Extended Abstract: Efficient Multi-unit Auction Design without Quasilinear Preferences

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Extended Abstract

The Vickrey-Clarke-Groves mechanism is celebrated as a major achievement in the field of mechanism design because it gives agents a dominant strategy to truthfully report their private information and implements a Pareto efficient allocation of resources. However, the Vickrey auction loses its desired incentive and efficiency properties without the quasilinearity restriction on bidder preferences.[1] This is relevant because there are many well-known environments where this restriction is violated: bidders may be risk averse, have wealth effects, face financing constraints or be budget constrained. Indeed, observed violations of quasilinearity - most notably the presence of budgets and financial constraints - are frequently cited as reasons for why we do not see multi-unit Vickrey auctions used in practice. [2]

This leads to my main research question: is there a mechanism that retains the desirable incentive and efficiency properties of the Vickrey auction when bidders have non-quasilinear preferences. I study this question the context of a multi-unit auction for homogenous goods where bidders have private values and multi-unit demands. Instead of assuming that bidders have quasilinear preferences, I only assume that bidders have weakly positive wealth effects - i.e. the goods being sold are normal goods. This nests cases where bidders have budgets, face financing constraints, or are risk averse.

I first study an environment where bidders have single dimensional types. This means that bidders who have relatively higher demands for their first unit also have relatively higher demands for later unit. I show that if bidders have single dimensional types, then we can construct a mechanism that is dominant strategy incentive compatible and ex-post Pareto efficient. More specifically, I use the taxation principle (see Rochet (1985)) to translate the problem of finding an efficient mechanism to a problem of finding an efficient menu of prices that ensures that an ex-post efficient allocation of resources. Thus, I show that there is an efficient mechanism by find a pricing vector p that states the marginal price of buying additional units. I assume bidder i faces a price vector that is independent of her reported

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[1]Ausubel's clinching auction similarly loses its desired incentive and efficiency properties without quasi-linearity. See Dobzinski, Lavi, and Nisan (2012).
[2]See Rothkopf (2007), Ausubel and Milgrom (2006).

type because we study mechanisms that are dominant strategy incentive compatible. I show that there is an efficient mechanism by defining a transformation that maps an arbitrary pricing vector to a more efficient pricing vector. I show that this transformation has a fixed point that defines an efficient pricing vector.

I then study an environment where bidders have multi-dimensional types. Multi-dimensional types implies that it is possible that there is a bidder who has relatively higher demand for her first unit of the good (versus her rivals) and a relatively lower demand for later units of the good. In other words, two bidders can have demand curves that cross. I show that it is impossible to construct a mechanism (1) subsidy-free, (2) dominant strategy incentive compatible, and (3) ex-post Pareto efficient. I show this in a case where bidder's have two-dimensional types. Since there is no mechanism that satisfies properties (1)-(3) in the two-dimensional types, it follows that there is no mechanism that satisfies properties (1)-(3) when we have a richer type space - the increase in dimensionality only adds to the number of incentive constraints that our mechanism must satisfy.

My results add to the developing literature on auctions with non-quasilinear preferences. Much of the recent work in this area makes function form restrictions on bidder preferences. For example, Dobzinski, Lavi, and Nisan (2012) study a similar question with budget constraints. Morimoto and Serizawa (2015) study the efficient design problem in a general preference domain, but under the assumption of single unit demand. Dasgupta and Maskin (2000) and Jehiel and Moldavanu (2001) study the related problem of designing efficient mechanisms in interdependent value environments. Like this paper, they show that efficient design is generally possible if and only if bidders have single dimensional private information. Here we obtain a similar result in a private value environment by relaxing the quasilinearity restriction.