for λ. Equation 27 implies

$$-\frac{\Delta u^2}{L\theta} - 1 - \lambda(-\frac{\Delta u^2}{2L^2\theta}\Delta W - 1) = 0$$

$$\Leftrightarrow \lambda = \frac{2L(\Delta u^2 + L\theta)}{\Delta u^2 \Delta W + 2L^2\theta}$$

and equation 26 implies

$$\frac{\Delta u^2}{L\theta} - 1 - \lambda(\frac{\Delta u^2 \Delta W}{2L^2\theta} - 1) = 0$$

$$\Leftrightarrow \lambda = \frac{2L(\Delta u^2 - L\theta)}{\Delta u^2 \Delta W - 2L^2\theta}.$$

Combining both of these expressions results in $\Delta W = 2L$. Furthermore, since $\lambda > 0$, the participation constraint has to bind:

$$\frac{\Delta u^2 \Delta W^2}{4L^2\theta} - W^U - W^L + R = 0$$

$$\Leftrightarrow \frac{\Delta u^2 \Delta W^2}{4L^2\theta} - 2W^U - \Delta W + R = 0$$

By replacing ΔW with 2L in the binding participation constraint, I get:

$$\frac{\Delta u^2}{\theta} - 2W^L - 2L + R = 0$$

$$\Leftrightarrow W^L = \frac{\Delta u^2 - 2L\theta + R\theta}{2\theta}.$$

Since $W^L \geq 0$ has to hold, $\Delta u \geq \sqrt{2L\theta - R\theta}$ has to hold. I can also compute W^U:

$$W^L + \Delta W = W^U$$

$$\Leftrightarrow W^U = \frac{\Delta u^2 + 2L\theta + R\theta}{2\theta}$$

I can now compute λ:

$$\lambda = \frac{2L(\Delta u^2 - L\theta)}{\Delta u^2 2L - 2L^2\theta} \Leftrightarrow \lambda = 1.$$

Therefore, a **solution** is $W^U = \frac{\Delta u^2 + 2L\theta + R\theta}{2\theta}$, $W^L = \frac{\Delta u^2 - 2L\theta + R\theta}{2\theta}$, $\mu = 0$ and $\lambda = 1$ if $\Delta u \geq \sqrt{2L\theta - R\theta}$.

Finally, a solution with $W^U = W^L = R$ is possible. From 26, it implies

$$\frac{\Delta u^2 - L\theta}{L\theta} - \lambda = 0 \Leftrightarrow \lambda = \frac{L\theta - \Delta u^2}{L\theta}.$$

From, 27, it also implies

$$\lambda + \mu = \frac{\Delta u^2 + L\theta}{L\theta}$$

$$\Leftrightarrow \mu = \frac{2\Delta u^2}{L\theta}.$$

Subsequently, a **solution** is $W^L = W^U = R$, $\lambda = \frac{L\theta - \Delta u^2}{L\theta}$ and $\mu = \frac{2\Delta u^2}{L\theta}$ if $\Delta u \leq \sqrt{L\theta}$.

To sum up, the feasible solutions are:

-If $\Delta u \leq \sqrt{L\theta}$, $W^L = W^U = R$.

- If $\Delta u \in [\sqrt{L\theta}, \sqrt{2L\theta - R\theta}]$ and $\Delta u \leq L\sqrt{\frac{\theta}{R}}$, $W^L = 0$, $W^U = \frac{2L}{\Delta u^2}(L\theta + \sqrt{L^2\theta^2 - \theta\Delta u^2 R})$.

-If $\Delta u \geq \sqrt{2L\theta - R\theta}$, $W^U = \frac{\Delta u^2 + 2L\theta + R\theta}{2\theta}$, $W^L = \frac{\Delta u^2 - 2L\theta + R\theta}{2\theta}$.

-If $\Delta u \geq \sqrt{L\theta}$ and $\Delta u \leq \frac{2L\theta}{\theta + R}$, $W^L = 0$ and $W^U = \frac{2L\theta}{\Delta u}$.

-If $\Delta u \geq \sqrt{L\theta}$ and $\Delta u \geq \frac{2L\theta}{\theta + R}$, $W^L = \frac{\Delta u(\theta + R) - 2L\theta}{2\Delta u}$ and $W^U = \frac{\Delta u(\theta + R) + 2L\theta}{2\Delta u}$.

Outside of $W^L = W^U = R$, the participation constraint binds for all solutions except for $W^L = 0$ and $W^U = \frac{2L\theta}{\Delta u}$. So, a sufficient condition for the participation constraint to bind is $\Delta u \geq \max \sqrt{L\theta}, \frac{2L\theta}{\theta + R}$. When the participation constraint binds, it is straightforward to show that an increase in R depresses profits.

Now, as to centralization, since the participation constraint does not bind when the principal induces a positive level of effort ($\Delta u \geq \frac{\theta}{2q^2}$), a marginal increase of the reservation utility starting from $R = 0$ will have no impact on centralized profits. The proposition is therefore proven.

Proof of proposition 5: Result i) is trivial and can be seen directly by $q^* = \frac{8}{3}\frac{\theta}{\Delta W} - (\frac{\theta}{\Delta W})^2 \frac{2\theta^P}{3\Delta u}$.

For result ii), first observe that the conditions for $-\frac{\partial q^*}{\partial \theta} > 0$ are the same as those for $\frac{\partial q^*}{\partial \Delta W} > 0$. It can be seen that

$$-\frac{\partial q^*}{\partial \theta} > 0$$

$$\Leftrightarrow \frac{8}{3\Delta W} - \frac{4\theta^P}{3\Delta u}(\frac{\theta}{\Delta W^2}) > 0$$

$$\Leftrightarrow \frac{\theta^P}{\Delta u} > \frac{2\Delta W}{\theta}$$

whereas

$$\frac{\partial q^*}{\partial \Delta W} > 0$$

$$\Leftrightarrow -\frac{\theta}{\Delta W^2}(\frac{8}{3}) + 2(\frac{\theta^2}{\Delta W^3})\frac{2\theta^P}{3\Delta u} > 0$$

$$\Leftrightarrow \frac{\theta^P}{\Delta u} > \frac{2\Delta W}{\theta}. \tag{33}$$

Then, by looking at equation 33, it can be seen that there exist a certain threshold $\frac{\Delta W}{\tilde{\theta}}$ that determines the direction of $\frac{\partial q^*}{\partial \Delta W}$ and $-\frac{\partial q^*}{\partial \theta}$.

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