Transparency and Delay in Bargaining*

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Abstract

This paper studies the Rubinstein bargaining game, in which both agents have reservation values. Agents are uncertain whether their opponents have high or low reservation values. Each agent tries to convince the other that he has a high reservation value, resulting in a unique war of attrition, as in the reputation literature. I analyze the information sensitivity of delay when agents publicly observe some noisy signal about their opponents' reservation values. A bargaining environment is said to be more transparent if the available information is more precise. I show that information disclosure *increases* delay, in the sense of first-order stochastic dominance, if transparency is not sufficiently high. Suppose, a mediator controls this transparency. Although full transparency is efficient, a bargaining environment is not likely to be fully transparent. I characterize the optimal transparency when a bargaining environment can be made transparent only to a limited extent. Also, given any transparency, I show that a mediator can strictly improve efficiency by disclosing information about an agent iff agents have sufficiently close bargaining strengths.

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1 Introduction

One of the most basic questions in economic theory is : How will two agents divide an economic surplus? In his seminal work, Rubinstein (1982) shows that when agents have complete information about the bargaining environment, they immediately reach an agreement. The exact terms of this agreement is unique and depends on how agents discount delay. However, when agents have some private information about themselves, they try to exploit this and convince their opponent that they should get a higher share of the surplus. This induces a war of attrition (WOA) and causes delay. Suppose that agents have some noisy information about their opponents and that such information is public.[1] A bargaining environment is more transparent if the information is more precise. Will disclosing this somewhat accurate information reduce delay? Contrary to popular expectations, this paper will show that disclosing information will increase delay, in the sense of first-order stochastic dominance, if the bargaining environment is not sufficiently transparent. This brings to mind the old saying - ‘A little learning is a dangerous thing’ (Alexander Pope: “An Essay on Criticism”, 1709).

In this paper, I will consider the Rubinstein (1982) bargaining game, which is the benchmark for non-cooperative bargaining theory. Two agents decide how to split a surplus by making offers and counteroffers alternately until they agree. Time is valuable and offers can be made frequently. Furthermore, each agent has some reservation value and will accept nothing below this value. For example, a buyer is bargaining with a seller over the price of a house. The seller’s valuation is 5, while the buyer’s valuation is 10 (in million dollar units, say). Thus, transferring the property will generate a surplus of 5. This is common knowledge, and agents decide how to split this surplus. Suppose that the seller has a neighbor who has sold a similar house for 6.5. The seller believes that his own place is slightly better, and so he will not sell the house for anything below 7. Thus, the seller has a reservation value of 2 while dividing a surplus of 5. Similarly, the buyer may have a friend who has bought a house last year somewhere nearby for 8.5. But the buyer thinks that this neighborhood is slightly better and so he is willing pay a little more but not more than 9 - i.e., he sets a reservation value of 1. If these reservation values are commonly known and compatible - i.e., the surplus can be divided in a way that will be agreeable to both agents - as Binmore, Shaked and

[1] Thus there is no higher-order uncertainty. Feinberg and Skrzypacz (2005) consider a seller who may or may not know the buyer’s valuation. Basak (2015) considers a simple model in which both the buyer and seller do not know what their opponents think of them.

Sutton (1989) show, that there is no delay, and agents get their reservation values if it is binding; otherwise, they get their Rubinstein shares.

Suppose that agents are uncertain about their opponents' reservation values. Nature draws a high or low reservation value for each agent independently from a common prior, and agents know only their own reservation values. Restricting to only two types, makes the information sensitivity analysis tractable. However, this is not without loss of generality.[2] I show that this results in a unique war of attrition (WOA). The high reservation type always insists on his high reservation share, while the low type randomizes between insisting and conceding to his opponent's high demand. Concession by either agent stops the game, and in the absence of concession, both agents become more convinced that their opponent has a high reservation value. In equilibrium, the beliefs evolve along a particular balanced path. Given the initial belief, whichever agent takes longer to convince his opponent that he is the high type has less-than-balanced *bargaining strength*. This agent is called the weak agent, and the other is the strong agent. The weak agent will concede immediately with positive probability such that, in the absence of no immediate concession, the updated belief reaches the balanced path. There is a time T when both agents become absolutely convinced that their opponent has high reservation value. Reputation building has been studied extensively in economic theory.[3] In the context of reputational bargaining, Abreu and Gul (2000) (henceforth AG) show that when offers can be made frequently, a unique WOA emerges. They assume that agents can be of certain behavioral types that always insist on a particular share of the surplus. Unlike the reputation literature, this paper does not assume behavioral insistent types. The agents are fully rational and, thus, could be threatened by beliefs in equilibrium. Nonetheless, I show that insistence is the unique rationalizable strategy for an agent with high reservation value. This establishes a link between the incomplete information and reputation bargaining literatures.

Agents do not know their opponents' reservation values. However, they are likely to have noisy information regarding what their opponents are comparing the house with and how much better or worse it is. In a more transparent bargaining environment, agents are more likely to get the right signals. Think of a fictitious mediator who controls this transparency. A mediator is a neutral third party appointed to facilitate a resolution to the conflict. In many bargaining environments the

[2]More types will start a step game, but uniqueness is not guaranteed without additional assumptions.

[3] The reputation literature started with Milgrom and Roberts (1982) and Kreps and Wilson (1982). We can see similar WOA in Kreps and Wilson (1982), when an incumbent and an entrant with multiple entry opportunities engage in a war to build their reputation.

mediator can be actual rather than fictional.[4] The mediator investigates each agent for the actual reservation value and publicly conveys the result to the other side. Because investigation is not fully accurate, it generates noisy public information. The mediator commits to the investigation with some known accuracy and cannot influence the result of her investigation; thus, she works as an information designer, [5] as defined in Kamenica and Gentzkow (2011). So transparency of a bargaining environment is equivalent to accuracy of the investigation. There is no delay in a fully transparent bargaining environment. However, a bargaining environment is usually not fully transparent and can be made transparent only to a limited extent. We are left with the question : Given any upper bound on transparency, what is the optimal choice of transparency that will make the bargaining outcome most efficient?

Suppose that information is available only about the a priori strong agent. I show that there is a threshold level of transparency such that : if the upper bound is below this threshold, a completely uninformative bargaining environment is optimal, and beyond this threshold, a maximum transparent bargaining environment is optimal. If there is public information about the a priori weak agent as well, then it can both enhance and decrease efficiency. Given the transparency on the a priori weak side, there are two thresholds of transparency on the a priori strong side (say, i-side). Below the lower threshold L, a completely uninformative signal on i is optimal, and beyond the upper threshold H, maximum transparency on the i-side is optimal. Unlike the case in which there is information leakage only on one side, these two thresholds are not necessarily the same. If these two thresholds are different, then there is a middle threshold M. When the upper bound is between L and M, the optimal transparency is maximum transparency, and when the upper bound is between M and H, the optimal transparency is the middle threshold transparency.

We see that appointing a mediator does not necessarily improve efficiency. So, the natural question is : Given the accuracy of the investigation, when should a mediator be appointed? Or, when does disclosing information about an agent reduce delay? I show that disclosing information about an agent strictly increases efficiency if and only if agents are sufficiently close in their bargaining strengths. If agents have close bargaining strength, then no agent gives up easily, and without any

[4] In the corporate world, a mediator is often hired to resolve conflicts between employees or departments quickly, so that agents can go back to their daily business. When a trade union is negotiating with management over its share of surplus, or an environmentalist group is negotiating with developers, a local authority may step in as a mediator.

[5] As opposed to a conventional mechanism designer. Hörner, Morelli and Squintani (2015) defines a mediator in a peace negotiation as a mechanism designer who can make only unenforceable recommendations. See the discussion section for details.

information disclosure, the negotiation may drag on for a long time.

What drives the above results is ingrained in how agents build their reputations, starting from the initial beliefs, and how transparency influences the distribution over the initial beliefs. After a high (low) signal is realized, the posterior belief that the agent is a high type increases (decreases). The mediator induces a distribution over the posterior beliefs. More accurate investigation or a more transparent bargaining environment induces more dispersed posterior beliefs. The posterior beliefs must satisfy *Bayesian consistency*.

After the signal realization, agents bargain to build reputation, starting with the posterior beliefs. In the WOA that follows, the strong agent determines how long it takes to build a reputation. Let us call this T. Accordingly, the weak agent concedes immediately with positive probability - say, μ. As bargaining continues, agents concede at a particular rate that makes the low types indifferent between conceding and insisting. These concession rates are independent of the transparency of the environment. The initial beliefs affect (1) the probability that no agreement will ever be reached; (2) how long it will take to build the reputation - T; and (3) the probability of immediate concession - μ. When agents have balanced strengths, no one gives up immediately - i.e., $\mu = 0$. If the initial belief that agent j is a high type decreases, or the initial belief that agent i is a high type increases, the bargaining strengths of the two agents diverge. However, these two changes affect delay quite differently. Suppose that agent j is less likely to be a high type; then, j has less than balanced strength. This linearly decreases the probability that agreement will never be reached. This will not affect T, but will linearly increase μ. All of these together increase efficiency in an affine way. Suppose that agent i becomes more likely to be a high type; then, i has more than balanced strength. This will linearly increase the probability that agreement will never be reached and convexly reduce T and concavely increase μ. All of these together will affect efficiency in a concave way.

To see what this affine-concave relation implies, suppose that information is disclosed about an a priori strong agent. Further, suppose that the transparency is not sufficiently high, so that the strong agent remains strong no matter what signal is realized. Since, information disclosure makes the posterior beliefs more dispersed and efficiency is concave in the posterior beliefs about the strong agent, the outcome is less efficient.

2 The Bargaining Game

$N = \{1, 2\}$ denotes the set of agents. These two agents bargain over splitting a fixed surplus 1. For interchangeability of the agents' identity, I will refer to the agents as i and j and $i \neq j$. Nature picks a state $\omega \in \mathcal{W} = \mathcal{W}_1 \times \mathcal{W}_2$, which specifies a pair of *reservation values* (ω_1, ω_2). Agents share a common prior $\pi \in \mathbb{P}(\mathcal{W})$, where $\mathbb{P}(.)$ denotes the set of probability distributions. In this paper, I will assume that the reservation values are drawn independently. A public signal $\tilde{\omega} = (\tilde{\omega}_i, \tilde{\omega}_j) \in \tilde{\mathcal{W}}$ is realized. I will assume that $\tilde{\mathcal{W}} = \mathcal{W}$. The public signals on two agents are conditionally independent. The public signal on agent i is denoted by the collection of conditional probability distributions over the signal $\tilde{\omega}_i$ given the state ω_i: $\{q_i(.|\omega)\}_{\omega \in \mathcal{W}}$, $i \in N$. The information structure is commonly known. Based on the signal that agent i gets, she forms her belief and higher-order beliefs, and this is captured in the type space Θ_i, $i \in N$. $\Theta = \Theta_1 \times \Theta_2$. Since this paper considers public information only, there is no higher-order uncertainty. So, $\theta_i = (\omega_i, \tilde{\omega}_j)$ - i.e., agent i of type θ_i has a reservation value ω_i and a public signal $\tilde{\omega}_j$ about his opponent's reservation value.

I will assume that one of the agents is picked randomly to make the first offer, and then an alternate offer bargaining protocol is followed. I will assume that the period length is Δ and I am interested in the limiting result when $\Delta \to 0$ - i.e., when offers can be made frequently. Let the action space at any time period t when agent i gets the chance to make a decision be A_{it}. $A_{i0} = [0, 1]$ and an agent i taking an action $x_i \in [0, 1]$ means that he demands x_i share of the surplus or offers $(1 - x_i)$ share to agent j. $A_{it} = [0, 1] \cup \{\mathcal{A}\}$ for any period $t > 0$. $\mathcal{A}$ stands for agreement, which means that agent i takes agent j's offer on the table. Thus, accepting gives an immediate payoff, while counteroffer leads to delay. For notational convenience, I write the action space for agent i as A_i, ignoring the fact that the action space is different at time 0. Suppose that no agent agrees until time period t and the history is $H^t = [0, 1]^t$. Let $H^0 = \emptyset$ be the initial history. A pure strategy assigns an action to any possible history. Let $S = S_1 \times S_2$ denote the set of pure strategy. A behavioral strategy of agent i is denoted by $\sigma_i : H \to \mathbb{P}(A_{it})$. Let Σ_i be the set of behavioral strategies and $\Sigma = \Sigma_1 \times \Sigma_2$. $\sigma_i(h)(\theta_i, \tilde{A}_i)$ denotes the probability that θ_i will play an action in $\tilde{A}_i$ after history h.

Let $\beta_j[\theta_i]$ be agent i of type θ_i's belief over states, agent j's type and action at any history. $\beta_j[\theta_i] : H \to \mathbb{P}(\mathcal{W} \times \Theta_j \times A_j)$. The marginal distribution $\alpha_j[\theta_i](h)(\tilde{\Theta}_j) = \beta_j[\theta_i](h)(\mathcal{W} \times \tilde{\Theta}_j \times A_j)$ for any $\tilde{\Theta}_j \subseteq \Theta_j$ denotes agent i of type θ_i's belief over agent j's type in $\tilde{\Theta}_j$, conditional on history

h. The marginal distribution $\hat{\sigma}_j[\theta_i](h)(\tilde{A}_j) = \beta_j[\theta_i](h)(\mathcal{W} \times \Theta_j \times \tilde{A}_j)$ for any $\tilde{A}_j \subseteq A_j$ denotes agent i of type θ_i's belief that j will take an action in $\tilde{A}_j$, conditional on history h. $\hat{\sigma}_j[\theta_i](h)(\theta_j, \tilde{A}_j) = \beta_j[\theta_i](h)(\mathcal{W} \times \{\theta_j\} \times \tilde{A}_j)$ denotes agent i of type θ_i's belief that θ_j will take an action in $\tilde{A}_j$, conditional on history h. If σ is played in equilibrium then $\hat{\sigma}_j[\theta_i](h)(\theta_j, \tilde{A}_j) = \sigma_j(h)(\theta_j, \tilde{A}_j)$. Note that when the available information is public, conditional on the signal realization $\tilde{\omega}$, different types vary only in terms of their reservation values and not their beliefs.

Let $\mathcal{C} = \mathcal{C}_1 \times \mathcal{C}_2$ be the space of consequence, where $(x_i, t) \in \mathcal{C}_i = [0,1] \times [0, \infty]$ means that the agent gets x_i share of the surplus after bargaining for time t. $t = \infty$ corresponds to no agreement. The payoff function for agent $i \in N$ is given by $u_i : \mathcal{C} \times \mathcal{W}_i \to R$, where $u_i(x_i, t, \omega_i) = e^{-r_i t} x_i$ if $x_i \geq \omega_i$ and $u_i(x_i, t, \omega_i) = -\epsilon$, $(\epsilon > 0)$ if $x_i < \omega_i$. Agent i discounts the share x_i that he gets in the future at a stationary rate r_i if the share is above his reservation value. Anything below the reservation value is not acceptable.[6] The reservation values ω_i, ω_j are said to be *compatible* if $\omega_i + \omega_j \leq 1$. I will assume that agents are expected utility maximizers. Suppose that agent i of type θ_i has reservation value ω_i. Let $\phi[\theta_i](\sigma)$ be his belief over consequence when the strategy profile is σ. The expected payoff of θ_i is $U_i(\sigma, \theta_i) = \int u_i(x_i, t, \omega_i) d\phi[\theta_i](\sigma)$. Let $U_i(\sigma, \theta_i | h)$ be the expected payoff of θ_i after history h, where σ describes the play thereafter.

Conditional Dominance

Let $\tilde{R} = \tilde{R}_1 \times \tilde{R}_2$, where $\tilde{R}_i \in \Sigma_i$, $i = 1, 2$ is a non-empty set of the strategy profile. Strategy σ_i is said to be conditionally dominated (CD) for agent i of type θ_i with respect to $\tilde{R}$ if $\exists\ \sigma_i' \in \Sigma_i$ such that $U_i(\sigma_i', \sigma_j, \theta_i | h) \geq U_i(\sigma_i, \sigma_j, \theta_i | h)$, $\forall\ \sigma_j \in R_j$ and $\forall\ h \in H$ and there exists some $h \in H$ such that $U_i(\sigma_i', \sigma_j, \theta_i | h) > U_i(\sigma_i, \sigma_j, \theta_i | h)$. A strategy is called *undominated* if it is not CD. Let us define $UD_i(\tilde{R}, \tilde{\Theta}) := \{\sigma_i \in \tilde{R}_i | \sigma_i$ is undominated w.r.t. R for some $\theta_i \in \tilde{\Theta}_i\}$. Let $UD(\tilde{R}, \tilde{\Theta}) = UD_1(\tilde{R}, \tilde{\Theta}) \times UD_2(\tilde{R}, \tilde{\Theta})$. Let $R = R_1 \times R_2$ be such that $UD(R, \Theta) = R$.

Complete Information

In the standard Rubinstein game, there are no reservation values. Agents agree immediately to unique terms of trade that give agent $i \in N$, $v_i^R = \frac{r_j}{r_i + r_j}$. Note that if agents discount delay equally,

[6] This is not the only way one can model reservation value. Reservation value can be interpreted as the status quo preference or the no-agreement payoff, which does not generate flow payoff. See the discussion section for alternative interpretations of reservation value.

they will divide the surplus equally. As offers can be made frequently, there is no first mover advantage. Suppose that each agent has some reservation value and will accept nothing below this value. The following proposition uniquely identifies the equilibrium in which there is complete information about the reservation values. If the reservation values are compatible, then agreement is reached immediately, and if they are not compatible, then agreement is never reached. Thus, the outcome is efficient.

Proposition 1 *(Binmore, Shaked and Sutton (1989)) When the reservation values ω are commonly known and are compatible, there exists a unique SPE. If for any $i \in N$, $\omega_i \geq v_i^R$, then agents immediately agree to a settlement that gives ω_i to agent i and $1 - \omega_i$ to agent j. If for all $i \in N$, $\omega_i \leq v_i^R$, then agents immediately agree to a settlement that gives v_i^R to any agent i.*

In a world with incomplete information, we see agents trying to convince each other that they should get a higher share of the surplus. They build their reputation by rejecting opponents' offers and, thus, cause delay. A more transparent bargaining environment has a distribution over the posterior beliefs that are more dispersed but the distribution and the posterior beliefs must satisfy Bayes consistency. After the signal realization, agents bargain to build reputation starting with the posterior beliefs. To analyze the relation between transparency and delay, I organize the paper in two parts. In the following section, I analyze the role of initial beliefs in this reputation formation. In the subsequent sections, I use this ex-post effect of posterior beliefs to characterize the role of transparency in the ex-ante distribution of delay.

3 Independent and Privately known Reservation Values

Suppose that any agent $j \in N$ can have either a high or a low reservation value : $\mathcal{W}_j := \{\bar{\omega}_j, \underline{\omega}_j\}$. Nature selects this reservation value for each type independently from a commonly known prior $\pi \in \mathbb{P}(\mathcal{W}_i) \times \mathbb{P}(\mathcal{W}_j)$. I will assume the reservation values satisfy the following :

Assumption 1 *$\bar{\omega}_j > v_j^R > \underline{\omega}_j$ for all $j \in N$ and for any $i, j \in N$, $\bar{\omega}_i + \underline{\omega}_j < 1$.*

The above assumption says that both agents have two reservation values. The high reservation value is binding (above Rubinstein share), while the low reservation value is not binding (below Rubinstein share). As we can see in Proposition 1, the reservation value influences the bargaining outcome under complete information only when the reservation value is binding. So the above assumption says that both agents have some reservation values that will influence the bargaining outcome under complete information. Thus, if both agents have high reservation values, then there is no feasible way to divide the surplus that will be agreeable to both agents. I am assuming that all other reservation values are compatible.

Lemma 1 *Any strategy in the set of undominated strategies R must satisfy the following two properties:* 1) *Agent i will never offer more than $\bar{\omega}_j$ to agent j; and* 2) *agent j will accept any offer above $\bar{\omega}_j$ with probability* 1 *and will never accept an offer below $1-\bar{\omega}_i$ with positive probability.*

Under complete information, we get a unique threshold below which an agent never accepts with positive probability and above which he accepts with probability 1. However, under incomplete information, we get only a bound. The upper bound is the best complete information scenario, and the lower bound is the worst complete information scenario for agent i. At any history for any belief, an agent accepts any offer with probability 1 above this upper threshold and never accepts an offer with positive probability below this lower threshold. Watson (1998) considers incomplete information about agents' discount rates and provide a similar result. Although the nature of incomplete information is different, the basic intuition remains the same.

Recall Myerson's notion of k insistent strategy for an agent. An agent is said to be k-insistent if he always demands k share of the surplus and rejects any offer that does not give him at least k. Let us formally define a k-insistent strategy as follows :

$$I_i(k) := \{\sigma_i \in R_i : \forall\ h \in H\ \sigma_i((h, x_j)(\mathcal{A}) = 1 \text{ for } x_j \leq 1-k \\ \text{and } \sigma_i((h, x_j))(\mathcal{A}) = 0 \text{ for } x_j > 1-k\}.$$

Note that the above insistent strategy does not specify what agent i demands when he rejects an offer - i.e., $\sigma_i((h, x_j))(\mathcal{A}) = 0$. Lemma 1 says any undominated strategy for the high type of agent i is to accepts any offer above $\bar{\omega}_i$ with probability 1. Also, by definition, he never accepts an offer below $\bar{\omega}_i$ with positive probability. Therefore, $I_i(\bar{\omega}_i)$ is the unique rationalizable strategy for the high type of agent i, for any $i \in N$. Unlike the reputation literature, here, this insistence is not a behavioral assumption. The high types are fully rational and Bayesian optimizers and, thus, can be threatened by belief. Nonetheless, it is the unique rationalizable strategy for them to insist. I will refer to type $\bar{\omega}_j$ as the $\bar{\omega}_j$ insistent type. So, if agent i demands less than $\bar{\omega}_i$ at some history h, it is revealed that $\theta_i \neq \bar{\omega}_i$. Following Myerson (1991), (when offers can be made frequently), we get the following Coasian result :

Lemma 2 *In any sequential equilibrium of the subgame where agent i has revealed that he is not the high type, but agent j has not done so, agent j gets $\bar{\omega}_j$ and i gets $1 - \bar{\omega}_j$.*

The above lemma is more generally referred to as Coase conjecture in the literature. Under one-sided incomplete information, agreement is reached immediately and the informed agent takes away all the additional surplus. Note that although both agents are making offers, the informed agent's action is essentially an accept or reject decision. Acceptance stops the game, while rejection means insistence. Only insisting on the high reservation value after rejecting an offer can be thought of as a serious demand. Lemma 1 shows that any higher demand has no positive chance of getting accepted. I am implicitly assuming that such non-serious offers do not affect opponent's belief in any favorable way. As long as agent i assigns positive probability (however small) that agent j is a high type, the game cannot continue forever. If it is known that the game is going to end in the next ϵ time then any θ_j would insist like $\bar{\omega}_j$. So, i would have preferred to concede now rather than wait for ϵ more time. This gives us the above result.

So, $\theta_i = \underline{\omega}_i$ can either insist like $\theta_i = \bar{\omega}_i$ or reveal his type. Insistence will give $\bar{\omega}_i$ to agent i of type $\theta_i = \underline{\omega}_i$ after some time t, while non-insistence will give her $1 - \bar{\omega}_j$. So, an outside observer can see that if an agreement is reached after some time $t \geq 0$, then it either gives (1) $\bar{w}_1$ to agent 1 and $1 - \bar{w}_1$ to agent 2 or (2) $\bar{w}_2$ to agent 2 and $1 - \bar{w}_2$ to agent 1. Thus, the incomplete information bargaining problem boils down the reputation formation, as in AG. AG assumes that agent i has irrational types who insist on a particular share of the surplus. The difference is that this is not an irrational type; rather, the unique rationalizable strategy for the high reservation value type is to insist on $\bar{\omega}_i$ like the irrational type.

AG show that the outcome of the discrete time bargaining problem converges in distribution to the continuous time unique war of attrition equilibrium. Let us define

$$F_i[\theta_i](t) := P(\{\sigma_i(h) \in I_i(\bar{\omega}_i) \text{ for any } h \text{ until time } t,\ \sigma_i(h) \notin I_i(\bar{\omega}_i) \text{ for any } h \text{ after time } t\})$$

$F_i[\theta_i](t)$ denotes the probability that agent i of type θ_i will insist until time t before she concedes.

We have $F_i[\bar{\omega}_i](t) = 0$ for all $t \geq 0$ - i.e., $\theta_i = \bar{\omega}_i$ never concedes. Let $T_i[\theta_i] := \{t | F_i[\theta_i](t) = \lim_{t\to\infty} F_i[\theta_i](t)\}$ be the time by which θ_i must have conceded. So, $T_i[\bar{\omega}_i] = \infty$. Let us define $T_i := \max_{T_i[\theta_i]<\infty} T_i[\theta_i]$. Therefore, after time T_i, it is revealed that agent i will never concede. With only two types, $T_i = T_i[\underline{\omega}_i]$. It takes agent i T_i time to build his reputation for being the high type. Since, in equilibrium, there is only one type of history until there is an agreement, time t is a sufficient statistic for history h. So, I will abuse notation by replacing h with t in the argument of belief. Let us define $G_j[\theta_i](t)$ as agent i of type θ_i's belief that j will concede by time t. All types θ_i have the same belief, $G_j[\theta_i](t) = \alpha_j[\theta_i](\emptyset)(\underline{\omega}_j)F_j[\underline{\omega}_j](t) = G_j(t)$.

Recall that $\theta_i = \bar{\omega}_i$ will never concede. Following Hendricks, Weiss and Wilson (1988), we know that $F_i[\underline{\omega}_i]$ is continuously increasing for $t > 0$. There can be mass probability of concession at $t = 0$. Agent i takes T_i time to build his reputation of being the high type. Since the low type will concede immediately if he learns that his opponent is never going to concede, we have $T_i = T_j =: T$. Thus, in equilibrium, both agents take the same time T to build their reputation for being the high type. Although there can be mass probability of concession at $t = 0$, if $\alpha_j[\theta_i](\emptyset)(\theta_j) > 0$ -i.e., θ_i believes that agent j can be of type θ_j, then θ_i and θ_j both cannot concede immediately with positive probability -i.e., $F_i[\theta_i](0)F_j[\theta_j](0) = 0$.

Let $C_j(s)$ be the consequence when j concedes at time s. Suppose agent i of type θ_i has reservation value ω_i. Let us define $u_i[\theta_i](t)$ as the expected utility (abusing notation) of agent i of type θ_i who concedes at time t if j has not conceded by then.

$$u_i[\theta_i](t) := \int_{s=0}^{t} u_i(C_j(s), \omega_i) dG_j[\theta_i](s) + (1 - G_j[\theta_i](t))u_i(C_i(t), \omega_i),$$

where $u_i(C_j(s), \underline{\omega}_i) = e^{-r_i s}\bar{\omega}_i$ and $u_i(C_i(s), \underline{\omega}_i) = e^{-r_i s}(1 - \bar{\omega}_j)$. Let $\mathbb{A}_i[\theta_i] := \{t : u_i[\theta_i](t) = max_s u_i[\theta_i](s)\}$. So, $(F_i[\theta_i] | \theta_i \in \Theta_i, i \in N)$ is a sequential equilibrium of the bargaining game if $Supp(F_i[\underline{\omega}_i]) \subseteq \mathbb{A}_i[\underline{\omega}_i]$ for all $i \in N$.

In equilibrium, agent i of type $\underline{\omega}_i$ is randomizing between conceding and insisting. This implies that $\underline{\omega}_i$ must be indifferent between the two options. To keep him indifferent, agent j must be conceding at a rate

$$\hat{\lambda}_j := \frac{r_i(1 - \bar{\omega}_j)}{\bar{\omega}_i + \bar{\omega}_j - 1} \tag{1}$$

This follows from $u_i'[\underline{\omega}_i](t) = 0$. This is the total concession rate of agent j, not just conditional on agent j being type $\underline{\omega}_j$. Thus, in the absence of concession, both agents become more convinced that their opponent is a high type. Since both these beliefs reach 1 at the same time T, we can solve backwards for the path of belief :

$$\alpha_j[\underline{\omega}_i](t)(\bar{\omega}_j) = e^{-\hat{\lambda}_j(T-t)}. \tag{2}$$

In the absence of concession until time t, the updated belief is such that the relative bargaining strength of any agent j

$$L_j(\alpha) = \frac{\alpha_j[\underline{\omega}_i](\bar{\omega}_j)^{\hat{\lambda}_i}}{\alpha_i[\underline{\omega}_j](\bar{\omega}_i)^{\hat{\lambda}_j}} \tag{3}$$

is 1. We will call the path of belief the *Balanced Path*, which keeps agents' bargaining strength balanced - i.e., $L_j(\alpha) = 1$. Formally,

$$B(\alpha) := \{\alpha | L_j(\alpha) = 1 \ \forall j \in N\}. \tag{4}$$

Let $\alpha_j(\emptyset)$ denote the prior belief and $\alpha_j(0)$ denote the updated belief at time $t = 0$ in the absence of immediate concession. If the initial belief $\alpha(\emptyset)$ is not on the balanced path, then agents concede immediately with positive mass probability such that, in the absence of immediate concession, $\alpha(0)$ reaches the balanced path. Recall that only one agent can concede immediately. Thus, the probability of immediate concession is uniquely identified. Although the types are defined very differently, the argument for reputation formation remains similar to that in AG. The above reputation formation equilibrium is formalized in the following preposition :

Proposition 2 *There is a unique sequential equilibrium in which any agent i of type $\bar{\omega}_i$ insists on $\bar{\omega}_i$, while type $\underline{\omega}_i$ either mimics type $\bar{\omega}_i$ or concedes to agent j's demand $\bar{\omega}_j$. In the absence of concession, the updated beliefs evolve along the balanced path. If the initial belief $\alpha(\emptyset)$ is not on the balanced path - say, $L_j(\alpha(\emptyset)) < 1$ - then agent j concedes immediately with positive probability (cipp) $\mu_j = 1 - L_j(\alpha(\emptyset))^{\frac{1}{\hat{\lambda}_i}}$, so that in the absence of immediate concession, the updated belief $\alpha(0)$ reaches the balance path.*

Recall that the initial beliefs are denoted by π. Abusing notation, I will assume $\alpha_i[\underline{\omega}_j](\emptyset)(\bar{\omega}_i) = \pi_i$ for all $i \in N$. There is $\pi_i\pi_j$ probability that agents have incompatible reservation values $(\bar{\omega}_i, \bar{\omega}_j)$, and so agreement is never reached. However, they will wait until time T, hoping that the opponent may be a lower reservation value type. Unlike the low type, the high type does not concede if the game reaches T without concession. If the agents are $(\bar{\omega}_i, \underline{\omega}_j)$ types, then agreement can happen only at $(x_i = \bar{\omega}_i, x_j = 1 - \bar{\omega}_i)$. There can be immediate agreement if $L_j(\alpha(\emptyset)) < 1$. By time T, agent j will become convinced that the opponent is $\bar{\omega}_i$ type and will concede. If both agents have low reservation values, then each agent will try to convince the other that he has a high reservation value. Agreement happens when either agent concedes to the other agent's demand. Agreement is reached by time T. If $\pi_i = \pi_j^{\frac{\hat{\lambda}_i}{\hat{\lambda}_j}}$, then $L_j(\alpha(\emptyset)) = 1$ - i.e., agents have balanced strength and no one cipp. If $\pi_i > \pi_j^{\frac{\hat{\lambda}_i}{\hat{\lambda}_j}}$, then agent i is said to be the strong (s) agent and agent j the weak (w) agent. From equation 2, we can solve for T, the time it takes to build reputation.

$$T(\pi) := -\frac{1}{\hat{\lambda}_s} ln(\pi_s). \tag{5}$$

Note that T depends only on the initial belief about the strong agent. The weak agent cipp $= \mu$, such that he can build reputation in the same T time. Therefore, $\frac{\pi_w}{1-\mu} = e^{-\hat{\lambda}_w T(\pi)}$. This implies that

$$\mu(\pi) = 1 - \pi_w \pi_s^{-\frac{\hat{\lambda}_w}{\hat{\lambda}_s}}. \tag{6}$$

Similar to Kambe (1999), when the probability that agents have high reservation value vanishes, there is no delay.

Corollary 1 *Suppose that* $(\pi_i, \pi_j) \downarrow (0, 0)$, *such that* $\max\{\frac{\pi_i}{\pi_j}, \frac{\pi_j}{\pi_i}\} \leq k$ *for some* $k > 1$. *If* $\hat{\lambda}_i > \hat{\lambda}_j$ *then agent* j *concedes immediately.*

Note that as the probability that agents have high reservation vanishes, the agents do not split the surplus, as Rubinstein proposes. The strong agent gets his reservation value and the weak agent takes the rest.

4 Public Information: Transparency and Delay

Agents know their own reservation values but receive some noisy public signals about their opponent's reservation values. Based on the signal received, agents update their beliefs. Since the signals are public, the posterior beliefs are commonly known. Agents bargain to build reputation, starting with the posterior beliefs. The previous section characterized the unique equilibrium, given any initial beliefs.

4.1 The Fundamentals

Public Information

Recall that a high or low reservation value for any agent $i \in N$ is drawn independently from a commonly known prior $\pi_i \in \mathbb{P}(\mathcal{W}_i)$. I will abuse notation and call $\pi_i = \pi_i(\omega_i = \bar{\omega}_i)$ - i.e., the ex-ante probability that agent i has a high reservation value. $\tilde{\mathcal{W}}_i = \mathcal{W}_i$ denotes the set of signal realizations and $q_i : \mathcal{W}_i \to \mathbb{P}(\tilde{\mathcal{W}}_i)$ is the noisy signal structure about agent i. Let $\tilde{\omega} = (\tilde{\omega}_i, \tilde{\omega}_j)$ denote the signal realization. Then,$q(\tilde{\omega} = \omega|\omega)$ captures the accuracy of signals. I will assume that

$$q(\tilde{\omega}|\omega) = q_i(\tilde{\omega}_i|\omega_i)q_j(\tilde{\omega}_j|\omega_j) \text{ where } q_i(\tilde{\omega}_i = \bar{\omega}_i|\omega_i) := \begin{cases} \bar{q}_i & \text{if } \omega_i = \bar{\omega}_i \\ 1 - \underline{q}_i & \text{if } \omega_i = \underline{\omega}_i \end{cases};$$

i.e., the signals are conditionally independent. For any $i \in N$, $\bar{q}_i$ ($\underline{q}_i$) captures the probability of receiving the right signal when the reservation value is high (low). I will call $q_i \equiv (\bar{q}_i, \underline{q}_i)$ for any $i \in N$, the *transparency* of the i-side and $q = (q_i, q_j)$ the transparency of the bargaining environment. Note that, if $q_i \in q_i^0 := \{q_i : \bar{q}_i + \underline{q}_i = 1\}$, the public signal disclosing i's reservation value is completely uninformative. A bargaining environment is said to be *fully opaque* if the signal on both sides is completely uninformative. If there is no difference in transparency across states, then a fully opaque bargaining environment on i-side has $\bar{q}_i = \underline{q}_i = \frac{1}{2}$ - i.e., any signal is equally likely to be realized for any ω_i.

Let us define $q_i' := \pi_i\bar{q}_i + (1-\pi_i)(1-\underline{q}_i) = P(\tilde{\omega}_i = \bar{\omega}_i)$ as the probability that a public information $\bar{\omega}_i$ is realized. Let us define $\bar{\pi}_i' := \frac{\pi_i\bar{q}_i}{q_i'} = P(\omega_i = \bar{\omega}_i|\tilde{\omega}_i = \bar{\omega}_i)$ as the updated probability that agent i's reservation value is high when public information says so, and $1 - \underline{\pi}_i' := \frac{\pi_i(1-\bar{q}_i)}{1-q_i'} = P(\omega_i = \bar{\omega}_i|\tilde{\omega}_i = \underline{\omega}_i)$ as the updated probability that agent i's reservation value is high when public information says otherwise. After public signal $\bar{\omega}_i$(or $\underline{\omega}_i$) is realized, the initial belief $\alpha_i(\emptyset)$ is captured by $\bar{\pi}_i'$(or $(1-\underline{\pi}_i')$). Abusing notation, I will refer to this posterior belief as π'. $\bar{\pi}_i' \geq \pi_i \geq 1-\underline{\pi}_i'$, and the equality holds when the i-side of the bargaining environment is completely opaque. Thus, given the prior π, a noisy information structure with transparency q, induces a distribution q' over posterior beliefs π'. The posterior beliefs must satisfy the Bayesian consistency

$$q_i'\bar{\pi}_i' + (1-q_i')(1-\underline{\pi}_i') = \pi_i\bar{q}_i + \pi_i(1-\bar{q}_i) = \pi_i. \tag{7}$$

Increasing transparency of the bargaining environment implies that when the public signal realization is $\bar{\omega}_i$ (or $\underline{\omega}_i$), the posterior belief that $\omega_i = \bar{\omega}_i$ (or $\omega_i = \underline{\omega}_i$) is higher. So, the posterior belief $\bar{\pi}_i'$ increases and $1-\underline{\pi}_i'$ falls, but equation 7 holds true. Note that the distribution over the posteriors, denoted by q_i', also changes as transparency changes.

Mediator

Suppose a neutral third-party mediator is appointed to facilitate a resolution to the conflict. The mediator investigates the agent for his underlying reservation value and conveys the result of her investigation to the other agent publicly. Because the investigation is not fully accurate, it generates noisy public information. Let q_i denote the accuracy of her investigation on agent i. The accuracy of her investigation is the transparency of the bargaining environment. If the mediator is actual rather than fictional, we would like to know if appointing a mediator will always reduce delay. If not, then when should we appoint a mediator?[7]

Bargaining Strength

Given the concession rate $\hat{\lambda}_i$ and the initial belief π', it will take $T_i(\pi') := -\frac{1}{\hat{\lambda}_i} ln(\pi_i')$ time for agent i to convince his opponent that he has a high reservation value. The less T_i is, the stronger agent i is. Thus, $\pi_i'^{\frac{1}{\hat{\lambda}_i}} = exp(-T_i(\pi'))$ is a measure of agents i's strength. The agent with a lower $T_i(\pi')$ is said to be the strong agent, denoted by s. Suppose that $T_i(\pi') < T_j(\pi')$ - i.e., $\pi_i'^{\frac{1}{\hat{\lambda}_i}} > \pi_j'^{\frac{1}{\hat{\lambda}_j}}$. Then the relative bargaining strength of agent j, $L_j(\pi') < 1$ - i.e., agent i is stronger than agent j. So, agent j cipp= $\mu(\pi') = 1 - L_j(\pi')^{\frac{1}{\hat{\lambda}_i}} = 1 - \pi_j \pi_i^{-\frac{\hat{\lambda}_j}{\hat{\lambda}_i}}$.

I use s to denote the strong agent in general. Abusing notation, I will use $s(\tilde{\omega})$ to denote the strong agent when the realized public signal is $\tilde{\omega}$. Let us define the strength vector s as the vector

[7] This paper focuses on the role of transparency in delay. In a companion paper, I explore the role of transparency when the mediator wants to implement a fair solution.

denoting which agent is strong for which signal realization :

$$s \equiv (s(\tilde{\omega}))_{\tilde{\omega}\in\tilde{\mathcal{W}}} \equiv (s(\bar{\omega}_i, \bar{\omega}_j), s(\bar{\omega}_i, \underline{\omega}_j), s(\underline{\omega}_i, \bar{\omega}_j), s(\underline{\omega}_i, \underline{\omega}_j)).$$

Given the transparency of the bargaining environment q, let $s(q)$ denote the strength vector and the region of transparency that result in a strength vector s being $S_s := \{q|s(q) = s\}$. Given the signal realization $\tilde{\omega}_j$, let $s(\tilde{\omega}_j)$ be the sub-strength vector denoting which agent is strong if $\bar{\omega}_i$ or $\underline{\omega}_i$ is realized (in that order).

Distribution of Delay

In their attempt to convince their opponent that they have high reservation values, agents cause delay in reaching an agreement. Let $\mathcal{D}$ be a random variable denoting delay. Let $\psi'(\pi')$ denote the distribution of delay given the initial belief π'. Further, from Proposition 2, we know that, given the initial belief π', there exists a time $T(\pi')$ such that if agreement is not reached by time $T(\pi')$, agreement is never reached. Recall that s and w denote the strong and the weak agent, respectively, and $T(\pi') := -\frac{1}{\hat{\lambda}_s} ln(\pi'_s)$. Therefore, $Supp(\psi'(\pi')) = [0, T(\pi')] \cup \{\infty\}$, where ∞ denotes the event that agreement is never reached. Recall from Proposition 2, that the belief about the strong agent s alone determines $T(\pi')$, and only the weak agent w concedes immediately with positive probability (cipp) $\mu(\pi') = 1 - \pi'_w \pi_s'^{-\frac{\hat{\lambda}_w}{\hat{\lambda}_s}}$. Also, recall that in equilibrium, agent i concedes at a constant hazard rate

$$\frac{g_i(\pi')(t)dt}{1 - G_i(\pi')(t)} = \hat{\lambda}_i dt,$$

where $G_i(\pi')(t)$ denotes agent j's belief that agent i will concede by time t. Solving this, we get

$$G_i(\pi')(t) = \begin{cases} 1 - (1 - G_i(\pi')(0))e^{-\hat{\lambda}_i t} & \text{if } t \leq T(\pi') \\ 1 - \pi'_i & \text{if } t = \infty \end{cases}. \quad (8)$$

The probability that the bargaining will cause, at most, t delay is the probability that at least one of the agents concedes by time t. Given independence, we have $\psi'(\pi')(t) = 1 - (1 - G_i(\pi')(t))(1 -$

$G_j(\pi')(t))$. This implies that

$$\psi'(\pi')(t) = \begin{cases} 1-(1-\mu(\pi'))e^{-(\hat{\lambda}_i+\hat{\lambda}_j)t} & \text{if } t \leq T(\pi') \\ 1-\pi_i'\pi_j' & \text{if } t=\infty \end{cases}. \quad (9)$$

Given the prior π and transparency q, we have a distribution q' over the posteriors π'. Accordingly, the distribution of delay before the signal is realized, denoted by $\psi(\pi, q)$, is defined as[8]

$$\begin{aligned} \psi(\pi,q)(t) &= \sum_{\tilde{w},w} P(\tilde{w},w)P(\mathcal{D}\leq t|\tilde{w},w) = \sum_{\tilde{w}} P(\tilde{w}) \sum_{w} P(w|\tilde{w})P(\mathcal{D}\leq t|\tilde{w},w) \\ &= \sum_{\pi'} q'(\pi')\left(1-(1-G_i(\pi')(t))(1-G_j(\pi')(t))\right) = \sum_{\pi'} q'(\pi')\psi'(\pi')(t). \end{aligned}$$

The question this paper addresses is : How does transparency q affect the distribution of delay, $\psi(\pi, q)$?

Almost Full Transparency

A bargaining environment converges to full transparency as $q \to \mathbf{1}$. In such a bargaining environment, the right signal will almost surely be realized, and, upon receiving a signal, agents are almost sure that the state is what the signal says. Suppose that the signal says that the reservation values are $(\bar{\omega}_i, \underline{\omega}_j)$; then, agent j is weak and cipp= 1. If the signal says that the state is $(\bar{\omega}_i, \bar{\omega}_j)$, then agreement is never reached. The argument is not trivial when the signals says that the state is $(\underline{\omega}_i, \underline{\omega}_j)$. The posterior beliefs $(1-\underline{\pi}_i', 1-\underline{\pi}_j') \downarrow (0,0)$. From Corollary 1, we know that[9] the weak agent will concede immediately. However, unlike the complete information case, agents do not split the surplus, as Rubinstein proposes. The strong agent gets his reservation value and the weak agent takes the rest. Thus, as the bargaining environment becomes almost fully transparent, if agreement can be reached, it is reached immediately,. Alternatively, we can say if the mediator can make her investigation almost fully accurate, she can avoid delay. However, we do not expect a bargaining environment to be fully transparent or the investigation to be fully accurate.

[8] To reduce notation burden, I use $P(E)$ to denote the probability of an event E in general, where the underlying state and Borel algebra are well understood.

[9] Except for the knife edge case, where agents have balanced strength as $(1-\underline{\pi}_i', 1-\underline{\pi}_j') \downarrow (0,0)$

4.2 Partial Transparency

There can be substantial delay in a partially transparent bargaining environment. The following lemma shows that if information disclosure reduces the probability of immediate concession, then it increases delay in the sense of first-order stochastic dominance (FSD).

Lemma 3 *$\psi(\pi, q) \succeq_{FSD} \psi'(\pi)$ if and only if $E_{q'|q}\ \mu(\pi') < \mu(\pi)$.*

This lemma follows from two features of the distribution of delay. First, the concession rate is constant, and, second, from Bayesian consistency (see equation 7), we know that the probability of no agreement is $\pi_i\pi_j$, irrespective of the level of transparency. Given the posterior beliefs π', from equation 6, we can say that the probability that the negotiation will end immediately is

$$\mu(\pi_i', \pi_j') = \begin{cases} 1 - \pi_j'\pi_i'^{-\frac{\hat{\lambda}_j}{\hat{\lambda}_i}} & \text{if } \pi_j'^{\hat{\lambda}_i} \leq \pi_i'^{\hat{\lambda}_j} \\ 1 - \pi_i'\pi_j'^{-\frac{\hat{\lambda}_i}{\hat{\lambda}_j}} & \text{if } \pi_j'^{\hat{\lambda}_i} \geq \pi_i'^{\hat{\lambda}_j} \end{cases}. \tag{10}$$

This relation is shown in Figure 1. The relation between $\mu(.)$ and posterior beliefs is like a 'valley.' The prior belief is a weighted average of the two posterior beliefs (see equation 7). Since the relation is not strictly convex, we cannot say that improving transparency will improve $E\mu$. When the agents

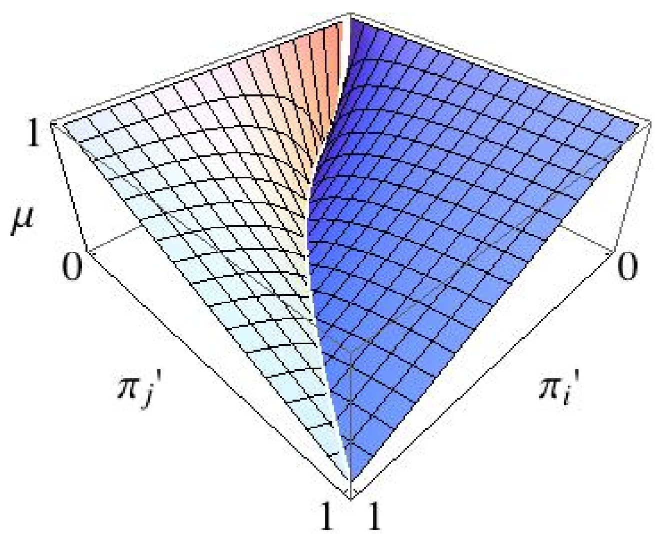

Figure 1: Probability of immediate concession and posterior beliefs

have balanced strength in the WOA, nobody gives up immediately. This can drag the negotiation on for a long time. As the strengths diverge, the probability of immediate concession increases.

If a mediator wants to increase the probability of immediate concession, she should try to make the strong agent stronger and the weak agent weaker. To understand how transparency affects $E_{q'|q}\ \mu(\pi')$, first, I will consider the case when information is available only on one side. Then, I will show what changes as information becomes available on both sides.

4.2.1 One-sided Public Information

Suppose that public information is available only regarding agent i. Let us suppose that the prior belief that j has a high reservation value, π_j, is fixed. From equation 10, we can see that given π_j, if agent i is weak - i.e., $\pi_i' \leq \pi_j^{\frac{\hat{\lambda}_i}{\hat{\lambda}_j}}$ - then $\mu(.)$ increases as π_i' decreases and in an affine way. On the other hand, if i is strong - i.e., $\pi_i' \geq \pi_j^{\frac{\hat{\lambda}_i}{\hat{\lambda}_j}}$ - then $\mu(.)$ increases as π_i' increases and in a concave fashion. This is shown in Figure 2.

This relation is crucial for understanding the relation between transparency and delay. First, consider the prior about the a priori weak agent π_w. From the reputation formation equilibrium, we know that this does not affect the time it takes to build the reputation, T. The weak agent cipp $= \mu$ such that he takes the same time to build the reputation, $T_w(\frac{\pi_w}{1-\mu}) = T$. Given that T is independent of π_w, the probability of immediate concession μ is affine in π_w. However, the prior belief about the a priori strong agent π_s reduces T (see equation 5) in a convex fashion. Consequently, the weak agent cipp $= \mu$ such that

$$T_w(\frac{\pi_w}{1-\mu}) = -\frac{1}{\hat{\lambda}_w} ln(\frac{\pi_w}{1-\mu}) = T = -\frac{1}{\hat{\lambda}_s} ln(\pi_s).$$

This implies that μ increases in a concave fashion with π_s. Recall that disclosing information about agent i will spread the posterior belief π_i' around π_i. Given transparency q_i, one of the following three situations may arise: (1) $1 - \underline{\pi}_i' \geq \pi_j^{\frac{\hat{\lambda}_i}{\hat{\lambda}_j}}$ - i.e., $s = (s(\bar{\omega}_i), s(\underline{\omega}_i)) = (i,i)$; (2) $\bar{\pi}_i' \leq \pi_j^{\frac{\hat{\lambda}_i}{\hat{\lambda}_j}}$ - i.e., $s = (j,j)$; and (3) $1-\underline{\pi}_i' \leq \pi_j^{\frac{\hat{\lambda}_i}{\hat{\lambda}_j}} \leq \bar{\pi}_i'$ - i.e., $s = (i,j)$. The following lemma identifies the information sensitivity of $E_{q'}\mu(\pi')$ in the interior of each of these cases.

Lemma 4 *If $s = (s(\bar{\omega}_i), s(\underline{\omega}_i)) = (i,j)$, then an increase in transparency will increase $E_{q'}\mu(\pi')$. If $s = (j,j)$, then an increase in transparency has no effect on $E_{q'}\mu(\pi')$. If $s = (i,i)$, then an increase in transparency will reduce $E_{q'}\mu(\pi')$.*

The relation between $E_{q'}\mu(\pi')$ and q_i for $s = (i,i)$ follows from concavity, and $s = (j,j)$ fol-

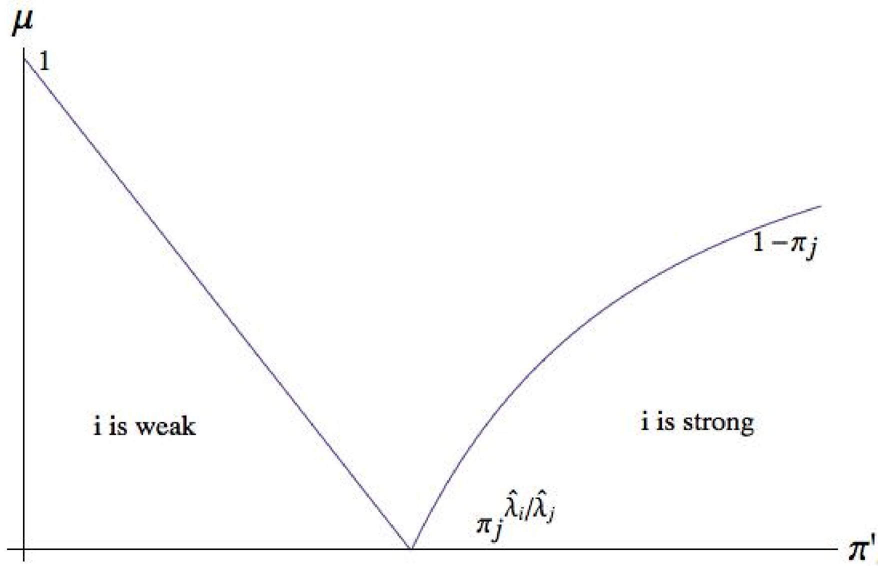

Figure 2: Probability of immediate concession and π_i' given π_j

lows from linearity (See Kamenica and Gentzkow (2011)). For $s = (i, j)$, increasing transparency increases μ for both signal realizations. However, the distribution over the posterior belief also changes, making the effect ambiguous. Nonetheless, the above lemma shows that increasing transparency increases $E_{q'}\mu(\pi')$.

A bargaining environment is said to be insufficiently transparent on the i-side if q_i is below some threshold level. If there is no state-wise difference in transparency - i.e., $\bar{q}_i = \underline{q}_i = q_i$ - then this means that $q_i \in [\frac{1}{2}, q_i^L]$, for some boundary q_i^L. If $\bar{q}_i \neq \underline{q}_i$, then this means that $q_i \in \{q_i : \bar{q}_i + \underline{q} \geq 1, \text{ and } \underline{q}_i \leq q_i^L(\bar{q}_i)\}$. To avoid notational burden, I will denote an insufficiently transparent bargaining environment simply by $q_i < q_i^L$.

Corollary 2 *Suppose that information is disclosed about the a priori strong agent i; then, there exists a threshold q_i^L such that: for $q_i < q_i^L$, $E_{q'}\mu(\pi') < \mu(\pi)$ and for $q_i \geq q_i^L$, $E_{q'}\mu(\pi')$ is increasing in q_i.*

Corollary 3 *Suppose that information is disclosed about the a priori weak agent j; then, there exists a threshold q_j^L such that: for $q_j < q_j^L$, $E_{q'}\mu(\pi') = \mu(\pi)$ and for $q_j \geq q_j^L$, $E_{q'}\mu(\pi')$ is increasing in q_j.*

Suppose that $\pi_i \geq \pi_j^{\frac{\hat{\lambda}_i}{\hat{\lambda}_j}}$. Under a fully opaque bargaining environment, agent i is strong. Suppose that for some q, $1 - \underline{\pi}_i' \geq \pi_j^{\frac{\hat{\lambda}_i}{\hat{\lambda}_j}}$; then, $s = (i, i)$ for any transparency less than q. Using

Lemma 4, we can say that $E_{q'}\mu(\pi') < \mu(\pi)$. Suppose that transparency is larger than q such that $1 - \underline{\pi}'_i \leq \pi_j^{\frac{\hat{\lambda}_i}{\hat{\lambda}_j}}$; then, from Lemma 4, we know that $E_{q'}\mu(\pi')$ is increasing in transparency. However, $E_{q'}\mu(\pi') < \mu(\pi)$ until some threshold level of transparency. If transparency can be beyond this threshold, then $E_{q'}\mu(\pi')$ increases in q_i. On the other hand, if $\pi_i \leq \pi_j^{\frac{\hat{\lambda}_i}{\hat{\lambda}_j}}$, then following the same argument, we can say that if q is not sufficiently high such that $\bar{\pi}'_i \leq \pi_j^{\frac{\hat{\lambda}_i}{\hat{\lambda}_j}}$, then $E_{q'}\mu(\pi') = \mu(\pi)$. However, if q is sufficiently high such that $\bar{\pi}'_i \geq \pi_j^{\frac{\hat{\lambda}_i}{\hat{\lambda}_j}}$, then $E_{q'}\mu(\pi')$ is increasing in q_i. From Lemma 3 and Corollary 2 and 3, the following proposition follows :

Proposition 3 *Given any prior π, there exists a threshold transparency q^L such that : if information is disclosed about the a priori strong agent i only, then for any $q_i < q_i^L$, $\psi(\pi, q) \succeq_{FSD} \psi'(\pi)$. If information is disclosed about a priori weak agent j only, for any $q_j < q_j^L$, the distribution of delay remains unchanged.*

Contrary to popular expectations, we see here that disclosing information about the a priori strong agent actually increases delay when the bargaining environment is not sufficiently transparent. Note that the result is very strong. From FSD, it follows that information disclosure not only increases expected delay (adjusting for no agreement), but it also increases the expected cost of delay for any bounded non-decreasing cost of delay. This brings to mind the old saying - ‘A little learning is a dangerous thing’ (Alexander Pope: “An Essay on Criticism,” 1709).

4.2.2 Two-sided Public Information

Disclosing information about the a priori strong agent and the a priori weak agent alone has different effects. Nonetheless, $E_{q'}\mu(\pi')$ is weakly below $\mu(\pi)$ in either case unless the bargaining environment is sufficiently transparent. Suppose that the mediator has the flexibility to choose whether or not she would investigate an agent. Then, if transparency on either side exceeds the respective thresholds, the mediator can guarantee that $E_{q'}\mu(\pi') > \mu(\pi)$ by disclosing information about one of the agents. Having the option of investigating both agents means that the mediator may be able to make $E_{q'}\mu(\pi') > \mu(\pi)$ by disclosing information about both, even when either side is not sufficiently transparent. However, if the mediator does not have the flexibility and has to investigate both agents, then it is not necessary that $E_{q'}\mu(\pi') > \mu(\pi)$, even when the bargaining environment is sufficiently transparent.

$$
\begin{aligned}
E_{q'}\mu(\pi) :=& q'_i q'_j \mu(\bar{\pi}'_i, \bar{\pi}'_j) + (1-q'_i)q'_j\mu(1-\underline{\pi}'_i, \bar{\pi}'_j) \\
&+ q'_i(1-q'_j)\mu(\bar{\pi}'_i, 1-\underline{\pi}'_j) + (1-q'_i)(1-q'_j)\mu(1-\underline{\pi}'_i, 1-\underline{\pi}'_j) \\
=& q'_i E_{q'_j}\mu(\bar{\pi}'_i, \pi_j) + (1-q'_i)E_{q'_j}\mu(1-\underline{\pi}'_i, \pi_j) \text{ for any } i \in N.
\end{aligned}
$$

Similar to the one-sided information case, given the posterior belief π'_j, there are three possible

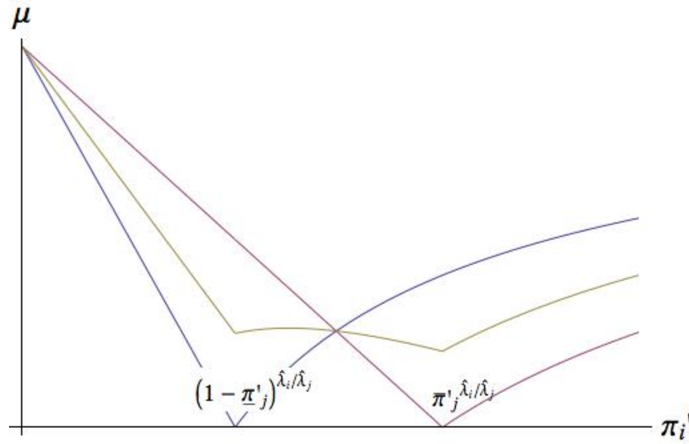

Figure 3: $\mu(\pi'_i, \bar{\pi}'_j)$, $\mu(\pi'_i, 1-\underline{\pi}'_j)$ and $E_{q'_j}\mu(\pi'_i, \pi_j)$

situations. Moreover, there are two possible posterior beliefs, $\bar{\pi}'_j$ and $1-\underline{\pi}'_j$. Accordingly, there are six possible situations that can be categorized in the following three cases:

1. For some $i \in N$, $s(\tilde{\omega}) = i$ for all $\tilde{\omega} \in \mathcal{W}$. Let $R_i^1 = S_{(i,i,i,i)}$ denote the set of transparency when this is the case.

2. For some $i \in N$, $s(\bar{\omega}_i, \underline{\omega}_j) = s(\underline{\omega}_i, \underline{\omega}_j) = i$ and $s(\underline{\omega}_i, \bar{\omega}_j) = s(\bar{\omega}_i, \bar{\omega}_j) = j$. Let $R_j^2 = S_{(i,j,i,j)}$ denote the set of transparency when this is the case.

3. For some $i \in N$, $s(\tilde{\omega}) = i$ for all $\tilde{\omega} \in \tilde{\mathcal{W}} \setminus \{(\underline{\omega}_i, \bar{\omega}_j)\}$ and $s(\underline{\omega}_i, \bar{\omega}_j) = j$. Let $R_i^3 = S_{(i,i,j,i)}$ be the set of transparency when this is the case.

Given the prior π, we can partition the space of transparency into the above regions. If the i-side is sufficiently transparent relative to the j side, then the signal about agent i determines the strength vector - i.e., $q \in R_i^2$ and vice versa. If transparency on both sides is close, then one agent's signal cannot solely determine the strength vector. Then, $q \in R_i^3$ for some i. If transparency on both sides is sufficiently small, then the a priori strong agent, say agent i, remains strong no matter what signal is realized - i.e., $q \in R_i^1$.

Using Lemma 4, we can say how q_i, q_j will affect $E_{q'}\mu(\pi)$. For example, suppose that $q \in R_i^2$. Then, $s(\bar{\omega}_j) = (i, j)$, which implies that increasing q_i will increase $E_{q_i'}\mu(\pi_i, \bar{\pi}_j') = q_i'\mu(\bar{\pi}_i', \bar{\pi}_j') + (1 - q_i')\mu(1 - \underline{\pi}_i', \bar{\pi}_j')$. Also, $s(\underline{\omega}_j) = (i, j)$, which implies that increasing q_i will increase $E_{q_i'}\mu(\pi_i, 1 - \underline{\pi}_j') = q_i'\mu(\bar{\pi}_i', 1 - \underline{\pi}_j') + (1 - q_i')\mu(1 - \underline{\pi}_i', 1 - \underline{\pi}_j')$. Since the weights to $E_{q_i'}\mu(\pi_i, \bar{\pi}_j')$ and $E_{q_i'}\mu(\pi_i, 1 - \underline{\pi}_j')$ are independent of q_i, the average of the two will also increase. Such a monotonic relation holds in all but for q_i when $q \in R_i^3$. This can be seen nicely in Figure 3. Since $s(\bar{\omega}_j) = (i, j)$, increasing q_i increases the probability of immediate concession, but if $s(\underline{\omega}_j) = (i, i)$, then q_i has just the opposite effect. The combined effect can go either way. I show that $E_{q'}\mu(\pi)$ is concave in q_i when $q \in R_i^3$ (See Lemma 9 in the appendix). Suppose that agent i is strong a priori. Using Lemma 3, we get the following proposition :

Proposition 4 *For any $q_i \leq q_i^L$, there exists $\tilde{q}_j^L(q_i) \in (q_j^0, q_j^L]$ such that for $q = (q_i, q_j)$ with $q_j < \tilde{q}_j^L(q_i)$, disclosing information about the strong agent or both agents will increase delay, in the sense of FSD and disclosing information about the weak agent alone does not affect delay.*

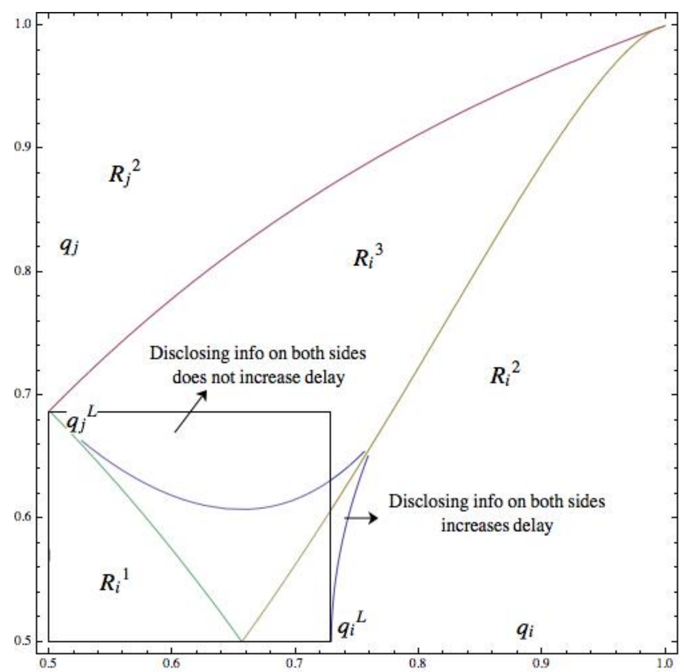

Figure 4: Transparency on both sides: Assuming $\bar{q}_i = \underline{q}_i = q_i$ for all $i \in N$

Recall from one-sided public information that if $q_i < q_i^L$, then disclosing information on agent i alone makes $E_{q'}\mu(\pi) < \mu(\pi)$. At $q = (q_i^L, q_j^0)$, $E_{q'}\mu(\pi) = \mu(\pi)$ and agent i's signal determines which agent is strong. Hence, $(q_i^L, q_j^0) \in R_i^2$. Using Lemma 4, we can argue that when $q \in R_i^2$, disclosing information on j will reduce $E_{q'}\mu(\pi)$, while disclosing information on i will increase $E_{q'}\mu(\pi)$. Suppose that the transparency on i-side is q_i^L, and the transparency on the j-side is marginally above q_j^0. Therefore, if information is revealed on both sides, $E_{q'}\mu(\pi) < \mu(\pi)$. So, to restore the equality, the information on i has to be more precise.

Corollary 4 *Given $q_j < q_j^L$, there exists $q_i > q_i^L$ such that if information is disclosed about the strong agent alone, delay does not increase, and disclosing information about the weak agent does not affect delay, but disclosing information about both agents increases delay.*

Also, recall that if $q_j \leq q_j^L$, then $E_{q'}\mu(\pi) = \mu(\pi)$. Consider q_i marginally above q_i^0; then, $q = (q_i, q_j^L) \in R_i^3$. I will show that for such q, $E_{q'}\mu(\pi) > \mu(\pi)$. Using Lemma 4, we can show that $E_{q'}\mu(\pi)$ is increasing in q_j. Therefore, to restore the equality, the transparency on the j side needs to be lower.

Corollary 5 *There is q ($q_i \leq q_i^L \; \forall i \in N$) such that although disclosing information about a priori strong agent alone increases delay, if information is disclosed about both agents, then delay does not increase.*

5 Optimal Transparency

A bargaining environment is not likely to be fully transparent or the mediator's investigation is not likely to be fully accurate. It is natural that the bargaining environment can be made transparent only to a limited extent. This is equivalent to saying that there is an upper bound on the accuracy of the investigations. Given such an upper bound, should a mediator make the bargaining environment as transparent as possible? We have learned that, if these upper bounds are sufficiently tight, disclosing information about one agent will increase delay in the sense of FSD. Disclosing information may increase the probability of immediate concession. Note that from Lemma 3, we can only say that if $E_{q'}\mu(\pi) > \mu(\pi)$, then $\psi(\pi, q) \not\succeq_{FSD} \psi'(\pi)$. This does not mean delay increases in the sense of FSD. Consider disclosing information about an a priori strong agent. If transparency is sufficiently high, we saw in the previous section that the probability of immediate concession when information is disclosed - i.e, $E_{q'_s}\mu(\pi')$ is higher than when no information is disclosed - i.e., $\mu(\pi)$. However, when a low signal is realized, the strength of the strong agent falls so much that he becomes the weak agent. Consequently, the bargaining can drag on longer. Therefore, we cannot rank $\psi(\pi, q)$ and $\psi'(\pi)$ in the sense of FSD.

Suppose that the mediator discounts delay at a stationary rate r_M. Let $Q_i = (\bar{Q}_i, \underline{Q}_i)$ denote the feasible set of transparency, where $Q_i := \{q_i : \bar{q}_i + \underline{q}_i \geq 1, \bar{q}_i \leq \bar{Q}_i, \underline{q}_i \leq \underline{Q}_i\}$. So, the mediator's objective is to

$$\max_{q \in Q} E_{\psi(\pi,q)} e^{-r_M t}.$$

Recall that there is positive mass probability $\pi_i \pi_j$ that the bargaining may never be settled and mass probability $E_{q'}\mu(\pi')$ that agreement will be reached immediately. I will normalize $r_M = 1$. Evaluating the expectation, we can say that the mediator's objective is equivalent to maximizing $E_{q'}\mu_1(\pi')$, where

$$\mu_1(\pi') = \mu(\pi') + (\hat{\lambda}_i + \hat{\lambda}_j)\xi(\pi'), \text{ where } \xi(\pi') = -\pi'_i \pi'_j e^{-T(\pi')} \tag{11}$$

Let us define $q_i^* := \arg\max_{q_i \in Q_i} E_{q'}\mu_1(\pi')$.

Welfare

Recall that, given any initial beliefs π', in the WOA, high types on both sides always insist. However, the low types only pretend to be insistent, and their equilibrium strategy depends on the initial belief π'. Thus, only the low types are manipulable. In equilibrium, these manipulable agents are indifferent between conceding at any time between $(0, T(\pi')]$. Since the weak manipulable agent concedes immediately with positive probability, his equilibrium payoff is $V(\underline{\omega}_w) = (1 - \bar{\omega}_s)$. The strong manipulable agent, on the other hand, does not concede immediately, but with probability $\mu(\pi')$, he is conceded to. After no immediate concession from the weak agent, he is indifferent between conceding at any time between $(0, T(\pi')]$. Thus, in equilibrium, he gets $V(\underline{\omega}_s) = \mu(\pi')\bar{\omega}_s + (1 - \mu(\pi'))(1 - \bar{\omega}_w)$. Therefore, the aggregate payoff of the two manipulable agents is $(\bar{\omega}_i + \bar{\omega}_j - 1)\mu(\pi') + (1 - \bar{\omega}_i) + (1 - \bar{\omega}_j)$. Taking expectation over all posteriors, we can then say maximizing $E_{q'}\mu(\pi')$ is equivalent to maximizing the aggregate payoff of the manipulable agents.

The insistent types also insist until $T(\pi')$, but unlike the manipulable types, they cannot give up when they see no concession until time $T(\pi')$. Therefore,

$$V_i(\bar{\omega}_i, \pi') = \int_0^T (\pi') e^{-r_i t} \bar{\omega}_i dG_j(s) = V_i(\underline{\omega}_i, \pi') - \pi'_j e^{-r_i T(\pi')}(1 - \bar{\omega}_j).$$

Suppose that the agents have equal discount rates, which are the same as the mediator's discount rate (normalized to 1). Substituting the low types' equilibrium payoff and aggregating the equilibrium payoff of all types weighted by their ex-ante probability, we can see maximizing welfare, $W(\pi, q) := \sum_{i,\omega_i} \pi'_i(\omega_i) V_i(\omega_i, \pi')$, is equivalent to maximizing $E_{q'}\mu_1(\pi')$.

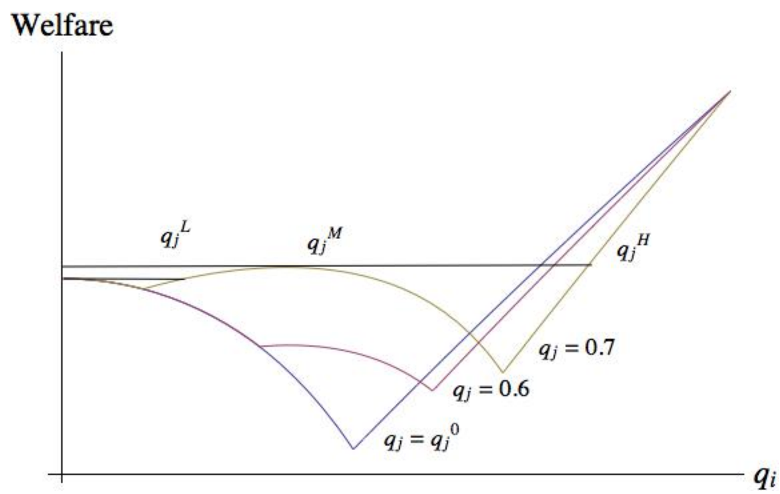

Figure 5: Optimal Transparency ($\pi_i = 0.5$, $\pi_j = 0.15$)

Proposition 5 *Suppose that only the a priori strong agent i is investigated. Then, $q_i^* = q_i^0$ if $Q_i < q_i^L$ and $q_i^* = (\bar{Q}_i, \underline{Q}_i)$, otherwise. If only an a priori weak agent j is investigated, then $q_j^* = Q_j$ if $Q_j < q_j^L$ and $q_j^* = (\bar{Q}_j, \underline{Q}_j)$, otherwise.*

As shown in equation 11, the mediator's objective μ_1 has two parts: 1) the probability of immediate concession, μ; and 2) $\xi = -\pi'_i \pi'_j e^{-T(\pi')}$, where $T(\pi')$ is the time it takes to build reputation, and $\pi'_i \pi'_j$ is the mass probability that agreement will never be reached. In the previous section, we studied the role of information revelation on μ. The above threshold result is no surprise if the mediator was maximizing $E_{q'}\mu(\pi')$.[10] However, the objective is to maximize $E_{q'}\mu_1(\pi')$. Recall that

[10] This is equivalent to maximizing the sum of payoff of the manipulable types of agents only.

the driving force behind this threshold rule for μ is (lemma 4) that μ is affine in π'_w and concave in π'_s. $T(\pi')$ does not depend on π'_w, and ξ is negative and linearly decreasing in π'_w. Substituting $T(\pi')$, we get $\xi = -{\pi'_s}^{1+\frac{1}{\lambda_s}}\pi'_w$. Thus, ξ is decreasing and concave in π'_s. Therefore, μ_1 is affine in π'_w and concave in π'_s. Thus, we can extend Lemma 4 for μ_1 (See Lemma 7 in the appendix).

When information is available on both sides, as we have seen before, whether or not disclosing information about an agent will help reduce delay depends on the transparency of the information on both sides. The relation may be complementary - i.e., improving transparency on one side alone, may increase delay but on both sides may reduce delay; or, substitutable - i.e., improving transparency on both sides may increase delay but only on one side may reduce delay. Thus, the simple threshold rule from one-sided information does not hold true.

To characterize the optimal transparency when both agents have noisy information about their opponent, I will simplify notation by assuming that $\bar{q}_i = \underline{q}_i = q_i$. Let us fix the transparency on the a priori weak side to some q_j and consider the transparency on the a priori strong side, $q_i \leq Q_i$.[11] There are two thresholds, q_i^H and q_i^L. As in the one-sided information case, if $Q_i \leq q_i^L$, then a fully opaque environment is most efficient, and if $Q_i \geq q_i^H$, a maximum transparent environment is most efficient. However, the two thresholds may not be equal. As Figure 5 shows, given q_j, some intermediate transparency of q_i improves efficiency or welfare because of the complementarity.

Proposition 6 *Given q_j, there exist q_i^L, q_i^M, q_i^H, with $\frac{1}{2} \leq q_i^L \leq q_i^M \leq q_i^H \leq 1$, such that*

$$\bar{q}_i^*(q_j; Q_i) = \begin{cases} \frac{1}{2} & \text{if } Q_i \leq q_i^L(q_j) \\ Q_i & \text{if } q_i^L(q_j) \leq Q_i \leq q_i^M(q_j) \\ q_i^M(q_j) & \text{if } q_i^M(q_j) \leq Q_i \leq q_i^H(q_j) \\ Q_i & \text{if } q_i^H(q_j,) \leq Q_i. \end{cases}$$

6 Appointing a Mediator

We have learned that given any prior belief π, there is a threshold rule for information disclosure when information is disclosed about one agent. If information can be disclosed about both agents, we saw that it is possible that disclosing information on both sides may do better or worse (see

[11] A similar result can be obtained for the opposite case. It is left for the readers.

Proposition 4). Clearly, appointing a mediator is not always going to improve efficiency. So, the natural question is : Given the accuracy of the investigation, when should we appoint a mediator?

For simplicity, I will focus on one-sided information disclosure - say, about agent i. Suppose that the objective is to maximize efficiency or welfare. Given the accuracy of the investigation on agent i, q_i, when can we say that $E_{q'}\mu_1(\pi') > \mu_1(\pi)$?

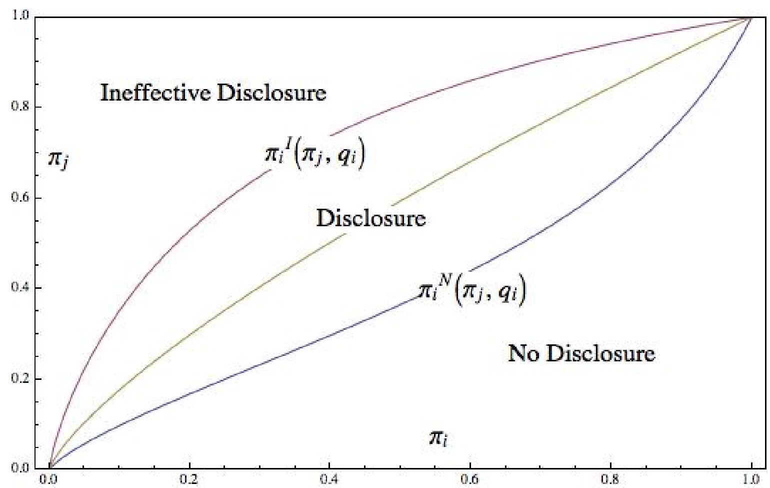

Figure 6: Information Disclosure about agent i

Proposition 7 *(Information Disclosure) Given* q_i *and* π_j*, there exist* $\pi_i^N(\pi_j, q_i)$ *and* $\pi_i^I(\pi_j, q_i)$ *satisfying* $1 \geq \pi_i^N(\pi_j, q_i) \geq \pi_j^{\frac{\hat{\lambda}_i}{\hat{\lambda}_j}} \geq \pi_i^I(\pi_j, q_i) \geq 0$*, such that (1) if* $\pi_i \leq \pi_i^I(\pi_j, q_i)$*, then disclosure is ineffective; (2) if* $\pi_i \in [\pi_i^I(\pi_j, q_i), \pi_i^N(\pi_j, q_i)]$*, then disclosure is optimal; and (3) if* $\pi_i \geq \pi_i^N(\pi_j, q_i)$ *then no disclosure is optimal.*

When agents have sufficiently close bargaining strengths, neither agent gives up easily in the WOA. The probability of immediate concession μ is small. Given any π_j, as π_i becomes higher(lower), i's bargaining strength becomes relatively larger (smaller). Suppose that π_i is sufficiently high, such that $1 - \underline{\pi}'_i \geq \pi_j^{\frac{\hat{\lambda}_i}{\hat{\lambda}_j}}$. Then, $s = (i, i)$ - i.e., no matter what signal is realized, agent i will remain the strong agent. From Lemma 7, we know that it is better not to disclose such information. The above proposition shows that there is a unique threshold such that if agent i's bargaining strength is above this threshold, then disclosing information reduces efficiency but otherwise enhances efficiency. Similarly, if π_i is sufficiently small such that $\bar{\pi}'_i \leq \pi_j^{\frac{\hat{\lambda}_i}{\hat{\lambda}_j}}$, then $s = (j, j)$. From Lemma 7, we know that disclosing information about i will have no effect on efficiency. If π_i is below this threshold, then $s = (i, j)$, and the concave closure of μ_1 is strictly above μ_1. This implies that disclosing information will increase efficiency.

7 Discussion

Reservation Value

The uniqueness result in this paper comes from the specific form of incomplete information. It is well established that without additional assumptions, there are a vast multiplicity of equilibria in two-sided incomplete information bargaining. So, surely, this model is with loss of generality. However, it capture many simple bargaining environments. For example, incomplete information about the reservation value seems very natural when it comes to buying and selling property such as housing. Loewenstein, Thompson and Bazerman (1989) have well documented that agents' preferences are often influenced by how others are doing. Thus, the price that a friend has paid for a similar property works as a reservation price. Similarly, how much money some less qualified employees are making elsewhere can be the reservation value for a labor union when it negotiates with the management over its share of the surplus.

Reservation value works as a status quo, as defined in Masatlioglu and Ok (2005). It makes some deals unacceptable based on the status quo but does not influence agents' preferences in any way. Also, note that reservation value is not a physical outside option that, when taken, stops the game. Let us suppose that two groups are negotiating over a surplus - e.g., a territorial dispute. The actual task of bargaining is usually delegated to a representative. The group sets a cutoff. If the representative fails to deliver a share above the cutoff he will be fired. [12] In territorial disputes, such cutoffs are probably chosen for some historical or political reasons. In labor dispute problems they are probably based on how other unions are doing. The opponent's cutoff is unknown to an agent, but he can get noisy information regarding the environment that has induced the cutoff.[13]

One can also think of reservation value as the value an agent assigns to never agreeing to trade. If such an option does not generate a flow payoff, then it fits our modeling assumptions. However,

[12] Li (2007) and Compte and Jehiel (2003) consider similar bargaining environment under complete information. However, the reservation value in these models are endogenous and depends on the past offers. They show delay can arise even under complete information.

[13] The cut-offs can also be thought of as a special case of Kőszegi and Rabin (2006)'s reference dependent preference. This is the case when agents are infinitely loss averse. Basak (2015) treats reservation value as a special case of psychological preference as in Geanakoplos, Pearce and Stacchetti (1989).

if such an outside option generates a flow payoff, then agents will discount only the net payoff. The standard two-sided incomplete information bargaining literature, such as Fudenberg and Tirole (1983), Cramton (1984), Cramton (1992), Chatterjee and Samuelson (1987) and Chatterjee and Samuelson (1988) consider this case. This is the crucial difference that drives the uniqueness result. I assume that it is commonly known that if the buyer buys the house, the trade will generate a $1 surplus, which they decide how to divide. However, these aforementioned models will assume that the value of the house v is unknown to the seller and that there is some cost c of selling the house that is unknown to the buyer. This will also give us reservation values - e.g., a high-cost seller will not sell the house below his cost. However, reservation values also influence agents' preferences. Suppose that the seller sells the house at price p after bargaining for time t. Then, his utility is $e^{-r_s t}(p - c)$. Thus, he discounts the net payoff. With such a model specification, the equilibrium under complete information will be quite different from that in Binmore, Shaked and Sutton (1989). Agents will first take their reservation value and then split the leftovers as Rubinstein proposes. This will not force the high type to insist on his high reservation value, and so uniqueness fails.

Similar to Watson (1998), suppose that the two types also differ in terms of their discount rates. In particular, suppose that the high type is also more patient, $r_i(\bar{\omega}_i) < r_i(\underline{\omega}_j)$. If assumption 1 holds true, then we can say that the patient type will always insist. However, the Rubinstein split will be different and the assumptions will have to be revised accordingly.

Uniqueness

Suppose, for contradiction, that there is an equilibrium in which the high reservation type of agent i rejects an offer above $\bar{\omega}_i$ with positive probability for some $i \in N$. Let M_i be such that there is no equilibrium in which agent i will reject an offer greater than M_i with positive probability. Hence, $M_i > \bar{\omega}_i$. Also, let m_j be such that there is no equilibrium in which agent j will ever take anything below m_j with positive probability. Therefore, $M_i \leq e^{-r_i\Delta}(1 - m_j)$ and $m_j \geq e^{-r_j\Delta}(1 - M_i)$. Combining these two inequalities and taking $\Delta \to 0$, we get $M_i \leq v_i^R$. Since $\bar{\omega}_i \geq v_i^R$, this contradicts the fact that $M_i > \bar{\omega}_i$. Thus, in any equilibrium, the high reservation type will always play the $\bar{\omega}_i$ insistent strategy - i.e., accepts an offer iff it is weakly better than $\bar{\omega}_i$.

If an agent ever plays an action not consistent with the insistent strategy of the high type, he reveals that he is the low type. If an agent reveals that he is the low type, while his opponent has not yet revealed so, following Myerson (1991), as $\Delta \to 0$, agreement is reached almost immediately, and the agent can get as much as he gets by conceding to his opponent's insistent demand. This result is commonly referred to as the Coasian result. [14] Thus, the game becomes a WOA - agents decide whether or not to concede. Once an agent concedes, the game stops, and by not conceding, an agent builds his reputation for being the high type. AG shows that equilibrium of the discrete time game converges in distribution to the unique continuous time equilibrium. Thus, we see that when incomplete information is about agents' reservation value, the limiting equilibrium is unique.

With two reservation values on each side, the game can be thought of as a staircase where agents are one step away. When there are more types, it may create more steps. Unlike AG, in which rational agents choose one behavioral type to imitate, here, agents choose a high step and gradually come down to lower steps. On each step, there is positive probability that an agent will not make any further concession. However, uniqueness is not guaranteed without additional assumption. Consider a subgame in which it has been revealed that agent i does not have a reservation value above v_i^R. Insistence is not the unique rationalizable strategy for this highest reservation value type left since the maximum he could have gotten had there been complete information, is not his reservation value (see the proof of lemma 1). We can get the insistence behavior if we assume that it is a common conjecture that agents never settle for a deal that gives them less than they could have gotten in the worst case complete information scenario.

Two-sided incomplete information bargaining

This paper does not claim to predict the equilibrium uniquely under any general form of incomplete information. Fudenberg and Tirole (1983) consider two-sided incomplete information with two periods, while Cramton (1984) extends this to the infinite horizon. He constructs an equilibrium in which the seller makes fully revealing offers. Thus, following the seller's offer, one-sided incomplete information bargaining ensues, where the uninformed party makes the offers, as in Sobel and Takahashi (1983). For this separating equilibrium, the low type must satisfy the incentive compatibility

[14] Gul and Sonnenschein (1988) establish that any delay in one-sided incomplete information bargaining is caused by the significant time interval between offers. If this time between offers vanishes, so does inefficiency. Gul, Sonnenschein and Wilson (1986) establishes the same result for a durable good monopolist. Also see Grossman and Perry (1986), Rubinstein (1985) for similar results on one sided incomplete information. The usual Coase conjecture argument assumes that only the uninformed agent makes the offers. In this model, the one sided incomplete information case is similar to Myerson (1991), where the informed agent is essentially choosing only whether to accept or reject (insist on the same demand) an offer made by the uninformed agent. Coase conjecture holds without any additional assumption.

of not pretending to be the high type. When reservation value does not influence the agent's intertemporal comparison, this can not be satisfied. Cramton (1992) extends this work, allowing for strategic delay. He constructs a separating equilibrium in which the agent's offer reveals his type. So, agents choose only when to make the offer. Suppose that the seller makes the offer at some time revealing his type. Following Admati and Perry (1987), impatient buyers will take the offer immediately rather than waiting, while patient buyers will wait until some time before they make the Rubinstein offer. In the absence of concession from either party, agents become more convinced that the gains from trade are lower. Chatterjee and Samuelson (1987) consider a similar problem, restricting the strategy space to only two actions, and show that there is a unique war of attrition equilibrium. Chatterjee and Samuelson (1988) extends this to many actions and show that the equilibrium can be generalized and uniqueness holds under off-path optimistic conjecture. Given that insistence is the unique rationalizable strategy, this is trivially satisfied.

Delay in bargaining

Under one-sided incomplete information, Ausubel and Deneckere (1989) show a folk theorem result when non-stationary equilibrium is considered and when there is no gap between the uninformed trader's valuation and the support of the informed trader's valuation. Admati and Perry (1987) show that there is a separating equilibrium with delay when the prior probability that the informed agent is the low type is sufficiently high, and if agents can strategically delay in making a counteroffer and, by doing so, prevent the opponent from revising his offer. Specific details of the bargaining environment can provide quite different incentives to agents, and, consequently, different from this paper, there can be inefficiency even under one-sided incomplete information. Vincent (1989) and Deneckere and Liang (2006) show that there can be delay when agents have interdependent values. Fuchs and Skrzypacz (2010) show that there is delay when new traders arrive over time. Board and Pycia (2014) show that Coase conjecture fails when there are outside options. Daley and Green (2012) consider a seller trying to sell an asset to an uninformed buyer, and information arrives stochastically over time. Agents may wait for the information to arrive rather than giving up, violating the Coase conjecture. The authors show that a higher quality of information may lead to a more inefficient outcome.

Reputation and bargaining

After Abreu and Gul (2000), Abreu and Pearce (2007) further develop reputation bargaining, sowing that the stationary behavioral type is a sufficiently rich class of perturbation to consider in stationary bargaining games. Compte and Jehiel (2002) show that the outside option can offset insistent behavior. Abreu, Pearce and Stacchetti (2012) consider a reputation bargaining model in which there is one-sided incomplete information about agents' discount rate. They show that non-Coasian result can occur when patient, rational agents can delay in making their initial demand to separate themselves from impatient agents.

Mediation

The mediator in the negotiation can commit only to the information structure and not to the signal realization. This is similar to Brocas and Carrillo (2007) and Kamenica and Gentzkow (2011), in which a sender tries to persuade a receiver by committing to an information structure. Hörner, Morelli and Squintani (2015) consider a static conflict game in which two countries decide whether to split a surplus or go to war. Each country is uncertain whether the other country is strong or weak. War partly destroys surplus, but a strong country can get a larger share by going to a war with a weak country. A mediator is defined as a mechanism designer who can only recommend nonenforceable resolutions - a division of the surplus and a probability of going to war. The authors show that the mediator does not fully reveal if a country is weak. Since, unmediated communication precludes such nondisclosure, it cannot achieve the same level of ex-ante welfare. It will be interesting to study what a mediator can do as an information designer in this conflict.

Information disclosure

The idea that more information can reduce agents' payoff in a strategic situation was first shown in Hirshleifer (1971). In auction theory, Milgrom and Weber (1982) show that disclosing public information that is affiliated to buyers' valuation reduces the buyers' fear of overbidding and, thus, increases revenue. Following their work, several papers focus on the agent's incentive to gather information in auction. Bergemann and Pesendorfer (2007) and Eso and Szentes (2007) show that when an auctioneer commits to an information structure, although more information increases efficiency, it also increases information rent. Thus, full disclosure is not optimal. Novshek and

Sonnenschein (1982), Clarke (1983), Vives (1984) and Gal-Or (1985) build the theory of information sharing in oligopoly. See Vives (1990) and Raith (1993) for a more general treatment. They show that firms facing a Cournot competition should not share their private information about market demand. Bergemann and Morris (2013) allow for a more general form of information sharing and show that disclosing private noisy information about the aggregate helps make the actions more correlated to the state, and so information disclosure may help.

In this model, the nature of incomplete information is very specific and more information can be interpreted as an agent having access to a more precise signal about his opponent's reservation value. Note that it also means that the opponent knows that the agent has more information. Blackwell (1951), Blackwell (1953) shows that in a single agent decision problem, more information is equivalent to higher payoff. A related literature extends Blackwell's notion of equivalence between more information and higher payoff in games. Gossner and Mertens (2001) establish the equivalence of the identity classes of the two relations in a zero sum game. Peski (2008) provides the necessary and sufficient condition for the equivalence of the relations in a zero sum game. Lehrer, Rosenberg and Shmaya (2010) consider games of common interest and characterize the situation in which one information structure is better than another with respect to different solution concepts.

8 Final Remarks

We saw a simple model of bargaining with two-sided incomplete information that results in a unique equilibrium. The equilibrium is similar to the one that we see in the reputation literature. The model provides some novel insights into the role of information in a reputation-formation equilibrium. It raises some follow-up questions, many of which can be answered using the same model. In a companion paper, I consider the case in which agents privately get noisy information about their opponent's reservation value. So, an agent knows his own reservation value but does not know what his opponent thinks about his reservation value. I also extend this paper to study the role of interdependence of reservation values in delay. This paper formalizes the idea of a mediator who has some control over the underlying information structure. To focus only on the role of transparency in delay, this paper remains silent about whether agents will give consent to appoint a mediator. Also, efficiency is not the only reasonable objective of the mediator. For example, a mediator may want to implement a fair division of the surplus. I leave these questions for future research.

9 Appendix

9.1 Bargaining with Reservation Values

Proof of lemma 1 Let Z_i be the set of offers i rejects with positive probability after some history while playing a strategy from the undominated set of strategies, R_i i.e.

$$Z_i := \{z \in [0,1] | \exists h \in H, x_j \in [0,1] \text{ and } \sigma_i \in R_i \text{ such that } x_j = 1 - z \text{ and } \sigma_i((h, x_j))(\mathcal{A}) < 1\}$$

Let us define $M_i := \sup Z_i$. Let Y_i be the set of offers i accepts with positive probability after some history while playing some strategy from R_i i.e.

$$Y_i := \{z \in [0,1] | \exists h \in H, x_j \in [0,1] \text{ and } \sigma_i \in R_i \text{ such that } x_j = 1 - z \text{ and } \sigma_i((h, x_j))(\mathcal{A}) > 0\}$$

Let us define $m_i := \inf Y_i$.

Lemma 5 *For $i \in N$, $\underline{\omega}_i \leq m_i$ and $\bar{\omega}_i \leq M_i$.*

Proof. Suppose for contradiction $\underline{\omega}_i > m_i$. Then by definition of m_i, there must exists some $x_i \in (m_i, \underline{\omega}_i)$ such that $x_i \in Y_i$ i.e. after some history if agent j offers x_i to agent i, she accepts it with positive probability. But by definition of $\underline{\omega}_i$, no θ_i would have ever accepted x with positive probability. Hence, contradiction.

Suppose for contradiction $\bar{\omega}_i > M_i$, then there exists $x_i \in (M_i, \bar{\omega}_i)$ such that i will accept x_i for sure after any history for any $\sigma_i \in R_i$. But there exists a strategy $\sigma_i \in R_i$ such that agent i with reservation value $\bar{\omega}_i$ does not accept x_i. Hence contradiction. ■

Given lemma 5, and by definition of m_i and M_j we get

$$m_i \geq e^{-r_i\Delta}(1 - \max\{M_j, \bar{\omega}_j\}) \tag{12}$$

Since any agent $i \in N$, can guarantee himself $e^{-r_i\Delta}(1-\max\{M_j, \bar{\omega}_j\})$. If he ever offers $\max\{M_j, \bar{\omega}_j\}+\epsilon$ (for ϵ however small) to agent j, agent j will accept the offer for sure. So, after no history i

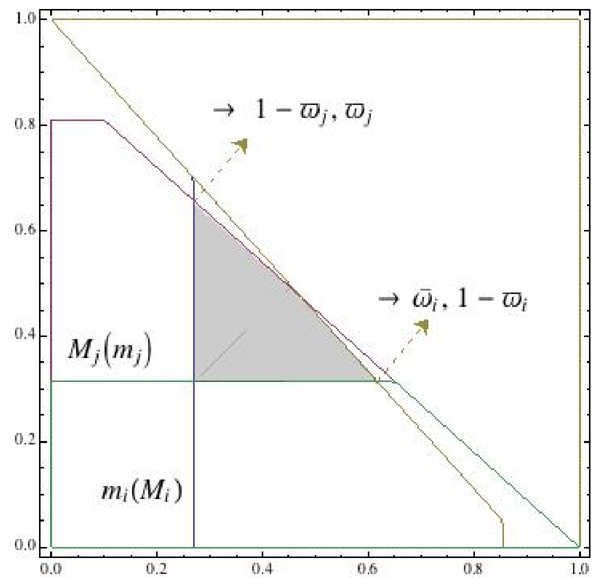

Figure 7: $m_i(M_i)$ and $M_j(m_j)$

would ever accept an offer which gives her anything less. To see this formally, suppose there exists $x_i < e^{-r_i\Delta}(1 - \max\{M_j, \bar{\omega}_j\})$ such that for some strategy $\sigma_i \in R_i$, i accepts x_i with positive probability after some history h. Suppose i rather rejects x_i and demands $x_i' = \frac{1-\max\{M_j,\bar{\omega}_j\}}{2} + \frac{x_i}{2e^{-r_i\Delta}}$. Since $1 - x_i' > \max\{M_j, \bar{\omega}_j\}$, agent j will surely accept this. Also, since $e^{-r_i\Delta}x_i' > x_i$, it gives i higher discounted value. Construct a strategy σ_i' such that $\sigma_i'(h') = \sigma_i(h')$ for all $h' \in H \setminus \{h\}$ and at history h, $\sigma_i'(h)$ behaves just like $\sigma_i(h)$ except the probability mass is shifted to demanding x_i' from accepting x. Then σ_i is conditionally dominated by σ_i'. Hence, $\sigma_i \notin R_i$.

Similarly, from Lemma 5, and by definition of m_i and M_j we also get

$$M_j \leq e^{-r_j\Delta}(1 - \max\{m_i, \underline{\omega}_i\}) \tag{13}$$

If any agent $j \in N$ offers i less than $\max\{m_i, \underline{\omega}_i\}$, then i will never accept it. So, j cannot get more than $e^{-r_j\Delta}(1 - \max\{m_i, \underline{\omega}_i\})$. So any strategy of j that rejects more than $e^{-r_j\Delta}(1 - \max\{m_i, \underline{\omega}_i\})$ is conditionally dominated by by a strategy that does not do so and otherwise same.

To see what these couple of inequalities 12 and 13 imply, let us first identify (m_i, M_j) satisfying both inequalities and then on the same diagram identify (M_i, m_j) that satisfy the two inequalities as in figure 7. Then we will use $M_i \geq m_i$ for any $i \in N$. When we consider (m_i, M_j) the corresponding reservation values are $(\bar{\omega}_j, \underline{\omega}_i)$ and when we consider (M_i, m_j) the corresponding reservation values are $(\underline{\omega}_j, \bar{\omega}_i)$.

Note that when there is no reservation values, the same two inequalities give us the unique Rubinstein solution. As $\Delta \to 0$, the solution converges to $(\frac{r_j}{r_i+r_j}, \frac{r_i}{r_i+r_j})$. Thus equal discount rate implies agents will divide the surplus equally. The first mover advantage vanishes when offers can be made very frequently. Also, if the reservation values are commonly known to be $\underline{\omega}_i$ on both sides, we can see the Rubinstein solution still remains the unique solution. However, if the reservation values are commonly known to be $(\bar{\omega}_i, \underline{\omega}_j)$, then the solution is unique but different than the Rubinstein solution. As $\Delta \to 0$, the solution converges to $(\bar{\omega}_i, 1 - \bar{\omega}_i)$. This proves **Proposition 1**.

When the reservation values are not commonly known and satisfies assumption 1, the two inequalities identifies only a range. This shows M_i for agent i cannot be higher that the maximum of all the complete information payoffs he can get. Therefore, for any undominated strategy agent i will accept any offer above $\bar{\omega}_i$ with probability 1. Therefore agent j will never offer agent i anything above $\bar{\omega}_i$. Also, m_i cannot be lower than the minimum of all complete information payoff an agent can get. Therefore, $m_i \geq 1 - \bar{\omega}_j$.

The high type agent i will always accept anything above $\bar{\omega}_i$ and will reject any offer below $\bar{\omega}_i$ by definition. Thus, insistence in the unique undominated strategy for the high type agents. Note the role of assumption 1. If $\bar{\omega}_i < v_i^R$, then v_i^R is the maximum of all complete information payoff agent i can get. By a similar argument as in this lemma, we get $M_i \leq v_i^R$. Thus, insistence is not the unique rationalizable strategy for the high type. □

Proof of Lemma 2

Step 1: The game must end in finite time with probability 1.

$\theta_j = \bar{\omega}_j$ always insists. Suppose, the negotiation has been going on for t_1 period. Let $h(t_1)$ denotes the history at t_1. If i agrees to j's demand of $\bar{\omega}_j$, then she gets $(1 - \bar{\omega}_j)$. Suppose agent i believes that j will behave like $\theta_j = \bar{\omega}_j$ for another t_2 time with probability $\psi_j(t_1, t_2)$. If i plays a strategy that will drag the game for until t_2 period with positive probability, then the maximum she can get is $(1 - \psi_j(t_1, t_2))1 + \psi_j(t_1, t_2)e^{-r_i(t_2 - t_1)}(1 - \bar{\omega}_j)$. Therefore, she will drag it only if

$$(1 - \bar{\omega}_j) \leq (1 - \psi_j(t_1, t_2))1 + \psi_j(t_1, t_2)e^{-r_i(t_2 - t_1)}(1 - \bar{\omega}_j)$$

$$\Rightarrow \psi_j(t_1, t_2) \leq \frac{1 - (1 - \bar{\omega}_j)}{1 - e^{-r_i(t_2 - t_1)}(1 - \bar{\omega}_j)} =: \delta < 1$$

Hence, there is K such that $\delta^K < \alpha_j(h(t_1))(\bar{\omega}_j)$. Since, we are considering independent and privately known reservation values, $\alpha_j[\theta_i](h(t_1))(\bar{\omega}_j) = \alpha_j(h(t_1))(\bar{\omega}_j)$ for all $\theta_i \in \Theta_i$. Repeating the argument K times we can find a sequence $0 = t_1, t_2, \dots t_{K+1}$ such that i plays a strategy that drags the game until t_{K+1} with positive probability only when i believes that j will behave like $\bar{\omega}_j$ with probability less than δ^K. But since i believes that $\theta_j = \bar{\omega}_j$ with probability $\alpha_j(h(t_1))(\bar{\omega}_j)$, we have a contradiction. So, i cannot possibly wait until t_{K+1} before she concedes.

Step 2: Agent i must concede immediately.

Suppose not. Let $\bar{t} > 0$ be the supremum of the time such that i has not conceded to j's demand $\bar{\omega}_j$. Consider the last ϵ time i.e. $(\bar{t} - \epsilon, \bar{t})$. Let x be the sup of i's payoff if j agrees to anything less than $\bar{\omega}_j$ in $(\bar{t} - \epsilon, \bar{t} - (1-b)\epsilon)$, $b \in (0,1)$. Let y be the sup of i's payoff if j does not do so. Let ξ be the probability i assigns to j not agreeing to anything less than $\bar{\omega}_j$ in $(\bar{t} - \epsilon, \bar{t} - (1-b)\epsilon)$. If i concedes she gets $(1 - \bar{\omega}_j)$. Since i is dragging it, it must be that

$$(1 - \bar{\omega}_j) \leq (1 - \xi)x + \xi y$$

$$\Rightarrow \xi \leq \frac{x - (1 - \bar{\omega}_j)}{x - y}, \text{ whenever } x - y > 0$$

At any t, θ_j can behave like $\bar{\omega}_j$ and guarantee herself $e^{-r_j(\bar{t}-t)}\bar{\omega}_j$. So the best i can get is $(1 - e^{-r_j(\bar{t}-t)}\bar{\omega}_j)$. [15] Therefore,

$$x \leq (1 - e^{-r_j\epsilon}\bar{\omega}_j)$$

$$y \leq e^{-r_i b\epsilon}(1 - e^{-r_j(1-b)\epsilon}\bar{\omega}_j)$$

Therefore, $y < (1 - \bar{\omega}_j)$ whenever

$$e^{-r_i b\epsilon}(1 - e^{-r_j(1-b)\epsilon}\bar{\omega}_j) < 1 - \bar{\omega}_j.$$

Or, (rearranging) whenever

$$\frac{\bar{\omega}_j(e^{r_i b\epsilon} - e^{-r_j(1-b)\epsilon})}{(e^{r_i b\epsilon} - 1)} < 1.$$

Taking limit of LHS as $\epsilon \to 0$ (using L'Hospital rule) we get $\dfrac{\bar{\omega}_j(r_i b + r_j(1-b))}{r_i b}$. This is less than 1 if

[15] Note that $1 - e^{-r_j(\bar{t}-t)}\bar{\omega}_j \geq \bar{\omega}_i$.

$b \in (\frac{\bar{\omega}_j r_j}{\bar{\omega}_j r_j + (1-\bar{\omega}_j) r_i}, 1)$. Therefore, for any b in the above interval and ϵ small enough $(1-\bar{\omega}_j) \geq y$ and since $(1-\bar{\omega}_j) \leq (1-\xi)x + \xi y$, we have $x \geq (1-\bar{\omega}_j)$. Since, $\frac{x-(1-\bar{\omega}_j)}{x-y}$ is increasing in x and y, we have

$$\xi \leq \lim_{\epsilon \to 0} \frac{\bar{\omega}_j(1-e^{-r_j\epsilon})}{1-e^{-r_j\epsilon}\bar{\omega}_j - e^{-r_i b\epsilon}(1-e^{-r_j(1-b)\epsilon}\bar{\omega}_j)} = \frac{1}{b}\frac{\bar{\omega}_j r_j}{\bar{\omega}_j r_j + (1-\bar{\omega}_j) r_i} < 1$$

So, for any b in the above interval and ϵ small enough, we can find $\delta_b < 1$ such that i will drag the negotiation only when she believes j will behave like $\bar{\omega}_j$ in $[\bar{t}-\epsilon, \bar{t}-(1-b)\epsilon)$ with probability less than δ_b. At time $\bar{t}-(1-b)\epsilon$, we can repeat the same argument for $[\bar{t}-(1-b)\epsilon, \bar{t}-(1-b)^2\epsilon)$ and so on. Therefore, i will drag the negotiation only if she believes j will not behave like $\bar{\omega}_j$ in $[\bar{t}-\epsilon, \bar{t}-(1-b)^k\epsilon)$ with probability less than δ_b^k. Take k such that $\delta_b^k < \alpha_j(h(\bar{t}-\epsilon))(\bar{\omega}_j)$, where $\alpha_j(h(\bar{t}-\epsilon))(\bar{\omega}_j)$ is agent i's belief that $\theta_j = \bar{\omega}_j$ at history h at time $\bar{t}-\epsilon$. Hence, contradiction. Note that, the result depends on $\Delta \to 0$. Otherwise, i may not have got the chance to make offers sufficiently close to $\bar{t}-(1-b)^m\epsilon$ for all $m = 1, 2 \ldots k$. □

Proof of Proposition 2 Since, $F_i[\underline{\omega}_i](t)$ is increasing $\mathbb{A}_i[\underline{\omega}_i]$ is dense in the $Supp(F_i[\underline{\omega}_i])$. Since $G_j[\underline{\omega}_i](t)$ is continuous, we have $u_i[\underline{\omega}_i](t)$ is constant in $Supp(F_i[\underline{\omega}_i])$. Therefore $u_i[\underline{\omega}_i](t)$ is differentiable and hence $G_j(t)$ is differentiable. $u_i'[\underline{\omega}_i](t) = 0$ implies that $G_j[\underline{\omega}_i](t)$ must have hazard rate

$$\lambda_j = \frac{g_j(t)}{1-G_j(t)} = \hat{\lambda}_j$$

After seeing j has not conceded by time t, θ_i's updated belief that agent j is of type θ_j is

$$\alpha_j[\theta_i](t)(\theta_j) = \frac{\alpha_j[\theta_i](\emptyset)(\theta_j)(1-F_j[\theta_j](t))}{1-G_j[\theta_i](t)}$$

Take log on both sides and then time derivative to get the continuous time analog of Bayesian updating as

$$\alpha_j[\underline{\omega}_i](t)(\bar{\omega}_j)' = -\alpha_j[\underline{\omega}_i](t)(\bar{\omega}_j)(\sigma_j[\bar{\omega}_j](t) - \hat{\lambda}_j)$$

where $\sigma_j[\bar{\omega}_j](t) := \frac{f_j[\bar{\omega}_j](t)}{1-F_j[\bar{\omega}_j](t)}$ is the hazard rate of concession of type $\theta_j = \bar{\omega}_j$. In this information structure we have $\sigma_j[\bar{\omega}_j](t) = 0$. So, $\alpha_j[\underline{\omega}_i](t)(\bar{\omega}_j)' = \alpha_j[\underline{\omega}_i](t)(\bar{\omega}_j)\hat{\lambda}_j$ and $\alpha_j(T) = 1$. Therefore,

$$\alpha_j[\underline{\omega}_i](t)(\bar{\omega}_j) = e^{-\hat{\lambda}_j(T-t)}.$$

Hence, at any $t \le T$, $\alpha_j[\underline{\omega}_i](t)(\bar{\omega}_j)^{\hat{\lambda}_i} = \alpha_i[\underline{\omega}_j](t)(\bar{\omega}_i)^{\hat{\lambda}_j}$. Thus, at any time α must be on the balanced path and gradually converges to $(1,1)$. If initial belief $\alpha(\emptyset)$ is such that $L_j(\alpha(\emptyset)) < 1$, then we will say j has less than balanced strength. We know only one of $\underline{\omega}_j$ can cipp. When j has less than balanced strength, $\underline{\omega}_j$ will cipp. Let μ_j be the total probability that agent j cipp such that upon seeing no concession from j, i's updated belief that j is of type $\bar{\omega}_j$ i.e. $\alpha_j[\underline{\omega}_i](0)(\bar{\omega}_j)$ is higher. μ_j is such that $\alpha(0)$ reaches the balanced path. $\alpha_j[\underline{\omega}_i](0)(\bar{\omega}_j) = \frac{\alpha_j[\underline{\omega}_i](t)(\emptyset)(\bar{\omega}_j)}{1-\mu_j}$ is updated belief of i that j is of type $\bar{\omega}_j$. So, if $L_j(\alpha(\emptyset)) < 1$, then

$$L_j(\alpha(0)) = (1-\mu_j)^{-\hat{\lambda}_i} L_j(\alpha(\emptyset)) = 1 \Rightarrow \mu_j = 1 - L_j(\alpha(\emptyset))^{1/\hat{\lambda}_i} = 1 - \frac{\alpha_j(\emptyset)(\bar{\omega}_j)}{(\alpha_i(\emptyset)(\bar{\omega}_i))^{\frac{\hat{\lambda}_j}{\hat{\lambda}_i}}}$$

□

Proof of Corollary 1 If $\hat{\lambda}_i > \hat{\lambda}_j$, then

$$L_j(\alpha(\emptyset)) = \frac{\pi_j^{\hat{\lambda}_i}}{\pi_i^{\hat{\lambda}_j}} = \left(\frac{\pi_j}{\pi_i}\right)^{\hat{\lambda}_i} \pi_i^{(\hat{\lambda}_i - \hat{\lambda}_j)} \to 0$$

Therefore, agent j cipp $= \mu = 1 - L_j^{\frac{1}{\hat{\lambda}_j}} \to 1$. □

9.2 Transparency and Delay

Proof of Lemma 3 The 'only if' direction follows from the definition of FSD. Let us consider the 'if' direction. From Bayesian consistency (see equation 7) we know the probability of no agreement remains unchanged - i.e., $\psi(\pi, q)(max_{\pi'} T(\pi')) = 1 - \pi_i\pi_j = \psi'(\pi)(T(\pi))$. Recall that $\psi(\pi, q)(t) = E_{q'}\psi'(\pi')$. Removing the upper bound on the distribution $\psi'(\pi')$, we get for any t

$$\psi(\pi, q)(t) \le \max\{1 - (1 - E_{q'}\mu(\pi'))e^{-(\hat{\lambda}_i + \hat{\lambda}_j)t}, 1 - \pi_i\pi_j\}$$

The equality holds for $t \leq min_{\pi'} T(\pi')$. If $\mu > E_{q'}\mu(\pi')$, then

$$\max\{1-(1-E_{q'}\mu(\pi'))e^{-(\hat{\lambda}_i+\hat{\lambda}_j)t}, 1-\pi_i\pi_j\} \leq \max\{1-(1-\mu(\pi))e^{-(\hat{\lambda}_i+\hat{\lambda}_j)t}, 1-\pi_i\pi_j\} = \psi'(\pi)(t)$$

Therefore, $\psi(\pi, q) \geq_{FSD} \psi'(\pi)$. $\square$

9.2.1 One-Sided Information

Proof of Lemma 4

Case 1: $1-\underline{\pi}'_i \geq \pi_j^{\frac{\hat{\lambda}_i}{\hat{\lambda}_j}}$

Agent i is strong no matter what signal is realized. This can happen only when agent i is strong in a fully opaque bargaining environment i.e. $\pi_i \geq \pi_j^{\frac{\hat{\lambda}_i}{\hat{\lambda}_j}}$. Suppose the transparency is not so high that $1-\underline{\pi}'_i$ falls below $\pi_j^{\frac{\hat{\lambda}_i}{\hat{\lambda}_j}}$. Since $\mu(\pi'_i)$ is concave in this region, by spreading the posterior beliefs, $E_{q'}\mu(\pi')$ will be lower. This is equivalent to saying that the concave closure of $\mu(\pi')$, denoted by $\hat{\mu}(\pi'_i)$, coincide with $\mu(\pi'_i)$ (see Kamenica and Gentzkow (2011)). Therefore an increase in transparency will reduce efficiency. A fully opaque bargaining environment , i.e. $\bar{\pi}'_i = 1-\underline{\pi}'_i = \pi_i$ maximizes $E\mu$.

Case 2: $\bar{\pi}'_i \leq \pi_j^{\frac{\hat{\lambda}_i}{\hat{\lambda}_j}}$

Agent i is weak no matter what signal is realized. This can happen only when agent i is weak in a fully opaque bargaining environment i.e. $\pi_i \leq \pi_j^{\frac{\hat{\lambda}_i}{\hat{\lambda}_j}}$. Suppose the transparency is not so high that $\bar{\pi}'_i$ exceeds $\pi_j^{\frac{\hat{\lambda}_i}{\hat{\lambda}_j}}$. Since $\mu(\pi'_i)$ is affine in this region the upper envelope $\hat{\mu}(\pi'_i)$ coincide with $\mu(\pi'_i)$. Notice that for any choice of Bayes possible $\bar{\pi}'_i$ and $1-\underline{\pi}'_i$ the welfare remains the same. So, transparency does not affect efficiency.

Case 3: $1-\underline{\pi}'_i \leq \pi_j^{\frac{\hat{\lambda}_i}{\hat{\lambda}_j}} \leq \bar{\pi}'_i$.

Agent i is strong when $\tilde{\omega}_i = \bar{\omega}_i$ is realized and i is weak when $\tilde{\omega}_i = \underline{\omega}_i$ is realized. This can happen either (1) when i is strong in a fully opaque bargaining environment but transparency in high enough to reduce $1-\underline{\pi}'_i$ below $\pi_j^{\frac{\hat{\lambda}_i}{\hat{\lambda}_j}}$ or (2) when i is weak in a fully opaque bargaining environment but transparency in high enough to increase $\bar{\pi}'_i$ above $\pi_j^{\frac{\hat{\lambda}_i}{\hat{\lambda}_j}}$. Improving transparency will increase $\bar{\pi}'_i$ and thus increase the strength of the strong agent while reduce $1-\underline{\pi}'_i$ and thus reduce the strength of the weak agent. However, the distribution over the posterior i.e. q_i' changes too. When $s = (i, j)$,

$$E_{q'}\mu(\pi') = 1 - \left(q_i' \frac{\pi_j}{(\bar{\pi}_i')^{\frac{\hat{\lambda}_j}{\hat{\lambda}_i}}} + (1 - q_i') \frac{1 - \underline{\pi}_i'}{(\pi_j)^{\frac{\hat{\lambda}_i}{\hat{\lambda}_j}}} \right)$$
$$= 1 - \pi_i \pi_j \left(\frac{\bar{q}_i}{(\bar{\pi}_i')^{1 + \frac{\hat{\lambda}_j}{\hat{\lambda}_i}}} + \frac{1 - \bar{q}_i}{(\pi_j)^{1 + \frac{\hat{\lambda}_i}{\hat{\lambda}_j}}} \right).$$

The last equality uses $q_i' \bar{\pi}_i' = \bar{q}_i \pi_i$ and $(1 - q_i')(1 - \underline{\pi}_i') = (1 - \bar{q}_i)\pi_i$. Since, agent i is strong when $\tilde{\omega}_i = \bar{\omega}_i$ is realized, $(\bar{\pi}_i')^{1 + \frac{\hat{\lambda}_j}{\hat{\lambda}_i}} \geq (\pi_j)^{1 + \frac{\hat{\lambda}_i}{\hat{\lambda}_j}}$. Taking partial derivative w.r.t. $\bar{q}_i$ and using this inequality and the fact that $\bar{\pi}_i'$ is increasing in $\bar{q}_i$, we have $D_{\bar{q}_i} E_{q'}\mu(\pi') > 0$. Also, $\bar{\pi}_i'$ is increasing in $\underline{q}_i$ and hence $D_{\underline{q}_i} E_{q'}\mu(\pi') > 0$. □

Proof of Corollary 2 Since agent i is a priori strong, $\pi_i \geq \pi_j^{\frac{\hat{\lambda}_i}{\hat{\lambda}_j}}$. When $q_i \in q_i^0$, $\tilde{\omega}_i$ is not informative and so agent i is strong no matter what signal is realized. As transparency increases ($\bar{q}_i$ and/or $\underline{q}_i$), the posterior belief $\bar{\pi}_i'$ increases and $1 - \underline{\pi}_i'$ falls. As long as $1 - \underline{\pi}_i' \geq \pi_j^{\frac{\hat{\lambda}_j}{\hat{\lambda}_i}}$, agent i remains strong no matter what signal is realized. Therefore, $E_{q'}\mu(\pi') < \mu(\pi)$ (from Lemma 4). If transparency increases further $s = (i, j)$. Form Lemma 4, we can say that $E_{q'}\mu(\pi')$ is increasing in q_i. □

Proof of Corollary 3 Since agent i is a priori weak, $\pi_i \leq \pi_j^{\frac{\hat{\lambda}_i}{\hat{\lambda}_j}}$. As transparency increases but as long as $\bar{\pi}_i' \leq \pi_j^{\frac{\hat{\lambda}_i}{\hat{\lambda}_j}}$ (note that $\bar{\pi}_i'$ is increasing in q_i), $E_{q'}\mu(\pi') = \mu(\pi)$. As transparency increases further $s = (i, j)$ and Lemma 4 says that $E_{q'}\mu(\pi')$ is increasing in q_i. □

Proof of Proposition 3 Suppose agent i is strong a priori. Let us define q_i^L to be the lower boundary of the set $\{q_i | s(q) = (i, j) \text{ and } E_{q'}\mu(\pi') \geq \mu(\pi)\}$. $s(q) = (i, j)$ only for sufficiently high q_i and given $s = (i, j)$, $E_{q'}\mu(\pi')$ is increasing in q_i (from Lemma 4). Given Corollary 2, we can then say that $E_{q'}\mu(\pi') < \mu(\pi)$ for $q_i < q_i^L$ and $E_{q'}\mu(\pi') \geq \mu(\pi)$ for $q_i \geq q_i^L$. The FSD then follows from Lemma 3.

Suppose agent j is weak a priori. Then let us define q_i^L as the lower boundary of $\{q_j | s(q_j) = (i,j)\}$. Note that $s(q) = (i,j)$ only when $q_j \geq q_j^L$. Given Corollary 3, we can say that $E_{q'}\mu(\pi') = \mu(\pi)$ for $q_i \leq q_i^L$ and $E_{q'}\mu(\pi') \geq \mu(\pi)$ for $q_i \geq q_i^L$. The FSD then follows from Lemma 3. $\square$

9.2.2 Two-Sided Information

Lemma 6 *(1) If $q \in R_i^1$ for some $i \in N$, then $E_{q'}\mu(\pi')$ decreases in q_i and not affected by q_j. (2) If $q \in R_j^2$ for some $j \in N$, then $E_{q'}\mu(\pi')$ increases in q_j and decreases in q_i. (3) If $q \in R_i^3$ for some $i \in N$, $E_{q'}\mu(\pi')$ increases in q_j but may increase or decrease in q_i.*

Proof.

Case 1: $q \in R_i^1$

If agent i is strong at a fully opaque bargaining environment , then agent i will also be strong when $\tilde{\omega} = (\bar{\omega}_i, \underline{\omega}_j)$ is realized. Consequently this case is possible only for agent i and not j. Suppose sufficiently transparent bargaining environment is not feasible such that $1 - \underline{\pi}_i' \geq (\bar{\pi}_j')^{\frac{\hat{\lambda}_i}{\hat{\lambda}_j}}$. Then for any signal realization agent i is strong and agent j is weak. For agent i, the posterior belief is such that $E_{q_j'}\mu(\pi_i', \pi_j')$ is concave in π_i' (see Figure 3). On the other hand, for agent j, the posterior beliefs are such that $E_{q_i'}\mu(\pi_i', \pi_j')$ is affine in π_j'. Hence, $E_{q'}\mu(\pi')$ decreases in q_i and not affected by q_j.

Case 2: $q \in R_j^2$

This represents the case when agent i is strong for two possible signal realization and so is agent j. However, agent j's signal determines which agent will be strong and not agent i's signal. This happens when the bargaining environment cannot be made much transparent on i-side but j-side is sufficiently transparent. Thus $\bar{\pi}_j'$ and $1 - \underline{\pi}_j'$ are widely spread while $\bar{\pi}_i'$ and $1 - \underline{\pi}_i'$ are close. In fact, $(1 - \underline{\pi}_j')^{\frac{\hat{\lambda}_i}{\hat{\lambda}_j}} \leq 1 - \underline{\pi}_i' \leq \bar{\pi}_i' \leq (\bar{\pi}_j')^{\frac{\hat{\lambda}_i}{\hat{\lambda}_j}}$. Thus for i, $\mu(\pi_i', \bar{\pi}_j')$ is concave while $\mu(\pi_i', 1 - \underline{\pi}_j')$ is affine. Therefore, $E_{q_j'}\mu(\pi_i', \pi_j')$ is concave in π_i'. Hence, $E_{q'}\mu(\pi')$ decreases in q_i. For j, on the other hand, given the posterior $\bar{\pi}_i'$ (or $1 - \underline{\pi}_i'$), j is strong if $\tilde{\omega}_j = \bar{\omega}_j$ and j is weak if $\tilde{\omega}_j = \underline{\omega}_j$. Using Lemma 4, we can say that increasing transparency on the j-side of the bargaining environment will increase both $E_{q_j'}\mu(\bar{\pi}_i', \pi_j')$ and $E_{q_j'}\mu(1 - \underline{\pi}_i', \pi_j')$. Therefore, $E_{q'}\mu(\pi')$ increases in q_j

Case 3: $q \in R_i^3$

Given q_j, q_i is neither too low (such that j's signal determines which agent is strong), nor too high (such the i's signal determines which agent is strong), then we have the above. Consider q_j first. Given the posterior belief $\bar{\pi}'_i$, j is weak no matter what signal is realized. From Lemma 4, we know that $E_{q'_j}\mu(\bar{\pi}'_i, \pi'_j)$ remains unchanged as q_j changes. On the other hand, given posterior $1 - \underline{\pi}'_i$, j is strong if $\bar{\omega}_j$ is realized and weak if $\underline{\omega}_j$ is realized. So, from Lemma 4, we know that $E_{q'_j}\mu(1 - \underline{\pi}'_i, \pi'_j)$ increases as q_j increases. Thus $E_{q'}\mu(\pi')$ increases as q_j increases. Now consider q_i. Given the posterior belief $\bar{\pi}'_i$, j is strong no matter what signal is realized. From Lemma 4, we know that $E_{q'_i}\mu(\pi'_i, \bar{\pi}'_j)$ decreases as q_i increases. On the other hand, given posterior $1 - \underline{\pi}'_j$, i is strong if $\bar{\omega}_i$ is realized and weak if $\underline{\omega}_i$ is realized. So, from Lemma 4, we know that $E_{q'_i}\mu(\pi'_i, 1 - \underline{\pi}'_j)$ increases as q_i increases. Thus increase in transparency q_i may increase or decrease welfare. ■

Proof of Proposition 4 Suppose agent i is strong a priori. Recall q_i^L and q_j^L as defined in Proposition 3. We saw disclosing information about the strong agent increases delay for $q_i \leq q_i^L$ and disclosing information about the weak agent does not affect delay if $q_j \leq q_j^L$.

Case 1: Consider $q_i \leq q_i^L$ and sufficiently small such that at (q_i, q_j^0), $s(\underline{\omega}_i) = i$.

Consider Figure 3 with π'_j in the horizontal axis. This corresponds to the case where π_j is in the left linear portion of $E_{q'_i}\mu$. As q_j increases, but not sufficiently high, then $s(\underline{\omega}_i, \bar{\omega}_j) = i$, which implies $q \in R_i^1$. Therefore q_j has no effect on $E_{q'}\mu(\pi')$ (from Lemma 6). Since, $s(\underline{\omega}_i, \bar{\omega}_j) = j$ at $q = (q_i^0, q_j^L)$, we have $s(\underline{\omega}_i, \bar{\omega}_j) = j$ for any (q_i, q_j^L) with $q_i \geq q_i^0$. If q_j becomes sufficiently high (but $\leq q_i^L$) then $s(\underline{\omega}_i, \bar{\omega}_j) = j$. Note that for such q, $s(\underline{\omega}_i, \underline{\omega}_j) = i = s(\bar{\omega}_i, \underline{\omega}_j)$. Also, since $s(\underline{\omega}_i, \bar{\omega}_j) = i$ at $q = (q_i^0, q_j^L)$ (balanced strength), we have $s(\bar{\omega}_i, \bar{\omega}_j) = j$ for such q. Hence, $q \in R_i^3$ and so from Lemma 6, we know that $E_{q'}\mu(\pi')$ increases in q_j. Since at (q_i, q_j^0), $E_{q'}\mu(\pi') > \mu(\pi)$, there is a threshold $\tilde{q}_j^L(q_i) \in (0, q_j^L]$ such that if information is disclosed about the strong agent or the both agents the immediate probability of concession will be less than $\mu(\pi)$. Using Lemma 3, we get our result.

Case 2: If q_i is sufficiently large but $q_i \leq q_i^L$, such that at (q_i, q_j^0), $s(\underline{\omega}_i) = j$.

Then $q \in R_i^2$ and so from Lemma 6, we know that increase in q_j will reduce $E_{q'}\mu(\pi')$. Therefore, if $(q_i, q_j^L) \in R_i^2$ then $\tilde{q}_j^L(q_i) = q_j^L$. If $(q_i, q_j^L) \in R_i^3$, then for sufficiently high q_j, $q \in R_i^3$ and hence

$E_{q'}\mu(\pi')$ is increasing in q_j. Thus, the same argument applies as in case 1. $\square$

Proof of Corollary 4 Consider q_i marginally above q_i^L and $q_j = q_j^0$. By definition of q_i^L, $s(\bar{\omega}_j) = s(\underline{\omega}_j) = (i, j)$ - i.e., $q \in R_i^2$. From Lemma 6, we know that $E_{q'_j}\mu(\pi')$ is decreasing on q_j and decreasing in q_i. By definition of q_i^L, when $q = (q_i^L, q_j^0)$, $E_{q'_j}\mu(\pi') = \mu(\pi)$. Hence for q_i marginally above q_i^L, there exists q_j marginally above q_j^0 such that $E_{q'_j}\mu(\pi') < \mu(\pi)$. $\square$

Proof of Corollary 5 When the transparency of information on j is q_j^L, for π'_i marginally above π_i, agent i is strong irrespective of $\tilde{\omega}_j$. Hence, $\mu(\pi')$ is concave in π'_i, which implies $E_{q'_j}\mu(\pi')$ is concave in π'_i. For π'_i marginally below π_i, $s(\bar{\omega}_j, \underline{\omega}_j) = (j, i)$. Therefore, $\mu(\pi')$ is concave in π'_i when $\pi'_j = 1 - \underline{\pi}'_j$ and affine in π'_i when $\pi'_j = \bar{\pi}'_j$. Therefore, $E_{q'_j}\mu(\pi')$ is concave in π'_i.

Since, the affine relation is negative, and all the other relations are positive, for π'_i marginally below π_i the slope of $E_{q'_j}\mu(\pi')$ is smaller than for π'_i marginally above π_i.

Let f be piece-wise concave above and below some x. Take $x_1 < x_2$ such that for $a \in (0, 1)$, $ax_1 + (1 - a)x_2 = x$. Also, $f'(x_2) > f'(x_1)$. Then $af(x_1) + (1 - a)f(x_2) > f(x)$. Suppose, for contradiction, $af(x_1) + (1 - a)f(x_2) \leq f(x)$. Then $(1 - a)(f(x_2) - f(x)) \leq a(f(x) - f(x_1))$. Also, $(1 - a)(x_2 - x) = a(x - x_1)$. After dividing we then get $\frac{f(x_2)-f(x)}{x_2-x} \leq \frac{f(x)-f(x_1)}{x-x_1}$. Take $x_2 \to x_1$, then $x \to x_1$ (since $x \in (x_1, x_2)$). Therefore $f'(x_2) \leq f'(x_1)$. Hence, contraction.

Replacing f by $E_{q'_j}\mu(\pi')$ we can then conclude that a marginal increase in q_i or spreading π_i in a Bayes consistent way, will increase $E_{q'}\mu(\pi')$. We have seen in the proof of Proposition 4 that, when q_i is marginally above q_i^0 and $q_j = q_j^L$, $q \in R_i^3$. From Lemma 6, we know that $E_{q'_j}\mu(\pi')$ increasing in q_j when $q \in R_i^3$. By definition, when $q = (q_i^0, q_j^L)$, $E_{q'}\mu(\pi') = \mu(\pi)$. Hence, for q_i marginally above q_i^0 there exists $q_j < q_j^L$ such that $E_{q'}\mu(\pi') > \mu(\pi)$. $\square$

9.3 Optimal Transparency

Proof of Proposition 5 Recall that

$$\begin{aligned}
\mu_1(\pi') &= \mu(\pi') + (\hat{\lambda}_i + \hat{\lambda}_j)\xi(\pi') \\
&= \mu(\pi') - (\hat{\lambda}_i + \hat{\lambda}_j)\pi'_i\pi'_j e^{-T(\pi')} \\
&= 1 - \pi'_w {\pi_s}^{-\frac{\hat{\lambda}_w}{\hat{\lambda}_s}} - (\hat{\lambda}_i + \hat{\lambda}_j)\pi'_w {\pi'_s}^{1+\frac{1}{\hat{\lambda}_s}}
\end{aligned}$$

Similar to Lemma 4, when there is one sided information, we have

Lemma 7 *If $s = (s(\bar{\omega}_i), s(\underline{\omega}_i)) = (i, j)$ then increase in transparency will increase $E_{q'}\mu_1(\pi')$. If $s = (j, j)$ then increase in transparency has no effect on $E_{q'}\mu_1(\pi')$. If $s = (i, i)$ then increase in transparency will reduce $E_{q'}\mu_1(\pi')$.*

Proof. $\mu(\pi')$ is linear in π'_w and so is $\xi(\pi')$. Hence, q_i has no effect on $E_{q'}\mu_1(\pi')$ when $s = (j, j)$. $\mu(\pi')$ is concave in π'_s and so is $\xi(\pi')$. Hence, increasing q_i reduces $E_{q'}\mu_1(\pi')$ when $s = (i, i)$. Suppose $s = (i, j)$

$$E_{q'_i}\mu_1(\pi') = 1 - \pi_i\pi_j\left(\bar{q}_i(\bar{\pi}'_i)^{-(1+\frac{\hat{\lambda}_j}{\hat{\lambda}_i})} + (1-\bar{q}_i){\pi'_j}^{-(1+\frac{\hat{\lambda}_i}{\hat{\lambda}_j})}\right) - (\hat{\lambda}_i + \hat{\lambda}_j)\pi_i\pi_j\left(\bar{q}_i(\bar{\pi}'_i)^{\frac{1}{\hat{\lambda}_i}} + (1-\bar{q}_i){\pi'_j}^{\frac{1}{\hat{\lambda}_j}}\right)$$

Note that

$$D_l\bar{\pi}'_i = \frac{\bar{\pi}'_i}{q'_i}(1-\pi_i) \text{ and } D_h\bar{\pi}'_i = \frac{\bar{\pi}_i}{\bar{q}_i}(1-\bar{\pi}'_i),$$

where subscript h and l denote partial derivative w.r.t. $\bar{q}_i$ and $\underline{q}_i$ respectively. Differentiating $E_{q'_i}\mu_1(\pi')$ w.r.t $\underline{q}_i$ and substituting $D_l\bar{\pi}'_i$ we get

$$D_lE_{q'_i}\mu_1(\pi') = \left(\frac{\hat{\lambda}_i + \hat{\lambda}_j}{\hat{\lambda}_i}\right)\frac{1-\pi_i}{q'_i}\left((\bar{\pi}'_i)^{-(1+\frac{\hat{\lambda}_j}{\hat{\lambda}_i})} - (\bar{\pi}'_i)^{\frac{1}{\hat{\lambda}_j}}\right) > 0$$

Differentiating $E_{q'_i}\mu_1(\pi')$ w.r.t $\bar{q}_i$ and substituting $D_h\bar{\pi}'_i$ we get

$$\begin{aligned} D_hE_{q'_i}\mu_1(\pi') = &\left(\frac{\hat{\lambda}_i + \hat{\lambda}_j}{\hat{\lambda}_i}\right)(1-\bar{\pi}'_i)\left((\bar{\pi}'_i)^{-(1+\frac{\hat{\lambda}_j}{\hat{\lambda}_i})} - (\bar{\pi}'_i)^{\frac{1}{\hat{\lambda}_j}}\right) \\ &- \left(\left((\bar{\pi}'_i)^{-(1+\frac{\hat{\lambda}_j}{\hat{\lambda}_i})} + (\hat{\lambda}_i + \hat{\lambda}_j)(\bar{\pi}'_i)^{\frac{1}{\hat{\lambda}_j}}\right) - \left({\pi_j}^{-(1+\frac{\hat{\lambda}_j}{\hat{\lambda}_i})} + (\hat{\lambda}_i + \hat{\lambda}_j){\pi_j}^{\frac{1}{\hat{\lambda}_j}}\right)\right) \end{aligned}$$

Define $g(x) := x^{-k} + kx$, then $g'(x) < 0$. Take $k = (\hat{\lambda}_i + \hat{\lambda}_j)$ and we can say $g((\bar{\pi}'_i)^{\frac{1}{\hat{\lambda}_j}}) < g(\pi_j^{\frac{1}{\hat{\lambda}_j}})$. Therefore $D_hE_{q'_i}\mu_1(\pi') > 0$. ■

Given this lemma, same argument applies as in Corollary 2 and Corollary 3. Hence, the threshold rule applies. □

Lemma 8 *(1) If $q \in R_i^1$ for some $i \in N$, then $E_{q'}\mu_1(\pi')$ decreases in q_i and not affected by q_j. (2)*

If $q \in R_j^2$ for some $j \in N$, then $E_{q'}\mu_1(\pi')$ increases in q_j and decreases in q_i. (3) If $q \in R_i^3$ for some $i \in N$, $E_{q'}\mu_1(\pi')$ increases in q_j but may increase or decrease in q_i.

Proof. This lemma claims that μ_1 has the same properties of μ as in Lemma 6. I have already shown that (Lemma 7) μ_1 has the same properties as μ (Lemma 4) under one-sided information. By the same argument as in Lemma 6 and replacing Lemma 4 by Lemma 7, we get this lemma. ■

Lemma 9 *If $q \in R_i^3$ for some $i \in N$, then $E_{q'}\mu(\pi')$ is concave in q_i.*

Proof. When $q \in R_i^3$, we have have,

$$E_{q'}\mu(\pi') = 1 - \left(q_i'q_j' \frac{\bar{\pi}_j'}{(\bar{\pi}_i')^{\frac{\hat{\lambda}_j}{\hat{\lambda}_i}}} + q_i'(1-q_j') \frac{1-\underline{\pi}_j'}{(\bar{\pi}_i')^{\frac{\hat{\lambda}_j}{\hat{\lambda}_i}}} + (1-q_i')q_j' \frac{1-\underline{\pi}_i'}{(\bar{\pi}_j')^{\frac{\hat{\lambda}_i}{\hat{\lambda}_j}}} + (1-q_i')(1-q_j') \frac{1-\underline{\pi}_j'}{(1-\underline{\pi}_i')^{\frac{\hat{\lambda}_j}{\hat{\lambda}_i}}} \right)$$

Let us define $\kappa : [0,1]^2 \times [0,1] \times R \to R$ as

$$\kappa(\bar{q}, \underline{q}; \pi, k) := L(\bar{q}, \underline{q}; \pi) \cdot N(\bar{q}, \underline{q}; \pi, k) \text{ where}$$

$$L(\bar{q}, \underline{q}; \pi) := 1 - \pi\bar{q} - (1-\pi)(1-\underline{q}) \text{ and } N(\bar{q}, \underline{q}; \pi, k) := \left(\frac{\pi(1-\bar{q})}{1 - \pi\bar{q} - (1-\pi)(1-\underline{q})} \right)^{-k}$$

Then,

$$\begin{aligned} E_{q'}\mu(\pi') =& 1 - \left(q_j'\bar{\pi}_j' + (1-q_j')(1-\bar{\pi}_j') \right) \kappa(1-q_i; \pi_i, \frac{\hat{\lambda}_j}{\hat{\lambda}_i}) - \\ & (1-\underline{\pi}_i')(1-q_i')\kappa(1-q_j; \pi_j, \frac{\hat{\lambda}_i}{\hat{\lambda}_j}) - (1-\underline{\pi}_j')(1-q_j')\kappa(q_i; \pi_i, \frac{\hat{\lambda}_j}{\hat{\lambda}_i}) \\ =& 1 - \left(\pi_j \kappa(1-q_i; \pi_i, \frac{\hat{\lambda}_j}{\hat{\lambda}_i}) + \pi_i(1-\bar{q}_i)\kappa(1-q_j; \pi_j, \frac{\hat{\lambda}_i}{\hat{\lambda}_j}) + \pi_j(1-\bar{q}_j)\kappa(q_i; \pi_i, \frac{\hat{\lambda}_j}{\hat{\lambda}_i}) \right) \end{aligned}$$

Lemma 10 *$\kappa(\bar{q}, \underline{q}; .)$ satisfies the following properties:*

1. *(Convexity) $D^2\kappa(\bar{q}, \underline{q}; .)$ is positive semidefinite*

2. $\kappa_{hh}(1-q_i, .)\kappa_{ll}(q_i, .) + \kappa_{hh}(q_i, .)\kappa_{ll}(1-q_i, .) - 2\kappa_{hl}(1-q_i, .)\kappa_{hl}(q_i, .) = 0$

where subscript h and l denotes the partial derivatives w.r.t. $\bar{q}$ and $\underline{q}$ respectively.

Proof. Differentiating $L(q) = 1 - \pi\bar{q} - (1-\pi)(1-\underline{q}) = 1 - q'$ w.r.t $\bar{q}$ and $\underline{q}$ respectively, we get $L_h(q) = -\pi$ and $L_l(q) = (1-\pi)$. Note that $N(q)^{-\frac{1}{k}} = \dfrac{\pi(1-\bar{q})}{L(q)} = 1 - \underline{\pi}'$. Therefore,

$$N_h(q) = k\frac{\pi(1-(1-\underline{\pi}'))}{(1-\underline{\pi}')(1-q')}N(q), \quad N_l(q) = k\frac{(1-\pi)}{(1-q')}N(q)$$

$$N_{hh}(q) = \frac{\pi}{(1-q')}\left(\frac{(1-(1-\underline{\pi}'))}{1-\underline{\pi}'}(k+1)+2\right), \quad N_h(q)N_{ll}(q) = (k-1)\frac{(1-\pi)}{(1-q')}N_l(q)$$

$$(1-q')N_{hl}(q) + L_h(q)N_l(q) + L_l(q)N_h(q) = (k+1)(1-\pi)N_h(q)$$

Therefore,

$$\begin{aligned}\kappa_h(q) =& L(q)N_h(q) + L_h(q)N(q)\\ =&\pi\left(k\frac{(1-(1-\underline{\pi}'))}{1-\underline{\pi}'}-1\right)N(q)\end{aligned}$$

$$\kappa_l(q) = (k+1)(1-\pi)N(q)$$

Taking second order partial derivatives we get,

$$\begin{aligned}\kappa_{hh}(q) =& (1-q')N_{hh}(q) + 2L_h(q)N_h(q)\\ =&\pi\left(\frac{(1-(1-\underline{\pi}'))}{1-\underline{\pi}'}(k+1)\right)N_h(q) > 0\\ \kappa_{ll}(q) =& (1-q')N_{ll}(q) + 2L_l(q)N_l(q)\\ =&(k+1)(1-\pi)N_l(q) > 0\end{aligned}$$

$$\kappa_{hl} = (1-q')N_{hl}(q) + L_h(q)N_l(q) + L_l(q)N_h(q) = (k+1)(1-\pi)N_h(q)$$

Therefore,

$$\begin{aligned}\kappa_{hh}\kappa_{ll} - \kappa_{hl}^2 =& (1-\pi)(1+k)^2N_h(q)\left(\pi\frac{(1-(1-\underline{\pi}'))}{1-\underline{\pi}'}N_l(q) - (1-\pi)N_h(q)\right)\\ =&(1-\pi)(1+k)^2N_h(q)\left(\pi\frac{(1-(1-\underline{\pi}'))}{1-\underline{\pi}'}k\frac{(1-\pi)}{(1-q')}N(q) - (1-\pi)k\frac{\pi(1-(1-\underline{\pi}'))}{(1-\underline{\pi}')(1-q')}N(q)\right)\\ =&0\end{aligned}$$

Also,

$$\kappa_{hh}(q)\kappa_{ll}(1-q) + \kappa_{hh}(1-q)\kappa_{ll}(q) - 2\kappa_{hl}(q)\kappa_{hl}(1-q)$$
$$= (1-\pi)(1+k)^2 N_h(1-q)\left(\pi\frac{(1-\bar{\pi}')}{\bar{\pi}'}N_l(q) - (1-\pi)N_h(q)\right)$$
$$+(1-\pi)(1+k)^2 N_h(q)\left(\pi\frac{(1-(1-\underline{\pi}'))}{1-\underline{\pi}'}N_l(1-q) - (1-\pi)N_h(1-q)\right) = 0$$

■

Taking partial derivatives of $E_{q'}\mu(\pi')$ w.r.t $\bar{q}_i$ (h) and $\underline{q}_i$ (l) we get,

$$\begin{aligned}
D_l E_{q'}\mu(\pi') =& \pi_j\kappa_l(1-q_i;\pi_i,\frac{\hat{\lambda}_j}{\hat{\lambda}_i}) - \pi_j(1-\bar{q}_j)\kappa_l(q_i;\pi_i,\frac{\hat{\lambda}_j}{\hat{\lambda}_i}) \\
=& \pi_j\left(\kappa_l(1-q_i;\pi_i,\frac{\hat{\lambda}_j}{\hat{\lambda}_i}) - \kappa_l(q_i;\pi_i,\frac{\hat{\lambda}_j}{\hat{\lambda}_i})\right) + \pi_j\bar{q}_j\kappa_l(q_i;\pi_i,\frac{\hat{\lambda}_j}{\hat{\lambda}_i}) \\
D_h E_{q'}\mu(\pi') =& \pi_j\kappa_h(1-q_i;\pi_i,\frac{\hat{\lambda}_j}{\hat{\lambda}_i}) - \pi_j(1-\bar{q}_j)\kappa_h(q_i;\pi_i,\frac{\hat{\lambda}_j}{\hat{\lambda}_i}) + \pi_i\kappa(1-q_j;\pi_j,\frac{\hat{\lambda}_i}{\hat{\lambda}_j}) \\
=& \pi_j\left(\kappa_l(1-q_i;\pi_i,\frac{\hat{\lambda}_j}{\hat{\lambda}_i}) - \kappa_l(q_i;\pi_i,\frac{\hat{\lambda}_j}{\hat{\lambda}_i})\right) + \pi_j\bar{q}_j\kappa_l(q_i;\pi_i,\frac{\hat{\lambda}_j}{\hat{\lambda}_i}) \\
&+ \pi_i\kappa(1-q_j;\pi_j,\frac{\hat{\lambda}_i}{\hat{\lambda}_j})
\end{aligned}$$

Taking the second order derivatives we get,

$$\begin{aligned}
D_{ll}E_{q'}\mu(\pi') =& -\pi_j\kappa_{ll}(1-q_i;\pi_i,\frac{\hat{\lambda}_j}{\hat{\lambda}_i}) - \pi_j(1-\bar{q}_j)\kappa_{ll}(q_i;\pi_i,\frac{\hat{\lambda}_j}{\hat{\lambda}_i}) < 0 \\
D_{hh}E_{q'}\mu(\pi') =& -\pi_j\kappa_{hh}(1-q_i;\pi_i,\frac{\hat{\lambda}_j}{\hat{\lambda}_i}) - \pi_j(1-\bar{q}_j)\kappa_{hh}(q_i;\pi_i,\frac{\hat{\lambda}_j}{\hat{\lambda}_i}) < 0 \\
D_{lh}E_{q'}\mu(\pi') =& -\pi_j^2(\kappa_{hh}(1-q_i,.)\kappa_{ll}(1-q_i,.) - \kappa_{hl}(1-q_i,.)^2) - \pi_j^2(\kappa_{hh}(q_i,.)\kappa_{ll}(q_i,.) - \kappa_{hl}(q_i,.)^2) \\
&-\pi_j^2(\kappa_{hh}(1-q_i,.)\kappa_{ll}(q_i,.) + \kappa_{hh}(q_i,.)\kappa_{ll}(1-q_i,.) - 2\kappa_{hl}(1-q_i,.)\kappa_{hl}(q_i,.)) = 0
\end{aligned}$$

The first two inequalities follow from Lemma 10.1. The last equality follows from Lemma 10.1 and 10.2. Therefore $E_{q'}\mu(\pi')$ is concave in q_i. ■

Proof of Proposition 6 Let us simplify $\bar{q}_i = \underline{q}_i = q_i$ for all $i \in N$.

Step 1: If $q \in R_i^3$ for some $i \in N$, then $E_{q'}\mu_1(\pi')$ is concave in q_i. To see this note that when $q \in R_i^3$,

$$E_{q'}\mu_1(\pi') = E_{q'}\mu(\pi') - \pi_i\pi_j \left(q_i(\bar{\pi}_i')^{\frac{1}{\hat{\lambda}_j}} + (1-q_i)\left[q_j(\bar{\pi}_j')^{\frac{1}{\hat{\lambda}_j}} + (1-q_j)(1-\underline{\pi}_i')^{\frac{1}{\hat{\lambda}_i}} \right] \right).$$

We have

$$\frac{\partial \bar{\pi}_i'}{\partial q_i} = \frac{\bar{\pi}_i'}{q_i'}\frac{1-\pi_i}{q_i} \text{ and } \frac{\partial(1-\underline{\pi}_i')}{\partial q_i} = -\frac{1-\underline{\pi}_i'}{1-q_i'}\frac{1-\pi_i}{1-q_i}$$

Taking partial derivatives of $q_i(\bar{\pi}_i')^{\frac{1}{\hat{\lambda}_j}}$ w.r.t. q_i and substituting $\frac{\partial \bar{\pi}_i'}{\partial q_i}$ we get

$$\begin{aligned} \frac{\partial}{\partial q_i} q_i(\bar{\pi}_i')^{\frac{1}{\hat{\lambda}_j}} &= (\bar{\pi}_i')^{\frac{1}{\hat{\lambda}_j}} \left(1 + \frac{1-\pi_i}{q_i'}\frac{1}{\hat{\lambda}_i} \right) \\ \frac{\partial^2}{\partial q_i^2} q_i(\bar{\pi}_i')^{\frac{1}{\hat{\lambda}_j}} &= \frac{(\bar{\pi}_i')^{\frac{1}{\hat{\lambda}_j}}}{(q_i')^2}\frac{(1-\pi_i)^2}{q_i}\frac{1}{\hat{\lambda}_i}(1+\frac{1}{\hat{\lambda}_i}) > 0 \end{aligned}$$

Similarly, taking partial derivatives of $(1-q_i)(1-\underline{\pi}_i')^{\frac{1}{\hat{\lambda}_j}}$ w.r.t. q_i and substituting $\frac{\partial(1-\underline{\pi}_i')}{\partial q_i}$ we get

$$\begin{aligned} \frac{\partial}{\partial q_i}(1-q_i)(1-\underline{\pi}_i')^{\frac{1}{\hat{\lambda}_j}} &= -(1-\underline{\pi}_i')^{\frac{1}{\hat{\lambda}_j}} \left(1 + \frac{1-\pi_i}{1-q_i'}\frac{1}{\hat{\lambda}_i} \right) \\ \frac{\partial^2}{\partial q_i^2}(1-q_i)(1-\underline{\pi}_i')^{\frac{1}{\hat{\lambda}_j}} &= \frac{(1-\underline{\pi}_i')^{\frac{1}{\hat{\lambda}_j}}}{(1-q_i')^2}\frac{(1-\pi_i)^2}{1-q_i}\frac{1}{\hat{\lambda}_i}(1+\frac{1}{\hat{\lambda}_i}) > 0 \end{aligned}$$

Given that $E_{q'}\mu(\pi')$ is concave in q_i when $q \in R_i^3$ (see Lemma 9) we have $\frac{\partial^2}{\partial q_i^2}E_{q'}\mu_1(\pi') < 0$.

Step 2: If q_i is sufficiently high such that i's signal determines which agent is stronger, i.e., $q \in R_i^2$. Using Lemma 8, we can say that an increase in q_i will increase $E_{q_j'}\mu_1(\pi')$. Hence, there exists $q_i^H(q_j) \leq 1$ such that the optimal choice of transparency is maximum transparency when q_i exceeds this threshold.

If q_i is sufficiently low, either $q \in R_i^1$ (if q_j is is sufficiently small such that $\bar{\pi}_j' \geq \pi_i^{\frac{\hat{\lambda}_j}{\hat{\lambda}_i}}$) or $q \in R_j^2$ (is q_j is large such that $\bar{\pi}_j' \leq \pi_i^{\frac{\hat{\lambda}_j}{\hat{\lambda}_i}}$). In either case $E_{q'}\mu_1(\pi')$ is decreasing in q_i (see Lemma 8). Hence, there exists a threshold q_i^L such that below this threshold the optimal choice of transparency is the minimum transparency.

Recall that under one sided public information or $q_j = q_j^0$, $E_{q'}\mu_1(\pi')$ first falls and then increase in q_i. Hence these two threshold are the same. However, when there is public information on both sides these two thresholds may not be the same. If q_i is neither too high nor too low, then it is possible that $q \in R_i^3$. From step 1, we know welfare is concave in q_i. If $E_{q'}\mu_1(\pi')$ in decreasing at the minimum q_i, then it is a decreasing function in this region. Consequently, $q_i^H(q_j) = q_i^L(q_j)$. Similarly if $E_{q'}\mu_1(\pi')$ in increasing at the maximum q_i, then it is an increasing function in this region. Consequently, $q_i^H(q_j) = q_i^L(q_j)$. However, if $E_{q'}\mu_1(\pi')$ in increasing in q_i at the minimum q_i and decreasing in q_i at the maximum q_i, then from concavity of $E_{q'}\mu_1(\pi')$ w.r.t q_i (see Lemma 8), we know there exists an intermediate q_i, say $q_i^M(q_j)$, where $E_{q'}\mu_1(\pi')$ in locally maximum. If $E_{q'}\mu_1(\pi')$ at $(q_i^M(q_j), q_j)$ exceeds the $\mu_1(\pi)$, then $q_i^M(q_j)$ is this intermediate value and $q_i^L(q_j) \leq q_i^M(q_j) \leq q_i^H(q_j)$. Also, if q_j is sufficiently high, then it is possible that for the intermediate value of q_i, $q \in R_j^3$. From Lemma 8. we know welfare in increasing in q_i. Therefore, $q_i^H(q_j) = q_i^L(q_j)$. □

9.4 Appointing a Mediator

Proof of Proposition 7

Case 1: $\pi_i \geq \pi_j^{\frac{\hat{\lambda}_i}{\hat{\lambda}_j}}$.

When agents have exactly same bargaining strength, $\pi_i = \pi_j^{\frac{\hat{\lambda}_i}{\hat{\lambda}_j}}$, for any q_i, $s = (i, j)$. From Lemma 7 we know that $E_{q'}\mu_1$ is increasing in q_i and hence disclosing information about i will increase $E_{q_i'}\mu_1(\pi')$. Thus, $E_{q'}\mu_1(\pi') > \mu_1(\pi)$ (since the RHS corresponds to $q_i = q_i^0$). Also, when π_i is sufficiently high such that $1 - \underline{\pi}_i' \geq \pi_j^{\frac{\hat{\lambda}_i}{\hat{\lambda}_j}}$, we have $s = (i, i)$. From Lemma 7, we know $E_{q'}\mu_1$ is decreasing in q_i and hence disclosing information about i will decrease $E_{q_i'}\mu_1(\pi')$. Thus, $E_{q'}\mu_1(\pi') < \mu_1(\pi)$.

The welfare difference of no information disclosure as compared to disclosed information $\mu_1 - E\mu_1$, is negative when agents have balanced strength and positive when i has sufficiently high relative strength. I will show that there is a unique $\pi^N \in [\pi_j^{\frac{\hat{\lambda}_i}{\hat{\lambda}_j}}, 1]$ such that $\mu_1 - E\mu_1 < 0$ for $\pi_i \in [\pi_j^{\frac{\hat{\lambda}_i}{\hat{\lambda}_j}}, \pi^N]$ and $\mu_1 - E\mu_1 > 0$ for $\pi_i \in [\pi^N, 1]$. This follows from the following Lemma.

Lemma 11 *Given $s = (i, j)$, if $q_i' \geq \bar{q}_i^{1+\hat{\lambda}_i}$, then $\mu_1 - E\mu_1$ is increasing in π_i and if $q_i' < \bar{q}_i^{1+\hat{\lambda}_i}$, then $\mu_1 - E\mu_1$ is concave in π_i.*

Proof. As defined in equation 11, $\mu_1 - E\mu_1 = (\mu - E\mu) + (\hat{\lambda}_i + \hat{\lambda}_j)(\xi - E\xi)$. For convenience, I will use

$$A = -\pi_i^{-\frac{\hat{\lambda}_j}{\hat{\lambda}_i}} \pi_j < 0,\ B = \left(1 - (\bar{q}_i)^{-\frac{\hat{\lambda}_j}{\hat{\lambda}_i}} (q_i')^{1+\frac{\hat{\lambda}_j}{\hat{\lambda}_i}}\right) > 0$$
$$C = -\pi_i^{1+\frac{1}{\hat{\lambda}_i}} \pi_j < 0,\ D = \left(1 - (\bar{q}_i)^{1+\frac{1}{\hat{\lambda}_i}} (q_i')^{-\frac{1}{\hat{\lambda}_i}}\right) < (>) 0$$
$$L_\mu = (1 - \underline{q}_i)\pi_i \pi_j^{-\frac{\hat{\lambda}_i}{\hat{\lambda}_j}},\ L_\xi = (1 - \underline{q}_i)\pi_i \pi_j^{1+\frac{1}{\hat{\lambda}_j}}$$

Note that if $q_i' \geq \bar{q}_i^{1+\hat{\lambda}_i}$ then $D < 0$ and vice versa. A', A'' will denote the first order and the second order derivative of A w.r.t. π_i and similarly for the other terms. Replacing $\bar{\pi}_i'$ and $1 - \underline{\pi}_i'$, we have $\mu - E\mu = AB + L_\mu$ and $\xi - E\xi = CD + L_\xi$. Both L_μ and L_ξ are linearly increasing in π_i.

$$A' = \left(\frac{\hat{\lambda}_j}{\hat{\lambda}_i}\right)\left(\frac{-A}{\pi_i}\right)\pi_j > 0,\ B' = -\left(1 + \frac{\hat{\lambda}_j}{\hat{\lambda}_i}\right)(\bar{q}_i)^{-\frac{\hat{\lambda}_j}{\hat{\lambda}_i}} (q_i')^{\frac{\hat{\lambda}_j}{\hat{\lambda}_i}} (\bar{q}_i + \underline{q}_i - 1) < 0$$
$$C' = \left(1 + \frac{1}{\hat{\lambda}_i}\right)\left(\frac{C}{\pi_i}\right)\pi_j < 0,\ D' = \left(\frac{1}{\hat{\lambda}_i}\right)(\bar{q}_i)^{1+\frac{1}{\hat{\lambda}_i}} (q_i')^{-(1+\frac{1}{\hat{\lambda}_i})} (\bar{q}_i + \underline{q}_i - 1) > 0$$

Therefore $(AB)' = A'B + AB' > 0$, which implies $\mu - E\mu$ is increasing. However, $C'D + CD'$ is not necessarily positive. Thus, $\xi - E\xi$ may not be increasing. Note that

$$AB' + (\hat{\lambda}_i + \hat{\lambda}_j)CD' = \left(\frac{\hat{\lambda}_i + \hat{\lambda}_j}{\hat{\lambda}_i}\right)(\bar{q}_i + \underline{q}_i - 1)((\bar{\pi}_i')^{-\frac{\hat{\lambda}_j}{\hat{\lambda}_i}} - (\bar{\pi}_i')^{-1+\frac{1}{\hat{\lambda}_i}}) > 0$$

Therefore, when $D < 0$, $\mu_1 - E\mu_1$ is increasing in π_i. If $D > 0$, this may not be true. Note that

$$A'' = \left(\frac{\hat{\lambda}_j}{\hat{\lambda}_i}\right)\left(1 + \frac{\hat{\lambda}_j}{\hat{\lambda}_i}\right)\left(\frac{A}{\pi_i^2}\right)\pi_j < 0,\ B'' = -\left(1 + \frac{\hat{\lambda}_j}{\hat{\lambda}_i}\right)\left(\frac{\hat{\lambda}_j}{\hat{\lambda}_i}\right)(\bar{q}_i)^{-\frac{\hat{\lambda}_j}{\hat{\lambda}_i}} (q_i')^{\frac{\hat{\lambda}_j}{\hat{\lambda}_i}-1} (\bar{q}_i + \underline{q}_i - 1)^2 < 0$$
$$C'' = \left(1 + \frac{1}{\hat{\lambda}_i}\right)\left(\frac{1}{\hat{\lambda}_i}\right)\left(\frac{C}{\pi_i^2}\right)\pi_j < 0,\ D'' = -\left(\frac{1}{\hat{\lambda}_i}\right)\left(1 + \frac{1}{\hat{\lambda}_i}\right)(\bar{q}_i)^{1+\frac{1}{\hat{\lambda}_i}} (q_i')^{-(2+\frac{1}{\hat{\lambda}_i})} (\bar{q}_i + \underline{q}_i - 1)^2 < 0$$

Substitute these expressions, one gets $(AB)'' = A''B + 2A'B' + AB'' < 0$ and when $D > 0$, $(CD)'' = C''D + 2C'D' + CD'' < 0$. Let us first consider $(CD)''$. Replacing C', C'' and given $C < 0$, we get $(CD)'' < 0$ iff

$$\pi_i^2 D'' + 2\left(1 + \frac{1}{\hat{\lambda}_i}\right)\pi_i D' + \left(1 + \frac{1}{\hat{\lambda}_i}\right)\left(\frac{1}{\hat{\lambda}_i}\right) D > 0$$

Substituting D, D', D'', the condition becomes

$$(\bar{q}_i)^{1+\frac{1}{\hat{\lambda}_i}}(q_i')^{-\frac{1}{\hat{\lambda}_i}}\left(1-\frac{\pi(\bar{q}_i+\underline{q}_i-1)}{q_i'}\right)^2<1$$

$$(\bar{q}_i)^{1+\frac{1}{\hat{\lambda}_i}}(q_i')^{-\frac{1}{\hat{\lambda}_i}}\left(\frac{1-\underline{q}_i}{q_i'}\right)^2<1$$

Since the last term in parentheses is less than 1, a sufficient condition for concavity is $D>0$ or $q_i' < \bar{q}_i^{1+\hat{\lambda}_i}$. Following similar steps we get $(AB)''<0$ when

$$(\bar{q}_i)^{-\frac{\hat{\lambda}_j}{\hat{\lambda}_i}}(q_i')^{1+\frac{\hat{\lambda}_j}{\hat{\lambda}_i}}\left(\frac{1-\underline{q}_i}{q_i'}\right)^2<1$$

which is always true. Therefore, when $D>0$, $\mu_1 - E\mu_1$ is concave in π_i. ■

Suppose, for contradiction there are multiple solutions to $\mu_1 - E\mu_1 = 0$. If $\mu_1 - E\mu_1$ is increasing then this not possible. Also, even if it is not increasing, from concavity and the fact that $\mu_1 - E\mu_1 < 0$ at balanced strength and $\mu_1 - E\mu_1 > 0$ when π_i is such that $1-\underline{\pi}_i' = \pi_j^{\frac{\hat{\lambda}_i}{\hat{\lambda}_j}}$, $\mu_1 - E\mu_1$ must be first increasing and then decreasing. If there are multiple solutions then $\mu_1 - E\mu_1 \le 0$ when π_i is such that $1-\underline{\pi}_i' = \pi_j^{\frac{\hat{\lambda}_i}{\hat{\lambda}_j}}$. Hence contradiction.

Case 2: $\pi_i \le \pi_j^{\frac{\hat{\lambda}_j}{\hat{\lambda}_i}}$.

If π_i falls but $\bar{\pi}_i' \ge \pi_j^{\frac{\hat{\lambda}_j}{\hat{\lambda}_i}}$ (note that $\bar{\pi}_i'$ is increasing in π_i) then the concave closure of μ_1 is always above μ_1. Therefore, disclosing information about i will increase $E_{q_i'}\mu_1(\pi')$. If π_i falls further then such disclosure will have no effect since i will be weak no matter what signal is realized. This gives us a threshold $\pi_i^I(\pi_j, q_i)$ such that disclosure is the optimal strategy whenever $\pi_i \in [\pi_i^I(\pi_j, q_i), \pi_j^{\frac{\hat{\lambda}_i}{\hat{\lambda}_j}}]$ and disclosure is ineffective whenever $\pi_i \in [0, \pi_i^I(\pi_j, q_i)]$. □

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