Signalling, Productivity and Investment

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April 15, 2016

Abstract

This article studies how investment varies with productivity in a simple credit market with asymmetric information and signalling. When the incentive constraint is slack, investment is continuously increasing in productivity. Nonetheless, when the incentive constraint is binding, then the high-type over- or under-invests, compared to the first-best, to signal her type. In this range of parameters, investment is constant and features a profound discontinuity. Implications of this result are discussed regarding the amplification of small productivity shocks.

KEYWORDS: Signalling, productivity, investment

JEL CLASSIFICATION: D82, E22, E32

1 INTRODUCTION

In a world of perfect capital markets, investment varies continuously with productivity. After a small productivity shock, a company can always slightly adjust its borrowing to make the net expected marginal return of investment zero. Nonetheless, in the presence of imperfect capital markets this result might not hold as it is highlighted in important contributions such as Bernanke and Gertler (1989) and Kiyotaki and Moore (1997). Small productivity shocks may be amplified through a financial accelerator. In this paper, I examine how investment varies with productivity in a simple credit market with asymmetric information and signalling.

The model features an entrepreneur with a risky, variable-investment project, who wishes to raise capital from the financial market. The entrepreneur can be one of two types and this is her private information. A high-type has a higher probability to succeed than a low-type for a given amount of investment. Since the market is unable to observe the true type of the entrepreneur, an adverse selection problem arises similar to Akerlof

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(1970). Contrary to Akerlof (1970), the entrepreneur can signal her type using as sorting devices the amount of capital it raises as well as the schedule of repayments. Trade in the financial market is modelled as a signalling game similar to Spence (1973).

I characterise the least-costly- separating equilibrium. I show that, as expected, there is a distortion, compared to the first-best, in the equilibrium investment of the high-type. Interestingly, the nature of the distortion depends on the ratio of productivities of the high- over the low-type. If this is strictly greater than unity, there might be over-investment (compared to the first best) in equilibrium. If this is strictly less than unity, there might be under-investment in equilibrium. Over-investment occurs due to a "rat race" in which a high- type entrepreneur tries to signal her type by undertaking excessive investment. Under-investment is the opposite of a rat race; to signal her type, a high-type entrepreneur raises less capital than it is required to make the net marginal return zero. I argue that this result has important implications regarding the behaviour of investment as a function of this ratio and hence the amplification of small productivity shocks on investment.

Regarding the related literature, Stiglitz andt Weiss (1981) is the seminal paper in credit markets with asymmetric information. They show that debt financing can lead to credit rationing in equilibrium if banks have no signalling device other than the interest rate. DeMeza and Webb (1987) show that by slightly changing the fundamentals in the Stiglitz and Weiss (1981) model, the opposite result prevails; there is too much investment in equilibrium.[1] Bester (1985, 1987) highlights the role of collateral as a signalling device to alleviate the asymmetries information between borrowers and lenders. The present paper differs from those mentioned above because it allows for variable-investment project and hence the nature of equilibrium is different. More specifically, all types raise capital which might be more or less than it is socially efficient.

Closely related papers, at least in terms of modelling, are these by Milde and Riley (1988) and Martin (2009). Milde and Riley (1988) also consider variable-investment projects and signalling in the credit market. Their main result is that high-types may select larger rather than smaller loans. Compared to Milde and Riley (1988), I provide further comparative statics since I fully characterise how investment varies as a function of productivity. Martin (2009) shows how entrepreneurial wealth affects economy's aggregate investment. In his model there may be a non-monotonic relationship between entrepreneurial wealth and investment. Intuitively, this happens because entrepreneurial wealth can be used as collateral by some types and therefore switch the equilibrium from pooling to separating causing a discontinuous change in investment. There are several important differences between this paper and Martin (2009). First, in this paper, I consider a different model of trade of loan contracts than Martin (2009). I assume that it is the informed enterepreneur who offers a contract as opposed to the uninformed finan-

[1] In particular, Stiglitz and Weiss (1981) show that the market equilibrium will entail under-investment compared to the first best if projects are ranked in a SOSD sense. On the contrary, DeMeza and Webb (1987) show how over-investment may arise when projects are ranked in a FOSD sense.

cial intermediaries in Martin (2009). In other words, Martin considers a screening model, as this was proposed by Hellwig (1987), whereas I consider a signalling model.[2] This is especially important because in this paper, I consider only the least costly separating equilibrium compared to Martin (2009) who considers both separating and pooling equilibria. In terms of the results, I show how small productivity shocks affect investment, a result that is not analysed in Martin (2009).

Similarly, the amplification of small shocks on aggregate investment has been highlighted in macroeconomic models with imperfect capital markets. Bernanke and Gertler (1989) and Kiyotaki and Moore (1997) show how small productivity shocks can be amplified in the economy through a financial accelerator. In Bernanke and Gertler (1989), lenders can enforce repayments only through costly monitoring, and, in Kiyotaki and Moore (1997), only through collateralised assets. Both papers draw similar conclusions: small productivity shocks are exacerbated and investment is positively correlated with net worth. In this paper, the investment policy of a company with a high-quality project is discontinuous to marginal productivity. This implies that a small, positive or negative productivity shock can lead to a large discontinuous fall in investment as the regime switches from under- to over-investment. However, the mechanism that drives the result is different. In Bernanke and Gertler (1989) and Kiyotaki and Moore (1997) asymmetric information is only ex post. On the contrary, in this paper there is ex ante asymmetric information and signalling.

The remainder of the paper is organised as follows. In Section 2, I present the model. In Section 3, I characterise the first-best investment policy; i.e. the investment policy under symmetric information. In Section 4, I characterise the least-costly separating equilibrium under asymmetric information. In Section 5, I provide comparative statics which are the main results of the paper. In Section 6, I discuss the model and the assumptions that drive the results.

2 THE MODEL

There are two dates: today and tomorrow. There is an entrepreneur who can be either low- or high-type, $i = L, H$. Today, the entrepreneur has a project. The prior probability that the entrepreneur is of type i is λ_i. Only the entrepreneur knows her type. Let $\tilde{z}_i = (z_i, 0)$ with distribution $(\pi_i, 1 - \pi_i)$. I assume that $\pi_L < \pi_H$. By investing x units, the project returns $\tilde{z}_i f(x)$, where $f(x)$ is a standard neoclassical decreasing-returns-to-scale production function, i.e. $f'(x) > 0$, $f''(x) < 0$, $\lim_{x \to 0} f'(x) = \infty$, $\lim_{x \to \infty} f'(x) = 0$. These regularity assumptions guarantee the existence of an interior maximum and make the problem well-behaved.

[2] Even though a screening model describes well the functioning of insurance markets, I believe that the signalling model is closer to how financial markets operate. In financial markets, one usually observes firms applying for financing to investors, being financial intermediaries or the market, rather than vice versa.

The entrepreneur is cashless but can borrow from a representative investor ("the financial market").[3] The investor is risk-neutral and has unlimited wealth. I abstract from questions related to capital structure and I assume that the entrepreneur always issues risky debt and is protected by limited liability.[4] A debt contract is denoted as $c = (x, R) \in \mathbb{R}^2_+$, where x is the amount of capital, and R is the gross interest rate. The expected payoff of an entrepreneur with project i from contract c is $U_i(c) = \pi_i(z_i f(\phi W + x) - Rx)$. The entrepreneur applies for a debt contract to the representative investor who either accepts or rejects.[5] If the investor accepts, the entrepreneur acquires the necessary funds. If he rejects, then the entrepreneur does not acquire any funds and the investment opportunity lapses.

3 First-Best

As a benchmark, let us characterise the equilibrium contracts when the type of the entrepreneur is perfectly observable. I continue to assume that the game played is as described above but now the investor after observing a contract knows the type of the entrepreneur who offered this contract. In the following proposition, I characterise the subgame perfect Nash equilibrium (SPNE) pair of contracts:

Proposition 3.1. *Under symmetric information, the SPNE pair of contracts is (c_H^*, c_L^*), where $c_i^* = (x_i^*, R_i^*)$ is such that: (i) $f'(x_i^*) = 1/\pi_i z_i$, and, (ii) $\pi_i R_i^* = 1$*

Proof. Consider (c_H, c_L) such that $U_i(c_i) \neq U_i(c_i^*)$ for some i. Because c_i^* solves the maximisation program:

$$\text{PROGRAM } T_i: \quad \max_{x,R} \quad \pi_i(z_i f(x) - r) \qquad \textit{subject to}$$

$$\pi_i R - 1 \geq 0$$

for every c such that $U_i(c) > U_i(c_i^*)$, it is true that $\pi_i R - x < 0$. Therefore, no equilibrium can be sustained at which $U_i(c) > U_i(c_i^*)$. Consider then some (c_i) such that $U_i(c_i) < U_i(c_i^*)$. Let type-i play $(\tilde{c}_i)$; the solution to program:

$$\text{PROGRAM } T_i(\delta): \quad \max_{x,R,\phi} \quad \pi_i(z_i f(x) - Rx) \qquad \textit{subject to}$$

$$\pi_i R - 1 \geq \delta$$

[3] Because the market is modelled as a signalling game (see below), there is no loss of generality to assume that there is a representative investor. This also greatly simplifies notation.

If there is no confusion, I use a feminine pronounce for the entrepreneur and a masculine pronounce for the investor.

[4] Because of the simplicity of the environment, i.e. success or failure, there is no loss of generality in considering only debt contracts. For more details see Chapter 3 of Tirole (2006).

[5] See Dosis (2016) for a discussion about this modelling choice.

for some $\delta > 0$ arbitrarily small. Because the decision of the investor is sequentially rational, he has to accept contract $\tilde{c}_i$. Because of the completeness axiom of the real number system, there exists δ such that $U_i(\tilde{c}_i) > U_i(c_i)$. By playing $\tilde{c}_i$ type-i can attain a higher payoff and hence c_i cannot be supported as an equilibrium. □

Note that f' is a strictly decreasing, continuously differentiable function and hence invertible with the invert continuously differentiable and invertible. For every z_i, there exists a unique $x_i^*(z_i)$. The only implication of Proposition (3.1) is that investment by type-i is continuously strictly increasing in z_i, or $dx_i^*(z_i)/dz_i > 0$. If $\pi_H z_H > \pi_L z_L$, type-H invests more than type-L and vice versa.

4 Asymmetric Information and Signalling

A. Definitions of PBE and the Intuitive Criterion

Let c_i denote a pure strategy for type-i. For every c, the investor updates his beliefs about the type of the entrepreneur. Let $q_i : \mathbb{R}_+^2 \to [0,1]$ for $i = L, H$ denote the belief that the entrepreneur is of type i, if the investor observes contract c, where $\sum_{i=L,H} q_i(c) = 1$. A strategy, i.e. decision, for the investor is denoted as d where $d : \mathbb{R}_+^2 \to [0,1]$. $d(c)$ simply represents the probability that the investor accepts contract c and hence $1 - d(c)$ the probability he rejects it. I am interested in the perfect Bayesian equilibria (PBE) of the game and especially those that pass the intuitive criterion of Cho and Kreps (1987).

Definition 4.1. *A PBE consists of a profile of strategies and a system of beliefs* $\left((\bar{c}_i)_i, \bar{d}, (\bar{q}_i)_i\right)$ *such that:*

$$\bar{c}_i \in \arg\max_{c} \quad \bar{d}(c)U_i(c) \;\; \forall \;\; i = L, H \tag{4.1}$$

$$\bar{d}(c) \in \arg\max_{\gamma \in [0,1]} \; \gamma \sum_{i=L,H} \bar{q}_i(c)(\pi_i R x - x) \;\; \forall \;\; c \in \mathbb{R}_+^2 \tag{4.2}$$

$$\bar{q}_i(c) = \begin{cases} 1, & \textit{if} \;\; c = \bar{c}_i, \;\; c \neq \bar{c}_{-i} \\ p_i, & \textit{if} \;\; c = \bar{c}_i = \bar{c}_{-i} \\ u \in [0,1], & \textit{if} \;\; c \neq \bar{c}_i, \;\; c \neq \bar{c}_{-i} \end{cases} \qquad \forall \;\; i = L, H \tag{4.3}$$

Equation (4.1) states that the strategy of every type needs to be best response in the strategy of the investor. Equation (4.2) states that the strategy of the investor needs to be sequentially rational based on his beliefs about the type of the entreperneur when he observes a contract application. Lastly, Equation (4.3) states that beliefs of the investor need to be consistent in equilibrium with the players' equilibrium strategies.

One important caveat of the PBE is that it is too lax regarding the determination of beliefs off-the-equilibrium path. For this reason, I focus on the PBE that pass the intuitive criterion of Cho and Kreps (1987). The intuitive criterion selects those equilibria that are not supported by unduly pessimistic beliefs. Loosely speaking, the receiver is able to compare a signal he might observe with the equilibrium signals and hence correctly infer the identity of the sender who sent the signal. A formal definition follows:

Definition 4.2. *If for a PBE $\left((\bar{c}_i)_i, \bar{d}, (\bar{q}_i)_i\right)$, there exists $c \in \mathbb{R}^2_+$ such that:*

- $U_i(c) > U_i(\bar{c}_i)$
- $U_{-i}(c) < U_{-i}(\bar{c}_{-i})$
- $\pi_i R \geq 1$
- $\bar{d}(c) < 1$

then $\left((\bar{c}_i)_i, \bar{d}, (\bar{q}_i)_i\right)$ fails to pass the intuitive criterion.

B. Characterisation of the PBE that Passes the Intuitive Criterion

There are two types of equilibria: separating and pooling. In a separating equilibrium the two types play different strategies. In a pooling equilibrium, the two types play the same strategy. In this section, I characterise, in a series of lemmas, the unique equilibrium that passes the intuitive criterion.

To begin with, note that in every equilibrium, there is a lower bound in the payoff of both types. In particular,

Lemma 4.3. *In every PBE, $U_L(c_L) \geq U_L(c^*_L)$ and $U_H(c_H) \geq \max\{0, U_H(c^*_L)\}$, where c^*_L is the solution of Program T_L.*

Proof. Suppose that there exists an equilibrium $\left((\bar{c}_i, \bar{\phi}_i)\right)_i, \bar{d}, (\bar{q}_i)_i)$ such that $U_L(\bar{c}_L) < U_L(c^*_L)$. By Definition (4.1), for every c either $U_L(c) < U_L(\bar{c}_L)$ or $\bar{d}(c) = 0$. Let $\tilde{c}$, where $\tilde{c} = (x^*_L, 1/\pi_L + \varepsilon)$ and x^*_L satisfies Proposition (3.1) above. Note that $x^*_L/\pi_L + \varepsilon > 0$ and hence $1/\pi_H + \varepsilon > 0$ because $\pi_H > \pi_L$. Therefore, regardless the beliefs, from (4.2), $\bar{d}(\tilde{c}) = 1$, otherwise the equilibrium fails to be sequentially rational. This, nevertheless, means that $U_L(\tilde{c}) = U_L(c^*_L) - \pi_1\varepsilon$, which for ε arbitrarily small is greater than $U_L(\bar{c}_L)$. Because this is true for every ε, a lower bound in the payoff of type-L and type-H respectively are $U_L(c^*_L)$ and $\max\{0, U_H(c^*_L)\}$. □

In words, Lemma (4.3) states that the payoff of every type is weakly higher than the payoff from contract c^*_L. This is so because an entrepreneur of any type can offer a contract arbitrarily close to c^*_L, which has to be accepted by the investor regardless his beliefs. Given that this is true for every contract arbitrarily close to c^*_L, both types can guarantee in equilibrium the payoff from c^*_L.

Below I show that no pooling equilibrium passes the intuitive criterion.

Lemma 4.4. *No pooling PBE passes the intuitive criterion.*

Proof. Consider a pooling PBE $\left((\bar{c},\bar{c}),\bar{d},(\bar{q}_i)_{i=L,H}\right)$ such that $U_L(\bar{c}) > U_L(c_L^*)$, $\pi_L\bar{R}-1<0$, $\pi_2\bar{R}-1>0$ and $\sum_i \lambda_i(\pi_i\bar{R}-1)\geq 0$. By Definition (4.1), for every c such that $U_2(c)\geq U_2(\bar{c})$, $\bar{d}(c)=0$. Nonetheless, due to the single-crossing property, there exists $\tilde{c}$ arbitrarily close to $\bar{c}$ such that $U_L(\tilde{c})<U_L(\bar{c})$, $U_H(\tilde{c})>U_H(\bar{c})$ and $\pi_H\tilde{R}-1>0$ which means that this, or any other pooling PBE, does not pass the intuitive criterion. □

The intuition behind Lemma (4.4) is that for every pooling equilibrium, there exists a contract that can be offered by type-H which makes strictly positive profits if the investor is indeed convinced that the entrepreneur who offered it is type-H. The result relies on the single-crossing property.

Lemma 4.5. *In every separating PBE $\bar{c}_L = c_L^*$.*

Proof. Because of Lemma (4.3), $U_L(c_L)\geq U_L(c_L^*)$. By Definition (4.1), in any separating equilibrium , $\bar{c}_L\neq\bar{c}_H$ and $\bar{q}_i(\bar{c}_i)=1$ for every $i=L,H$. Because c_L^* solves Program T_L, for every c such that $U_L(c)>U_L(c_L^*)$, it is true that $\pi_L R-1<0$. Therefore, $\bar{d}(c)=0$. □

The following maximisation program is key in the characterisation of the equilibrium that passes the intuitive criterion:

$$\text{PROGRAM LCS:}\quad \max_{x,R}\quad \pi_H(z_Hf(x)-Rx)\qquad \textit{subject to}$$

$$U_L(c_L^*)\geq \pi_L(z_Lf(x)-Rx) \tag{IC}$$

$$\pi_HR-1\geq 0 \tag{PP}$$

The following lemma characterises the solution of Program LCS.

Lemma 4.6. *In the solution of Program LCS, (PP) is binding.*

Proof. Denote as $\hat{c}$ the solution of Program LCS. Suppose first that neither (PP) nor (IC) are binding. Consider $(\hat{x},\hat{R}-\varepsilon)$ for $\varepsilon>0$ small enough such that both (PP) and (IC) are satisfied. Clearly, $\pi_H(z_Hf(\hat{x})-(\hat{R}-\varepsilon)x)>\pi_H(z_Hf(\hat{x})-\hat{R}\hat{x})$ and hence $\hat{c}$ does not solve Program LCS; a contradiction.

Now suppose that (PP) is not binding but (IC) is binding. Consider (x',R') such that $\pi_HR'-1\geq 0$ and

$$z_Lf(x')-R'x'=z_Lf(\hat{x})-\hat{R}\hat{x} \tag{4.4}$$

It is clear that (x',R') satisfies (IC). Equation (4.4) can be rewritten as:

$$z_Hf(x')-R'x'=z_Hf(\hat{x})-\hat{R}\hat{x}+(z_2-z_1)(f(x')-f(\hat{x})) \tag{4.5}$$

Note however that if $z_H > z_L$, $x' > \hat{x}$ and $R'x' > \hat{R}\hat{x}$, then

$$z_H f(x') - R'x' > z_H f(\hat{x}) - \hat{R}\hat{x}$$

Similarly, if $z_H < z_L$, $x' < \hat{x}$ and $R'x' < \hat{R}\hat{x}$, then

$$z_2 f(x') - R'x' > z_2 f(\hat{x}) - \hat{R}\hat{x}$$

In both cases (x', R') provides higher payoff than $(\hat{x}, \hat{R})$ to type-H and hence $(\hat{x}, \hat{R})$ cannot be a solution to Program LCS. Therefore (PP) is binding. □

One can now show that there is a unique equilibrium that passes the intuitive criterion and, as expected, this corresponds to the least-costly separating equilibrium. This result is formally stated and proven in the following lemma.

Lemma 4.7. *The only PBE that passes the intuitive criterion is the one such that $\bar{c}_L = c_L^*$ and $\bar{c}_H$ is the solution to Program LCS.*

Proof. It was shown in Lemma (4.4) that no pooling equilibrium passes the intuitive criterion. It is now shown that $\bar{c}_L = c_L^*$ and $\bar{c}_H$, the solution to Program LCS, passes the intuitive criterion. Let $\big((\bar{c}_i)_i, \bar{d}, (\bar{q}_i)_i\big)$ such that: (a) $\bar{c}_L$ and $\bar{c}_H$ are as specified above, (b) $\bar{d}(c) = 1$ for every c such that $U_L(c) \leq U_L(c_L^*)$ or $U_H(c) \leq U_H(\bar{c}_H)$, and $\bar{d}(c) = 0$ otherwise, (c) $\bar{q}_L(c_L^*) = \bar{q}_H(\bar{c}_H) = 1$ and $\bar{q}_L(c) = 1$, $\bar{q}_H(c) = 0$ for every c such that $U_L(c) \geq U_L(c_L^*)$ and $U_H(c) < U_H(\bar{c}_H)$ and $\bar{q}_L(c) = 0$, $q_H(c) = 1$ for every c such that $U_L(c) < U_L(c_L^*)$ and $U_H(c) \geq U_H(\bar{c}_H)$. By definition, for every c such that $U_H(c) > U_H(\bar{c}_H)$ and $U_L(c) < U_L(c_L^*)$, $\pi_H R - 1 < 0$ and for every c such that $U_L(c) > U_L(c_L^*)$, $\pi_L R - 1 < 0$. Hence, this PBE passes the intuitive criterion. A straightforward extension of the argument given in in Lemma (4.4) establishes that no other separating equilibrium passes the intuitive criterion. □

The intuition behind this result is as follows: the least-costly separating equilibrium consists of a pair of contracts that are the only ones maximising the payoff of the two types among all the incentive compatible pairs of contracts that make positive profits for all possible beliefs of the investor. Hence, there exists no other contract that if introduced can attract only one of the types and make strictly positive profits.

5 Comparative Statics

Following the analysis of the previous section, it is evident that the key equation is the one that characterises the equilibrium level of investment for a type-H entrepreneur. The first question is the following: when is constraint (IC) binding? To answer this question consider again constraint (IC) as an equality, or:

$$\pi_L z_L f(x) - \frac{\pi_L}{\pi_H} x - U_L(c_L^*) = 0 \tag{5.1}$$

Because $f'' < 0$, there are exactly two solutions of Equation (5.1). Denote these two solutions as $\underline{x}$ and $\bar{x}$ respectively, where $\underline{x} < \bar{x}$. It is easy to verify that when $\pi_H z_H f'(\underline{x}) < 1$ or $\pi_H z_H f'(\bar{x}) > 1$ constraint (IC) is not binding in Program LCS. This is because by decreasing (increasing) x slightly one can still satisfy the constraint and increase the payoff of type-H.

What if neither $\pi_H z_H f'(\underline{x}) < 1$ nor $\pi_H z_H f'(\bar{x}) > 1$? Then constraint (IC) is binding. By definition

$$\pi_L z_L f(\underline{x}) - \frac{\pi_L}{\pi_H}\underline{x} = \pi_L z_L f(\bar{x}) - \frac{\pi_L}{\pi_H}\bar{x} \tag{5.2}$$

Equation (5.2) can be rewritten as

$$\pi_H z_L f(\underline{x}) - \underline{x} = \pi_H z_L f(\bar{x}) - \bar{x} \tag{5.3}$$

Equation (5.3) is equivalent to

$$\pi_H z_H f(\underline{x}) - \underline{x} = \pi_H z_H f(\bar{x}) - \bar{x} + \pi_H(z_L - z_H)(f(\bar{x}) - f(\underline{x})) \tag{5.4}$$

Note that $\pi_H z_H f(x) - x$ is the payoff of type-H from contract $(x, 1/\pi_H)$. It is only straightforward to verify that for $z_L > z_H$, $\pi_H z_H f(\underline{x}) - \underline{x} > \pi_H z_H f(\bar{x}) - \bar{x}$. Equivalently, when $z_L < z_H$, $\pi_H z_H f(\underline{x}) - \underline{x} < \pi_H z_H f(\bar{x}) - \bar{x}$. The result has the following intuitive explanation: Under the maintained Assumptions, Equation (5.1) has two solutions. The ratio z_H/z_L determines which is the one that solves Program LCS. When $z_H/z_1 > 1$ it is the largest one. Intuitively, when $z_H/z_1 > 1$, there is a "*rat race*" for type-H; she over-invests in equilibrium to signal her type. On the contrary, when $z_H/z_L < 1$, there is the opposite of a *rat race*; type-H under-invests to signal her type. At $z_H = z_L$, there is a discontinuity as the investment switches from one solution to the other.

Let us denote as I_i, the equilibrium investment level of type-i. As I argued in Lemma (4.3), I_L is independent of z_H. Nonetheless, I_H depends both on z_H as this is implied by Program LCS. As such, let us denote as $I_H(z_H)$ the equilibrium investment level of type-H as a function of these two parameters. The following proposition is the main result of the paper.

Proposition 5.1. *For a given z_L, $I_H(z_H)$ features the following behaviour:*

$$I_H(z_H) = \begin{cases} x_H^*, & \text{if } \; z_H < \frac{1}{\pi_H f'(\underline{x})} \\ \underline{x}, & \text{if } \; \frac{1}{\pi_H f'(\underline{x})} \leq z_H < z_L \\ \bar{x}, & \text{if } \; z_L < z_H \leq \frac{1}{\pi_H f'(\bar{x})} \\ x_H^*, & \text{if } \; z_H > \frac{1}{\pi_H f'(\bar{x})} \end{cases} \tag{5.5}$$

The main implication of Proposition (5.1) is that $I_H(z_H)$ cannot be continuous and strictly increasing everywhere. In particular, it might be constant and it is characterised by a discontinuity exactly at $z_H = z_L$. This discontinuity implies some sort of a financial accelerator. More specifically, an infinitesimal small negative productivity shock can cause a large discontinuous fall in investment for type-H.

6 Numerical Examples

Assume that $f(x) = 2x^{1/2}$. It is straightforward to verify that $x_i^*(z_i) = (\pi_i z_i)^2$ and $U_i(c_i^*) = (\pi_i z_i)^2$. Equation (5.1) becomes:

$$2x^{1/2} - \frac{1}{\pi_H z_L} x - \pi_L z_L = 0$$

This is equivalent to the quadratic:

$$\frac{1}{(\pi_H z_L)^2} x^2 + 2(\frac{\pi_L}{\pi_H} - 2)x + (\pi_L z_L)^2 = 0$$

The discriminant of this quadratic is $4(\frac{\pi_L}{\pi_H} - 2)^2 - 4\frac{(\pi_L z_L)^2}{(\pi_H z_L)^2} = 16\frac{\pi_H - \pi_L}{\pi_H}$. The two solutions then are:

$$\underline{x} = \big(-(\frac{\pi_L}{\pi_H} - 2) - 2(\frac{\pi_H - \pi_L}{\pi_H})^{1/2}\big)(\pi_H z_L)^2, \qquad \bar{x} = \big(-(\frac{\pi_L}{\pi_H} - 2) + 2(\frac{\pi_H - \pi_L}{\pi_H})^{1/2}\big)(\pi_H z_L)^2$$

Assume now that $\pi^H = 1/3$, $\pi_L = 1/4$ and $z_L = 4$. The only variable is z_H. Straightforward algebra leads to $\underline{x} = 4/9$ and $\bar{x} = 4$. The investment function is given in Equation (6.1) and is depicted in Figure (1).

$$I_H(z_H) = \begin{cases} \frac{z_H^2}{9}, & \text{if} \quad z_H < 2 \\ \frac{4}{9}, & \text{if} \quad 2 \leq z_H < 4 \\ 4, & \text{if} \quad 4 < z_H \leq 6 \\ \frac{z_H^2}{9}, & \text{if} \quad z_H > 6 \end{cases} \tag{6.1}$$

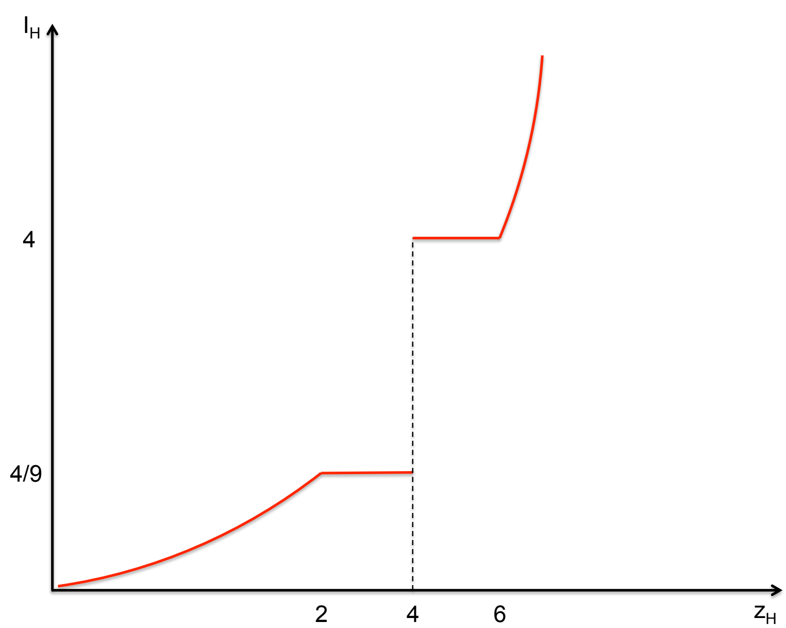

Figure 1: Investment Under Asymmetric Information

7 Conclusion

In this paper, I studied how investment varies with productivity in a credit market with asymmetric information and signalling. The model featured an entrepreneur with a risky, variable-investment project who wished to raise capital in the financial market. The entrepreneur could be one of two possible types. The two possible types differed both with respect to the probability of success as well as the marginal productivity in case of success. I showed that investment could not be a continuous and strictly increasing function of productivity everywhere. In particular when the incentive constraint was binding, then investment was constant and featured a profound discontinuity. I argued that this might have implications regarding the amplification of small productivity shocks. A small negative (positive) productivity shock for the high type could cause a large discontinuous fall (rise) in investment. Equivalently, a small negative (positive) productivity shock for the low type could cause a large discontinuous fall (rise) in investment.

The model I studied was a very parsimonious one with many simplifying assumptions. A plausible avenue for future research is to extend the model to more than two possible types, preserving the rest of the assumptions. Another possibility is to embed the model in a dynamic, perhaps, overlapping generations model. This model might be more appropriate to draw macroeconomic conclusions that are difficult to be obtained in a static model. I leave these questions for future research.

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