Intermediated Surge Pricing

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April 2016

Abstract

I study a market in which a profit-maximizing intermediary facilitates trade between buyers and sellers. The intermediary sets prices for the buyers and sellers, with the difference being her fee. Optimal prices increase with demand and, under plausible conditions, the optimal percent fee decreases with demand. However, if the intermediary keeps a constant percent fee regardless of demand, as is the case for some intermediaries, the price paid by buyers during high (low) demand increases (decreases) even further; that is, surge pricing is amplified.

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1 Introduction

Intermediaries abound in markets, facilitating trade between buyers and sellers. The services they provide include reduction in search costs, information exchange, access to inventory, and diversification of risk. While the digital economy has diminished the need for intermediaries in some areas it has also created new markets intermediated by middlemen.

Some of these new intermediated markets are for goods or services for immediate delivery. Uber, the online transportation company, provides a motivating example. The firm is an intermediary between car drivers and passengers. Its superior matching technology reduces search costs on both sides of the market. Uber sets a price (per mile) that passengers pay and takes a fixed percent fee from each transaction it mediates. Uber's software system allows it to monitor local demand and supply conditions in real time. While Uber responds to sharp increases in demand by raising price, it keeps the same percent fee, usually 20%. Thus, the payment that a car driver receives is a fixed percent of the payment made by the passenger. Conceivably, if Uber were to reduce its percent fee during high demand, it may increase profits by enticing more drivers to enter the supply pool. Further, the flexibility afforded by not having the payment by a passenger and the payment to a driver in lock step may moderate price increases during periods of high demand.

These questions are explored in a simple market model with a monopolist intermediary who sets prices for buyers and sellers. The focus is on how optimal prices change with changes in demand, especially when there is some inflexibility in the prices set by the intermediary.

Consider a market with an intermediary who facilitates trade between a large number of buyers and sellers. Search costs for buyers and sellers are larger than the gains to trade; hence without the intermediary there is (essentially) no trade. This is the case for several markets that have experienced extraordinary growth after the advent of the internet and digital communication enabled intermediaries to reduce search costs. In the model, each buyer's value and each seller's cost for the good is private information. Consequently, the intermediary does not have the ability to price discriminate between buyers or between sellers. The intermediary sets a price for all buyers and a price for all sellers to maximize her profit. As there are a large number of buyers and sellers, it is optimal for each to act as a price-taker. The intermediary's profit is the difference between the buyer price and seller price multiplied by the number of units traded.

The technological advances that have reduced search costs and enabled better matches between buyers and sellers have also made it easier for the intermediary to monitor demand and supply and adjust prices accordingly. Therefore, changes in optimal prices with changes in market conditions are a focus of this paper. Of particular interest is the revenue share[1] of the intermediary at optimal prices and changes in this optimal share with changes in demand and supply. To capture changes in market conditions in a tractable manner, in the model there is a continuum of buyers and a continuum of sellers rather than a large finite number of each.[2] An increase in the mass of buyers corresponds to an increase in demand relative to supply.

It is shown that there exists a unique set of optimal prices at which the intermediary's profit is maximized. As the mass of buyers increases, the optimal price charged to buyers and paid to sellers increase – also known as *surge pricing*. However, the optimal share (i.e., percent fee) of the intermediary may increase or decrease.

The intermediary is a monopolist in its interactions with buyers and a monopsonist in its interactions with sellers. The marginal cost of the intermediary-monopolist is determined by the intermediary-monopsonist, while the marginal revenue of the intermediary-monopsonist is determined by the intermediary-monopolist. The change in the optimal share of the intermediary due to an increase in the mass of buyers depends on the elasticities of demand and supply through their effect on the gross mark-up of the intermediary monopolist and on the gross mark-down of the intermediary monopsonist.

For a sufficiently large increase in buyer mass, the optimal share of the intermediary decreases. During periods of high demand, raising the seller price draws more

[1] The percent of buyer price that is taken by the intermediary as fees.

[2] Myerson and Satterthwaite [8] obtain the optimal mechanism for a profit-maximizing intermediary with one buyer and one seller.

sellers in, thereby increasing the number of trades. If the increase in demand is large enough, it is optimal for the intermediary to capture a smaller share of this larger pie. However, firms such as Uber and Lyft extract the same share, charging a constant percent fee of 20% of buyer price, regardless of the level of demand. This inflexibility in pricing results in a match between demand and supply that is less than optimal for the intermediary.

There may be a legal rationale for Uber's constant percent fee. Currently, the firm is a defendant in lawsuits that question its standing as an intermediary. The plaintiffs are sellers (Uber car drivers), who claim that they should be considered employees of Uber and are entitled to various benefits under labor regulations. An important consideration in the lawsuit is who controls the prices and fees.[3] Plausibly, if Uber were to exercise greater flexibility in its pricing, such as attune its percent fee to changes in demand, then it might weaken its legal claim that it is an intermediary. This imposes a constraint in its pricing decisions.

I show that a constant percent fee exacerbates surges in buyer prices when there is a sharp increase in demand. Recall that in the absence of a constant-fee constraint, it is optimal for the intermediary to reduce its percent fee when the demand increase is large. In this scenario, under a constant percent fee constraint the intermediary will charge a fee that is smaller than optimal during low demand and greater than optimal during high demand. This in turn increases the surge in buyer prices. Specifically, a constant percent fee increases the buyer price during high demand and decreases the buyer price during low demand, compared to optimal prices when the intermediary's fee is flexible.[4] Thus, while a constant percent fee may strengthen Uber's position in pending litigation, it is costly for two reasons. Not only does a constant percent fee reduce profits, it also amplifies surge pricing leading to a loss in buyer goodwill.[5]

The question addressed in this paper – the impact of some rigidity in pricing on surge pricing – has not been asked in the literature. Several papers, notably Armstrong [1], Caillaud and Jullien [4], and Rochet and Tirole [9, 10], analyze two-sided

[3]See Douglas O'Connor et al. v. Uber Technologies et al., 2015 [5]. Similar lawsuits have been filed against other intermediaries including Lyft, Postmates, Instacart, GrubHub, Shyp, and Washio.

[4]Under these conditions, a constant percent fee also diminishes the surge in seller prices.

[5]See Grubb [6], for instance.

markets, where two types of agents interact on a platform provided by a third party. Indirect network externalities play a significant role in such markets. In particular, costs on one side of the market cannot be easily passed through to the other side. Consequently, the terms of trade depend not only on the total amount that the two parties to a transaction pay but also on the division of the payments.

An older literature investigates equilibrium and efficiency in markets with intermediaries. Rubinstein and Wolinsky [11] obtain a steady-state equilibrium in a market with buyers, sellers, and intermediaries, highlighting the relationship between trading rules and the endogenous terms of trade. Stahl [13], Spulber [12], and Yanelle [14] explore conditions under which competition between intermediaries leads to (in)efficient outcomes. Yavas [15] examines the welfare-improving role of a monopolist intermediary in a market with search costs, while Biglaiser [2] shows that an expert intermediary can overcome an adverse selection problem between buyers and sellers.

The paper is organized as follows. The model is presented in the next section. In Section 3, optimal prices are derived and as well as comparative statics for the optimal fee for the intermediary with respect to changes in demand. Pricing under a constant-fee constraint is investigated in Section 4. Section 5 concludes. All proofs are in an appendix.

2 The model

There are (potential) gains to trade between buyers and sellers. Each buyer has utility for one unit of a homogenous object and each seller has one unit to sell. I assume that there is a continuum of buyers and sellers, with each buyer and seller being infinitesimal. There is a mass μ of buyers and a unit mass of sellers.[6] Changes in demand are captured by changes in μ. Each buyer's valuation v has cumulative distribution function F_b with strictly positive and continuous density function f_b on support $[0, 1]$. Each seller's cost c has cumulative distribution function F_s with strictly positive and continuous density function f_s on support $[0, 1]$.

[6]The assumption of a unit mass of sellers is without loss of generality as μ may be viewed as the mass of buyers per unit mass of sellers.

Following Bulow and Roberts [3], the demand curve is $q = \mu(1 - F_b(v))$ and the supply curve is $q = F_s(c)$.

The search costs for buyers and sellers are assumed to be prohibitively high. An intermediary with superior matching technology enables trade between the two sides. The intermediary knows μ, F_b, and F_s. However, a buyer's value or a seller's cost are not known to the intermediary. Thus, perfect price discrimination is not an option for the intermediary. Instead, she selects a price p_b for buyers and a price p_s for sellers so as to maximize profits. Through the intermediary, any buyer with value $v \geq p_b$ will purchase a unit and any seller with cost $c \leq p_s$ will sell a unit. To simplify the notation, the intermediary's marginal costs per transaction are assumed to be zero. The intermediary's fixed cost are also assumed to be zero.

The intermediary in our model is a matchmaker who does not trade but simply matches buyers with sellers. She is not a market maker who buys and holds inventory. This role as a matchmaker but not a market maker is appropriate in markets for immediate delivery of perishable goods.

An assumption that is maintained throughout is that the distributions F_b and F_s are *regular* in the sense of Myerson [7]. That is, the virtual utility of buyers, $v - \frac{1-F_b(v)}{f_b(v)}$, is strictly increasing in v and the virtual cost of sellers, $c + \frac{F_s(c)}{f_s(c)}$, is strictly increasing in c.[7]

3 Optimal intermediation

At price $p_b = v$, buyers demand $\mu(1 - F_b(v))$ units and at price $p_s = c$, sellers are willing to supply $F_s(c)$ units. Thus, $q = \min\{\mu(1-F_b(v), F_s(c)\}$ is the amount traded. If $\mu(1-F_b(v)) < F_s(c)$ then the intermediary can lower the seller price p_s slightly below c and still trade q units. Similarly, if $\mu(1-F_b(p_b)) > F_s(p_s)$ then the intermediary can raise the buyer price p_b slightly above v and still trade q units. Hence, intermediary profit-maximization implies that optimal prices $p_b = v$ and $p_s = c$ are such that

[7]Most of the results in the paper can be proved if the virtual utility and virtual cost functions are weakly increasing. However, the proofs are simpler when these functions are assumed to be strictly increasing.

demand equals supply:

$$q \;=\; \mu[1 - F_b(v)] \;=\; F_s(c) \tag{1}$$

Thus, $v = F_b^{-1}(1 - \frac{q}{\mu})$, $c = F_s^{-1}(q)$ and the intermediary's profit as a function of q is

$$\begin{aligned}
\Pi_I(q) &= q\,[v - c] \\
&= q\,[F_b^{-1}(1 - \frac{q}{\mu}) - F_s^{-1}(q)] \\
\Longrightarrow \qquad \frac{d\Pi_I}{dq} &= F_b^{-1}(1 - \frac{q}{\mu}) - F_s^{-1}(q) + q\frac{dF_b^{-1}(1 - \frac{q}{\mu})}{dq} - q\frac{dF_s^{-1}(q)}{dq} \\
&= F_b^{-1}(1 - \frac{q}{\mu}) - F_s^{-1}(q) - \frac{q}{\mu f_b(F_b^{-1}(1 - \frac{q}{\mu}))} - \frac{q}{f_s(F_s^{-1}(q))} \\
&= v - c - \frac{1 - F_b(v)}{f_b(v)} - \frac{F_s(c)}{f_s(c)} \qquad (2)
\end{aligned}$$

As $\frac{d\Pi_I}{dq} = 0$ at optimal prices (c^*, v^*) we have

$$\Longrightarrow \qquad v^* - \frac{1 - F_b(v^*)}{f_b(v^*)} \;=\; c^* + \frac{F_s(c^*)}{f_s(c^*)} \tag{3}$$

Thus, a necessary condition for optimality is that the virtual utility of the marginal buyer equals the virtual cost of the marginal seller. This necessary condition states that marginal revenue equals marginal cost (see Bulow and Roberts [3]).

Note that (3) is independent of μ. There are many solutions to (3), one for each μ; equation (1) pins down a unique solution as shown below.

As the densities f_s and f_b are strictly positive and continuous on their support the locus of points (c^*, v^*) satisfying (3) is a continuous curve. This is the blue curve, labeled 'MR=MC,' in Figure 1. This curve is a positively-sloped function in the regular case. To see this, start at any point (c^*, v^*) on the 'MR=MC' curve. If the cost is increased from c^* to $c^* + \Delta c$, then the right-hand side of (3) increases by regularity. To restore equality in (3) the buyer's value must be increased from v^*, once again by regularity. Hence, there is a $\Delta v > 0$ such that $(c^* + \Delta c, v^* + \Delta v)$ is on the 'MR=MC' curve. The points $v_0 > 0$ and $c_1 < 1$ are obtained from $v_0 - \frac{1-F_b(v_0)}{f_b(v_0)} = 0$ and $c_1 + \frac{F_s(c_1)}{f_s(c_1)} = 1$. Buyers with $v < v_0$ and sellers with $c > c_1$ do not trade. Note also that $MR = v - \frac{1-F_b(v)}{f_b(v)} > c + \frac{F_s(c)}{f_s(c)} = MC$ above the 'MR=MC' curve and $MR < MC$ below this curve.

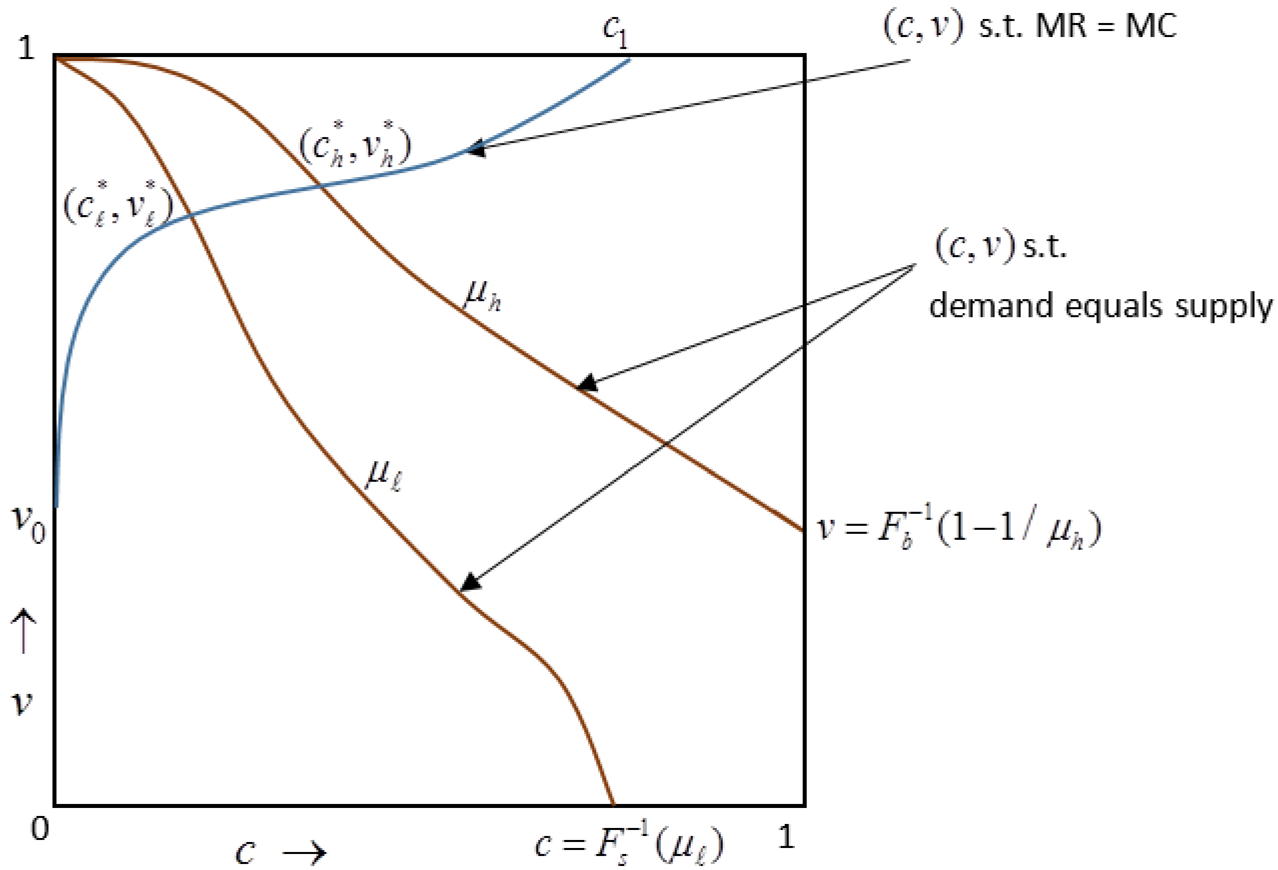

Figure 1

The two negatively-sloped brown curves, labeled demand equals supply, represent the points (c, v) that satisfy (1) for buyer mass μ_ℓ and μ_h, respectively. In the figure, $\mu_\ell < 1 < \mu_h$. The intersection of the blue curve with a brown curve yields the optimal (c^*, v^*) for the buyer mass corresponding to the brown curve. This is proved next.

Proposition 1 *Assume that F_b and F_s are regular and the mass of buyers is μ. There exists a unique set of optimal prices $(c^*(\mu), v^*(\mu))$ at which the intermediary's profit is maximized. Moreover, $\frac{dc^*(\mu)}{d\mu} > 0$ and $\frac{dv^*(\mu)}{d\mu} > 0$.*

The following example illustrates the equilibrium.

EXAMPLE 1: If F_b and F_s are uniformly distributed on $[0, 1]$ then (1) and (3) are

$$\begin{aligned} \mu(1-v^*) &= c^*, \qquad 2v^* - 1 = 2c^* \\ \Longrightarrow \qquad v^* &= 1 - \frac{1}{2(1+\mu)}, \qquad c^* = \frac{\mu}{2(1+\mu)}, \qquad \Pi_I = \frac{\mu}{4(1+\mu)} \end{aligned}$$

The distributions in this example are regular. Proposition 1 implies that optimal prices increase with μ. This can be verified directly as,

$$\frac{dv^*}{d\mu} = \frac{1}{2(1+\mu)^2} > 0, \qquad \frac{dc^*}{d\mu} = \frac{1}{2(1+\mu)^2} > 0$$

The optimal percent fee of the intermediary (expressed as a fraction of the buyer price), α^* is

$$\alpha^* \equiv \frac{v^* - c^*}{v^*} = \frac{1+\mu}{1+2\mu}$$

As $dv^*/d\mu = dc^*/d\mu$, α^* decreases with μ

$$\frac{d\alpha^*}{d\mu} = \frac{1}{(1+2\mu)^2} < 0$$

□

In the example, as buyer mass μ increases, the intermediary's optimal percent fee, α^*, decreases. However, in general α^* may increase or decrease with μ. The interest in the comparative statics of α^* stems from the rigid nature of pricing by some intermediaries. When demand increases substantially, firms such as Uber and Lyft practice what they call surge pricing. That is, the price for buyers increases appreciably above the normal price. However, both Uber and Lyft keep the same percent (usually 20%) of the buyer price. Conceivably, reducing α^* in periods of very high demand may lead to a better match between supply and demand, yielding greater profit for the intermediary. As shown next, changes in $\alpha^*(\mu)$ with μ are determined by the elasticities of the demand and supply curves.

The intermediary is both a monopolist and a monopsonist. It acts as a monopolist in its interactions with buyers, with its marginal cost determined by the equilibrium in the sellers' market. The intermediary also acts as a monopsonist in its interactions with sellers, with its marginal revenue from a unit of input determined by the equilibrium in the buyers' market. With this in mind, define price elasticities of demand and supply

$$\begin{aligned}\eta_b(v) &= \frac{v}{q}\frac{dq}{dv} = -v\frac{f_b(v)}{1-F_b(v)} \\ \eta_s(c) &= \frac{c}{q}\frac{dq}{dc} = c\frac{f_s(c)}{F_s(c)}\end{aligned}$$

where we use $q = \mu(1-F_b(v)) = F_s(c)$, and note that v and c are prices along the demand curve and supply curve, respectively.

From price theory we know that the price v for his product charged by a monopolist with constant marginal cost c satisfies

$$\frac{v}{c} = \frac{1}{1+\frac{1}{\eta_b(v)}}$$

Call $\frac{1}{1+\frac{1}{\eta_b(v)}}$ the *gross mark-up* for our intermediary monopolist who faces a constant marginal cost of c. Further, a monopsonist with constant marginal revenue product of a unit of input equal to v will set price for the input at c such that

$$\frac{v}{c} = 1+\frac{1}{\eta_s(c)}$$

Call $1+\frac{1}{\eta_s(c)}$ the *gross mark-down* for our intermediary monopsonist with a constant marginal revenue of v from a unit of input.

As μ increases, $v^*(\mu)$ and $c^*(\mu)$ increase, by Proposition 1. The product of the gross mark-up and the gross mark-down determines whether the intermediary's optimal percent fee increases or decreases with μ.

Proposition 2 *The intermediary's percent fee,* $\alpha^*(\mu) = \frac{v^*(\mu)-c^*(\mu)}{v^*(\mu)}$ *decreases with* μ *if and only if the product of the gross mark-up of the intermediary monopolist and the gross mark-down of the intermediary monopsonist*

$$\frac{1+\frac{1}{\eta_s(c^*)}}{1+\frac{1}{\eta_b(v^*)}}$$

is decreasing with μ.

Recall that as μ increases, so do the optimal prices $c^*(\mu)$ and $v^*(\mu)$. It is reasonable that $|\eta_b(v)|$ increases (i.e., demand becomes more elastic) as v increases. This is certainly true if the hazard rate $\frac{f_b(v)}{1-F_b(v)}$ of F_b increases with v, i.e. $1 - F_b(v)$ is log concave. If, in addition, $\eta_s(c)$ increases, or at least does not decrease too fast, as c increases, then $\alpha^*(\mu)$ decreases with μ. Note that a decreasing reversed hazard rate $\frac{f_s(c)}{F_s(c)}$, i.e. log concave $F_s(c)$, limits the rate at which $\eta_s(c)$ might decrease as c increases.

Another necessary and sufficient condition for $\frac{d\,\alpha^*(\mu)}{d\,\mu} < 0$ is that $\frac{c^*}{v^*}\frac{dv^*}{dc^*}$, the elasticity of v^* with respect to c^* along the locus of points satisfying equation (3), is less than 1. This is established next.

Proposition 3 *The intermediary's share $\alpha^*(\mu) = \frac{v^*-c^*}{v^*}$, expressed as a fraction of the buyer's price, decreases with μ if and only if*

$$\frac{dv^*}{dc^*} < \frac{v^*}{c^*}$$

In particular, if $\frac{dv^*}{d\mu} \leq \frac{dc^*}{d\mu}$ then $\frac{d\alpha^*}{d\mu} < 0$.

The necessary and sufficient condition in Proposition 3 admits a geometric interpretation. This condition states that, at a specific value of μ, the slope of the 'MR=MC' curve in Figure 1 (the locus of points satisfying (3)) is less than the slope of the line from the origin to the point on the 'MR=MC' curve.

While Propositions 2 and 3 are stated for local changes in buyer mass μ, they are readily applicable to large changes in μ. Let $\mu_h > \mu_\ell$. Then $\alpha^*(\mu_\ell) > \alpha^*(\mu_h)$ if and only if

$$\frac{1 + \frac{1}{\eta_s(c^*(\mu_\ell))}}{1 + \frac{1}{\eta_b(v^*(\mu_\ell))}} > \frac{1 + \frac{1}{\eta_s(c^*(\mu_h))}}{1 + \frac{1}{\eta_b(v^*(\mu_h))}}$$

if and only if

$$\frac{v^*(\mu_h) - v^*(\mu_\ell)}{c^*(\mu_h) - c^*(\mu_\ell)} \leq \frac{v^*(\mu_\ell)}{c^*(\mu_\ell)}$$

The proof is omitted.

As shown next, the optimal percent fee for the intermediary decreases for a sufficiently large increase in buyer mass.

Lemma 1 *There exist* $\underline{\mu}$ *and* $\overline{\mu}$, $\underline{\mu} \leq \overline{\mu}$, *such that for any* $\mu_\ell < \underline{\mu}$ *and any* $\mu_h \geq \overline{\mu}$,

$$\alpha^*(\mu_h) \;<\; \alpha^*(\mu_\ell)$$

That $\underline{\mu} = \overline{\mu}$ is possible in Lemma 1 follows from Example 1 where it was directly established that $\frac{d\,\alpha^*(\mu)}{d\,\mu} < 0$ for each μ. In this example, the locus of points satisfying (3) is $v^* = c^* + 0.5$. Thus, $\frac{d\,v^*}{d\,c^*} = 1 < \frac{v^*}{c^*}$, verifying the necessary and sufficient condition of Proposition 3. Alternatively, note that in this example,

$$\begin{aligned} \eta_b(v) \;&=\; -\frac{v}{1-v} \qquad \eta_s(c) \;=\; 1 \\ \Longrightarrow \qquad \frac{1+\frac{1}{\eta_s(c^*)}}{1+\frac{1}{\eta_b(v^*)}} \;&=\; \frac{2}{1-\frac{1-v^*}{v^*}} \;=\; \frac{2v^*}{2v^*-1} \end{aligned}$$

which decreases with v^* and hence with μ. This verifies the necessary and sufficient condition of Proposition 2.

Comparison with a benchmark monopolist

It is useful to compare pricing by the intermediary with pricing by a monopolist who acquires the productive assets and services of all the sellers who contract with the intermediary. I make this comparison under the assumption that the monopolist's total cost of providing goods remain unchanged.[8]

It is easy to show that the marginal cost of the monopolist at $q = F_s(c)$ units is c, which is less than $c + \frac{F_s(c)}{f_s(c)}$, the marginal cost at q units for the intermediary. The marginal revenue of the monopolist is the same as that of the intermediary. Consequently, the monopolist's optimal buyer price is lower and the quantity higher than the corresponding optimal values for the intermediary. Monopoly prices, (c_m, v_m), are less inefficient than intermediary prices, (c^*, v^*), as can be seen from

$$v_m - c_m \;=\; \frac{1-F_b(v_m)}{f_b(v_m)} \;<\; \frac{1-F_b(v_m)}{f_b(v_m)} + \frac{F_s(c_m)}{f_s(c_m)}$$

[8]From the resistance of some intermediaries to classify sellers as employees rather than as independent contractors it appears that this may not be a tenable assumption. That is, total costs of this monopolist would be higher. Nevertheless, it is instructive to consider this benchmark monopolist with the same cost of providing service as the sellers collectively.

Thus, while (c^*, v^*) is on the 'MR=MC' curve in Figure 1, (c_m, v_m) is below this curve. Consequently, $v^* - c^* > v_m - c_m$ imply that the monopoly outcome is more efficient.

Observe that if the hazard rate of F_b is increasing in v, i.e., $1-F_b(v)$ is log concave, then $\frac{1-F_b(v)}{f_b(v)}$ decreases with v. Therefore, $v_m - c_m$ decreases as μ increases and so does the monopolist's percent fee $\alpha_m = \frac{v_m - c_m}{v_m}$.

4 Surge pricing under constrained intermediation

As mentioned earlier, intermediaries such as Uber and Lyft keep a constant fraction of the buyer price regardless of demand conditions. A possible reason for this practice may be perceived legal constraints. I show below that, under reasonable conditions, if an intermediary operates under the constraint that its percent fee is constant then as demand increases the magnitude of surge in buyer price is amplified and the magnitude of surge in seller price is muted.

It is sufficient to consider two possible levels of demand, high or low. The mass of buyers is μ_h during high demand and μ_ℓ during low demand, with $\mu_h > \mu_\ell$. The fraction of time that demand is high is r. Alternatively, r may be viewed as the probability that demand is high at any given moment. The constraint is that the intermediary keeps the same fraction α of the buyer price regardless of the level of demand.[9] Thus, rather than choose any prices v_h, c_h during high demand and any prices v_ℓ, c_ℓ during low demand, the intermediary is constrained to choose v_h, v_ℓ and α and set $c_h = (1-\alpha)v_h$, $c_\ell = (1-\alpha)v_\ell$. That is, the optimal prices satisfy

$$\frac{1}{1-\alpha} = \frac{v_h}{c_h} = \frac{v_\ell}{c_\ell} \tag{4}$$

for some $\alpha \in (0,1)$. I refer to (4) as the constant-fee constraint.

Let $(\hat{c}_\ell, \hat{v}_\ell)$ and $(\hat{c}_h, \hat{v}_h)$ be the optimal prices at μ_ℓ and at μ_h respectively under

[9] Equivalently, the intermediary adds a constant mark-up, $\frac{\alpha}{1-\alpha}$, to the seller price to obtain the buyer price.

the constant-fee constraint. Let

$$\hat{\alpha} \;=\; \frac{\hat{v}_\ell - \hat{c}_\ell}{\hat{v}_\ell} \;=\; \frac{\hat{v}_h - \hat{c}_h}{\hat{v}_h}$$

be the optimal fee of the intermediary under this constraint.

As already mentioned, the object being sold is perishable and cannot be stored. Thus, optimality implies that demand must equal supply under each of the two demand scenarios. That is, (1) is satisfied at μ_ℓ and at μ_h. Therefore, $(\hat{c}_k, \hat{v}_k)$ lies on the demand equals supply curve for μ_k, $k = \ell, h$ in Figure 1. A consequence is that

$$\hat{c}_h > \hat{c}_\ell \quad \text{and} \quad \hat{v}_h > \hat{v}_\ell \tag{5}$$

To see this, note that the constant-fee constraint states that the straight line through $(\hat{c}_\ell, \hat{v}_\ell)$ and $(\hat{c}_h, \hat{v}_h)$ passes through the origin and has slope $\frac{1}{1-\hat{\alpha}} > 0$. As the demand equals supply curve for μ_h lies above the demand equals supply curve for μ_ℓ (see proof of Proposition 1), (5) follows.

Let $v_k^* = v^*(\mu_k)$, $c_k^* = c_k^*(\mu_k)$, and $\alpha_k^* = \alpha_k^*(\mu_k)$ be the (unconstrained) optimal prices and intermediary share at buyer mass μ_k, $k = \ell, h$. In Figure 1, the slope of the line from the origin to the intersection of the 'MR=MC' and the demand equals supply curve for μ_k is $v_k^*/c_k^* = 1/[1 - \alpha_k^*]$, $k = \ell, h$. The next proposition says that $\hat{v}_\ell/\hat{c}_\ell$ (which is equal to $\hat{v}_h/\hat{c}_h$) lies in between the smaller and larger of v_k^*/c_k^*, $k = \ell, h$.

Proposition 4 *Under the constant-fee constraint, (4), the optimal fee for the intermediary, $\hat{\alpha}$, satisfies*

$$\min\{\alpha_\ell^*, \alpha_h^*\} \;\leq\; \hat{\alpha} \;\leq\; \max\{\alpha_\ell^*, \alpha_h^*\}, \tag{6}$$

with strict inequalities if $\alpha_\ell^* \neq \alpha_h^*$. Further, optimal prices $(\hat{c}_\ell, \hat{v}_\ell)$, $(\hat{c}_h, \hat{v}_h)$ and an optimal fee $\hat{\alpha} = \frac{\hat{v}_k - \hat{c}_k}{\hat{v}_k}$ exist.

The main result is that if the unconstrained optimal share of the intermediary is lower at a higher demand, then the buyer price increase in high demand is greater under the constant-fee constraint.

Proposition 5 *If $\mu_\ell < \mu_h$ is such that $\alpha^*_\ell > \alpha^*_h$, then as the mass of buyers increases from μ_ℓ to μ_h the surge in buyer prices is greater and the surge in seller prices is smaller under the constant-fee constraint than at the unconstrained optimal prices. That is,*

$$\begin{aligned} \hat{v}_h &> v^*_h > v^*_\ell > \hat{v}_\ell \\ c^*_h &> \hat{c}_h > \hat{c}_\ell > c^*_\ell \end{aligned}$$

From Lemma 1 we know that there exist $\underline{\mu} \leq \overline{\mu}$ such that $\alpha^*_\ell > \alpha^*_h$ for any $\mu_\ell < \underline{\mu}$ and any $\mu_h \geq \overline{\mu}$. Together with Proposition 5 we have

Corollary 1 *There exist $\underline{\mu}$ and $\overline{\mu}$, $\underline{\mu} \leq \overline{\mu}$, such that for any $\mu_\ell < \underline{\mu}$ and any $\mu_h \geq \overline{\mu}$, as the mass of buyers increases from μ_ℓ to μ_h the surge in buyer prices is greater and the surge in seller prices is smaller under the constant-fee constraint than at the unconstrained optimal prices.*

Similarly, from Propositions 2 and 5 we have

Corollary 2 *If the product of the gross mark-up of the intermediary monopolist and the gross mark-down of the intermediary monopsonist decreases as the mass of buyers increases from μ_ℓ to μ_h the surge in buyer prices is greater and the surge in seller prices is smaller under the constant-fee constraint than at the unconstrained optimal prices.*

A necessary condition for optimality under the constant-fee constraint is presented below.

Lemma 2 *Under the constant-fee constraint, (4), the optimal prices $(\hat{c}_\ell, \hat{v}_\ell)$ at μ_ℓ and $(\hat{c}_h, \hat{v}_h)$ at μ_h satisfy*

$$\left[\hat{v}_h - \frac{1-F_b(\hat{v}_h)}{f_b(\hat{v}_h)}\right] - \left[\hat{c}_h + \frac{F_s(\hat{c}_h)}{f_s(\hat{c}_h)}\right] = \frac{\lambda \hat{c}_\ell}{r}\frac{1-F_b(\hat{v}_h)}{f_b(\hat{v}_h)} + \frac{\lambda \hat{v}_\ell}{r}\frac{F_s(\hat{c}_h)}{f_s(\hat{c}_h)} \tag{7}$$

$$\left[\hat{v}_\ell - \frac{1-F_b(\hat{v}_\ell)}{f_b(\hat{v}_\ell)}\right] - \left[\hat{c}_\ell + \frac{F_s(\hat{c}_\ell)}{f_s(\hat{c}_\ell)}\right] = -\frac{\lambda \hat{c}_h}{1-r}\frac{1-F_b(\hat{v}_\ell)}{f_b(\hat{v}_\ell)} - \frac{\lambda \hat{v}_h}{1-r}\frac{F_s(\hat{c}_\ell)}{f_s(\hat{c}_\ell)} \tag{8}$$

where λ is a Lagrangian multiplier.

Suppose that $\alpha_\ell^* > \alpha_h^*$. This is the case depicted in Figure 1, where the line from the origin to (c_ℓ^*, v_ℓ^*) is steeper than the line from the origin to (c_ℓ^*, v_ℓ^*). From Proposition 4, we know that $\alpha_\ell^* > \hat{\alpha} > \alpha_h^*$, and therefore at the optimal constrained prices we have $\frac{v_\ell^*}{c_\ell^*} > \frac{\hat{v}_\ell}{\hat{c}_\ell} = \frac{\hat{v}_h}{\hat{c}_h} > \frac{v_h^*}{c_h^*}$. Imagine a straight line with slope $\frac{1}{1-\hat{\alpha}}$ through the origin in Figure 1; this line intersects the 'MR=MC' curve in the segment between (c_ℓ^*, v_ℓ^*) and (c_h^*, v_h^*) . Both $(\hat{c}_\ell, \hat{v}_\ell)$ and $(\hat{c}_h, \hat{v}_h)$ are on this line, at the intersection points with the respective demand equals supply curves. It is clear that $(\hat{c}_\ell, \hat{v}_\ell)$ is below the 'MR=MC' curve along the demand equals supply curve for μ_ℓ and $(\hat{c}_h, \hat{v}_h)$ is above the 'MR=MC' curve along the demand equals supply curve for μ_h. Hence, $MR < MC$ at buyer mass μ_ℓ and $MR > MC$ at buyer mass μ_h. Further, $(\hat{c}_\ell, \hat{v}_\ell)$ is southeast of (c_ℓ^*, v_ℓ^*) and $(\hat{c}_h, \hat{v}_h)$ is northwest of (c_h^*, v_h^*) along the respective demand equals supply curves. This provides a geometric proof for Proposition 5. Moreover, the Lagrange multiplier λ in (7) and (8) is positive when $\alpha_\ell^* > \alpha_h^*$.

Observe that (7) and (8) may be written as

$$\begin{aligned}\Big[\hat{v}_h - \frac{1-F_b(\hat{v}_h)}{f_b(\hat{v}_h)}\Big] - \Big[\hat{c}_h + \frac{F_s(\hat{c}_h)}{f_s(\hat{c}_h)}\Big] &= \frac{\lambda(1-\hat{\alpha})\hat{v}_\ell\hat{v}_h}{r}\Big[\frac{1}{|\eta_b(\hat{v}_h)|} + \frac{1}{\eta_s(\hat{c}_h)}\Big] \\ \Big[\hat{v}_\ell - \frac{1-F_b(\hat{v}_\ell)}{f_b(\hat{v}_\ell)}\Big] - \Big[\hat{c}_\ell + \frac{F_s(\hat{c}_\ell)}{f_s(\hat{c}_\ell)}\Big] &= -\frac{\lambda(1-\hat{\alpha})\hat{v}_\ell\hat{v}_h}{1-r}\Big[\frac{1}{|\eta_b(\hat{v}_\ell))|} + \frac{1}{\eta_s(\hat{c}_\ell)}\Big]\end{aligned}$$

Thus, the divergence between the marginal revenue and marginal cost at $(\hat{c}_k, \hat{v}_k)$ increases as (i) the elasticity of demand at $\hat{v}_k$ decreases or (ii) the elasticity of supply at $\hat{c}_k$ decreases or (ii) the fraction of time that buyer mass is μ_k decreases.

Example 2: Consider Example 1 under the constant-fee constraint. The buyer mass is $\mu_\ell = 1$ half the time and $\mu_h = 2$ half the time. Note that for F_b and F_s uniformly distributed on [0,1], $\eta_b(v) = -\frac{v}{1-v}$ and $\eta_s(c) = 1$. Substituting in the first-order conditions above, we obtain

$$\begin{aligned}2\hat{\alpha}\hat{v}_h - 1 &= 2\lambda(1-\hat{\alpha})\hat{v}_\ell \\ 1 - 2\hat{\alpha}\hat{v}_\ell &= 2\lambda(1-\hat{\alpha})\hat{v}_h\end{aligned}$$

Moreover, the demand equals supply condition implies that

$$\hat{v}_k = \frac{\mu_k}{1-\hat{\alpha}+\mu_k}, \quad k = \ell, h$$

Substituting $\hat{v}_k$ in the first-order conditions we have two equations in two unknowns: $\hat{\alpha}$ and λ. For $\mu_\ell = 1$, $\mu_h = 2$, the optimal solution is

$$\hat{\alpha} \;=\; 0.6316, \quad (\hat{c}_\ell, \hat{v}_\ell) = (0.2692, 0.7308), \quad (\hat{c}_h, \hat{v}_h) = (0.3111, 0.8444)$$

The unconstrained optimal solutions at high and low demand are

$$\alpha_\ell^* = 0.6667,\; (c_\ell^*, v_\ell^*) = (0.25, 0.75), \qquad \alpha_h^* \; = 0.6,\; (c_h^*, v_h^*) = (0.3333, 0.8333)$$

Note the amplification of the surge in buyer prices and the reduction of the surge in seller prices under the constant-fee constraint, as established in Proposition 5. As the quantity sold equals the seller price in this uniform distribution example, it is easy to see that $\hat{c}_h < c_h^*$ implies that a less than optimal quantity of trades are completed during high demand. Similarly, compared to the unconstrained solution, too many trades are completed during low demand. □

The impact of a constant fee on efficiency is mixed. Under the constant-fee constraint, efficiency deteriorates (improves) in the demand condition with the lower (higher) unconstrained percent fee. To see this, observe that at the efficient outcome the intermediary's profit is zero: the efficient outcome with buyer mass μ_k, $k = \ell, h$ is $v_k^e = c_k^e$ obtained by the intersection of the diagonal in Figure 1 with the demand equals supply curve for μ_k. The unconstrained optimal prices are inefficient as $v_k^* > c_k^*$, i.e., (c_k^*, v_k^*) is above the diagonal. Any movement from (c_k^*, v_k^*) towards (away from) the diagonal along the demand equals supply curve increases (decreases) the gains from trade. In Figure 1, $\alpha_h^* < \alpha_\ell^*$, and thus $\alpha_h^* < \hat{\alpha} < \alpha_\ell^*$ by Proposition 4. Consequently, the constrained optimal prices $(\hat{c}_h, \hat{v}_h)$ are further away from the diagonal than (c_k^*, v_k^*) and the constant-fee constraint decreases efficiency in the high demand setting. Similarly, $\alpha_h^* < \alpha_\ell^*$ implies that efficiency increases in the low demand setting under the constant-fee constraint.

5 Concluding Remarks

The simple model of intermediation considered here shows that charging a constant fee reduces intermediary profits and, surprisingly, may also magnify the surge in buyer prices and attenuate the surge in seller prices during high demand periods. In the model, the intermediary does not hold inventory, which is also the case in several markets. Relaxing this assumption would reduce price swings, but would not change the main insight that a constant fee for the intermediary increases the surge in buyer price.

That the intermediary has market power in its interactions with buyers and sellers has implications for anti-trust policy. One's initial intuition might be that if an intermediary hires its sellers, an issue in pending litigation, then its marginal costs would decrease, thereby reducing the distortions of the intermediary-monopsonist. The comparison with a benchmark monopolist in Section 3 supports this view. However, this argument assumes that the total cost would remain unchanged. This may not be the case. For instance, many car drivers sell their services to both Uber and Lyft, switching between the two depending on who provides their closest next ride. This ability might be curtailed if they were to become employees of one of the two firms. Moreover, the intermediary's marginal costs might be reduced only if it bought the productive assets of the sellers along with their services; this would reduce the scale of operations of the intermediary due to capital constraints which in turn may reduce the value of the service to buyers. These considerations are also germane to anti-trust policy.

6 Appendix: Proofs

Proof of Proposition 1: First, I show that the necessary conditions for optimality, (1) and (3), are satisfied at exactly one set of prices.

If (c, v), (c', v'), satisfy (1) then $v < v'$ if and only if $c > c'$. That is, a demand equals supply curve in Figure 1 has strictly negative slope. To see this, note that $v < v'$ implies $F_b(v) < F_b(v')$, as f_b is strictly positive on its support. Thus, $F_s(c) = \mu(1 - F_b(v)) > \mu(1 - F_b(v')) = F_s(c')$. Hence, $c > c'$. Reversing this argument we have $c > c'$ implies $v < v'$.

It was argued earlier that the locus of points (c^*, v^*) that satisfy (3) has strictly positive slope. Thus, as the locus of points that satisfy (1) has strictly negative slope, there is a unique $(c^*(\mu), v^*(\mu))$ that satisfies the two necessary conditions for optimality: the demand equals supply condition (1) and the marginal revenue equals marginal cost condition (3).

As the intermediary's profit function is maximized on a compact domain $\{0 \leq c \leq 1,\ 0 \leq v \leq 1,\ v \geq c\}$, a maximum exists. Further, this maximum must occur in the interior of the domain because the intermediary profit is zero at any point on the boundary of the domain while it is strictly positive at any point in the interior. As the two necessary conditions are satisfied at this interior maximum, this maximum must occur at the unique $(c^*(\mu), v^*(\mu))$ that satisfies (1) and (3).

As the locus of points (v^*, c^*) that satisfy (3) has strictly positive slope, either (i) $\frac{dc^*(\mu)}{d\mu} > 0$ and $\frac{dv^*(\mu)}{d\mu} > 0$ or (ii) $\frac{dc^*(\mu)}{d\mu} < 0$ and $\frac{dv^*(\mu)}{d\mu} < 0$. To rule out (ii), it is enough to show that the locus of points satisfying (1) at $\mu + \Delta\mu$ is above the locus of points satisfying (1) at μ, $\Delta\mu > 0$. If $\mu(1 - F_b(v)) = F_s(c)$ then $(\mu + \Delta\mu)(1 - F_b(v)) > F_s(c)$. hence there exists a $\Delta v >$ such that $(\mu + \Delta\mu)(1 - F_b(v + \Delta v)) = F_s(c)$. Hence, if (c, v) satisfies (1) at μ then there exists $\Delta v > 0$ such that $(c, v + \Delta v)$ satisfies (1) at $\mu + \Delta\mu$. □

Proof of Proposition 2: Equation (3) may be written as

$$v^* \left[1 - \frac{1}{v^*} \frac{1 - F_b(v^*)}{f_b(v^*)}\right] = c^* \left[1 + \frac{1}{c^*} \frac{F_s(c^*)}{f_s(c^*)}\right]$$

$$\iff \quad v^*\left[1+\frac{1}{\eta_b(v^*)}\right] = c^*\left[1+\frac{1}{\eta_s(c^*)}\right]$$

$$\iff \quad \frac{1}{1-\alpha^*(\mu)} = \frac{v^*}{c^*} = \frac{1+\frac{1}{\eta_s(c^*)}}{1+\frac{1}{\eta_b(v^*)}}$$

Observe that $\alpha^*(\mu)$ decreases with μ if and only if $\frac{1}{1-\alpha^*(\mu)}$ decreases with μ. □

Proof of Proposition 3: We have

$$\frac{d\alpha^*}{d\mu} = \frac{d}{d\mu}\left[\frac{v^*-c^*}{v^*}\right] < 0$$

$$\iff \quad v^*\left[\frac{dv^*}{d\mu}-\frac{dc^*}{d\mu}\right]-\frac{dv^*}{d\mu}[v^*-c^*] = c^*\frac{dv^*}{d\mu}-v^*\frac{dc^*}{d\mu} < 0$$

$$\iff \quad \frac{c^*}{v^*}\frac{dv^*/d\mu}{dc^*/d\mu} < 1$$

$$\iff \quad \frac{c^*}{v^*}\frac{dv^*}{dc^*} < 1$$

$$\iff \quad \frac{dv^*}{dc^*} < \frac{v^*}{c^*}$$

As $c^* < v^*$, this inequality is satisfied if $\frac{dv^*}{d\mu} \leq \frac{dc^*}{d\mu}$. □

Proof of Lemma 1: As $\mu \to 0$, $v^*(\mu) \to v_0 > 0$, $c^*(\mu) \to 0$ and thus, $\alpha^*(\mu) \to 1$. Next, as $\mu \to \infty$, $v^*(\mu) \to 1$, $c^*(\mu) \to c_1$ and thus, $\alpha^*(\mu) \to 1 - c_1$.

Thus $1 = \alpha^*(0) > 1 - c_1 = \alpha^*(\infty)$. By continuity of the (c^*, v^*) curve, there exist $\underline{\mu} \leq \overline{\mu}$ for any $\mu_\ell < \underline{\mu}$ and any $\mu_h \geq \overline{\mu}$, we have $\alpha^*(\mu_h) < \alpha^*(\mu_\ell)$.[10] □

Proof of Proposition 4: Because the good cannot be stored, at any optimal solution the demand equals supply constraint must be met during both the high demand and the low demand periods; the argument is the same as in the unconstrained case. Thus, under the constant-fee constraint, only $((c_\ell, v_\ell), (c_h, v_h))$ that are at the intersections of a straight line from the origin and the two demand equals supply curves in Figure 1 need be considered.

Hence, if quantity q_k is sold at buyer mass μ_k then the buyer price is $v_k = F_b^{-1}(1-\frac{q_k}{\mu_k})$ and the seller price is $c_k = F_s^{-1}(q_k)$, $k = \ell, h$. The intermediary's expected profit

[10] Note that there are many selections of $\underline{\mu} \leq \overline{\mu}$ for which the lemma holds.

under the constant-fee constraint is

$$\Pi_I(q_h, q_\ell) \;=\; r\, q_h\Big[F_b^{-1}(1-\frac{q_h}{\mu_h}) - F_s^{-1}(q_h)\Big] + (1-r)\, q_\ell\Big[F_b^{-1}(1-\frac{q_\ell}{\mu_\ell}) - F_s^{-1}(q_\ell)\Big]$$

s.t.

$$F_s^{-1}(q_\ell)F_b^{-1}(1-\frac{q_h}{\mu_h}) \;=\; F_s^{-1}(q_h)F_b^{-1}(1-\frac{q_\ell}{\mu_\ell})$$

The profit may be written as the integral of its derivative obtained in (2)

$$r\int_{v_h}^{1}\int_{0}^{c_h}\Big(v-\frac{1-F_b(v)}{f_b(v)}-[c+\frac{F_s(c)}{f_s(c)}]\Big)f_s(c)f_b(v)\,dc\,dv$$
$$+(1-r)\int_{v_\ell}^{1}\int_{0}^{c_\ell}\Big(v-\frac{1-F_b(v)}{f_b(v)}-[c+\frac{F_s(c)}{f_s(c)}]\Big)f_s(c)f_b(v)\,dc\,dv$$

subject to the constant-fee constraint.

The argument below is followed in Figure 1. Note that if $\hat\alpha$ is the optimal fee, then the optimal prices $(\hat c_k, \hat v_k)$ are at the intersection of a straight line through the origin with slope $\frac{1}{1-\hat\alpha}$ and the demand equals supply curve for μ_k.

If $\hat\alpha < \min\{\alpha_\ell^*, \alpha_h^*\}$ then, in Figure 1, the straight line through the origin with slope $\frac{1}{1-\hat\alpha}$ is less steep than each of the two straight lines from the origin to (c_k^*, v_k^*), $k=\ell, h$.[11] Each $(\hat c_k, \hat v_k)$ lies below the 'MR=MC' curve of Figure 1, the region where MR$= v-\frac{1-F_b(v)}{f_b(v)} < c+\frac{F_s(c)}{f_s(c)}$ =MC. Thus, selling a little less by decreasing c_k and increasing v_k slightly, while maintaining the constant-fee constraint and the demand equals supply condition, will increase profit in each of the two states ℓ and h and thereby increase $\Pi_I(q_h, q_\ell)$. This contradicts the optimality of $\hat\alpha$.

Similarly, if $\hat\alpha > \min\{\alpha_\ell^*, \alpha_h^*\}$ then each $(\hat c_k, \hat v_k)$ is above the 'MR=MC' curve of Figure 1, the region where MR>MC. Selling a little more will increase profit in each the two states.

Thus, (6) must hold.

Suppose that $\alpha_\ell^* \neq \alpha_h^*$ and that it is optimal to set $\hat\alpha = \alpha_\ell^*$. Therefore, $(\hat c_\ell, \hat v_\ell) = (c_\ell^*, v_\ell^*)$. Thus, at low demand the prices are unconstrained optimal but not at high demand (as $\alpha_\ell^* \neq \alpha_h^*$). Consequently, marginal revenue equals marginal cost at low demand but not at high demand. Because marginal revenue and marginal cost are

[11]These lines have slope $\frac{1}{1-\alpha_k^*}$, $k=\ell, h$. Further, $\frac{1}{1-\hat\alpha_k^*} > \frac{1}{1-\hat\alpha}$ if and only if $\alpha_k^* > \hat\alpha$.

continuous functions, the intermediary's profits are greater if the constant percent fee is set at $\hat{\alpha}+\epsilon$ rather than at $\hat{\alpha}$, where $|\epsilon|$ is small and $\epsilon > 0$ if $\alpha_h^* > \alpha_\ell^*$ and $\epsilon < 0$ if $\alpha_h^* < \alpha_\ell^*$. This contradicts the assumption that $\hat{\alpha}$ is optimal. An identical argument implies that $\hat{\alpha} \neq \alpha_h^*$. Thus, each of the two inequalities in (6) must be strict when $\alpha_\ell^* \neq \alpha_h^*$.

To prove that an optimal fee and prices exist first note that for each α there exist a unique set of prices $(c_k(\alpha), v_k(\alpha))$ that satisfies the demand-supply constraint:

$$\mu_k(1-F_b(v_k)) \;=\; F_s(c_k) \;= F_s((1-\alpha)v_k)$$

As v_k is increased from 0, the left-hand side decreases from μ_k and the right-hand side increases from 0, with equality at a unique point $(c_k(\alpha), v_k(\alpha))$ where $c_k(\alpha) = (1-\alpha)v_k(\alpha)$. Hence, the profit function may be written as a function of α:

$$\begin{aligned}\Pi_I(\alpha) \;=\; & r\int_{v_h(\alpha)}^{1}\int_0^{c_h(\alpha)}\Big(v-\frac{1-F_b(v)}{f_b(v)}-[c+\frac{F_s(c)}{f_s(c)}]\Big)f_s(c)f_b(v)\,dc\,dv \\ & +(1-r)\int_{v_\ell(\alpha)}^{1}\int_0^{c_\ell(\alpha)}\Big(v-\frac{1-F_b(v)}{f_b(v)}-[c+\frac{F_s(c)}{f_s(c)}]\Big)f_s(c)f_b(v)\,dc\,dv\end{aligned}$$

where the constant-fee constraint is satisfied as $c_k(\alpha) = (1-\alpha)v_k(\alpha)$. The domain for continuous function $\Pi_I(\alpha)$ is a compact set $[0,1]$.[12] Hence, there exists an $\hat{\alpha}$ at which $\Pi_I(\alpha)$ is maximized. □

Proof of Proposition 5: As $\alpha_\ell^* > \alpha_h^*$, Proposition 4 implies that $\hat{\alpha}$, the constrained optimal share for the intermediary, satisfies $\alpha_h^* \leq \hat{\alpha} \leq \alpha_\ell^*$. Thus, the constrained optimal prices satisfy

$$\frac{1}{1-\alpha_h^*} = \frac{v_h^*}{c_h^*} \;\leq\; \frac{\hat{v}_h}{\hat{c}_h} = \frac{1}{1-\hat{\alpha}} = \frac{\hat{v}_\ell}{\hat{c}_\ell} \;\leq\; \frac{v_\ell^*}{c_\ell^*} = \frac{1}{1-\alpha_\ell^*}$$

That is, $(\hat{c}_\ell, \hat{v}_\ell)$ is below the 'MR=MC' curve (and on the demand equals supply line for μ_ℓ) and, similarly, $(\hat{c}_h, \hat{v}_h)$ is above 'MR=MC' curve; this can be seen in Figure 1 where $\alpha_\ell^* > \alpha_h^*$. As the demand equals supply curve is negatively sloped (see proof of Proposition 1), it follows immediately that $\hat{v}_h > v_h^*$ and $v_\ell^* > \hat{v}_\ell$. That $v_h^* > v_\ell^*$ follows from Proposition 1.

[12]In fact, the search for an optimal α may be restricted to the compact set $[\min\{\alpha_\ell^*, \alpha_h^*\}, \max\{\alpha_\ell^*, \alpha_h^*\}]$.

The same argument implies that $c_h^* > \hat{c}_h$ and $\hat{c}_\ell > c_\ell^*$. That $\hat{c}_h > \hat{c}_\ell$ follows from (5). □

Proof of Lemma 2: Noting that $v_* = F_b^{-1}(1 - \frac{q_*}{\mu_*})$ and $c_* = F_s^{-1}(q_*)$, the intermediary's expected profit under the constant share constraint is

$$\Pi_I(q_h, q_\ell) \;=\; r\, q_h\Big[F_b^{-1}(1 - \frac{q_h}{\mu_h}) - F_s^{-1}(q_h)\Big] + (1-r)\, q_\ell\Big[F_b^{-1}(1 - \frac{q_\ell}{\mu_\ell}) - F_s^{-1}(q_\ell)\Big]$$

s.t.

$$F_s^{-1}(q_\ell)F_b^{-1}(1 - \frac{q_h}{\mu_h}) \;=\; F_s^{-1}(q_h)F_b^{-1}(1 - \frac{q_\ell}{\mu_\ell})$$

The Lagrangian for the intermediary profit-maximization problem is

$$\begin{aligned} Ł_I(q_h, q_\ell, \lambda) \;=\; & r\, q_h\Big[F_b^{-1}(1 - \frac{q_h}{\mu_h}) - F_s^{-1}(q_h)\Big] + (1-r)\, q_\ell\Big[F_b^{-1}(1 - \frac{q_\ell}{\mu_\ell}) - F_s^{-1}(q_\ell)\Big] \\ & +\lambda\Big[F_s^{-1}(q_\ell)F_b^{-1}(1 - \frac{q_h}{\mu_h}) - F_s^{-1}(q_h)F_b^{-1}(1 - \frac{q_\ell}{\mu_\ell})\Big] \end{aligned}$$

$$\begin{aligned} \Longrightarrow \qquad \frac{\partial Ł_I}{\partial q_h} \;=\; & r\Big[F_b^{-1}(1 - \frac{q_h}{\mu_h}) - F_s^{-1}(q_h)\Big] + r q_h \frac{dF_b^{-1}(1 - \frac{q_h}{\mu_h})}{dq_h} - r q_h \frac{dF_s^{-1}(q_h)}{dq_h} \\ & +\lambda\Big[F_s^{-1}(q_\ell)\frac{dF_b^{-1}(1 - \frac{q_h}{\mu_h})}{dq_h} - F_b^{-1}(1 - \frac{q_\ell}{\mu_\ell})\frac{dF_s^{-1}(q_h)}{dq_h}\Big] \\ =\; & r\Big[F_b^{-1}(1 - \frac{q_h}{\mu_h}) - F_s^{-1}(q_h)\Big] - r\frac{q_h}{\mu f_b(F_b^{-1}(1 - \frac{q_h}{\mu_h}))} - r\frac{q_h}{f_s(F_s^{-1}(q_h))} \\ & -\lambda F_s^{-1}(q_\ell)\frac{q_h}{\mu f_b(F_b^{-1}(1 - \frac{q_h}{\mu_h}))} - \lambda F_b^{-1}(1 - \frac{q_\ell}{\mu_\ell})\frac{q_h}{f_s(F_s^{-1}(q_h))} \\ =\; & r(v_h - c_h) - (r + \lambda c_\ell)\frac{1 - F_b(v_h)}{f_b(v_h)} - (r + \lambda v_\ell)\frac{F_s(c_h)}{f_s(c_h)} \end{aligned}$$

As $\frac{\partial Ł_I}{\partial q_h} = 0$ at optimal prices $(\hat{c}_\ell, \hat{v}_\ell)$ and $(\hat{c}_h, \hat{v}_h)$, we have

$$\hat{v}_h - \frac{1 - F_b(\hat{v}_h)}{f_b(\hat{v}_h)} - \frac{\lambda \hat{c}_\ell}{r}\frac{1 - F_b(\hat{v}_h)}{f_b(\hat{v}_h)} \;=\; \hat{c}_h + \frac{F_s(\hat{c}_h)}{f_s(\hat{c}_h)} + \frac{\lambda \hat{v}_\ell}{r}\frac{F_s(\hat{c}_h)}{f_s(\hat{c}_h)} \qquad (9)$$

Similarly, $\frac{\partial Ł_I}{\partial q_\ell} = 0$ at optimal prices $(\hat{c}_\ell, \hat{v}_\ell)$ and $(\hat{c}_h, \hat{v}_h)$ implies

$$\Longrightarrow \qquad \hat{v}_\ell - \frac{1 - F_b(\hat{v}_\ell)}{f_b(\hat{v}_\ell)} + \frac{\lambda \hat{c}_h}{1-r}\frac{1 - F_b(\hat{v}_\ell)}{f_b(\hat{v}_\ell)} \;=\; \hat{c}_\ell + \frac{F_s(\hat{c}_\ell)}{f_s(\hat{c}_\ell)} - \frac{\lambda \hat{v}_h}{1-r}\frac{F_s(\hat{c}_\ell)}{f_s(\hat{c}_\ell)} \qquad (10)$$

Equations (9) and (10) imply (7) and (8). □

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