Information Provision in a Directed Search Model*

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Abstract

Two capacity-constrained sellers post prices and provide information about characteristics of their products to a finite number of buyers. After observing information, buyers select a seller to visit without coordination. We find that sellers provide all available information in any equilibrium. We also characterize sellers' pricing strategy in the symmetric equilibrium.

Keywords: information provision; directed search; horizontal differentiation

JEL Classification Numbers: C72, D43, D83

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1 Introduction

In many economic environments, sellers provide information about characteristics of their products to potential buyers. This is especially the case for new or sophisticated products. Consider the examples of free samples, movie trailers and free-trial computer games. By choosing how much information that free samples contain about the products, a seller controls how much potential buyers know about the characteristics of the seller's product. However, a buyer's valuation often depends on both the characteristics of the product and the buyer's idiosyncratic taste or need. Usually the buyer knows more about his taste or need than the seller does, who only knows the distribution of the buyer's possible valuations.

In situations where the buyer's possible tastes are diverse, the distribution of the buyer's posterior expected valuation becomes more dispersed when the seller provides more information. Therefore, the seller faces a trade-off in providing information to buyers. More information raises the posterior expected valuation of buyers whose taste matches with the characteristics of the seller's good. This allows the seller to charge a higher price to extract part of this extra surplus. However, more information lowers the posterior expected valuation of buyers whose taste does not match with the characteristics and leads to a lower market share for the seller.

This paper studies this trade-off in a model where two capacity-constrained sellers decide their prices and the amount of information provided to potential buyers. After observing prices and the provided information, buyers select a seller to visit without coordination. The number of buyers visiting a seller may exceed the latter's capacity so that only some of them are able to trade successfully. We identify the amount of information that sellers provide in any equilibrium and their pricing strategy in the symmetric equilibrium.

We find that, regardless of the opponent's choice of information provision and price, if a seller does not provide all information, she always has a profitable deviation to providing all available information and charging a higher price. The improvement in information and the price increase are such that buyers who observe the high signal realization receive the same expected payoff from trading with the seller as they do before the deviation. When the seller deviates, buyers who observe the low signal realization no longer select the seller, which implies that high-signal buyers can trade with the seller with a higher probability. Therefore, high-signal buyers visit the seller with an equal or higher probability. Given that at least one half of buyers who visit the seller under partial information are high-signal buyers, the probability that the seller can sell under full information is at least one half of that under partial information. On the other hand, the seller's price under full information is at least twice as high as her price under partial information. Therefore, the seller's expected profit under full information must be greater than that under partial information.

To the best of our knowledge, the present paper is the first study on sellers' competition in information provision and price setting in a model with coordination friction among buyers. It is at the intersection of several literatures. The first one is sellers' competition in information provision and price setting for a representative buyer. Damiano and Li (2007) consider a two-stage game in which two sellers first choose how much information to provide a representative buyer about the characteristics of their goods. Then, after knowing each other's information structure, sellers decides on prices. It is found that there exists a symmetric equilibrium where both sellers provide all available information. Ivanov (2013) studies a case with more than two sellers where sellers simultaneously decides how much information to provide and what price to charge. It is found that sellers provide all available information in the unique symmetric equilibrium when the number of sellers is sufficiently large.

The present article is different from a representative-buyer model in that there is competition among buyers for sellers' goods. In real life, we can often observe such competition. Customers of restaurants compete with each other for the seats and job seekers compete for vacancies. A model with a representative buyer fails to capture the interaction among buyers and its implication for sellers' equilibrium price setting and information provision strategy.

The paper is also related to the directed search literature. Peters (1984) studies the matching game between a group of sellers and buyers in which sellers publicly post their respective prices. After observing all sellers' prices, buyers decide which seller to select without coordination (see also Burdett et al., 2001). Lester (2011) studies a situation when some buyers are uninformed about sellers' prices and randomly select a seller.[1] Unlike these above papers, this paper considers sellers' information provision about buyers' private valuations.

The paper is organized as follows. Section 2 describes the setup of the

[1]See Huang (2016) for a study on equilibrium price dispersion in Lester's model.

model. Section 4 identifies the equilibrium information provision strategy of sellers. Section 5 characterizes the equilibrium pricing strategy. Section 6 concludes and discusses future works.

2 Model

Players, preferences and information: There are two sellers, A and B, and a finite number of buyers. The set of buyers is $\mathcal{I} = \{1, ..., I\}$ where $2 \leq I < +\infty$. Each seller has a single unit of indivisible good for which her[2] reservation value is 0. Buyer i's valuation of seller j's good is $u_{i,j}$. When buyer i trades with seller j at price p_j, the buyer's payoff is $u_{i,j} - p_j$ and the seller's profit is p_j.

Sellers' goods are horizontally differentiated and buyers have idiosyncratic tastes or needs. Each buyer is initially uncertain about his valuations due to the uncertainty regarding the characteristics of sellers' goods. Assume that $u_{i,j}$'s are identically and independently distributed over $\{0, 1\}$ with equal probabilities.[3] Each buyer's ex ante expected valuation of any seller's good is $1/2$ and sellers' goods are ex ante homogeneous.

Information provision: A seller can provide buyers some information about the characteristics of her good. After knowing the characteristics, a buyer can refine the estimate of his valuation. We assume that seller j provides

[2] Throughout this paper, we use "she" to denote a seller and "he" to a buyer.

[3] The model can be seen as the reduced form of a fully specified model where each seller's type is either L or R and each buyer's type is either l or r. A player can be either type with equal probabilities and players' types are independent from each other's. A buyer's valuation of a seller's good is 1 only when the seller's type matches the buyer's. Otherwise, the valuation is 0. A player knows his or her own type but is uncertain about the other players' types.

buyer i a noisy signal about the buyer's valuation.[4] Following Damiano and Li (2007), we consider the case where the seller chooses from a class of signals with symmetric and binary signal structures. The realization of the signal, $v_{i,j}$, is either 0 or 1. The signal structure is represented by a probability $\alpha_{i,j}$, defined as

$$\alpha_{i,j} = Pr(v_{i,j} = 1 | u_{i,j} = 1) = Pr(v_{i,j} = 0 | u_{i,j} = 0).$$

Without loss of generality, we restrict our attention to cases where $\alpha_{i,j} \in [1/2, 1]$. As $\alpha_{i,j}$ increases, the signal is more informative about $u_{i,j}$ in the sense of Blackwell (1953). The signal is perfectly informative when $\alpha_{i,j} = 1$, while perfectly uninformative when $\alpha_{i,j} = 1/2$. We say that seller j provides all information to buyer i when $\alpha_{i,j} = 1$.

Although seller j can choose the structure of the signal, we assume that she does not observe its realization. This reflects a situation where a buyer's valuation depends on his idiosyncratic need or taste, which is privately known to the buyer. We abstract from the possibility that a seller may discriminate among buyers by offering them signals with different informativeness. Thus, it is assumed that $\alpha_{i,j} = \alpha_j$ for all $i \in I$.[5]

Given the signal structure, the buyer's expected valuation conditional on signal realization 1 is $E\left(u_{i,j} | \alpha_j, v_{i,j} = 1\right) = \alpha_j$, while that conditional on realization 0 is $E\left(u_{i,j} | \alpha_j, v_{i,j} = 0\right) = 1 - \alpha_j$. The distribution of the buyer's

[4]Alternatively, a seller can provide information about her type to the buyer. As long as different types of sellers choose the same signal structure so that a buyer cannot infer the seller's type from the signal structure, this alternative model is isomorphic to the one that we consider here.

[5]Li and Shi (2015) study discriminatory information provision by a monopolist seller who faces a buyer that may have different likelihoods of having high valuation for the seller's good.

ex post expected valuation is more dispersed when the seller's signal is more informative.

Matching with frictions: Buyers and sellers play a game with three stages. In stage one, sellers simultaneously and independently post prices and signal structures. Let p_j and α_j denote seller j's price and signal structure respectively.

In stage two, buyers observe the profile of sellers' prices and signal structures. After privately observing signal realizations from both sellers, buyers simultaneously and independently choose at most one seller to visit without coordination. Given the profile of sellers' prices and signal structures $(\alpha_j, p_j, \alpha_{-j}, p_{-j})$, a buyer's strategy is a pair of probabilities of visiting sellers under all possible profiles of signals,

$$\left\{\left(\pi^A_{s,t}(\alpha_j, p_j, \alpha_{-j}, p_{-j}), \pi^B_{s,t}(\alpha_j, p_j, \alpha_{-j}, p_{-j})\right)\right\}_{(s,t)\in\{0,1\}^2}, \tag{1}$$

where $\pi^j_{s,t}(\alpha_j, p_j, \alpha_{-j}, p_{-j})$ is the probability of visiting seller $j \in \{A, B\}$ when observing signal realization s from seller j and t from the other seller. Since any profile of signal realization happens with equal probabilities, the probability that the buyer visits seller j before he observes the signals is

$$\pi^j(\alpha_j, p_j, \alpha_{-j}, p_{-j}) := \sum_{(s,t)\in\{0,1\}^2} \pi^j_{s,t}(\alpha_j, p_j, \alpha_{-j}, p_{-j})/4.$$

In stage three, matches are realized and trade happens. If a buyer is the only one who visits a seller, the buyer trades with the seller at the seller's posted price. If more than one buyer visits a seller, the seller randomly picks one with equal probabilities to trade at the posted price. The other buyers cannot visit any other seller. If no buyer visits a seller, the seller keeps the good.

For the brevity of notation, we surpress prices and signal structures in the buyer's strategy when there is no possibility of confusion. Suppose that every buyer follows the same strategy as in (1). For a buyer, the number of the other buyers who visit seller j follows the Binomial distribution with parameters $I-1$ and π^j. Therefore, the conditional probability that the buyer can trade with seller j conditional on that he visits her is

$$\mathcal{H}(\pi^j) := \sum_{l=0}^{I-1} \binom{I-1}{l} \left(\pi^j\right)^l \left(1-\pi^j\right)^{I-1-l} \frac{1}{l+1} = \frac{1-(1-\pi^j)^I}{I\pi^j},$$

and his expected payoff from visiting her is

$$\mathcal{H}(\pi^j) \left[E\left(u_{i,j}|\alpha_j, v_{i,j}\right) - p_j\right].$$

When every buyer visits seller j with probability π^j, the probability that the seller can sell her good is

$$\mathcal{M}(\pi^j) := 1 - \left(1-\pi^j\right)^I,$$

which is the probability that at least one buyer visits the seller. The seller's expected profit is $\mathcal{M}(\pi^j)p_j$.

We study subgame perfect Nash equilibria of the above game where buyers play symmetric strategy in visiting sellers (as is standard in the directed search literature) [6]. In equilibrium, when a buyer visits a seller with a positive probability after observing some profile of signals, the buyer's expected

[6]Despite the fact that buyers have private information, we use subgame perfect Nash equilibria rather than perfect Bayesian equilibria because buyers are passive in the sense that they don't have a chance to change sellers' beliefs by revealing their private information.

payoff from visiting the seller after observing the signal profile is no less than that from visiting elsewhere. In addition, for any signal profile under which a buyer's expected payoff from visiting some seller is nonnegative, the probabilities that the buyer visits two sellers must sum up to one. We have the following definition.

Definition 1 *(Buyers' symmetric equilibrium) Given* (α_A, p_A) *and* (α_B, p_B)*, a symmetric equilibrium among buyers is a pair of visiting probabilities under all possible profiles of signals,* $\left\{\left(\pi_{s,t}^{A*}, \pi_{s,t}^{B*}\right)\right\}_{(s,t)\in\{0,1\}^2}$*, satisfying that: (1)* $\mathcal{H}(\pi^{j*})\left[E(u_{i,j}|\alpha_j, s, t) - p_j\right] \geq \max\left\{\mathcal{H}(\pi^{-j*})\left[E(u_{i,-j}|\alpha_{-j}, s, t) - p_{-j}\right], 0\right\}$ *for any* $(s,t) \in \{0,1\}^2$ *and* $j \in \{A, B\}$ *such that* $\pi_{s,t}^{j*} > 0$ *and, (2)* $\pi_{s,t}^{j*} + \pi_{s,t}^{-j*} = 1$ *for any* $(s,t) \in \{0,1\}^2$ *such that* $E(u_{i,j}|\alpha_j, s, t) \geq p_j$ *for at least one* $j \in \{A, B\}$*.*

Given buyers' symmetric equilibrium strategy, seller j's expected profit is

$$\Pi^j(\alpha_j, p_j, \alpha_{-j}, p_{-j}) := \mathcal{M}(\pi^{j*}(\alpha_j, p_j, \alpha_{-j}, p_{-j}))p_j.$$

We consider equilibria between sellers in possibly mixed strategies. When the other seller follows a mixed strategy in signal structures and prices whose distribution function is $G^{-j}(.)$, a seller's expected profit by choosing signal structure α_j and price p_j is

$$\Pi^j(\alpha_j, p_j; G^{-j}(.)) := \int_{\alpha_{-j}\in[1/2,1], p_{-j}\in[0,1]} \Pi^j(\alpha_j, p_j, \alpha_{-j}, p_{-j}) dG^{-j}(\alpha_{-j}, p_{-j}).$$

We can define an equilibrium between sellers as follows.

Definition 2 *An equilibrium between sellers is a pair of distributions* $G^A(.)$ *and* $G^B(.)$ *such that for any* (α_j, p_j) *in the support of* $G^j(.)$*,*

$$\Pi^j(\alpha_j, p_j; G^{-j}(.)) \geq \max_{\alpha'_j \in [1/2,1], p'_j \in [0,1]} \Pi^j(\alpha'_j, p'_j; G^{-j}(.)),\ for\ j \in \{A, B\}.$$

In the next section, we characterize the symmetric equilibrium among buyers for any profile of prices and signal structures chosen by sellers.

3 Buyers' symmetric equilibrium

Standard argument, such as that in Peters (1984) or Camera and Kim (2013), can establish that there exists a unique symmetric equilibrium among buyers for any profile of signal structures and prices. Figure 1 summarizes buyers' equilibrium strategy for different values of α_A, p_A, α_B and p_B such that $p_B > 1 - \alpha_B$. Figure 2 shows the equilibrium strategy given α_B and p_B such that $p_B \leq 1 - \alpha_B$.

In the next section, we study sellers' equilibrium strategy in choosing signal structures, given buyers' symmetric equilibrium strategy.

4 Full information provision

This section establishes that sellers provide all available information in equilibrium. To prove this, we claim that if a seller's signal structure is not perfectly informative, she can increase her expected profit by deviating to the perfectly informative one. To establish such a claim, we consider two cases.

In the first case, the probability that the seller sells her good is strictly positive when she chooses a signal structure that is imperfectly informative. We show that the seller can increase her expected profit by deviating to the

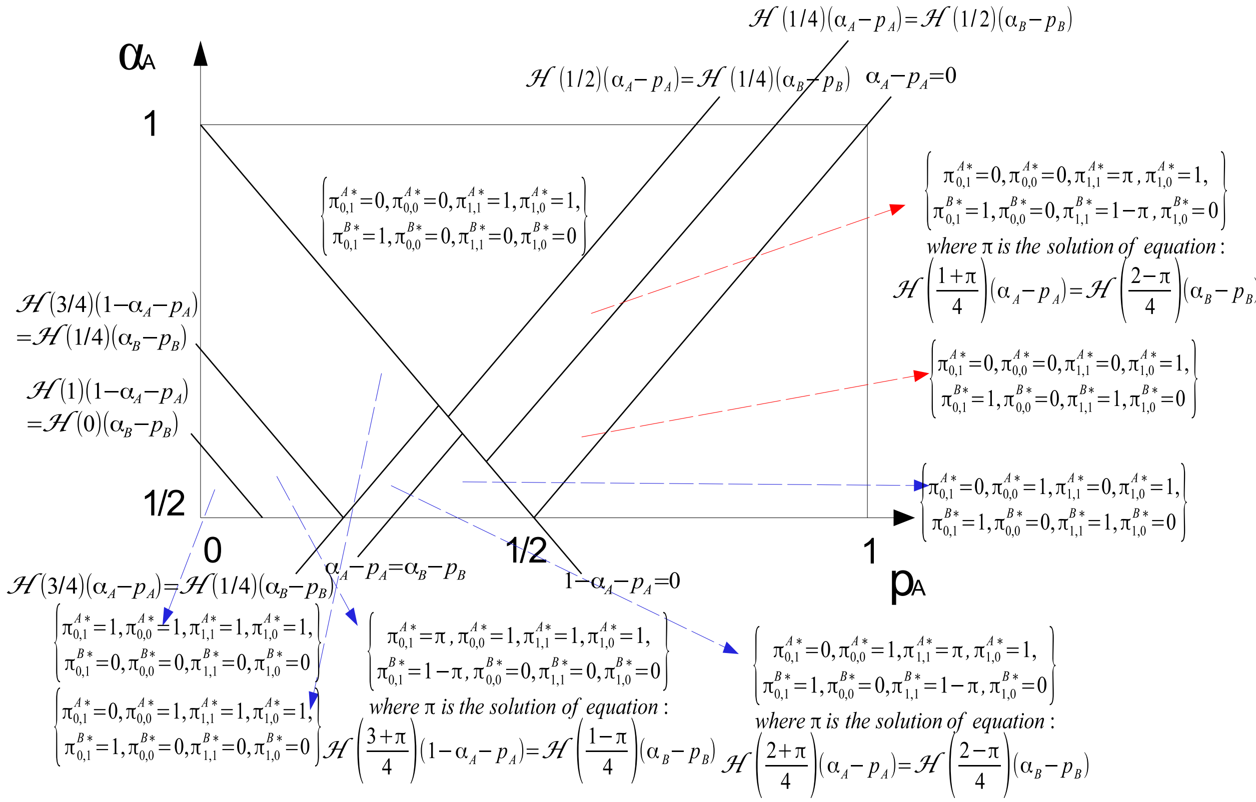

Figure 1: Symmetric equilibrium among buyers when $p_B > 1 - \alpha_B$

perfectly informative signal structure and a higher price. The price increase is equal to the increase in buyers' expected payoff from trading with the seller when observing signal realization 1 from her.

Lemma 3 *For any pair of signal structures and prices such that* $\alpha_j < 1$ *and* $\mathcal{M}(\pi^{j*}(\alpha_j, p_j, \alpha_{-j}, p_{-j})) > 0$ *for some seller* $j \in \{A, B\}$, $\Pi^j(1, p_j + 1 - \alpha_j, \alpha_{-j}, p_{-j}) > \Pi^j(\alpha_j, p_j, \alpha_{-j}, p_{-j})$.

To see why the above inequality holds, first consider the case where buyers

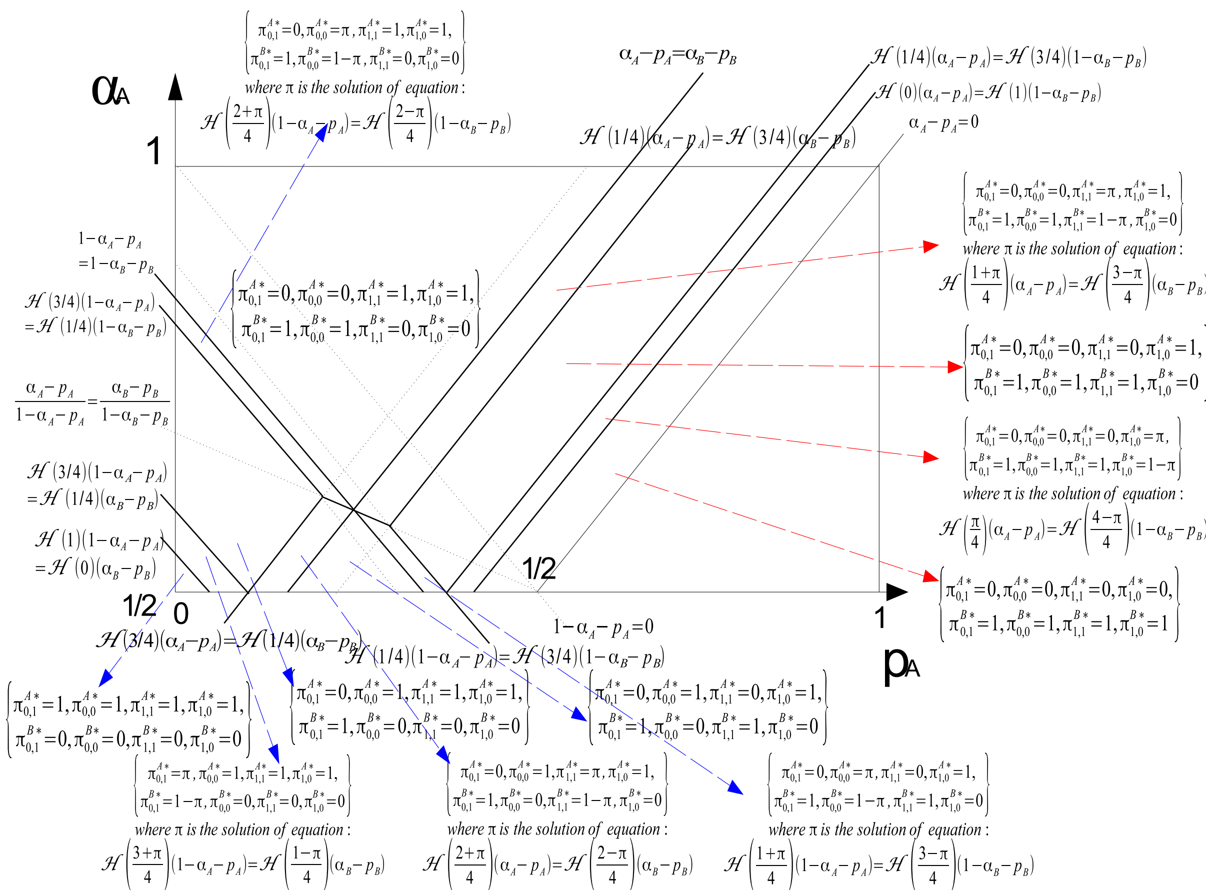

Figure 2: Symmetric equilibrium among buyers when $p_B \leq 1 - \alpha_B$

do not visit seller j under signal structure α_j and price p_j when observing signal realization 0 from the seller. Call them low signal buyers. Notice that after the seller deviates to the perfectly informative signal structure, these low-signal buyers do not visit the seller either, since their expected payoff from visiting her is negative. For buyers who observe signal realization 1 (or high-signal buyers), since their expected payoff from trading with the seller after she deviates the perfectly informative signal structure is the same as before the deviation, their expected payoff from visiting the seller is the same as before. As a result, they select the seller with an equal probability. This implies that the seller can sell her good with an equal probability as before. When the initial probability that the seller can sell her good is strictly positive, after she deviates to the perfectly informative signal structure, her expected profit becomes strictly greater, since the probability for her to sell her good is the same as before while her price becomes strictly greater.

Now consider the case where low-signal buyers initially visit seller j with a strictly positive probability. For this to be true, we must have that $p_j < 1-\alpha_j$. This implies that the price after the seller deviates to full information provision is greater than twice of the seller's initial price. On the other hand, since high-signal buyers are more likely to visit the seller than low-signal buyers, at most one half of buyers who initially visit her are low-signal buyer. Therefore, when the seller deviates to the perfectly informative signal structure, since low-signal buyers no longer visit her, the probability that she can sell her good decrease by at most one half. Given that the price is at least twice as high and the probability of selling is at most one half as low, the expected profit of the seller must increase.

In the second case, the initial probability that the seller can sell her good is zero. We show that the seller can earn a strictly positive expected profit by deviating to the perfectly informative signal structure and a sufficiently small price.

Lemma 4 *For any pair of signal structures and prices such that* $\alpha_j < 1$ *and* $\mathcal{M}(\pi^{j*}(\alpha_j, p_j, \alpha_{-j}, p_{-j})) = 0$ *for some seller* $j \in \{A, B\}$*,* $\Pi^j(1, \varepsilon, \alpha_{-j}, p_{-j}) > 0$ *when* $\varepsilon > 0$ *is sufficiently small.*

To see why the above result holds, notice that seller j's probability of selling under $(\alpha_j, p_j, \alpha_{-j}, p_{-j})$ is zero only when $p_{-j} \leq 1 - \alpha_{-j}$. Otherwise, buyers who receive signal realization 1 from seller j and 0 from the other seller will visit seller j with probability 1, which implies that seller j can sell her good with a strictly positive probability. We show that seller j can earn a strictly positive expected profit by deviating to the perfectly informative signal structure and a sufficiently small price $\varepsilon > 0$. By contradiction, suppose that $\pi^{j*}(1, \varepsilon, \alpha_{-j}, p_{-j}) = 0$. In other words, every buyer visits the seller with probability 0 for any profile of signal realizations from two sellers. For a buyer who observes signal realization 1 from seller j and 0 from the other seller, his expected payoff from visiting seller j is $\mathcal{H}(\pi^{j*}(1, \varepsilon, \alpha_{-j}, p_{-j}))(1 - \varepsilon) = 1 - \varepsilon$, while that from visiting the other seller is $\mathcal{H}(1 - \pi^{j*}(1, \varepsilon, \alpha_{-j}, p_{-j}))(1 - \alpha_{-j} - p_{-j}) = (1 - \alpha_{-j} - p_{-j})/I$. Since $1 - \alpha_{-j} - p_{-j} < 1/2$, for a sufficiently small ε, $1 - \varepsilon$ is strictly greater than $(1 - \alpha_{-j} - p_{-j})/I$. This implies that the equilibrium probability that the buyer visits seller j is strictly positve, which contradicts with the supposition that $\pi^{j*}(1, \varepsilon, \alpha_{-j}, p_{-j}) = 0$. Therefore, when

ε is sufficiently small, the probability that seller j can sell her good under the perfectly informative signal structure and price ε must be strictly positive.

Given Lemma 3 and 4, a seller can always increase her expected profit by deviating to the perfectly informative signal structure regardless of the opponent seller price and signal structure. This establishes that both sellers choose the perfectly informative signal structure in any equilibrium where buyers follow their symmetric equilibrium strategies.

Proposition 1 *Each seller chooses the perfectly informative signal structure in any equilibrium between sellers.*

The proof of the above result can be found in the appendix. The next section characterizes the equilibrium pricing strategy of sellers when both of them choose the perfectly informative signal structure.

5 Equilibrium pricing strategy under full information provision

The model under full information provision is isomorphic to the one in Lester (2011) (see also Huang, 2016). Under full information provision, buyers who observe signal 1 from one seller and 0 from the other have valuation 1 for the first seller and 0 for the second one. Therefore, they can be seen as "uninformed buyers" in the model of Lester (2011), in the sense that their decisions on which seller to visit are independent from the prices charged by two sellers. On the other hand, buyers who observe signal realization 1 from both sellers can be seen as "informed buyers" in Lester (2011).

Using an argument similar to the one in Lester (2011), we can indentify the condition under which there exists an equilibrium between sellers in pure price-setting strategies. Denote by p_c the candidate for a pure-strategy equilibrium in setting prices. To be an equilibrium, a seller's expected profit must reach its local maximum at p_c when the other seller charges p_c. This condition can be arranged as

$$\frac{p_c}{1-p_c} = -\frac{\mathcal{M}(3/8)}{\mathcal{M}'(3/8)}\frac{2\mathcal{H}'(3/8)}{\mathcal{H}(3/8)}. \tag{2}$$

Another necessary condition for p_c to be an equilibrium pricing strategy is that under full information provision, a seller's expected profit by charging p_c is no less than that by charging 1 when the other seller charges p_c. This can be written as

$$\mathcal{M}(3/8)p_c \geq \mathcal{M}(1/4). \tag{3}$$

It can be verified that when $I \geq 7$, p_c as the solution of equation (2) satisfies inequality (3). This implies that there exists an equilibrium in setting prices where each seller charges p_c when $I \geq 7$. On the other hand, when $I \leq 6$, (3) is not satisfied by p_c. This implies that there does not exist a pure strategy equilibrium in setting prices when $I \leq 6$. Following the steps in Huang (2016), we can show that there exists an equilibrium in setting prices where sellers use a mixed strategy. The support of the equilibrium pricing strategy consists of a countable number of prices characterized by the first-order difference equation,

$$\frac{1}{p_{k+1}}+\frac{\mathcal{M}(1/2)-\mathcal{M}(3/8)}{\mathcal{M}(3/8)}\frac{p_c}{1-p_c}\frac{1-p_{k+1}}{p_{k+1}^2}=\frac{1}{p_k}-\frac{\mathcal{M}(3/8)-\mathcal{M}(1/4)}{\mathcal{M}(3/8)}\frac{p_c}{1-p_c}\frac{1-p_k}{p_k^2}, \tag{4}$$

with the initial condition being

$$\frac{1}{p_1}+\frac{\mathcal{M}(1/2)-\mathcal{M}(3/8)}{\mathcal{M}(3/8)}\frac{p_c}{1-p_c}\frac{1-p_1}{p_1^2}=\frac{\mathcal{M}(1/2)}{\mathcal{M}(1/4)}. \tag{5}$$

The equilibrium probability of charging each price is given by

$$x_k=\frac{\mathcal{M}(1/4)}{\mathcal{M}(3/8)}\frac{p_c}{1-p_c}\frac{1-p_k}{p_k^2}. \tag{6}$$

In addition, we can prove the following result.

Proposition 2 *When $I \leq 6$, there exists a symmetric equilibrium between sellers where each seller chooses the perfectly informative signal structure and randomizes over a countable number of prices. The equilibrium pricing strategy is characterized by difference equation (4) and (5), with the probability of charging each price given by (6). When $I \geq 7$, there exists a symmetric equilibrium where each seller chooses the perfectly informative signal structure and price p_c as defined by (2).*

The proof of the above result can be found in the appendix.

6 Concluding remarks

This paper investigates information provision by capacity-constrained sellers when they can also choose the prices of their goods, in a model where buyers visit sellers without coordination. The main result is that the perfectly informative signal structure is always more profitable for a seller than the partially informative one is, regardless of the other seller's choice of signal structure and price.

As can be seen in the proof, this result is associated with the model's stylized assumptions. One is that each seller's signal can only take binary realizations. Another one is the symmetric nature of sellers' signal structures. A natural question to investigate is whether the perfectly informative signal structure is still more profitable than the partially informative one in a more general setup. Another possible extension is to consider more general trading mechanisms of sellers. For example, instead of posting take-it-or-leave-it prices, sellers can post auctions.[7]

References

David Blackwell. Equivalent comparisons of experiments. *Annals of Mathematical Statistics*, 24(2):265–272, 1953.

Kenneth Burdett, Shouyong Shi, and Randall Wright. Pricing and matching with frictions. *Journal of Political Economy*, 109(5):1060–1085, 2001.

[7] See Troncoso-Valverde (2015) for a model where sellers compete in setting reserve prices of second-price auctions and providing information about potential bidders' valuations.

Gabriele Camera and Jaehong Kim. Buyer's equilibrium with capacity constraints and restricted mobility: A recursive approach. *Economics Letters*, 118:321–323, 2013.

Ettore Damiano and Hao Li. Information provision and price competition. Mimeo, University of Toronto, 2007.

Ce Huang. Price dispersion in a directed search model. Forthcoming in *Canadian Journal of Economics*, 2016.

Maxim Ivanov. Information revelation in competitive markets. *Economic Theory*, 52:337–365, 2013.

Benjamin Lester. Information and prices with capacity constraints. *American Economic Review*, 101(4):1591–1600, 2011.

Hao Li and Xianwen Shi. Discriminatory information disclosure. Mimeo, University of British Columbia, 2015.

Michael Peters. Bertrand equilibrium with capacity constraints and restricted mobility. *Econometrica*, 52(5):1117–1127, 1984.

Cristian Troncoso-Valverde. Information provision in competing auctions. Mimeo, Universidad Diego Portales, 2015.

A Proof of Lemma 3

Note that seller j's price increases when she deviates to full information provision. Therefore, if the probability that the seller can sell her good after she deviates is no less than her initial probability of selling, her expected profit is strictly greater after the deviation. So the remaining proof focuses on the case where the seller's probability of selling under $(1, p_j + 1 - \alpha_j, \alpha_{-j}, p_{-j})$ is strictly less than that under $(\alpha_j, p_j, \alpha_{-j}, p_{-j})$. For the brevity of notation, use $\pi^{j*}_{s,t}$ to denote $\pi^{j*}_{s,t}(\alpha_j, p_j, \alpha_{-j}, p_{-j})$, $\pi^{j*'}_{s,t}$ to denote $\pi^{j*}_{s,t}(1, p_j + 1 - \alpha_j, \alpha_{-j}, p_{-j})$, π^{j*} to denote $\pi^{j*}(\alpha_j, p_j, \alpha_{-j}, p_{-j})$ and $\pi^{j*'}$ to denote $\pi^{j*}(1, p_j+1-\alpha_j, \alpha_{-j}, p_{-j})$. Notice that $\pi^{j*'} < \pi^{j*}$. We need the following result to proceed.

Lemma 5 *When $p_{-j} \leq 1 - \alpha_{-j}$, or $p_j > 1 - \alpha_j$ and $p_{-j} > 1 - \alpha_{-j}$, if the probability that seller j sells her good under $(1, p_j + 1 - \alpha_j, \alpha_{-j}, p_{-j})$ is strictly less than that under $(\alpha_j, p_j, \alpha_{-j}, p_{-j})$, the probability that the other seller sells her good under $(1, p_j + 1 - \alpha_j, \alpha_{-j}, p_{-j})$ is no less than that under $(\alpha_j, p_j, \alpha_{-j}, p_{-j})$.*

Proof. When $p_{-j} \leq 1 - \alpha_{-j}$, the expected payoff of a buyer who visits seller $-j$ is nonnegative. This means that $\pi^{j*}_{s,t} + \pi^{-j*}_{s,t} = 1$ for all (s,t) and $\pi^{j*} + \pi^{-j*} = 1$. By the same argument, we have $\pi^{j*'} + \pi^{-j*'} = 1$. If the probability that seller j can sell her good under $(1, p_j + 1 - \alpha_j, \alpha_{-j}, p_{-j})$ is strictly less than that under $(\alpha_j, p_j, \alpha_{-j}, p_{-j})$, which implies that $\pi^{j*'} < \pi^{j*}$, we have that $\pi^{-j*'} > \pi^{-j*}$. This means that the probability that seller $-j$ can sell her good under $(1, p_j + 1 - \alpha_j, \alpha_{-j}, p_{-j})$ is greater than that under $(\alpha_j, p_j, \alpha_{-j}, p_{-j})$.

When $p_j > 1-\alpha_j$ and $p_{-j} > 1-\alpha_{-j}$, a buyer's expected payoff from visiting a seller is strictly negative when observing signal realization 0 from her. This implies that the buyer visits either seller with probability zero in equilibrium when observing signal realization 0 from both sellers, i.e., $\pi^{j*}_{0,0} = \pi^{-j*}_{0,0} = 0$.

On the other hand, if the buyer observes signal realization 1 from at least one seller, he receives a nonnegative payoff from visiting at least one seller and the sum of the equilibrium probabilities that he visits two sellers is equal to 1. In other words, $\pi_{1,0}^{j*} + \pi_{1,0}^{-j*} = 1$, $\pi_{0,1}^{j*} + \pi_{0,1}^{-j*} = 1$ and $\pi_{1,1}^{j*} + \pi_{1,1}^{-j*} = 1$. Therefore, we have that $\pi^{j*} + \pi^{-j*} = 3/4$. Similarly, we can show that $\pi^{j*'} + \pi^{-j*'} = 3/4$. By the same argument as the one when $p_{-j} \leq 1 - \alpha_{-j}$, we know that the statement of Lemma 5 is true. ■

Case 1. $p_j \leq 1 - \alpha_j$ and $p_{-j} \leq 1 - \alpha_{-j}$.

In this case, we consider several subcases.

Subcase 1.1 $\pi_{1,1}^{j*} > 0$.

We first prove that $\pi_{1,0}^{j*} > 0$. A buyer's expected utility from visiting seller j when observing signal realization $(1,1)$ is equal to that under signal realization $(1,0)$. On the other hand, the buyer's expected utility from visiting seller $-j$ under signal realization $(1,1)$ is no less than that under signal realization $(1,0)$ because the buyer's expected payoff from trading with seller $-j$ under signal realization $(1,1)$ is no less than that under signal realization $(1,0)$. Given $\pi_{1,1}^{j*} > 0$ and visiting seller j is equally attractive under signal realization $(1,0)$ as that under signal realization $(1,1)$ while visiting seller $-j$ is less attractive, the equilibrium probability that the buyer visits seller j under signal realization $(1,0)$ must also be positive, i.e., $\pi_{1,0}^{j*} > 0$.

Next, from $\pi_{1,1}^{j*} > 0$ and $\pi_{1,0}^{j*} > 0$, we can show that $\pi_{1,1}^{j*'} = 1$ and $\pi_{1,0}^{j*'} = 1$. Under $(1, p_j + 1 - \alpha_j, \alpha_{-j}, p_{-j})$, a buyer's expected payoff from trading with seller j upon observing signal realization $(1,1)$ is the same as that upon under $(\alpha_j, p_j, \alpha_{-j}, p_{-j})$. From $\pi^{j*'} < \pi^{j*}$, the probability that the buyer can trade with seller j is strictly greater under $(1, p_j + 1 - \alpha_j, \alpha_{-j}, p_{-j})$ than under $(\alpha_j, p_j, \alpha_{-j}, p_{-j})$. Therefore, the buyer's expected payoff from visiting j when observing signal realization $(1,1)$ is strictly greater under $(1, p_j + 1 - \alpha_j, \alpha_{-j}, p_{-j})$ than under $(\alpha_j, p_j, \alpha_{-j}, p_{-j})$. Similarly, from $\pi^{-j*} \leq \pi^{-j*'}$, the buyer's expected payoff from visiting $-j$ when observing signal realization $(1,1)$ is strictly less under $(1, p_j + 1 - \alpha_j, \alpha_{-j}, p_{-j})$ than under $(\alpha_j, p_j, \alpha_{-j}, p_{-j})$. Given $\pi^{j*}_{1,1} > 0$, we have that $\pi^{j*'}_{1,1} = 1$. By following the same argument as before, $\pi^{j*'}_{1,1} = 1$ implies that $\pi^{j*'}_{1,0} = 1$.

Thus $\pi^{j*'} = (\pi^{j*'}_{1,1} + \pi^{j*'}_{1,0})/4 = 1/2$, and the probability that seller j sells her good under $(1, p_j + 1 - \alpha_j, \alpha_{-j}, p_{-j})$ is $1 - (1 - 1/2)^I \geq 3/4$. Even if the probability that j sells her good under $(\alpha_j, p_j, \alpha_{-j}, p_{-j})$ is 1, the probability that the seller can sell under $(1, p_j + 1 - \alpha_j, \alpha_{-j}, p_{-j})$ is still greater than one half of the probability of selling under $(\alpha_j, p_j, \alpha_{-j}, p_{-j})$. On the other hand, from $p_j \leq 1 - \alpha_j$, we know that $p_j + 1 - \alpha_j \geq 2p_j$. This means that j's expected profit under $(1, p_j + 1 - \alpha_j, \alpha_{-j}, p_{-j})$ is strictly greater than that under $(\alpha_j, p_j, \alpha_{-j}, p_{-j})$.

Subcase 1.2 $\pi^{j*}_{1,1} = 0$.

This implies that $\pi^{j*}_{0,1} = 0$, which in turn implies that $\pi^{j*} \leq 1/2$. Therefore, the probability that seller j can sell her good under $(\alpha_j, p_j, \alpha_{-j}, p_{-j})$ is no greater than $1 - (1 - 1/2)^I$. There are two possibilities to consider. One possibility is that $\pi^{j*}_{1,0} > 0$. From the fact that $\pi^{j*}_{1,0} > 0$, together with $\pi^{j*} > \pi^{j*'}$ and $\pi^{-j*} \leq \pi^{-j*'}$, we have that $\pi^{j*'}_{1,0} = 1$. This implies that $\pi^{j*'} \geq 1/4$. Another possibility is that $\pi^{j*}_{1,0} = 0$, which implies that $\pi^{j*}_{0,0} = 0$ and $\pi^{j*} = 0$. This is the case that is ruled out because we assume that the probability that seller j sells her good is strictly positive under $(\alpha_j, p_j, \alpha_{-j}, p_{-j})$.

Case 2. $p_j \leq 1 - \alpha_j$ and $p_{-j} > 1 - \alpha_{-j}$.

In this case, we know that $\pi^{j*}_{1,0} = 1$, $\pi^{-j*}_{1,0} = 0$, $\pi^{j*}_{0,0} = 1$ and $\pi^{-j*}_{0,0} = 0$. In addition, $\pi^{j*}_{0,1} = 1 - \pi^{-j*}_{0,1}$ and $\pi^{j*}_{1,1} = 1 - \pi^{-j*}_{1,1}$. Therefore, $\pi^{j*} = (2 + \pi^{j*}_{0,1} + \pi^{j*}_{1,1})/4$ and $\pi^{-j*} = (2 - \pi^{j*}_{0,1} - \pi^{j*}_{1,1})/4$. Also, we know that $\pi^{j*'}_{1,0} = 1$, $\pi^{-j*'}_{1,0} = 0$, $\pi^{j*'}_{0,0} = 0$ and $\pi^{-j*'}_{0,0} = 0$. In addition, $\pi^{j*'}_{0,1} = 0$, $\pi^{-j*'}_{0,1} = 1$ and $\pi^{j*'}_{1,1} = 1 - \pi^{-j*'}_{1,1}$. Therefore, $\pi^{j*'} = (1 + \pi^{j*'}_{1,1})/4$ and $\pi^{-j*'} = (2 - \pi^{j*'}_{1,1})/4$. There are two subcases to consider.

Subcase 2.1 $\pi^{j*}_{0,1} > 0$.

By following a similar argument as before, we can establish that $\pi^{j*}_{1,1} = 1$ and $\pi^{-j*}_{1,1} = 0$. Therefore, $\pi^{j*} = (3 + \pi^{j*}_{0,1})/4$ and $\pi^{-j*} = (1 - \pi^{j*}_{0,1})/4$. From $\pi^{j*} = (3 + \pi^{j*}_{0,1})/4$ and $\pi^{j*'} = (1 + \pi^{j*'}_{1,1})/4$, we have that $\pi^{j*} > \pi^{j*'}$. In addition, from $\pi^{-j*} = (1 - \pi^{j*}_{0,1})/4$ and $\pi^{-j*'} = (2 - \pi^{j*'}_{1,1})/4$, we have that $\pi^{-j*} \leq \pi^{-j*'}$. From $\pi^{j*} > \pi^{j*'}$ and $\pi^{-j*} \leq \pi^{-j*'}$, we have that $\pi^{j*'}_{1,1} \geq \pi^{j*}_{1,1}$. Since $\pi^{j*}_{1,1} = 1$, we know that $\pi^{j*'}_{1,1} = 1$. Therefore, $\pi^{j*'} = 1/2$.

Subcase 2.2 $\pi^{j*}_{0,1} = 0$.

In this subcase, we know that $\pi^{j*} = (2 + \pi^{j*}_{1,1})/4$, $\pi^{j*'} = (1 + \pi^{j*'}_{1,1})/4$, $\pi^{-j*} = (2 - \pi^{j*}_{1,1})/4$ and $\pi^{-j*'} = (2 - \pi^{j*'}_{1,1})/4$. First, we can show that $\pi^{j*}_{1,1} \leq \pi^{j*'}_{1,1}$. By contradiction, suppose that $\pi^{j*}_{1,1} > \pi^{j*'}_{1,1}$. This implies that $\pi^{-j*} < \pi^{-j*'}$ since $\pi^{-j*} = (2 - \pi^{j*}_{1,1})/4$ and $\pi^{-j*'} = (2 - \pi^{j*'}_{1,1})/4$. From $\pi^{j*} > \pi^{j*'}$ and $\pi^{-j*} < \pi^{-j*'}$, by following our previous argument, we can prove that $\pi^{j*}_{1,1} \leq \pi^{j*'}_{1,1}$, which contradicts with our supposition that $\pi^{j*}_{1,1} > \pi^{j*'}_{1,1}$. Therefore, we must have $\pi^{j*}_{1,1} \leq \pi^{j*'}_{1,1}$. Thus, $\pi^{j*'} = (1 + \pi^{j*'}_{1,1})/4 \geq (1 + \pi^{j*}_{1,1})/4 = \pi^{j*} - 1/4$.

Case 3. $p_j > 1 - \alpha_j$.

In this case, we know that $\pi_{0,1}^{j*} = \pi_{0,1}^{j*'} = \pi_{0,0}^{j*} = \pi_{0,0}^{j*'} = 0$. If $\pi^{j*} > \pi^{j*'}$, by Lemma 5, we know that $\pi^{-j*} \leq \pi^{-j*'}$. Following the same argument as before, we can show that $\pi_{1,1}^{j*} \leq \pi_{1,1}^{j*'}$ and $\pi_{1,0}^{j*} \leq \pi_{1,0}^{j*'}$. Therefore, we know that $\pi^{j*} \leq \pi^{j*'}$, which is a contradiction. Thus, π^{j*} is no greater than $\pi^{j*'}$.

B Proof of Proposition 1

By contradiction, suppose that there exists an equilibrium in which seller j chooses a signal structure $\alpha_j < 1$ and some price p_j with a strictly positive probability. According to Definition 2, we know that

$$\Pi^j(\alpha_j, p_j; G^{-j}(.)) \geq \Pi^j(1, p_j + 1 - \alpha_j; G^{-j}(.)),$$

where $G^{-j}(.)$ is the other seller's equilibrium strategy in choosing signal structures and prices. The above inequality can be written as

$$\begin{aligned} &\int_{(\alpha_{-j}, p_{-j}) \in \overline{G^{-j}}} \Pi^j(\alpha_j, p_j, \alpha_{-j}, p_{-j}) dG^{-j}(\alpha_{-j}, p_{-j}) \geq \\ &\qquad \int_{(\alpha_{-j}, p_{-j}) \in \overline{G^{-j}}} \Pi^j(1, p_j + 1 - \alpha_j, \alpha_{-j}, p_{-j}) dG^{-j}(\alpha_{-j}, p_{-j}), \end{aligned} \tag{7}$$

where $\overline{G^{-j}}$ is the support of $G^{-j}(.)$.

However, from Lemma 3, we know that as long as the probability that seller j can seller her good under $(\alpha_j, p_j, \alpha_{-j}, p_{-j})$ is strictly positive,

$$\Pi^j(\alpha_j, p_j, \alpha_{-j}, p_{-j}) < \Pi^j(1, p_j + 1 - \alpha_j, \alpha_{-j}, p_{-j}).$$

On the other hand, if the probability that seller j can sell her good under $(\alpha_j, p_j, \alpha_{-j}, p_{-j})$ is equal to zero, which implies that $\Pi^j(\alpha_j, p_j, \alpha_{-j}, p_{-j}) = 0$, we have that

$$\Pi^j(\alpha_j, p_j, \alpha_{-j}, p_{-j}) \leq \Pi^j(1, p_j + 1 - \alpha_j, \alpha_{-j}, p_{-j}).$$

Therefore, inequality (7) can hold only when seller j sells her good with zero probability by choosing (α_j, p_j) for any $(\alpha_{-j}, p_{-j}) \in \overline{G^{-j}}$. From Lemma 4, we know that the seller can earn a higher expected profit by choosing $(1, \varepsilon)$ when $\varepsilon > 0$ is sufficiently small. This implies that

$$\Pi^j(\alpha_j, p_j; G^{-j}(.)) < \Pi^j(1, \varepsilon; G^{-j}(.)),$$

which contradicts with the supposition that the seller chooses (α_j, p_j) with a positive probability in equilibrium.

C Proof of Proposition 2

First, we consider the case of $I \leq 6$. Use $\{p_k\}_{k=1}^{\infty}$ to denote the price sequence characterized by the difference equation (4) and (5) and use x_k to denote the probability of charging p_k as characterized by (6). Suppose that one seller provides all information and randomizes over $\{p_k\}_{k=1}^{\infty}$ with the probability of charging p_k being x_k. A seller's expected profit by choosing signal structure α_j and price p_j is

$$\sum_{k=1}^{\infty} \Pi^j(\alpha_j, p_j, 1, p_k) x_k.$$

Since $0 < p_k < 1$ for all $k \in \mathbb{N}$, we know that seller j can sell her good with a strictly positive probability under $(\alpha_j, p_j, 1, p_k)$ for all $k \in \mathbb{N}$. From Lemma 3, we know that for all $k \in \mathbb{N}$,

$$\Pi^j(\alpha_j, p_j, 1, p_k) \leq \Pi^j(1, p_j + 1 - \alpha_j, 1, p_k),$$

which implies that

$$\sum_{k=1}^{\infty} \Pi^j(\alpha_j, p_j, 1, p_k)x_k \leq \sum_{k=1}^{\infty} \Pi^j(1, p_j + 1 - \alpha_j, 1, p_k)x_k. \tag{8}$$

From Huang (2016), we know that when $I \leq 6$ and both sellers choose the perfectly informative signal structure, there exists a symmetric equilibrium in price setting where both sellers randomize over $\{p_k\}_{k=1}^{\infty}$ with the probability of charging p_k being x_k. This implies that for any $l \in \mathbb{N}$, seller j's expected profit by charging p_l is no less than that by charging any other price, when the other seller follows the equilibrium pricing strategy. This implies that for any $l \in \mathbb{N}$,

$$\sum_{k=1}^{\infty} \Pi^j(1, p_j + 1 - \alpha_j, 1, p_k)x_k \leq \sum_{k=1}^{\infty} \Pi^j(1, p_l, 1, p_k)x_k. \tag{9}$$

Inequalities (8) and (9) together imply that for any $l \in \mathbb{N}$,

$$\sum_{k=1}^{\infty} \Pi^j(\alpha_j, p_j, 1, p_k)x_k \leq \sum_{k=1}^{\infty} \Pi^j(1, p_l, 1, p_k)x_l.$$

The above inequality implies that it is more profitable for seller j to choose the perfectly informative signal structure and price p_l than to choose any other signal structure and price, given that the other seller chooses the perfectly informative signal structure and randomizes over prices $\{p_k\}_{k=1}^{\infty}$ with the probability of charging p_k being x_k. Therefore, when $I \leq 6$, choosing the perfectly informative signal structure and randomizing over $\{p_k\}_{k=1}^{\infty}$ is a symmetric equilibrium between sellers.

Similarly, we can show that when $I \geq 7$, there exists a symmetric equilibrium where each seller chooses the perfectly informative signal structure and price p_c as defined by (2).