m-Proper Equilibria[1]

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We consider a class of equilibrium refinements for finite games in strategic form. The refinements in this family are indexed from least to most restrictive. Proper equilibrium is obtained as a special case within the class; all other concepts are stronger than trembling-hand perfection and weaker than proper equilibrium, so they provide a collection of intermediate refinements. We argue theoretically and illustrate by examples that in some applications, the intermediate refinement concepts are preferable to either trembling-hand or proper equilibrium.

Note: In collaboration with Erkut Ozbay (U. Maryland), we are currently (April 2016) about to run experiments testing the predictions of our concept, Milgrom and Mollner's Test Set equilibrium and Cognitive Hierarchies, against the classic concepts in variations of the games below. We expect to have these results ready by June 2016.

Consider a symmetric two player game in normal form with payoffs given by the following matrix:

		Player 2			
		s^1	s^2	s^3	s^4
Player 1	s^1	$8,8$	$5,8$	$8,0$	$0,1$
	s^2	$8,5$	$5,5$	$0,8$	$1,0$
	s^3	$0,8$	$8,0$	$5,5$	$8,0$
	s^4	$1,0$	$0,1$	$0,8$	$8,8$

This game has two pure Nash equilibria: (s^1, s^1) and (s^4, s^4). Both of these are also trembling-hand perfect equilibria (Selten 1975). However, once we consider trembles, the two equilibria do not appear equally plausible. In equilibrium (s^1, s^1), the best tremble, which causes no utility loss to the deviating player, is to tremble to s^2, but if players occasionally tremble to s^2, the equilibrium strategy s^1 is still a best response against a mix of the equilibrium strategy s^1 and the best tremble s^2. By contrast, (s^4, s^4) is more fragile:

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the best tremble (the least costly in terms of payoff) is to tremble to s^3, and as soon as a player trembles to s^3 with any probability, s^4 is not a best response given this tremble, the other player prefers to actually deviate to s^3 all the time, and the equilibrium (s^4, s^4) breaks down.

Proper equilibrium (Myerson 1978) was conceived to account for precisely this intuition that trembling agents are more likely to tremble toward less costly deviations from equilibrium play, than toward costlier deviations. However, the proper equilibrium refinement turns out to be too restrictive to help us in our example: neither (s^1, s^1) and (s^4, s^4) satisfy its requirements, and there is no pure proper equilibrium in this game. In fact, even van Damme's (1991) concept of a weakly proper equilibrium is too strong: in this example, there is no pure weakly proper equilibrium.

We would like to identify a refinement of trembling-hand perfection that satisfies the following:

a) it selects all proper equilibria, so it inherits the good existence properties of the proper refinement.

b) it is easier, or at least not harder to compute, than existing concepts.

c) it recognizes that equilibria that do well against the most plausible trembles (such as (s^1, s^1) in our motivating example) are qualitatively different from equilibria that perform poorly against the most plausible trembles (such as (s^4, s^4) in our example) and should be selected.

We describe a family of concepts satisfying these desiderata.

Definitions

Consider a finite game, in which the set of players is N of size n, the set of feasible pure strategies for each player $i \in N$ is S_i and $\max\{|S_1|, ..., |S_n|\} = M$, so that each player has at most M pure strategies to choose from.

Let $S = \prod_{i=1}^{n} S_i$ be the set of all feasible pure strategy profiles. Let $\Delta(S)$ be the set of all possible mixed strategy profiles. Let $\sigma \in \Delta(S)$ be a mixed strategy profile. For each $i \in N$, let $\sigma_i \in \Delta(S_i)$ be a mixed strategy for player i. Let σ also be represented as $\sigma = (\sigma_i, \sigma_{-i})$ so that $\sigma_{-i} \in S_{-i} = \prod_{j \in N \setminus \{i\}} S_j$ is a list with a mixed strategy for every other player except i.

For each $i \in N$, for any $s_i \in S_i$ and for any $\sigma_{-i} \in \Delta(S_{-i})$, let $u_i(s_i, \sigma_{-i})$ be the expected utility for player i given that i plays s_i and the other players play σ_{-i}.

For each $i \in N$, and for any $\sigma_{-i} \in \Delta(S_{-i})$, let $BR_i^1(\sigma_{-i}) \subseteq S_i$ be the set of first best responses by player i to σ_{-i}; that is, $BR_i^1(\sigma_{-i}) \equiv \{s_i \in S_i : u_i(s_i, \sigma_{-i}) \geq u_i(s_i', \sigma_{-i}) \; \forall s_i' \in S_i\}$.

We define recursively second best responses as strategies that are best responses among those that are not a first best response; third best responses as strategies that are a best response when restricted to using those that are neither first nor second best responses; and more generally m-th best responses as the best responses among the remaining strategies that weren't best responses at any previous first to $(m-1)$-th level.

Definition 1 *For any integer $m \in \{2, ..., M\}$, for each $i \in N$, and for any $\sigma_{-i} \in \Delta(S_{-i})$, let $BR_i^m(\sigma_{-i}) \subset S_i$ be the set of m-th best responses to σ_{-i}, defined by*

$$BR_i^m(\sigma_{-i}) \equiv \{s_i \in S_i : u_i(s_i, \sigma_{-i}) \geq u_i(s_i', \sigma_{-i}) \forall s_i' \in S_i \setminus \bigcup_{h=1}^{m-1} BR_i^h(\sigma_{-i})\}.$$

Informally, the set of $m-th$ best responses is composed of the strategies that are a best response among those that were not an $h-th$ best response for any $h < m$.

Trembling-hand perfection (Selten 1975) and properness (Myerson 1978) both require that an equilibrium strategy profile $\sigma \in \Delta(S)$ be approximated by a sequence of totally mixed strategy profiles $\{\sigma^k\}_{k=1}^{\infty}$ with $\lim_{k \longrightarrow \infty} \sigma^k = \sigma$, and they both impose some restrictions on this sequence. Trembling-hand requires that along the sequence, any strategy that is not a best response to σ_{-i}^k be played with a strictly positive probability less than ε^k by player i, where $\{\varepsilon^k\}_{k=1}^{\infty}$ converges to zero. Properness requires that for any $s_i, s_i' \in S_i$, if $u_i(s_i, \sigma_{-i}^k) < u_i(s_i', \sigma_{-i}^k)$, then $0 < \sigma_i^k(s_i) \leq \varepsilon^k \sigma_i^k(s_i')$. In our notation, properness requires that for any $s_i, s_i' \in S_i$ such that $s_i \in BR_i^m(\sigma_{-i}^k)$ and $s_i' \in BR_i^{m'}(\sigma_{-i}^k)$, if $m' < m$, then $0 < \sigma_i^k(s_i) \leq \varepsilon^k \sigma_i^k(s_i')$. An indexed family of intermediate concepts suggests itself.

Definition 2 *For any integer $m \in \{1, ..., M-1\}$, a mixed strategy profile $\sigma \in \Delta(S)$ is an $\varepsilon-$(**m-proper**) equilibrium if: i) σ is totally mixed, and ii) for any integers $l, k, h \in \{1, ..., M\}$ such that $l < k \leq m < h$ and for any player i and any $s_i, s_i', s_i'', s_i''' \in S_i$ s.t. $s_i \in BR_i^l(\sigma_{-i})$,*

$s_i' \in BR_i^k(\sigma_{-i})$, $s_i'' \in BR_i^m(\sigma_{-i})$ and $s_i''' \in BR_i^h(\sigma_{-i})$, it follows that $\sigma_i(s_i') \leq \varepsilon\sigma_i(s_i)$ and $\sigma_i(s_i''') \leq \varepsilon\sigma_i(s_i'')$.

As in Myerson's (1978) proper equilibrium, the weights assigned by the totally mixed strategies should be ε times smaller for any strategy that yields a lower payoff than another, but this nuanced relative order only applies to those strategies that are among the first to $m-th$ best responses; all other strategies can receive any weights as long as these weights are positive but less than ε times the weight of any $m-th$ best response. Since any strategy except the worst one must be a m-th best response for some $m \in \{1, ..., M-1\}$, the extreme case $m = M-1$ takes us back to Myerson's (1978) definition of an $\varepsilon-$**proper** equilibrium.

Definition 3 *For any integer $m \in \{1, ..., M-1\}$, a mixed strategy profile $\sigma \in \Delta(S)$ is an **m-proper** equilibrium if it is a limit of a sequence of $\varepsilon-$(**m-proper**) equilibria with ε converging to zero along the sequence.*

An **m-proper** equilibrium requires that players assign arbitrarily greater weight to strategies that are better over those that are not as good, among the top m tiers of strategies. In this it coincides with proper. It coincides with trembling hand perfection in letting the relative weights of all other strategies, which are not among the m-th best responses, be arbitrarily assigned as long as they are small. It follows that the concept of an **(M–1)-proper** equilibrium coincides with the concept of proper equilibrium (Myerson 1978), whereas, at the other end, the concept of trembling hand perfection is slightly weaker than **1-proper** since trembling hand perfection does not require other first best responses (aside from the one sustaining the equilibrium) to be assigned greater weight than non-best responses, whereas **1-proper** adds this restriction.

Observe that each level m introduces a new restriction in the admissible sequences, and thus tightens the solution concept. For each integer $m \in \{1, ..., M\}$, let $E^m \subseteq \Delta(S)$ be the set of strategy profiles that are **m-proper** equilibrium; let E^0 denote the trembling-hand equilibria and note that E^{M-1} is the set of proper equilibria.

Remark 1 $E^{M-1} \subseteq E^{M-2} \subseteq ... \subseteq E^1 \subseteq E^0$.

Since the set of **m-proper** equilibria is contained by the set of trembling-hand perfect equilibria, and it contains the set of proper equilibria, it follows from the existence of proper equilibria in finite games that an **m-proper** equilibrium exists as well for any $m \in \{1, ..., M-1\}$.

It also follows that we can compare **m-proper** equilibria to extensive form solution concepts: Proposition 1 in Reny (1992) together with Remark 1 imply that given any game in extensive form, and given any **m-proper** equilibrium $\sigma \in \Delta(S)$ of the normal form associated with such extensive form game, there exists a system of beliefs such that σ and this system of beliefs satisfy weak sequential rationality; while Proposition 1 in Mailath, Samuelson and Swinkels (1997) together with Remark 1 imply that any strategy profile that is quasiperfect (van Damme 1984) in the extensive form is **m-proper** in the associated normal form game.

Discussion and Examples

The appeal of this family of concepts is grounded on at least three reasons, one axiomatic, and two practical for applied work.

1. **Minimize Restrictions.** Given two solution concepts that make the same sharp prediction for a given application, the weaker concept is preferred. Govindan and Wilson (2008) express this scientific norm thus: "*The prevailing practice in the social sciences is to invoke the weakest refinement that suffices for the game being studied. This reflects a conservative attitude about using unnecessarily restrictive refinements. If, say, there is a unique sequential equilibrium that uses only admissible strategies, then one refrains from imposing stronger criteria.*" According to this norm, in any application in which there are multiple trembling-hand perfect equilibria so we need a refinement to make a sharper solution, and in which there is a unique **m-proper** equilibrium for some $m < M-1$, the **m-proper** equilibrium is a more appropriate concept to make a unique prediction than proper equilibrium.

2. **Existence and uniqueness in pure strategies.** In many applications, researchers restrict their equilibrium analysis to pure strategies. In cases in which there exist multiple trembling-hand perfect pure equilibria, no pure proper equilibrium, and a unique **m-proper**

equilibrium for some $m < M - 1$, using the **m-proper** equilibrium concept allows the researcher to make a unique prediction in pure strategies, which was impossible if we only consider trembling-hand perfection (too weak) or proper equilibrium (too restrictive).

3. **Ease of computation.** The proper equilibrium refinement has been used only seldom in practice, because constructing the sequence of trembles and of $\varepsilon - proper$ equilibria that converges to the proper equilibrium (or proving that no such sequence exists) is cumbersome. There are many constraints on the trembles, and checking whether they are all satisfied is difficult. By loosening restrictions, **m-proper** equilibria with $m < M - 1$ become easier to check. The difficulty of computation increases in m, so incentives align with those dictated by the normative prescription of minimizing restrictions: more restrictive and harder to compute solution concepts (**m-proper** equilibria with a higher m) should be used only if less restrictive and easier to compute concepts do not yield a sufficiently sharp prediction. Proper equilibria (the highest of the **m-proper**) should then be used only as a last resort, within this family of restrictions.

We illustrate the advantages of this family of refinements through examples. First we return to our initial motivating game.

Example 1 *Assume* $N = \{1, 2\}$, $S_1 = S_2 = \{s^1, s^2, s^3, s^4\}$ *and payoffs are given by the following matrix, where player* 1 *is the row player and player* 2 *is the column player.*

		Player 2			
		s^1	s^2	s^3	s^4
Player 1	s^1	$8, 8$	$5, 8$	$8, 0$	$0, 1$
	s^2	$8, 5$	$5, 5$	$0, 8$	$1, 0$
	s^3	$0, 8$	$8, 0$	$5, 5$	$8, 0$
	s^4	$1, 0$	$0, 1$	$0, 8$	$8, 8$

The set of pure Nash equilibria is $\{(s^1, s^1), (s^4, s^4)\}$. *The set of pure trembling-hand perfect Nash equilibria coincides with the set of Nash equilibria. There is no pure proper equilibria. Strategy profile* (s^1, s^1) *is not a proper equilibrium because* $u_i(s^4, \sigma_{-i}) > u_i(s^3, \sigma_{-i})$ *for any* σ_{-i} *sufficiently close to* $(1, 0, 0, 0)$*; thus, being a proper equilibrium would require that for some sequence of totally mixed strategies* $\{\sigma_{-i}^n\}_{n=1}^{\infty}$ *that converges to the pure strategy* s^1, *if n is sufficiently large, $\sigma_i^n(s^3) \leq \varepsilon \sigma_i^n(s^4)$, in which case $u_i(s^2, \sigma_{-i}^n) > u_i(s^1, \sigma_{-i}^n)$ and then it must be $\sigma_i^n(s^1) \leq \varepsilon \sigma_i^n(s^2)$, which is a contradiction. A similar argument shows that (s^4, s^4) also fails to satisfy the proper equilibrium refinement.*

Since $u_i(s^4, s^1) > u_i(s^3, s^1)$, in order for (s^1, s^1) to be a weakly proper equilibrium (van Damme 1991), there must be some sequence of totally mixed strategies $\{\sigma_{-i}^n\}_{n=1}^{\infty}$ that converges to the pure strategy s^1 such that $\sigma_i^n(s^3) \leq \varepsilon \sigma_i^n(s^4)$, in which case $u_i(s^2, \sigma_{-i}^n) > u_i(s^1, \sigma_{-i}^n)$ so s^1 is not a best response to σ_{-i}^n and thus (s^1, s^1) is not a weakly proper equilibrium. A similar argument shows that (s^4, s^4) also fails to satisfy the weakly proper equilibrium refinement. We are thus unable to make a unique prediction in pure strategies using trembling-hand perfection, weakly proper equilibrium or proper equilibrium.

We show that (s^1, s^1) is the unique ***2-proper*** *equilibrium in pure strategies. Let $\sigma_i^n = (1 - \frac{\varepsilon^n}{2} - \varepsilon^{2n}, \frac{\varepsilon^n}{2}, \frac{\varepsilon^{2n}}{2}, \frac{\varepsilon^{2n}}{2})$ for each $i \in \{1, 2\}$ and some small $\varepsilon > 0$. Then $\{\sigma_i^n\}_{n=1}^{\infty}$ converges to s^1 as $n \longrightarrow \infty$, $BR_i^1(\sigma_{-i}^n) = s^1$, and $BR_i^2(\sigma_{-i}^n) = s^2$ for any $n \in \mathbb{N}$ and thus the weights of σ_i^n respect this partial order (s^1 is played at least $\frac{1}{\varepsilon}$ more often than s^2, which is played $\frac{1}{\varepsilon}$ more often than anything else). So (s^1, s^1) is a* ***2-proper*** *equilibrium. Finally, note that (s^4, s^4) is not a* ***2-proper*** *equilibrium. For any σ_{-i}^n sufficiently close to $(0, 0, 0, 1)$, $u_i(s^3, \sigma_{-i}^n) > \max\{u_i(s^1, \sigma_{-i}^n), u_i(s^2, \sigma_{-i}^n)\}$ and $BR_i^1(\sigma_{-i}^n) \bigcup BR_i^2(\sigma_{-i}^n) = \{s^3, s^4\}$ so $\sigma_i^n(s^1) \leq \varepsilon \sigma_i^n(s^3)$, but then $BR_i^1(\sigma_{-i}^n) = s^3$ so $\sigma_i^n(s^4) \leq \varepsilon \sigma_i^n(s^3)$ a contradiction.*

Therefore, the concept of ***2-proper*** *equilibrium obtains a unique prediction in pure strategies, which was unattainable using trembling-hand perfection or proper equilibrium. In fact, we can show that none of the following existing refinements refines the set of pure strategy equilibria: according to the notions of Weakly Proper (van Damme 1991), Proper (Myerson 1998), Extended Proper (Milgrom and Mollner 2014), Risk Dominance (Harsanyi and Selten 1988), Cardinal Generalized Risk Dominance (Peski 2010), Generalized Half-Dominance (Ijima 2015), p-Dominance for any $p \in (0, 1)$ (Morris, Rob and Shin 1995) or Evolutionary stability (Maynard Smith 1982), the set of pure strategy equilibria that satisfy any one of these refinements is empty; whereas, both pure Nash equilibria are Trembling Hand-Perfect (Selten 1975) and Ordinal Generalized Risk Dominant (Peski 2010).[4] On the other hand,*

[4] A proof of these claims is available from the author.

(s^1, s^1) *is the equilibrium selected by Test-Set equilibria (Milgrom and Mollner 2014) and is the unique strategy profile chosen by all types with* $k \geq 2$ *in the level-k cognitive hierarchy model of Camerer, Ho and Chong (2004).*

According to the cognitive hierarchy model, each player's level of thinking is a probabilistic type, distributed according to a discrete Poisson distribution. Level 0 types randomize uniformly over all strategies, and for any $k \in \mathbb{N}$, level k types best respond to the play of all level 0 to level $k-1$ types of other players. For any parameter λ of the Poisson distribution of types, the cognitive hierarchies solution then yields a distribution of play. We can use this model as a more comparable refinement by taking the limit as $\lambda \longrightarrow \infty$, so that for any k, the probability that a player's level of thinking is more than k approaches 1.

It is easy to find games in which **m-proper** leads to a different prediction from level-k cognitive hierarchies. For instance, consider games in which players seek to coordinate to be close to each other, but not necessarily at exactly the same location, just close. Applications that fit this description include economies of aglomeration in urban development; opinion formation in a group that punishes sharp dissent; and fashion and product development with preferences for comformity.

Example 2 (Approximate Coordination Game with 4 players) *Consider a game with set of players* $N = \{1, 2, 3, 4\}$*. Each player* i *simultaneously chooses a location* $s_i \in S = \{1, 2, ..., l\}$*, where* l *is an integer greater than 3. Players obtain a payoff equal to the number of agents located within one unit of distance, i.e. adjacent to them or at the same location.*

The set of pure Nash equilibria of this game is $NE = \{s \in S^4 : \max_{i \in N} s_i - \min_{i \in N} s_i \leq 1\}$*, that is, in any pure Nash equilibrium all four players coordinate to all be located in* $\{x, x+1\}$ *for some* $x \in \{1, 2, ..., l-1\}$*. All players attain their highest payoff (3) in any pure Nash equilibrium. The set of pure trembling hand perfect equilibria is* $THP = \{s \in S^4 : s_i \in \{x, x+1\}$ *for some* $x \in \{2, 3, ..., l-2\}\}$*, that is, the NE in which no one is located at a corner location (corners are weakly dominated in this game). The set of pure strategy Nash equilibria that can be approximated as the limit of a Cognitive Hierarchies distribution of play as* $\lambda \longrightarrow \infty$ *(denote it* CH*) includes all trembling hand perfect pure equilibria (*$THP \subset CH$*)*

but it also includes in its predictions strategy profiles with coordination failures in which each player locates at an isolated location, $\{s \in S^4 : |s_i - s_j| > 1 \forall i,j \in N\} \subset CH$, *or in which two pairs of agents coordinate separately,* $\{s \in S^4 : \exists g,h,i,j \in N$ *and* $x \in \{2,3,...,l-3\}$ *s.t.* $s_g = x$, $s_h \in \{s_g, s_g+1\}$, $s_i \geq s_h + 2$, $s_j \in \{s_i, s_i+1\}$, $s_j \leq l-1\} \subset CH$, *even though these profiles are not Nash equilibria.*

Are equilibria in which three players coordinate perfectly while the fourth locates adjacent to them, such as $(2,3,3,3)$ *as plausible as equilibria in which all four players coordinate at the same location? according to THP, yes; according to* ***3-proper****, no. A sequence of trembles in which all players tremble randomly to any strategy supports equilibrium* $(2,3,3,3)$ *as THP: because all players are as likely to tremble to location 4 as to location 1, locations 2 and 3 are equally good. But not all trembles are equally costly. Consider the payoff vector for Player* i *as a function of* s_i *given* s_{-i}*. The first table is for Player 1, and the second one for any player* $i \in \{2,3,4\}$.

s_1	1	2	3	4	≥ 5	s_i	1	2	3	4	≥ 5
$u_1(s_1;3,3,3)$	0	3	3	3	0	$u_i(s_i;2,3,3)$	1	3	3	2	0

The union of the 1st, 2nd and 3rd Best responses for every player is the set of locations $\{2,3,4\}$*. Therefore, in a* ***3-proper*** *equilibrium, all players must tremble more frequently to location 4 than to location 1. And if players tremble more frequently to* 4 *than to* 1*, then* 2 *is no longer a best response: for every player, locating at 3 yields a strictly higher payoff than locating at 2.*

The set of pure ***3-proper*** *equilibria is the union of two subsets: either all four players coordinate to locate at the same (non-corner) spot, or players coordinate to locate two and two at each of two adjacent locations, neither of them a corner. Formally, the set of pure 3-proper equilibria is* $\{s \in S^4 : s_i \in \{x, x+1\}$ *for some* $x \in \{2,3,...,l-2\}$ *and* $|\{i \in N : s_i = x\}| \in \{0,2,4\}\}$. ***3-proper*** *eliminates trembling-hand equilibria selected as well by the cognitive hierarchy prediction, in which all players except one coordinate on a location while the fourth player chooses an adjacent location.*

In summary, we consider a family of equilibrium refinements that take increasingly more restrictive steps from trembling-hand perfection to proper equilibrium, and we show how in some examples, these intermediate concepts provide solutions that are more satisfying and easier to compute that if we only considered the two extreme concepts of trembling-hand perfection and proper equilibrium.

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