Contests with Foot-Soldiers

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In many-real world contests (*e.g., elections*), *contestants engage foot-soldiers to fight for them, by promising them alternative forms of compensation. This paper studies a bilateral contest where the contestants can recruit foot-soldiers by offering them* conditional compensations (*each contestant*'*s foot-soldiers get their promised rewards if and only if that contestant wins the contest*). *In our contest game, the two contestants – an* underdog *and an* overdog – *make simultaneous conditional offers to attract foot-soldiers. Each foot-soldier*'*s decision to join a contestant depends upon the offers, his relative closeness to the contestant, and his assessment about the contestants*' *chances of winning. Our current analysis focuses on two payoff structures*: *one in which the winner*'*s net prize depends* (*negatively*) *only on her offer amount, and the other in which it depends* (*negatively*) *on the total compensation to be paid to her foot-soldiers. Under the former payoff structure*, *the two candidates*' *offers are identical in every pure-strategy Nash equilibrium*; *and so, whenever this common offer is positive*, *the* overdog *increases her* probability lead *against the* underdog. *Under the latter payoff structure*, *the* underdog *offers a higher compensation than the* overdog; *nevertheless she remains an underdog in the contest* (*and indeed can become more so*).

1. Introduction

The literature on "contests" started with the seminal contributions of Tullock (1967, 1980) and Krueger (1974) who studied rent-seeking, and of Becker (1983) who studied the practice of lobbying. Since then, *contest theory* has been applied to improve our understanding of influence-activities within organizations (see, e.g., Rosen 1986), patent races (see, e.g., Reinganum 1989), prize-seeking (see, e.g., Nitzan 1994), political and electoral competition (see, e.g., Baron 1994), litigation (see, e.g., Hirshleifer and Osborne 2001), and conflicts and wars (see, e.g., Garfinkel and Skaperdas 2007).

It is important to recognize that in many contest scenarios, the contestants try to engage various "foot soldiers to fight on their behalf" by promising them alternative kinds of compensation.

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Two examples of such contests are: (*a*) political parties enticing unemployed youth (*cadres*) to bolster their election campaigns, in ethical ways (by organizing and attending political rallies) and/or in unethical ways (by voter-intimidation);[1] (*b*) competing multinational firms improving their chances of successfully entering a "new market" by trying to co-opt local supply chains and/or distribution channels *via* various kinds of side-contracts and deals.

In such scenarios where the contestants try to involve "foot soldiers" in the *contest process*, a set of interesting questions present themselves for analysis: What kind of a contestant – an "underdog" or an "overdog" – has a greater incentive to engage foot-soldiers? How does the public perception of the eventual success of one contestant *vs.* another influence the foot-soldiers' incentives to work for a particular contestant? What is the "net outcome effect" for the contestants when they can engage foot soldiers – does an *ex ante* underdog become more (or less) of an underdog in contests with foot-soldiers?[2] Would some (or all) of the contestants prefer an exogenously imposed ban on the hiring of foot-soldiers? This paper aims to address such issues by constructing and analyzing a class of models of "contests with foot-soldiers".

The rest of the current (preliminary) draft is organized as follows. Section 2 presents the basic contest model. Equilibrium characterization results for two specific "payoff structures" are presented in Section 3. Section 4 contains some concluding remarks. Formal proofs will be presented in an (as yet unprocessed) Appendix.

2. A Contest Model with Conditional Compensation Offers

In what follows, we present a stylized model of a "bilateral contest with foot soldiers". The defining feature of our model is that the contestants can entice foot-soldiers to join their campaigns by making '*conditional compensations offers*', where each contestant's foot-soldiers get their promised rewards if and only if that specific contestant wins the contest. This kind of a situation arises overwhelmingly in election contests where a substantial portion of a foot-soldier's "reward" is realized if and only if *his* party wins the elections. At this initial stage of our analysis, we study a model that incorporates *only* conditional compensation offers, precisely because we want to study a scenario where the foot-soldiers' *beliefs* about the eventual "winnability" of a contestant play a significant role in their determining which contestant to work for. In subsequent research, we plan to extend our analysis to the general case where each

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[1] For studies of various kinds of "unethical practices" in election contests see Chaturvedi (2005), Collier (2009), and Collier and Vicente (2012).

[2] This question is related to the broader issue of "allocative effects of contests"; see Corchón (2000).

contestant is in a position to make both *conditional and unconditional* compensation offers.[3]

There are two contestants – L and R – fighting for a prize; they are located at positions 0 and 1 respectively. There is a set Σ of continuum of agents of unit measure located on the line interval [0, 1]. The contestants compete in making conditional compensation offers to the agents in order to entice (some of) them to become foot-soldiers in their fight for the prize.

First, a publicly observed variable $\theta \in [-½, ½]$ provides public information about each contestant's *ex ante* chance of winning the prize. We take θ to be the *ex ante win probability difference* – i.e., the difference in the initial win probabilities of the contestants L and R. Then, defining p_i to be contestant i's 'prior win probability' (for $i = L, R$), we have:

$$p_L = 0.5(1+\theta) \in [0.25, 0.75], \text{ and } p_R = 0.5(1-\theta) \in [0.25, 0.75].$$

Here, if $\theta < 0$ (resp., $\theta > 0$) then contestant L (resp., R) will be the "*ex ante* underdog".[4]

After θ is publicly observed, each contestant i, in her attempt to recruit foot-soldiers for her cause, announces a conditional compensation offer g_i from a feasible set $[0, g^+]$. We assume that each contestant's announcement g_i is credible due to reputational concerns. Subsequently, we consider two different formulations of each contestant's "costs incurred" in keeping her promise.

An agent s, located at position $s \in [0, 1]$, decides to become a foot-solder of a specific contestant by making the following cost-benefit analysis. Having observed θ and the promised compensation vector $\{g_L, g_R\}$, the agents form *updated beliefs* $\{\pi_L, \pi_R \equiv (1-\pi_L)\}$ about each contestant's chances of winning the contest; we assume that these beliefs are identical across all agents. We define the 'distance' $d(s, i)$ between agent $s \in [0, 1]$ and contestant i as follows: $d(s, L) = s$, and $d(s, R) = 1-s$. We then posit that given $\{\pi_L, \pi_R\}$, if agent s becomes the foot-soldier of contestant i (for $i = L, R$), then his expected payoff is:

$$F_i(s, \pi_i \mid g_i) \equiv \pi_i.g_i + \beta.[1 - d(s, i)].$$

The above payoff specification incorporates the feature that each agent s gets a non-pecuniary benefit from becoming a foot-soldier of a specific contestant i, and the magnitude of this benefit – parameterized by $\beta > 0$ – depends on the 'closeness' of agent s to contestant i. As a result, given $\{g_L, g_R\}$ and $\{\pi_L, \pi_R\}$, agent s chooses to become a foot-soldier of some contestant i only if $F_i(s, \pi_i \mid g_i) \geq F_j(s, 1-\pi_i \mid g_j)\}$, for $i, j = L, R$, and $i \neq j$. We assume that: $\beta \geq g^+$; this implies that

[3] *Ex ante* (and *ex post*) budget constraints, as well as legal stipulations, can limit the amounts of each kind of compensation offer that a contestant can make.

[4] Note that we start from a situation where each contestant has a prior win probability of at least 25%.

an agent located at $s = 0$ will always strictly prefer to be the foot-soldier of contestant L, while an agent located at $s = 1$ will always strictly prefer to be the foot-soldier of contestant R.[5]

If $\{S_L, S_R\} \in (0, 1)\times(0, 1)$ is the vector of measures of foot-soldiers of the contestants, we posit that the '*ex post* win probability difference' between them will be $[\theta + \lambda\times(S_L - S_R)] \in (-1, 1)$. Here, $\lambda \in (0, 0.5)$ measures the effectiveness of foot-soldiers in raising win probabilities. Then, defining $P_i(S_i, S_j)$ to be contestant i's 'posterior win probability', we have:

$$P_L(S_L, S_R) = 0.5[1 + \theta + \lambda\times(S_L - S_R)] \in (0, 1) \text{ and } P_R(S_R, S_L) = 0.5[1 - \theta + \lambda\times(S_R - S_L)]] \in (0, 1).$$

If $P_i(S_i, S_j) < ½ < P_j(S_j, S_i)$, then we will refer to contestant i as the "*ex post* underdog."

We now determine the "rational-expectations equilibrium" choices made by the agents in joining the two contestants, given the publicly-observed θ, and the announced compensations $\{g_L, g_R\}$.

For any $\theta \in [-½, ½]$ and $s \in [0, 1]$, define: $\Pi(s, \theta) \equiv 0.5[1 + \theta + \lambda.(2s - 1)] \in (0, 1)$. Next, given $\{g_L, g_R\} \in [0, g^+]\times[0, g^+]$, define $Z(s, \theta) \equiv 2\beta.s - [g_L + g_R].\Pi(s, \theta)$. Then define $s^*(g_L, g_R; \theta) \in [0, 1]$ such that $Z(s^*(g_L, g_R; \theta), \theta) = [\beta - g_R]$. Note that if $s^*(g_L, g_R; \theta)$ exists in $(0, 1)$ and the agents hold the beliefs $\{\pi_L = w(s^*(g_L, g_R; \theta), \theta);\ \pi_R = [1 - w(s^*(g_L, g_R; \theta), \theta)]\}$, then an agent located at $s^*(g_L, g_R; \theta)$ will be indifferent between being a foot-soldier of either contestant.

Further, we argue that under the assumption: $\beta \geq g^+$, $s^*(g_L, g_R; \theta)$ is unique and belongs in $(0, 1)$. Note that $Z(0, \theta) < 0 \leq [\beta - g_R] < Z(1, \theta)$, with $Z_s(s, \theta) = 2\beta - [g_L + g_R].\lambda > 0$ for all $s \in [0, 1]$ and $\theta \in [-½, ½]$. The equation $\{Z(s, \theta) = [\beta - g_R]\}$ thus has a unique solution $s^*(g_L, g_R; \theta) \in (0, 1)$.

The above analysis proves the following result:

PROPOSITION 1. Starting from any $\theta \in [-½, ½]$, for an announced conditional compensation vector $\{g_L, g_R\} \in [0, g^+]\times[0, g^+]$, all agents s_L located in the interval $[0, s^*(g_L, g_R; \theta)]$ become foot-soldiers of contestant L, and all agents s_R located in the interval $[s^*(g_L, g_R; \theta), 1]$ become foot-soldiers of contestant R in the ensuing "rational-expectations equilibrium". As a result, the "equilibrium posterior win probability" of contestant L is $P_L^*(g_L, g_R; \theta) \equiv \Pi(s^*(g_L, g_R; \theta), \theta)$ and that of contestant R is $P_R^*(g_L, g_R; \theta) \equiv [1 - \Pi(s^*(g_L, g_R; \theta), \theta)]$.

Finally, from the above analysis, we can deduce that:

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[5] In our current formulation, we implicitly assume that each agent *will* choose to be somebody's foot-soldier as his 'outside option utility' is 0. We will consider a more general model in future where each agent's outside option utility will be $\bar{u} \geq 0$ (arising out of some alternate employment opportunity). Then an agent can indeed choose not to be anyone's foot-soldier; specifically, given $\{g_L, g_R\}$ and $\{\pi_L, \pi_R\}$ agent s will choose to be contestant i's foot-soldier only if: $F_i(s, \pi_i \mid g_i) \geq max\{\bar{u}, F_j(s, 1-\pi_i \mid g_j)\})\}$.

$$s^*(g_L, g_R;\, \theta) \;=\; 0.5\{1 + [(p_L.g_L - p_R.g_R)/(\beta - 0.5\lambda[g_L + g_R])]\}, \text{ and}$$

$$\Pi^*(g_L, g_R;\, \theta) \;\equiv\; \Pi(s^*(g_L, g_R;\, \theta), \theta) \;=\; p_L + 0.5\lambda[(p_L.g_L - p_R.g_R)/(\beta - 0.5\lambda[g_L + g_R])].$$

Thus $s^*(.)$ and $\Pi^*(.)$ are unique in (0, 1) and are continuous in all their arguments. This implies that in the "rational-expectations equilibrium" given $(g_L, g_R;\, \theta)$, the measure of foot-soldiers and the posterior win probability is uniquely determined for each contestant, and these variables are continuous in $(g_L, g_R;\, \theta)$.

We complete the description of our contest model by discussing alternative structures of the contestants' payoffs. We posit the following: If contestant i (for $i = L, R$) wins the contest then her payoff is $\boldsymbol{V^i} = V(g_i, S_i)$, while if she loses the contest then her payoff is zero. Here, we refer to $V(g_i, S_i)$ as the "size of the prize" to each contestant as a function of her conditional compensation offer and the measure of her foot-soldiers. In general, we will posit that $V(.)$ will be unambiguously falling in g_i, while it may or may not be falling in S_i. In the current draft, we consider the following two distinct specifications of a contestant's "size of the prize".

Payoff Structure I: $\boldsymbol{V}^i \equiv V(g_i)$ with $V'(.) < 0$ and $V''(.) \leq 0$. We can interpret this structure in the following manner: When contestant i announces g_i she essentially commits to a specific amount of reduction in the "size of the prize" that will accrue to her if (and only if) she wins the contest, while her foot-soldiers will collectively capture the remainder of the prize if she wins.[6]

Payoff Structure II: $\boldsymbol{V}^i \equiv V([g_i.S_i])$ with $V'(.) < 0$ and $V''(.) \leq 0$. We can interpret this payoff structure as follows: When contestant i announces g_i she commits to a 'total wage bill' $[g_i.S_i]$ to her foot-soldiers, and the reduction in her prize's size depends on this total wage bill.[7]

In either case, given a realized θ, each contestant i simultaneously announce $g_i \in [0, g^+]$ to maximize her expected payoff: $\boldsymbol{W}^i = P_i^*(g_i, g_j;\, \theta) \times \boldsymbol{V^i}$. Since $s^*(g_L, g_R;\, \theta)$ is unique in (0, 1) and continuous in all its arguments, the maximand of each contestant is a well-defined continuous function in the choice variables (g_i, g_j). These facts allow us to determine the 'best-response compensation offer functions' $g_L^{BR}(g_R)$ and $g_R^{BR}(g_L)$. Every intersection point of $g_L^{BR}(g_R)$ and $g_R^{BR}(g_L)$ will give us a vector $\{g_L^*, g_R^*\}$ that constitutes a *pure-strategy Nash equilibrium conditional compensation offer vector*. If $\boldsymbol{W}^L$ and $\boldsymbol{W}^R$ are quasi-concave in g_L and g_R respectively, existence of at least one such 'pure-strategy Nash equilibrium' will be guaranteed.

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[6] Thus, the size of the prize going to the winner depends only on her announced g_i, and not on the measure of her foot-soldiers S_i. In the context of an election, the announced g_i might ear-mark a set of revenue-collection sources only to be tapped by the foot-soldiers. Note that if $V(g_i) = v - (g_i)^n$ where the grand prize is v and reduction is $(g_i)^n$, then we will have $V'(.) < 0$ and $V''(.) \leq 0$ for all $n \geq 1$.

[7] If $V(g_i S_i) = v - \varphi(g_i.S_i)$, where $\varphi'(.) > 0$ and $\varphi''(.) \geq 0$, we will have $V'(.) < 0$ and $V''(.) \leq 0$.

3. Contest Equilibria

In this section, we characterize – for payoff structures I and II – the contestants' *pure-strategy* Nash equilibrium conditional compensation offers, and the resultant foot-soldier sizes and posterior win probabilities, for every initial win perception as parameterized by $\theta \in [-½, ½]$.

3.1 Equilibrium Outcomes for Payoff Structure I

Under payoff structure I, for a realized $\theta \in [-½, ½]$, contestants L and R simultaneously announce $g_L \in [0, g^+]$and $g_R \in [0, g^+]$ to maximize their respective expected payoffs:

$$\boldsymbol{W}^L = \Pi^*(g_L, g_R; \theta)\times V(g_L), \text{ and } \boldsymbol{W}^R = [1-\Pi^*(g_L, g_R; \theta)]\times V(g_R).$$

It can be verified that when $V'(.) < 0$, both the maximands are quasi-concave in their respective maximizers, giving us the following result:

PROPOSITION 2. Under payoff structure I, there exists at least one pure-strategy Nash equilibrium conditional compensation offer vector $\{g_L^*, g_R^*\}$.[8]

Next, note that for each contestant i,

$$\partial \boldsymbol{W}^i/\partial g^i = \{(p_i.\beta - 0.5\lambda g_i)/[\beta - 0.5\lambda(g_L + g_R)]^2\}\times\{[\beta - 0.5\lambda(g_i + g_j)]V'(g_i) + 0.5\lambda V(g_i)\}.$$

Recognize that the first bracketed term is unambiguously positive, while the second bracketed term is independent of the initial win probabilities. This special structure of the "marginal utility functions" of the two contestants leads to the following result:

PROPOSITION 3. Under payoff structure I, every pure-strategy Nash equilibrium is *symmetric*; i.e., if $\{g_L^*, g_R^*\}$ is a Nash equilibrium compensation vector then $g_L^* = g_R^*$.

Note that for any realized $\theta \neq 0$ (an event that occurs with probability one), the two contestants are not *ex ante* symmetric – one is the *ex ante* underdog while the other is the *ex ante* overdog. Proposition 3 asserts that when the contest winner's *net prize* depends only on her compensation offer, then the underdog and the overdog make *identical* offers in any pure-strategy equilibrium.

But that does not necessarily mean that the contestants "preserve" their initial win probabilities. To see this, consider the foot-soldier located at $s = ½$. If he receives identical positive

[8] Existence of a pure-strategy Nash equilibrium is also guaranteed for the following payoff structure: $\boldsymbol{V}^i \equiv V(g_i)$ with $V'(.) < 0$ and $V''(.) \geq 0$. In this case, the maximands are quasi-convex (generating corner equilibria), and it can be shown that exists at least one equilibrium offer vector $\{g_L^*, g_R^*\}$ where $g_i^* \in \{0, g^+\}$ for $i = L$ and R.

compensation offers from the two contestants, he will work for the one who has a higher *ex ante* probability of winning. Thus, when the two contestants make identical positive compensation offers, the *ex ante* underdog will manage to hire a "smaller" group of foot-soldiers, and as a result, will secure an even lower *ex post* probability of winning the contest.

This will not be the case only when the symmetric equilibrium involves each contestant i making the "null offer" $g_i = 0$. In that case, half of the foot-soldiers will work for one contestant and half for the other, and as a result, each contestant will indeed preserve her initial win probability.[9] We summarize the above arguments in the following result:

PROPOSITION 4. Under payoff structure I, consider the case where $\theta \neq 0$, and denote contestant U (resp., contestant O) to be that contestant for whom $p_U < ½$ (resp., $p_O > ½$). In the 'null equilibrium' $\{g_U^* = g_O^* = 0\}$, we will have $\{S_U^* = S_O^* = ½\}$ and $\{P_U^* = p_U < ½ < p_O = P_O^*\}$. In contrast, in any other pure-strategy equilibrium – where $\{g_U^* = g_O^* > 0\}$ necessarily – we will have $\{S_U^* < S_O^*\}$ and $\{P_U^* < p_U < ½ < p_O < P_O^*\}$.[10]

The above result states that in any (pure-strategy) equilibrium in which the contestants make positive conditional compensation offers, the *ex ante* overdog manages to "extend his lead". The only scenario in which she fails to do so is when the contestants are situated in the "null equilibrium" with $\{g_L^* = g_R^* = 0\}$. Given that, a natural question to ask is: For what parameter configurations must any (pure-strategy) equilibrium involve the contestants making positive compensation offers? Intuitively, it is clear that larger the overall prize from winning the contest, and the greater the effectiveness of foot-soldiers in raising win probabilities, the greater will be each contestant's incentive to make a positive compensation offer. Our next result confirms this intuition in the context of a specific form of the "size of the prize" function $V(.)$. The result also recognizes that an *ex ante* underdog will necessarily prefer the "null outcome" $\{g_L = g_R = 0\}$ to any "non-null equilibrium" with $\{g_U^* = g_O^* > 0\}$ because in the latter equilibrium she commits to spending resources for its foot-soldiers but ends up with a lower chance of winning.

PROPOSITION 5. Consider the case where $V(g_i) = v - (g_i)^n$, for $n \geq 1$ and $v > (g^+)^n$. In this case, $\{g_L^* = g_R^* = 0\}$ cannot be a Nash equilibrium under either of the following parameter

[9] In our formal model, this happens because all agents necessarily become foot-soldiers of some contestant. Note however, that even if we had posited that a positive compensation offer is required for hiring a foot-soldier, the following result would still be valid: "The contestants' posterior win probabilities will equal their prior win probabilities when $g_L = g_R = 0$."

[10] In our contest model, there can exist multiple pure-strategy Nash equilibria for different parameter configurations. For example when $V(g_i) = (v - g_i)$ with $2\beta - \lambda g^+ < \lambda v < 2\beta$, then $\{g_U^* = g_O^* = 0\}$ and $\{g_U^* = g_O^* = g^+\}$ are both equilibrium compensation vectors.

configurations: (i) $n > 1$; (ii) $n = 1$ and $[\lambda.v] > [2\beta]$. In either of these parameter configurations, at least one of the two contestants – the *ex ante* underdog – will strictly prefer an exogenously imposed ban on the hiring of foot-soldiers.

3.2 Equilibrium Outcomes for Payoff Structure II

Under payoff structure II, for a realized $\theta \in [-\frac{1}{2}, \frac{1}{2}]$, contestants L and R simultaneously announce $g_L \in [0, g^+]$and $g_R \in [0, g^+]$ to maximize their respective expected payoffs:

$$\boldsymbol{W}^L = \Pi^*(g_L, g_R; \theta)\times V(g_L.S_L), \text{ and } \boldsymbol{W}^R = [1-\Pi^*(g_L, g_R; \theta)]\times V(g_R.S_R).$$

It can be verified that when $V'(.) < 0$ and $V''(.) \leq 0$, both the maximands are quasi-concave in their respective maximizers, giving us the following result:

PROPOSITION 6. In the contest game under payoff structure II, there exists at least one pure-strategy Nash equilibrium conditional compensation offer vector $\{g_L^*, g_R^*\}$.

We now establish that when the total wage bill to the foot-soldiers determine the winning contestant's size of the prize, it is the *ex ante* underdog who will announce a higher conditional compensation in any (pure-strategy) equilibrium.

PROPOSITION 7. Under payoff structure II, consider the case where $\theta \neq 0$, and denote contestant U (resp., contestant O) to be that contestant for whom $p_U < \frac{1}{2}$ (resp., $p_O > \frac{1}{2}$). In any pure-strategy Nash equilibrium we will have $g_U^* \geq g_O^*$, with the inequality being necessarily strict whenever $g_U^* \in (0, g^+)$ or $g_O \in (0, g^+)$ or both.

The difference in the implications of Propositions 3 and 7 is to be understood as follows. Starting from a symmetric situation where both contestants offer the same compensation, the "incremental cost" of raising the compensation amount for the *ex ante* underdog is smaller relative to that of the overdog under payoff structure II than under payoff structure I. This is because when both contestants offer similar compensation amounts, a larger number of foot-soldiers join the overdog, thus raising her promised total wage bill; and it is the wage bill $[g_i.S_i]$ (rather than only the offer $[g_i]$) that determines the "cost of keeping one's promise" under payoff structure II. Thus, relative to the overdog, the underdog has a greater incentive to raise her offer under payoff structure II than under payoff structure I.

Proposition 7 implies that there can be the following distinct types of pure-strategy Nash equilibrium offers under payoff structure II: the "null equilibrium offers" $\{g_L^* = g_R^* = 0\}$; the "max equilibrium offers" $\{g_L^* = g_R^* = g^+\}$; and the "asymmetric equilibrium offers" where the *ex* *ante* underdog offers a strictly greater compensation as compared to the *ex ante* overdog.[11] Our final result clarifies the nature of the contest outcomes in these different kinds of equilibria.

PROPOSITION 8. Under payoff structure II, consider the case $\theta \neq 0$, and denote contestant U (resp., contestant O) to be that contestant for whom $p_U < ½$ (resp., $p_O > ½$). In the null equilibrium $\{g_U{*}= g_O{*} = 0\}$, we will have $\{S_U{*}= S_O{*} = ½\}$ and $\{ P_U{*}= p_U < ½ < p_O = P_O{*}\}$. In the 'max equilibrium' $\{g_U{*}= g_O{*} = g^+\}$, we will have $\{S_U{*} < S_O{*}\}$ and $\{P_U{*}< p_U < ½ < p_O < P_O{*}\}$. In *every* pure-strategy equilibrium (with $\{g_U{*}\geq g_O{*}\geq 0\}$ necessarily) we will have $P_U{*}< ½ < P_O{*}$.

The most noteworthy assertion in the above result is the following: In any (pure-strategy) Nash equilibrium of our contest model where the winning prize is reduced by the total wage bill promised to the contestants' foot-soldiers, the *ex ante* underdog remains an underdog *ex post* (i.e., $P_U{*}< ½ < P_O{*}$), even in equilibria where she offers a strictly higher conditional compensation than the *ex ante* overdog.

Given that unambiguous result, can we be assured that by offering a higher compensation, the *ex ante* underdog manages to "narrow the probability gap" between her and the *ex ante* overdog? Not necessarily. In some specific parameter configuration, we can have an equilibrium offer vector of the following kind: $\{g_U{*}= g^+, g_O{*}= g^+ – \varepsilon\}$, where ε is an arbitrarily small positive number. In such a situation, if it is also true that in terms of initial probabilities, p_U is sufficiently smaller than p_O, then the overdog might indeed attract more foot-soldiers than the underdog. If that is the case, then we will have the 'probability gap' widening: $g_U{*}> g_O{*}$ and $P_U{*}< p_U < ½ < p_O < P_O{*}$. When situated in such an equilibrium, the *ex ante* underdog will strictly prefer an imposition of a ban on the recruitment of foot-soldiers.

4. Concluding Remarks

This paper has initiated a study of "contests with foot-soldiers". We have considered a contest game where two contestants – an *underdog* and an *overdog* – simultaneously offer 'conditional compensation amounts' to attract foot-soldiers. The foot-soldiers' decisions depend upon these offers, their relative closeness to the contestants, and their assessment about the contestants' chances of winning. Our current analysis has focused on two payoff structures: one in which the

[11] Two comments about such equilibria under payoff structure II are in order. Firstly, there can exist multiple pure-strategy Nash equilibria for different parameter configurations. Secondly, the null outcome $\{g_L= g_R = 0\}$ will not constitute a Nash equilibrium for sufficiently high values of the overall prize and of λ. Specifically, if $V(g_i.S_i) = v – \varphi(g_i.S_i)$, where $\varphi(0) = 0$, $\varphi'(.) > 0$ for positive arguments and $\varphi''(.) \geq 0$, then the null equilibrium $\{g_L{*}= g_R{*} = 0\}$ will exist if and only if $[\lambda.v] \leq \beta.\varphi'(0)$.

winner's net prize depends (negatively) only on her offer amount, and the other in which it depends (negatively) on the total wage bill to be paid to her foot-soldiers. Under the former payoff structure, the offers of the two candidates are identical in every pure-strategy Nash equilibrium; and so, whenever this common offer is positive, the overdog enhances her "probability lead" against the underdog. Under the latter payoff structure, the underdog offers a higher compensation that the overdog, but she remains an underdog in the contest.

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